Appendix Four

Properties of Moist Air

A4.1 Methods of Specifying Moisture Content

(a) *The vapor concentration*, or absolute humidity ρ_v is the mass of vapor per unit volume of moist air.

(b) *The specific humidity* q is the mass of vapor per unit mass of moist air:

$$q = \rho_v/\rho. \tag{A4.1}$$

(c) *The mixing ratio* r is the ratio of the mass of vapor to the mass of dry air:

$$r = q/(1 - q). \tag{A4.2}$$

(d) *The vapor pressure* e' of water vapor in moist air is defined as the function of p and q given by (3.1.12)

$$e'/p = q/(\epsilon + (1 - \epsilon)q) = r/(\epsilon + r) = r/(0.62197 + r). \tag{A4.3}$$

If air were an ideal gas mixture, e' would be exactly equal to the partial pressure e of water vapor. In practice it will be slightly different.

(e) *The relative humidity* U is the ratio of r to the saturation mixing ratio r_w of moist air relative to a plane water surface:

$$U = r/r_w = q(1 - q_w)/(q_w(1 - q)) \tag{A4.4}$$

It is usually expressed as a percentage.

A4.2 Saturation Vapor Pressure

(a) The *saturation vapor pressure* $e_w(T)$ of pure water vapor over a plane water surface is discussed in Section 3.4. Values are given in Table 94 of the Smithsonian Meteorological Tables. For temperatures t between $\pm 40°C$, the value in millibars is given correct to 1 part in 500 by

$$\log_{10} e_w(t) = (0.7859 + 0.03477t)/(1 + 0.00412t). \qquad (A4.5)$$

(b) In air, the partial pressure e'_w of water vapor at saturation is not exactly e_w but is given by

$$e'_w = f_w e_w(T), \qquad (A4.6)$$

where f_w lies between 1 and 1.006 for observed atmospheric conditions, values being given in Table 89 of the Smithsonian Meteorological Tables. The value of f_w is given correct to 2 parts in 10^4 by

$$f_w = 1 + 10^{-6}p(4.5 + 0.0006t^2), \qquad (A4.7)$$

where p is the pressure in millibars. Values of r_w and q_w follow from (A4.3) with $e' = e'_w$, and are shown as functions of p and T in Fig. 3.6a.

(c) The saturation vapor pressure $e_i(T)$ of pure water vapor over ice is given in Table 96 of the Smithsonian Meteorological Tables and satisfies (correct to 3 parts in 1000 for $0 > t > -40$)

$$\log_{10} e_i(t) = \log_{10} e_w(t) + 0.00422t. \qquad (A4.8)$$

(d) The saturation partial pressure e'_i in moist air is f_i times e_i. Values of f_i are given in Table 90 of the Smithsonian Meteorological Tables, and f_i is given correct to 1 part in 10^4 by (A4.7).

(e) The saturation vapor pressure over a salt solution is less than over fresh water. For seawater, the reduction is about 2% (Kraus, 1972, p. 46).

A4.3 Further Quantities Related to Moisture Content

(a) *Dew point* T_d is the temperature to which air must be cooled at constant pressure and constant mixing ratio to reach saturation with respect to a plane water surface. The *frost point* is the equivalent with respect to a plane ice surface.

(b) *Lifting condensation level* is the level at which a parcel of moist air lifted adiabatically becomes saturated.

(c) *Wet-bulb temperature* T_w is the temperature to which a parcel of air is cooled by evaporating water into it gradually, adiabatically, and at constant pressure until it is saturated. It is measured directly by a thermometer whose bulb is covered by a moist cloth over which air is drawn.

(d) From the above definitions, it follows that for a parcel with pressure p the θ contour through (p, T), the θ_e^* or θ_w^* contour through (p, T_w), and the r_w or q_w contour through (p, T_d) all intersect at the lifting condensation level (Normand's rule).

A4.4 Latent Heats

The latent heat of vaporization L_v is given by (3.4.6), i.e.,

$$L_v(t) = 2.5008 \times 10^6 - 2.3 \times 10^3 t \quad \mathrm{J\ kg^{-1}}, \tag{A4.9}$$

where t is the temperature (in degrees Celsius). The latent heat of sublimation L_s is given by

$$L_s(t) = 2.839 \times 10^6 - 3.6(t + 35)^2 \quad \mathrm{J\ kg^{-1}}. \tag{A4.10}$$

A4.5 Lapse Rates

The dry adiabatic lapse rate is given to within 0.3% by

$$\Gamma = g/c_p. \tag{A4.11}$$

The saturation adiabatic lapse rate (for liquid water) is given approximately by

$$\begin{aligned}\Gamma_s = {} & 6.4 - 0.12t + 2.5 \times 10^{-5}t^3 \\ & + [-2.4 + 10^{-3}(t - 5)^2](1 - p/p_r) \quad \mathrm{K\ km^{-1}},\end{aligned} \tag{A4.12}$$

where $p_r = 1000$ mb and t is the temperature (in °C). The maximum error in the range $|t| < 40$ and $500 < p < 1000$ is 0.2 K km^{-1}. Accurate values of Γ_s are given in Table 79 of the Smithsonian Meterological Tables, whereas Table 80 gives values for the ice stage.