Q1

$$\begin{aligned}
\frac{1}{a+bi} &= \frac{1}{a+bi} * \frac{a-bi}{a-bi} \\
&= \frac{a-bi}{a^2+abi-abi+b^2} \\
&= \frac{a-bi}{a^2+b^2} \\
c+di &= \frac{a}{a^2+b^2} - \frac{bi}{a^2+b^2}
\end{aligned}$$

therefore $c = \frac{a}{a^2+b^2}$ and $d = \frac{-b}{a^2+b^2}$

Q2

$$\left(\frac{-1+\sqrt{3}i}{2}\right)^3 = \frac{1}{8} * \left(-1+\sqrt{3}i\right)^3$$

using the identity $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ where $a = -1$ and $b = \sqrt(3)i$ we have

$$\begin{aligned}
\frac{1}{8} * \left(-1+\sqrt{3}i\right)^3 &= (-1)^3 + 3*(-1)^2*(\sqrt{3}i) + 3*(-1)*(\sqrt{3}i)^2 + (\sqrt{3}i)^3 \\
&= \frac{1}{8} * \left(-1 + 3*\sqrt{3}i + 9 - 3*\sqrt{3}i\right) \\
&= \frac{8}{8} \\
&= 1
\end{aligned}$$

Q3

$-\sqrt{i}$ and $+\sqrt{i}$ are two distinct square roots of i

Q4

To show:

$$\alpha + \beta = \beta + \alpha \ \forall \alpha{,}\beta \in \mathbb{C}$$

let $\alpha = a + bi$ and $beta = c + di$
by definition of addition of complex numbers

$$(a+bi) + (c+di) = (a+c) + (b+d)i \tag{1}$$

by definition of complex numbers we know a,b,c,d $\in \mathbb{R}$
so addition is commutative in $\mathbb{R}$

as a consequence $a+b=b+a$ and $c+d=d+c$
there (1) becomes

$$
\begin{aligned}
(a+bi)+(c+di) &= (a+c)+(b+d)i \\
&= (c+a)+(d+b)i \\
&= (c+di)+(a+bi) \text{ (from bidirectionality of how complex number addition is defined)} \\
&= \beta+\alpha
\end{aligned}
$$

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