My formalization project

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Chapter 1

Basic Definitions and Lemmas

1.1 Vectors

Lemma 1.1.1. *For any vector* $v : \mathrm{Fin}\, 2 \to X$*, we have* $v = ![v(0), v(1)]$.

Proof. $\square$

Lemma 1.1.2. *For any function* $f : X \to Y$ *and elements* $x_1, x_2 \in X$*, we have* $f \circ ![x_1, x_2] = ![f(x_1), f(x_2)]$.

Proof. $\square$

1.2 Theories

Lemma 1.2.1. *Models of the theory of dense linear orders realize the reflexivity sentence.*

Proof. $\square$

Lemma 1.2.2. *Models of the theory of dense linear orders realize the transitivity sentence.*

Proof. $\square$

Lemma 1.2.3. *Models of the theory of dense linear orders realize the antisymmetry sentence.*

Proof. $\square$

Lemma 1.2.4. *Models of the theory of dense linear orders realize the totality sentence.*

Proof. $\square$

Lemma 1.2.5. *Models of the theory of dense linear orders realize the densely ordered sentence.*

Proof. $\square$

Lemma 1.2.6. *Models of the theory of dense linear orders realize no minimum sentence.*

Proof. $\square$

Lemma 1.2.7. *Models of the theory of dense linear orders realize no maximum sentence.*

Proof. $\square$

Chapter 2

Model Theory and The Rest of Mathlib

2.1 Instances

Definition 2.1.1. *Any structure of the language of orders is an ordered set.*

Lemma 2.1.2. *The structure induced from $\leq$ which is induced from another structure is equal to the original structure.*

Proof. □

Definition 2.1.3. *Models of the theory of dense linear orders are preorders.*

Definition 2.1.4. *Models of the theory of dense linear orders are linear orders.*

2.2 Homomorphisms

Lemma 2.2.1. *A function which is an order embedding is also an embedding in the model theoretic sense.*

Proof. □

Lemma 2.2.2. *A function which is an order isomorphism is also an isomorphism in the model theoretic sense.*

Proof. □

Chapter 3

Main Result

Theorem 3.0.1. *The theory of dense linear orders without endpoints is $\aleph_0$-categorical.*

Proof. $\square$