CS 321
For Thursday 14 Jan
Hand In at Class

Homework #1

1. This is about Sudoku puzzles. There is an example on the attached file Sudoku.pdf. This example uses *letters* but the more common form uses the digits 1 to 9.

   (a) Solve the attached Sudoku puzzle and discover the hidden word. On your homework hand in the **hidden word**, but **DON'T** hand in the filled in grid.

   (b) Argue (informally) that there is an algorithm for the standard 9×9 Sudoku which *either* finds a correctly filled-in grid *or* shows that no correctly filled-in grid is possible.
   When describing an *algorithm* remember to state what the input looks like and what the output looks like.

   (c) Generalize Sudoku from the standard 9×9 grid to $n \times n$ grids. Show (again informally) that there is an algorithm for these generalized $n \times n$ Sudoku's.

2. The Fibonacci numbers are defined by

$$f_0 = 0, \quad f_1 = 1 \qquad \text{and} \quad f_n = f_{n-1} + f_{n-2} \quad \text{for } n \geq 2.$$

   (a) Use the Euclidean algorithm to find

$$\mathbf{gcd}(\, f_8,\, f_7\,).$$

   (b) Let

$$g(n) \;=\; \mathbf{gcd}(\, f_n,\, f_{n-1}\,).$$

   Give a FAST algorithm to compute $g(n)$.

3. A Boolean expression $B(\,x_1,\,x_2,\,\ldots\,x_n\,)$ is **Satisfiable iff** there is an assignment of **True**'s and **False**'s to the variables (the x's) so that the result of evaluating the expression B with this assignment is **True**.

   (a) Give an algorithm which takes as input a Boolean expression $B(\,x_1,\,x_2,\,\ldots\,x_n\,)$
   and outputs
   True if the expression is Satisfiable,
   False if the expression is **not** Satisfiable.

   (b) DETAILS: We don't want "gory" details, so don't code this in a programming language. To avoid problems of syntax, assume that you already have an algorithm EVAL(B, $\vec{v}$) which takes as input a Boolean expression B and vector $\vec{v}$ of Boolean values and outputs the value of the expression with the assignment $\vec{v}$.

4. A **Graph** G consists of a set of vertices V and a set of edges E, where E consists of some **un**ordered pairs of vertices.
   There is a **Path** between vertices v and w if there is a sequence of edges
   $$(v, x_1)\ (x_1, x_2)\ \ldots\ (x_k, w).$$

   (a) Path Problem
   INPUT: A graph G and a two vertices v and w.
   QUESTION: Is there a path between v and w ?

   (b) Show that there is a reasonable algorithm for this problem by giving an informal description of such an algorithm, **AND** commenting on how much "time" and "space" might be needed to execute your algorithm.

5. Archimedes, the man who famously exclaimed "Eureka!", said that the Roman number system was *perfect* because he could use it to represent arbitrarily large numbers.
   Explain *briefly* why you agree or disagre with Archimedes.