CS 321
Thursday, 3 Mar

Homework #7
NonDeterminism and Automata

1. Describe a **DETERMINISTIC** PushDown Automaton which accepts the set of well-formed strings of parentheses.
HINT: Look at the algorithm in the little RED book and show how to execute that algorithm using a pushdown automaton.

2. Without building the grammar, SHOW that there is a context sensitive grammar that generates the set of PRIMES in unary.
PRIMES $= \{ a^p \mid \; p \text{ is a prime } \} = \{ aa,\ aaa,\ aaaaa,\ \ldots \}$.

3. Again without building the grammar, SHOW that there is a context sensitive grammar which generates the BOOLEAN Tautologies.
A BOOLEAN Tautology is a well-formed BOOLEAN expression (e.g. matching parentheses, no two adjacent operators, etc.) which is TRUE for each and every assignment of TRUE and FALSE to the operands.

4. Show that the following program is a nondeterministic *acceptor* for the set of all non-empty strings (of B's and W's)in which the number of B's equals the number of W's or is twice the number of W's.

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For each character in the Input string
     Put the correspondingly colored ball in the bag.

Pick one Ball and replace it in the bag
               If the ball was W   GO TO L1
               If the ball was B   GO TO L2

   L1:  WHILE there are at least 2 balls in the bag
            Pick 2 balls
                     IF B and W  THEN Discard ELSE Put back in bag
        EOW
           IF 0 balls THEN HALT  ELSE  GO TO L1

   L2:  WHILE there are at least 3 balls in the bag
            Pick 3 balls
                     IF  2 B's and  one W  THEN Discard ELSE Put back in bag
        EOW
           IF 0 balls THEN HALT  ELSE  GO TO L2
```

5. Use the idea of <u>fork</u> to show that:
   (a) $\mathcal{N}-\mathbf{RE} \ = \ \mathbf{RE}$
   (b) $\mathcal{N}-\mathbf{coRE} \ = \ \mathbf{coRE}$
   (c) $\mathcal{N}-\mathbf{Recursive} \ = \ \mathbf{Recursive}$.

6. In the last HW, you used grammars to show that all CLASSes in the Chomsky Hierarchy are closed under UNION. This time use the NONDETERMINISM of the associated classes of automata to show that all CLASSes in the Chomsky Hierarchy are closed under UNION.

7. Use Automata to show that MOST CLASSes in the Chomsky Hierarchy are closed under INTERSECTION.
   (I.E., If $S1 \in C$ and $S2 \in C$ then $S1 \cap S2 \in C$ where $S1$ and $S2$ are sets of strings and C is a class in the Chomsky Hierarchy.)
   NOTE: One class is NOT closed under intersection, don't try to show that it is.

8. The following is a Turing machine which is a nondeterministic acceptor.

$$q_0 \, a \, q_1 \, b \, L$$

$$q_0 \, b \, q_1 \, a \, L$$

$$q_1 \, a \, q_0 \, a \, R$$

$$q_1 \, b \, q_1 \, a \, L$$

$$q_1 \, b \, q_0 \, b \, R$$

$$q_1 \, c \text{ ACCEPT}$$

   Assume that this acceptor starts in state q_0, at the right hand end of the input tape.
   (a) Show that the string $c\,a\,b$ is ACCEPTED.
   (b) Show that the string $c\,a\,a$ is NOT ACCEPTED.
   (c) Is the string $c\,a\,a\,b\,c\,a\,a\,b$ accepted???.

9. From the nondeterministic **M** given below, construct a deterministic **DM** with only the states reachable from $\{ q_0 \}$. How many states does this **DM** have? Describe in words the set recognized by **M** (and **DM**) if q_3 is the only accepting state.

M	0	1
q_0	$q_1,\ q_2$	q_1
q_1	q_1	q_3
q_2	q_2	q_3
q_3	q_3	q_3

10. Show that the set of all binary strings with a 1 in the K^{th} position from the end can be recognized by a nondeterministic **MK** machine with $O(K)$ states.

    (a) For $K = 2$, give both **M2** and the corresponding deterministic **DM2** and compare the number of states in these two machines.

    (b) For $K = 3$, give both **M3** and the corresponding deterministic **DM3** and compare the number of states in these two machines.

    (For specifity, 010 has a 1 in the 2^{nd} position from the end, but 1100 has a 1 in the 3^{nd} and the 4^{th} positions from the end.)