A Simplified Adaptive Kalman Filter Algorithm for Carrier Recovery of M-QAM Signals

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***Abstract*— The paper proposes a carrier synchronization procedure for high-order QAM modulated signals that uses a decision-directed extended Kalman filtering (EKF) technique. The proposed method combines the Kalman filter with a lock detector to establish the status of the synchronization process, modifying adaptively the Kalman filter parameters. As compared with the complete EKF filter, the proposed synchronization algorithm simplifies the Kalman measurement model to a single scalar equation reducing significantly the computational complexity of the algorithm. Simulations show that the simplified Kalman algorithm has equivalent synchronization performances with the complete EKF filter.**

I. INTRODUCTION

Carrier synchronization is a very important operation in digital communication systems that aims to remove any phase or frequency offsets that exists between the transmitter and receiver oscillators in order to allow a coherent demodulation [1]. High-order quadrature amplitude modulation (M-QAM) systems are widely used in modern communication systems especially in the field of cable communication. For these types of modulations, a precise carrier recovery at the receiver is an essential problem.

The most common approach to carrier synchronization problem in QAM demodulation is to use decision-directed (DD) Phase-Locked Loop (PLL) techniques [1-3]. QAM receivers require fast convergence rates and small steady-state phase tracking error. It is difficult for a PLL circuit to simultaneously satisfy both requirements since a fast acquisition rate means a large loop bandwidth while a smaller loop bandwidth leads to a better steady-state performance but with a long acquisition time. An optimal behavior from a PLL might be theoretically possible by changing the loop bandwidth parameters, depending the stage of the convergence process is [1, 4].

An alternative to PLL carrier recovery is the use of Kalman filter algorithms [5-7]. The Kalman filter is known as the optimally linear recursive filter in the minimum mean-square error sense. It is advantageous to use a Kalman filter algorithm for carrier recovery under high dynamic situations. The Kalman filter can adaptively cope with the system environment changes more better than a PLL. Extended Kalman Filters based on the carrier phase dynamic model were successfully used in carrier recovery of QAM signals [8, 9]. By using the Kalman filter algorithm, the system model incorporates as variables, the phase and the frequency of the incoming signal. Filter parameters are modified when system status changes from acquisition to tracking mode, thereby achieving a faster acquisition and a lower phase noise in tracking mode [9]. This adaptive DD-EKF algorithm adopt the architecture proposed by Matsuo and Namiki [1, 10] which rely on a coarse frequency acquisition mode and a fine phase tracking mode, selected by a lock detection module.

In this paper, we propose a simplified carrier synchronization technique which applies the EKF algorithm. The main idea in the proposed scheme is the observation model reduction to a single scalar equation to relieve computing load by replacing the time-expensive matrix inversion by a simple scalar division. Simulations show good acquisition and frequency tracking performances of the proposed simplified EKF scheme and no significant degradation relatively to the complete algorithm [9].

II. THE CARRIER SYNCHRONIZATION SYSTEM ARCHITECTURE

Assuming perfect timing synchronization and adequate gain control, the receiver in Fig. 1 [9] observes the discrete time down-converted signal

$$r(n) = m(n)e^{j(\Omega n+\varphi)} + v(n) = m(n)e^{j\theta(n)} + w(n) \qquad (1)$$

where $m(n) = a(n) + jb(n)$ is the n^{th} transmitted complex QAM symbol, φ and Ω are the carrier phase and frequency offset and $w(n)$ is a zero-mean complex Gaussian noise with distribution $N(0,\sigma_w^2)$.

The EKF filter estimates the total phase of the incoming signal carrier $\hat{\theta}(n) = \hat{\Omega}(n)n + \hat{\varphi}(n)$ and brings to the input of hard-decision QAM Detector the signal

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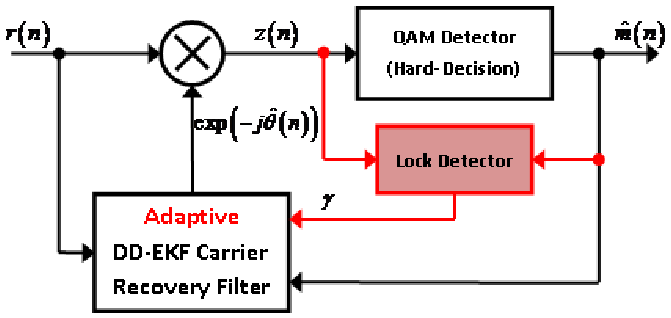

Figure 1 The adaptive decision-directed EKF filter carrier synchronization system

$$z(n)=m(n)e^{j(\theta(n)-\hat{\theta}(n))}+v_z(n)=m(n)e^{j\Delta\theta(n)}+w_z(n) \quad (2)$$

where $\Delta\theta(n)$ is a phase error and $w_z(n)$ is complex Gaussian noise with the same power as $w(n)$. The carrier synchronization aim is to cancel $\Delta\theta(n)$.

A hard decision is made on $z(n)$ by QAM Detector to generate the detected complex symbol $\hat{m}(n)$. The action of Lock Detector from Fig. 1 will be presented in Section 4.

III. The DD-EKF Algorithm

A. *The Complete Two-State Modeling Approach in the State-Space [9]*

In order to estimate the phase and frequency of the carrier of a M-QAM modulated signal by EKF filtering, it is necessary to model the incoming wave in the state space. The parameters of the model, phase $\theta(n)$ and frequency $\Omega(n)$, are related by:

$$\Omega(n)=\theta(n)-\theta(n-1) \quad (3)$$

To synchronize the model with the received signal, the evolution of $\Omega(n)$ is driven by a random walk model

$$\Omega(n)=\Omega(n-1)+v(n) \quad (4)$$

where $v(n)$ is a sequence of independent and identically distributed random scalars with distribution $N(0,\gamma^2)$. The rate of evolution of the signal frequency estimation is controlled by γ.

Including eq. (3) in eq. (4), the state evolution equation of Kalman filter is written as:

$$\begin{bmatrix}\theta(n)\\ \Omega(n)\end{bmatrix}=\mathbf{F}\begin{bmatrix}\theta(n-1)\\ \Omega(n-1)\end{bmatrix}+\mathbf{G}v(n) \text{ with } \mathbf{F}=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} \text{ and } \mathbf{G}=\begin{bmatrix}0\\ 1\end{bmatrix} \quad (5)$$

The 2×1 observations vector consists of In-Phase and Quadrature components of the received signal:

$$\mathbf{r}[n]=\left[\mathrm{Re}(r(n)) \quad \mathrm{Im}(r(n))\right]^T \quad (6)$$

In these conditions, eq. (1) expresses the observations vector as a nonlinear function of state vector $\mathbf{x}[n]=\left[\theta(n) \quad \Omega(n)\right]^T$:

$$\mathbf{r}[n]=\mathbf{h}(\mathbf{x}[n])+\mathbf{w}[n] \quad (7)$$

where the 2×1 nonlinear function $\mathbf{h}(\mathbf{x}(n))$ is written as:

$$\mathbf{h}(\mathbf{x}[n])=\begin{bmatrix}a(n)\cos\theta(n)-b(n)\sin\theta(n)\\ a(n)\sin\theta(n)+b(n)\cos\theta(n)\end{bmatrix} \quad (8)$$

The correlation matrix of the complex noise vector $\mathbf{w}(n)$ is:

$$\mathbf{Q}=\frac{\sigma_w^2}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad (9)$$

In order to use the EKF algorithm, we apply the first order linearization procedure to $\mathbf{h}(\mathbf{x}[n])$ in (8) around the estimation of the state vector $\hat{\mathbf{x}}[n|n-1]$:

$$\mathbf{h}(\mathbf{x}[n])=\mathbf{h}(\hat{\mathbf{x}}[n|n-1])+\left.\frac{\delta\mathbf{h}}{\delta\mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}[n|n-1]}(\mathbf{x}[n]-\hat{\mathbf{x}}[n|n-1]) \quad (10)$$

with

$$\mathbf{H}[n]=\left.\frac{\delta\mathbf{h}}{\delta\mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}[n|n-1]}=\begin{bmatrix}-a(n)\sin\hat{\theta}(n|n-1)-b(n)\cos\hat{\theta}(n|n-1) & 0\\ -b(n)\sin\hat{\theta}(n|n-1)+a(n)\cos\hat{\theta}(n|n-1) & 0\end{bmatrix} \quad (11)$$

To calculate $\mathbf{H}[n]$ it is necessary to know the values of transmitted symbol $a(n)+jb(n)$ which are unknown at the receiver. If the receiver works with a sufficiently low error rate, then, in place of transmitted symbol, can be used the output of the Hard-Decision device $\hat{m}(n)=\hat{a}(n)+j\hat{b}(n)$. Therefore, in the decision-directed EKF synchronization algorithm $\mathbf{H}[n]$ is replaced by:

$$\hat{\mathbf{H}}[n]=\begin{bmatrix}-\hat{a}(n)\sin\hat{\theta}(n|n-1)-\hat{b}(n)\cos\hat{\theta}(n|n-1) & 0\\ -\hat{b}(n)\sin\hat{\theta}(n|n-1)+\hat{a}(n)\cos\hat{\theta}(n|n-1) & 0\end{bmatrix} \quad (12)$$

Similarly, the nonlinear function $\mathbf{h}(\hat{\mathbf{x}}[n|n-1])$ is calculated as:

$$\hat{\mathbf{h}}(\hat{\mathbf{x}}[n|n-1])=\begin{bmatrix}\hat{a}(n)\cos\hat{\theta}(n|n-1)-\hat{b}(n)\sin\hat{\theta}(n|n-1)\\ \hat{a}(n)\sin\hat{\theta}(n|n-1)+\hat{b}(n)\cos\hat{\theta}(n|n-1)\end{bmatrix} \quad (13)$$

B. *The Complete EKF Algorithm*

Assume that the initial state $\mathbf{x}[1]$, the observation noise $\mathbf{w}[n]$ and the state noise $v[n]$ are jointly Gaussian and mutually independent. Let $\hat{\mathbf{x}}\left[n|n-1\right]$ and $\mathbf{R}\left[n|n-1\right]$ be the conditional mean and the conditional variance of $\hat{\mathbf{x}}[n]$ given the observations $\mathbf{r}[1],\ldots,\mathbf{r}[n-1]$ and let $\hat{\mathbf{x}}\left[n|n\right]$ and $\mathbf{R}\left[n|n\right]$ be the conditional mean and conditional variance of $\hat{\mathbf{x}}[n]$ given the observations $\mathbf{r}[1],\ldots,\mathbf{r}[n]$. Finally, the 2×2 matrix $\mathbf{K}[n]$ is the Kalman gain matrix at moment n. The EKF algorithm is expressed in Table I.

TABLE I. THE EKF ALGORITHM

A. Measurement Update Equations
$\mathbf{K}[n]=\mathbf{R}\left[n|n-1\right]\hat{\mathbf{H}}^T[n]\left(\hat{\mathbf{H}}[n]\mathbf{R}\left[n|n-1\right]\hat{\mathbf{H}}^T[n]+\mathbf{Q}[n]\right)^{-1}$
$\hat{\mathbf{x}}\left[n|n\right]=\hat{\mathbf{x}}\left[n|n-1\right]+\mathbf{K}[n]\left(\mathbf{r}[n]-\hat{\mathbf{h}}\left(\hat{\mathbf{x}}\left[n|n-1\right]\right)\right)$
$\mathbf{R}\left[n|n\right]=\mathbf{R}\left[n|n-1\right]-\mathbf{K}[n]\mathbf{H}[n]\mathbf{R}\left[n|n-1\right]$
B. Time Update Equations
$\hat{\mathbf{x}}\left[n+1|n\right]=\mathbf{F}\hat{\mathbf{x}}\left[n|n\right]$
$\mathbf{R}\left[n+1|n\right]=\mathbf{F}\mathbf{R}\left[n|n\right]\mathbf{F}^T+\gamma^2\mathbf{G}\mathbf{G}^T$

C. The Simplified EKF algorithm

A reduction in the computational load of the extended Kalman filter can be attained by discarding one component either the In-Phase or the Quadrature component, from the two-dimensional measurement equation in (7). For example, by keeping the In-Phase element in the measurement equation, the nonlinear observation function (8) simplifies to

$$h\left(\mathbf{x}[n]\right)=a(n)\cos\theta(n)-b(n)\sin\theta(n) \tag{14}$$

or

$$h\left(\mathbf{x}[n]\right)=a(n)\sin\theta(n)+b(n)\cos\theta(n) \tag{15}$$

Likewise, the new dimensions of square matrices $\mathbf{K}[n]$, $\hat{\mathbf{H}}[n]$ and $\mathbf{Q}[n]$ reduce to 2×1, 1×2 respectively 1×1. Most important consequence of these reductions consists in the replacement of the matrix inversion from Kalman algorithm in TABLE I. by a simple scalar division.

From simulations results that the performance degradation of the simplified algorithm, due to the loss of certain observations information, is not significant.

IV. ADAPTIVE CARRIER SYNCHRONIZATION

Carrier synchronization process consists of two distinct stages: acquisition and tracking. Both stages involve different modes of action of the synchronization system in order to achieve both fast carrier synchronization in the acquisition stage and low phase error in the tracking stage. To meet these antagonistic requirements firstly, it is necessary to detect the synchronization moment and then, the EKF filter parameters should be dependent on the stage where the synchronization process is found. The model in Fig. 1 fulfills these requirements, using a lock detector to estimate the status of the synchronization process and an adaptive EKF filter [1,9].

Both signals $z(n)$ and $\hat{m}(n)$ are used by the Lock Detector to sense the state of the synchronization process in order to detect whether or not the output constellation is phase-locked. The lock detector generates the binary signal [1]

$$y(n)=\begin{cases}1, & \text{if } \left|z(n)-\hat{m}(n)\right|<\lambda\\ 0, & \text{otherwise}\end{cases} \tag{16}$$

When the system is phase-locked, $y(n)$ will be equal to 1 in majority. Every N_1 symbols, the Lock Detector compares the average value of $y(n)$ taken on N_2 symbols with a threshold value β to determine if the system has acquired or not synchronization.

Since the use of a single value for the variance of estimated carrier frequency as in [3] cannot provide both a fast convergence rate in acquisition mode and a low frequency error in tracking mode, the adaptive carrier synchronization system model uses the lock detector to toggle the estimated frequency variance γ^2 of EKF filter from a large value in acquisition mode to a very low value in tracking mode:

$$\gamma=\begin{cases}\gamma_1, & \text{in acquisition mode}\\ \gamma_2, & \text{in tracking mode}\end{cases},\quad \gamma_1\gg\gamma_2 \tag{17}$$

V. SIMULATION RESULTS

Simulations using the carrier synchronization system of Fig. 1 were made both with complete and simplified DD-EKF algorithm, aiming to certify the performances of the new simplified algorithm and to highlight its advantages. Each simulation trial consists of 50,000 DVB-C symbols of 64-QAM constellation and symbol rate Ω_S. The modulated signal propagates through a communication channel with additive white Gaussian noise (AWGN) having zero-mean and variance σ_w^2. The received signal applied at the input of decision-directed synchronization systems has a carrier frequency offset Ω, perfect timing synchronization and perfect gain control, being sampled with one sample per symbol.

Viewing the four-quadrant symmetry of M-QAM signals, the action of the algorithm is confined to a carrier frequency offset between $-\Omega_S/8$ and $\Omega_S/8$, where Ω_S is the symbol rate. For an initial frequency offset near to $\Omega_S/8$, the algorithm can converge to $\pm\Omega_S/4+\Omega$ instead of Ω, the real value. This “false-lock” situation can be avoided by imposing to $\hat{\Omega}(n)$, the estimated frequency, the following limiting condition:

$$-\Omega_S/8\le\hat{\Omega}(n)\le\Omega_S/8 \tag{18}$$

The estimated phase variance γ^2 of EKF filter controls how the filter responds to phase variations of the input signal: fast in the acquisition mode respectively slow in tracking mode. Consequently, by mode of operation, it takes two distinct

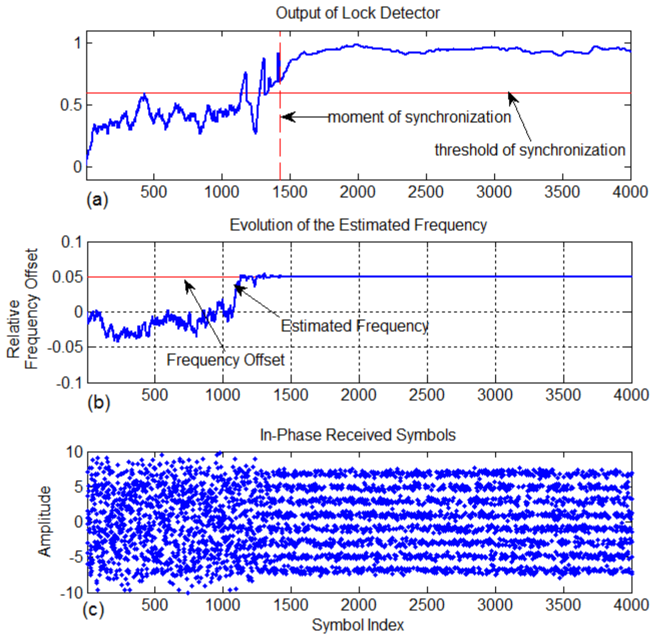

Figure 2 Description of a typical synchronization process in a simplified DD-EKF algorithm: (a) Output of Lock Detector, (b) Evolution of the estimated frequency, (c) In-Phase input signal to Hard-Decision detector.

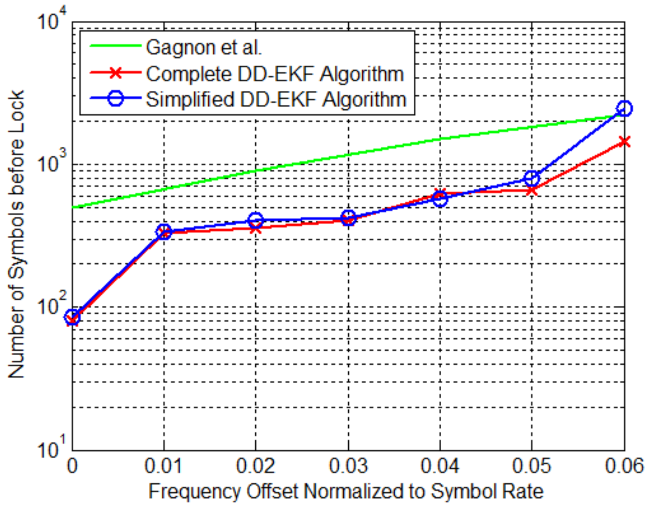

Figure 3 Acquisition performances of the carrier synchronization systems for 64-QAM. For the PLL carrier synchronization loop, the results are taken directly from Gagnon et al. [1].

values as reveals eq. (17). The values were determined experimentally and are identical both for the complete and the simplified algorithm: $\gamma_1 = 2 \cdot 10^{-2} \Omega_S$ in acquisition mode and $\gamma_2 = 5 \cdot 10^{-5} \Omega_S$ in tracking mode.

The Lock Detector establishes when the synchronization system must change from acquisition to tracking mode. The values of this block parameters have been set as in [1] and [9]: the synchronization threshold λ in eq. (16) is 0.7, the threshold of averages of $y(n)$ is $\beta = 0.6$. As was stated in the previous Section, the averaging parameters of Lock Detector, N_1 and N_2 change when the system switches. They are

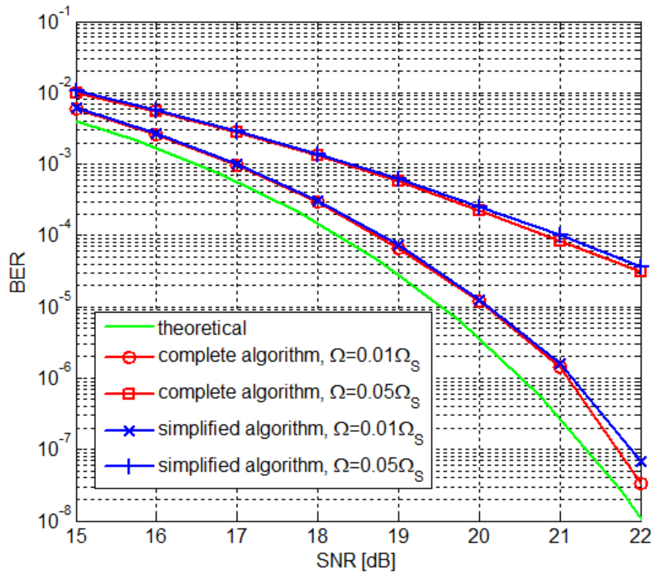

Figure 4 The dependence of BER on SNR at various initial carrier frequency offsets for complete and simplified adaptive DD-EKF algorithms.

lower in acquisition mode: $N_1 = 16$ and $N_2 = 64$ and higher in tracking mode: $N_1 = 64$ and $N_2 = 256$.

A typical description of the way the carrier synchronization process works is made in Fig. 2. The simulation used the following parameters: SNR = 20 dB and $\Omega = 0.05\,\Omega_S$, where Ω is the carrier frequency offset. A first conclusion, taken from Fig. 2, concerns the utility of the lock detector to determine precisely the moment of transition from acquisition to tracking.

Fig. 3 shows the acquisition time performances, given in symbols, for various initial carrier frequency offsets. for various initial carrier frequency offsets with a signal-to-noise ratio SNR = 30 dB. The curves are obtained by averaging the results of 100-run simulations. Lock is considered achieved when the rms value of the phase error $\theta(n) - \hat{\theta}(n)$ over the last 256 symbols is less than 1° rms. The simulations were carried out both on complete and simplified adaptive DD-EKF algorithms and compared with the results reported by Gagnon et al. [1] for a very fast PLL carrier synchronization loop in terms of acquisition speed. Practically, the two algorithms perform alike and much better than a PLL carrier synchronization system.

The best way to characterize the performance of a synchronization system in tracking mode is to determine the Bit-Error-Rate (BER) of output binary signal of the QAM Detector in Fig. 1. Fig. 4 compares the performance of complete and simplified DD-EKF algorithms with the theoretical curve of 64-QAM. For moderate frequency offsets ($\Omega = 0.01\Omega_S$), the difference between the performance of both algorithms and the theoretical curve is lower than 1 dB. Again, these results show, that even the complete algorithm performs slightly better, this advantage is canceled by the lower computational load of the simplified adaptive DD-EKF algorithm.

VI. CONCLUSIONS

The objective of this paper is to introduce and characterize the simplified variant of the adaptive decision directed extended Kalman filtering (DD-EKF) algorithm that we introduced in a previous paper [9]. The algorithm uses a very efficient Lock Detector to assess whether or not the system attained synchronization. The Kalman filter modifies its parameters when system status changes from acquisition to tracking mode, thereby permitting a faster acquisition time and a lower phase noise in tracking mode.

As compared with its complete version, the simplified algorithm has a significantly reduced computational complexity but maintains almost unchanged the synchronization and tracking performances.

REFERENCES

[1] Gagnon, G., et al., A simple and fast carrier recovery algorithm for high-order QAM. IEEE Trans. Commun. Lett., Oct. 2005. 49(4): p. 918-920.

[2] Best, R.E., Phase-Locked Loops: Design, Simulation, and Applications. 5 ed2003, New York: McGraw-Hill.

[3] Ke., C.N., C.Y. Huang., and C.P. Fan, An adaptive carrier synchronizer for M-QAM cable receiver. IEEE Trans. Consum. Electron., Nov. 2003. vol. 49(4): p. 983-989.

[4] Patapoutian, A., On phase-locked loops and Kalman filters. IEEE Trans. Commun. , May 1999. vol. 47(5): p. 670-672.

[5] Aghamohammadi, A., H. Meyr, and G. Ascheid, Adaptive synchronization and channel parameter estimation using an extended Kalman filter. IEEE Trans. on Communications, Nov. 1989. Vol. 37(1212-1219).

[6] Patapoutian, A., Application of Kalman filters with a loop delay in synchronization. IEEE Trans. on Communications, May 2002. Vol. 50(No. 5): p. 703-706.

[7] Kim, K.-H., G.-I. Jee, and J.-H. Song, Carrier tracking loop using adaptive two-stage Kalman filter for high dynamic situations. International Journal of Control, Automation and Systems, Dec. 2008. vol. 6(6): p. 948-953.

[8] Lin, W.-T. and D.-C. Chang, Adaptive carrier synchronization using Decision-Aided Kalman filtering algorithms. IEEE Trans.on Consumer Electronics, Nov. 2007 53(4): p. 1260-1267.

[9] Câmpeanu, A. and J. Gál. High-Order QAM Fast Carrier Synchronization by an Adaptive Decision-Directed EKF Algorithm. in Proceedings of 34th International Conference on Telecommunications and Signal Processing (TSP). August 18-20, 2011. Budapest, Hungary.

[10] Matsuo, Y. and J. Namiki, Carrier recovery systems for arbitrarily mapped APK signals IEEE Trans. on Communications, 1982. 30(10): p. 2385 - 2390