Extended Kalman Filter for Channel and Carrier Frequency Offset Estimation

Pham Hong Lien, Nguyen Duc Quang and Luu Thanh Tra

***Abstract* - In this paper, the algorithms of combining channel estimation with CFO estimation are studied for mobile OFDM system. The comb-type pilot is used for channel estimation . There are some known algorithms for channel estimation such as LS (Least Square), Minimum Mean Square Error (MMSE), some adaptive estimators such as Kalman Filter, Extended Kalman Filter... To enhance performance, we modified the algorithm of channel estimation by jointly estimating the channel frequency response and CFO (Channel Frequency Offset) from the transmission of known pilot symbols. This new estimator had good performance in fading environments for vehicular model with high mobile speed. Bit Error Rate (BER) performance of the proposed algorithm is verified by computer simulation.**

***Keywords* - Extended Kalman Filter, estimation channel, mobile OFDM.**

I. INTRODUCTION

This article focuses on channel estimation system for mobile OFDM (Orthogonal Frequency Division Modulation) system. The OFDM has been chosen to be used for current 4G such as LTE, WiMAX advanced and near future mobile systems since it shows many advantages [6]. The channel estimation can be performed by inserting pilot tones into OFDM symbol [7], [8]. Comb type pilot system has good performance in fading environment [9]. LS (Least Square), MMSE (Minimum Mean Squared Error) are popular estimators and used for mobile WiMAX system [10].

In [1], [2], adaptive Estimators based Kalman algorithm are researched and developed. However, Kalman estimator only had good performance in fading environment with low speed and had poor performance in high speed condition. This is because Kalman estimator is computed adaptively from changes of the previous state and Kalman algorithm keeps good estimator if the next state does not changed rapidly or in slow fading environment. When the next state changes rapidly in condition of high speed, Kalman computation cannot estimate next state from the previous state or Kalman parameters are not converged or optimized. For example, in [15], [16], a modified Kalman Filter is proposed for OFDM channel estimation where the time-varying channel is modelled as an Auto Regression (AR) process and the parameters of the AR process are assumed real and within the range [0.98, 1] for slow fading channels.

However, in the high mobility environment, these parameters are relative large (e.g. in the 200km/h environment, they are complex values with magnitudes varying in [0, 1.5]) representing a fast fading channels. In this paper, to extend this issue, we developed an Extended Kalman Filter (EKF) to jointly estimate the channel frequency response and Carrier Frequency Offset from the transmission of known pilot symbols. Our algorithm gave better performance than traditional estimators and Extended Kalman estimator in [1].

These results are proved in section VI with computer simulation and this algorithm is proposed in this paper.

II. SYSTEM MODEL

OFDM converts serial data stream into parallel blocks of size N and modulates these blocks using inverse fast Fourier Transform (IFFT). Time domain samples of an OFDM symbol can be obtained from frequency domain data symbols as Fig. 1 shows a typical block diagram of OFDM system with pilot signal assisted. The binary information data are grouped and mapped into multi-amplitude multi-phase signals.

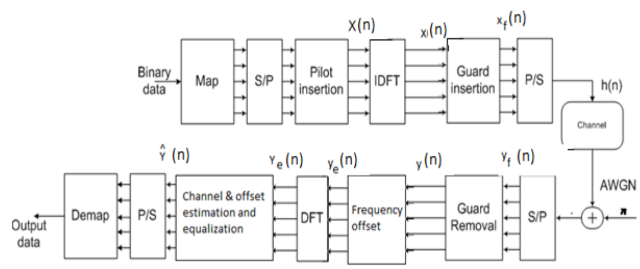

Figure 1. OFDM system model with CFO estimation

The binary information is first grouped and mapped according to the modulation in "signal mapper". After inserting pilots either to all sub-carriers with a specific period or uniformly between the information data sequence, IFFT block is used to transform the data sequence of length N {X(k)} into time domain signal {x(n)} with the following equation:

$$x(n) = IFFT\{X(k)\} = \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi kn}{N}} \qquad n = 0,1,2,\ldots N-1 \quad (1)$$

Where N is FFT length

Following IFFT block, guard time, which is chosen to be larger than the expected delay spread, is inserted to prevent inter-symbol interference. This guard time includes the cyclically extended part of OFDM symbol in order to eliminate inter-symbol interference (ISI). The total OFDM symbol is given as follows:

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$$x_f(n) = \begin{cases} x(N+n), & n = -N_g, -N_g+1, \cdots, -1 \\ x(n), & n = 0,1,\cdots N-1 \end{cases} \quad (2)$$

where N_g is the length of the guard interval.

Signal after the conversion from parallel to serial will be passed through fading channel and added noise. At the receiver, the receiver signal is

$$y_f(n) = x_f(n) \otimes h(n) + w(n) \quad (3)$$

With matrix equation, we obtain:

$$y(n) = \tilde{h}(n)x(n) + w(n) \quad (4)$$

$$y(n) = [y(0)\ x(1) \dots y(N-1)]^T \quad (5)$$

$$x(n) = [x(0)x(1) \dots x(N-1)]^T \quad (6)$$

$$w(n) = [w(0)\ \ w(1) \dots w(N-1)]^T \quad (7)$$

$\tilde{h}(n)$ is circulant matrix of $h(n)$.

Assume that the channel response is constant in a OFDM period. Frequency channel response is calculated:

$$F_N^H \tilde{h}(n) F_N = H(n) \quad (8)$$

Receiver signal in time domain with carrier offset frequency effect is calculated as:

$$y_e(n) = C_e \tilde{h}\ (n)x(n) + w(n) \quad (9)$$

After DFT (FFT) transforming, we obtain:

$$Y_e(n) = C_e F_N^H \tilde{h}(n) F_N X(n) + W(n) \quad (10)$$

With $W(n) = F_N{}^H w(n)$ is frequency response of noise matrix

$$W(n) = [W(0)\ W(1) \dots W(N-1)]^T \quad (11)$$

Where $C_e = diag\,(e^{j*2*pi*\varepsilon*\frac{k}{N}})$ is diagonal matrix represents the carrier frequency offset , k = 0,1,… N-1.

Where ε is carrier frequency offset between the transmitter and the receiver.

Where : $z_n = [\,h^T(k)\,,\ \varepsilon(k)]^T$,(with k = 0,1,…N -1, N is number of sub-carrier). N is FFT length.

Transtion equation is established as below:

$$z(n) = Fz(n-1) + v_1(n) \quad (12)$$

Where F is transition matrix and selected approximately unit matrix. Then F is unit matrix $v_1(n)$ is zero mean white noise process with correlation matrix. We have measurement equation:

$$y(n) = C_e \widetilde{h(n)} x(n) + w(n) \text{or}\ y(n) = C(z(n)) + v_2(n) \quad (13)$$

Where $z(n) = [\,h^T(n)\,,\ \varepsilon(n)]^T$ và $C(z(n)) = C_e \tilde{h}(n)$ (14)

$h^T(n)$: channel response in time domain at the n[th] OFDM symbol.

$\varepsilon(n)$: is carrier frequency offset between the transmitter and the receiver at the n[th] OFDM symbol.

III. PROPOSED ALGORIHM OF CHANNEL ESTIMATION COMBINED WITH CFO ESTIMATION

Part 1 (Training process with EKF):

+ At the first OFDM symbol:

In this symbol, we use LS algorithm and linear interpolation so that we rebuild the first channel response so that other OFDM symbols can base on this information.

$$H_0 = \frac{Y_0}{X_0} \quad (15)$$

With H_0, X_0, Y_0 is Channel Frequency Response, Transmitted signal and Received signal at pilot frequency in first OFDM symbol

Then, we interpolate frequency channel response at data–subcarrier by interpolation algorithms. In this paper, we use linear interpolation algorithm. Finally, after this step, we already has channel response in all sub-carriers and this is channel response in frequency domain.

$$H_e(k) = H_e(mL + l) \qquad 0 \le l < L$$

$$= (H_p(m+1) - H_p(m))\frac{l}{L} + H_p(m) \quad (16)$$

Where L =3 :

After we calculated all channel response in frequency domain, we perform change back to the time domain through IFFT to transform our response time domain channels as follows :

$$h(n) = IFFT(H(n)); \quad (17)$$

where $h(n)$ is channel reponse in time domain at the *n-th* OFDM symbol

Where $H(n)$ is channel reponse in frequency domain at the n-th OFDM symbol

Then, We use zero forcing equalizer to get the approximate input signal as follows : $\hat{X}(n) = Y(n)/H(n)$

Where $\hat{X}(n)$ is the approximate signal signals in the frequency domain through the process of estimating and crude equalizer.

$Y(n)$ is the signal at the receiver in the frequency domain

Then, we get an approximate transmitter signal through IFFT : $\hat{x}(n) = IFFT(\hat{X}(n))$.

+ At other OFDM symbols:

We determine channel estimation from Extended Kalman estimator as below

Step 1: Prediction (before receiving a OFDM symbol): $\hat{z}_{k|k-1} = f(z_{k-1}) = \begin{bmatrix} \hat{a}_{k-1} \\ \hat{A}_{k-1}\hat{h}_{k-1} \end{bmatrix}$ (18)

$$P_{k|k-1} = F_{k-1}P_{k-1}F_{k-1}^H + Q_u = \begin{vmatrix} P_{a,k|k-1} & P_{ah,k|k-1} \\ P_{ah,k|k-1} & P_{h,k|k-1} \end{vmatrix} \quad (19)$$

Where $Q_u = \begin{bmatrix} Q_\epsilon & 0 \\ 0 & Q_v \end{bmatrix}$ is the covariance of noises

Step 2: Correction (once the reception of the OFDM symbol has completed):

$$K_k = P_{k|k-1} \begin{bmatrix} 0 \\ X_k^H \end{bmatrix} ([0 \ X_k] P_{k|k-1} [0 \ X_k]^H + Q_w)^{-1} \quad (20)$$

$$= \begin{bmatrix} P_{ah,k|k-1} \\ P_{h,k|k-1} \end{bmatrix} X_k^H (X_k P_{h,k|k-1} X_k^H + Q_w)^{-1} \hat{z}_k = \hat{z}_{k|k-1} + K_k(y_k - [0 \ X_k] \hat{z}_{k|k-1}) \quad (21)$$

$$= \hat{z}_{k|k-1} + K_k(y_k - X_k \hat{h}_{k|k-1})$$

$$P_k = P_{k|k-1} - K_k [0 \ X_k] P_{k|k-1} \quad (22)$$

with K_k is Gain Kalman,

Step 3: We determine h_k (this is frequency channel response at pilot-subcarrier) from the upper information.

$$h_k = [0 \ I_{Np}] z_k \quad (23)$$

Part 2 (CFO estimation and channel estimation with EKF): Adaptive estimate channel response time domain and the frequency shift so that interference cancellation ICI

+ *At the first OFDM symbol:* From the training process, we get the channel response in frequency domain through IFFT, we have the channel reponse in time domain: *h(1)=IFFT(H(1)).*

Frame synchronization process is taking place at the beginning of each transmission frame to calculate the frequency shift, so we also know exactly suppose the frequency shift boot for the first OFDM symbols ε (1). Thus we have the initial conditions of the model extended Kalman from equation 12:

$$z(1) = [h^T(1), \varepsilon(1)]^T \quad (24)$$

+ ***At other OFDM symbols(n > 1)*** *:*

Step 1: Extended Kalman algorithm is performing iterative process to calculate the estimate of z (n)

Transition matrix is unit matrix. Measurement matrix is calculated as below

$$F(n+1,n) = I_d \quad (25)$$

$$C(n) = \frac{\partial C}{\partial z} = [\frac{\partial(C)}{\partial h}, \frac{\partial(C)}{\partial \varepsilon}] = [C_e \tilde{x}(n), C'_e \tilde{h}(n) x(n)] \quad (26)$$

Note : *C(n)* Is the first order deriviation of $C(z(n))$

C_e' is the first order deriviation of C_e

$$C_e' = diag\,(j2 * pi * \frac{k}{N} * e^{j*2*pi*\epsilon*\frac{k}{N}}) \quad (27)$$

With k = 0,1,…N-1

Step 2: We continue to calculate other equations in EKF algorithm.

$$G_f(n) = K(n,n-1) C^H(n) [C(n) K(n,n-1) C^H(n) + Q_2(n)]^{-1} \quad (28)$$

$$\alpha(n) = y(n) - C(n, z(n)) \quad (29)$$

K(n)= [I - G_f(n)C(n)]K(n,n-1) (4.19)$K(n+1,n) = F(n+1,n) K(n) F^H(n+1,n) + Q_1(n)$ (30)

Step 3: We calculate frequency response H and the offset with the following equation:

$$\hat{z}(n) = F(n+1,n)\hat{z}(n-1) + (K(n)\alpha(n) \quad (31)$$

$$h(n) = [I_N \ \ 0]\hat{z}(n) \quad (32)$$

$$\varepsilon(n) = [0_N \ 1] \ \hat{z}(n) \quad (33)$$

With I_N is *NxN* unit matrix

Step 4: Based on calculation of h*(n)* and *ε(n), we calculate the total offset at the nth OFDM symbol*

$$\varepsilon = real(\varepsilon(n)) + \varepsilon(n-1), \quad (34)$$

Then, we have the output signal with some equation:

$$Y_e(n) = y(n) * C_e(-\varepsilon) \quad (35)$$

$$\hat{X}(n) = Y_e(n)/H(n) \quad (36)$$

With $Y_e(n)$ is frequency response of $y_e(n)$, $H(n)$) is frequency response of *h(n)* through FFT transforming,

Where *w(n)* is additive white Gaussian noise and *h(n)* is the channel impulse response.

V. SIMULATION RESULTS

A. Simulation parameters

The paper uses parameters of IEEE 802.16e to simulate mobile OFDM system. In this paper, we use modulation schemes 4-QAM for simulation.

TABLE I. SIMULATION PARAMETERS

Simulation Parameter	
Number of Sub-carrier	128
FFT size	128
Modulation	4-QAM
Number of pilot-carrier	32
ITU-R channel B	Vehicular
Fixed Guard Interval	32
License frequency	2.5 GHz
Bandwidth	1.25 MHz

The ITU-R wideband channel, is described based on a tapped delay line model, with a maximum number of 6 taps [14]. Channel model ITU-R for 3 environments: indoor, pedestrian, vehicular and empirical model (channel B) is used to describe multi-path channel.

TABLE 2. CHANNEL PROFILE

Tap	Channel A		Channel B	
	Delay(ns)	Power (dB)	Delay (ns)	Power (dB)
1	0	0	0	0
2	0.4	-1	0.8	-1
3	0.8	-9	1.6	-9
4	1.2	-10	2.2	-10
5	1.8	-15	3.6	-15
6	2.6	-20	5.2	-20

B. Simulation results

In this section, we simulate the proposed algorithm with LS, Kalman and EKF. Known channel is the BER result when we know exactly the fading channel and offset. This result is considered as the lower border for other algorithms.

Figure 2 is the BER result with speed of 60 km/h without offset. We can see that our proprosed algorithm has better performance from 20 dB.

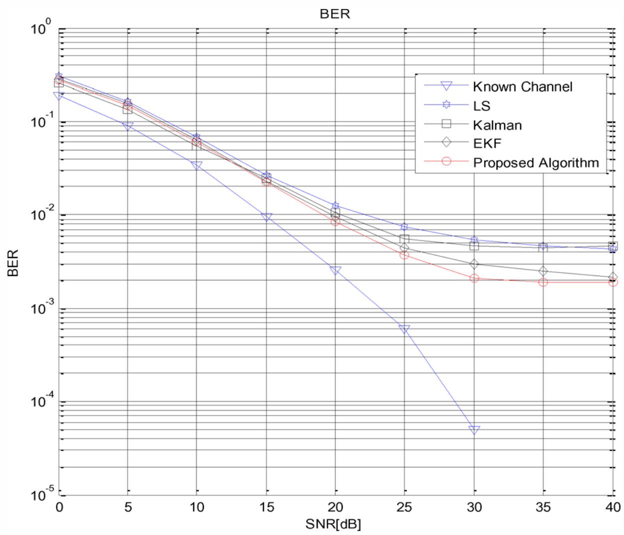

Figure 2. BER result for vehicular model with mobile speed of 60 km/h, no offset

Figure 3 is the BER result with speed of 60 km/h with offset 0.1. We can see that our proprosed algorithm still has the best perfromance with CFO estimation while other algorithms do not have CFO estimation.

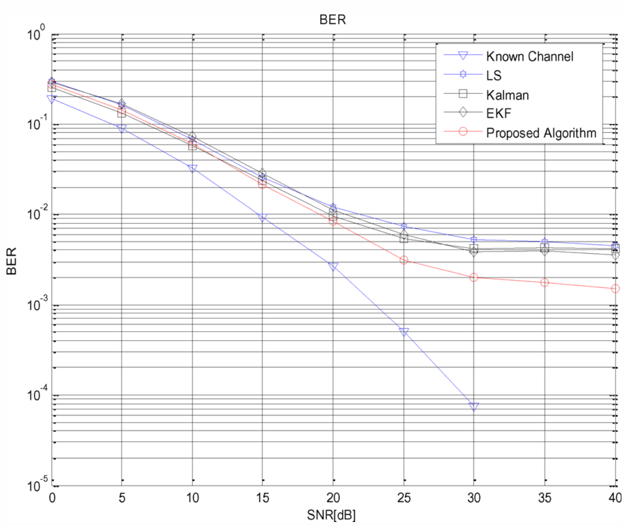

Figure 3. BER result with mobile speed of 60km/h in vehicular model with offset 0.1

To analyze the algorithm, Figure 6 is simulation of alpha value (equation 29) . This value shows stability of the algorithm. The lower value is, the more stable the algorithm is. With simulation of figure 4, we can see the proposed algorithm is more stable than Kalman and EKF. This helps the algorithm to have better performance

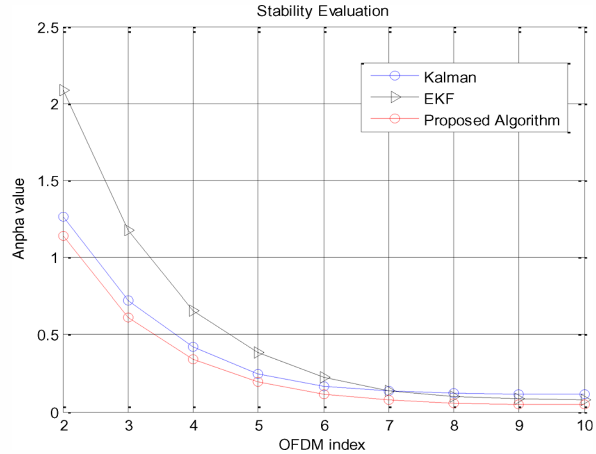

Figure 4. BER result with mobile speed of 60 km/h

Figure 5 shows CFO estimation. When we change the offset, we can see estimated offset is nearly the same as the standard offset.

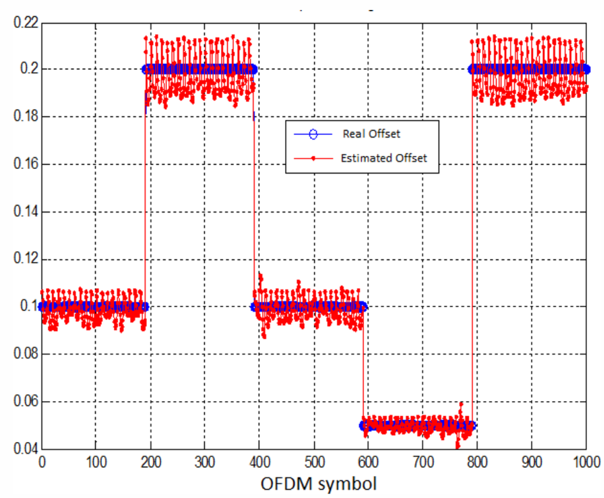

Figure 5.CFO estimation

However, The proposed alorithm is more complexcity than other alorithms. Table 3 shows the complexity of algorithms.

TABLE 3. COMPLEXITY COMPARISON

Algorithm	**Complexity**
Kalman	O(N/4)3
EKF	O(N/2)3
Proposed algorithm	O(N+1)3

V. CONCLUSION

The paper evaluates performance of channel estimation based on our proposed algorithm. Simulation parameters used in the paper are followed by mobile OFDM standard. Paper showed issues of estimators based on Extended Kalman Filter in fading enviroment and the paper proposed a new algorithm to extend these issues. Estimator based on EKF with jointly estimating CFO is proposed in this paper.

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