Introduction

There is considerable public interest in election forecasting, a task that is complicated by the numerous mechanisms that may lead to a divergence between voting intentions as measured by pre-election polls and election results. Polling error refers to the accuracy of pre-election polls in terms of predicting election results but this is bound to be affected lead, referring to the time between the administration of the poll and the election [@jennings2020election]. Simply put, we should expect that the fewer days that remain until the election is held, the more accurate the polls and election forecasts will become, although even in the final days of the campaign considerable error, far exceeding sampling error alone, can be expected [@jennings2016timeline; @jennings2018election]. @jennings2020election suggest that a lead of two or three months before an election occurs is often sufficient for creating accurate election forecasts.

However, the accuracy and lead of election forecasts may vary from country to country, as specific polling error mechanisms may be at work in some contexts but not in others. Survey methodologists rely on the Total Survey Error framework to decompose error sources which cause survey statistics to diverge from population parameters. Of these error sources, sampling error is almost always accounted for, but non-sampling errors such as coverage, nonresponse, measurement, and estimation errors often are not, although each can cause both systematic and variable errors [@biemer2010total; @groves2010total]. @prosser2018twilight find that measurement error ("shy" voters) plays a limited role in polling error. Instead, the fact that voting intention is dynamic and problems with gathering representative samples and weighting are the primary causes of polling error. This suggest that nonresponse and estimation errors play important roles in polling error.

Nonresponse bias is the prime suspect in analyses of polling error and can be caused by various mechanisms. On the one hand, If the same characteristic predicts both response propensities and target variables (i.e. voting intentions), corresponding to a missing at random mechanism, bias can be adjusted in estimation by weighting on the characteristic, provided that it has been observed. On the other hand, if a target variable is the true cause of variation in response propensities, corresponding to a missing not at random mechanism, analyses of the target variable will be biased by nonresponse bias [@little2020statistical; @groves2006nonresponse]. The latter scenario is easy to envisage in polls, where voters of specific parties may be more engaged in politics even when weighting adjustments for other characteristics are applied.

To address the issue of polling error, election forecasters often adjust for "house effects", where some pollsters systematically over- or underestimate support for specific parties, relative to other pollsters [@jennings2018election]. Here, we add to the literature on election forecasting by adjusting for industry effects, referring to cases where polling error has been observed repeatedly and the direction in which it occurs as it relates to specific parties is consistent. This scenario would reflect an industry wide inability to recruit representative samples or to identify suitable auxiliary information to adjust for nonresponse bias as it relates to voting intention [@kalton2003weighting; @groves2006nonresponse].

We put our assumptions to an empirical test by publishing an election forecast prior to the Icelandic parliamentary election of 2024. Response rates in Icelandic surveys have declined significantly over time but remain high in the international contexts **(Hvaða Einarsson er hvað í bibtex?)** (Einarsson et al., n.d.; Einarsson and Helgason, n.d.). However, most Icelandic pre-election polls are conducted using probability-based online panels. Online panels have mixed records when it comes to polling error but probability-based ones have been found to outperform non-probability ones [@kennedy2016evaluation; @callegaro2014online]. However, the repeated selection of respondents carries significant risks in terms of panel conditioning [@struminskaya2021panel] and attrition [@frankel2014looking]. Therefore, online panels often rely heavily on model-based (rather than design-based) inference [@little2004model], which will only be successful in the case that a suitable source of auxiliary information is identified.

Icelandic politics were characterised by remarkable stability prior to the 2008 financial crisis but have seen high electoral volatility since [@onnudottir2021electoral; @helgason2022electoral] . In fact, only one of the previous four governments has been able to serve a full four-year term, causing early elections in each instance, for a total of five elections in the span of 11 years. Each election has been associated with polling errors which can be attributed to the high degree of electoral volatility. Despite this, there has been little variation in voting intention estimates between polling houses and biases at the party level have been similar election-to-election, with right-wing parties being underestimated **(Hvaða bibtex er þetta?)** (Einarsson and Helgason, n.d.). This suggests that pollsters are relying on similar methods but failing to address the issue of polling error, i.e. industry effects may be at play.

As our model comes with significant assumptions regarding the direction of polling error, it is plausible that it may not improve the prediction of election outcomes. For example, if pollsters have identified new recruitment methods or identified weighting characteristics correlated both with the propensity to respond and target variables [@little2005weighting], our adjustments will introduce bias rather than adjust it. If, however, the same industry wide problems remain, our model will provide a more accurate picture of the voting intentions of the Icelandic electorate than an unadjusted polling average would.

Methods

We develop a Bayesian hierarchical model to forecast the 2024 Icelandic Parliamentary Elections by combining:

- A dynamic linear model for polling data that captures:
  - Temporal evolution of party support
  - House-specific and industry-wide polling biases
  - Increased volatility during the government coalition change leading up to the election
  - Cross-party correlations in support changes

- A fundamentals model incorporating:
  - Historical election results
  - Incumbency status and duration
  - Economic indicators (inflation and GDP growth)

Model Structure

Statistical Framework

The model combines polling data $y_{n,p}$ and fundamentals data through a hierarchical structure:

$$
\begin{aligned}
y_n &\sim \text{Dirichlet-Multinomial}(\pi_n, \phi_{h[n]}) \\
\pi_n &= \text{softmax}(\eta_n) \\
\eta_{n,p} &= \beta_{p,t[n]} + \gamma_{p,h[n]}
\end{aligned}
$$

where $\phi_{h[n]}$ is house-specific overdispersion, $t[n]$ indexes the time of poll n, and $h[n]$ indexes the polling house.

The fundamentals component predicts election results through:

$$
\begin{aligned}
y_d^{(f)} &\sim \text{Dirichlet-Multinomial}(\pi_d^{(f)}, \phi_f) \\
\pi_d^{(f)} &= \text{softmax}(\mu_d) \\
\mu_{d,p} &= \alpha_p + \beta_{\text{lag}} x_{p,d} + \beta_{\text{years}} \log(I_{p,d}) + \beta_{\text{vnv}} v_{p,d} + \beta_{\text{growth}} g_{p,d}
\end{aligned}
$$

The two components are linked through the election day prediction μ_{pred}, which is calculated from the fundamentals model using current economic conditions and previous election results. This prediction serves as a prior for the election day support:

$$
\beta_T \sim \mathcal{N}(\mu_{\text{pred}}, \tau_f \cdot \sigma)
$$

The relative weight between polling and fundamentals is controlled by τ_f, which is computed so that the fundamentals component has a desired percentage weight in the prediction at a specified time before the election (see Appendix for details).

Polling Component

The polling model tracks latent party support $\beta_{p,t}$ through time using a multivariate random walk with party-specific volatility

$$
\begin{aligned}
\log(\sigma_p) &\sim \text{Normal}(\mu_{\log\sigma}, \tau_{\log\sigma}) \\
\mu_{\log\sigma} &\sim \text{Normal}(-4, 1) \\
\tau_{\log\sigma} &\sim \text{Exponential}(2),
\end{aligned}
$$

and correlation structure Ω between parties:

$$
\beta_t = \begin{cases} \text{Normal}\left(\beta_{t+1}, (1+\tau_s)\sqrt{\Delta_t}\Sigma\right) & \text{after government split} \\ \text{Normal}\left(\beta_{t+1}, \sqrt{\Delta_t}\Sigma\right) & \text{otherwise} \end{cases}
$$

where:

- $\Sigma = \text{diag}(\sigma_1, \dots, \sigma_P)\Omega\text{diag}(\sigma_1, \dots, \sigma_P)$ is the covariance matrix
- σ_p captures party-specific volatility scales
- Ω is the correlation matrix between the party-specific innovations
- τ_s allows for increased volatility after the government split
- Δ_t is the number of days between polls

The observed polling data follows a Dirichlet-multinomial distribution with house-specific overdispersion parameters ϕ_h:

$$
\begin{aligned}
y_n &\sim \text{Dirichlet-Multinomial}(\pi_n, \phi_{h[n]}) \\
\pi_n &= \text{softmax}(\beta_{t[n]} + \gamma_{h[n]})
\end{aligned}
$$

where γ_h captures house-specific biases in reported support levels, with the election day house effect fixed to zero ($\gamma_1 = 0$). The other house effects are modeled hierarchically with explicit sum-to-zero constraints:

$$
\begin{aligned}
\gamma_{p,h} &= \begin{cases} 0 & \text{if } h = 1 \\ \mu_{\gamma,p} + \sigma_{\gamma,p}\tilde{\gamma}_{p,h} & \text{if } h > 1 \end{cases} \\
\gamma_{P,h} &= -\sum_{p=1}^{P-1} \gamma_{p,h} \\
\tilde{\gamma}_{p,h} &\sim \text{Normal}(0, 1)
\end{aligned}
$$

where $h = 1$ represents the election (reference category), and the final party's house effects are determined by the sum-to-zero constraint.

Fundamentals Component

The fundamentals model predicts party vote shares using economic and political variables:

$$\mu_{d,p} = \alpha_p + \beta_{\text{lag}}(x_{p,d} - \alpha_p) + \beta_{\text{inc}} \log(I_{p,d}) + \beta_{\text{vnv}} v_{p,d} + \beta_{\text{growth}} g_{p,d}$$

where:

- α_p are party-specific intercepts that sum to zero ($\sum_p \alpha_p = 0$)
- x are previous election vote shares
- I are years in government for incumbent parties
- V is inflation on an annual basis six months before the election
- G is economic growth on an annual basis six months before the election
- β_{lag} captures persistence in party support
- β_{inc} measures the effect of time spent in government on incumbent parties
- β_{vnv} captures the impact of inflation on incumbent parties
- β_{growth} captures the impact of growth on incumbent parties

Model Integration

The fundamentals prediction serves as a prior for the election day vote shares β_T:

$$\beta_T \sim \mathcal{N}(\mu_{\text{pred}}, \tau_f \cdot \sigma)$$

where μ_{pred} is the fundamentals prediction and τ_f controls how much weight is given to the fundamentals versus polling data at some point before the elections (see Appendix for details on choosing τ_f).

Constituency Component

The model extends the national-level polling model to incorporate constituency-level effects through a hierarchical structure that properly accounts for constituency weights and ensures identifiability through carefully constructed sum-to-zero constraints.

Constituency Effects Structure

For each constituency poll, the linear predictor combines national trends, house effects, and constituency-specific deviations:

$$
\begin{aligned}
y_k &\sim \text{Dirichlet-Multinomial}(\pi_k, \phi_{h[k]}) \\
\pi_k &= \text{softmax}(\eta_k) \\
\eta_{k,p} &= \beta_{p,t} + \gamma_{p,h} + \delta_{p,k}
\end{aligned}
$$

where $\delta_{p,k}$ represents the constituency-specific deviation for party p in constituency k.

The constituency effects follow a hierarchical structure:

$$
\begin{aligned}
\delta_{p,k} &= \sigma_{\delta,p}\tilde{\delta}_{p,k} \text{ for } k < K \\
\delta_{p,K} &= -\sum_{k=1}^{K-1} w_k \delta_{p,k} / w_K \\
\tilde{\delta}_{p,k} &\sim \text{Normal}(0, 1)
\end{aligned}
$$

where:

- $\sigma_{\delta,p}$ is the party-specific scale of constituency variation
- w_k represents constituency weights based on historical voting patterns
- The final constituency's effect is determined by the weighted sum-to-zero constraint

This structure ensures that:

1. Constituency effects are properly weighted by constituency size
2. The sum-to-zero constraint holds when accounting for constituency weights
3. Each party can have different patterns of geographic variation ($\sigma_{\delta,p}$)
4. Information is shared across constituencies through the hierarchical prior

Integration with National Model

The constituency model is integrated with the national model through:

1. Shared national-level support trajectories ($\beta_{p,t}$)
2. Common house effects ($\gamma_{p,h}$)
3. Joint estimation of dispersion parameters (ϕ_h)

This allows constituency-level polls to inform national-level estimates while accounting for systematic geographic differences in party support.

Prediction

The model generates both national and constituency-level predictions:

$$
\begin{aligned}
y^{\text{rep}}_{\text{national}} &\sim \text{Dirichlet-Multinomial}(\text{softmax}(\beta_T), \phi_1) \\
y^{\text{rep}}_{k} &\sim \text{Dirichlet-Multinomial}(\text{softmax}(\beta_T + \delta_k), \phi_1)
\end{aligned}
$$

These predictions incorporate uncertainty in: - National support trends - Constituency-specific deviations - House effects - Overdispersion at both national and constituency levels

Prediction

The model generates both national-level predictions and constituency-specific predictions:

- National: $y^{\text{rep}}_{\text{national}}$ represents overall party support
- Constituency: $y^{\text{rep}}_{\text{constituency}}$ gives constituency-level predictions

These predictions incorporate the combined uncertainty in:

- National trends
- Constituency-specific effects
- House effects
- Overdispersion in both national and constituency-level polls

Results

Parameter Estimates

Polling Component

House Effects

- Plot of house-specific biases (γ_h) with uncertainty intervals
- Table of significant house effects by party
- Analysis of industry-wide bias patterns

Volatility Parameters

- Estimates of party-specific volatility (σ_p)
- Impact of government split ($\tau_{\text{stjornarslit}}$)
- Systematic shifts after split ($\beta_{\text{stjornarslit}}$)

Cross-Party Correlations

- Heatmap of correlation matrix (Ω)
- Discussion of strongest party-to-party relationships
- Clustering analysis of correlated party groups

Fundamentals Component

Economic Effects

- Coefficient plot for β_{growth}
- Analysis of GDP growth impact on incumbent parties

Political Effects

- Coefficient estimates for:
  - Previous vote share effect (β_{lag})
  - Incumbency duration (β_{inc})
  - Government participation (β_{vnv})
- Discussion of which factors most strongly predict party support

Model Validation

Historical Performance

- Out-of-sample predictions for previous elections
- Comparison with simple polling averages
- Analysis of when/where model performs best/worst

Uncertainty Calibration

- Coverage of prediction intervals
- Comparison of predicted vs. actual volatility
- Assessment of overdispersion estimates (ϕ_h)

2024 Election Prediction

Point Predictions

- Table of predicted vote shares with 95% intervals
- Comparison to latest polls
- Discussion of largest predicted changes

Coalition Scenarios

- Probability of different majority combinations
- Most likely government formations
- Key parties for coalition formation

Prediction Evolution

- Plot showing how predictions changed over time
- Impact of recent polling data
- Effect of economic/political developments

Discussion

Appendix

Model Specification Details

Notation

Input Data

Polling Data

- P: Number of political parties *(including the Other category)*
- T: Number of time points (dates), $1, \ldots, T$, where T is the date of the next election
- H: Number of polling houses
- N: Number of observations (polls)
- $y_{n,p}$: Count of responses for party p in poll n
- Δ_t: The time difference between polls at $t-1$ and t in days

Fundamentals Data

- D_f: Number of past elections
- P_f: Number of parties in historical data
- $y_{p,d}^{(f)}$: Vote share for party p in election d
- $x_{p,d}$: Previous vote share for party p in election d
- $I_{p,d}$: Years party p has been incumbent at election d
- $v_{p,d}$: Inflation rate (%) for party p at election d (if incumbent)
- $g_{p,d}$: GDP growth rate (%) for party p at election d (if incumbent)

Data Preprocessing

The economic variables are transformed to better capture their effects:

- Incumbent years are log-transformed: $\log(I_{p,d})$
- Excess inflation is calculated as deviation from 2% target and log-transformed: $\log(1 + v_{p,d} - 0.02)$
- GDP growth is log-transformed: $\log(1 + g_{p,d}/100)$

Both economic variables (inflation and growth) are only included for incumbent parties by multiplying them with the incumbency indicator.

Government Split Effects

The model accounts for potential changes in voting patterns during government coalition changes through two mechanisms: 1. Increased volatility scaled by $\tau_{\text{stjornarslit}}$ 2. Potential systematic shifts in party support through $\beta_{\text{stjornarslit}}$

Cross-Party Correlations

Rather than treating each party's support as independent, the model captures correlations in support changes through a Cholesky decomposition of the correlation matrix Ω. This allows the model to account for situations where increased support for one party typically corresponds to decreased support for specific other parties.

Choosing τ_f

To choose how we want to calculate the standard deviation for prior on β_T, we can frame our model as a Gaussian-Gaussian conjugate problem where:

- The prior (fundamentals prediction) is: $\beta_T \sim \mathcal{N}(\mu_{\text{pred}}, \tau_f \cdot \sigma)$
- The likelihood (polling prediction from time t) is: $\beta_T \sim \mathcal{N}(\beta_t, V(t) \cdot \sigma)$

where $V(t)$ represents the accumulated variance from time t to election time T:

- For $t \leq 47$ (after government split): $V(t) = t \cdot (1 + \tau_{\text{stjornarslit}})^2$
- For t $>$ 47 (before government split): $V(t) = (t - 47) + 47 \cdot (1 + \tau_{\text{stjornarslit}})^2$

Using standard Gaussian-Gaussian conjugate formulas:

$$\begin{aligned}\text{Posterior precision} &= \frac{1}{(\tau_f \cdot \sigma)^2} + \frac{1}{V(t) \cdot \sigma^2} \\ &= \left(\frac{1}{\tau_f^2} + \frac{1}{V(t)}\right) \cdot \frac{1}{\sigma^2}\end{aligned}$$

For a desired fundamentals weight w at time t:

$$w = \frac{1/\tau_f^2}{1/\tau_f^2 + 1/V(t)}$$

Solving for τ_f:

$$\tau_f = \sqrt{V(t) \cdot (1 - w)/w}$$

where $V(t)$ depends on $\tau_{\text{stjornarslit}}$ as defined above. As an example, if we choose $w = \frac{1}{3}$ and $t = 180$, we get:

$$\begin{aligned}V(180) &= (180 - 47) + 47 \cdot (1 + \tau_{\text{stjornarslit}})^2 \\ &= 133 + 47 \cdot (1 + \tau_{\text{stjornarslit}})^2\end{aligned}$$

Then:

$$
\begin{aligned}
\tau_f &= \sqrt{V(180) \cdot (1 - \frac{1}{3}) / \frac{1}{3}} \\
&= \sqrt{(133 + 47 \cdot (1 + \tau_{\text{stjornarslit}})^2) \cdot 2} \\
&= \sqrt{266 + 94 \cdot (1 + \tau_{\text{stjornarslit}})^2}
\end{aligned}
$$

This shows how τ_f adapts to the estimated value of $\tau_{\text{stjornarslit}}$ in our model, increasing when there is more uncertainty during the government split period.