Statistische Physik im Gleichgewicht

WS 2023/2024 – Blatt 7

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Problem 13: Finite-size scaling (*10 Points*)

In the last exercise, we created a program for simulating the 2D Ising model with importance sampling. Here we also encountered the issue of finite-size effects. Using finite-size scaling helps us to still gather information from these systems. It is helpful to compare the size of the system with the correlation length of the infinite system, which gives us the quantity L/ξ:

(a) Justify why one can write

$$L/\xi \propto L|T - T_C|^{\nu}.$$

We now assume that $\langle m \rangle$ in the finite system depends on this ratio. Since we prefer a linear dependence on temperature, we write:

$$\langle m \rangle = L^q f\Big(L^{1/\nu}(T - T_C)\Big),$$

with the exponent q and the function f still unknown.

(b) Justify that $f(r) \propto r^{\beta}$ far below the critical point, and that $q = -\frac{\beta}{\nu}$. Here, β is a critical exponent and $r = L^{1/\nu}(T - T_C)$.

(c) Choose three values $L = 10$, $L = 20$, and $L = 30$ and ten values of T each, so that you stay in the range $-5 < (T - Tc)L^{1/\nu} < 5$. Simulate these systems with your existing program.

(d) With the insights from before, we now expect

$$\langle m \rangle = L^{-\beta/\nu} f\Big(L^{1/\nu}(T - T_C)\Big),$$

Using $\nu = 1$, find the exponent β, such that all the data collapses on a common curve, as well as possible (you can use trial-and-error for that). What is the value of β? You can assume $T_C = \frac{2}{\ln(\sqrt{2}+1)} \approx 2.27$.
Hint: Before plotting the data, think about which term should be on the abscissa, and which on the ordinate.

Feedback:

Roughly how much time did you spend on this problem set?