A paper

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somewhere

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- Question:
  - What weight $\mathbf{w} \in \left\{\mathbf{w} \in \mathbb{R}^N : \sum_{i\in[N]} w_i = 1\right\}$ minimizes the forecast error variance $E\left[(y_{t+1} - \mathbf{w}'\mathbf{f}_t)^2\right]$

Empirical Example (Macro)

- European Central Bank's surveys of professional forecasters. CPI, 1-year-ahead or 2-year-ahead. Data: 1999Q1–2018Q4 (20 years), about 120 forecasters. Unbalanced, 30 forecasters of complete record. 40-quarter rolling window.

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Section 1

Problem and Algorithm

Bates and Granger (1969)

- Forecast error $\mathbf{e}_t = (e_{1t}, \ldots, e_{Nt})'$ with $e_{it} = y_{t+1} - f_{it}$.
- Sample variance-covariance (VC): $\widehat{\Sigma} := T^{-1} \sum_{t=1}^{T} \mathbf{e}_t \mathbf{e}_t'$.

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- Bates and Granger: (Problem-BG)

$$\min_{\mathbf{w} \in \mathbb{R}^N} \frac{1}{2} \mathbf{w}' \widehat{\Sigma} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}' \mathbf{1}_N = 1.$$

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- When $\widehat{\Sigma}$ is invertible,

$$\widehat{\mathbf{w}}^{\mathrm{BG}} = \widehat{\Sigma}^{-1} \mathbf{1}_N \, / \, \mathbf{1}_N' \widehat{\Sigma}^{-1} \mathbf{1}_N.$$

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- When N is close to T, the inverse is ill-conditioned.
- When $N > T$, sample VC is singular.

Summary

- Constrained quadratic optimization formulates many economic and financial questions.

- We propose L2-relaxation to stabilize the numerical solution.
- A new machine learning algorithm to improve the estimation.
- Asymptotic optimality under group structure.