Statistische Physik im Gleichgewicht

WS 2023/2024 – Blatt 4

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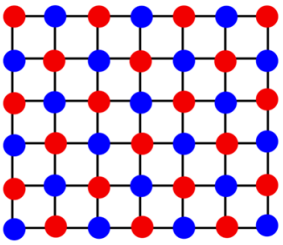
Problem 11: Antiferromagnetism (*10 Points*)

An antiferromagnet is described by the Hamiltonian

$$H = J \sum_{\langle ij \rangle} S_i S_j - h \sum_i S_i$$

with $h, J > 0$ and $S_i = \pm 1$.



(a) This model system is often represented as two grids A and B that pervade each other (see sketch). Why is this a sensible description? What states do A and B correspond to?

(b) Use the variational approach to arrive at the following self-consistent equations

$$m_A = \tanh\left(-\beta J m_B z + h\right)$$
$$m_B = \tanh\left(-\beta J m_A z + h\right)$$

where $m_{A,B}$ are the magnetizations of subsystems A and B.

(c) Explain the role of the state $m_A = -m_B = m$ for the system. How are the spins structured?

(d) The critical temperature of the antiferromagnet is called the *Néel temperature*, T_N. Above the Néel temperature the system is isotropic, and below antiferromagnetic. Calculate the magnetic susceptibilities $\chi_{A,B} = \partial m_{A,B}/\partial h|_{h\to 0}$ and the total susceptibility $\chi_{\text{tot}} = (\chi_A + \chi_B)/2$. Show that χ_{tot} above the transition point has the form

$$\chi_{\text{tot}} = \frac{1}{T + T_N}.$$

Hint: At some point, you should assume $m = m_A = -m_B$ here as well. The critical point should be clear, but you can also calculate it again.

(e) Calculate the susceptibility below T_N. Discuss what happens at $T = T_N$ in comparison to a ferromagnet. Sketch the susceptibility in the vicinity of T_N.

(f) We now couple the order parameter with a magnetic field in a different way by introducing a (unphysical) staggered magnetic field, that creates a field $+h$ on system A and $-h$ on system B. Adjust the self-consistent equations and then calculate the susceptibilities $\chi_A = \partial m_A/\partial h|_{h\to 0}$ and $\chi_B = \partial m_B/\partial(-h)|_{h\to 0}$. What happens now for χ_{tot} at $T = T_N$?

Feedback:

Roughly how much time did you spend on this problem set?