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An Improved Frequency Offset Estimation Algorithm for OFDM System

YANG Tiejun
School of Information Science and Engineering
Henan University of Technology
Zhengzhou, China
tjyanghlyu@126.com

HU Lei
School of Information Science and Engineering
Henan University of Technology
Zhengzhou, China
hulya@126.com

***Abstract*—In Orthogonal Frequency Division Multiplexing (OFDM) system, Carrier frequency offsets (CFOs) between the transmitter and the receiver destroy the orthogonality between carriers and degrade the system performance significantly. On the basis of traditional frequency offset estimation algorithm, an improved carrier frequency offset estimation algorithm is presented in this paper, which uses one complex training sequence, can effectively estimate fractional frequency offset in the time domain and the integer frequency offset in the frequency domain. Simulation results show that the improved carrier frequency offset estimation algorithm performs better than conventional P. Moose and Schmidl algorithm, which can effectively improve the frequency estimation accuracy and provides a wide acquisition range for the carrier frequency offset with low complexity.**

Keywords-carrier frequency offset; frequency offset estimation; training sequence; OFDM

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a promising multi-carrier modulation scheme that shows high spectral efficiency and robustness to frequency selective channels. In OFDM, a frequency-selective channel is divided into a number of parallel frequency-flat sub channels, thereby reducing the receiver signal processing of the system. OFDM has been adopted in many wireless standards such as digital audio broadcasting (DAB), HIPERLAN/2, IEEE 802.11a wireless local area networks (WLAN) , IEEE 802.16a metropolitan area network (MAN), and a potential candidate for fourth-generation (4G) mobile wireless systems.

A major disadvantage of the OFDM system is its sensitivity to frequency offset errors due to Doppler shifts and differences in the oscillator frequencies at the transmitter and the receiver. The carrier frequency offset causes a net shift of the signal spectrum which may damage the subcarrier orthogonality and increase the noise level due to ICI, which degrades system performance. Therefore, frequency offset estimation in OFDM systems is essential to improve communication system performance.

Many methods for carrier frequency offset estimation in OFDM systems have been developed, which can be divided into two categories: the data-aided methods which use the training sequences or pilot signals or the non-data-aided methods which use the cyclic prefix and blind estimation methods. Frequency offset estimation method which based training sequences was first proposed by P. Moose [1] which used two identical OFDM symbols transmitted in succession to calculate the subcarrier frequency offset, but its estimation range is less than half subcarrier spacing. Schmidl and Cox [2] perform rapid synchronization with a relatively simplified computation in the time domain and an extended range for the acquisition of the carrier frequency offset, which had larger frequency offset estimation scope and higher estimation accuracy, but it needs two symbols for synchronization. The joint maximum-likelihood (ML) estimator proposed by Sandell, Börjesson, and van de Beek in [5] [6] exploits the redundancy already present in the cyclic prefix appended to every OFDM symbol, the synchronization parameters can be estimated using a single symbol. Blind estimation methods [4] [8] including MUSIC algorithm and ESPRIT algorithm, are either only effective for small frequency offsets or with considerable computational complexity.

In this paper, an improved frequency offset estimation method is proposed by using a simple complex training sequence. Fractional and integer frequency offset are estimated and corrected in the time domain and the frequency domain. This method has high estimation accuracy and low complexity.

This paper is organized as follows. Section II describes the OFDM system model. Section III briefly introduces conventional frequency estimation methods. Section IV presents the improved frequency estimation algorithm which implements fractional frequency offset in the time domain and integer frequency in the frequency. In Section V, we present simulation results illustrating the performance of the offset estimation algorithm and concluding remarks can be found in Section Ⅵ.

II. THE BASIC PRINCIPLE OF OFDM

Fig. 1 shows the block diagram of OFDM transceiver. OFDM system is very sensitive to frequency offset, which may be introduced in the radio channel, so accurate frequency offset synchronization is essential. Especially for burst mode data transmission in wireless LAN applications, we must keep the overhead, namely, the number of pilot symbols required for the synchronization, as low as possible.

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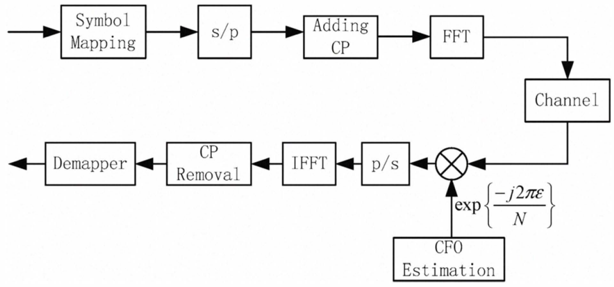

Figure 1. OFDM block diagram

We consider an OFDM system implemented by the inverse fast Fourier transform (IFFT) and fast Fourier transform (FFT). X (k) is the modulated data on the kth subcarrier. N is the FFT length. The OFDM samples at the output of IFFT are given by

$$\mathrm{x}(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) \exp(\frac{j2\pi kn}{N}) \quad (1)$$

At the receiver, when the normalized frequency offset is ε under ideal time synchronization, the receiver signal in AWGN channel is

$$\mathrm{r}(n) = x(n) e^{\frac{j2\pi\varepsilon n}{N}} + \mu(n) \quad (2)$$

Where $\mu(n)$ is white Gaussian noise with zero mean and variance $\sigma_\mu^2 = E\left(\left|\mu(n)\right|^2\right)$, normalized frequency offset is

$$\varepsilon = \varepsilon_I + \varepsilon_F = \frac{N\Delta f}{f_S} \quad (3)$$

Where ε_I is the integer frequency offset, ε_F is the decimal multiple frequency offset, Δf is the carrier frequency offset between the transmitter and the receiver, f_s is the sampling frequency, f_s/N is the frequency separation between the subcarriers in the OFDM signal.

Each subcarrier is shifted in frequency by a constant amount of $2\pi\varepsilon/N$, the peaks of the spectra on each subcarrier are no longer aligned with the sampled frequencies of the DFT. Additionally. The adjacent subcarrier spectra are non-zero at the sampled frequencies. Hence, carrier frequency offsets have the cumulative effect of reducing the signal energy and increasing the interference. Moose demonstrated that a lower-bound on the effective SNR due to carrier frequency offset can be derived as

$$SNR_\varepsilon = \frac{SNR}{1 + 0.5947 SNR \sin^2(\pi\varepsilon)} \frac{\sin^2 \pi\varepsilon}{(\pi\varepsilon)^2} \quad (4)$$

It has been proved that the frequency accuracy of 1-2% of the subcarrier spacing in Rayleigh channel and 4% in AWGN channel is prerequisite when the influence of frequency offset can be negligible.

III. CONVENTIIONAL CARRIER FREQUENCY OFFSET ESTIMATION ALGORITHM

In scheme suggested by Schmidl, two symbols with identical data are used to estimate the frequency offset. The synchronization is performed on a training sequence of two OFDM symbols. The frame timing is implemented using one unique OFDM symbol as the first symbol, which has a repetition within half a symbol period. The carrier frequency offset is estimated in two steps. First, the fractional part is detected and compensated for. Then the integer part is estimated and corrected.

Consider the first training symbol where the first half is identical to the second half, except for the progressive phase shift caused by the frequency offset. By multiplying the conjugate of the sample from the first half with the corresponding sample from the second half, the sum of products can be expressed in the following equation:

$$P(d) = \sum_{k=0}^{N/2-1} (r_{d+k}^* r_{d+k+N/2}) \quad (5)$$

The sum of the correlation value is normalized with the received energy from the second half of the first training sequence. This energy is calculated as

$$T(d) = \frac{|P(d)|^2}{R(d)^2} \quad (6)$$

The main difference between the two halves of the first training symbol will be a phase difference of $\pi T\Delta f$, which can be estimated as

$$\hat{\varphi} = angle(P(d)) \quad (7)$$

The frequency offset estimate is

$$\hat{\Delta f} = \hat{\varphi} / (\pi T) \quad (8)$$

As a second step, the integer part of the frequency offset is estimated. The second training symbol, which contains a PN sequence modulated differentially with respect to the first training symbol, is used. This PN sequence can be retrieved and compared with a reference sequence.

IV. CARRIER FREQUENCY OFFSET ESTIMATION ALGORITHM OF OFDM

A. *Fractional carrier frequency offset estimation*

The frequency offset estimation algorithm based on complex training sequence use a novel sequence C_n. It is a periodic sequence with the length N, N is the carrier number.

$$\mathrm{C}(n) = \sqrt{2} \exp(\frac{j\pi n}{4}) \quad (9)$$

In the receiver, after adding frequency offset and channel noise, the received time domain training sequence signal can be expressed as

$$\mathrm{r}(n) = C(n) e^{\frac{j2\pi\varepsilon_F n}{N}} + \mu(n) \quad (10)$$

The correlation operation is implemented using the known training sequence and received training sequences. The correlation values are expressed in the following equation

$$\begin{aligned} R &= \sum_{n=0}^{N-1} C(n) * r(n)^* + \mu(n) \\ &= \sum_{n=0}^{N-1} 2\exp(-j2\pi n\varepsilon_F / N) + \sum_{n=0}^{N-1} \sqrt{2} \exp(j\pi n/4) \mu(n)^* \end{aligned} \quad (11)$$

Ignoring the impact of noise, equation (11) can be simplified as

$$\begin{aligned} R &= \sum_{n=0}^{N-1} 2 * \exp(-j2\pi n\varepsilon_F / N) \\ &= 2 * \frac{1-\exp(-j2\pi\varepsilon_F)}{1-\exp(-j2\pi\varepsilon_F / N)} \\ &= 2 * \frac{\exp(-j\pi\varepsilon_F)}{\exp(-j\pi\varepsilon_F / N)} * \frac{(\exp(j\pi\varepsilon_F)-\exp(-j\pi\varepsilon_F))}{(\exp(j\pi\varepsilon_F / N)-\exp(-j\pi\varepsilon_F / N))} \\ &= 2 * \exp(j\frac{1-N}{N}\pi\varepsilon_F) * \frac{\sin(\pi\varepsilon_F)}{\sin(\pi\varepsilon_F / N)} \end{aligned} \quad (12)$$

When N is large enough

$$\frac{\sin(\pi\varepsilon_F)}{\sin(\pi\varepsilon_F / N)} \approx \frac{\pi\varepsilon_F}{\pi\varepsilon_F / N} = N \quad (13)$$

Equation (4) can be simplified as

$$R = 2 * N * \exp(j\frac{1-N}{N}\pi\varepsilon_F) \quad (14)$$

The fractional frequency offset would seriously destroy the orthogonality among subcarriers, and the integer frequency offset will cause the shift of the frequency domain signal. We can first estimate the normalized fractional frequency offset ε_F as

$$\hat{\varepsilon}_F = \frac{N}{\pi(1-N)} angle(\frac{R}{2N}) \quad (15)$$

The estimation range of the fractional frequency offset is $|\varepsilon_F| < 1$.

After the fractional frequency offset estimation is estimated, the received symbols are frequency corrected by multiplying the samples by $\exp(-j2\pi \hat{\varepsilon}_F n / N)$ to compensate the fractional frequency offset. So the received training sequence became

$$y(n) = r(n)\exp(\frac{-j2\pi\varepsilon_F n}{N}) \quad (16)$$

B. *Integer carrier frequency offset estimation*

After FFT, we will use frequency domain pilot to estimate the integer frequency offset. The received frequency domain OFDM training sequence signal is Y (k).

$$Y(k) = FFT(y(n)) \quad (17)$$

In the following, $\{P(k), k \in C\}$ are the pilot signals, and C is the pilot signal sub-carrier index set. Suppose the fractional frequency offset correction has been completed, m is the integer frequency offset. We have

$$Y(k) = X((k-m) \bmod N) \quad (18)$$

Define the intermediate variable

$$M(k) = \sum_{k \in C} Y((k-m) \bmod N) P(k) \quad (19)$$

The normalized integer frequency offset estimation m will be estimated as the following

$$\hat{m} = \arg\max_{m \in (-\frac{N}{2}, \frac{N}{2})} (| M(m) |) \quad (20)$$

The correction range is $|m| < N/2$.

So, the total value of frequency offset estimation is

$$\hat{\varepsilon} = \frac{N}{\pi(1-N)} angle(R / 2N) + \hat{m} \quad (21)$$

V. Simulation Results

We verify the performance of our presented algorithms by simulation and compare the results with P.Moose and Schimidl algorithm. OFDM systems with 128 sub-carriers and QPSK modulation over frequency selective fading channel, which is the exponential decaying Rayleigh fading channel model proposed for the IEEE 802.11-98/156r2, was investigated.

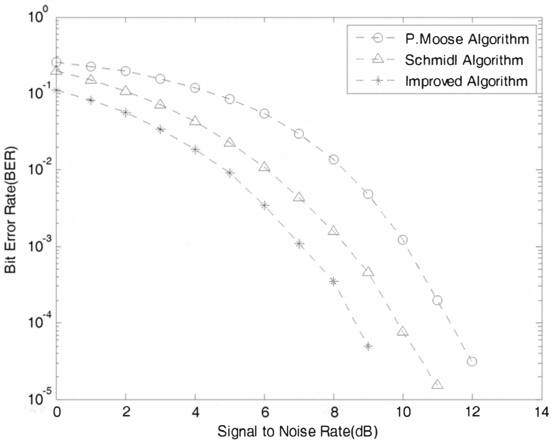

Figure 2. Bit Error Rate curves of three methods when ε=0.39

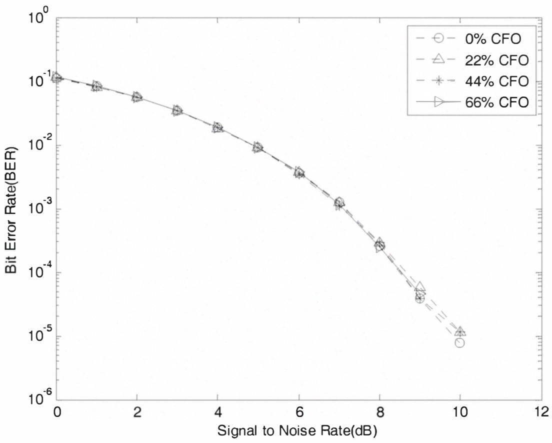

Figure 3. Bit Error Rate with different carrier frequency offset

Fig. 2 show the BER results of P.Moose algorithm, Schimidl algorithm and the frequency offset estimation algorithm based on complex training sequence over frequency selective fading channel. We see that the BER of the improved algorithm presented in this paper was better than conventional P.Moose algorithm and Schimidl algorithm. It has 1.5db performance advantage compared with Schimidl algorithm and 2.5db with P.Moose algorithm at a BER of 10^{-5}. The algorithm based on complex training sequence improves the accuracy of frequency offset estimation.

Figure 3 shows the BER results of the improved frequency offset estimation algorithm with the different frequency offset. We sec that the improved algorithm can effectively correct different frequency offset, which provides a wide acquisition range for the carrier frequency offset.

VI. CONCLUSIONS

In this paper, an improved frequency offset estimation algorithm based on complex training sequence was presented, which implements fractional frequency offset in the time domain and integer frequency in the frequency, and compared with the conventional P. Moose algorithm and Schmidl algorithm. Simulation results show that the improved algorithm performs better than P. Moose algorithm and Schmidl algorithm, which has the lower SNR cost of nearly 1.5 dB than the Schmidl algorithm, and nearly 2.5 dB than the P. Moose algorithm at a BER of 10^{-5}. The improved frequency offset estimation algorithm can effectively improve the accuracy of frequency offset estimation and provides a wide acquisition range for the carrier frequency offset. Furthermore, it requires only one training sequence, can save the system overhead and improve the efficiency of spectrum.

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