EEE 321 LAB2 OFFLAB ASSIGMENT

Name/Surname: Muhammet Melih Çelik ID:22003836

Part 1: Fourier Series Expansion

1. The signal x(t) given in the first part was sketched by hand. Handmade sketch can be seen in Figure 1 below:

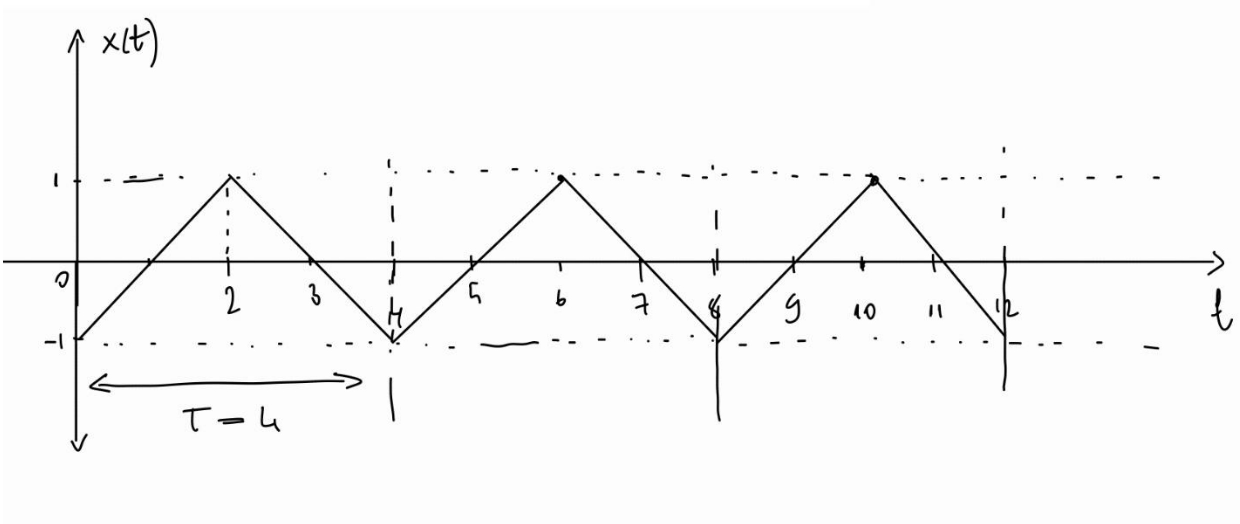

Figure 1: The Signal Plot Sketched by Hand

2. Three periods of $x(t)$ was plotted in MATLAB. The plot can be seen below:

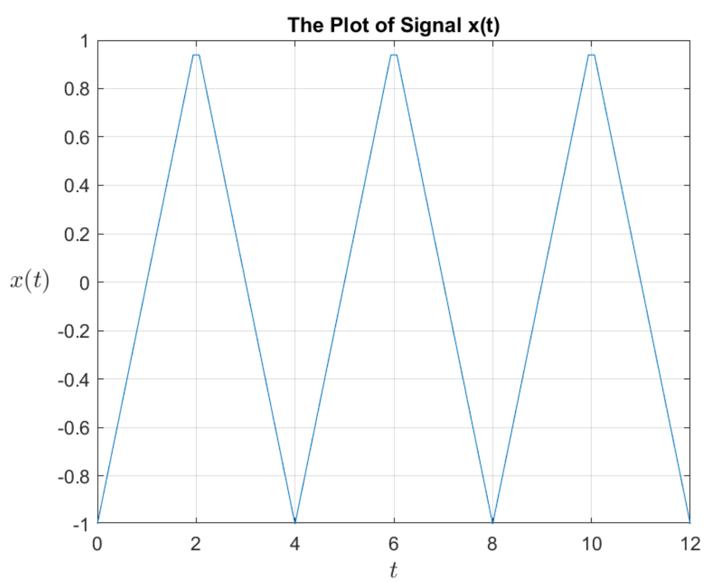

Figure 2: The Triangle Wave Plot

3. The Fourier Series coefficients can be found by the analysis equation in an analytical manner.

$$a_k = \frac{1}{T}\int_0^4 x(t)\, e^{-jk\frac{2\pi}{4}t}dt = \frac{1}{4}\left[\int_0^2 (t-1)\, e^{-jk\frac{\pi}{2}t}dt + \int_2^4 (3-t)\, e^{-jk\frac{\pi}{2}t}dt\right]$$

$$= \frac{1}{4}\left[\int_0^2 t\, e^{-jk\frac{\pi}{2}t}dt - \int_0^2 e^{-jk\frac{\pi}{2}t}dt + 3\int_2^4 e^{-jk\frac{\pi}{2}t}dt - \int_2^4 t\, e^{-jk\frac{\pi}{2}t}dt\right]$$

$$\int_0^2 t\, e^{-jk\frac{\pi}{2}t}dt, \quad m = -jk\frac{\pi}{2} \qquad = \frac{t e^{mt}}{m}\Big]_0^2 - \int_0^2 \frac{e^{mt}}{m}dt = \frac{2e^{mt}}{m} - \frac{e^{mt}}{m^2}\Big]_0^2$$

$$u = t \qquad v = \frac{e^{mt}}{m}$$

$$du = dt \qquad dv = e^{mt}dt \qquad = \boxed{\frac{2e^{2m}}{m} - \frac{e^{2m}}{m^2} + \frac{1}{m^2}}$$

$$\int_0^2 e^{mt}dt = \frac{e^{mt}}{m}\Big]_0^2 = \boxed{\frac{e^{2m}}{m} - \frac{1}{m}}$$

$$m = -j\frac{k\pi}{2}$$

$$\frac{1}{\cancel{4}}\cdot\frac{\left(1-(-1)^k\right)^2}{-\frac{(k\pi)^2}{\cancel{4}}}$$

$$\int_2^4 e^{mt}dt = \frac{e^{mt}}{m}\Big]_2^4 = \boxed{\frac{e^{4m} - e^{2m}}{m}}$$

$$\left(-j\frac{k\pi}{2}\right)^2$$

$$\int_2^4 t e^{mt}dt = \frac{t e^{mt}}{m}\Big]_2^4 - \int_2^4 \frac{e^{mt}}{m}dt = \boxed{\frac{4e^{4m} - 2e^{2m}}{m} - \left(\frac{e^{4m}}{m^2} - \frac{e^{2m}}{m^2}\right)}$$

$$\cancel{(-j)^2} - \frac{(k\pi)^2}{4}$$

$$\cancel{\boxed{\frac{2e^{2m}}{m}}} - \frac{e^{2m}}{m^2} + \frac{1}{m^2} \cancel{\boxed{-\frac{e^{2m}}{m}}} + \frac{1}{m} + \frac{3e^{4m}}{m} \cancel{\boxed{-\frac{3e^{2m}}{m}}} - \frac{4e^{4m}}{m} + \cancel{\boxed{\frac{2e^{2m}}{m}}} + \frac{e^{4m}}{m^2} - \frac{e^{2m}}{m^2}$$

$$\frac{1}{m} - \frac{e^{4m}}{m} + \frac{1}{m^2} - \frac{2e^{2m}}{m^2} + \frac{e^{4m}}{m^2} = \frac{(1-e^{2m})\cancel{(1+e^{2m})}}{\cancel{m}} + \frac{(1-e^{2m})^2}{m^2} \qquad \frac{\left(1-e^{-jk\pi}\right)^2}{(k\pi)^2}$$

$$a_k = \begin{cases} 0, & k = 2n \quad n\in\mathbb{Z} \\ \frac{-4}{(k\pi)^2}, & k = 2n+1 \quad n\in\mathbb{Z} \end{cases} \qquad a_k = -\frac{\left(1-(-1)^k\right)^2}{(k\pi)^2}$$

4. In this part the square sum of the Fourier coefficients can be found by the Parseval's equation.

$$\frac{1}{T}\int_0^T |x(t)|^2 dt = \sum_{-\infty}^{+\infty} |a_k|^2$$

$$|x(t)| = \begin{cases} 1-t, & 0 \le t < 1 \\ t-1, & 1 \le t < 2 \\ 3-t, & 2 \le t < 3 \\ t-3, & 3 \le t < 4 \end{cases}$$

$$\frac{1}{T}\int_0^T |x(t)|^2 dt = \frac{1}{4}\left[\int_0^1 (1-t)^2 dt + \int_1^2 (t-1)^2 dt + \int_2^3 (3-t)^2 dt + \int_3^4 (t-3)^2 dt\right]$$

$$I = \frac{1}{4}[I_1 + I_2 + I_3 + I_4]$$

$$I_1 = \int_0^1 (1-t)^2 dt = \int_0^1 (t^2 - 2t + 1) dt = \frac{t^3}{3} - t^2 + t\Big]_0^1 = \frac{1}{3}$$

$$I_2 = \int_1^2 (t-1)^2 dt = \int_1^2 (t^2 - 2t + 1) dt = \frac{t^3}{3} - t^2 + t\Big]_1^2 = \frac{1}{3}$$

$$I_3 = \int_2^3 (3-t)^2 dt = \int_2^3 (t^2 - 6t + 9) dt = \frac{t^3}{3} - 3t^2 + 9t\Big]_2^3 = 3 - \frac{8}{3}$$

$$I_4 = \int_3^4 (t-3)^2 dt = \int_3^4 (t^2 - 6t + 9) dt = \frac{t^3}{3} - 3t^2 + 9t\Big]_3^4 = \frac{64}{3} - 21$$

$$Total\ Energy = \frac{1}{4}[I_1 + I_2 + I_3 + I_4] = \frac{1}{3} = 0.3333$$

The N number was found with an iterative MATLAB program, which is given below:

```
%The Iterative program to find the number N.
I = 1/3;

sum = 0; %initial sum.
k = 1; %starting index
per = sum/I;
lim = 0.9995;

while per < lim
    sum = sum + 2*(((1-(-1)^k)^4)/((k*pi)^4));
    per = sum/I;
    if per >= lim
        break
    end
    k = k + 1;
end
disp(sum)
disp(per)
disp(k)
```

Output:

```
Command Window
    0.3332

    0.9997

     7

fx >>
```

Part 2: Sum of Complex Exponentials

The $x_a(t)$ signal was synthesized by summing the complex exponentials, whose specifications were given in the off-lab assignment. The plot of the signal can be seen in figure below:

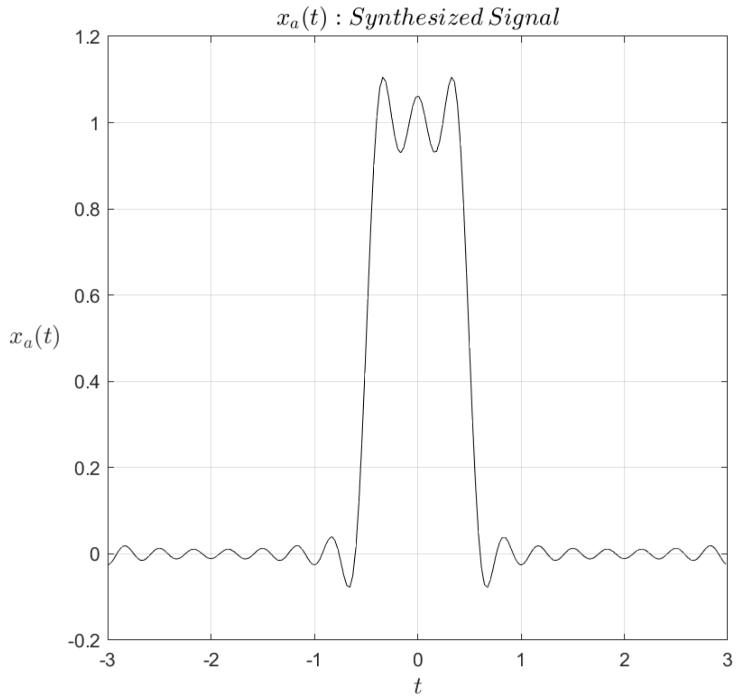

Figure 3: The Synthesized Signal with Complex Exponentials

If the coefficient number was increased as it is asked in the off-lab assignment; in the synthesized signal, the Gibbs phenomenon becomes easy to observe as it is the case in Figure 4.

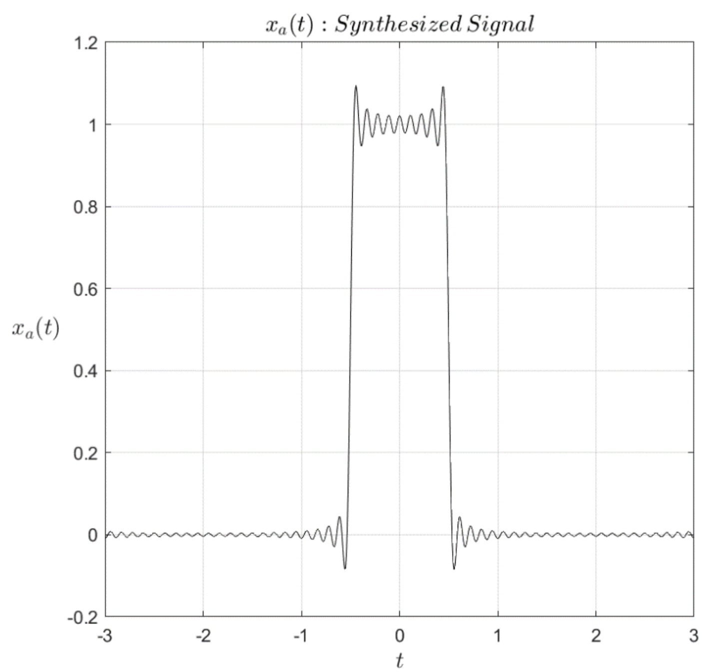

Figure 4: The Synthesized Signal with İncreased Number of Complex Exponentials

Part 3: Fourier Series Approximation

By using the iterative MATLAB program, four different Ns were found and 4 different signals were generated with those founded Ns. These signals were plotted can be seen in Figure 5. It is clear that when the coefficient number decreases, the signal starts to demolish to its basis signals, which is the case when N = 3 in third signal.

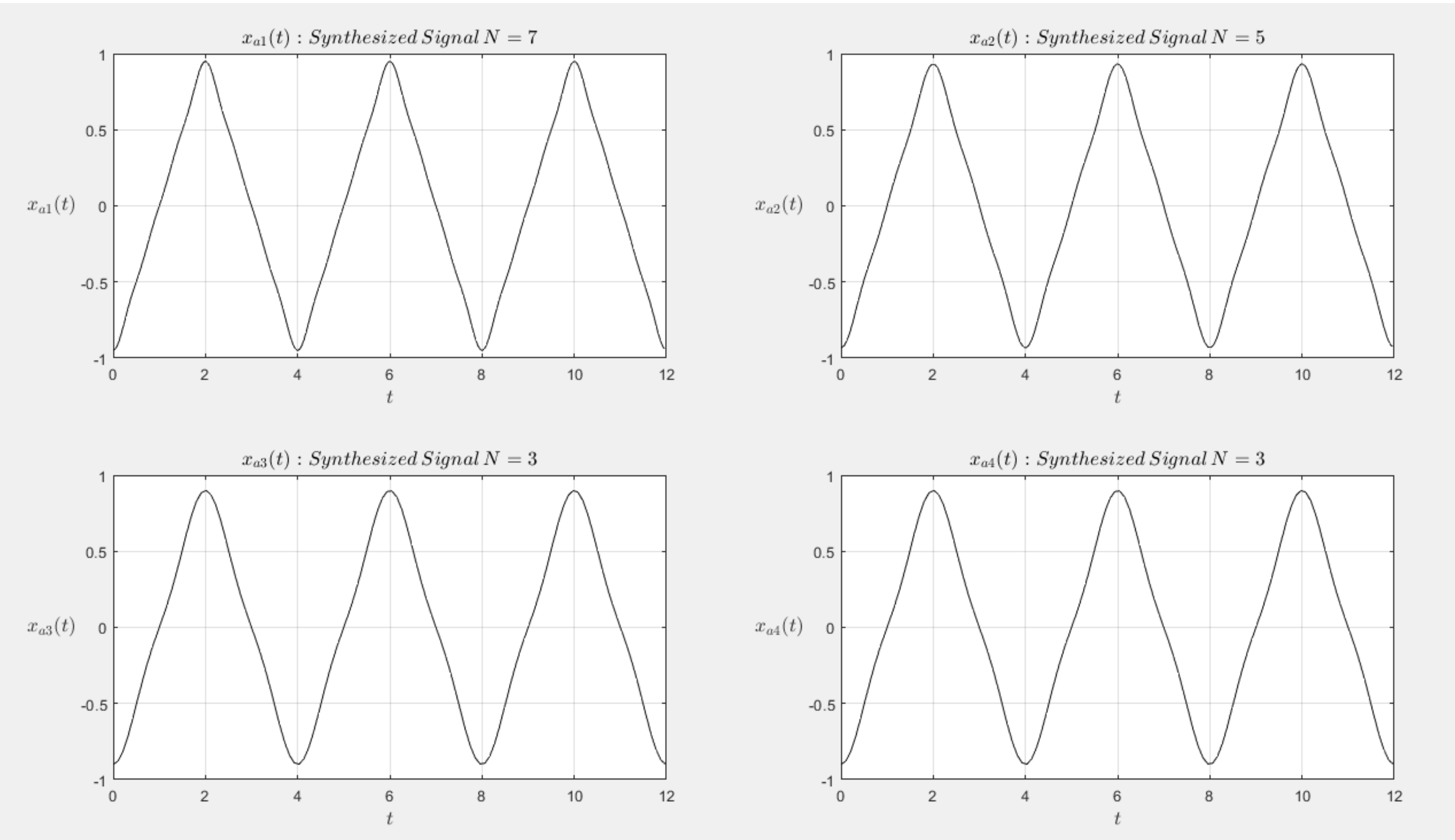

Figure 5: Signals Which Has Different Number of Complex Exponentials

EEE 321 LAB2 ONLAB ASSIGMENT

Name/Surname: Muhammet Melih Çelik ID:22003836

1. The signal $\hat{x}(t)$ was plotted in Figure 1 below for $t \in [0,3T)$.

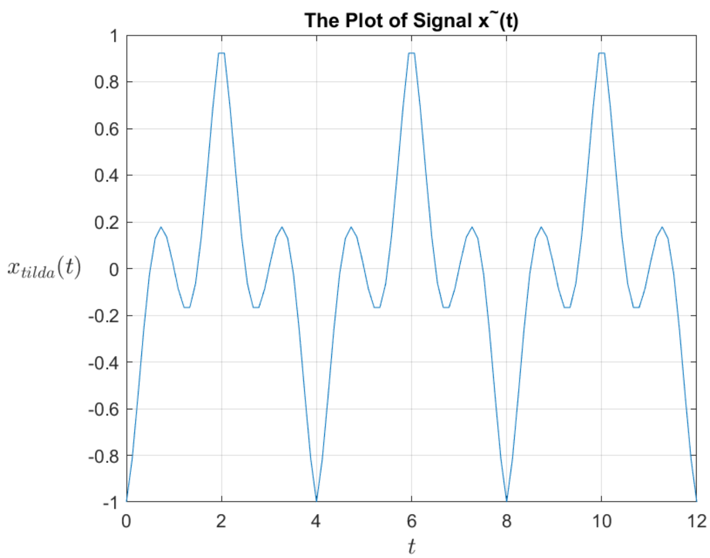

Figure 6: The Signal for the On-lab Assignment

2. The Fourier Coefficients of the $\hat{x}(t)$ was calculated analytically with the frequency shifting property.

$$\hat{x}(t) = \cos(\pi t)\, x(t), \qquad x(t) = \begin{cases} t-1, 0 \le t < 2 \\ 3-t, 2 \le t < 4 \end{cases}$$

As the Fourier coefficients of $x(t)$ were known as:

$$a_k = -\frac{(1-(-1)^k)^2}{(k\pi)^2}$$

The a_{ks} were calculated by using the a_k above.

$$e^{jM\left(\frac{2\pi}{T}\right)t} x(t) \leftrightarrow \mathcal{F}.S \leftrightarrow a_{k-M}$$

$$\hat{x}(t) = \cos(\pi t)\, x(t) = \frac{1}{2} e^{j2\left(\frac{2\pi}{4}\right)t} x(t) + \frac{1}{2} e^{-j2\left(\frac{2\pi}{4}\right)t}$$

$$\hat{x}(t) \leftrightarrow \mathcal{F}.S \leftrightarrow a_{ks} = \frac{1}{2} a_{k+2} + \frac{1}{2} a_{k-2}$$

$$a_{ks} = -\frac{1}{2}\left[\frac{(1-(-1)^{k+2})^2}{\left((k+2)\pi\right)^2} + \frac{(1-(-1)^{k-2})^2}{(\,k-2)\pi\,)^2}\right]$$

```matlab
%-------------------------EEE 321 LAB2 OFFLAB--------------------------------
%PART 1: Fourier Series Expansion
T = 4;  %period of the signal.
t = linspace(0,3*T,100);
triangle_wave = sawtooth(2*pi*t/T,0.5);

%plotting
plot(t,triangle_wave);
grid on;
ylabel ('$x(t)$','Interpreter','latex','FontSize',13);
xlabel ('$t$','Interpreter','latex','FontSize',13);
title 'The Plot of Signal x(t)'
ax = gca;
ax.YLabel.Rotation = 360;

%The Iterative program to find the number N.
I = 1/3;

sum = 0; %initial sum.
k = 1; %starting index
per = sum/I;
lim = 0.99;

while per < lim
    sum = sum + 2*(((1-(-1)^k)^4)/((k*pi)^4));
    per = sum/I;
    if per >= lim
        break
    end
    k = k + 1;
end

%The Stem plot of the Fourier Coefficients

k = -7:1:7;
a_k = -(((1-(-1).^k).^2)./((k*pi).^2));
stem(k,a_k,"k");
grid on;
title ('$a_k:Fourier \, Series \, Coefficients$',...
    'Interpreter','latex','FontSize',13);
ylabel('$a_k$','Interpreter','Latex','FontSize',13);
xlabel('$k$','Interpreter','Latex','FontSize',13);
ax = gca;
ax.YLabel.Rotation = 360;

%PART 2:Sum of Complex Exponentials

ID = 22003836;
D_15 = mod(ID,15);
N = D_15+30;
T = 4;
fmax = 2*pi*N/T;
f_sampling = 2*fmax; %Nyquist sampling theorem
```

```
samp_int = 1/f_sampling;

k_pos = 1:1:N;
k_neg = -(N):1:-1;
A_pos = sin((pi*k_pos)/4)./(pi*k_pos);
A_zero= 1/4;
A_neg = sin((pi*k_neg)/4)./(pi*k_neg);
A = [A_neg A_zero];
A= [A A_pos];   %final condition of coefficient array.

t = -3:samp_int:3;
the_signal = FourierSum(T,N,A,t);

plot(t,the_signal,"k");
grid on;
title ('$x_a(t):Synthesized \, Signal$',...
    'Interpreter','latex','FontSize',13);
ylabel('$x_a(t)$','Interpreter','Latex','FontSize',13);
xlabel('$t$','Interpreter','Latex','FontSize',13);
ax = gca;
ax.YLabel.Rotation = 360;

%PART 3: Fourier Series Approximation

Ns = [7,5,3,3];
T = 4;

[x1,t1,samp_int1] = signalGenerator(Ns(1,1),T);
[x2,t2,samp_int2] = signalGenerator(Ns(1,2),T);
[x3,t3,samp_int3] = signalGenerator(Ns(1,3),T);
[x4,t4,samp_int4] = signalGenerator(Ns(1,4),T);

subplot(2,2,1);
plot(t1,x1,"k");
grid on;
title ('$x_{a1}(t):Synthesized \, Signal \, N = 7$',...
    'Interpreter','latex','FontSize',13);
ylabel('$x_{a1}(t)$','Interpreter','Latex','FontSize',13);
xlabel('$t$','Interpreter','Latex','FontSize',13);
ax = gca;
ax.YLabel.Rotation = 360;

subplot(2,2,2);
plot(t2,x2,"k");
grid on;
title ('$x_{a2}(t):Synthesized \, Signal \, N = 5$',...
    'Interpreter','latex','FontSize',13);
ylabel('$x_{a2}(t)$','Interpreter','Latex','FontSize',13);
xlabel('$t$','Interpreter','Latex','FontSize',13);
ax = gca;
ax.YLabel.Rotation = 360;

subplot(2,2,3);
plot(t3,x3,"k");
```

```
grid on;
title ('$x_{a3}(t):Synthesized \, Signal \, N = 3$',...
    'Interpreter','latex','FontSize',13);
ylabel('$x_{a3}(t)$','Interpreter','Latex','FontSize',13);
xlabel('$t$','Interpreter','Latex','FontSize',13);
ax = gca;
ax.YLabel.Rotation = 360;

subplot(2,2,4);
plot(t4,x4,"k");
grid on;
title ('$x_{a4}(t):Synthesized \, Signal \, N = 3$',...
    'Interpreter','latex','FontSize',13);
ylabel('$x_{a4}(t)$','Interpreter','Latex','FontSize',13);
xlabel('$t$','Interpreter','Latex','FontSize',13);
ax = gca;
ax.YLabel.Rotation = 360;
```

Published with MATLAB® R2023b

```
%----------------------EEE 321 LAB2 ONLAB CODE-----------------------------
clf;
T = 4;  %period of the signal.
t = linspace(0,3*T,100);
triangle_wave = sawtooth(2*pi*t/T,0.5);

x_tilda = cos(pi*t).*triangle_wave;
plot(t,x_tilda);
grid on;
ylabel ('$x_{tilda}(t)$','Interpreter','latex','FontSize',13);
xlabel ('$t$','Interpreter','latex','FontSize',13);
title 'The Plot of Signal x^~(t)'
ax = gca;
ax.YLabel.Rotation = 360;
```

Published with MATLAB® R2023b