

## Properties of Moist Air

### A4.1 Methods of Specifying Moisture Content

- (a) *The vapor concentration*, or absolute humidity  $\rho_v$  is the mass of vapor per unit volume of moist air.
- (b) *The specific humidity*  $q$  is the mass of vapor per unit mass of moist air:

$$q = \rho_v/\rho. \quad (\text{A4.1})$$

- (c) *The mixing ratio*  $r$  is the ratio of the mass of vapor to the mass of dry air:

$$r = q/(1 - q). \quad (\text{A4.2})$$

- (d) *The vapor pressure*  $e'$  of water vapor in moist air is defined as the function of  $p$  and  $q$  given by (3.1.12)

$$e'/p = q/(\epsilon + (1 - \epsilon)q) = r/(\epsilon + r) = r/(0.62197 + r). \quad (\text{A4.3})$$

If air were an ideal gas mixture,  $e'$  would be exactly equal to the partial pressure  $e$  of water vapor. In practice it will be slightly different.

- (e) *The relative humidity*  $U$  is the ratio of  $r$  to the saturation mixing ratio  $r_w$  of moist air relative to a plane water surface:

$$U = r/r_w = q(1 - q_w)/(q_w(1 - q)) \quad (\text{A4.4})$$

It is usually expressed as a percentage.

#### A4.2 Saturated Vapor Pressure

##### A4.4 Latent Heats

The latent heat of vaporization  $L_v(t)$  is given by

$$L_v(t) = \frac{1}{T_s} \left( \frac{1}{T_s} - \frac{1}{T} \right)^{-1}$$

where  $T_s$  is the saturation adiabatic lapse rate.

The dry adiabatic lapse rate

$$T_s = 6.4 \times 10^{-6} t + 25$$

where  $p_s = 1000 \text{ mb}$  and  $t$  is stage.

79 of the Smithsonian Meteorological Tables and satishes (correct to 3 parts in 1000 for  $0 < t < -40^\circ\text{C}$ )

(A4.5)  $\log_{10} e_w(t) = 0.7859 + 0.03477t/(1 + 0.00412t)$

Table 96 of the Smithsonian Meteorological Tables and satishes (correct to 3 parts in 1000 for  $0 < t < -40^\circ\text{C}$ )

(A4.6)  $e_w = f_w e_w(T)$

where  $f_w$  lies between 1 and 1.006 for observed atmospheric conditions, values being given in Table 89 of the Smithsonian Meteorological Tables. The value of  $f_w$  is given

where  $e_w$  is the pressure in millibars. Values of  $r_w$  and  $q_w$  follow from (A4.3) with  $e_w = e_w$ , and are shown as functions of  $p$  and  $T$  in Fig. 3.6a.

(A4.7)  $f_w = 1 + 10^{-6} p(4.5 + 0.0006t^2)$

Table 96 of the Smithsonian Meteorological Tables and satishes (correct to 3 parts in 1000 for  $0 < t < -40^\circ\text{C}$ )

(A4.8)  $\log_{10} e_i(t) = \log_{10} e_w(t) + 0.00422t$

(d) The saturation partial pressure  $e_i$  in moist air is  $f_i e_w$ . Values of  $f_i$  are given in Table 90 of the Smithsonian Meteorological Tables, and  $f_i$  is given correct to 1 part in  $10^4$  by (A4.7).

(e) The saturation vapor pressure over a salt solution is less than over fresh water. For seawater, the reduction is about 2% (Kraus, 1972, p. 46).

(A4.9)  $Dew point$   $T_d$  is the temperature to which air must be cooled at constant pressure and constant mixing ratio to reach saturation with respect to a plane water surface. The dew point is the equivalent temperature to which a parcel of moist air lifted adiabatically becomes saturated.

(b) Lifting condensation level is the level at which a parcel of moist air lifted adiabatically becomes saturated.

(c) Wet-bulb temperature  $T_w$  is the temperature to which a parcel of air is cooled by evaporating water into it gradually, adiabatically, and at constant pressure until it is saturated. It is measured directly by a thermometer whose bulb is covered by a moist cloth over which air is drawn.

(d) From the above definitions, it follows that for a parcel with pressure  $p$  contour through  $(p, T_d)$  all intersect at the lifting condensation level (Normand's rule).

(e) Dew point  $T_d$  or  $q_w$  contour through  $(p, T_w)$ , and the  $r_w$  or  $q_w$  contour through  $(p, T_d)$  all intersect at the lifting condensation level (Normand's rule).

#### A4.3 Further Quantities Related to Moisture Content

The latent heat of vaporization  $L_v(t)$  is given by

(A4.10)  $L_v(t) = A_4 \rho_{air} c_p (T_s - T)$

where  $T_s$  is the temperature of the air at saturation.

(A4.11)  $A_4 = \frac{1}{T_s} \left( \frac{1}{T_s} - \frac{1}{T} \right)^{-1}$

(A4.12)  $\rho_{air} = \rho_{air}(T)$

(A4.13)  $c_p = c_p(T)$

(A4.14)  $T_s = T_w + \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.15)  $L_v(t) = A_4 \rho_{air} c_p (T_w - T)$

(A4.16)  $A_4 = \frac{1}{T_w} \left( \frac{1}{T_w} - \frac{1}{T} \right)^{-1}$

(A4.17)  $\rho_{air} = \rho_{air}(T)$

(A4.18)  $c_p = c_p(T)$

(A4.19)  $T_w = T_s - \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.20)  $L_v(t) = A_4 \rho_{air} c_p (T_s - T_w)$

(A4.21)  $A_4 = \frac{1}{T_s} \left( \frac{1}{T_s} - \frac{1}{T_w} \right)^{-1}$

(A4.22)  $\rho_{air} = \rho_{air}(T_w)$

(A4.23)  $c_p = c_p(T_w)$

(A4.24)  $T_s = T_w + \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.25)  $L_v(t) = A_4 \rho_{air} c_p (T_w - T)$

(A4.26)  $A_4 = \frac{1}{T_w} \left( \frac{1}{T_w} - \frac{1}{T} \right)^{-1}$

(A4.27)  $\rho_{air} = \rho_{air}(T)$

(A4.28)  $c_p = c_p(T)$

(A4.29)  $T_w = T_s - \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.30)  $L_v(t) = A_4 \rho_{air} c_p (T_s - T_w)$

(A4.31)  $A_4 = \frac{1}{T_s} \left( \frac{1}{T_s} - \frac{1}{T_w} \right)^{-1}$

(A4.32)  $\rho_{air} = \rho_{air}(T_w)$

(A4.33)  $c_p = c_p(T_w)$

(A4.34)  $T_s = T_w + \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.35)  $L_v(t) = A_4 \rho_{air} c_p (T_w - T)$

(A4.36)  $A_4 = \frac{1}{T_w} \left( \frac{1}{T_w} - \frac{1}{T} \right)^{-1}$

(A4.37)  $\rho_{air} = \rho_{air}(T)$

(A4.38)  $c_p = c_p(T)$

(A4.39)  $T_w = T_s - \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.40)  $L_v(t) = A_4 \rho_{air} c_p (T_s - T_w)$

(A4.41)  $A_4 = \frac{1}{T_s} \left( \frac{1}{T_s} - \frac{1}{T_w} \right)^{-1}$

(A4.42)  $\rho_{air} = \rho_{air}(T_w)$

(A4.43)  $c_p = c_p(T_w)$

(A4.44)  $T_s = T_w + \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.45)  $L_v(t) = A_4 \rho_{air} c_p (T_w - T)$

(A4.46)  $A_4 = \frac{1}{T_w} \left( \frac{1}{T_w} - \frac{1}{T} \right)^{-1}$

(A4.47)  $\rho_{air} = \rho_{air}(T)$

(A4.48)  $c_p = c_p(T)$

(A4.49)  $T_w = T_s - \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.50)  $L_v(t) = A_4 \rho_{air} c_p (T_s - T_w)$

(A4.51)  $A_4 = \frac{1}{T_s} \left( \frac{1}{T_s} - \frac{1}{T_w} \right)^{-1}$

(A4.52)  $\rho_{air} = \rho_{air}(T_w)$

(A4.53)  $c_p = c_p(T_w)$

(A4.54)  $T_s = T_w + \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.55)  $L_v(t) = A_4 \rho_{air} c_p (T_w - T)$

(A4.56)  $A_4 = \frac{1}{T_w} \left( \frac{1}{T_w} - \frac{1}{T} \right)^{-1}$

(A4.57)  $\rho_{air} = \rho_{air}(T)$

(A4.58)  $c_p = c_p(T)$

(A4.59)  $T_w = T_s - \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.60)  $L_v(t) = A_4 \rho_{air} c_p (T_s - T_w)$

(A4.61)  $A_4 = \frac{1}{T_s} \left( \frac{1}{T_s} - \frac{1}{T_w} \right)^{-1}$

(A4.62)  $\rho_{air} = \rho_{air}(T_w)$

(A4.63)  $c_p = c_p(T_w)$

(A4.64)  $T_s = T_w + \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.65)  $L_v(t) = A_4 \rho_{air} c_p (T_w - T)$

(A4.66)  $A_4 = \frac{1}{T_w} \left( \frac{1}{T_w} - \frac{1}{T} \right)^{-1}$

(A4.67)  $\rho_{air} = \rho_{air}(T)$

(A4.68)  $c_p = c_p(T)$

(A4.69)  $T_w = T_s - \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.70)  $L_v(t) = A_4 \rho_{air} c_p (T_s - T_w)$

(A4.71)  $A_4 = \frac{1}{T_s} \left( \frac{1}{T_s} - \frac{1}{T_w} \right)^{-1}$

(A4.72)  $\rho_{air} = \rho_{air}(T_w)$

(A4.73)  $c_p = c_p(T_w)$

(A4.74)  $T_s = T_w + \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.75)  $L_v(t) = A_4 \rho_{air} c_p (T_w - T)$

(A4.76)  $A_4 = \frac{1}{T_w} \left( \frac{1}{T_w} - \frac{1}{T} \right)^{-1}$

(A4.77)  $\rho_{air} = \rho_{air}(T)$

(A4.78)  $c_p = c_p(T)$

(A4.79)  $T_w = T_s - \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.80)  $L_v(t) = A_4 \rho_{air} c_p (T_s - T_w)$

(A4.81)  $A_4 = \frac{1}{T_s} \left( \frac{1}{T_s} - \frac{1}{T_w} \right)^{-1}$

(A4.82)  $\rho_{air} = \rho_{air}(T_w)$

(A4.83)  $c_p = c_p(T_w)$

(A4.84)  $T_s = T_w + \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.85)  $L_v(t) = A_4 \rho_{air} c_p (T_w - T)$

(A4.86)  $A_4 = \frac{1}{T_w} \left( \frac{1}{T_w} - \frac{1}{T} \right)^{-1}$

(A4.87)  $\rho_{air} = \rho_{air}(T)$

(A4.88)  $c_p = c_p(T)$

(A4.89)  $T_w = T_s - \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.90)  $L_v(t) = A_4 \rho_{air} c_p (T_s - T_w)$

(A4.91)  $A_4 = \frac{1}{T_s} \left( \frac{1}{T_s} - \frac{1}{T_w} \right)^{-1}$

(A4.92)  $\rho_{air} = \rho_{air}(T_w)$

(A4.93)  $c_p = c_p(T_w)$

(A4.94)  $T_s = T_w + \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.95)  $L_v(t) = A_4 \rho_{air} c_p (T_w - T)$

(A4.96)  $A_4 = \frac{1}{T_w} \left( \frac{1}{T_w} - \frac{1}{T} \right)^{-1}$

(A4.97)  $\rho_{air} = \rho_{air}(T)$

(A4.98)  $c_p = c_p(T)$

(A4.99)  $T_w = T_s - \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.100)  $L_v(t) = A_4 \rho_{air} c_p (T_s - T_w)$

(A4.101)  $A_4 = \frac{1}{T_s} \left( \frac{1}{T_s} - \frac{1}{T_w} \right)^{-1}$

(A4.102)  $\rho_{air} = \rho_{air}(T_w)$

(A4.103)  $c_p = c_p(T_w)$

(A4.104)  $T_s = T_w + \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.105)  $L_v(t) = A_4 \rho_{air} c_p (T_w - T)$

(A4.106)  $A_4 = \frac{1}{T_w} \left( \frac{1}{T_w} - \frac{1}{T} \right)^{-1}$

(A4.107)  $\rho_{air} = \rho_{air}(T)$

(A4.108)  $c_p = c_p(T)$

(A4.109)  $T_w = T_s - \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.110)  $L_v(t) = A_4 \rho_{air} c_p (T_s - T_w)$

(A4.111)  $A_4 = \frac{1}{T_s} \left( \frac{1}{T_s} - \frac{1}{T_w} \right)^{-1}$

(A4.112)  $\rho_{air} = \rho_{air}(T_w)$

(A4.113)  $c_p = c_p(T_w)$

(A4.114)  $T_s = T_w + \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.115)  $L_v(t) = A_4 \rho_{air} c_p (T_w - T)$

(A4.116)  $A_4 = \frac{1}{T_w} \left( \frac{1}{T_w} - \frac{1}{T} \right)^{-1}$

(A4.117)  $\rho_{air} = \rho_{air}(T)$

(A4.118)  $c_p = c_p(T)$

(A4.119)  $T_w = T_s - \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.120)  $L_v(t) = A_4 \rho_{air} c_p (T_s - T_w)$

(A4.121)  $A_4 = \frac{1}{T_s} \left( \frac{1}{T_s} - \frac{1}{T_w} \right)^{-1}$

(A4.122)  $\rho_{air} = \rho_{air}(T_w)$

(A4.123)  $c_p = c_p(T_w)$

(A4.124)  $T_s = T_w + \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.125)  $L_v(t) = A_4 \rho_{air} c_p (T_w - T)$

(A4.126)  $A_4 = \frac{1}{T_w} \left( \frac{1}{T_w} - \frac{1}{T} \right)^{-1}$

(A4.127)  $\rho_{air} = \rho_{air}(T)$

(A4.128)  $c_p = c_p(T)$

(A4.129)  $T_w = T_s - \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.130)  $L_v(t) = A_4 \rho_{air} c_p (T_s - T_w)$

(A4.131)  $A_4 = \frac{1}{T_s} \left( \frac{1}{T_s} - \frac{1}{T_w} \right)^{-1}$

(A4.132)  $\rho_{air} = \rho_{air}(T_w)$

(A4.133)  $c_p = c_p(T_w)$

(A4.134)  $T_s = T_w + \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.135)  $L_v(t) = A_4 \rho_{air} c_p (T_w - T)$

(A4.136)  $A_4 = \frac{1}{T_w} \left( \frac{1}{T_w} - \frac{1}{T} \right)^{-1}$

(A4.137)  $\rho_{air} = \rho_{air}(T)$

(A4.138)  $c_p = c_p(T)$

(A4.139)  $T_w = T_s - \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.140)  $L_v(t) = A_4 \rho_{air} c_p (T_s - T_w)$

(A4.141)  $A_4 = \frac{1}{T_s} \left( \frac{1}{T_s} - \frac{1}{T_w} \right)^{-1}$

(A4.142)  $\rho_{air} = \rho_{air}(T_w)$

(A4.143)  $c_p = c_p(T_w)$

(A4.144)  $T_s = T_w + \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.145)  $L_v(t) = A_4 \rho_{air} c_p (T_w - T)$

(A4.146)  $A_4 = \frac{1}{T_w} \left( \frac{1}{T_w} - \frac{1}{T} \right)^{-1}$

(A4.147)  $\rho_{air} = \rho_{air}(T)$

(A4.148)  $c_p = c_p(T)$

(A4.149)  $T_w = T_s - \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.150)  $L_v(t) = A_4 \rho_{air} c_p (T_s - T_w)$

(A4.151)  $A_4 = \frac{1}{T_s} \left( \frac{1}{T_s} - \frac{1}{T_w} \right)^{-1}$

(A4.152)  $\rho_{air} = \rho_{air}(T_w)$

(A4.153)  $c_p = c_p(T_w)$

(A4.154)  $T_s = T_w + \frac{L_v(t)}{A_4 \rho_{air} c_p}$

(A4.155)  $L_v(t) = A_4 \rho_{air} c_p (T_w - T)$

(A4.156)  $A_4 = \frac{1}{T_w} \left( \frac{1}{T_w} - \frac{1}{T} \right)^{-1}$

(A4.157)  $\rho_{air} = \rho_{air}(T)$

(A4.158)  $c_p = c_p(T)$

**A4.4 Latent Heats**

The latent heat of vaporization  $L_v$  is given by (3.4.6), i.e.,

$$L_v(t) = 2.5008 \times 10^6 - 2.3 \times 10^3 t \text{ J kg}^{-1}, \quad (\text{A4.9})$$

where  $t$  is the temperature (in degrees Celsius). The latent heat of sublimation  $L_s$  is given by

$$L_s(t) = 2.839 \times 10^6 - 3.6(t + 35)^2 \text{ J kg}^{-1}. \quad (\text{A4.10})$$

**A4.5 Lapse Rates**

The dry adiabatic lapse rate is given to within 0.3% by

$$\Gamma = g/c_p. \quad (\text{A4.11})$$

The saturation adiabatic lapse rate (for liquid water) is given approximately by

$$\begin{aligned} \Gamma_s &= 6.4 - 0.12t + 2.5 \times 10^{-5}t^3 \\ &\quad + [-2.4 + 10^{-3}(t - 5)^2](1 - p/p_r) \text{ K km}^{-1}, \end{aligned} \quad (\text{A4.12})$$

where  $p_r = 1000 \text{ mb}$  and  $t$  is the temperature (in  $^{\circ}\text{C}$ ). The maximum error in the range  $|t| < 40$  and  $500 < p < 1000$  is  $0.2 \text{ K km}^{-1}$ . Accurate values of  $\Gamma_s$  are given in Table 79 of the Smithsonian Meteorological Tables, whereas Table 80 gives values for the ice stage.

$$0412t. \quad (\text{A4.5})$$

Saturation is not exactly  $e_w$

$$(A4.6)$$

conditions, values being  
The value of  $f_w$  is given

$$(A4.7)$$

follow from (A4.3) with

Vapor over ice is given in  
isfies (correct to 3 parts

$$(A4.8)$$

times  $e_i$ . Values of  $f_i$  are  
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2, p. 46).

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respect to a plane water  
lane ice surface.

parcel of moist air lifted

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t constant pressure until  
ose bulb is covered by a

a parcel with pressure  $p$   
( $p, T_w$ ), and the  $r_w$  or  $q_w$   
n level (Normand's rule).