EEE 321 LAB6 OFFLAB ASSIGNMENT REPORT

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PART 1: DT-LTI Systems Characterized by

Linear Constant Coefficient Difference Equations

For equation (1) below:

$$y[n] = \sum_{l=1}^{N} a[l]\, y[n-l] + \sum_{k=0}^{M} b[k]x[n-k] \quad (1)$$

y[0] and y[1] found as follows:

as $x[n] = 0, y[n] = 0\ ; n < 0$

$$y[0] = \sum_{l=1}^{N} a[l]y[-l] + \sum_{k=0}^{M} b[k]\, x[-k] = b[0]x[0]$$

$$y[1] = \sum_{l=1}^{N} a[l]y[1-l] + \sum_{k=0}^{M} b[k]x[1-k]$$

$$= a[1]y[0] + b[0]x[1] + b[1]x[0]$$

$$= a[1](b[0]x[0]) + b[0]x[1] + b[1]x[0]$$

Hence,

$$y[0] = b[0]x[0]$$

$$y[1] = a[1]b[0]x[0] + b[0]x[1] + b[1]x[0]$$

To find H(z) as division of two polynomials, Z-transform of both sides of equation (1) was taken as follows:

$$Y(z) = \sum_{l=1}^{N} a[l]Y(z)z^{-l} + \sum_{k=0}^{M} b[k]X(z)z^{-k}$$

$$Y(z)\left(1 - \sum_{l=1}^{N} a[l]z^{-l}\right) = X(z)\sum_{k=0}^{M} b[k]z^{-k}$$

$$H(z) = \frac{\sum_{k=0}^{M} b[k]z^{-k}}{(1 - \sum_{l=1}^{N} a[l]z^{-l})} = \frac{\sum_{k=0}^{M} b[k]z^{-k}}{\sum_{l=0}^{N} -a[l]z^{-l}}$$

Hence,

$$P = M, C_n[p] = b[k];$$
$$Q = N, C_d[q] = -a[l].$$

PART 2: Impulse Response Analysis of DTLTI Systems

The DTLTI function was written as below:

```
%DTLTI function

function [y]=DTLTI(a,b,x,Ny)

    for n=0:Ny-1
        value1 = 0;
        value2 = 0;

        for l=1:length(a)-1
            if (n-l) < 0
                y_n_var = 0;
                value1 = value1 + a(1,l+1)*y_n_var;
            elseif (n-l) >= 0
                value1 = value1 + a(1,l+1)*y(1,n-l+1);
            end
        end

        for k=0:length(b)-1
            %disp(k)
            if (n-k) < 0 || (n+1) > numel(x)
                x_n_var = 0;
                value2 = value2 + b(1,k+1)*x_n_var;
            elseif (n-k) >= 0 && (n+1) <= numel(x)
                value2 = value2 + b(1,k+1)*x(1,n-k+1);
            end
                y(1,n+1) = value1 + value2;
        end

    end

end
```

i- Impulse response of the system was obtained as below:

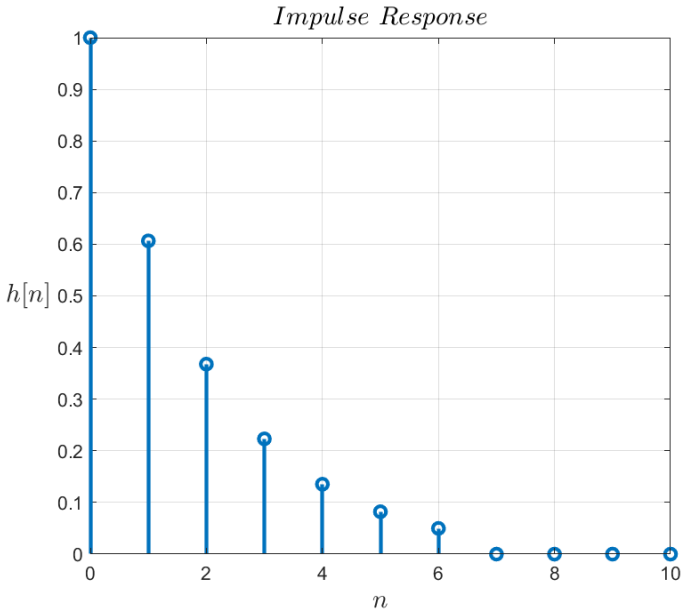

Figure 1: Impulse Response of The System

ii- Impulse response is same with coefficients array b[k].

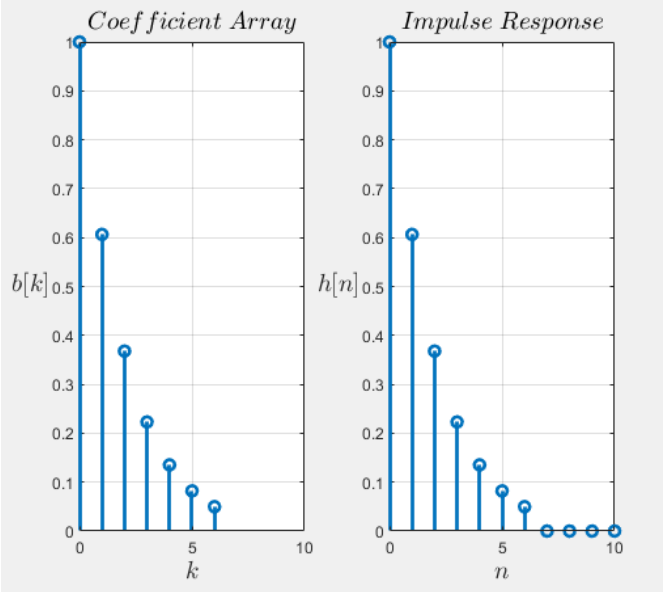

Figure 2: Coefficient Array vs Impulse Response

iii- System is FIR as its impulse response have finite duration. Length of impulse response is10. M was 7 in my case; hence length of impulse response is larger than M.

iv- Calculations for subsection (d) was performed as in the next page.

$$h[n] = e^{-n}, 0 \leq n < \infty$$

$$H(z) = \sum_{-\infty}^{+\infty} e^{-n} z^{-n} = \sum_{n=0}^{+\infty} (e^{-1} z^{-1})^n = \frac{1}{1 - e^{-1} z^{-1}}$$

As Z-transform is equal to DTFT on the unit circle of Z-plane, $z = e^{j\omega}$:

$$H(z) = \frac{1}{1 - e^{-1} z^{-1}} \leftrightarrow h[n] = e^{-n} u[n]$$

v- Magnitude response of the system:

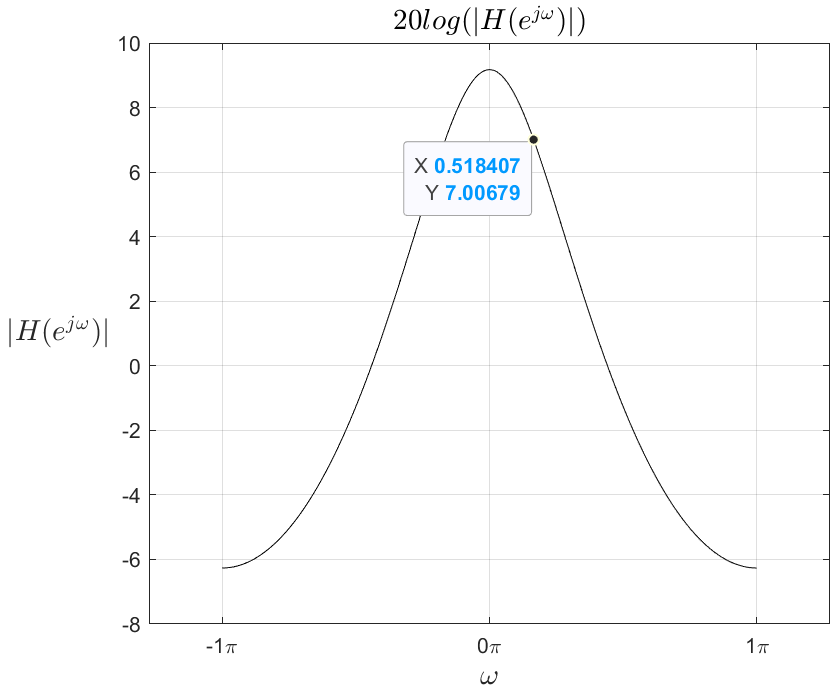

Figure 3: Magnitude Response of The System

This system behaves like a lowpass filter. 3dB bandwidth is a range where signal's amplitude attenuation is equal or lower than -3dB. Cut-Off frequency is the frequencies which corresponds to end point of 3dB bandwidth at which signal's power becomes half.

Cut-Off frequency of this system is $\omega_c = 0.62$ rad/s, $\omega_c / 2\pi = 0.098$ Hz.

Frequency response of the system by linear chirping with different L values:

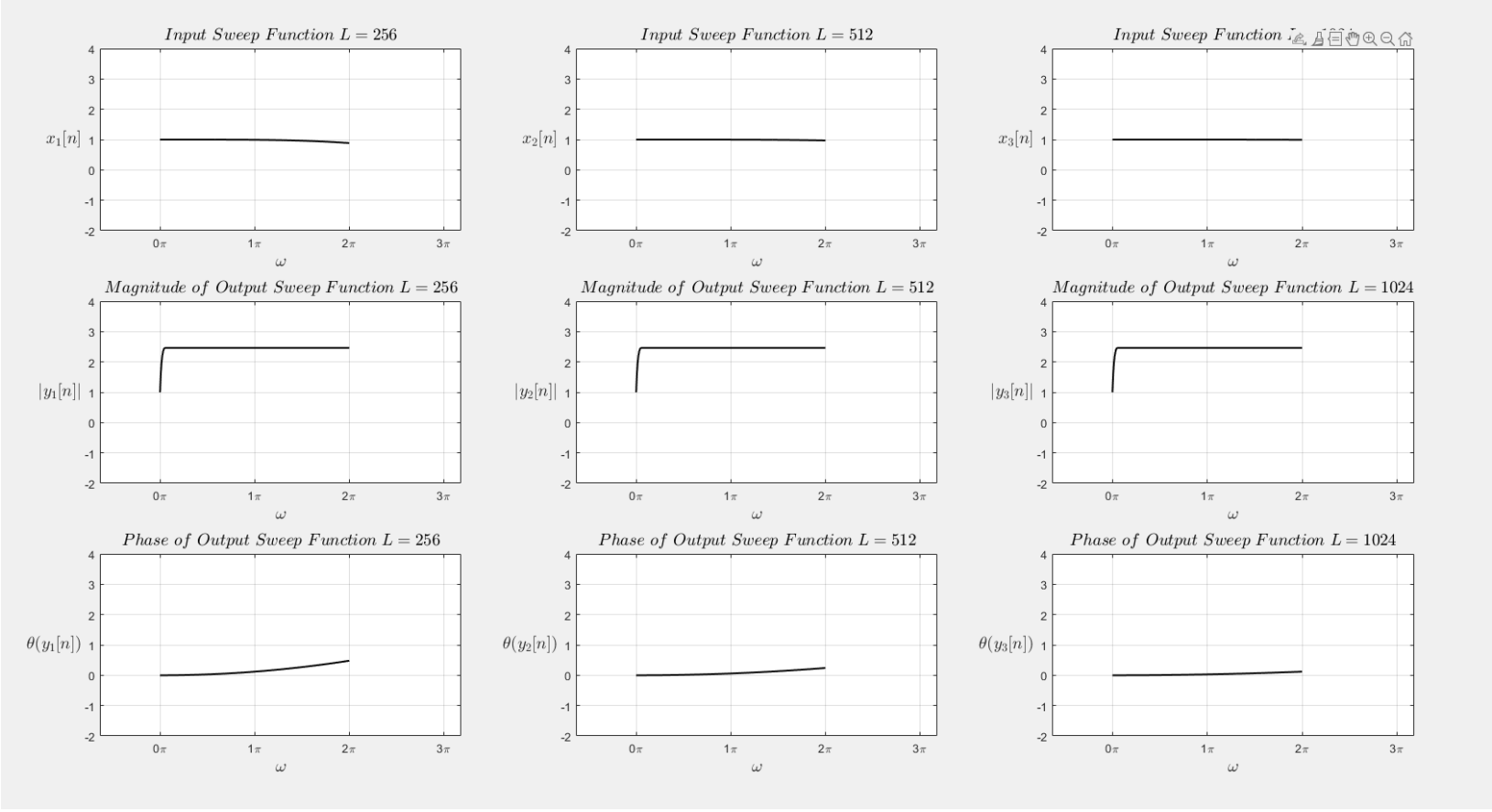

Figure 4: Sweep Functions

Even though there are sudden jumps at n = 0, the output chirp shows a steady behavior.

PART 3: Filter Design

The filter design was conducted as below:

i- The filter should have causal impulse response → $h[n] = 0, n < 0$

ii- $H(z) = \frac{z - z_1}{(z - p1)(z - p_2)}$;

$[n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8]; n_i = ID(i) + 3, ID = 22003836$

$[n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8] =$ [5,5,3,3,6,11,6,9];

$$z_1 = \frac{5 + j9}{\sqrt{25 + 81}}, \qquad p_1 = \frac{3 + j11}{\sqrt{1 + 9 + 121}}, \qquad p_2 = \frac{3 + j6}{\sqrt{1 + 9 + 36}}$$

1) The Z-transform of the filter was given below:

$$z_1 = 0.485 + j0.874, \qquad p_1 = 0.252 + j0.961, \qquad p_2 = 0.442 + j0.884$$

$$H(z) = \frac{z - z_1}{(z - p1)(z - p_2)} = \frac{z^{-1}(1 - z_1 z^{-1})}{(1 - p_1 z^{-1})(1 - p_2 z^{-1})}$$

$$= \frac{z^{-1}[1 - (0.485 + j0.874)z^{-1}]}{[1 - (0.252 + j0.961)z^{-1}][1 - (0.442 + j0.884)z^{-1}]}$$

2) $H(z) = \frac{z^{-1}(1-z_1 z^{-1})}{(1-p_1 z^{-1})(1-p_2 z^{-1})} = \frac{A}{1-p_1 z^{-1}} + \frac{B}{1-p_2 z^{-1}}$

$$A = [(1 - p_2 z^{-1})H(z)]_{z=p1} = \frac{p_1 - z_1}{p_1(p_1 - p_2)}$$
$$B = [(1 - p_2 z^{-1})H(z)]_{z=p2} = \frac{p_2 - z_1}{p_2(p_2 - p_1)}$$

The values for A and B obtained from MATLAB is:

$$A = 0.342 - j1.172, B = -0.127 + j0.177$$

Hence, the impulse response is:

$$h[n] = (0.342 - j1.172)(0.252 + j0.961)^n + (-0.127 + j0.177)(0.442 + j0.884)^n$$

3) The region of convergence were shown in the figures in the next with the zero and poles.
4) This system is stable as ROC contains the unit circle which means that DTFT is convergent and present for the system and hence, response is absolutely summable which means that system is stable.
5) This system is IIR because output depends on its past output values as well.
6) $H(z), z = e^{j\omega} \rightarrow H(e^{j\omega}) = \frac{e^{-j\omega} - z_1 e^{-j2\omega}}{(1-p_1 e^{-j\omega})(1-p_2 e^{-j\omega})}$; DTFT of the frequency response was plot as in Figure 6. The filter shows a bandpass behavior as it passes frequencies within narrow range.
7) The magnitude and phase of chirp output were given in Figures 7 and 8 respectively.

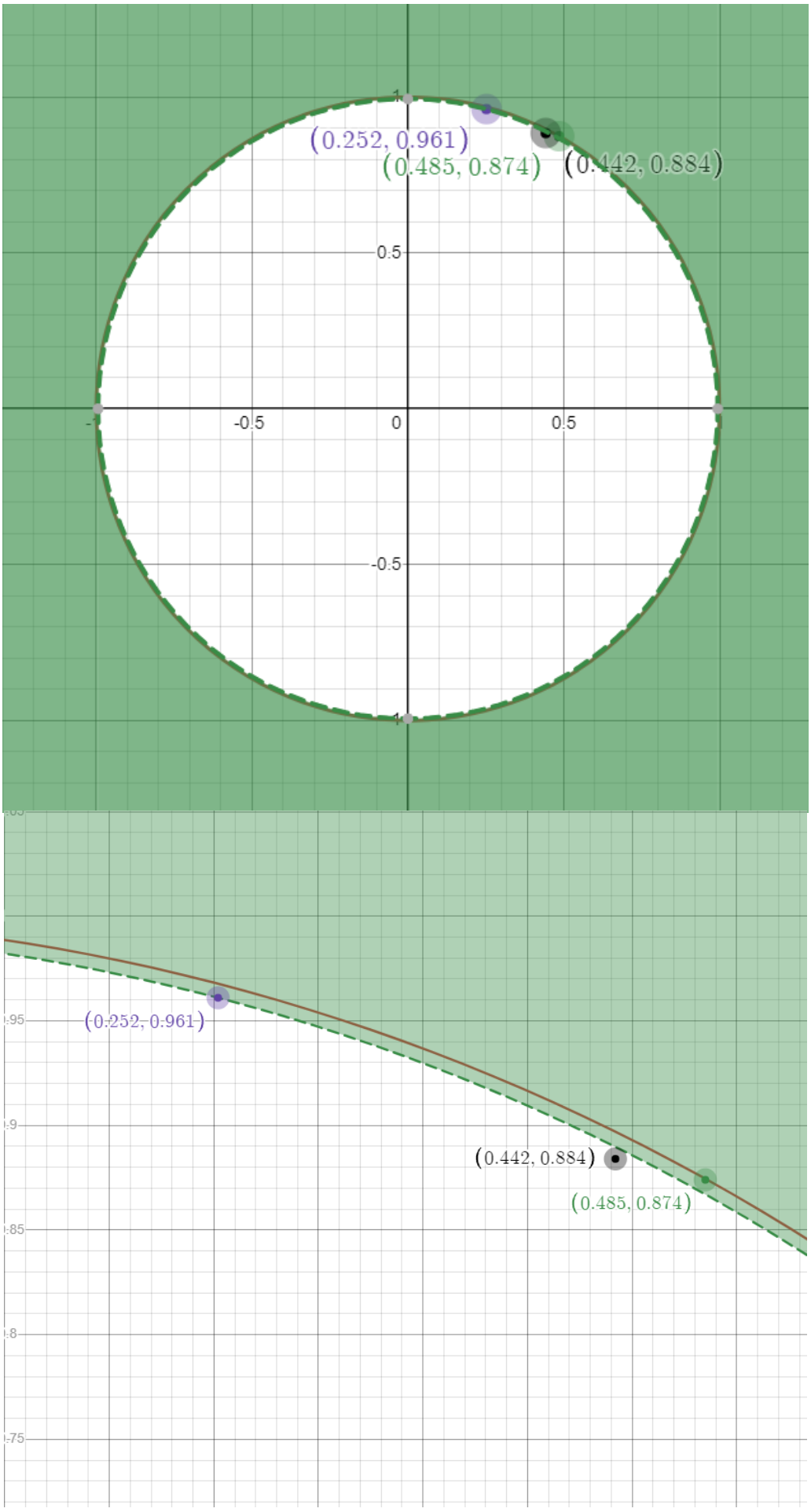
(0.252, 0.961)
(0.485, 0.874)
(0.442, 0.884)
1
0.5
-0.5
0
0.5
-1
(0.252, 0.961)
(0.442, 0.884)
(0.485, 0.874)
.95
.9
.85
.8
.75

Figure 5: ROC

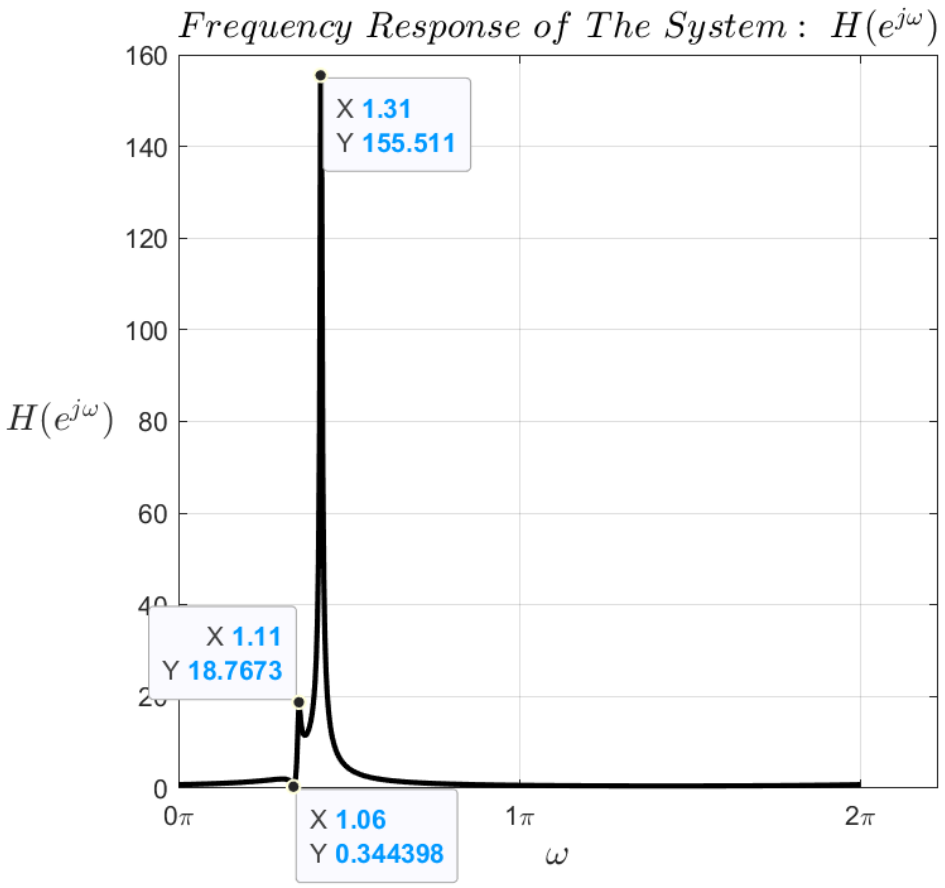

Figure 6: Frequency Response of The System

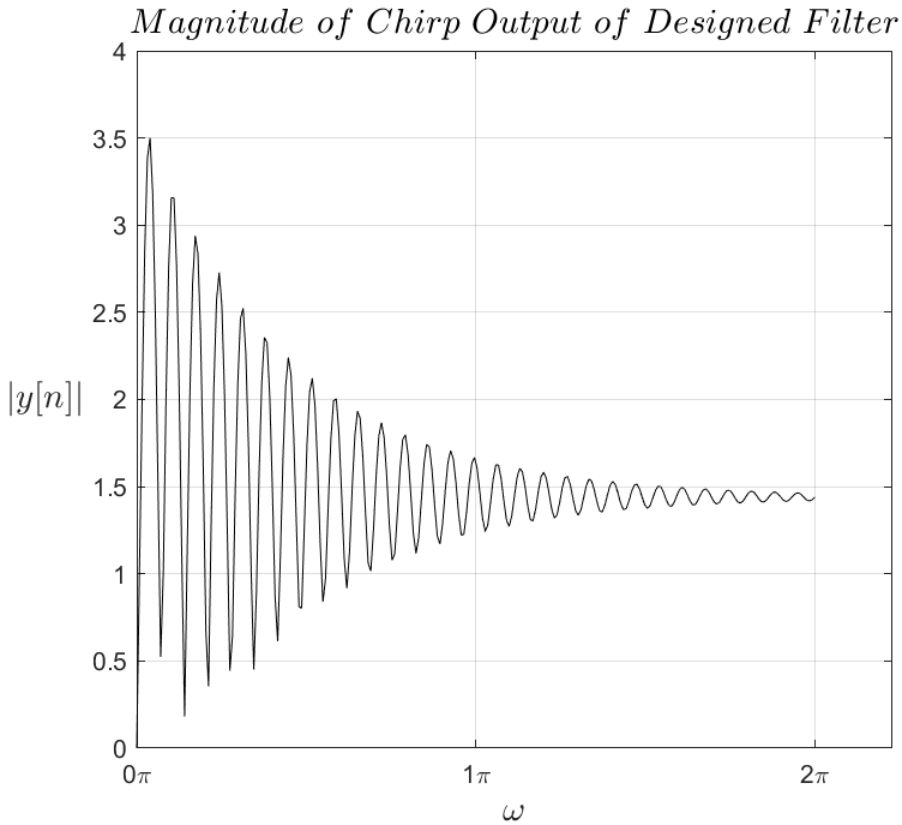

Figure 7: Chirp Output of Designed Filter

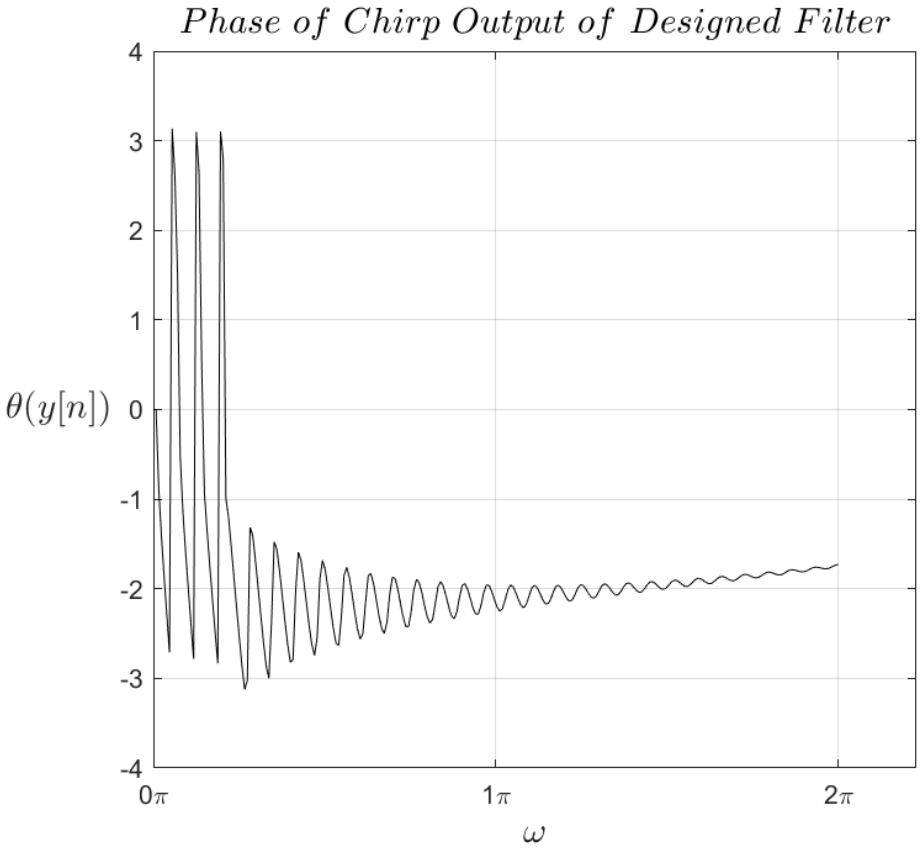

Figure 8: Phase of Chirp Output

```
%---------------------EEE321 LAB6 OFFLAB CODE------------------------------
%PART2:mpulse Response Analysis of DTLTI Systems
D = 22003836;
D_4 = mod(D,5);
M = 6 + D_4;

k = 0:1:M-1;
n = 0:1:10;

x = zeros(1,11);
a = zeros(1,10);
b = exp(-k/2);

for i=1:11
    if i == 1
        x(1,i) = 1;
    else
        x(1,i) = 0;
    end
end

h = DTLTI(a,b,x,11);

clf;
stem(n,h,LineWidth=2);
grid on;
title('$Impulse\,\,Response$','Interpreter','latex','FontSize',14)
ylabel('$h[n]$','Interpreter','latex','FontSize',14);
xlabel('$n$','Interpreter','latex','FontSize',14);
xlim([0,10]);

ax = gca;
ax.YLabel.Rotation = 360;

subplot(1,2,1);
stem(k,b,LineWidth=2);
grid on;
title('$Coefficient\,\,Array$'...
    ,'Interpreter','latex','FontSize',14);
ylabel('$b[k]$','Interpreter','latex','FontSize',14);
xlabel('$k$','Interpreter','latex','FontSize',14);
xlim([0,10]);

ax = gca;
ax.YLabel.Rotation = 360;

h = DTLTI(a,b,x,11);

subplot(1,2,2);
stem(n,h,LineWidth=2);
grid on;
title('$Impulse\,\,Response$'...
    ,'Interpreter','latex','FontSize',14);
```

```
ylabel('$h[n]$','Interpreter','latex','FontSize',14);
xlabel('$n$','Interpreter','latex','FontSize',14);

ax = gca;
ax.YLabel.Rotation = 360;

%-------DTFT of system's response-----------------------------------------
clf;
j = sqrt(-1);
w = -pi:0.01:pi;
H = abs(1./(1-exp(-1)*exp(-j*w)));
db = 20*log(H);

plot(w,db,"k");
grid on
title ('$20log(|H(e^{j\omega})|)$','Interpreter','latex','FontSize',14);
ylabel('$|H(e^{j\omega})|$','Interpreter','latex','FontSize',14);
xlabel('$\omega$','Interpreter','latex','FontSize',14);
tick_positions = -pi:pi:pi;
tick_labels = arrayfun(@(x) ...
    sprintf('%.0f\\pi', x/pi), tick_positions, 'UniformOutput', false);
ax = gca;
ax.XTick = tick_positions;
ax.XTickLabel = tick_labels;
ax.YLabel.Rotation = 360;

%-----Sweep Signal--------------------------------------------------------
L1 = 256;
L2 = 512;
L3 = 1024;

n = linspace(0,2*pi,L1);
M = 6 + D_4;
k = 0:1:M-1;
a = zeros(1,10);
b = exp(-k/2);

x1 = exp(j*(pi/L1)*(n.^2));
y1 = DTLTI(a,b,x1,length(n));
y1_mag = abs(y1);
y1_pha = angle(y1);

x2 = exp(j*(pi/L2)*(n.^2));
y2 = DTLTI(a,b,x2,length(n));
y2_mag = abs(y2);
y2_pha = angle(y2);

x3 = exp(j*(pi/L3)*(n.^2));
y3 = DTLTI(a,b,x3,length(n));
y3_mag = abs(y3);
y3_pha = angle(y3);

%Plots
clf;
%1-----------------------------------------------------------------------
```

```
subplot(3,3,1);
plot(n,x1,"k",LineWidth=1.5);
grid on
title ('$Input\,\,Sweep\,\,Function\,\,L = 256$',...
    'Interpreter','latex','FontSize',14);
ylabel('$x_1[n]$','Interpreter','latex','FontSize',14);
xlabel('$\omega$','Interpreter','latex','FontSize',14);
ylim([-2 4]);
xlim([-2 10]);
tick_positions = -pi:pi:3*pi;
tick_labels = arrayfun(@(x) ...
    sprintf('%.0f\\pi', x/pi), tick_positions, 'UniformOutput', false);
ax = gca;
ax.XTick = tick_positions;
ax.XTickLabel = tick_labels;
ax.YLabel.Rotation = 360;

subplot(3,3,4);
plot(n, y1_mag, 'k', 'LineWidth', 1.5);
grid on;
title('$Magnitude\,\,of\,\,Output\,\,Sweep\,\,Function\,\,L = 256$', ...
    'Interpreter', 'latex', 'FontSize', 14);
ylabel('$|y_1[n]|$', 'Interpreter', 'latex', 'FontSize', 14);
xlabel('$\omega$', 'Interpreter', 'latex', 'FontSize', 14);
ylim([-2 4]);
xlim([-2 10]);
tick_positions = -pi:pi:3*pi;
tick_labels = arrayfun(@(x) ...
    sprintf('%.0f\\pi', x/pi), tick_positions, 'UniformOutput', false);
ax = gca;
ax.XTick = tick_positions;
ax.XTickLabel = tick_labels;
ax.YLabel.Rotation = 360;

subplot(3,3,7);
plot(n, y1_pha, 'k', 'LineWidth', 1.5);
grid on;
title('$Phase\,\,of\,\,Output\,\,Sweep\,\,Function\,\,L = 256$', ...
    'Interpreter', 'latex', 'FontSize', 14);
ylabel('$\theta(y_1[n])$', 'Interpreter', 'latex', 'FontSize', 14);
xlabel('$\omega$', 'Interpreter', 'latex', 'FontSize', 14);
ylim([-2 4]);
xlim([-2 10]);
tick_positions = -pi:pi:3*pi;
tick_labels = arrayfun(@(x) ...
    sprintf('%.0f\\pi', x/pi), tick_positions, 'UniformOutput', false);
ax = gca;
ax.XTick = tick_positions;
ax.XTickLabel = tick_labels;
ax.YLabel.Rotation = 360;

%2-------------------------------------------------------------------------
subplot(3,3,2);
plot(n,x2,"k",LineWidth=1.5);
```

```
grid on
title ('$Input\,\,Sweep\,\,Function\,\,L = 512$',...
    'Interpreter','latex','FontSize',14);
ylabel('$x_2[n]$','Interpreter','latex','FontSize',14);
xlabel('$\omega$','Interpreter','latex','FontSize',14);
ylim([-2 4]);
xlim([-2 10]);
tick_positions = -pi:pi:3*pi;
tick_labels = arrayfun(@(x) ...
    sprintf('%.0f\\pi', x/pi), tick_positions, 'UniformOutput', false);
ax = gca;
ax.XTick = tick_positions;
ax.XTickLabel = tick_labels;
ax.YLabel.Rotation = 360;

subplot(3,3,5);
plot(n, y2_mag, 'k', 'LineWidth', 1.5);
grid on;
title('$Magnitude\,\,of\,\,Output\,\,Sweep\,\,Function\,\,L = 512$', ...
    'Interpreter', 'latex', 'FontSize', 14);
ylabel('$|y_2[n]|$', 'Interpreter', 'latex', 'FontSize', 14);
xlabel('$\omega$', 'Interpreter', 'latex', 'FontSize', 14);
ylim([-2 4]);
xlim([-2 10]);
tick_positions = -pi:pi:3*pi;
tick_labels = arrayfun(@(x) ...
    sprintf('%.0f\\pi', x/pi), tick_positions, 'UniformOutput', false);
ax = gca;
ax.XTick = tick_positions;
ax.XTickLabel = tick_labels;
ax.YLabel.Rotation = 360;

subplot(3,3,8);
plot(n, y2_pha, 'k', 'LineWidth', 1.5);
grid on;
title('$Phase\,\,of\,\,Output\,\,Sweep\,\,Function\,\,L = 512$', ...
    'Interpreter', 'latex', 'FontSize', 14);
ylabel('$\theta(y_2[n])$', 'Interpreter', 'latex', 'FontSize', 14);
xlabel('$\omega$', 'Interpreter', 'latex', 'FontSize', 14);
ylim([-2 4]);
xlim([-2 10]);
tick_positions = -pi:pi:3*pi;
tick_labels = arrayfun(@(x) ...
    sprintf('%.0f\\pi', x/pi), tick_positions, 'UniformOutput', false);
ax = gca;
ax.XTick = tick_positions;
ax.XTickLabel = tick_labels;
ax.YLabel.Rotation = 360;

%3-------------------------------------------------------------------------
subplot(3,3,3);
plot(n,x3,"k",LineWidth=1.5);
grid on
title ('$Input\,\,Sweep\,\,Function\,\,L = 1024$',...
```

```
    'Interpreter','latex','FontSize',14);
ylabel('$x_3[n]$','Interpreter','latex','FontSize',14);
xlabel('$\omega$','Interpreter','latex','FontSize',14);
ylim([-2 4]);
xlim([-2 10]);
tick_positions = -pi:pi:3*pi;
tick_labels = arrayfun(@(x) ...
    sprintf('%.0f\\pi', x/pi), tick_positions, 'UniformOutput', false);
ax = gca;
ax.XTick = tick_positions;
ax.XTickLabel = tick_labels;
ax.YLabel.Rotation = 360;

subplot(3,3,6);
plot(n, y3_mag, 'k', 'LineWidth', 1.5);
grid on;
title('$Magnitude\,\,of\,\,Output\,\,Sweep\,\,Function\,\,L = 1024$', ...
    'Interpreter', 'latex', 'FontSize', 14);
ylabel('$|y_3[n]|$', 'Interpreter', 'latex', 'FontSize', 14);
xlabel('$\omega$', 'Interpreter', 'latex', 'FontSize', 14);
ylim([-2 4]);
xlim([-2 10]);
tick_positions = -pi:pi:3*pi;
tick_labels = arrayfun(@(x) ...
    sprintf('%.0f\\pi', x/pi), tick_positions, 'UniformOutput', false);
ax = gca;
ax.XTick = tick_positions;
ax.XTickLabel = tick_labels;
ax.YLabel.Rotation = 360;

subplot(3,3,9);
plot(n, y3_pha, 'k', 'LineWidth', 1.5);
grid on;
title('$Phase\,\,of\,\,Output\,\,Sweep\,\,Function\,\,L = 1024$', ...
    'Interpreter', 'latex', 'FontSize', 14);
ylabel('$\theta(y_3[n])$', 'Interpreter', 'latex', 'FontSize', 14);
xlabel('$\omega$', 'Interpreter', 'latex', 'FontSize', 14);
ylim([-2 4]);
xlim([-2 10]);
tick_positions = -pi:pi:3*pi;
tick_labels = arrayfun(@(x) ...
    sprintf('%.0f\\pi', x/pi), tick_positions, 'UniformOutput', false);
ax = gca;
ax.XTick = tick_positions;
ax.XTickLabel = tick_labels;
ax.YLabel.Rotation = 360;

%PART3

z1 = 0.485000000000000 + 0.874000000000000i;
p1 = 0.252000000000000 + 0.961000000000000i;
p2 = 0.442000000000000 + 0.884000000000000i;

w = 0:0.01:2*pi;
```

```
H = (exp(-j*w)-z1*exp(-j*2*w))./((1-p1*exp(-j*w)).*(1-p2*exp(-j*w)));

plot(w,abs(H),"k",LineWidth=2);
grid on;
title('$Frequency\,\,Response\,\,of\,\,The\,\,System:\,\,H(e^{j\omega})$',...
      'Interpreter','latex','FontSize',14);
ylabel('$H(e^{j\omega})$','Interpreter','latex','FontSize',14);
xlabel('$\omega$','Interpreter','latex','FontSize',14);
tick_positions = 0:pi:2*pi;
tick_labels = arrayfun(@(x) ...
    sprintf('%.0f\\pi', x/pi), tick_positions, 'UniformOutput', false);
ax = gca;
ax.XTick = tick_positions;
ax.XTickLabel = tick_labels;
ax.YLabel.Rotation = 360;

n = linspace(0,2*pi,L1);
x = exp(j*(pi/L1)*(n.^2));
a = [p1+p2 -p1*p2];
b = [0 1 -z1];

y = DTLTI(a,b,x,length(n));
y_mag = abs(y);
y_pha = angle(y);

clf;
plot(n,y_mag,"k");
grid on;
title('$Magnitude\,\,of\,\,Chirp\,\,Output\,\,of\,\,Designed\,\,Filter$',...
      'Interpreter','latex','FontSize',14);
ylabel('$|y[n]|$','Interpreter','latex','FontSize',14);
xlabel('$\omega$','Interpreter','latex','FontSize',14);
tick_positions = 0:pi:2*pi;
tick_labels = arrayfun(@(x) ...
    sprintf('%.0f\\pi', x/pi), tick_positions, 'UniformOutput', false);
ax = gca;
ax.XTick = tick_positions;
ax.XTickLabel = tick_labels;
ax.YLabel.Rotation = 360;

clf;
plot(n,y_pha,"k");
grid on;
title('$Phase\,\,of\,\,Chirp\,\,Output\,\,of\,\,Designed\,\,Filter$',...
      'Interpreter','latex','FontSize',14);
ylabel('$\theta(y[n])$','Interpreter','latex','FontSize',14);
xlabel('$\omega$','Interpreter','latex','FontSize',14);
tick_positions = 0:pi:2*pi;
tick_labels = arrayfun(@(x) ...
    sprintf('%.0f\\pi', x/pi), tick_positions, 'UniformOutput', false);
ax = gca;
ax.XTick = tick_positions;
ax.XTickLabel = tick_labels;
ax.YLabel.Rotation = 360;
```

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```
%DTLTI function
function [y]=DTLTI(a,b,x,Ny)

    for n=0:Ny-1
        value1 = 0;
        value2 = 0;

        for l=1:length(a)-1
            if (n-l) < 0
                y_n_var = 0;
                value1 = value1 + a(1,l+1)*y_n_var;
            elseif (n-l) >= 0
                value1 = value1 + a(1,l+1)*y(1,n-l+1);
            end
        end

        for k=0:length(b)-1
            %disp(k)
            if (n-k) < 0 || (n+1) > numel(x)
                x_n_var = 0;
                value2 = value2 + b(1,k+1)*x_n_var;
            elseif (n-k) >= 0 && (n+1) <= numel(x)
                value2 = value2 + b(1,k+1)*x(1,n-k+1);
            end
                y(1,n+1) = value1 + value2;
        end

    end

end
```

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