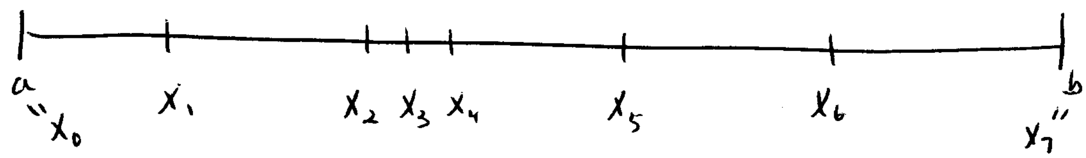
Overview of BVP_SOLVER

- Solves differential equations: $\underline{y}' = \underline{f}(x, \underline{y})$, $\underline{g}_a(\underline{y}(a)) = 0$, $g_b(y(b)) = 0$.

- Given a mesh that partitions $[a,b]$, $\{x_i\}_{i=0}^{N}$,

e.g.



the differential equation is approximated by a set of nonlinear equations, whose solution gives approximation to $y(t)$ at the mesh points: $Y = [y_0, y_1, \ldots y_N]$ (Discrete Solution)
$\underline{y_i \approx y(x_i)}$

- The nonlinear system is $\underline{F}(\underline{Y}) = \underline{0}$ and it is solved using Newton's method. Sometimes the Newton iteration fails to converge, and we must try a new mesh. Otherwise, we can extend Y to obtain a <u>continuous solution approximation</u>, $\underline{u}(x) \approx \underline{y}(x)$ on $[a,b]$

- We check the quality of $\underline{u}(x)$ by estimating its <u>defect</u> $\underline{\delta}(x) \equiv \underline{u}'(x) - \underline{f}(x, \underline{u}(x))$. How well does $\underline{u}(x)$ satisfy the differential equation?

- If $\underline{\delta}(x)$ is too big, $\underline{u}(x)$ is not acceptable and we retry with a new mesh. $(|\underline{\delta}(x)| < 0.1)$

- If $\underline{\delta}(x)$ is reasonable small, (< 0.1), we then see if $\delta(x)$ is less than the user tolerance. If not we retry with a new mesh.