Methods Camp

UT Austin, Department of Government

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2023-08-10

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Class schedule

Date	Time	Location
Thurs, Aug. 10	9:00 AM - 4:00 PM	RLP 1.302E
Fri, Aug. 11	9:00 AM - 4:00 PM	RLP 1.302E
Sat, Aug. 12	No class	-
Sun, Aug. 13	No class	-
Mon, Aug. 14	9:00 AM - 4:00 PM	RLP 1.302E
Tues, Aug. 15	9:00 AM - 4:00 PM	RLP 1.302E
Weds, Aug. 16	9:00 AM - 4:00 PM	RLP 1.302E

On class days, we will have a lunch break from 12:00-1:00 PM. We'll also take short breaks periodically during the morning and afternoon sessions as needed.

Description

Welcome to Introduction to Methods for Political Science, aka "Methods Camp"! In the past our incoming students have told us their math skills are rusty and they would like to be better prepared for UT's methods courses. Methods Camp is designed to give everyone a chance to brush up on some skills in preparation for the Stats I and Formal Theory I courses. The other goal of Methods Camp is to allow you to get to know your cohort. We hope that struggling with matrix algebra and the dreaded chain rule will still prove to be a good bonding exercise.

As you can see from the above schedule, we'll be meeting on Thursday, August 10th and Friday, August 11th as well as from Monday, August 14th through Wednesday, August 16th. Classes at UT begin the start of the following week on Monday, August 22nd. Below is a tentaive schedule outlining what will be covered in the class, although we may rearrange things a bit if we find we're going too slowly or too quickly through any of the material.

Course outline

1 Thursday morning: R and RStudio

- Introductions
- RStudio (materials are on the website as zipped RStudio projects)
- Objects (vectors, matrices, data frames)
- Basic functions (`mean()`, `length()`, etc.)

2 Thursday afternoon: `tidyverse` basics I

- Packages: installation and loading (including the tidyverse)
- Data wrangling with dplyr (basic verbs, including the new `.by =` syntax)
- Data visualization basics with `ggplot2`
- Data loading (.csv, .rds, .dta, .xlsx)
- Quarto fundamentals

3 Friday morning: Matrices

- Matrices
- Systems of linear equations
- Matrix operations (multiplication, transpose, inverse, determinant).
- Solving systems of linear equations in matrix form (and why that's cool)
- Introduction to OLS

4 Friday afternoon: `tidyverse` basics II

- Data merging and pivoting (`*_join()`, `pivot_*()`)
- Value recoding (`if_else()`, `case_when()`)
- Missing values
- Data visualization extensions: facets, text annotations

5 Monday morning: Functions and loops

- Functions
- For-loops and `lapply()`
- Finding R help (help files, effective Googling, ChatGPT)

6 Monday afternoon: Calculus

- Limits (not sure how to teach this in an R-centric way yet, but there must be a way)
- Derivatives (symbolic, numerical, automatic)
- Integrals

7 Tuesday morning: Probability

- Concepts: probability, random variables, etc.
- PMF, PDF, CDF, etc.
- Distributions (binomial, normal; different functions in R and how to use them)
- Expectation and variance

8 Tuesday afternoon: Simulations

- Simulations (ideas, seed setting, etc.)
- Sampling
- Bootstrapping

9 Wednesday morning: Text analysis

- String manipulation with `stringr`
- Simple text analysis (counts, tf-idf, etc.) with `tidytext` and visualization

10 Wednesday afternoon: Wrap-up

- Project management fundamentals (RStudio projects, keeping raw data, etc.)
- Self-study resources and materials
- Other software (Overleaf, Zotero, etc.)
- Methods at UT

Contact info

If you have any questions during or outside of methods camp, you can contact Andrés at andres.cruz@utexas.edu and Matt at mjmartin@utexas.edu.

1 R and RStudio

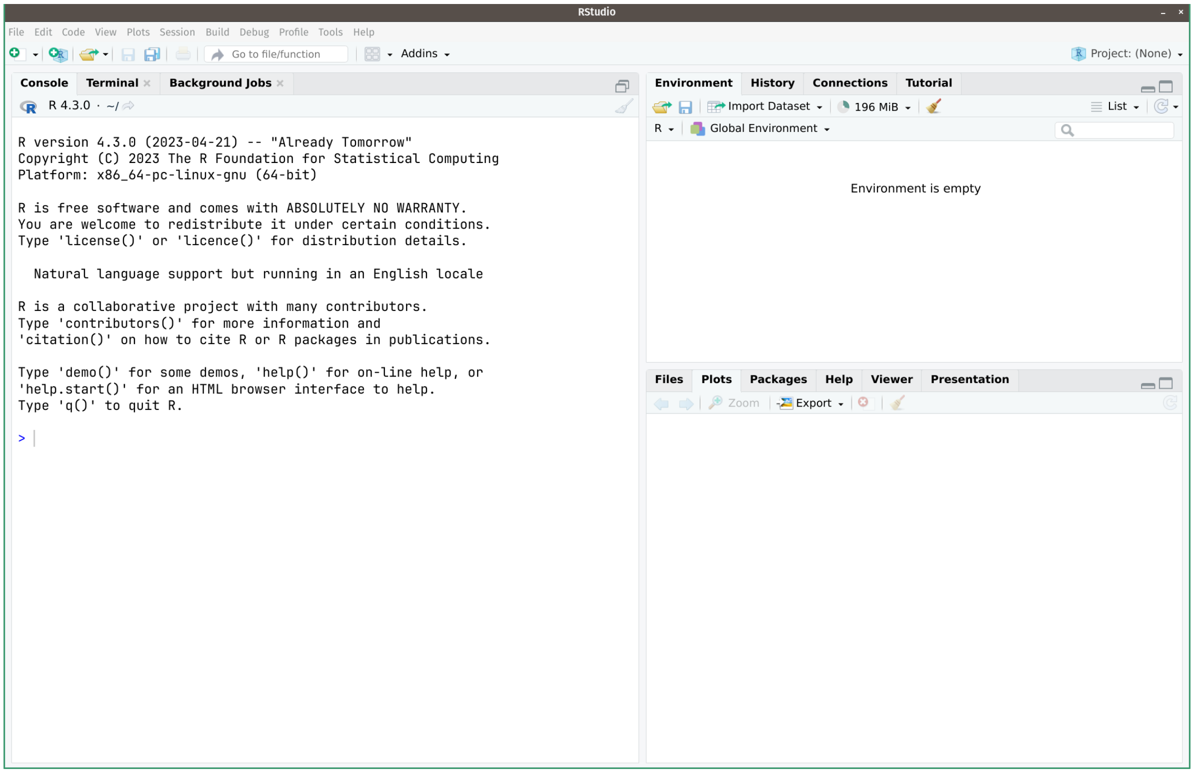
1.1 Installing R and RStudio

R is a programming language optimized for statistics and data analysis. Most people use R from RStudio, a graphical user interface (GUI) that includes a file pane, a graphics pane, and other goodies. Both R and RStudio are open source, i.e., free as in beer and free as in freedom!

Your first steps should be to install R and RStudio, in that order (if you have installed these programs before, make sure that your versions are up-to-date—if not, follow the instructions below):

1. Download and install R from the official website, CRAN. Click on "Download R for <Windows/Mac>" and follow the instructions. If you have a Mac, make sure to select the version appropriate for your system (Apple Silicon for newer M1/M2 Macs and Intel for older Macs).
2. Download and install RStudio from the official website. Scroll down and select the installer for your operating system.

After these two steps, you can open RStudio in your system, as you would with any program. You should see something like this:



That's it for the installation! We also *strongly* recommend that you change a couple of RStudio's default settings:[1]

- `Tools > Global Options > General > Uncheck "Restore .RData into workspace at startup"`
- `Tools > Global Options > General > Save workspace to .RData on Exit > Select "Never"`
- `Tools > Appearance` to change to a dark theme, if you want! Pros: better for night sessions, hacker vibes...

1.2 Setting up for Methods Camp

TODO: how to download the materials and open the RStudio project (include discussion on .zip files). How to make sure that everything's fine (see top-right for RStudio project).

[1]The idea behind these settings is to force R to start from scratch with each new session. No lingering objects avoids misunderstandings and helps with reproducibility!

1.3 Short intro to R

TODO: objects and common functions. Explain how to run Quarto code blocks, and how to render a document to html.

2 `tidyverse` basics I

3 Matrices

3.1 Introduction

3.1.1 Scalars

- One number (12, for example) is referred to as a scalar.
- Each scalar in a matrix is an *element* of that matrix.

$$\begin{bmatrix} 12 \end{bmatrix}$$

> **Note**
>
> This is also called a 1 x 1 ("one by one") matrix.

3.1.2 Vectors

- We can put several scalars together to make a vector.
- Here is an example:

$$\begin{bmatrix} 12 \\ 14 \\ 15 \end{bmatrix} = b$$

- Since this is a column of numbers, we cleverly refer to it as a *column vector*.
- Here is another example of a vector:

$$\begin{bmatrix} 12 & 14 & 15 \end{bmatrix} = d$$

- This, in contrast, is called a *row vector*.

3.2 Operators

3.2.1 Summation

- The summation operator, $\sum$, lets us perform an operation on a sequence of numbers, which is often but not always a vector.

$$x = \begin{bmatrix} 12 & 7 & -2 & 0 & 1 \end{bmatrix}$$

- We can then calculate the sum of the first three elements of the vector, which is expressed as follows:

$$\sum_{i=1}^{3} x_i$$

- Then, we do the following math:

$$12 + 7 + -2 = 17$$

3.2.2 Product

- The product operator, $\prod$, can also perform operations over a sequence of elements in a vector.

$$z = \begin{bmatrix} 5 & -3 & 5 & 1 \end{bmatrix}$$

- We can then calculate the calculate the product of the four elements in the vector, which is expressed as follows:

$$\prod_{i=1}^{4} z_i$$

- Then, we do the following math:

$$5 * -3 * 5 * 1 = -75$$

3.3 Matrices

3.3.1 Basics

- We can append vectors together to form a matrix:

$$\begin{bmatrix} 12 & 14 & 15 \\ 115 & 22 & 127 \\ 193 & 29 & 219 \end{bmatrix} = A$$

- The number of rows and columns of a matrix constitute the dimensions of the matrix.
- The first number ("r") is the number of rows and the second number ("c" here) is the number of columns in the matrix.

! Important

Find a way to remember "r x c" *permanently*. The order of the dimensions never changes.

- The matrix A above, for example, is a $3x3$ matrix.
- We often use capital letters (sometimes also **bold-faced**) to represent matrices.

3.3.2 Structure

- How do we refer to specific elements of the matrix?
- Matrix A is an $m \times n$ matrix where $m = n = 3$
- More generally, matrix B is an $m \times n$ matrix where the elements look like this:

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & \dots & b_{1n} \\ b_{21} & b_{22} & b_{23} & \dots & b_{2n} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ b_{m1} & b_{m2} & b_{m3} & \dots & b_{mn} \end{bmatrix}$$

- Thus b_{23} refers to the second unit down and third across.

Reminder

When trying to identify a specific element, the first subscript is the element's row and the second subscript is the element's column (*always* in that order).

3.4 Matrix operations

3.4.1 Addition and subtraction

- Addition and subtraction are straightforward operations.
- Matrices must have *exactly* the same dimensions for both of these operations.
- We add or subtract each element with the corresponding element from the other matrix.
- This is expressed as follows:

$$A \pm B = C$$

$$c_{ij} = a_{ij} \pm b_{ij} \;\forall i, j$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \pm \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$
$$=$$
$$\begin{bmatrix} a_{11} \pm b_{11} & a_{12} \pm b_{12} & a_{13} \pm b_{13} \\ a_{21} \pm b_{21} & a_{22} \pm b_{22} & a_{23} \pm b_{23} \\ a_{31} \pm b_{31} & a_{32} \pm b_{32} & a_{33} \pm b_{33} \end{bmatrix}$$

Practice

$$A = \begin{bmatrix} 1 & 4 & 2 \\ -2 & -1 & 0 \\ 0 & -1 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 1 & 0 \\ 2 & -1 & 0 \\ 7 & 1 & 2 \end{bmatrix}$$

Calculate $A + B$

Practice

$$A = \begin{bmatrix} 6 & -2 & 8 & 12 \\ 4 & 42 & 8 & -6 \\ -14 & 5 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 18 & 42 & 3 & 7 \\ 0 & -42 & 15 & 4 \\ -7 & 0 & 21 & -18 \end{bmatrix}$$

Calculate $A - B$

3.4.2 Scalar multiplication

- Scalar multiplication is very intuitive.
- As we know, a scalar is a single number, or a 1 x 1 matrix.
- We multiply each value in the matrix by the scalar to perform this operation.
- This is expressed as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$cA = \begin{bmatrix} ca_{11} & ca_{12} & ca_{13} \\ ca_{21} & ca_{22} & ca_{23} \\ ca_{31} & ca_{32} & ca_{33} \end{bmatrix}$$

Practice

$$A = \begin{bmatrix} 1 & 4 & 2 \\ 8 & -1 & 3 \\ 0 & -2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} -15 & 1 & 5 \\ 2 & -42 & 0 \\ 7 & 1 & 6 \end{bmatrix}$$

Calculate $2 \times A$ and $-3 \times B$

3.4.3 Matrix multiplication

- Two matrices must be *conformable* for them to be multiplied together.
- This means that the number of columns in the first matrix equals the number of rows in the second.
- When multiplying $A \times B$, if A is $m \times n$, B must have n rows.

> **!** Important
>
> The conformability requirement *never* changes. Before multiplying anything, check to make sure the matrices are indeed conformable.

- The resulting matrix will have the same number of rows as the first matrix and the number of columns in the second.
- For example, if A is $i \times k$ and B is $k \times j$, then $A \times B$ will be $i \times j$.

Which of the following can we multiply? What will be the dimensions of the resulting matrix?

$$b = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} M = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 4 \\ 2 & 3 & 2 \end{bmatrix} L = \begin{bmatrix} 6 & 5 & -1 \\ 1 & 4 & 3 \end{bmatrix}$$

> Warning
>
> When multiplying matrices, *order matters*.

- Why can't we multiply in the opposite order?

3.4.4 Multiplication steps

- Multiply each row by each column, summing up each pair of multiplied terms.

> Note
>
> This is sometimes to referred to as the "dot product," where we multiply matching members, then sum up.

- The element in position ij is the sum of the products of elements in the ith row of the first matrix (A) and the corresponding elements in the jth column of the second matrix (B).

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

3.4.5 Example

- Suppose a company manufactures two kinds of furniture: chairs and sofas.
  - A chair costs \$100 for wood, \$270 for cloth, and \$130 for feathers.
  - Each sofa costs \$150 for wood, \$420 for cloth, and \$195 for feathers.

	Chair	Sofa
Wood	100	150
Cloth	270	420
Feathers	130	195

- The same information about unit cost (C) can be presented as a matrix.

$$C = \begin{bmatrix} 100 & 150 \\ 270 & 420 \\ 130 & 195 \end{bmatrix}$$

> **Note**
>
> Note that each of the three rows of this 3 x 2 matrix represents a material (wood, cloth, or feathers), and each of the two columns represents a product (chair or coach). The elements are the unit cost (in USD).

- Now, suppose that the company will produce 45 chairs and 30 sofas this month.
- This production quantity can be represented in the following table, and also as a 2 x 1 matrix (Q).

Product	Quantity
Chair	45
Sofa	30

$$Q = \begin{bmatrix} 45 \\ 30 \end{bmatrix}$$

- The “total expenditure” is equal to the “unit cost” times the “production quantity” (the number of units).
- The total expenditure (E) for each material this month is calculated by multiplying these two matrices.

$$E = CQ = \begin{bmatrix} 100 & 150 \\ 270 & 420 \\ 130 & 195 \end{bmatrix} \begin{bmatrix} 45 \\ 30 \end{bmatrix} = \begin{bmatrix} (100)(45) + (150)(30) \\ (270)(45) + (420)(30) \\ (130)(45) + (195)(30) \end{bmatrix} = \begin{bmatrix} 9,000 \\ 24,750 \\ 11,700 \end{bmatrix}$$

- Multiplying the 3 x 2 Cost matrix (C) times the 2 x 1 Quantity matrix (Q) yields the 3 x 1 Expenditure matrix (E).
- As a result of this matrix multiplication, we determine that this month the company will incur expenditures of:
  - $9,000 for wood
  - $24,750 for cloth
  - $11,700 for feathers.

3.4.6 Properties

- Addition and subtraction:
  - Associative: $(A \pm B) \pm C = A \pm (B \pm C)$
  - Communicative: $A \pm B = B \pm A$
- Multiplication:
  - $AB \neq BA$
  - $A(BC) = (AB)C$
  - $A(B + C) = AB + AC$
  - $(A + B)C = AC + BC$

3.4.7 Special matrices

Square matrix:

- The **diagonal** of a square matrix is a set of numbers consisting of the elements on the line from the upper-left-hand to the lower-right-hand corner of the matrix. Only a square matrix has a diagonal.
- The **trace** of a matrix is simply the sum of the diagonal elements of the matrix. So, then, a matrix must be square to have a trace.

Diagonal matrix:

- In a **diagonal matrix**, all of the elements of the matrix that are not on the diagonal are equal to zero.

Scalar matrix:

- A **scalar matrix** is a diagonal matrix where the diagonal elements are all equal to each other. In other words, we're really only concerned with one scalar (or element) held in the diagonal.

Identity matrix:

- The **identity matrix** is a scalar matrix with all of the diagonal elements equal to one.
- All of the off-diagonal elements are equal to zero.
- The capital letter I is reserved for the identity matrix.

3.4.8 Transpose

Original Matrix (s x t)		Transpose Matrix (t x s)
Row 1	*becomes* $\longrightarrow$	Column 1
Row 2	*becomes* $\longrightarrow$	Column 2
Row 3	*becomes* $\longrightarrow$	Column 3
$\vdots$	$\vdots$	$\vdots$
Row s	*becomes* $\longrightarrow$	Column s

...which is the same as...

Column 1	*becomes* $\longrightarrow$	Row 1
Column 2	*becomes* $\longrightarrow$	Row 2
Column 3	*becomes* $\longrightarrow$	Row 3
$\vdots$	$\vdots$	$\vdots$
Column t	*becomes* $\longrightarrow$	Row t

$$\underset{3 \times 5}{J} = \begin{bmatrix} 4 & 5 & 3 & 7 & -9 \\ 3 & 0 & -1 & 2 & 4 \\ 7 & -2 & 12 & 0 & 1 \end{bmatrix} \qquad \underset{5 \times 3}{J'} = \underset{5 \times 3}{J^{T}} = \begin{bmatrix} 4 & 3 & 7 \\ 5 & 0 & -2 \\ 3 & -1 & 12 \\ 7 & 2 & 0 \\ -9 & 4 & 1 \end{bmatrix}$$

3.4.9 Inverse

4 `tidyverse` basics II

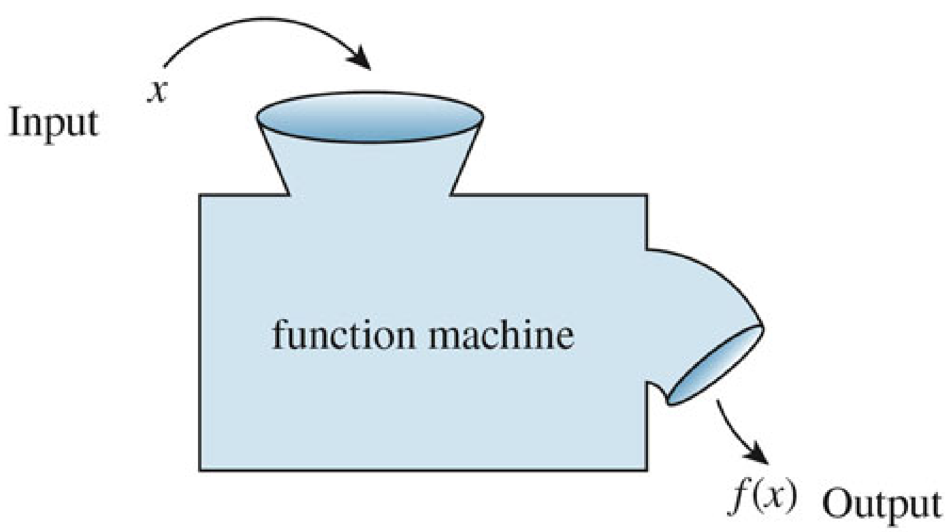
5 Functions and loops

5.1 Basics

5.1.1 What is a function?

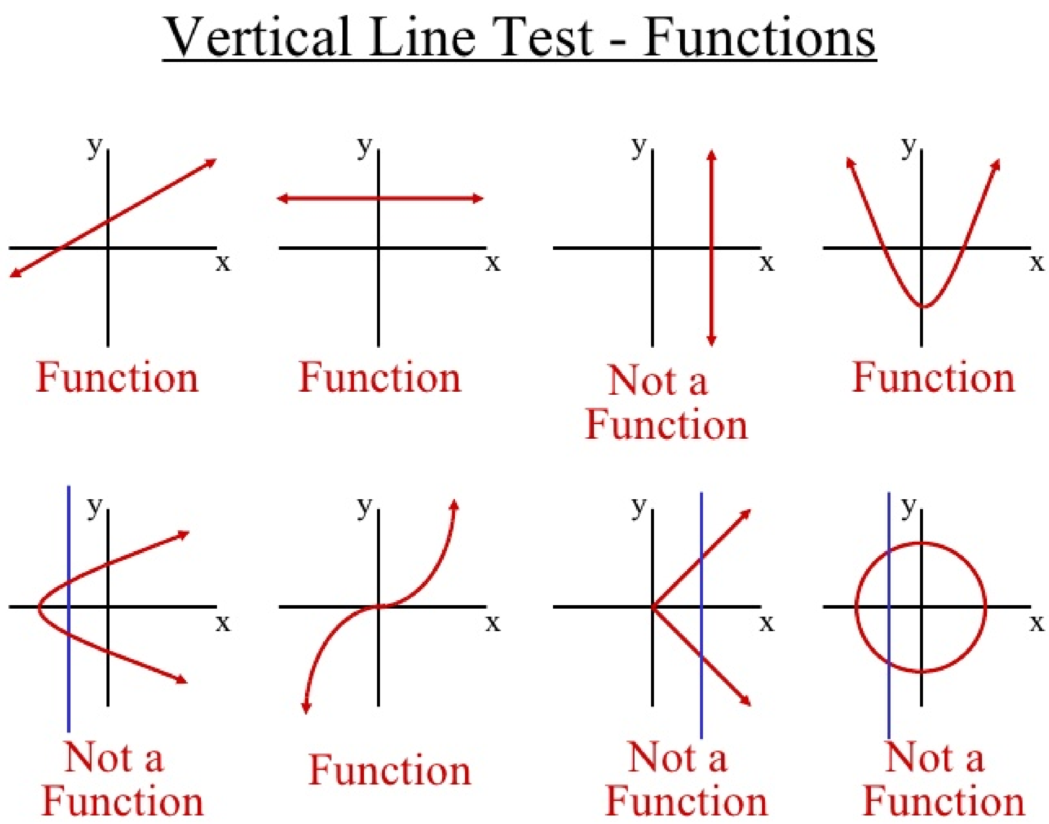
- Anything that takes input(s) and gives one defined output.
- They assign a unique value in its range (y values) for each value in its domain (x values).
- In math, this usually looks something like $f(x) = 3x + 4$.
  - x is the *argument* that the function takes.
  - For any x, multiply x by 3 and then add 4
  - Alternative but equivalent notation: $y = 3x + 4$
  - y is "a function of" x, so $y = f(x)$
- We describe functions with both equations and graphs.

5.1.2 Function machine



5.1.3 Visualization

When graphed, we can't draw vertical line through a function. Why not?



5.2 Types of functions

5.2.1 Linear functions

- We can easily make a function that describes a line.

$$y = mx + b$$

- m is the slope (for every one unit increase in x, y increases m units).

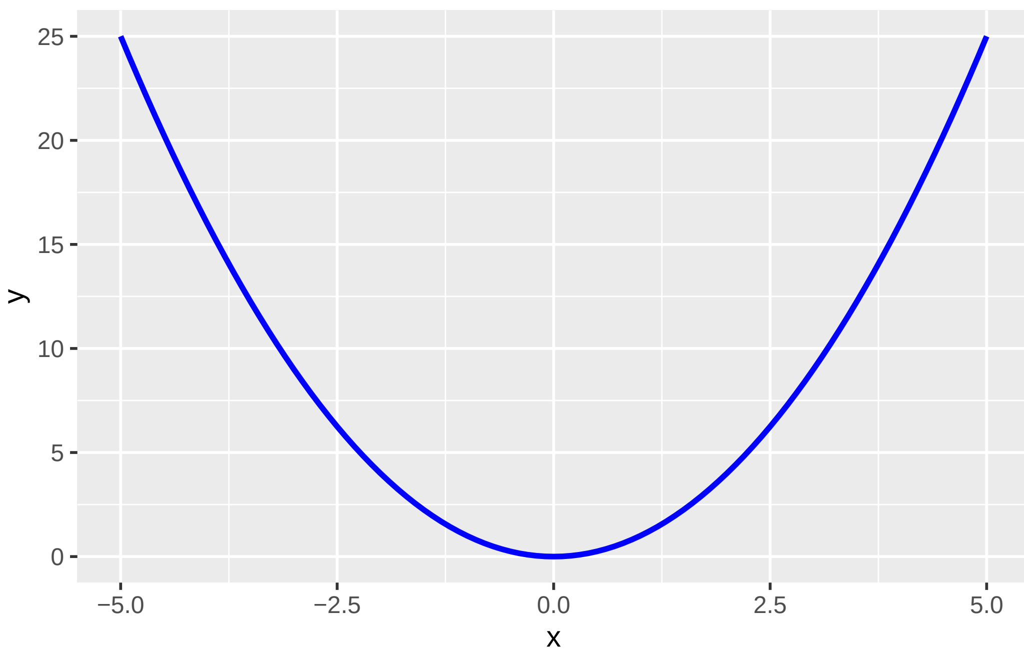
- b is the y-intercept: the value of y when $x = 0$.
- More generally, $y = a + bx$ - a is the intercept and b is the slope.

5.2.2 Quadratic

- These lines have one curve.

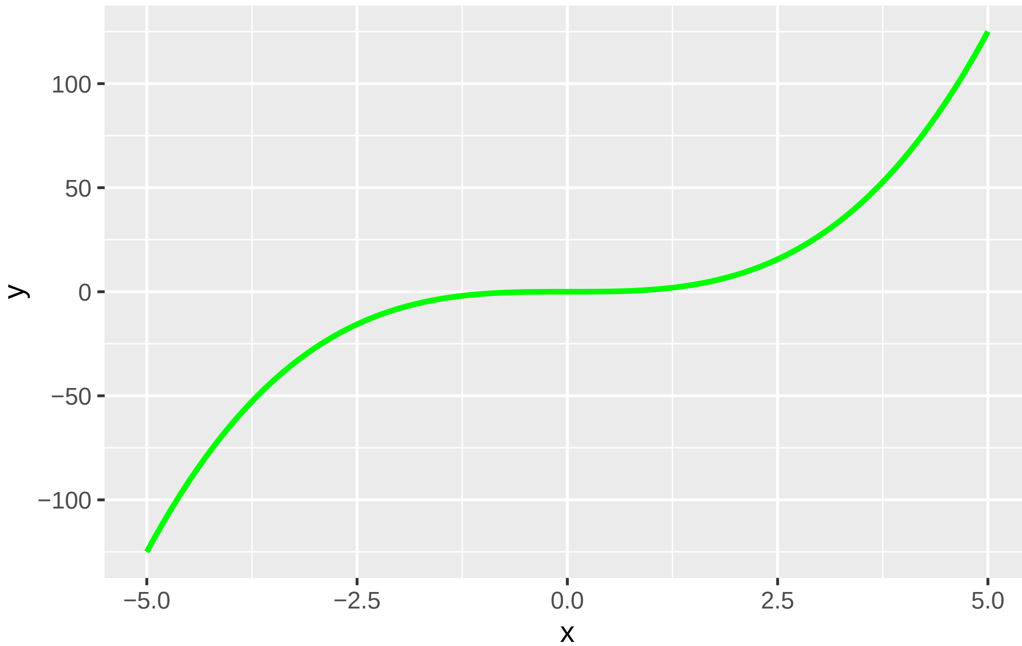
$$y = ax^2 + bx + c$$

- a, b, and c don't have well-defined meanings here.
- If a is negative, the function opens downward; if a is positive,it opens upward.
- Note that x^2 always returns positive values.



5.2.3 Cubic

- These lines (generally) have two curves (inflection points).
- $y = ax^3 + bx^2 + cx + d$
- a, b, c, and d don't have well-defined meanings here.



5.2.4 Polynomial

$$y = ax^n + bx^{n-1} + ... + c$$

- These functions have (maximum) $n - 1$ changes in direction (turning points). - They also have (maximum) n x-intercepts. - They can be made arbitrarily precise.

5.2.5 Exponential

$$y = ab^x$$

or

$$f(x) = ab^x$$

- Here our independent variable, or input (x), is the exponent.

5.2.6 Trigonometric functions

- These functions include sine, cosine, and tangent.
- They are interesting (to some), but not usually useful for social science.

5.3 Logarithms and exponents

5.3.1 Logarithms

- Logarithms are basically the opposite (inverse) of exponents.
- They ask how many times you must raise the base to get x.
- $log_a(b) = x$ is asking "a raised to what power x gives b?
- $\log_3(81) = 4$ because $3^4 = 81$
- Logarithms can be undefined.
- The base cannot be 0, 1, or negative.

5.3.2 Relationships

If,

$$log_a x = b$$

then,

$$a^{log_a x} = a^b$$

and

$$x = a^b$$

5.3.3 Basic rules

$$\frac{\log_x n}{\log_x m} = \log_m n$$

$$\log_x(ab) = \log_x a + \log_x b$$

$$\log_x \left(\frac{a}{b}\right) = \log_x a - \log_x b$$

$$\log_x a^b = b \log_x a$$

$$\log_x 1 = 0$$

$$log_x x = 1$$

$$m^{\log_m(a)} = a$$

5.3.4 Natural logarithms

- We most often use natural logarithms.
- This means $\log_e(x)$, often written $\ln(x)$.
- $e \approx 2.7183$.
- ln(x) and its exponent opposite, e^x, have nice properties when we hit calculus.

5.3.5 Definition of e

- Imagine you invest \$1 in a bank and receive 100% interest for one year, and the bank pays you back once a year:

$$(1+1)^1 = 2$$

- When it pays you twice a year with compound interest:

$$(1+1/2)^2 = 2.25$$

- If it pays you three times a year:

$$(1+1/3)^3 = 2.37...$$

- What will happen when the bank pays you once a month? Once a day?

$$(1+\frac{1}{n})^n$$

- However, there is limit to what you can get

$$\lim_{n\to\infty}(1+\frac{1}{n})^n = 2.7183... = e$$

- For any interest rate k and number of times the bank pays you t:

$$\lim_{n\to\infty}(1+\frac{k}{n})^{nt} = e^{kt}$$

- e is important for defining *exponential growth*. Since $ln(e^x) = x$, the natural logarithm helps us turn exponential functions into linear ones.

Practice

Solve the problems below, simplifying as much as you can.

$$log_{10}(1000)$$

$$log_2(\frac{8}{32})$$

$$10^{log_{10}(300)}$$

$$ln(1)$$

$$ln(e^2)$$

$$ln(5e)$$

5.4 Functions of functions

5.4.1 Basics

- Functions can take other functions as arguments.
- This means that outside function takes output of inside function as its input.
- This is typically written as $f(g(x))$.
- Say we have the exterior function f(x)=x^2 and the interior function g(x)=$x - 3$.
- Then if we want f(g(x)), we would subtract 3 from any input, and then square the result.
- We write this $(x - 3)^2$, NOT $x^2 - 3$.

5.4.2 PMF, PDF, and CDF

- PMF - probability mass function
  - This gives the probability that a discrete random variable is exactly equal to some value.
- PDF - probability density function
  - This gives the probability that a continuous random variable falls within a particular range of values.
- CDF - cumulative distribution function
  - This gives the probability that a random variable X takes a value less than or equal to x.

6 Calculus

6.1 Theory

- Calculus is about dealing with infinitesimal values.
- We are going to focus on two big ideas:
  - Derivatives
  - Integrals

6.1.1 Derviative

- "Derivative" is just a fancy term for slope.
- Slope is the rate of change $\frac{\delta y}{\delta x}$ or $\frac{dy}{dx}$.
- Specifically, the derivative is the *instantaneous* rate of change.
- We need slope for our statistics, which are all about fitting lines.
- We also need slope for taking maxima and minima.
- The equation for a line is $y = mx + b$. What is its slope?

6.1.2 Calculating derivatives

- Slope is rise over run, which is $\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$
- To see why, consider the slope of a line connecting two points:

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

- We can define $x_2 = x_1 + \Delta x$ (or equivalently $\Delta x = x_2 - x_1$)

$$m = \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

- As we've seen, for a curve, we need to be infinitely close for our line's defining points, yielding

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

- This gives us this instantaneous slope (rate of change) of a function at every point on its domain. The above equation is the definition of the derivative.

6.1.3 Notation

- $\frac{d}{dx} f(x)$ is read "The derivative of f of x with respect to x." - You can also say "The instantaneous rate of change in f of x with respect to x."
- If $y = f(x)$, $\frac{dy}{dx}$ is "The derivative of y with respect to x". - Warning: Do not try to cancel out the d's, no matter how tempting it is.
- There is the advantage of always specifying the variable with respect to which we're differentiating (it's the one in the denominator).

Lagrange's prime notation: - $f'(x)$ (read: "f prime x") is the derivative of $f(x)$. - This is useful when it is clear which variable were are referring to (e.g., when there's only one).

- What is $\dfrac{d(x^2)}{dx}$? - x^2 - $2x^{2-1}$ -$2x$
- What is $\frac{d(4x^3)}{dx}$?
  - $4x^3$
  - $4 * 3x^{3-1}$
  - $12x^2$

Practice

Take the derivative of each of these.

$$x^3$$

$$3x^2$$

$$60x^{11}$$

$$x$$

$$\frac{4}{x^2}$$

$$9\sqrt{x}$$

$$6x^{5/2}$$

$$11,596,232$$

Evaluate the derivatives at $x = 2$ and $x = -1$

$$x^3$$

$$3x^2$$

$$60x^{11}$$

$$x$$

$$\frac{4}{x^2}$$

$$9\sqrt{x}$$

$$6x^{5/2}$$

$$11,596,232$$

Practice

Take the derivative of each of these.

$$x^3$$

$$3x^2$$

$$60x^{11}$$

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$$6x^{5/2}$$

$$11,596,232$$

Evaluate the derivatives at $x = 2$ and $x = -1$

$$x^3$$

$$3x^2$$

$$60x^{11}$$

$$x$$

$$\frac{4}{x^2}$$

$$9\sqrt{x}$$

$$6x^{5/2}$$

$$11,596,232$$

6.1.4 Special functions

A few functions have particular rules:

- $\frac{d(ln(x))}{dx} = \frac{1}{x}$
- $\dfrac{d(log_b(x))}{dx} = \dfrac{1}{x * ln(b)}$
- $\frac{d(e^x)}{dx} = e^x$
- $\frac{d(a^x)}{dx} = a^x ln(a)$
- $\frac{dy}{dx}c = 0$
- $\frac{d(x^x)}{dx} = x^x(1 + ln(x))$

6.1.5 Derivatives with addition and substraction

- Easiest rule to remember:

$$\frac{d(f(x) \pm g(x))}{dx} = f'(x) \pm g'(x)$$

Practice

Take the derivative of each of these

$$x^2 + x + 5$$

$$x^4 - 4x^3 + 5x^2 + 8x - 6$$

$$3x^5 - 6x^2$$

$$5x^2 + 8\sqrt{x} - \frac{1}{x}$$

$$ln(x) + 5e^x - 4x^3$$

6.2 Advanced rules

6.2.1 Product rule

- A little more complicated:

$$\frac{d(f(x) \times g(x))}{dx} = f'(x)g(x) + g'(x)f(x)$$

- Example: $2x \times 3x$

Practice

Take the derivative of each of these:

$$x^3 * x$$

$$e^x * x^2$$

$$ln(x) * x^{-3}$$

Remember, $\frac{d(f(x)*g(x))}{dx} = f'(x)g(x) + g'(x)f(x)$.

6.2.2 Quotient rule

$$\frac{d\frac{f(x)}{g(x)}}{dx} = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

If you're having trouble with this, just apply the product rule to:

$$\frac{d[f(x) * g^{-1}(x)]}{dx}$$

Remember, $\frac{d\frac{f(x)}{g(x)}}{dx} = \frac{f'(x)g(x)-g'(x)f(x)}{[g(x)]^2}$.

6.2.3 Chain rule

$$\frac{d[f(g(x))]}{dx} = f'(g(x)) * g'(x)$$

Let's take the derivative of a function of a function:

$$\frac{d[ln(x^2)]}{dx}$$

$$f(x) = ln(x), g(x) = x^2$$

$$f'(x) = \frac{1}{x}, g'(x) = 2x$$

$$\frac{1}{x^2} * 2x = \frac{2}{x}$$

Practice

Take the derivative of each of these:

$$(3x^4 - 8)^2$$

$$e^{x^2}$$

Remember, $\frac{d(f(g(x))}{dx} = f'(g(x)) * g'(x)$.

6.2.4 Second derivative

- Same process as taking single derivative, except input for second derivative is output from first.
- Second derivative tells us whether the slope of a function is increasing, decreasing, or staying the same at any point x on the function's domain.
- Example: driving a car.
  - $f(x)$ = distance traveled at time x
  - $f'(x)$ = speed at time x
  - $f''(x)$ = acceleration at time x

Graph $f(x) = x^2$, $f'(x)$, and $f''(x)$.

$$\frac{d^2(x^4)}{dx^2} = f''(x^4)$$

- First, we take the first derivative:

$$f'(x^4) = 4x^3$$

- Then we use that output to take the second derivative:

$$f''(x^4) = f'(4x^3) = 12x^2$$

Practice

Take the second derivative of the following functions:

$$x^5$$

$$6x^2$$

$$4ln(x)$$

$$3x$$

$$4x^{3/2}$$

6.3 Differentiable and Continuous Functions

- Informally: A function is continuous at a point if its graph has no holes or breaks at that point
- Formally: A function is continuous at a point a if:

$$\lim_{x \to a} f(x) = f(a)$$

Continuity requires 3 conditions to hold:

- $f(a)$ is defined (a is in the domain of f)
- $\lim_{x \to a} f(x)$ exists
- $\lim_{x \to a} f(x) = f(a)$ (the value of f equals the limit of f at a)

Differentiable:

- If $f'(x)$ exists, f is differentiable at x.
- If f is differentiable at every point of an open interval I, f is differentiable on I.
- Graph must have a (non-vertical) tangent line at each point, be relatively smooth, and not contain any breaks, bends, or cusps.

- If a function is differentiable at a point, it is also continuous at that point.
- If a function is continuous at a point, it is *not* necessarily differentiable at that point.

6.3.1 When is f not differentiable?

When does $f'(x)$ not exist?

- When the function is discontinuous at that point.
  - Jump or break in the graph.
- There are different slopes approaching the point from the left and from the right.
  - Corner point
- When the graph of the function has a vertical tangent line at that point.
  - Cusp
  - Vertical inflection point

6.4 Extrema and optimization

Optimization lets us find the minimum or maximum value a function takes.

- Formal theory
  - Utility maximization, continuous choices
- Ordinary Least Squares (OLS)
  - Focuses on *minimizing* the squared errors between observed data and values predicted by a regression
- Maximum Likelihood Estimation (MLE)
  - Focuses on *maximizing* a likelihood function, given observed values

6.4.1 Extrema

Informally, a maximum is just the highest value a function takes, and a minimum is the lowest value.

- Easy to identify extrema (maxima or minima) intuitively by looking at a graph of the function.
  - Maxima are high points ("peaks")
  - Minima are low points ("valleys")
- Extrema can be *local* or *global*.

6.4.2 Identifying extrema

The derivative of a function gives the rate of change. - When the derivative is zero (or fails to exist), the function has *usually* reached a (local) maximum or minimum.

- Why?

At a maximum, the function must be increasing before the point and decreasing after it.

At a minimum, the function must be decreasing before the point and increasing after it.

So we'll start by identifying points where this is the case ("critical points" or "stationary points").

A technical note:

A point where $f'(x) = 0$ or $f'(x)$ does not exist is called a *critical point* (or *stationary point*). Local extrema occur at critical points, but not all critical points are extrema. For instance, sometimes the graph is changing between concave and convex ("inflection points"). Sometimes the function is not differentiable at that point for other reasons, as discussed earlier.

So we can find the local maxima and/or minima of a function by taking the derivative, setting it equal to zero, and solving for x (or whatever).

$$f'(x) = 0$$

This gives us the first-order condition (FOC).

6.4.3 Minimum or maximum?

BUT we don't know if we've found a maximum or minimum, or even if we've found an extremum or just an inflection point.

6.4.4 Second derivatives

The second derivative gives us the rate of change of the rate of change of the original function. So it tells us whether the slope is getting larger or smaller.

$$f(x) = x^2$$
$$f'(x) = 2x$$
$$f''(x) = 2$$

Second Derivative Test - Start by identifying $f''(x)$

- Substitute in the stationary points (x^*) identified from the FOC
- $f''(x^*) > 0$ we have a local minimum
- $f''(x^*) < 0$ we have a local maximum
- $f''(x^*) = 0$ we (may) have an inflection point - need to calculate higher-order derivatives (don't worry about this now)

Collectively these give use the Second-Order Condition (SOC).

6.4.5 Local vs. Global Extrema

To find the minimum/maximum on some interval, compare the local min/max to the value of the function at the interval's endpoints.

- To find the global minimum/maximum, check the function's limits as it approaches $+\infty$ and $-\infty$.
- Extreme value theorem: if a real-valued function f is continuous on the closed interval [a,b], then f must attain a (global) maximum and a (global) minimum.

6.5 Partial derivatives

- Can take derivative with respect to different variables
- Notation: For a function $fy = (x, z) = xz$, we might want to know how the function changes with x:

$$\frac{\partial}{\partial_x} f(x, y) = \frac{\partial_y}{\partial_x} = \partial_x f$$

- Treat all other variables as constants and take derivative with respect to the variable of interest (here x).

How do we take a partial derivative?

Treat all other variables as *constants* and take *derivative* with respect to the variable of interest.

From our earlier example:

$$y = f(x, z) = xz$$
$$\frac{\partial_y}{\partial_x} = ?$$

$$y = f(x, z) = xz$$
$$\frac{\partial_y}{\partial_x} = z$$

Why? Because the partial derivative of xz with respect to x treats z as a constant.

What is $\frac{\partial_y}{\partial_z}$?

6.5.1 Application

- $\frac{\partial(x^2y+xy^2-x)}{\partial x}$
- We apply the addition rule to take the derivative of each term with respect to x.
- $\frac{\partial(x^2y)}{\partial x}+\frac{\partial(xy^2)}{\partial x}+\frac{\partial(-x)}{\partial x}$
- $2xy + y^2 - 1$

- $\frac{\partial(x^2y+xy^2-x)}{\partial y}$
- We apply the addition rule to take the derivative of each term with respect to y
- $\frac{\partial(x^2y)}{\partial y}+\frac{\partial(xy^2)}{\partial y}+\frac{\partial(-x)}{\partial y}$
- $x^2 + 2xy$

Practice

Take the partial derivative with respect to x and to y of the following functions. What would the notation for each look like?

$$3xy - x$$

$$ln(xy)$$

$$x^3 + y^3 + x^4y^4$$

$$e^{xy}$$

6.6 Integrals

6.6.1 Area under a curve

Often we want to find the area under a curve. - Net effect of change - Cumulative density functions (CDFs) - Expected values and utilities

Sometimes this is easy. What's the area under the curve between $x = -1$ and $x = 1$ for this function?

$$f(x) = \begin{cases} \frac{1}{3} & \text{for } x \in [0, 3] \\ 0 & \text{otherwise} \end{cases}$$

Hint: We can draw this and look at the graph. Remember $Area = \ell * w$

Sometimes (usually) finding the area under a curve is harder. But this is basically the question behind integration.

6.6.2 Integrals as summation

You're familiar with summation notation.

$$\sum_{i=1}^{n} i$$

But this only works when we have discrete values to add. When we need to add continuously, we have to use something else. Specifically, integrals.

6.6.3 Definite integrals

Let's say we have a function

$$y = x^2$$

And we want to find the area under the curve from $x = 0$ to $x = 1$. To find the area we're interested in here, we can use the definite integral.

Generally speaking, the notation looks like this:

$$\int_{x=a}^{b} f(x), dx$$

Here a is the lower limit of integration, b is the upper limit of integration, our function $f(x)$ is our integrand, and x is our variable of integration.

For our question, we're looking for

$$\int_{x=0}^{1} f(x)dx$$

Which will give us a real number denoting the area under the curve of our function ($y = x^2$) between $x = 0$ and $x = 1$.

If f is continuous on $[a, b]$ or bounded on $[a, b]$ with a finite number of discontinuities, then f is integrable on $[a, b]$.

6.6.4 Indefinite integrals

The *indefinite* integral, or *anti-derivative*, $F(x)$ is the inverse of the function $f'(x)$.

$$F(x) = \int f(x)\ dx$$

This means if you take the derivative of $F(x)$, you wind up back at $f(x)$.

$$F' = f \text{ or } \frac{dF(x)}{dx} = f(x)$$

This process is called anti-differentiation, or indefinite integration.

While the definite integral gives us a real number (the total area under a curve), the indefinite integral gives us a function.

We need the concept of indefinite integrals to help us solve definite integrals.

6.6.5 Solving definite integrals

$$\int_a^b f(x)\ dx = F(b) - F(a) = F(x)\Big|_a^b$$

Constant of integraton

A quick note:

C in the following slides is the called the "constant of integration." We need to add it when we define *all* antiderivatives (integrals) of a function because the anti-derivative "undoes" the derivative.

Remember that the derivative of any constant is zero. So if we find an integral $F(x)$ whose derivative is $f(x)$, adding (or subtracting) any constant will give us another integral $F(x)+C$ whose derivative is *also* $f(x)$.

6.6.6 Rules of integration

$$\int_a^a f(x)\ dx = 0$$

$$\int_a^b f(x)\ dx = -\int_b^a f(x)dx$$

$$\int a\ dx = ax + C \text{ where } a \text{ is a constant}$$

$$\int af(x)dx = a\int f(x)\ dx \text{ where } a \text{ is a constant}$$

6.6.7 More rules

$$\int (f(x) + g(x))\ dx = \int f(x)dx + \int g(x)dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad \forall n \neq -1$$

$$\int x^{-1} dx = \ln|x| + C$$

6.6.8 Solving the problem

Remember our function $y = x^2$ and our goal of finding the area under the curve from $x = 0$ to $x = 1$.

- Find the indefinite integral, $F(x)$
  - $\int x^2 \, dx$
  - $\frac{x^3}{3} + C$

- Evaluate at our lowest and highest points, $F(0)$ and $F(1)$.
  - $F(0) = 0$
  - $F(1) = \frac{1}{3}$
  - Technically $0 + C$ and $\frac{1}{3} + C$, but the C's will fall out in the next step
- Calculate $F(1) - F(0)$
  - $\frac{1}{3} - 0 = \frac{1}{3}$

Practice — indefinite integrals

$$\int x^2 \, dx$$

$$\int 3x^2 \, dx$$

$$\int x \, dx$$

$$\int 3x^2 + 2x - 7 \, dx$$

$$\int \frac{2}{x} \, dx$$

Practice — definite integrals

$$\int_1^7 x^2 \ dx$$

$$\int_1^{10} 3x^2 \ dx$$

$$\int_7^7 x \ dx$$

$$\int_1^5 3x^2 + 2x - 7 \ dx$$

$$\int_1^e \frac{2}{x} \ dx$$

6.6.9 Integration by parts

- What if we want to integrate the product of two functions? How to evaluate $\int [f(x)g(x)]dx$?
- There's a formula for that: integration by parts.
- We can derive the formula from the product rule for derivatives, which you already know.

$$\frac{d(f(x)g(x))}{d(x)} = f^{'}(x)g(x) + g^{'}(x)f(x)$$

$$\int \frac{d(f(x)g(x))}{d(x)}dx = \int [f^{'}(x)g(x) + g^{'}(x)f(x)]dx$$

$$f(x)g(x) = \int f^{'}(x)g(x)dx + \int g^{'}(x)f(x)dx$$

- Note that we can't just plug in our two original functions into this formula. We have some work to do first.
- Board example:

$$\int x\sqrt{x}dx$$

7 Probability

7.1 What is probability?

- Frequency with which an event occurs.
  - Typically:

$$Pr(A) = P(A) = \pi(A) = \frac{\text{Number of ways an event can occur}}{\text{Total number of possible outcomes}}$$

- Probability predicts real-world events using theoretical quantities.
  - Formally, it assigns a likelihood of occurrence to each event in sample space
  - We use the probability space triplet (Ω, S, P), which are the sample space, event space, and probability mapping, respectively.
- We can consider probability as a function that maps $\Omega \to \mathbb{R}$.
- We can conceive it in terms of relative frequency or subjective belief.

7.2 Kolmogorov's axioms

- $Pr(S_i) \in \mathbb{R},\ \ 1 \geq Pr(S_i) \geq 0 \qquad \forall S_i \in S$
  - Where S is the event space, S_i are events.
  - Probabilities must be non-negative.
- $Pr(\Omega) = 1$
  - Where Ω is the sample space.
  - Something has to happen.
  - Probabilities sum/integrate to 1.
- $Pr\left(\bigcup_{i=1}^{\infty} S_i\right) = \sum_{i=1}^{\infty} Pr(S_i) \iff Pr(S_i \cap S_j) = 0 \ \ \forall i \neq j$
  - The probability of disjoint (mutually exclusive) sets is equal to the sum of their individual probabilities.

7.3 Some definitions

- **Random variable**: a variable whose value is determined by the outcome of a random process.
  - Sometimes also called a *stochastic variable*.
  - May be discrete or continuous.
- **Distribution** (of a random variable): the set of values the variable might take.
  - Probability mass function / probability density function defines the probability with which each value occurs.
  - Always sums / integrates to 1.
- **Realization** (of a random variable): a particular value taken by the variable.

- **Population**: the entire set of objects (people, cases, etc.) in which we are interested.
  - Often denoted N.
- **Sample**: a subset of the population we can observe, from which we try to make generalizations about the population.
  - Often denoted n.
- **Frequency distribution**: a count of how often a variable takes each of its possible values.
  - The number of members of a sample that take each value of a variable.
- **Independent random variables**: two variables are statistically independent if the value of one does not affect the value of the other.
  - Formally, $Pr(A \cap B) = Pr(A)Pr(B)$

7.4 Discrete probability

- A sample space in which there are a (finite or infinite) countable number of outcomes
- Each realization of random process has a discrete probability of occurring.
  - $f(X = x_i) = P(X = x_i)$ is the probability the variable takes the value x_i.

7.4.1 Probability Mass Function (PMF)

Probability of each occurrence encoded in probability mass function (PMF)

- $0 \leq f(x_i) \leq 1$
  - Probability of any value occurring must be between 0 and 1.
- $\sum_x f(x_i) = 1$
  - Probabilities of all values must sum to 1.

7.4.2 Discrete distribution

- What's the probability that we'll roll a 3 on one die roll:

$$Pr(y = 3) = \frac{1}{6}$$

- If one roll of the die is an "experiment."
- We can think of a 3 as a "success."
- $Y \sim Bernoulli\left(\frac{1}{6}\right)$
- Fair coins are $\sim Bernoulli(.5)$, for example.
- More generally, $Bernoulli(\pi)$.
  - π represents the probability of success.

- Drawing a specific card from a deck:

$$Pr(y = \text{ace of spades}) = \frac{1}{52}$$

- Drawing any card with a specific value from a deck:

$$Pr(y = ace) = \frac{4}{52}$$

- Getting a specific value on two dice rolls:

$$Pr(y = 8) = \frac{5}{36}$$

- We can express the probability mass function in tabular format or in a graph.

7.5 Continuous probability

- What happens when our outcome is continuous?
- There are an infinite number of outcomes.
- This makes the denominator of our fraction difficult to work with.
- The probability of the whole space must equal 1.
- Even if all events are equally likely, $\frac{1}{\infty} = 0$

7.5.1 Basics

- The domain may not span -∞ to ∞.
  - Even space between 0 and 1 is infinite.
- The domain is defined as the area under the probability density function.
- Two common examples are the uniform and bell curves.

7.5.2 Probability Density Function (PDF)

- Similar to PMF from before, but for continuous variables.
- Gives the probability a value falls within a particular interval
  - $P[a \leq X \leq b] = \int_a^b f(x)\,dx$
  - Total area under the curve is 1.
  - $P(a < X < b)$ is the area under the curve between a and b (where $b > a$).

7.6 Cumulative Density Function (CDF)

7.6.1 Discrete

- Cumulatve density function is probability X will take a value of x or lower.
- PDF is written $f(x)$, and CDF is written $F'(x)$.

$$F_X(x) = Pr(X \leq x)$$

- For discrete CDFs, that means summing up over all values.
- What is the probability of rolling a 6 or lower with two dice? $F(6) = ?$

7.6.2 Continuous

- We can't sum probabilities for continuous distributions (remember the 0 problem).
- Solution: integration

$$F_Y(y) = \int_{-\infty}^{y} f(y)dy$$

- Examples of uniform distribution.

7.7 Statistics

7.7.1 Introduction

- While probability allows us to make predictions about events using distributions, statistics uses events to make estimates about distributions and variables.
- It is the process of learning from data.
- A statistic is a summary of data, capturing some theoretically-relevant quantity.
- Broad categories of numerical and categorical.

7.7.2 Univariate statistics

- These measure a single variable.
- Readily expressed in graphical form.
- Common examples:
  - Central tendency (mean, median, and mode)
  - Variance

7.7.3 Examples of univariate statistics

- The mean ($\bar{x}$) is calculated by summing the data, then dividing by the number of observations:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- The median is found by ordering the observations from highest to lowest and finding the one in the middle.

- The mode is the most common number.

$$x = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 6 & 7 & 8 & 9 \end{bmatrix}$$

- What are the mean, median, and mode of x?

7.7.4 Measures of central tendency

- Mean balances values on either side.
- Median balances observations on either side.
- Mode finds the most typical observation.
- Which is the best? Like most of what you'll learn in statistics, it depends.

7.7.5 Deviations from central tendency

- Consider two data sets:

$$x = \begin{bmatrix}1 & 1.5 & 2 & 2.5 & 5.5 & 8.5 & 9 & 9.5 & 10\end{bmatrix}$$

$$y = \begin{bmatrix}4.5 & 4.8 & 5 & 5.3 & 5.5 & 5.7 & 6 & 6.2 & 6.5\end{bmatrix}$$

- What is the mean of each?
- What is the median of each?
- Are they similar distributions?

7.7.6 Variance

- We use variance to measure the spread of a single variable.
- Formally defined as the squared deviation from the mean (μ).
- For discrete random variables, it is written $Var(x) = \sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$
- For continuous random variables, it is written $Var(x) = \sigma^2 = \int (x - \mu)^2 f(x)\ dx$

7.7.7 Standard deviation

- Sometimes variance doesn't make sense, either mathematically or conceptually.
  - Not always clear how to interpret "squared deviation from the mean."
- Instead, will frequently see standard deviation, which is square root of variance.
- It is written σ.

7.8 Bivariate statistics

7.8.1 Covariance

- While measures of central tendency and variance/standard deviation provide useful summaries of a single variable, they don't provide insights into relationships between variables.
- For that, we need bivariate statistics.
- Most common and straightforward is covariance.

- Colloquially, can think of covariance as measure of linear deviation from mean.
- When values from one variable are above their mean, are values from the other above or below their mean?
- Put another way, if I told you the value of x was high, would you expect values of y to be high or low?
- Formally, it is written as:

$$cov(X, Y) = E(X - E(X))(Y - E(Y)) = E(XY) - E(X)E(Y)$$

- It is important to note that the magnitude is meaningless; only the direction is interpretable.

7.8.2 Correlation

- Correlation is a normalized measure of covariance
- It is calculated as:

$$\rho_{X,Y} = \frac{cov(X, Y)}{\sigma_X \sigma_Y}$$

- It varies between -1 and 1.
- What is correlation of two independent variables?

7.9 Regression

7.9.1 Ordinary least squares

- Ordinary least squares regression (OLS) is probably the most widely-used model in political science.
- It is all about drawing a line through data.

- This allows us to evaluate the relationship (the association) between x on y.
- The dependent variable, y, must be continuous, generally speaking.
- The main question is which line to draw.

Line and equation ($\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$) on board

7.9.2 Residuals

- In basically any set of data, no line can pass through every point (observation).
- We will always have make some error in predicting values.
- The error between the line and some point is referred to as the residual.
- If we refer to our predicted value as $\hat{y}$, then we can calculate the residual for each observation with the following equation:

$$e_i = y_i - \hat{y}_i$$

7.9.3 Finding the right line

- OLS determines the "best" line by minimizing the sum of squared residuals.
- Plug in all the values for the slope amd intercept and calculate the sum of squared residuals for these infinity combinations.
- That is a lot of work.
- The best solution turns out to be calculus.
- We want to minimize the sum of squared residuals with respect to our β's.

8 Simulations

9 Text analysis

10 Wrap up

10.1 Methods at UT

10.1.1 Required methods courses

- Scope and Methods of Political Science
  - Statistics I (Statistics/linear regression)
- Statistics II (Linear regression and more)
- Statistics III (Maximum likelihood estimation)
  - Only required if your major field is methods

10.1.2 Other methods courses

- **Statistics/econometrics:**
  - Bayesian Statistics
  - Causal Inference
  - Math Methods for Political Analysis
  - Time Series and Panel Data
  - Panel and Multilevel Analysis

10.1.3 More courses

- **Formal Theory**
  - Intro to Formal Political Analysis
  - Formal Political Analysis II
  - Formal Theories of International Relations
- **Everything else**
  - Conceptualization and Measurement
  - Experimental Methods in Political Science
  - Qualitative Methods

    - Network Analysis
    - Seminar in Field Experiments

10.1.4 Other departments at UT

You can also take courses through the Economics, Mathematics, or Statistics (Statistics and Data Science) departments.

- M.S. in Statistics

Software and Topic Short Courses - R, Python, Stata, etc.

- More info here.

10.1.5 Other resources

Summer programs at UT:

- Short courses in statistics
    - Department sometimes offers scholarships to cover part of the cost.

Summer programs outside UT:

- ICPSR (Inter-university Consortium for Political and Social Research)
    - Ann Arbor, Michigan
- EITM (Empirical Implications of Theoretical Models)
    - Houston and other locations (Michigan, Duke, Berkeley, Emory)
- IQMR (Institute for Qualitative and Multi-Method Research)
    - Syracuse, NY

References