CFO Estimation in GFDM Systems Using Extended Kalman Filter

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***Abstract*—Multi-carrier modulations are useful in communication systems because of their proper performance in multipath fading channels. The orthogonal frequency division multiplexing (OFDM) is one of the modulations which is used in the most communication systems; however, it does not work properly in many new applications like 5G networks. As a result, several new multi-carrier modulations are presented for 5G systems like the generalized frequency division multiplexing (GFDM). Despite having some advantages to OFDM, the GFDM has some similar problems to the OFDM. For example, the intercarrier interference (ICI) effect which occurs because of existing the carrier frequency offset (CFO). It is caused by dissimilarity of the transmitter and the receiver oscillators and Doppler shift effect. This paper studies the CFO effect on the GFDM performance and applies the extended Kalman filter (EKF) to decrease the CFO and to improve the performance of this system according to the symbol error rate (SER). It is shown that using the EKF can reduce SER more than one order of magnitude which helps GFDM to be practicable for 5G networks.**

Keywords-component; GFDM, CFO, EKF, UKF, Estimation, Generalized Frequency Division Multiplexing, Carrier Frequency Offset, Extended Kalman Filter, Optimal Filtering, 5G.

I. Introduction

Nowadays the multi-carrier modulations are seriously applied in communication systems because of their favorable performance in multipath fading channels. The orthogonal frequency division multiplexing (OFDM) is one of the modulations which is used in the most communication systems [1]. Recently by developing the wireless internet communications, many new ideas are presented about the internet of things. In addition, inefficient use of spectrum in OFDM has caused a new generation of communication (5G) whose requirements are not met with OFDM [2]. As a result, several new multi-carrier modulations are presented for 5G systems like generalized frequency division multiplexing (GFDM) which generalizes the concept of OFDM [3] [4].

In addition to having the advantages of OFDM, the GFDM has other benefits; for example: being suitable for cognitive radio networks [5], controlling out of band radiation, having a suitable peak to average power ratio (PAPR), ignoring orthogonality of subcarriers [3] and multiplexing in time and frequency. The main drawback of GFDM is intercarrier interference (ICI) problem. This effect is caused by two sources: one is ignoring the orthogonality of the subcarriers because of the flexibility of the pulse shaping [5] and the other is the existence of carrier frequency offset (CFO) [6]. Ignoring orthogonality of subcarriers by applying a pulse shaping filter e.g, root raised cosine (RRC) and raised cosine (RC) filter introduces a degree of freedom, reduces out of band radiation but might introduce ICI. To reduce this ICI, several solutions have been proposed. [7] uses match filter (MF), zero forcing (ZF) and minimum mean square error (MMSE) receivers to reduce the self-induced intercarrier interference. In [5], double sided ICI cancelling (DSIC) is used to reduce bit error rate (BER) of GFDM.

For ICI caused by the second source, some studies have been done for OFDM. However, these frequency synchronization methods can not be directly used in GFDM. Therefore, for GFDM a few methods have been presented; for example in [8], joint time and frequency offset estimation are considered based on the maximum-likelihood (ML) receiver. In [9], non data-aided method is proposed to estimate CFO and in [10], the CFO is reduced using a certain GFDM symbol as known information.

In [11] the authors have used extended Kalman filter to estimate CFO in OFDM. Therefore, to estimate CFO by considering the GFDM features, this synchronization mechanism can be used with some changes. In this paper, the EKF is used to estimate CFO in GFDM to show the effects of this filter on the SER of system. In this regard, we use the data sequences and apply them in EKF equations, then we estimate the normalized CFO and after that, we can detect the data. In fact, using the optimal filters, synchronization in the frequency domain is achieved and the performance of GFDM system improves significantly.

The rest of this paper is organized as follows: Section II brings the background about GFDM and explains its equations. Section III introduces extended Kalman filter. After that, Section IV presents CFO estimation using the EKF. Analysis and simulation are presented in section V and the last section presents conclusion of this study.

II. GFDM background

Although the GFDM has been built based on the OFDM, they have some differences. For example in the GFDM, each frame consists of MK data symbols which is transmitted in

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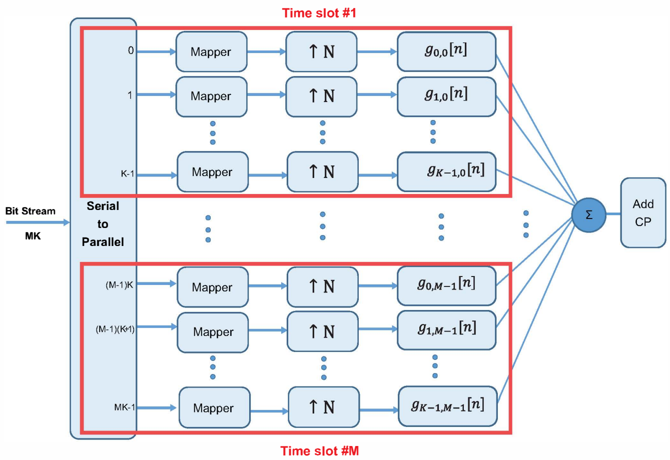

Fig. 1. Block diagram of GFDM transmitter

M time-slots using K subcarriers and each data symbol is formed by a pulse shape $g(t)$. However, in the OFDM, each frame has K data symbols which is transmitted in one time-slot using K subcarriers and each data symbol is formed by a rectangular pulse shape [12]. In GFDM modulator, a sequence of length N is divided into K groups with M symbols. Thus each symbol is introduced by $d_k[m]$, where k and m represent the subcarrier and the subsymbol indices, respectively that k varies from 0 to $K-1$ and m varies from 0 to $M-1$. Each $d_k[m]$ is sent after pulse shaping which is given by [10] as

$$g_{k,m}\left[n\right]=g\left(\left[n-mK\right]\ modN\right)e^{j2\pi\frac{kn}{N}}. \tag{1}$$

In fact, each of them is a circular shift of $g(n)$ filter in time and frequency. This sequence can also be represented as a column vector $\mathbf{g}_{k,m}$. The transmit signal $x[n]$ is obtained through the summation of all subcarriers and subsymbols signals according to

$$x\left[n\right]=\sum_{k=0}^{K-1}\sum_{m=0}^{M-1}d_k\left[m\right]g_{k,m}\left[n\right] \tag{2}$$

In the last stage, the cyclic prefix (CP) with the length N_{cp} is added to the block. Note that the CP is added after each M time slots and after each time slot in GFDM and OFDM respectively. To show the equations in a matrix form, the transmit signal $\mathbf{x}=\left(x\left[0\right],x\left[1\right],\ldots,x\left[N-1\right]\right)^T$ may be expressed as

$$\mathbf{x}=\mathbf{A}\mathbf{d} \tag{3}$$

with the data vector $\mathbf{d}$ and transmit matrix $\mathbf{A}$ as

$$\begin{aligned}&\mathbf{d}=\left(d_0\left[0\right],\ldots,d_0\left[M-1\right],\ldots,d_{K-1}\left[0\right],\ldots,d_{K-1}\left[M-1\right]\right)^T\\&\mathbf{A}=\left(\mathbf{g}_{0,0},\ldots,\mathbf{g}_{0,M-1},\mathbf{g}_{1,0},\ldots,\mathbf{g}_{1,M-1},\ldots,\mathbf{g}_{K-1,M-1}\right)\end{aligned} \tag{4}$$

In Fig. 1, the block diagram of GFDM transmitter is reperesented.

The received signal on the receiver side is

$$\mathbf{y}=\mathbf{h}*\tilde{\mathbf{x}}+\mathbf{v} \tag{5}$$

which $\mathbf{h}$ is multipath channel impulse response, $\mathbf{v}$ is additive white Gaussian noise, $\tilde{\mathbf{x}}$ is the transmission signal after adding CP and $*$ denotes convolution operator. Note that the length of CP is chosen to conquest the channel delay spread [10]. After receiving the data, the first synchronization begins in time domain. Then, the CP is removed and the synchronization in frequency domain is needed to reduce existing CFO. Finally, the channel effect is removed and the demodulated signal and $\hat{d}_k[m]$ is detected.

III. Extended Kalman Filters

Kalman filter finds the optimal estimation based on the MMSE method in linear programming. This filter has an iterative form which makes the algorithm faster than the other Bayesian estimators. If the nonlinear signal model is presented, the Kalman filter can not be used; however, the linearization methods make the Kalman techniques possible and present the extended Kalman filter for error covariance minimization of the estimation [13].

The EKF algorithm has two update stages which are named time and measurement updates. First, the Gauss-Markov dynamical system model is required which is nonlinear, like

$$\begin{aligned}X_{n+1}&=f_n\left(X_n,u_n,w_n\right)\\Z_n&=h_n(X_n,\nu_n)\end{aligned} \tag{6}$$

where X_n, Z_n, u_n, w_n, ν_n and n are state vector, observation vector, deterministic input vector, system noise, measurement noise and the number of EKF updating step, respectively. f_n and h_n are also transient state and observation functions, both of which are nonlinear. The system and the measurement noises are independent white Gaussian noise which are also independent of initial state X_0

$$\begin{aligned}E\{w_n w_l'\}&=Q_n\delta_{n-l}\\E\{\nu_n\nu_l'\}&=R_n\delta_{n-l}\end{aligned} \tag{7}$$

where Q_n and R_n are the system and measurement noise covariance matrices, respectively. E is expectation operator and δ is Dirac delta function. By linearization of (6) around the predicted state $\hat{X}_{n|n-1}$,which means the estimate of X at time n given observations set $Z_{n-1}=\left\{z_0,z_1,...,z_{n-1}\right\}$, and updated state $\hat{X}_{n|n}$ in step n, the linearized Gauss-Markov equations are presented as follows

$$\begin{aligned}X_{n+1}&=f_n\big(\hat{X}_{n|n},u_n,0\big)+F_n\big(X_n-\hat{X}_{n|n}\big)+G_nw_n+\ldots\\Z_n&=h_n\big(\hat{X}_{n|n-1},0\big)+H_n'\big(X_n-\hat{X}_{n|n-1}\big)+V_n'\nu_n+\ldots\end{aligned} \tag{8}$$

where

$$\begin{aligned}F_n&=\frac{\partial f_n}{\partial X_n}\Big|_{X_n=\hat{X}_{n|n},w_n=0}\\G_n&=\frac{\partial f_n}{\partial w_n}\Big|_{X_n=\hat{X}_{n|n},w_n=0}\end{aligned} \tag{9}$$

and

$$\begin{aligned} H_n' &= \frac{\partial h_n}{\partial X_n}\Big|_{X_n=\hat{X}_{n|n-1},\nu_n=0} \\ V_n' &= \frac{\partial h_n}{\partial \nu_n}\Big|_{X_n=\hat{X}_{n|n-1},\nu_n=0} \end{aligned} \tag{10}$$

According to (6)-(10) the time update equations of EKF are

$$\begin{aligned} \hat{X}_{n+1|n} &= f_n\left(\hat{X}_{n|n}, u_n, 0\right) \\ \Sigma_{n+1|n} &= F_n \Sigma_{n|n} F_n' + G_n Q_n G_n' \end{aligned} \tag{11}$$

and the measurement update equations are

$$\begin{aligned} \hat{X}_{n|n} &= \hat{X}_{n|n-1} + L_n\left(Z_n - h_n\left(\hat{X}_{n|n-1}, 0\right)\right) \\ \Sigma_{n|n} &= (I - L_n H_n')\,\Sigma_{n|n-1} \\ L_n &= \Sigma_{n|n-1} H_n \left(H_n' \Sigma_{n|n-1} H_n + V_n' R_n V_n\right)^{-1} \end{aligned} \tag{12}$$

where Σ denotes the covariance matrix of error estimation in each case. There is another type of Kalman filter which is named Unscented Kalman Filter (UKF) and it is used in strongly nonlinear problems. Unlike the EKF, the UKF does not use the linearized model but it uses the initial non-linear model and approximations of the state random variables distribution [14]. As a result, the UKF algorithm is absolutely different from the EKF algorithm. However, since our investigations show that this algorithm does not have significant superiority to the EKF in GFDM CFO reduction, we have not included its equations in this article.

IV. CFO ESTIMATION WITH EKF IN GFDM

CFO is caused by dissimilarity of the transmitter and the receiver oscillators. Also, the Doppler shift effect which is caused by the motion of users, is proportional to the frequency offset. Existing of CFO results in wrong decisions about the frequency of subcarriers and results in ICI. We assume that synchronization in time domain is done and our focus is just on the frequency offset problem. Simulations in [9] show that the maximum CFO which can be tolerated for GFDM is $\varepsilon = 0.03$ for QPSK and $\varepsilon = 0.01$ for 16 QAM. However, because of the smaller block length, the OFDM has a larger tolerance. This result emphasizes that in GFDM a good synchronization is needed. In this section we uses EKF to estimate the CFO in GFDM.

In the EKF algorithm a sequence with length N' is used. With each data, the equations are updated. EKF is an iterative algorithm in which $\hat{X}$ and Σ are updated and in each step the new values are used for the next step. To apply the EKF, adding an estimator block on the receiver side is enough. The received signal in GFDM is

$$z[n] = (\mathcal{H}[n] * x[n-\theta])\, exp(\frac{j2\pi n\varepsilon[n]}{N}) + \nu[n] \tag{13}$$

which is also the measurement equation in EKF. In (13), $z[n]$ is the received signal sample, $\mathcal{H}$ is the channel impulse response, θ is time deviation between the transmitter and the receiver which is assumed to be zero in this article, $\nu[n]$ is addative white Gaussian noise and $\varepsilon = \frac{\Delta f}{f_{sc}}$ is the normalized CFO [9]. By the definition

$$\begin{aligned} \tilde{x}[n] &\triangleq \mathcal{H}[n] * x[n-\theta] \\ h(\varepsilon[n]) &\triangleq \tilde{x}[n].exp(\frac{j2\pi n\varepsilon[n]}{N}) \end{aligned} \tag{14}$$

then the received signal can be expressed as

$$z[n] = h(\varepsilon[n]) + \nu[n]\,. \tag{15}$$

We assume that the channel is non-frequency-selective in one symbol duration, so the frequency offset will be constant approximately and the optimum estimation of the channel is available at the receiver.

$$\begin{aligned} &\varepsilon[n] = \varepsilon[n-1] \\ &z[n] \approx h(\hat{\varepsilon}[n-1]) + H'(\hat{\varepsilon}[n-1])(\varepsilon[n] - \hat{\varepsilon}[n-1]) + \nu[n] \end{aligned} \tag{16}$$

where $\hat{\varepsilon}[n-1] = \hat{\varepsilon}_{n|n-1} = \hat{\varepsilon}_{n-1|n-1}$ and

$$\begin{aligned} H'(\hat{\varepsilon}[n-1]) &= \frac{\partial h(\varepsilon[n])}{\partial \varepsilon[n]}\Big|_{(\varepsilon[n]=\hat{\varepsilon}[n-1])} \\ &= j\frac{2n\pi}{N}x[n]exp(j\frac{2n\pi\hat{\varepsilon}[n-1]}{N}). \end{aligned} \tag{17}$$

Therefore, the estimation algorithm steps are

1) Determining the initial value for $\hat{\varepsilon}[0]$ and error covariance $P[0] = \Sigma_{0|0}$
2) Computing the value of $H[n]$ at the point $\hat{\varepsilon}[n-1]$ which is the frequency offset estimation in the previous step
3) Computing Kalman filter gain $K[n]$ using $P[n-1]$ $(= \Sigma_{n|n-1} = \Sigma_{n-1|n-1})$ and $H[n]$
$$K[n] = P[n-1]\,H[n]\,(H'[n]\,P[n-1]\,H[n] + R[n])^{-1} \tag{18}$$
4) Computing the value of $\hat{H}[n]$ using $x[n]$ and $\hat{\varepsilon}[n-1]$. Then determining the error between $z[n]$ and $\hat{h}[n]$
$$\hat{h}[n] = (\mathcal{H}[n] * x[n])\, e^{\frac{j2\pi n\hat{\varepsilon}[n-1]}{N}} \tag{19}$$
5) Updating the value of $\hat{\varepsilon}[n]$ using
$$\hat{\varepsilon}[n] = \hat{\varepsilon}[n-1] + K[n]\left(z[n] - \hat{h}[n]\right) \tag{20}$$
6) Then updating the new value of error covariance $P[n]$
$$P[n] = (1 - K[n]\,H'[n])\,P[n-1] \tag{21}$$
7) Returning to step 1.

V. RESULTS

GFDM signal is generated using a RRC filter with the roll-off factor of 0.1. It is assumed that $M = 5$ and $K = 128$ which are the number of time slots and subcarriers respectively. Also, 16QAM mapper is used and 10 blocks of data are sent. The GFDM system is simulated and CFO is estimated at the receiver. Then the effect of CFO is removed from other blocks of data.

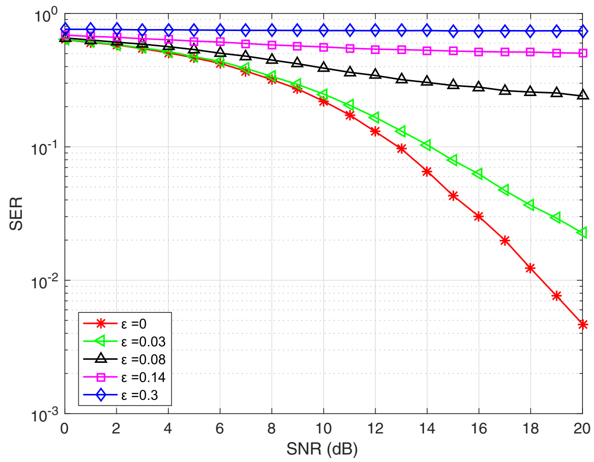

Fig. 2. Effect of CFO on GFDM system from the perspective of SER

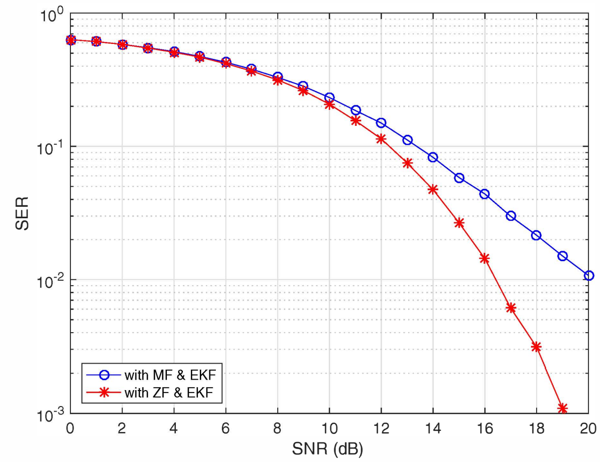

Fig. 4. SER comparison between MF and ZF detector in GFDM system using EKF block, $\varepsilon = 0.08$

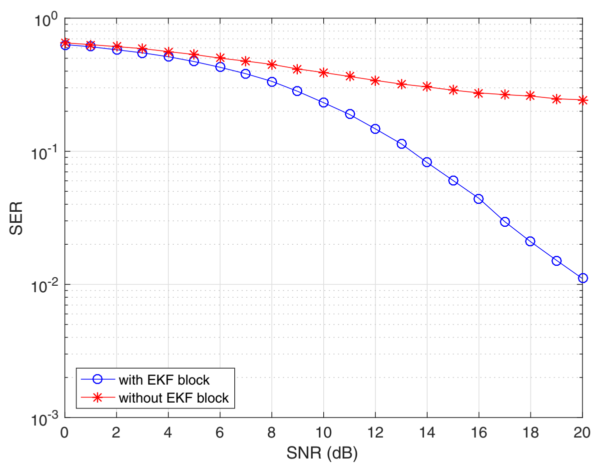

Fig. 3. Comparing SER of GFDM system with and without of EKF block for $\varepsilon = 0.08$ and MF receiver

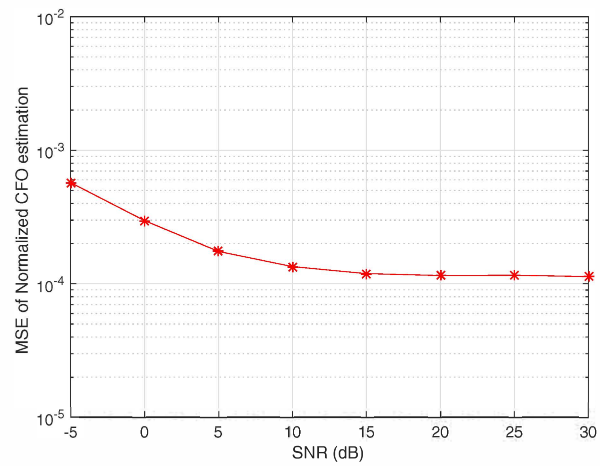

Fig. 5. The mean squared error of EKF for estimation of CFO

Fig. 2 shows the effect of CFO on the SER of GFDM system with differnet values of ε. It is seen that when the CFO offset increases, the performance of the system degrades. In Fig. 3 the advantage of existing EKF block is shown from the perspective of SER. In that simulation, the channel is AWGN and the detector at the receiver is MF. It can be observed that using the EKF block has significantly improved the performance especially at high SNR. For example, approximately one decade reduction of SER is achieved at SNR=17dB.
Also, the ZF detector can be used. According to [7] the ZF receiver can reduce the self-created interference more than MF receiver in GFDM. The comparison between ZF and MF detectors in this method also is shown in Fig. 4. The mean squared error (MSE) of this algorithm is shown in Fig. 5. From this simulation, in an AWGN channel, the best performance of this synchronization method with an accuracy of MSE=10^{-4} can be achieved.
The numerical evaluation of EKF and UKF algorithms are depicted in Fig. 6. It is seen that the performance of UKF has no significant difference with that of EKF, although it is a bit better.

VI. Conclusion

In this paper, a method for CFO estimation based on EKF algorithm was proposed. The Gauss-Markov dynamical system model was used and EKF algorithm was applied. EKF is an iterative algorithm in which after each step, the last estimated CFO is fed back to the estimator. The performance of this method was investigated by MATLAB simulations based on SER of the system. Simulation results showed that estimating CFO by EKF block, at the receiver side, increases the performance of GFDM system and reduces its SER.

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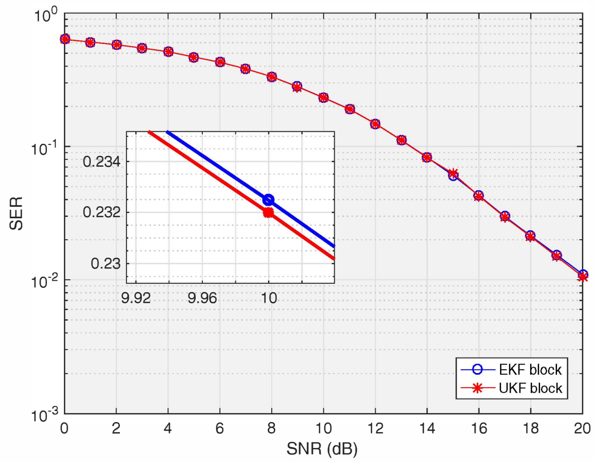

Fig. 6. SER comparison between UKF and EKF, $\varepsilon = 0.08$

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