

Inverse Wishart Degrees of Freedom

Standard Inverse Wishart

$$X \sim W^{-1}(\Psi, df)$$

$$\mathbb{E}[X] = \frac{\Psi}{df-n-1}$$

$$Var(X_{i,i}) = \frac{2 \cdot \Psi_{i,i}^2}{(df-n-1)^2(df-n-3)}$$

$$CV(X_{i,i})^2 = \frac{Var(X_{i,i})}{\mathbb{E}[X_{i,i}]^2}$$

$$CV(X_{i,i})^2 = \frac{\frac{2 \cdot \Psi_{i,i}^2}{(df-n-1)^2(df-n-3)}}{\frac{\Psi_{i,i}^2}{(df-n-1)^2}}$$

$$CV(X_{i,i})^2 = \frac{2}{df-n-3}$$

$$df - n - 3 = 2 \cdot CV(X_{i,i})^{-2}$$

$$df = 2 \cdot CV(X_{i,i})^{-2} + n + 3$$

Assuming NONMEM parameterizes the inverse Wishart as:

$$\Omega \sim W^{-1}(df \cdot \Omega_{prior}, df)$$

$$\mathbb{E}[\Omega] = \frac{df \cdot \Omega_{prior}}{df-n-1}$$

$$\Omega_{prior} = \frac{df-n-1}{df} \cdot \mathbb{E}[\Omega]$$