

Q1

$$\begin{aligned}\frac{1}{a+bi} &= \frac{1}{a+bi} * \frac{a-bi}{a-bi} \\ &= \frac{a-bi}{a^2+abi-abi+b^2} \\ &= \frac{a-bi}{a^2+b^2} \\ c+di &= \frac{a}{a^2+b^2} - \frac{bi}{a^2+b^2}\end{aligned}$$

therefore $c = \frac{a}{a^2+b^2}$ and $d = \frac{-b}{a^2+b^2}$

Q2

$$\left(\frac{-1+\sqrt{3}i}{2}\right)^3 = \frac{1}{8} * (-1+\sqrt{3}i)^3$$

using the identity $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ where $a = -1$ and $b = \sqrt{3}i$ we have

$$\begin{aligned}\frac{1}{8} * (-1+\sqrt{3}i)^3 &= (-1)^3 + 3 * (-1)^2 * (\sqrt{3}i) + 3 * (-1) * (\sqrt{3}i)^2 + (\sqrt{3}i)^3 \\ &= \frac{1}{8} * (-1 + 3 * \sqrt{3}i + 9 - 3 * \sqrt{3}i) \\ &= \frac{8}{8} \\ &= 1\end{aligned}$$

Q3

$-\sqrt{i}$ and $+\sqrt{i}$ are two distinct square roots of i

Q4

To show:

$$\alpha + \beta = \beta + \alpha \quad \forall \alpha, \beta \in \mathbb{C}$$

let $\alpha = a + bi$ and $\beta = c + di$

by definition of addition of complex numbers

$$(a+bi) + (c+di) = (a+c) + (b+d)i \tag{1}$$

by definition of complex numbers we know $a, b, c, d \in \mathbb{R}$

so addition is commutative in \mathbb{R}

as a consequence $a + b = b + a$ and $c + d = d + c$
there (1) becomes

$$\begin{aligned}(a + bi) + (c + di) &= (a + c) + (b + d)i \\ &= (c + a) + (d + b)i \\ &= (c + di) + (a + bi) \text{ (from bidirectionality of how complex number addition is defined)} \\ &= \beta + \alpha\end{aligned}$$

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Q5

To prove:

$$\alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma \quad \forall \alpha, \beta, \gamma \in \mathbb{C}$$

Assuming $\alpha = a + bi$, $\beta = c + di$, and $\gamma = e + fi$
evaluating $\alpha + (\beta + \gamma)$

$$\begin{aligned}\alpha + (\beta + \gamma) &= (a + bi) + ((c + di) + (e + fi)) \\ &= (a + bi) + ((c + e) + (d + f)i) \text{ (From addition of complex numbers)} \\ &= (a + (c + e)) + (b + (d + f))i \text{ (From addition of complex numbers)} \\ &= ((a + c) + e) + ((b + d) + f)i \text{ (from associativity of real numbers)} \\ &= ((a + c) + (b + d)i) + (e + fi) \text{ (Bidirectionality of addition of complex numbers)} \\ &= ((a + bi) + (c + di)) + (e + fi) \text{ (Bidirectionality of addition of complex numbers)} \\ &= (\alpha + \beta) + \gamma\end{aligned}$$

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Q6