
Statistische Physik im Gleichgewicht

WS 2023/2024 – Blatt 10

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Problem 16: 1D Ising model

(5 Points)

In the lecture, we studied the 1D-Ising model via renormalization group for a scaling factor $b = 2$. We now consider a general value of the scaling factor b . For simplicity, we will now assume that there is no external field, i.e. $h = 0$.

- (a) Show that the renormalization group equation then becomes:

$$u' = u^b,$$

where $u = \tanh K$.

Hint: To generalize to the scaling factor b , you will need to take the trace over S_2, S_3, \dots, S_b to find the effective coupling constant between S_1 and S_{b+1} . It will be useful to take the trace over S_2 first, S_3 next, and so on.

- (b) From part a), it is clear that $T = 0$ ($u = 1$) is a fixed point. What happens for $T > 0$? Is this fixed point stable or unstable?
- (c) Find the correlation length $\xi(u)$. Recall that the correlation length should be transformed as $\xi(u') = \frac{1}{b}\xi(u)$. What happens in the low temperature limit ($K \rightarrow \infty$)?

Hint: Use the expansion $\tanh K \approx 1 - 2e^{-2K}$.

Feedback:

Roughly how much time did you spend on this problem set?