Data Science Analysis Lab

Lecture Notes

Metin Oktay Yılmaz

Systems Design Master
FH Kärnten
Austria

Contents

1 General Notes

02.03.2023 - Lecture 1 - Seminarraum 11

- only two sessions will be blended learning. their exercises are mandatory (affects %10 the final mark). Final exam: required to score %50 at least.
- 5 matlab exercises, each of which affects the final mark %5.
- all the dates are announced.

2 Implementation of Discrete-Time Systems

Review of Discrete-time Systems: input output behavior of discrete time systems can be described by difference equation, with constant coeffs a_k and b_k.

$$y[n] = \sum_{k=1}^{N} a_k * y[n-k] + \sum_{k=0}^{M} b_k * x[n-k]$$

where $x[n]$ are the current input samples, $x[n-k]$ are the previous input samples. Term on the left represents previous output samples, terms on the right reps input values.

Implementation of Discrete-Time Systems

Impulse reponse $h[n]$ is the output for $x[n] = \delta[n]$.

Step response $s[n]$ is the output for unit pulse sequence $x[n] = \sigma[n]$.

Step response and impulse response are connected:

$$s[n] = \sum_{k=-\infty}^{n} h[k]$$

Frequency response $H[\Omega]$: The horizontal axis on the frequency-amplitude graph is the ratio between sampling frequency and the frequency of the applied signal. It is only showed up to ratio of 0.5 because there should not be any useful info after this frequency. It is advantageous to use logarithmic scaling in such graphs. Following was written along with text *normalized frequency.*

$$\Omega = 2\pi \frac{f}{f_s}$$

$$h[n] \rightarrow FFT \rightarrow H[\Omega]$$

System transfer function of discrete time systesm are obtained by *z-transform*: (check the supplementary material for z-transform.)

$$h[n] \rightarrow z\ transform \rightarrow H[z]$$

Example 1:
Using the following difference equation:

$$y[n] = b_0 * x[n] + b_1 * x[n-1] + a_1 * y[n-1]$$

Determine:

- Impulse response
- System transfer function

Solution:

- Recall the impulse function. Input signal $x[n] = \delta[n]$

$$\begin{aligned} y[-1] &= b_0 * x[-1] + b_1 * x[-1-1] + a_1 * y[-2] = 0 \\ y[0] &= b_0 * x[0] + b_1 * x[-1] + a_1 * y[-1] = 0 \\ y[1] &= b_0 * x[1] + b_1 * x[0] + a_1 * y[0] = b_1 + a_1 * b_0 \end{aligned}$$

- Develop the transfer function from the given difference equation.

$$\begin{aligned} Y(z) &= b_0.X(z) + b_1.X(z).z^{-1} + a_1.Y(z).z^{-1} \\ Y(z)(1 - a_1.z^{-1}) &= X(z)(b_0 + b_1 z^{-1}) \\ H(z) &= \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} \end{aligned}$$

Example 2:
Using the following difference equation:

$$y[n] = 3 * x[n] - x[n-1] + 0.5 * y[n-1]$$

Determine:

- Impulse response
- Step response
- System transfer function

Solution:

- Recall the impulse function. Input signal $x[n] = \delta[n]$

$$
\begin{aligned}
y[-1] &= 3*0 - 0 + 0.5*0 = 0 \\
y[0] &= 3*1 - 0 + 0.5*0 = 3 \\
y[1] &= 3*0 - 1*1 + 0.5*3 = 0.5 \\
y[2] &= 3*0 - 1*0 + 0.5*0.5 = 0.25 \\
y[3] &= 3*0 - 1*0 + 0.5*0.25 = 0.125 \\
y[4] &= 0.5*y[3] = 0.0625 \\
y[n] &= 0.5*y[n-1]
\end{aligned}
$$

- Recall the step function. Input signal $x[n] = \sigma[n]$

$$
\begin{aligned}
y[-1] &= 3*0 - 0 + 0.5*0 = 0 \\
y[0] &= 3*1 - 0 + 0.5*0 = 3 \\
y[1] &= 3*1 - 1*1 + 0.5*3 = 3.5 \\
y[2] &= 3*1 - 1*1 + 0.5*3.5 = 3.75 \\
y[3] &= 3*1 - 1*1 + 0.5*3.75 = 3.875 \\
y[n] &= 2 + 0.5*y[n-1]
\end{aligned}
$$

- System transfer function:

$$
H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} = \frac{3 - z^{-1}}{1 - 0.5z^{-1}}
$$

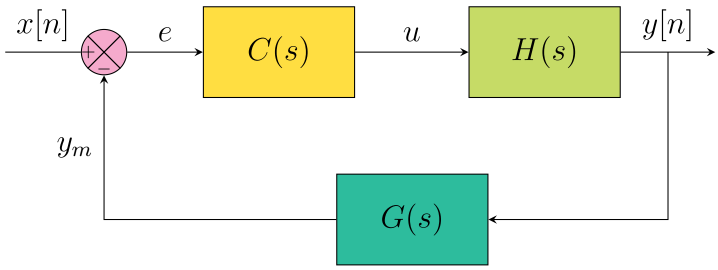
3 Block Diagram Representation

07.03.2023 - Lecture 2 - Seminarraum 11

This lecture started with block diagram representation of mathematical representations of discrete systems.

Example: Draw the block diagram of the below respresentation.

$$y[n] = 3x[n] - x[n-1] + 0.5y[n-1]$$



4 Implementation of Discrete-time Systems

IIR (Infinite impulse response). Due to the feedback part, the impulse response can be considered infinitely long.

Because of the feedback, IIR systems are also called recursive systems (filters (does it also mean they are filters?)). It is also possible that such systems can be instable. THerefore stability criteria has to be considered in the design process.

Order of the system is equal to the number of delayed output elements.

Good filter characteristics are achievabe with low filter orders, which also mean higher operational speeds. The disadvantage is that there is no linear phase characteristics.

The system function for an Nth order IIR-System is as follows:

$$\boxed{H(z) = \frac{\displaystyle\sum_{k=1}^{M} b_k z^{-k}}{1 - \displaystyle\sum_{k=1}^{N} a_k z^{-k}} = \frac{B(z)}{A(z)} = H_1(z) * H_2(z)}$$

$H_1 = B(z)$ is the feedforward part, $H_2 = \frac{1}{A(z)}$ is the feedback part.

5 Cascade and Parallel Form

09.03.2023 - Lecture 3 - Seminarraum 12

To develop a new way to represent which also achives...

- Splitting of higher order system in a aseries of systems of lower order
- Factoring the numerator and denominator polynomials

$$H(z) = \frac{\prod_{k=1}^{M_1}(1 - f_k z^{-1}) \prod_{k=1}^{M_2}(1 - g_k z^{-1})(1 - g_k^* z^{-1})}{\prod_{k=1}^{N_1}(1 - c_k z^{-1}) \prod_{k=1}^{N_2}(1 - d_k z^{-1})(1 - d_k^* z^{-1})}$$

- Desirable to implement with minimum of storage and computation
- Modular structure combining pairs of real factors and complex conjugate pairs into second-order factors, expressed as:

$$H(z) = \prod_{k=1}^{M_1} \frac{b_0 k + b_{1k} z^{-1} + b_{2k} z^{-1}}{1 - a_{1k} z^1 - a_{2k} z^2}$$

  where (assuming $M \leq N$):

$$N(s) = (N + 1) + 2$$

Example:

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

To illustrate the cascade structure, we will split the second-order system incascade of first-order systems.

- Determine poles and zeros:

  $B(z) = 0 \rightarrow$ Numerator polynomial
  $A(z) = 0 \rightarrow$ Denominator polynomial

  Multiply polynomials to get rid of negative powers. Then it is just akin to solving a polynomial.

Zeros:

$$z^2 + 2z + 1 = 0$$
$$z_{0,1} = -1$$

Poles:

$$z^2 - 0.75z + 0.125 = 0$$
$$z_\infty = \frac{1}{2}, \frac{1}{4}$$

- Determine $H_1(z)$ and $H_2(z)$

$$H(z) = \frac{(1 + z^{-1})(1 + z^{-1})}{(1 - 0.5z^{-1})(1 - 0.25z^{-1})}$$

$$= \frac{1 + z^{-1}}{1 - 0.5z^{-1}} \cdot \frac{1 + z^{-1}}{1 - 0.25z^{-1}}$$

The fraction on the left is $H_1(z)$ and on the right is $H_2(z)$

- Direct form 1
- Direct form 2

6 Cascade of Second Order Sections (SOS) Structures (aka biquads)

Number of multiplications is in general 5. By extracting the leading coefficient: 4 multp. + 1 coefficient.

A rational system transfer function H(z) can be expressed by partial fraction expansion in the form of sum of:

- first order system with real poles
- second order systems with a pair of complex conjugate poles
- scaled delay element (e.g. $c_0 z^{-1} + c_1 z^{-2}$)

$H(z)$ can be interpreted as parallel combination of first- and second-order IIR systems with possibly N_p simple scaled delay paths.

Example: Consider again:

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

By applying partial fraction expansion, develop a parallel structure.

- Perform long division, if $M \geq N$: M is the degree of the denominator, N is the degree of numerator. In this case, they are both 2, so $M \geq N$

$$(z^{-2} + 2z^{-1} + 1) : (0.125z^{-2} - 0.75z^{-1} + 1) = 8$$

  Here went on some interesting stuff. The right hand polynomial was multiplied by 8 and the result was subtracted from the left hand polynomial and it resulted $8z^{-1} - 7$.

$$H(z) = 8 + \frac{8z^{-1} - 7}{(1 - 0.5z^{-1})(1 - 0.25z^{-1})}$$

$$H(z) = 8 + \frac{18}{1 - 0.5z^{-1}} + \frac{-25}{1 - 0.25z^{-1}}$$

- Draw the parallel structure with first order systems.

7 Transposition and Reversal of Systems

This section is too visual to not fall terribly behind. Check the handwritten notes.