

Carrier Frequency Offset Estimation Using Extended Kalman Filter in Uplink OFDMA Systems

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Abstract—This paper presents a new carrier frequency offset (CFO) estimation algorithm with low-complexity for uplink OFDMA systems. Extended Kalman Filter (EKF) is employed in time domain, while the multiple access interference (MAI) cancellation strategy is combined in every recursion. In addition, an adaptive noise variance estimator is contrived for EKF, and a robust EKF method is derived for CFO estimation with data symbols. The proposed algorithm is suitable for online estimation, and can be applied to both preamble and data symbols. Simulation results show that it has a large estimation range, while approaching the Cramer-Rao Bound (CRB) closely with quite reduced complexity.

Index Terms—OFDMA, carrier frequency offset, Extended Kalman Filter.

I. INTRODUCTION

Orthogonal frequency-division multiple access (OFDMA) is being considered as a modulation and multiple access method for the 4th generation wireless networks. In OFDMA, different users simultaneously transmit their own data by modulating an exclusive set of orthogonal sub-carriers, thus each user's signal can be separated easily in the frequency domain. Besides, robustness to narrowband interference and dynamic channel assignment are other two advantages of OFDMA systems.

Synchronization error is one of the most challenging problems in OFDMA systems. As different users' signals are mixed together at the receiver, the carrier frequency offset (CFO) will cause inter-carrier interference both from the user itself and from all the other users. Recently, several synchronization algorithms for uplink OFDMA systems have been proposed [1]-[3]. The algorithm proposed in [1] estimates the offsets by assuming that all users are already synchronized in time and frequency except for one new user. However, this assumption may not hold in practice. A decision-feedback tracking loop is employed for nonpilot-aided synchronization in [2], which performs well but is only suitable for the case where the offset is small. A maximum likelihood estimator is employed in [3], where the Expectation Maximization algorithm is combined. However, an one-dimensional searching problem for the optimal solution should be involved in the algorithm, which makes it quite complex.

Generally, the estimation of CFO in OFDM systems can be seen as a nonlinear least squares problem [4], and the Extended Kalman Filter (EKF) has been employed to solve

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this problem [5]. As a recursive MMSE algorithm, the EKF estimator always performs quite well. However, this method is only designed for single-user OFDM systems. In uplink OFDMA systems, the CFOs of all users may differ from each other, and the interference from other users will degrade the estimator's performance significantly. One possible solution is to combine all users' CFOs into the state vector in Kalman filter. But as this is a high-dimensional nonlinear state-space model, the estimator is likely difficult to converge.

In this paper, a new CFO estimator for uplink OFDMA systems is proposed, which uses an one-dimensional EKF for every user. In each recursion, the results of these independent EKF estimators are utilized together for multiple access interference (MAI) cancellation. Meanwhile, an adaptive algorithm is contrived to track the noise variance in EKF, and a robust EKF method is also derived for CFO estimation in data symbols. Since every user's CFO is estimated by an one-dimensional EKF, the proposed algorithm has a low complexity and is suitable for online estimation. Moreover, both preamble and data symbols can be employed for estimation. It is verified that the proposed algorithm has a large estimation range, and can approach the Cramer-Rao Bound (CRB) closely.

This paper is organized as follows. The uplink OFDMA signals with CFOs are described in Section II, while the EKF estimator is introduced in Section III. Improved methods are proposed in Section IV, including MAI cancellation strategy, adaptive noise variance estimator and robust EKF method. Simulation results are discussed in Section V and, finally, conclusion is offered in Section VI.

II. SYSTEM DESCRIPTION

We consider an OFDMA system that consists of M users, where every user experiences an independent multi-path channel as shown in Fig. 1. There are N sub-carriers in total and one sub-carrier can only be allocated to one user. Let Γ_i denotes the index set of the sub-carriers assigned to the i th user. Then, we have $\bigcup_{i=1}^M \Gamma_i = \Pi$ and $\Gamma_i \cap \Gamma_j = \emptyset$, if $i \neq j$. Here, Π denotes the index set of all the usable sub-carriers. Note that these sub-carriers may be allocated interleaved or non-interleaved [2].

Similarly to [8], assume that two types of OFDMA symbols are utilized in the uplink. For every user, one preamble is transmitted first, in which all the usable sub-carriers are

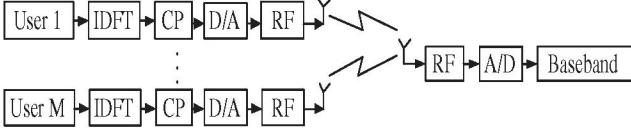


Fig. 1. OFDMA Uplink Model.

assigned with pilots. Then several data symbols are transmitted sequentially, where scattered pilots are inserted among data sub-carriers.

Let $X_i(k)$ denote the k th complex data of the i th user in frequency domain, where $k \in \Gamma_i$. Then, by operation of IDFT and cyclic prefix insertion, the n th transmitted time domain sample of the i th user can be expressed as

$$s_i(n) = \frac{1}{\sqrt{N}} \sum_{k \in \Gamma_i} X_i(k) \exp(j \frac{2\pi n k}{N}) \quad -N_g \leq n \leq N-1 \quad (1)$$

where N_g is the length of cyclic prefix.

Moreover, assume that the channel impulse response of the i th user is

$$\mathbf{h}_i = [h_i(0) \ h_i(1) \ \dots \ h_i(P-1)]^T \quad (2)$$

where P is the length of the maximum channel delay spread and $P \leq N_g$. Let ε_i denote the i th user's normalized CFO at the receiver, $z_i(n)$ the additive white Gaussian noise. Then, the received signal of the i th user is given by

$$r_i(n) = y_i(n) \exp(j \frac{2\pi n \varepsilon_i}{N}) + z_i(n) \quad (3)$$

where $y_i(n) = \sum_{l=0}^{P-1} s_i(n-l) h_i(l)$. Finally, the total signal at the receiver can be written as

$$r(n) = \sum_{i=1}^M r_i(n) = \sum_{i=1}^M y_i(n) \exp(j \frac{2\pi n \varepsilon_i}{N}) + z(n), \quad (4)$$

where $z(n)$ presents the channel noise at the receiver and $z(n) = \sum_{i=1}^M z_i(n)$.

After dropping the cyclic prefix and applying N-point DFT, the received signal in frequency domain can be written as

$$Y(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} r(n) \cdot \exp(-j2\pi nk/N), \quad k = 0, 1, \dots, N-1. \quad (5)$$

Due to the existence of the CFOs, the received signal $Y(k)$ includes interference both from the same user and from all the other users, which was analyzed in [2].

III. EXTENDED KALMAN FILTER ESTIMATOR

It is reasonable to assume that all users' preambles are known at the receiver, and they are employed for CFO estimation. Meanwhile, assume that different users' channels have been estimated perfectly. Thus, $y_i(n)$ in (4) is known at the

receiver. Generally, the CFOs can be estimated by minimizing the following cost function:

$$\min_{\omega} \left\{ \sum_{n=0}^{N-1} \left| r(n) - \sum_{i=1}^M y_i(n) \exp(j \frac{2\pi n \tilde{\varepsilon}_i}{N}) \right|^2 \right\} \quad (6)$$

where $\omega = [\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \dots, \tilde{\varepsilon}_M]^T$. However, this multi-dimensional optimization problem is quite complex to solve, especially when the number of the subscribers is large.

Instead of directly solving the optimization problem in (11), this paper adopts an Extended Kalman Filter (EKF) to simplify the estimation procedure. Specifically, the CFO is estimated by a recursive process, where the state vector and measurement vector are all one-dimensional. Thus, the optimal solution searching problem is avoided, and the EKF estimator has a low complexity. Besides, it is also applicable for online estimation.

Let $\hat{r}_i(n)$ denotes the i th user's estimated component from signal $r(n)$. Then, we have

$$\hat{r}_i(n) = y_i(n) \exp(j \frac{2\pi n \varepsilon_i}{N}) + w_i(n), \quad 0 \leq n \leq N-1 \quad (7)$$

where $w_i(n)$ includes both channel noise term, $z(n)$, and interference from other users. The variance of $w_i(n)$ is assumed to be σ_i^2 . Note that $\hat{r}_i(n)$ may be obtained by an MAI cancellation process, which will be introduced in Section IV.

Consequently, a state-space model is employed to estimate the CFO as follows:

state equation:

$$\varepsilon_i(n) = \varepsilon_i(n-1) \quad (8)$$

measurement equation:

$$\hat{r}_i(n) = y_i(n) \exp\left(j \frac{2\pi n \cdot \varepsilon_i(n)}{N}\right) + w_i(n) \quad (9)$$

where n is the recursive index.

Similarly to [5], we apply the Extended Kalman Filter to the proposed model. Assuming that the CFO is in the range of $(-\tau, \tau]$, the EKF estimator is then described as below:

- 1) Initialization: $\sigma_i^2, P_i(-1), \hat{\varepsilon}_i(-1), n = 0$,

$$g(x) = \begin{cases} \tau, & \text{if } x > \tau \\ x, & \text{if } \tau \geq x \geq -\tau \\ -\tau, & \text{if } x < -\tau \end{cases} \quad (10)$$

- 2) Calculate the Kalman Gain $K_i(n)$:

$$K_i(n) = P_i(n-1) H_i^*(n) \cdot [H_i(n) P_i(n-1) H_i^*(n) + \sigma_i^2]^{-1} \quad (11)$$

$$H_i(n) = \frac{\partial r_i(n)}{\partial \varepsilon_i} \Big|_{\varepsilon_i=\hat{\varepsilon}_i(n-1)} = \frac{j2\pi n}{N} \exp\left(j \frac{2\pi n \hat{\varepsilon}_i(n-1)}{N}\right) \cdot y_i(n) \quad (12)$$

3) Update the estimate:

$$\hat{\varepsilon}_i(n) = g \{ \hat{\varepsilon}_i(n-1) + I_{update}(i, n) \} \quad (13)$$

$$I_{update}(i, n) = Re \left\{ K_i(n) \left[\hat{r}_i(n) - y_i(n) \exp(j \frac{2\pi n \hat{\varepsilon}_i(n-1)}{N}) \right] \right\} \quad (14)$$

4) Calculate the estimation variance:

$$P_i(n) = [I - K_i(n) H_i(n)] P_i(n-1) \quad (15)$$

5) If $n < N - 1$, then $n = n + 1$ and return to step 2.

After N times of recursions, $\hat{\varepsilon}_i(N-1)$ is used as the estimated value of the i th user's CFO, which is denoted as $\hat{\varepsilon}_i$.

The parameter τ in nonlinear function $g(x)$ represents the estimation range of the EKF algorithm. When the offset to be estimated is large, the error caused by linearization in EKF will affect the algorithm's performance. However, it is verified that the normalized CFO in the range larger than $(-0.5, 0.5]$ can be well estimated. Thus, with an initial estimation method to obtain the integer part of the normalized CFO, the EKF algorithm may be extended to the full range easily.

IV. IMPROVEMENT IN EKF ESTIMATION

In the uplink OFDMA systems, the performance of the basic EKF estimator will be affected by MAI seriously. In this section, several simple methods are proposed to mitigate the impact of the MAI. Compared with the basic EKF, the proposed methods will improve the estimation performance significantly with only a little increase in complexity.

A. MAI Cancellation Strategy

MAI cancellation is a crucial method in multi-user communication systems [6]-[7]. Generally, this method is employed when all the received data in one OFDM symbol have been processed. However, it will cause some iterations at the receiver, which needs more computations and more time for estimation.

In this paper, the MAI cancellation is implemented in each recursion, which is shown in Fig. 2. The results from the $(n-1)$ th recursion are employed to estimate and eliminate different users' signals in the n th recursion. Consider the n th recursion, the MAI cancellation method can be described as follows:

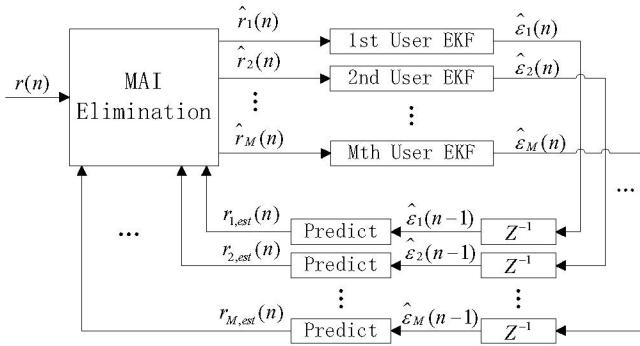


Fig. 2. Block diagram of the proposed EKF with MAI cancellation.

1) MAI Estimation: for $i = 1$ to M

$$r_{i,est}(n) = y_i(n) \exp \left(j \frac{2\pi n \cdot \hat{\varepsilon}_i(n-1)}{N} \right) \quad (16)$$

2) MAI Elimination: for $i = 1$ to M

$$\hat{r}_i(n) = r(n) - \sum_{j=1, j \neq i}^M r_{j,est}(n) \quad (17)$$

After certain recursions, the algorithm becomes convergent and the MAI can be eliminated efficiently. As the MAI cancellation and the recursion are processed simultaneously, no iteration is needed in this method. Thus, it has quite a low complexity, and can be applied for online estimation.

B. Adaptive Noise Variance Estimation

Noise variance σ_i^2 in EKF algorithm is a crucial parameter which affects the performance of estimator. Besides, it is also an essential information for soft demodulation and soft decoder. However, analysis in [2] shows that σ_i^2 includes both the channel noise and the multiple access interference, which makes it difficult to be predetermined. Besides, since the MAI cancellation is also combined, the power of noise σ_i^2 varies in the recursive estimation process. Thus, an adaptive noise variance estimation method is quite desirable here.

In particular, in the EKF algorithm, it turns out that

$$\begin{aligned} & E \left\{ \left| \hat{r}_i(n) - y_i(n) \exp \left(j \frac{2\pi n \cdot \hat{\varepsilon}_i(n-1)}{N} \right) \right|^2 \right\} \\ & \approx H_i(n) P_i(n-1) H_i^*(n) + \sigma_i^2. \end{aligned} \quad (18)$$

Accordingly, a noise variance estimator can be obtained by a nonlinear function $f(x)$:

$$\begin{aligned} \hat{\sigma}_i^2 = & \frac{1}{N} \sum_{n=0}^{N-1} f \left\{ \left| \hat{r}_i(n) - y_i(n) \exp \left(j \frac{2\pi n \hat{\varepsilon}_i(n-1)}{N} \right) \right|^2 \right. \\ & \left. - H_i(n) P_i(n-1) H_i^*(n) \right\} \end{aligned} \quad (19)$$

where

$$f(x) = \begin{cases} x, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{cases} \quad (20)$$

As the MAI cancellation is combined in every recursion, the estimates from the early recursions may have more MAI, while those from the late ones have less. Thus, a decay factor $\beta(n)$ is used to track such variation. That is,

$$\beta(n) = \frac{b^{N-1-n}(1-b)}{1-b^N}, 1 > b > 0 \quad (21)$$

$$\begin{aligned} \hat{\sigma}_i^2 = & \\ & \frac{1}{N} \sum_{n=0}^{N-1} \beta(n) \cdot f \left\{ \left| \hat{r}_i(n) - y_i(n) \exp \left(j \frac{2\pi n \hat{\varepsilon}_i(n-1)}{N} \right) \right|^2 \right. \\ & \left. - H_i(n) P_i(n-1) H_i^*(n) \right\}, \end{aligned} \quad (22)$$

or, in a sequential form which is given by

$$\begin{aligned}\hat{\sigma}_i^2(n) = & \left[1 - \frac{1-b}{1-b^{n+1}} \right] \cdot \hat{\sigma}_i^2(n-1) + \\ & \frac{1-b}{1-b^{n+1}} \cdot f \left\{ \left| \hat{r}_i(n) - y_i(n) \exp(j \frac{2\pi n \hat{\varepsilon}_i(n-1)}{N}) \right|^2 \right. \\ & \left. - H_i(n) P_i(n-1) H_i^*(n) \right\}, \quad 0 \leq n \leq N-1.\end{aligned}\quad (23)$$

At the end of the recursions, the interference from other users is eliminated, and the result of the proposed algorithm can be seen as the estimated variance of channel noise, $z(n)$.

C. Robust Estimation of CFO in Data Symbols

Generally, the proposed algorithm is accomplished based on the preamble. On the other hand, it is evident that the scattered pilots in data symbols may also be used in the estimation. Assume that these scattered pilots have been known at the receiver, then in the proposed algorithm, $y_i(n)$ is substituted by $\tilde{y}_i(n)$. That is,

$$\tilde{y}_i(n) = \sum_{l=0}^{P-1} \tilde{s}_i(n-l) h_i(l), \quad (24)$$

$$\begin{aligned}\tilde{s}_i(n) = & \frac{1}{\sqrt{N}} \sum_{k \in \hat{\Gamma}_i} X_i(k) \exp(j \frac{2\pi n k}{N}), \\ 0 \leq n \leq N-1.\end{aligned}\quad (25)$$

Here, $\hat{\Gamma}_i$ denotes the index set of the i th user's scattered pilots in data symbols, and $\hat{\Gamma}_i \subseteq \Gamma_i$.

However, since only scattered pilots can be utilized, the estimator is affected by interference from all users more easily. Here, we propose a robust EKF method for estimating the CFO with data symbols. In particular, the update of the estimate in EKF is substituted by

$$\begin{aligned}\hat{\varepsilon}_i(n) = & g \{ \hat{\varepsilon}_i(n-1) + u [I'_{update}(i, n), n] \}, \\ 0 \leq n \leq N-1\end{aligned}\quad (26)$$

$$u(x, n) = \begin{cases} \Gamma/(n+1), & \text{if } x \geq \Gamma/(n+1) \\ x, & \text{if } \Gamma/(n+1) > x > -\Gamma/(n+1) \\ -\Gamma/(n+1), & \text{if } x \leq -\Gamma/(n+1) \end{cases} \quad (27)$$

Consequently, the influence function of this robust method, $IF[x, y]$, can be derived as:

$$|IF[x, \hat{\varepsilon}_i(n-1)]| = |n \cdot [\hat{\varepsilon}_i(n) - \hat{\varepsilon}_i(n-1)]| \quad (28)$$

$$\leq |n \cdot u [I'_{update}(i, n), n]| < \Gamma, \quad (29)$$

where

$$\begin{aligned}I'_{update}(i, n) = & \\ Re \left\{ K_i(n) \left[x - y_i(n) \exp \left(j \frac{2\pi n \hat{\varepsilon}_i(n-1)}{N} \right) \right] \right\}.\end{aligned}\quad (30)$$

Since the influence function is bounded, the robustness of the estimator is ensured.

V. SIMULATION AND DISCUSSION

In computer simulations, the OFDMA system parameters are selected from [8], where the uplink bandwidth is 20MHz, $N=2048$, and $N_g=256$. All the usable sub-carriers are grouped into 32 sub-channels. We assume that there are 4 users and every user employs 8 sub-channels. One preamble is transmitted first in the uplink and QPSK modulation is used for data sub-carriers. The sub-carrier partition and the pilot assignment method are all the same as those in [8]. SUI-4 channel model with 3 paths [9] is considered. The proposed EKF algorithm is used, where MAI cancellation and adaptive noise variance estimator are utilized for preamble, while all the proposed improved methods are employed for data symbol.

We evaluate the performance of the proposed algorithm based on the simulation results for User 1. Define SNR as $SNR = \frac{\sigma_1^2}{\sigma_z^2}$, where σ_1^2 is the mean power of the received signal from User 1, and σ_z^2 is the variance of channel noise $z(n)$. Consequently, the CRB of $\hat{\varepsilon}_1$ is derived as:

$$CRB(\hat{\varepsilon}_1) \approx \frac{3\sigma_z^2}{8\pi^2 N \sigma_1^2} = \frac{3}{8\pi^2 N \cdot SNR} \quad (31)$$

First of all, we evaluate the convergence behavior of the proposed algorithm. The actual normalized CFOs are set to $[\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4] = [2, -2, 1, -1]$ and the SNR is set to 5dB. The average estimation result of ε_1 from 1000 Monte-Carlo runs is depicted in Fig. 3. It is observed that the proposed algorithm has a fast convergence speed, especially when the preamble is used. Besides, although less pilots are employed in data symbols, the estimator can still achieve the stable state in one OFDM symbol.

Fig. 4 shows the results of the adaptive noise variance estimator for $SNR=5$ dB. The true variance of channel noise $z(n)$ (dotted line) and the estimate provided by the proposed algorithm (solid line) are all depicted, where the variance is normalized by the power of received signal from User 1. Since the interference from other users is eliminated in recursions gradually, the estimated variance decreases in the recursive process. Specifically, after certain recursions, the estimator can track the true variance of channel noise with very small error.

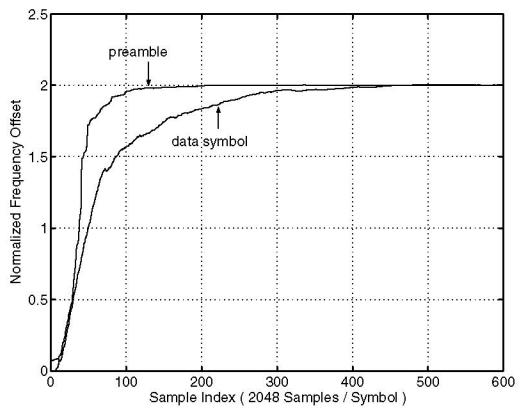


Fig. 3. Estimation performance of the proposed EKF algorithm. CFO to be estimated is 2 and SNR= 5dB.

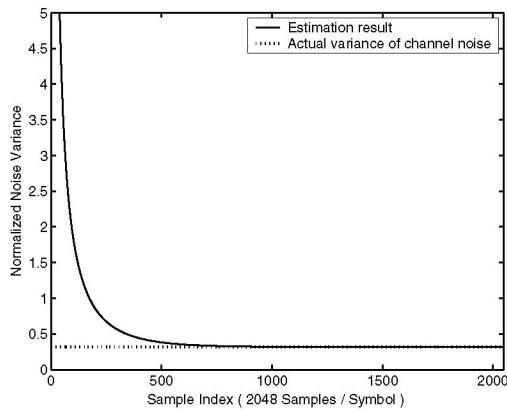


Fig. 4. Adaptive noise variance estimation for SNR= 5dB. One preamble is used.

Next, the performance of the proposed EKF is evaluated by mean square errors (MSE) of estimate $\hat{\varepsilon}_i$, which is $E\{|\varepsilon_i - \hat{\varepsilon}_i|^2\}$. For every user, a random normalized CFO is generated from the interval of $(-\varepsilon_{max}, \varepsilon_{max}]$. 1000 Monte-Carlo runs are performed for every situation. Here, only the estimation results of User 1 are provided.

Fig. 5 shows the MSE performance of the proposed algorithm when one preamble is utilized. The maximum normalized CFO is set to $\varepsilon_{max}=10$. For comparison, we also show the performance of the basic EKF without all the proposed improvement. It is clarified that the proposed algorithm approaches the CRB closely for quite a large estimation range, which is due to the adaptive tracking of noise variance and the MAI cancellation. As expected, the basic EKF estimator has a poor performance caused by the interference from other users, which may decrease the effective SNR to less than -4.7 dB.

Fig. 6 shows the MSE performance of the proposed algorithm in data symbols. The maximum normalized CFO is set to $\varepsilon_{max}=5$. The result of the EKF algorithm without the robust CFO estimation method is also depicted, labeled as “nonrobust EKF”. Although less pilots can be utilized in data symbols, the proposed algorithm can still obtain a moderate performance for a large enough range. However, without the robust estimation method, the performance of the algorithm decreases seriously.

VI. CONCLUSION

In this paper, a new algorithm using Extended Kalman Filter in time domain is proposed to estimate the CFOs in uplink OFDMA signals. The interference from other users is eliminated by the MAI cancellation in every recursion. Besides, an adaptive estimator is derived to track the variation of the noise in the recursions. Moreover, a robust EKF method is also contrived for estimating the CFOs in data symbols. Since only one-dimensional EKF is employed, the proposed algorithm has quite a low implementation complexity, and can be applied for online estimation. The performance of the estimator with only one preamble or one data symbol was demonstrated by simulations. It is worth to note that the proposed estimator has a large estimation range, and can

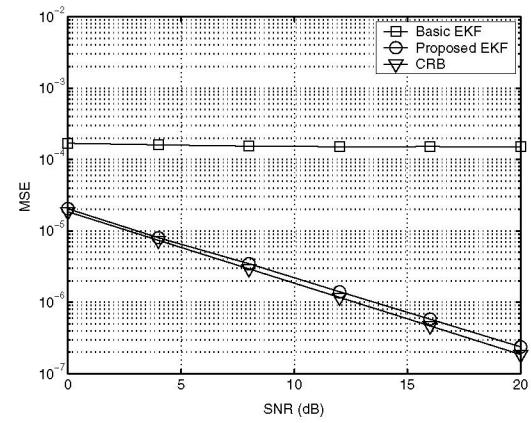


Fig. 5. MSE for CFO estimation versus SNR when one preamble is used.

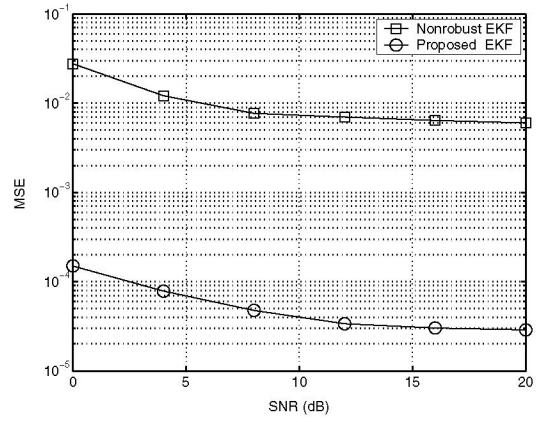


Fig. 6. MSE for CFO estimation versus SNR when one data symbol is used.

approach the Cramer-Rao Bound closely when preamble is used.

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