Inverse Wishart Distribution for an Omega Prior

Given a desired mean and coefficient of variation (CV) of a prior for an OMEGA matrix, we can derive the required parameters of the inverse Wishart distribution as follows. We define the inverse Wishart distribution in terms of two parameters: the degrees of freedom df and scale matrix $\mathbf{\Psi}$. The mean and diagonal elements of the variance of a $p \times p$ random variable $\mathbf{X}$ with an inverse Wishart distribution ($\mathbf{X} \sim W^{-1}(\mathbf{\Psi}, df)$) are as follows:

$$\mathbb{E}[\mathbf{X}] = \frac{\mathbf{\Psi}}{df - n - 1} \tag{1}$$

$$\mathrm{Var}(X_{i,i}) = \frac{2 \cdot {\Psi_{i,i}}^2}{(df - n - 1)^2 (df - n - 3)} \tag{2}$$

Given the required CV of the ith diagonal of $\mathbf{X}$ ($X_{i,i}$) we can derive the appropriate degrees of freedom for that component:

$$CV(X_{i,i})^2 = \frac{\mathrm{Var}(X_{i,i})}{\mathbb{E}[X_{i,i}]^2} \tag{3}$$

$$= \frac{\frac{2 \cdot {\Psi_{i,i}}^2}{(df-n-1)^2 (df-n-3)}}{\frac{{\Psi_{i,i}}^2}{(df-n-1)^2}} \tag{4}$$

$$= \frac{2}{df - n - 3} \tag{5}$$

$$df - n - 3 = 2 \cdot CV(X_{i,i})^{-2} \tag{6}$$

$$df = 2 \cdot CV(X_{i,i})^{-2} + n + 3 \tag{7}$$

Now, taking into account our target mean $\mathbb{E}[\mathbf{\Omega}]$, assuming that NONMEM parameterizes the inverse Wishart as $\mathbf{\Omega} \sim W^{-1}(df \cdot \mathbf{\Omega}_{\text{prior}}, df)$, we have

$$\mathbb{E}[\mathbf{\Omega}] = \frac{df \cdot \mathbf{\Omega}_{\text{prior}}}{df - n - 1} \tag{8}$$

$$\mathbf{\Omega}_{\text{prior}} = \frac{df - n - 1}{df} \cdot \mathbb{E}[\mathbf{\Omega}] \tag{9}$$