Reinforcement Learning

an introduction

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NTNU

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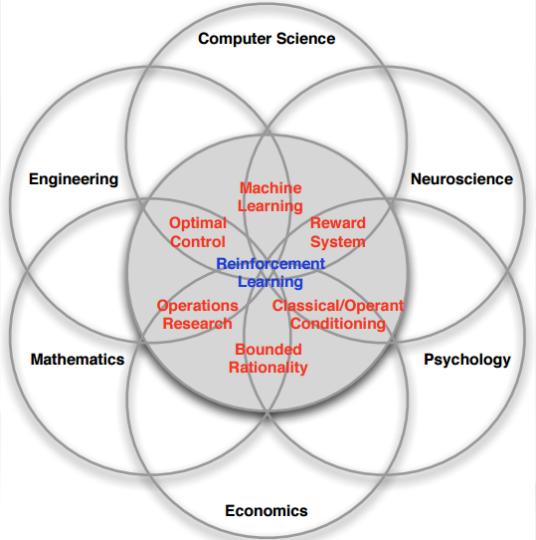
What is RL?


Computer Science
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What is RL?

Use cases:

- AlphaGo

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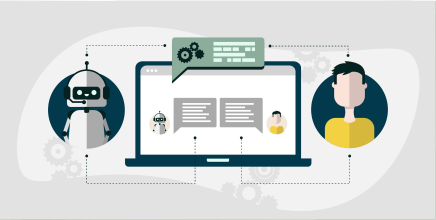
- AlphaGo
- Autonomous Cars

What is RL?

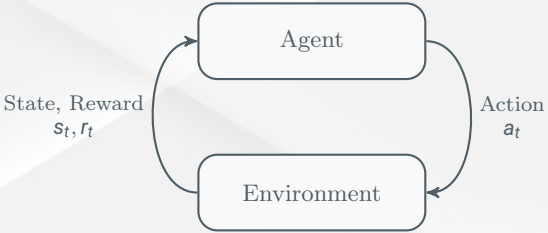


Use cases:

- AlphaGo
- Autonomous Cars
- Natural Language Processing

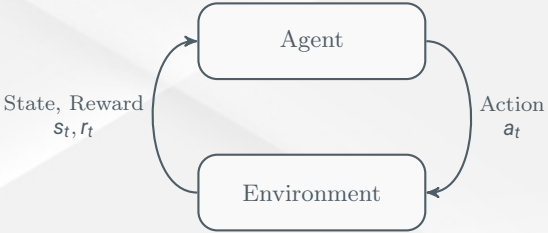
Agents and environments

- Agent
- Environment

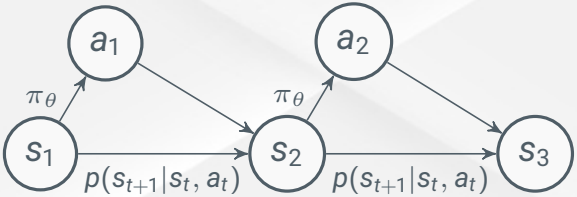


Agents and environments

- Agent
  - Learner and decision-maker.
- Environment
  - Everything outside the agent that the agent interacts with



Markov Decision Processes (MDPs)



Markov Decision Process

The RL objective

$$\theta^* = \arg\max_\theta \mathbb{E}_{\tau \sim p_\theta(\tau)} \left[\sum_t r(s_t, a_t) \right]$$

- Policy Gradient methods

The RL objective

$$\theta^* = \arg\max_{\theta} \mathbb{E}_{\tau \sim p_\theta(\tau)} \left[\sum_t r(s_t, a_t) \right]$$

- Policy Gradient methods
- Actor-Critic methods

The RL objective

$$\theta^* = \arg\max_\theta \mathbb{E}_{\tau \sim p_\theta(\tau)} \left[\sum_t r(s_t, a_t) \right]$$

- Policy Gradient methods
- Actor-Critic methods
- Value-based methods

The RL objective

$$\theta^* = \arg\max_\theta \mathbb{E}_{\tau \sim p_\theta(\tau)} \left[\sum_t r(s_t, a_t) \right]$$

- Policy Gradient methods
- Actor-Critic methods
- Value-based methods
- (Model-based methods)

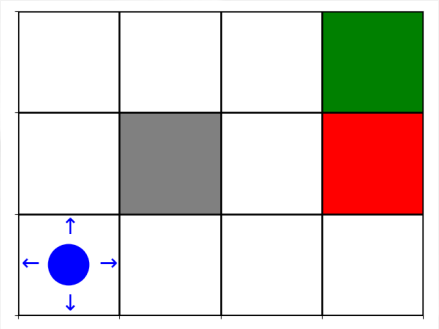
The RL objective

$$\theta^* = \arg\max_\theta \mathbb{E}_{\tau \sim p_\theta(\tau)} \left[\sum_t r(s_t, a_t) \right]$$

- Policy Gradient methods
- Actor-Critic methods
- Value-based methods
- (Model-based methods)
- (On-policy and off-policy)

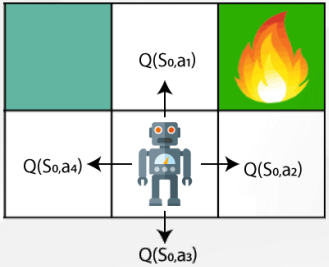
Gridworld environment

- Discrete state and action space.
  - States: 2D coordinates (x,y)
  - Actions: move (left, right, up, down)
- Markov property.
- Stochastic transition probability.
- An episode is finished when a terminal state is reached.
- Rewards:
  - Green: “Good" terminal state (reward = +1)
  - Red: “Bad" terminal state (reward = -1)
  - White: reward = 0
  - Gray: Inaccessible



The Gridworld environment

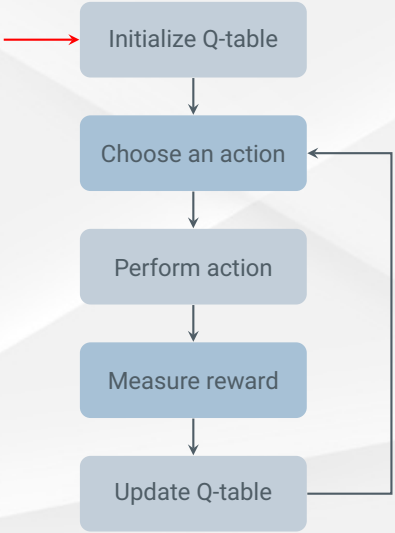
Q-learning



Q_table

		Actions			
		A_1	A_2	...	A_M
States	S_1	$Q(S_1, A_1)$	$Q(S_1, A_2)$	...	$Q(S_1, A_M)$
	S_2	$Q(S_2, A_M)$	$Q(S_2, A_M)$	...	$Q(S_2, A_M)$
	...	...	...	...	...
	S_N	$Q(S_N, A_1)$	$Q(S_N, A_2)$	...	$Q(S_N, A_M)$

Q-learning overview



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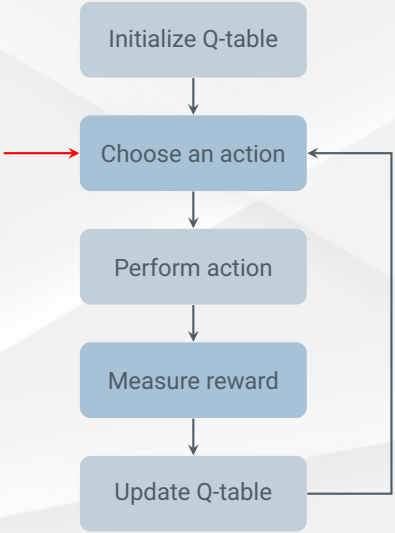

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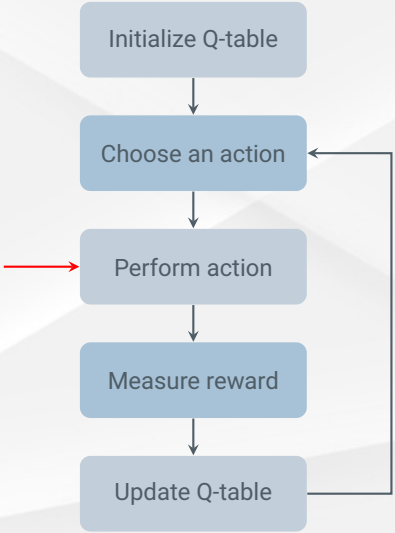

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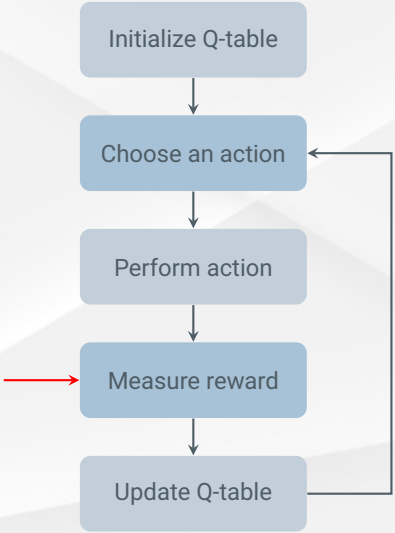

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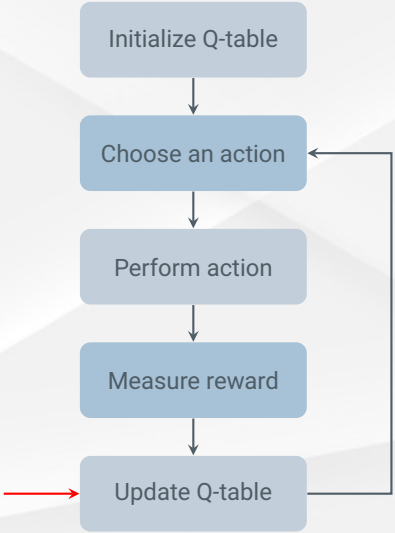
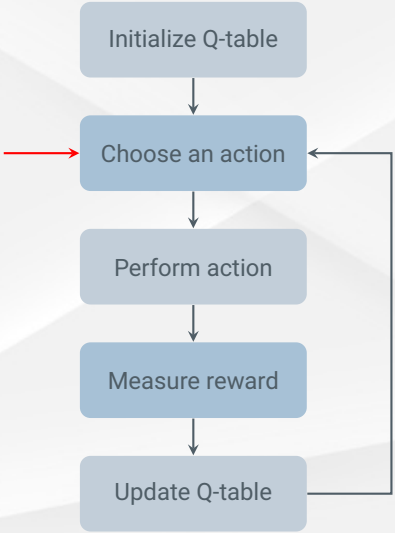


Q-learning overview



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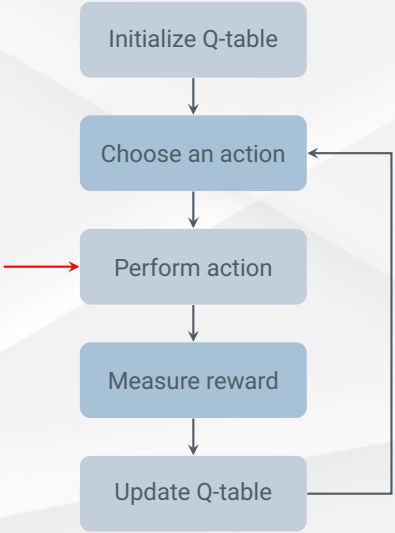

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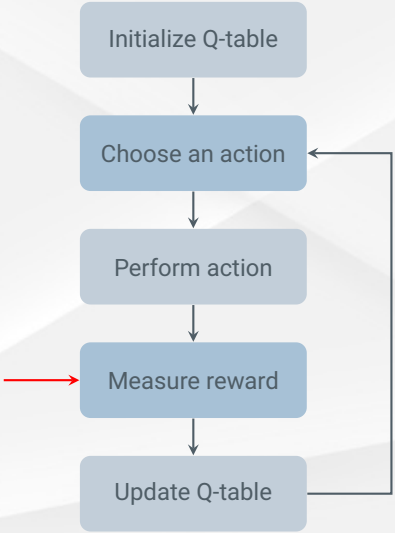

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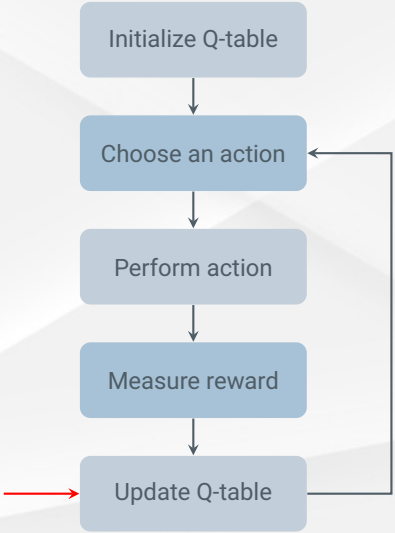

Q-learning algorithm

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\epsilon > 0$, $\gamma \in [0, 1]$

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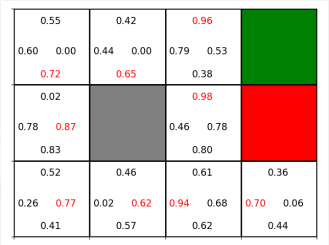
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$



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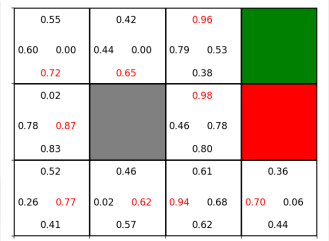
for episode = 1, M **do**

end



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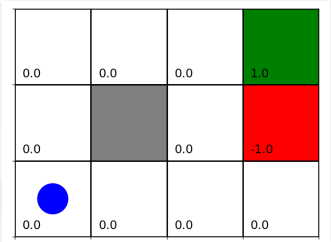
for episode = 1, M **do**
    Initialize $\mathcal{S}$

end



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  while $S \neq terminal$ **do**
  end
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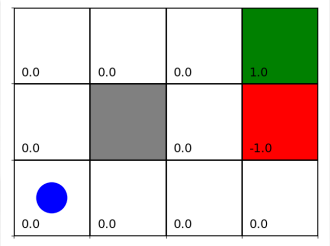

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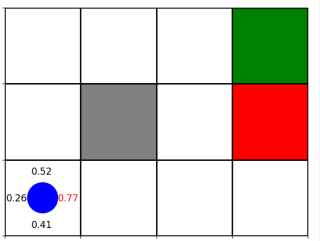

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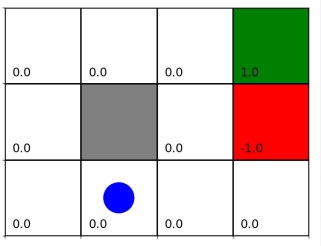

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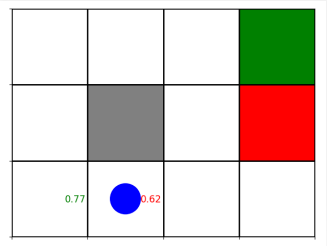
    $Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma \max_a Q(S', a) - Q(S, A)]$
  end
end



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  end
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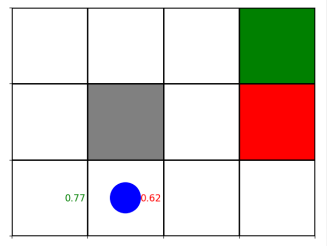

$$Q(bottom\_left, RIGHT) \leftarrow 0.77 + 0.005 \times [0 + 0.99 \times 0.62 - 0.77]$$

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  end
end

1.41 1.45 1.45 1.68	1.75 1.63 1.89 1.80	1.89 1.86 1.91 1.78	
1.61 1.64 1.63 1.69		1.75 1.72 0.16 1.67	
1.67 1.68 1.70 1.68	1.72 1.70 1.72 1.71	1.74 1.72 1.63 1.72	0.17 1.64 1.33 1.46

After 100k episodes

Q-learning algorithm

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    $S \leftarrow S'$
  end
end

1.69 1.70 1.86 1.73	1.86 1.78 1.88 1.86	1.89 1.86 1.90 1.77	
1.84 1.70 1.71 1.70		1.77 1.74 0.15 1.62	
1.71 1.69 1.72 1.69	1.70 1.70 1.73 1.70	1.74 1.71 1.64 1.72	0.14 1.63 1.50 1.61

After 200k episodes

Q-learning algorithm

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    $S \leftarrow S'$
  end
end

1.85 1.84 1.86 1.83	1.87 1.85 1.89 1.87	1.89 1.86 1.91 1.77	
1.84 1.82 1.82 1.81		1.78 1.71 0.09 1.69	
1.82 1.80 1.78 1.80	1.78 1.80 1.76 1.78	1.75 1.78 1.65 1.75	0.11 1.66 1.55 1.60

After 300k episodes

Q-learning algorithm

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  end
end

1.85 1.84 1.86 1.83	1.87 1.85 1.89 1.87	1.89 1.87 1.91 1.78	
1.84 1.83 1.83 1.81		1.77 1.75 0.13 1.68	
1.82 1.81 1.79 1.80	1.78 1.80 1.77 1.78	1.76 1.78 1.66 1.76	0.10 1.66 1.51 1.61

After 400k episodes

Q-learning algorithm

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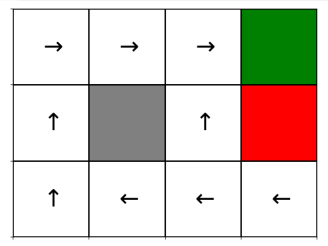
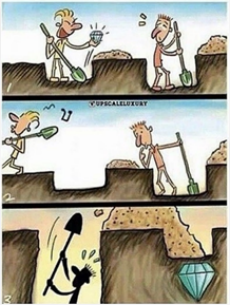
end

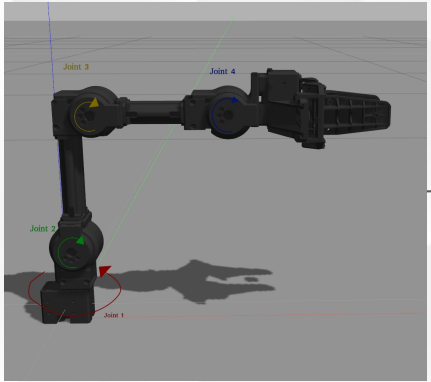


Final policy

Open Problems - Exploration vs. Exploitation



Open Problems - Transfer Learning



Open problems - Explainability

Final frame

Thank you for your attention!