Parallel Program Design Patterns and Strategies

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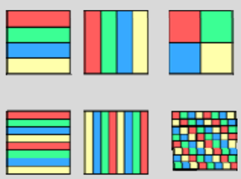
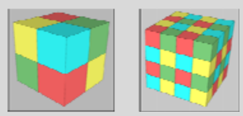
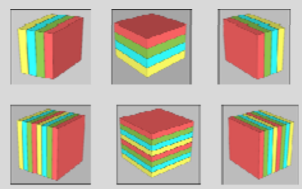
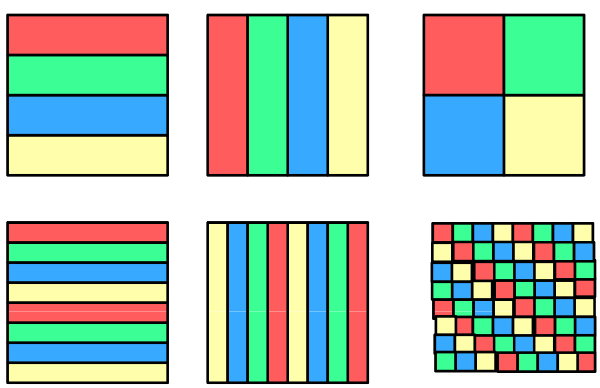






Data Decomposition

Suppose you are doing image processing operations on a grid of pixels using 4 cores. Each pixel needs to see its neighboring pixels to get the job done. How would you distribute the pixels among the 4 cores?



How we distribute data and then communicate among the distributions is called a **Design Pattern**.

Foster's Methodology: PCAM(R)

Partition the Problem
Think about how to break the problem up into its fundamental units of computing

Examine the Required **Communication**
Local: each task communicates with other tasks within a core – hopefully often
Global: each task communicates with a large number of other tasks between cores – hopefully seldom

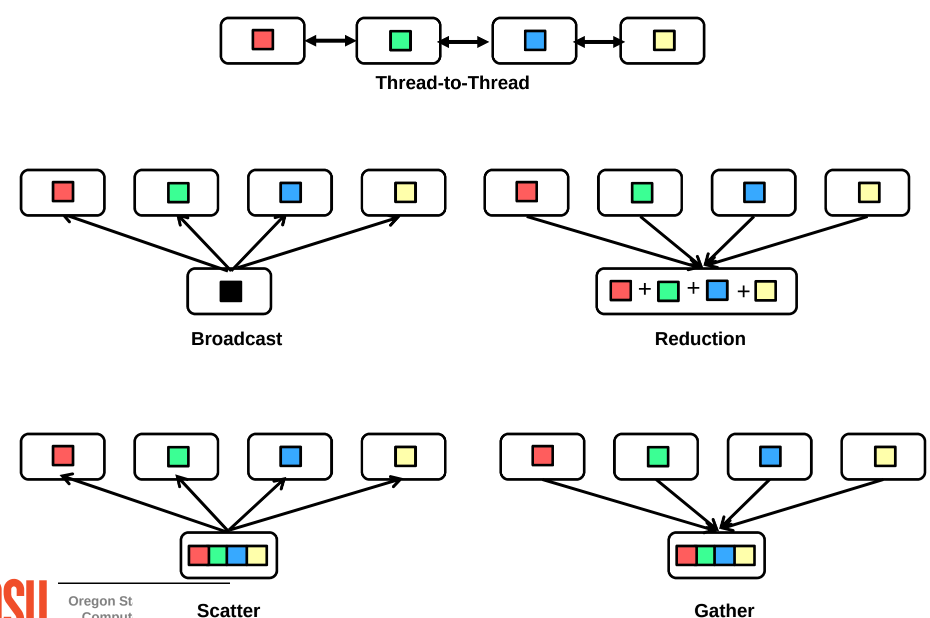
Agglomerate (or **Aggregate**)
Combine the small partitioned tasks into larger tasks

Map
Assign the larger tasks to cores or threads

Reduce
Combine multi-results into one result

Reference: Ian Foster, *Designing and Building Parallel Programs*, Addison-Wesley, 1995

Types of Parallel Communications

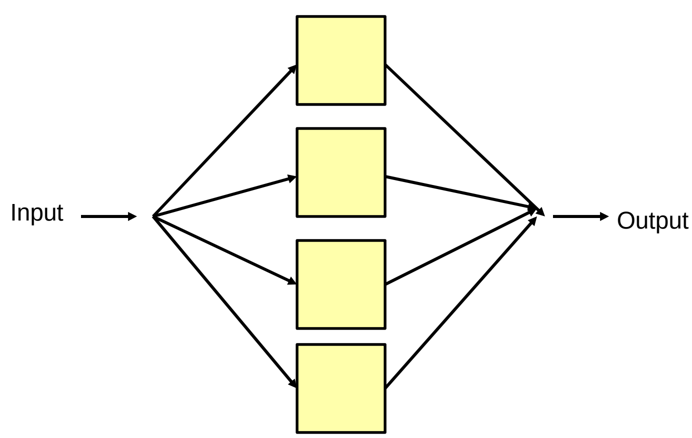


PCAMR Rules of Thumb

1. Focus effort on the most time-consuming computation
2. Focus effort on whatever data is accessed most frequently
3. Focus effort on maximizing the *Compute : Communicate Ratio*
4. Use agglomeration to reduce communication by increasing locality
5. If agglomeration replicates data, be sure this does not affect the scalability of the algorithm by restricting the range of problem sizes and processor costs
6. Does the number of tasks scale with the problem size? (Not the size of each task!)
7. Place tasks that can execute concurrently on different cores
8. Place tasks that communicate frequently on the same core to increase locality
9. Be sure the Manager is not a bottleneck
10. If you are using cyclic or probabilistic load balancing, be sure you have enough tasks to keep everyone busy

Paradigms for Task Scheduling / Mapping

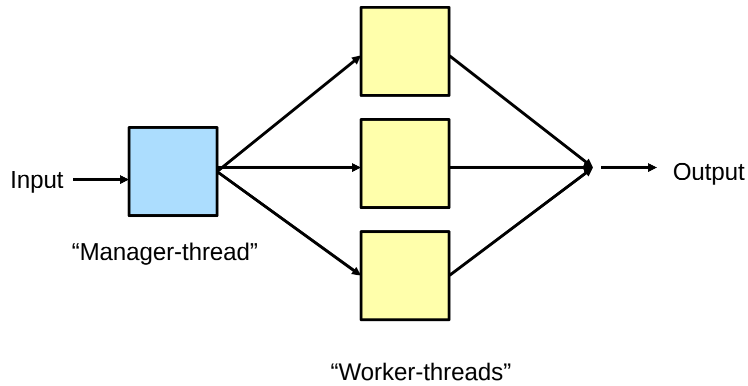
Decentralized (Peer)



“Peer-threads”

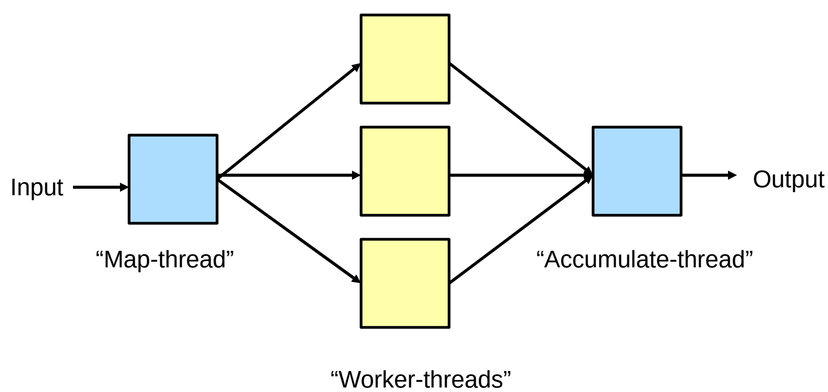
Paradigms for Task Scheduling / Mapping

Manager / workers



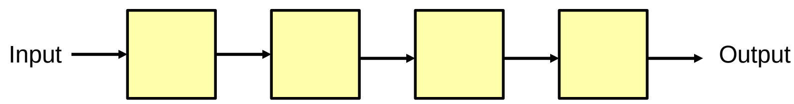
Paradigms for Task Scheduling / Mapping

Map-Reduce



Paradigms for Task Scheduling / Mapping

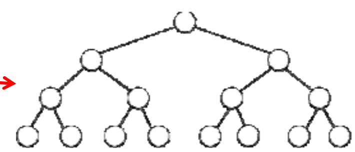
Pipeline



Requires some sort of queue between the stages

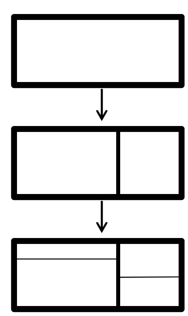
Load Balancing Strategies:
Assigning Portions of the Overall Task to the Threads

Recursive Equal Bisection



Recursive Unequal Bisection

Subdivisions don’t necessarily have to be equal in size



Local algorithms

Each core checks its load against its neighboring cores and adjusts what it is handling

Load Balancing Strategies: Assigning Portions of the Overall Task to the Threads

Cyclic mappings

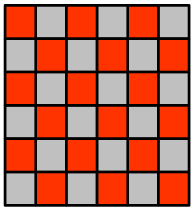
If there are N cores, allocate every Nth task to a particular core. This is like using **chunksize** in *omp parallel for.*

Probabilistic methods

Allocate each task to a randomly-chosen core.

Design Patterns

1. **Replicating computation**
2. **Red / Black (Even / Odd)**
3. **Divide-and-Conquer (Reduction)**
4. **Block Scheduling**



Red/Black or Even/Odd

```
#include <algorithm>
            . . .
for(  int i = 0;  i < NUMN;  i++ )
{
        int first = i % 2;       // 0 if i is 0, 2, 4, ...
                                 // 1 if i is 1, 3, 5, ...

        #pragma omp parallel for default(none),shared(A,first)

        for(  int j = first;  j < NUMN-1;  j += 2 )
        {
                if(  A[ j ]  >  A[ j+1 ]  )
                {
                        std::swap( A[ j ], A[ j+1 ] );
                }
        }
}
```

NUMN = 6

Step #

original	0	1	2	3	4	5
6	5	5	3	3	1	1
5	6	3	5	1	3	2
4	3	6	1	5	2	3
3	4	1	6	2	5	4
2	1	4	2	6	4	5
1	2	2	4	4	6	6

Design Patterns: Divide and Conquer

$$\sum_{i=0}^{2^N-1} = \sum_{i=0}^{2^{N-1}-1} + \sum_{i=2^{N-1}}^{2^N-1}$$

e.g., N = 4 :

$$\sum_{i=0}^{15}$$

$$\sum_{i=0}^{7} + \sum_{i=8}^{15}$$

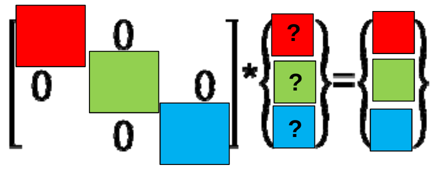
$$\sum_{i=0}^{3} + \sum_{i=4}^{7} \qquad \sum_{i=8}^{11} + \sum_{i=12}^{15}$$

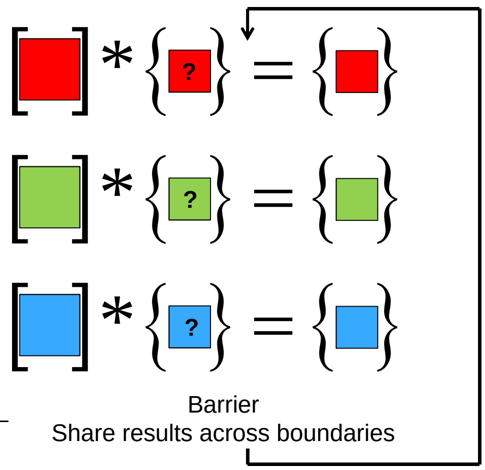
$$\sum_{i=0}^{1} + \sum_{i=2}^{3} \quad \sum_{i=4}^{5} + \sum_{i=6}^{7} \qquad \sum_{i=8}^{9} + \sum_{i=10}^{11} \quad \sum_{i=12}^{13} + \sum_{i=14}^{15}$$

Design Patterns: Block Scheduling

Example: A diagonally-dominant matrix solution

- Break the problem into blocks
- Solve within the block
- Handle borders separately after a Barrier



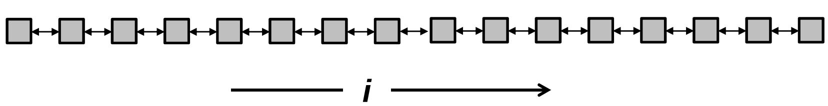


Another Block Schedule Example: 1D Heat Transfer Equation

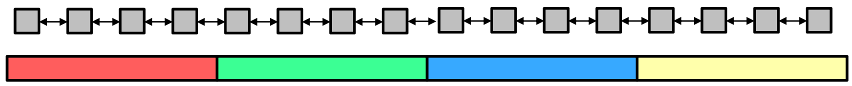
$$\rho C \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2}\right)$$

$$\downarrow$$

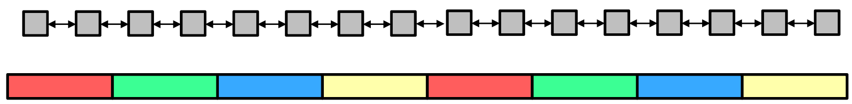
$$\frac{\Delta T}{\Delta t} = \frac{k}{\rho C}\left(\frac{\Delta^2 T}{\Delta x^2}\right) \longrightarrow \Delta T_i = \left(\frac{k}{\rho C}\right)\left(\frac{T_{i-1} - 2T_i + T_{i+1}}{(\Delta x)^2}\right)\Delta t$$



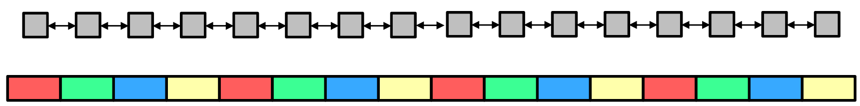
Partition: 1D Domain (Data) Decomposition



1D Block

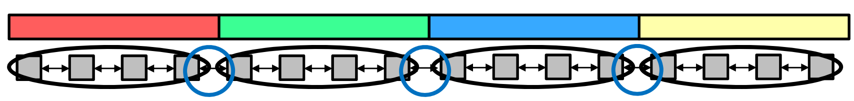


1D Cyclic



1D Cyclic

Communication, Agglomeration, Mapping: 1D Compute-to-Communicate Ratio



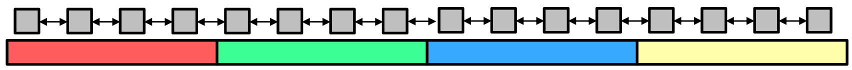
Intracore computing

Intercore communication

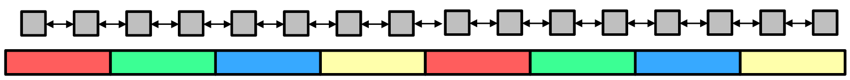
Compute : Communicate ratio = N : 2

where N is the number of compute cells per core

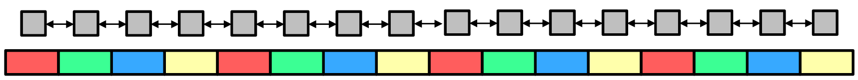
1D Domain (Data) Decomposition



Compute : Communicate = 4 : 2

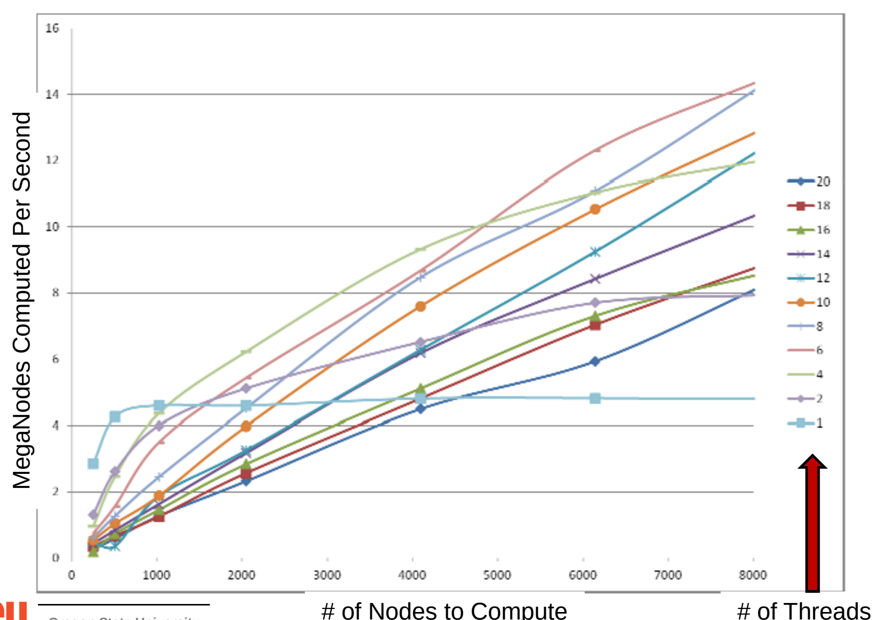


Compute : Communicate ratio = 2 : 2

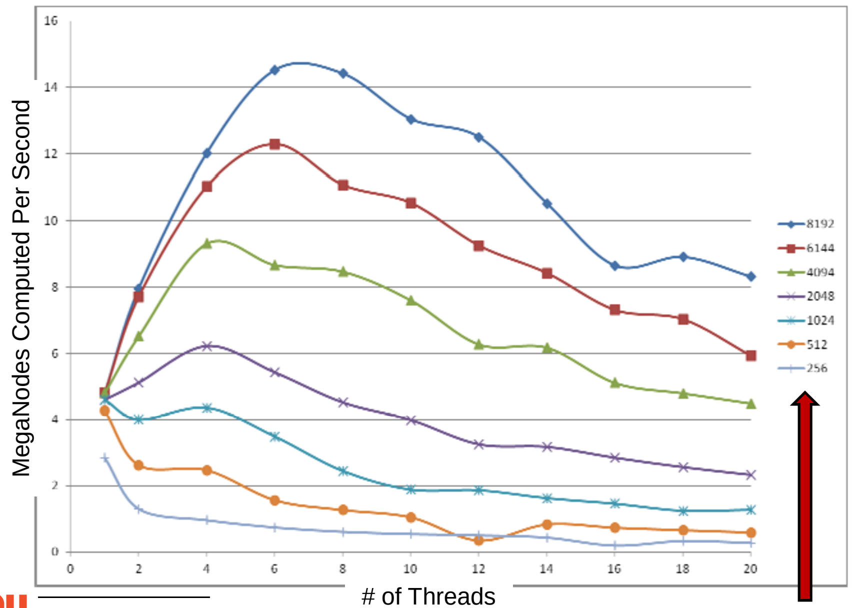


Compute : Communicate ratio = 1 : 2

Performance as a Function of Number of Nodes



Performance as a Function of Number of Threads

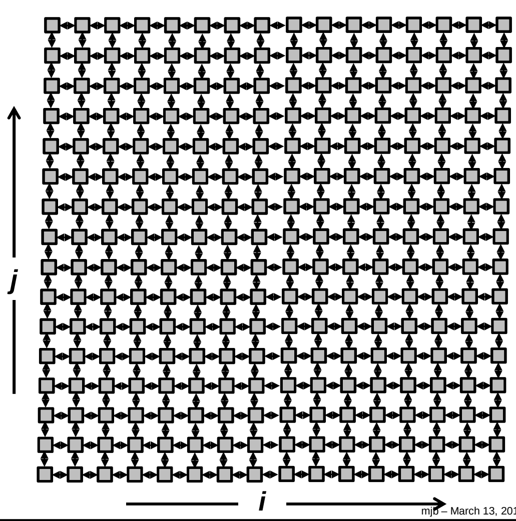


2D Heat Transfer Equation

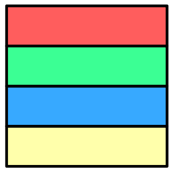
$$\rho C \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

$$\frac{\Delta T}{\Delta t} = \frac{k}{\rho C}\left(\frac{\Delta^2 T}{\Delta x^2} + \frac{\Delta^2 T}{\Delta y^2}\right)$$

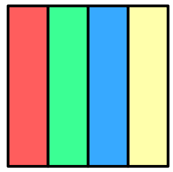
$$\Delta T_{i,j} = \left(\frac{k}{\rho C}\right)\left(\frac{T_{i-1,j} - 2T_{i,j} + T_{i+1,j}}{(\Delta x)^2} + \frac{T_{i,j-1} - 2T_{i,j} + T_{i,j+1}}{(\Delta y)^2}\right)\Delta t$$



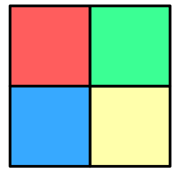
2D Domain (Data) Decomposition



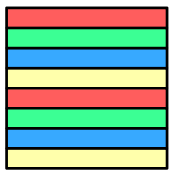
2D *,Block



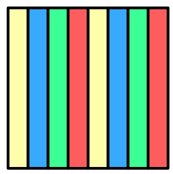
2D Block,*



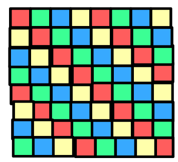
2D Block, Block



2D *,Cyclic



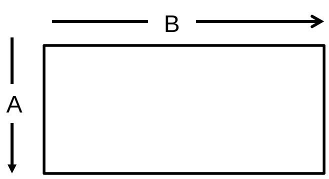
2D Cyclic,*



2D Cyclic, Cyclic

The Decomposition Order Matters (think cache)

float Array[A][B];

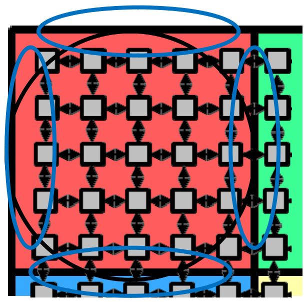


In 2D problems, this is often (but not always) thought of as:

float Array[NY][NX];

0	0
0	1
0	2
0	3
0	...
0	$B-1$
1	0
1	1
1	2
1	3
1	...
1	$B-1$
.	. .
$A-1$	0
$A-1$	1
$A-1$	2
$A-1$	3
$A-1$	...
$A-1$	$B-1$

2D Compute-to-Communicate Ratio



Intracore computing

Intercore communication

Compute : Communicate ratio = N^2 : 4N = N : 4

where N is the dimension of compute cells per core

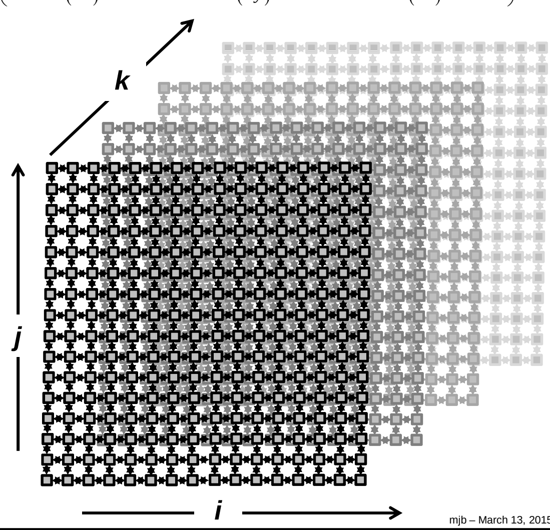
The 2D Compute : Communicate ratio is sometimes referred to as *Area-to-Perimeter*

3D Heat Transfer Equation

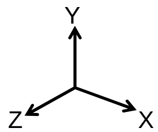
$$\rho C \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

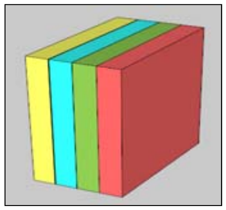
$$\frac{\Delta T}{\Delta t} = \frac{k}{\rho C}\left(\frac{\Delta^2 T}{\Delta x^2} + \frac{\Delta^2 T}{\Delta y^2} + \frac{\Delta^2 T}{\Delta z^2}\right)$$

$$\Delta T_{i,j,k} = \left(\frac{k}{\rho C}\right)\left(\frac{T_{i-1,j,k} - 2T_{i,j,k} + T_{i+1,j,k}}{(\Delta x)^2} + \frac{T_{i,j-1,k} - 2T_{i,j,k} + T_{i,j+1,k}}{(\Delta y)^2} + \frac{T_{i,j,k-1} - 2T_{i,j,k} + T_{i,j,k+1}}{(\Delta z)^2}\right)\Delta t$$

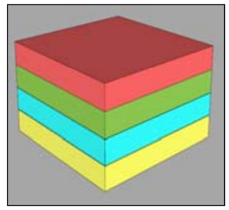


3D Domain (Data) Decomposition

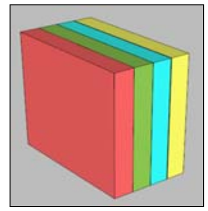




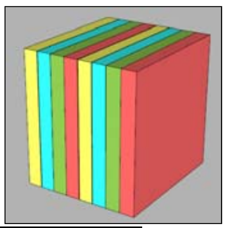
3D Block, *, *



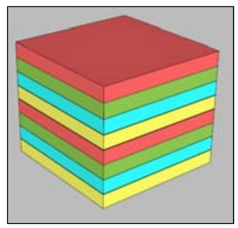
3D *,Block, *



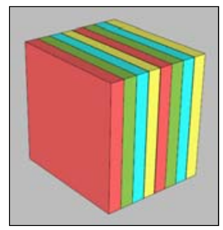
3D *,*,Block



3D Cyclic, *, *

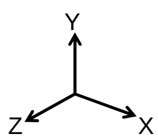


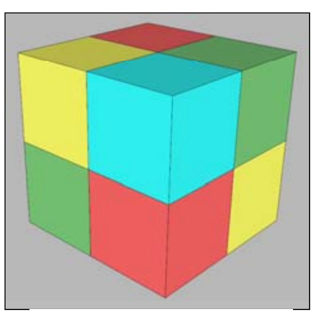
3D *,Cyclic, *



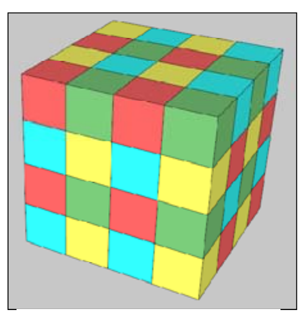
3D *,*,Cyclic

3D Domain (Data) Decomposition





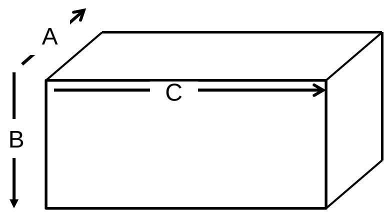
3D Cyclic, Cyclic, Cyclic



3D Cyclic, Cyclic, Cyclic

The Decomposition Order Matters (think cache)

float Array[A][B][C];



In 3D problems, this is often (but not always) thought of as:

float Array[NZ][NY][NX];

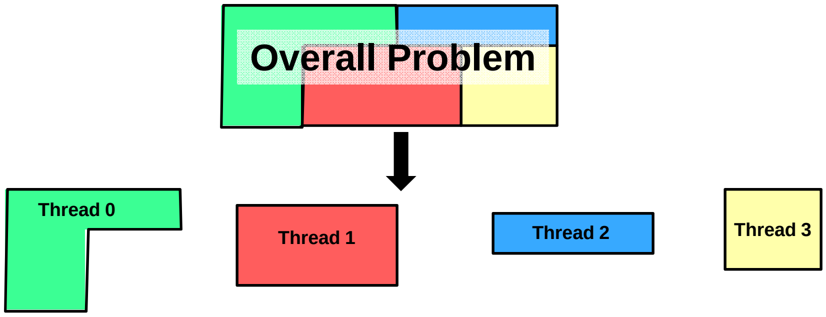
3D Compute-to-Communicate Ratio

Compute : Communicate ratio = $N^3 : 6N^2 = N : 6$

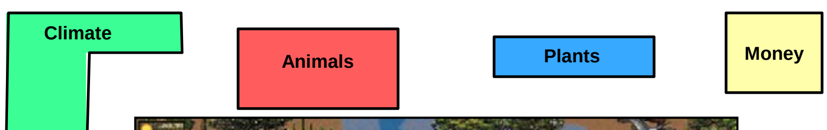
where N is the dimension of compute cells per core

In 3D the Compute : Communicate ratio is sometimes referred to as *Volume-to-Surface*

Functional (or Task) Decomposition



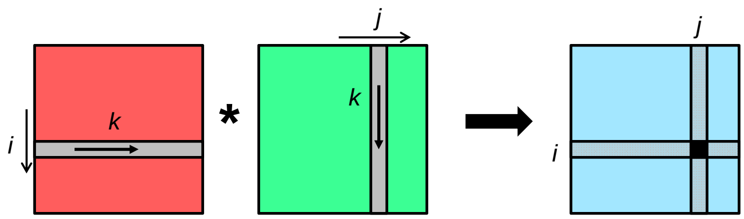
Functional (or Task) Decomposition



Credit: Maxis (Sim Park)

Matrix Multiply

The usual approach is multiplying the entire A row * entire B column
This is equivalent to computing a single dot product



Row i of A * Column j of B ➡ Element (i,j) of C

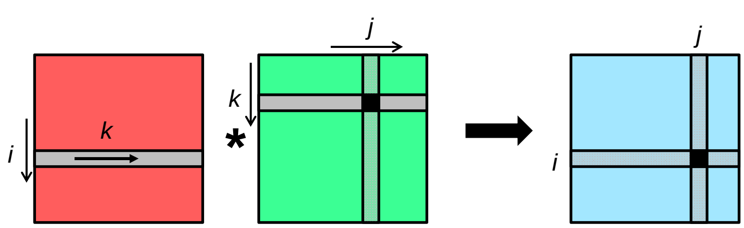
```
for( i = 0; i < SIZE; i++ )
    for( j = 0; j < SIZE; j++ )
        for( k = 0; k < SIZE; k++ )
```

∑ A[i][k] * B[k][j] —Sum and store➡ C[i][j]

Problem: Column j of the B matrix is not doing a unit stride

Matrix Multiply

Scalable Universal Matrix Multiply Algorithm (SUMMA)
Entire A row * one element of B row
Equivalent to computing one item in many separate dot products

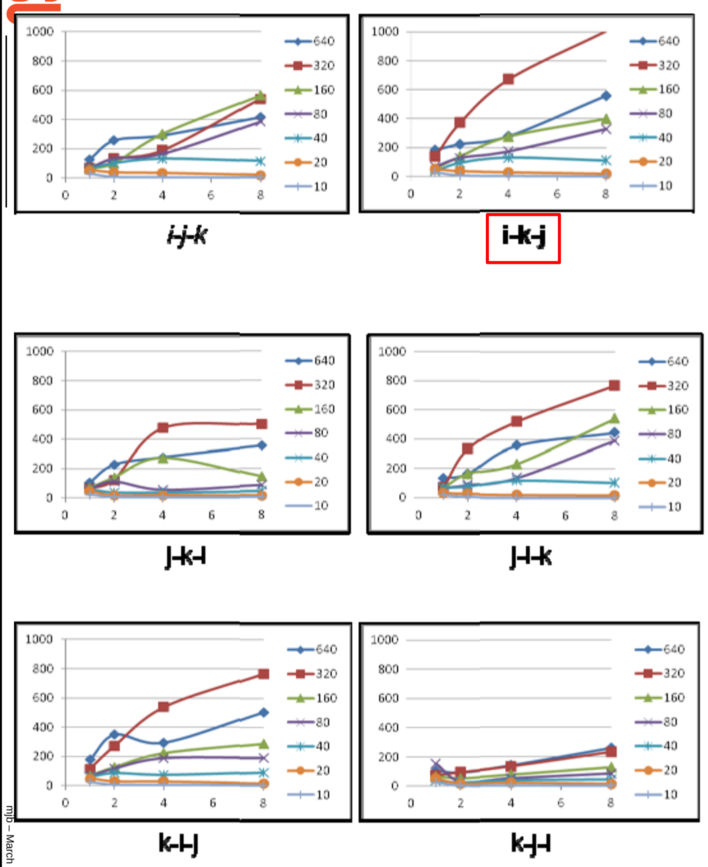


Row i of A Row k of B Element (i,j) of C

```
for( i = 0; i < SIZE; i++ )
    for( k = 0; k < SIZE; k++ )
        for( j = 0; j < SIZE; j++ )
```

A[i][k] * B[k][j] —Add to➡ C[i][j]

Performance vs. Number of Threads (MegaMultiplies / Sec)



Performance vs. Matrix Size (MegaMultiplies / Sec)