02.03.2023 - Lecture 1 - Seminarraum 11

General Notes

- only two sessions will be blended learning. their exercises are mandatory (affects %10 the final mark). Final exam: required to score %50 at least.
- 5 matlab exercises, each of which affects the final mark %5.
- all the dates are announced. **Implementation of Discrete-Time Systems**

  Review of Discrete-time Systems: input output behavior of discrete time systems can be described by difference equation, with constant coeffs a_k and b_k.

  $$y[n] = \sum_{k=1}^{N} a_k * y[n-k] + \sum_{k=0}^{M} b_k * x[n-k]$$

  where $x[n]$ are the current input samples, $x[n-k]$ are the previous input samples. Term on the left represents previous output samples, terms on the right reps input values.

  Implementation of Discrete-Time Systems

  Impulse reponse $h[n]$ is the output for $x[n] = \delta[n]$.

  Step response $s[n]$ is the output for unit pulse sequence $x[n] = \sigma[n]$.

  Step response and impulse response are connected:

  $$s[n] = \sum_{k=-\infty}^{n} h[k]$$

  Frequency response $H[\Omega]$: The horizontal axis on the frequency-amplitude graph is the ratio between sampling frequency and the frequency of the applied signal. It is only showed up to ratio of 0.5 because there should not be any useful info after this frequency. It is advantageous to use logarithmic scaling in such graphs. Following was written along with text *normalized frequency.*

  $$\Omega = 2\pi \frac{f}{f_s}$$

  $$h[n] \rightarrow FFT \rightarrow H[\Omega]$$

System transfer function of discrete time systesm are obtained by *z-transform*: (check the supplementary material for z-transform.)

$$h[n] \rightarrow z\ transform \rightarrow H[z]$$

Example 1:

Using the following difference equation:

$$y[n] = b_0 * x[n] + b_1 * x[n-1] + a_1 * y[n-1]$$

Determine:

- Impulse response
- System transfer function

Solution:

- Recall the impulse function. Input signal $x[n] = \delta[n]$

$$\begin{aligned} y[-1] &= b_0 * x[-1] + b_1 * x[-1-1] + a_1 * y[-2] = 0 \\ y[0] &= b_0 * x[0] + b_1 * x[-1] + a_1 * y[-1] = 0 \\ y[1] &= b_0 * x[1] + b_1 * x[0] + a_1 * y[0] = b_1 + a_1 * b_0 \end{aligned}$$

Homework: Continue on calculation.

- Develop the transfer function from the given difference equation.

$$\begin{aligned} Y(z) &= b_0.X(z) + b_1.X(z).z^{-1} + a_1.Y(z).z^{-1} \\ Y(z)(1 - a_1.z^{-1}) &= X(z)(b_0 + b_1 z^{-1}) \\ H(z) &= \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} \end{aligned}$$

Example 2:

Using the following difference equation:

$$y[n] = 3 * x[n] - x[n-1] + 0.5 * y[n-1]$$

Determine:

- Impulse response
- Step response

- System transfer function

  Solution:

- Recall the impulse function. Input signal $x[n] = \delta[n]$

$$\begin{aligned} y[-1] &= 3*0 - 0 + 0.5*0 = 0 \\ y[0] &= 3*1 - 0 + 0.5*0 = 3 \\ y[1] &= 3*0 - 1*1 + 0.5*3 = 0.5 \\ y[2] &= 3*0 - 1*0 + 0.5*0.5 = 0.25 \\ y[3] &= 3*0 - 1*0 + 0.5*0.25 = 0.125 \\ y[4] &= 0.5*y[3] = 0.0625 \\ y[n] &= 0.5*y[n-1] \end{aligned}$$

- Recall the step function. Input signal $x[n] = \sigma[n]$

$$\begin{aligned} y[-1] &= 3*0 - 0 + 0.5*0 = 0 \\ y[0] &= 3*1 - 0 + 0.5*0 = 3 \\ y[1] &= 3*1 - 1*1 + 0.5*3 = 3.5 \\ y[2] &= 3*1 - 1*1 + 0.5*3.5 = 3.75 \\ y[3] &= 3*1 - 1*1 + 0.5*3.75 = 3.875 \\ y[n] &= 2 + 0.5*y[n-1] \end{aligned}$$

- System transfer function:

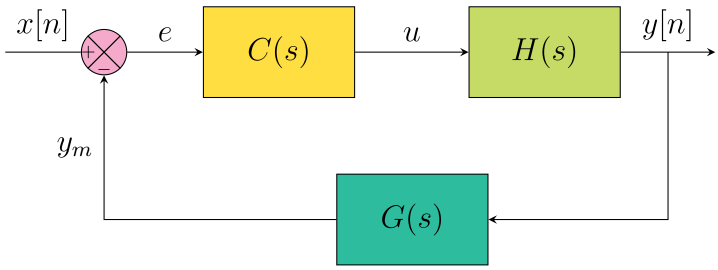
$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} = \frac{3 - z^{-1}}{1 - 0.5z^{-1}}$$

07.03.2023 - Lecture 2 - Seminarraum 11

This lecture started with block diagram representation of mathematical representations of discrete systems.

Example: Draw the block diagram of the below respresentation.

$$y[n] = 3x[n] - x[n-1] + 0.5y[n-1]$$



Implementation of Discrete-time Systems

IIR (Infinite impulse response). Due to the feedback part, the impulse response can be considered infinitely long.

Because of the feedback, IIR systems are also called recursive systems (filters (does it also mean they are filters?)). It is also possible that such systems can be instable. THerefore stability criteria has to be considered in the design process.

Order of the system is equal to the number of delayed output elements.

Good filter characteristics are achievabe with low filter orders, which also mean higher operational speeds. The disadvantage is that there is no linear phase characteristics.

The system function for an Nth order IIR-System is as follows:

$$\boxed{H(z) = \frac{\sum_{k=1}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \frac{B(z)}{A(z)} = H_1(z) * H_2(z)}$$

H1 = B(z) is the feedforward part, H2 = 1/A(z) is the feedback part.