# Simple 2-out-of-2 secret-sharing scheme:

$$\Sigma:$$

$$\mathcal{M} = \{0,1\}^{\ell} \qquad \frac{\mathsf{Share}(m):}{s_1 \leftarrow \{0,1\}^{\ell}}$$

$$t = 2$$

$$n = 2$$

$$Share(m):$$

$$s_1 \leftarrow \{0,1\}^{\ell}$$

$$s_2 := s_1 \oplus m$$

$$\mathsf{return}(s_1, s_2)$$

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### Claim:

 $\Sigma$  is a secure 2-out-of-2 secret-sharing scheme. That is,

$$\mathcal{L}^{\Sigma}_{\mathsf{tsss-L}} \equiv \mathcal{L}^{\Sigma}_{\mathsf{tsss-R}}.$$

We will **use** the fact that one-time pad has one-time security  $(\mathcal{L}_{ots-L}^{OTP} \equiv \mathcal{L}_{ots-R}^{OTP})$ .



 $\mathcal{L}_{\mathsf{tsss-L}}^{\Sigma}$   $\underline{\mathsf{QUERY}(m_L, m_R, U)}:$   $\mathsf{if} \ |U| \ge 2: \mathsf{return} \ \mathsf{err}$   $\boldsymbol{s} \leftarrow \Sigma.\mathsf{Share}(m_L)$   $\mathsf{return} \ (s_i)_{i \in U}$ 

Starting point is  $\mathcal{L}_{tsss-L}^{\Sigma}$ .

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\mathcal{L}_{\mathsf{tsss-L}}^{\Sigma}
\underline{\mathsf{QUERY}(m_L, m_R, U):}
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s \leftarrow \Sigma.\mathsf{Share}(m_L)
\mathrm{return} \ (s_i)_{i \in U}
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Starting point is  $\mathcal{L}_{tsss-L}^{\Sigma}$ . Fill in details of  $\Sigma$ 



 $\mathcal{L}^{\Sigma}_{tsss\text{-L}}$ QUERY $(m_L, m_R, U)$ : if  $|U| \ge 2$ : return err  $s_1 \leftarrow \{0,1\}^{\ell}$  $s_2 := s_1 \oplus m_L$ return  $(s_i)_{i \in U}$ 

Details of  $\Sigma$  filled in.



 $\mathcal{L}^{\Sigma}_{tsss\text{-L}}$ 

QUERY $(m_L, m_R, U)$ :

if  $|U| \ge 2$ : return err

 $s_1 \leftarrow \{0,1\}^{\ell}$ 

 $s_2 := s_1 \oplus m_L$ 

return  $(s_i)_{i \in U}$ 

Details of  $\Sigma$  filled in.

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QUERY(m_L, m_R, U):
  if |U| \ge 2: return err
  if U = \{1\}:
    s_1 \leftarrow \{0, 1\}^{\ell}
      s_2 := s_1 \oplus m_L
      return s<sub>1</sub>
  elsif U = \{2\}:
     s_1 \leftarrow \{0,1\}^{\ell}
      s_2 := s_1 \oplus m_I
      return s2
  else return null
```

Duplicate body for the 3 possible authorized sets:  $\{1\}, \{2\}, \emptyset$ .

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Recognize  $s_2$  as OTP encryption of  $m_L$ .



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     s_2 \leftarrow \text{QUERY}'(m_L, m_R)
      return s2
  else return null
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```
\frac{\text{QUERY}'(m_L, m_R):}{k \leftarrow \{0, 1\}^{\ell}}
    c := k \oplus m_l
    return c
```

Write it in terms of the "left" OTP security library.



```
QUERY(m_L, m_R, U):
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      s_2 \leftarrow \text{QUERY}'(m_L, m_R)
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$$\mathcal{L}_{\text{ots-L}}^{\text{OTP}}$$

$$\frac{\text{QUERY}'(m_L, m_R):}{k \leftarrow \{0, 1\}^{\ell}}$$

$$c := k \oplus m_L$$

$$\text{return } c$$

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QUERY'(m_L, m_R):
  k \leftarrow \{0,1\}^{\ell}
  c := k \oplus m_R
  return c
```

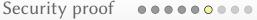
OTP security says we can replace  $\mathcal{L}_{ots-L}$  with  $\mathcal{L}_{ots-R}$ .



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$$\begin{array}{c}
\mathcal{L}_{\text{ots-R}}^{\text{OTP}} \\
\downarrow \\
k \leftarrow \{0,1\}^{\ell} \\
c := k \oplus m_R \\
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Inline the subroutine call.



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Three branches of if-statement can be unified.



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This happens to be  $\mathcal{L}_{tsss-R}^{\Sigma}$ .



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\mathcal{L}_{\mathsf{tsss-R}}^{\Sigma}
QUERY(m_L, m_R, U):
   if |U| \ge 2: return err
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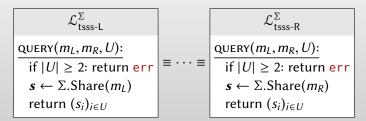
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## Summary

### We showed:



So  $\Sigma$  is a secure 2-out-of-2 secret-sharing scheme.