A High Efficiency Carrier Estimator for OFDM Communications *

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Abstract

In OFDM communications, the loss of orthogonality due to carrier offset must be compensated before DFT-based demodulation can be performed. In this paper, we present two blind carrier offset estimation algorithms by exploiting intrinsic structure information of OFDM signals. Both algorithms offer the accuracy of super resolution subspace methods, viz., MUSIC [7] and ESPRIT [4], without involving computationally intensive subspace decompositions. Adaptation rules for implementation of one of the proposed algorithm are provided. Also derived is the Cramèr-Rao bound of the carrier offset estimates with which performance of the new algorithms is evaluated.

1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) has received increasing attention in wireless broadcasting systems for its ability to mitigate the frequency-dependent distortion across the channel bandwidth [5]. Despite its promises, OFDM has been shown to be very sensitive to inaccurate frequency references [3]. A carrier offset at the receiver can cause loss of subcarrier orthogonality, and thus introduces interchannel interference (ICI) and severely degrades the system performance [3]. Accurate carrier offset estimation and compensation is more critical in OFDM than other modulation schemes.

Most existing carrier frequency offset estimation techniques rely on periodic transmission of reference symbols, which inevitably reduces the bandwidth efficiency. Schmidl and Cox [6] introduced a blind approach which is capable of estimating the carrier offset towards its closest subcarrier. However, the algorithm requires the constellation on each subcarrier to have points equally spaced in phase. Moreover, the length of the guard interval must be chosen from a subset of allowed values. The latest effort in blind estimation involves joint timing and frequency synchronization as in [8], however only flat fading channels are considered.

This paper proposes a new solution to the carrier offset estimation problem without using reference symbols, pilot carriers or excess cyclic prefix. The techniques developed here provide a high accuracy carrier estimate by taking advantage of the inherent orthogonality among OFDM subchannels. When the OFDM signal is distorted by an unknown carrier offset, the received signal possesses an algebraic structure, due to virtual carriers, which will be shown to be sufficient for blind carrier estimation. All derived estimators are in analytic form and the acquisition range is not limited to one-half the subcarrier spacing as is the case with some other algorithms. A salient feature of these algorithms is that they can offer the performance of super-resolution subspace algorithm, viz, MUSIC [7] and ESPRIT [4], without the costly subspace decomposition.

2 Problem Formulation

2.1 OFDM Principles

We begin with the data model of a discrete time baseband OFDM signal. Denote $\mathbf{s}(k) \stackrel{\text{def}}{=} [s_0(k) \cdots s_{N-1}(k)]^T$ as the kth block of data. In OFDM, a block of P symbols from the data stream is first padded with zeros before applied to an IDFT of size N; therefore, $s_P(k) = \cdots = s_{N-1}(k) = 0$. $N > P$ because sufficiently wide filter guard bands is needed for reliable communications. In the European video broadcasting system for example, $N = 2048$ and

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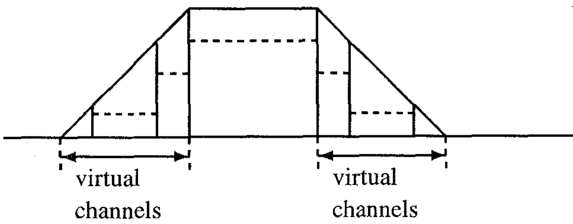

Figure 1. Transmit spectra of OFDM

$P = 1705$ [2]. The $N - P$ unused subcarriers shown in Figure 1 are often referred to as the virtual carriers [5, 1]. Note that it is not necessary to create the virtual carriers at the transmitter, oversampling at the receiver site creates the same effect [1]. Using matrix representation, the resulting N-point time domain signal is given by

$$\mathbf{x}(k) \stackrel{\text{def}}{=} [x_0(k)x_1(k)\cdots x_{N-1}(k)]^T = \mathbf{W}\mathbf{s}(k), \quad (1)$$

where $\mathbf{W}\mathbf{s}$ is the IDFT matrix. For DFT based OFDM, a L-point cyclic prefix is added to multiplexed output of the IDFT before it is transmitted through a fading channel to cope with the the FIR composite channel [5]. The cyclic extension is chosen to be longer than the impulse response to avoid interblock interference and preserve orthogonality of the subchannels.

Let $x_n(k) = x((k-1)(N+L)+n)$, $x(m)$ be the sample sequence to be transmitted, and $h(m)$ be the impulse response of the multipath channel. The input to the receiver is given by $y(m) = x(m) * h(m)$. The use of a cyclic prefix will transform the linear convolution of the channel to a cyclic convolution after removing the cyclic prefix [5]. We can show for the kth block:

$$\begin{aligned} \mathbf{y}(k) &= [y_L(k)\cdots y_{N+L-1}(k)]^T \\ &= \mathbf{W}\mathrm{diag}\left(H(0)\cdots H(N-1)\right)\mathbf{s}(k) \end{aligned} \quad (2)$$

where $H(i),\ i = 1\cdots N$ is the frequency response of the channel as obtained by the DFT of $\{h(m)\}$. Clearly, each subchannel can be recovered to a scalar ambiguity by applying a DFT to $\mathbf{y}(k)$:

$$\mathbf{W}_P^H\mathbf{y}(k) = \mathbf{H}\left[s_0(k)\cdots s_{P-1}(k)\right]^T. \quad (3)$$

where $\mathbf{W}_P$ consists of the first P columns of $\mathbf{W}$ and $\mathbf{H} = \mathrm{diag}(H(0)\cdots H(P-1))$. If $\{s_i(k)\}$ are differentially encoded, the information bearing symbols can be perfectly recovered even when the channel information is not available.

2.2 Carrier Offset

In the presence of a carrier offset, the received signal is modulated by a residual carrier $e^{jn\phi}$. Taking into account the removed prefix, the received N-point signal $\mathbf{y}(k)$ becomes:

$$\begin{aligned} \mathbf{y}(k) &= \mathbf{E}\mathbf{W}_P\underbrace{\mathbf{H}\left[s_0(k)\cdots s_{P-1}\right]^T e^{j\phi(k-1)(N+L)}}_{\stackrel{\text{def}}{=}\tilde{s}(k)} \\ &= \mathbf{E}\mathbf{W}_P\tilde{\mathbf{s}}(k). \end{aligned} \quad (4)$$

where $\mathbf{E} = \mathrm{diag}(1,\ e^{j\phi}, \cdots, e^{j(N-1)\phi})$ and L is the length of the prefix. Since $\mathbf{W}^H\mathbf{E}\mathbf{W} \neq \mathbf{I}$, the $\mathbf{E}$ matrix destroys the orthogonality among the subchannel carriers and thus introduces ICI. To recover $\{\mathbf{s}(k)\}$, the carrier offset, ϕ, needs to be estimated and compensated before performing the DFT. The demodulation can be described as the following concatenated matrix operations:

$$e^{-j\phi(k-1)(N+L)}\mathbf{W}^H\mathbf{E}^H\mathbf{y}(k) = \mathbf{W}^H\mathbf{E}^H\mathbf{E}\mathbf{W}\mathbf{s}(k) = \mathbf{s}(k).$$

The problem addressed in this paper is the estimation of the carrier offset, ϕ, from the receiver outputs, $\{\mathbf{y}(k)\}$, without the use of a training sequence or known input symbols.

3 Blind Carrier Offset Estimation

Two carrier offset estimators are presented in this section. The first one mimics the well-known subspace-based MUSIC [7] approach whereas the second one resembles the ESPRIT [4] algorithm. None of them however, requires the use of subspace decomposition and blind estimation is accomplished by exploiting the data structure directly. An adaptive version of the MUSIC-like algorithm is also derived. In all algorithms, the estimation of the carrier offset does not depend on the channel characteristics.

3.1 The MUSIC-like Approach

The MUSIC algorithm estimates the parameter of interest by first performing a subspace decomposition on the data observations and then constructing a cost function based on the so-called orthogonal subspace obtained from the decomposition. In principle this approach can be applied directly to OFDM for carrier offset estimation. However the computational cost of subspace decomposition may be prohibitively expensive for OFDM applications.

Collecting K sample vectors and arranging them in a matrix form,

$$\mathbf{Y} = \left[\mathbf{y}(0)\,\mathbf{y}(1)\cdots\mathbf{y}(K-1)\right], \quad (5)$$

our goal is to derive a high efficiency algorithm to estimate the carrier offset, ϕ, directly from $\mathbf{Y}$ without the use of any training sequence or known input symbols.

Notice that in OFDM, the *signature waveform* of the ith subcarrier, which is the ith DFT vector $\mathbf{w}_i$, is inherently orthogonal to other signature waveforms. $\mathbf{W}_P$ consists of a subset of orthonormal columns and its orthogonal complement, $\mathbf{W}^{\perp} = [\mathbf{w}_{P+1} \ldots \mathbf{w}_N]$, is known *a priori*. In the absence of the carrier offset, *i.e.*, $\phi = 0$ and for $i = 1 \cdots N-P$, the following equation always holds regardless of the fading channels,

$$\mathbf{w}_{P+i}^H \mathbf{Y} = \underbrace{\mathbf{w}_{P+i}^H \mathbf{W}_P}_{=0} [\bar{\mathbf{s}}(1) \cdots \bar{\mathbf{s}}(K)] = 0,$$

Such is not true when $\phi \neq 0$. However, if we let $\mathbf{Z} = \text{diag}(1,\ z,\ z^2, \cdots, z^{N-1})$, it can be easily shown that when $z = e^{j\phi}$,

$$\mathbf{w}_{P+i}^H \mathbf{Z}^H \mathbf{Y} = \mathbf{w}_{P+i}^H \mathbf{Z}^H \mathbf{E} \mathbf{W}_P [\bar{\mathbf{s}}(1) \cdots \bar{\mathbf{s}}(K)] = 0$$

In other words, the orthogonal subspace of $\mathbf{Y}$ has a *fixed* structure: $\mathbf{Z}(\phi)\mathbf{W}^{\perp}$. This observation suggests that we form a cost function as follows,

$$P(z) = \sum_{i=1}^{L} \left\| \mathbf{w}_{P+i}^H \mathbf{Z}^{-1} \mathbf{Y} \right\|^2, \tag{6}$$

where $L \leq N - P$. Clearly, $P(z)$ is zero when $z = e^{j\phi}$. Therefore one can find the carrier offset by evaluating $P(z)$ for all possible values of $\phi \in [0,\ 2\pi]$, as in the MUSIC algorithm in array signal processing. It is noted that $P(z)$ forms a polynomial of z with order $2(N-1)$. Such allows a closed-form estimate of ϕ through polynomial rooting. In particular, $e^{j\phi}$ can be identified as the root of $P(z)$ on the unit circle. The proposed MUSIC-like algorithm is summarized as follows:

1. Form the polynomial cost function as in (6) using the receiver outputs, $\{y_n(k)\}$.
2. Estimate the carrier offset as the null of $P(z)$ or the phase of the root of $P(z)$ on the unit circle. In the presence of noise, the carrier offset is estimated as the minima of $P(z)$ or the phase of the root of $P(z)$ closest to the unit circle.

3.2 ESPRIT-like Approach

The standard ESPRIT algorithm exploits a shift invariant structure available in the signal subspace and estimates the parameters of interest through subspace decomposition and eigenvalue calculation. We show in the following that in OFDM, the shift invariant structure that enables ESPRIT manifests itself directly in the received signal, allowing carrier offset estimation to be accomplished without subspace decomposition or eigenvalue estimation.

Given the kth block of the received signal in (2), one can form $N - P + 1$ blocks of P consecutive samples in both the forward and backward directions as follows,

$$\begin{array}{l} \mathbf{y}_F^i = [y_i, \cdots, y_{i+P-1}]^T \\ \mathbf{y}_B^i = [y_{N-i-1}, \cdots, y_{N-i-P}]^H \end{array} \quad i = 0 \cdots, N-P, \tag{7}$$

where $(\cdot)^H$ denotes conjugate transpose. From (4), it can be easily verified that

$$\mathbf{y}_F^i(k) = \bar{\mathbf{E}} \bar{\mathbf{W}} \Delta^i \bar{\mathbf{s}}(k), \tag{8}$$

where $\bar{\mathbf{W}}$ consists of the first P rows of $\mathbf{W}_P$, and

$$\bar{\mathbf{E}} = \text{diag}\left(1, e^{j\phi}, \cdots, e^{j(P-1)\phi}\right), \tag{9}$$

$$\Delta = \text{diag}\left(1, e^{j\omega+\phi}, \cdots, e^{j(P-1)\omega+\phi}\right). \tag{10}$$

Similarly the backward vector y_B^i is given by:

$$\begin{aligned} \mathbf{y}_B^i(k) &= \bar{\mathbf{E}} \bar{\mathbf{W}} \Delta^i \underbrace{e^{-j\phi(N-1)} \begin{bmatrix} 1 & 0 & \cdots \\ 0 & \ddots & \vdots \\ 0 & 0 & e^{j(P-1)(N-1)\omega} \end{bmatrix} \bar{\mathbf{s}}^*(k)}_{\stackrel{\text{def}}{=} \mathbf{r}(k)} \\ &= \bar{\mathbf{E}} \bar{\mathbf{W}} \Delta^i \mathbf{r}(k). \end{aligned} \tag{11}$$

Let us denote:

$$\begin{aligned} \mathbf{Y}^u(k) &= \left[y_F^0(k) \cdots y_F^{N-P-1}\ y_B^0(k) \cdots y_B^{N-P-1}(k)\right] \\ &\stackrel{\text{def}}{=} \bar{\mathbf{E}} \bar{\mathbf{W}} \mathbf{T}(k), \\ \mathbf{Y}^d(k) &= \left[y_F^1(k) \cdots y_F^{N-P}(k)\ y_B^1(k) \cdots y_B^{N-P}(k)\right] \\ &= \bar{\mathbf{E}} \bar{\mathbf{W}} \Delta \mathbf{T}(k). \end{aligned} \tag{12}$$

where $\mathbf{T}(k) = [\bar{\mathbf{s}}(k) \cdots \Delta^{N-P-1}\bar{\mathbf{s}}(k)\, \mathbf{r}(k) \cdots \Delta^{N-P-1}\mathbf{r}(k)]$. Then upon defining $\mathbf{A} = \bar{\mathbf{E}}\bar{\mathbf{W}}$, $\mathbf{Y}^u = \left[\mathbf{Y}^u(1) \cdots \mathbf{Y}^u(K)\right]$, $\mathbf{Y}^d = \left[\mathbf{Y}^d(1) \cdots \mathbf{Y}^d(K)\right]$, and $\mathbf{T} = \left[\mathbf{T}(1) \cdots \mathbf{T}(K)\right]$, we obtain

$$\mathcal{Y}^u = \mathbf{A}\mathbf{T}, \quad \mathcal{Y}^d = \mathbf{A}\Delta\mathbf{T}. \tag{13}$$

Consequently,

$$\begin{aligned} \Psi &\stackrel{\text{def}}{=} \mathcal{Y}^d (\mathcal{Y}^u)^{\dagger} = \mathbf{A}\Delta\mathbf{T}\,(\mathbf{A}\mathbf{T})^{\dagger} = \mathbf{A}\Delta\mathbf{T}\mathbf{T}^{\dagger}\mathbf{A}^{-1} \\ &= \mathbf{A}\Delta\mathbf{A}^{-1}. \end{aligned} \tag{14}$$

Here, $(\cdot)^{\dagger}$ represents the right pseudo-inverse.

The above gives a similarity transformation of Δ which preserves the eigenvalues of Δ. Since the trace of Δ is $e^{j\phi} \sum_{m=0}^{P-1} e^{jm\omega}$, we can estimate $e^{j\phi}$ from the sum of eigenvalues of Ψ, or simply its trace. In particular, ϕ can be calculated as:

$$exp(j\phi) = \frac{tr(\Psi)}{\sum_{m=0}^{P-1} exp(jm\omega)}, \omega = 2\pi/N. \tag{15}$$

3.3 Adaptive Solution

Adaptive algorithms are desirable for carrier tracking in a time-varying environment. The MUSIC-like searching algorithm can be implemented adaptively with simple modifications. One can iterate on the estimation on the ϕ by going against the gradient of $P(z)$ in (6) with respect to ϕ. If we proceed as follows:

1. Begin with an initial value, e.g. 0, for ϕ_{est} for the frequency offset, which provides and initial estimate to where the minimum point of the $P(z)$ may be located.

2. Using this initial estimate, compute the gradient vector evaluated with respect to ϕ.

3. Compute the next estimate for ϕ by making a change in the initial or present estimate in a direction opposite to that of the gradient vector.

4. Compute the gradient again for the newly found estimate of ϕ and return to the previous step.

Successive corrections to the estimate of ϕ in the directions of the negative of the gradient vector will eventually lead to the minimum value of $P(z)$ at which point the estimate of ϕ is closest to the actual ϕ. Let $\nabla P_k(z)$ denote the value of the gradient vector at time k. The updated value of the estimate of ϕ at time $k+1$ is computed by using the simple recursion relation

$$\phi_{k+1} = \phi_k - \mu \nabla P_k(z). \tag{16}$$

where μ is a positive-real valued constant.

4 Performance Analysis and Computer Simulations

The Cramèr-Rao bound has been derived to evaluate the performance of the proposed estimators. estimators. Computer simulations were performed for an OFDM system with $N = 32$ carriers and $P = 20$ data streams. The carrier offset is estimated using the algorithms described in the previous sections. The true frequency offset $\phi = 0.25\Delta\omega$, where $\Delta\omega = \frac{2\pi}{N}$ is the channel spacing.

Using noise-free data, the null spectrum and the locus of the roots of $P(z)$ in (6) for the MUSIC-like algorithm are illustrated in Figure 2 and 3 respectively. Clearly both spectrum-searching and polynomial rooting yield perfect carrier offset estimates in the absence of noise. However the accuracy of the carrier estimation of the searching approach is governed by the step size whereas the rooting algorithm and the ESPRIT-like approach do not suffer from this limitation.

Figure 4 shows the behavior of the two proposed algorithms under different SNR values. 2 symbol blocks were used in this simulation and the mean-square errors (MSEs) of the carrier offset estimate were obtained by running 500 independent trials. The MUSIC-like algorithm offers excellent performance as indicated by the small gap between its MSEs and the CR bound. The ESPRIT-like algorithm on the other hand, suffers from the signal cancellation problem introduced by the trace operation and has its MSE decrease at a much slower rate.

The next case studies how the performance of the new algorithms improves with the increase of the number of data samples. Under 15 dB SNR, we plot in Figure 5 the MSEs vs. the number of received symbols. As seen, the MUSIC-like algorithm provides, under reasonable SNR values, very good performance, whereas the ESPRIT-like algorithm has the inherent problem which cannot be cured by increasing the number of data blocks. Notice that both the CR bound and the MSEs of the MUSIC-like approaches flat out after the number of symbol blocks reaches 4, indicating that the new approach are highly data efficient and can accomplish high performance estimation with limited observations.

In the last example we investigate the transient behavior of the proposed adaptive estimator at 15 dB SNR. It is observed from Figure 6 that the algorithm converged satisfactory after 10 iterations and the excess error is acceptably small.

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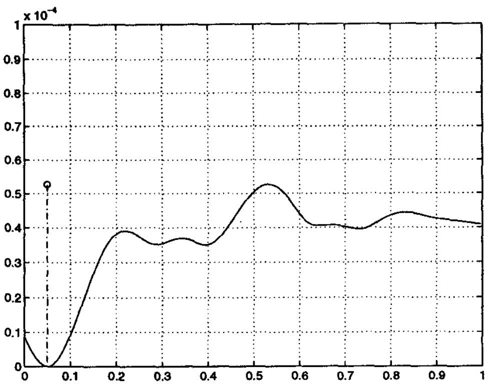

Figure 2. Null Spectrum for Noise-free Data

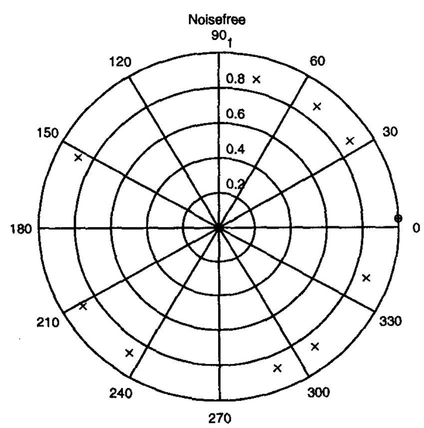

Figure 3. Root Locus for Noise-free data

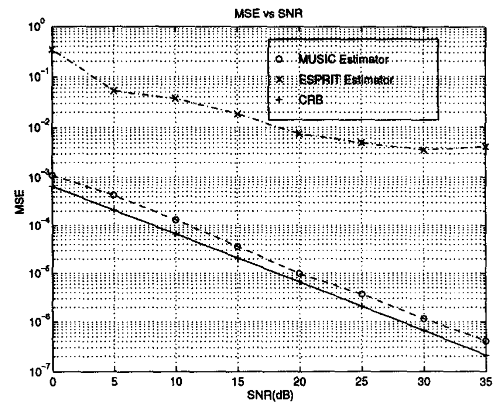

Figure 4. MSE vs SNR No. of Blocks=2

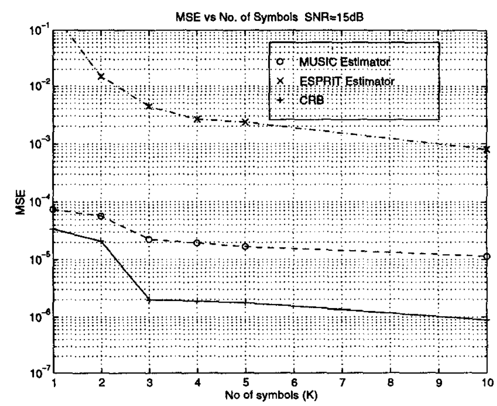

Figure 5. Batch Est. MSE vs No. of OFDM symbols SNR=15dB

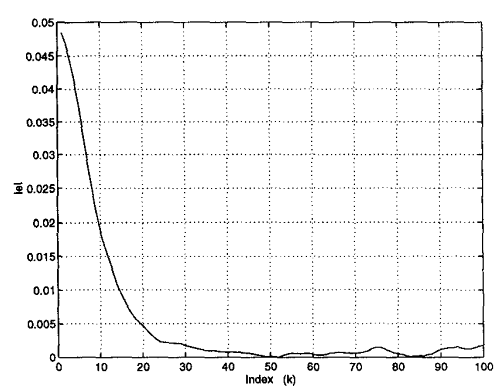

Figure 6. Adaptive Est. Error vs No. of Blocks SNR=15dB