Monte Carlo (1.2)

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[ ]:
```
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```

[1]:
```
import numpy as np
import pandas as pd
import time
from scipy.stats import norm
import matplotlib.pyplot as plt
```

1 Problem 1

1.1 (a) Monte Carlo Valuation of European Call Option

[13]:
```
def MCEU(K,T,S,sig,r,div,N,M,optype):

    start=time.time()

    dt=T/N

    # risk neutral drift
    nudt= (r-div-0.5*sig**2)*dt

    sigsdt=sig*np.sqrt(dt)

    lnS=np.log(S)


    sum_CT=0
    sum_CT2=0

    lnSt=np.zeros((M,N+1))

    #lnSt[0,0]= lnS

    ST=np.zeros((M,1))

    e=np.zeros((M,N))
```

```python
for  j in range(0,M):

    lnSt[j,0]=lnS


    for i in range(1,N+1):

        e=np.random.normal(0,1,1)

        lnSt[j,i]=lnSt[j,i-1]+ nudt+ sigsdt*e

    #return lnSt

    ST[j]= np.exp(lnSt[j,i])

    #return ST

    if optype== "c":

        CT= max(0,ST[j]-K)

    elif optype== "p":

        CT= max(0,K-ST[j])



    sum_CT= sum_CT+ CT

    sum_CT2= sum_CT2 + CT*CT

#return lnSt
#return ST

call_value= float(sum_CT/M*np.exp(-r*T))

SD= np.sqrt( (sum_CT2- sum_CT/M)*np.exp(-2*r*T)/ (M-1) )
```

```
    SE= float(SD/ np.sqrt(M))

    end=time.time()

    duration=end-start

    return call_value, SE, duration

    #return SD

    #return SE
```

1.1.1 Calculate call value

[14]:
```
a=MCEU(K=100,T=1,S=100,sig=0.2,r=0.06,div=0.03,N=10,M=100,optype="c")
a
```

[14]:
```
(10.034012917312175, 1.715842946565834, 0.014039754867553711)
```

[15]:
```
pd.DataFrame(a)
```

[15]:

	0
0	10.034013
1	1.715843
2	0.014040

[16]:
```
b=MCEU(K=100,T=1,S=100,sig=0.2,r=0.06,div=0.03,N=10,M=100,optype="p")
b
```

[16]:
```
(6.498896116354739, 1.1452016217745051, 0.013765811920166016)
```

[17]:
```
pd.DataFrame(b)
```

[17]:

	0
0	6.498896
1	1.145202
2	0.013766

1.2 (b) Monte Carlo Valuation

1.3 Antithetic Variance Reduction

[18]:
```
def MCAV(K,T,S,sig,r,div,N,M,optype):

    start=time.time()

    dt=T/N
```

```
nudt= (r-div-0.5*sig**2)*dt

sigsdt= sig*np.sqrt(dt)

lnS=np.log(S)

sum_CT=0

sum_CT2=0


lnSt1=np.zeros((M,1))

lnSt2=np.zeros((M,1))

St1=np.zeros((M,1))
St2=np.zeros((M,1))

#lnSt1[0,0]=lnS
#lnSt2[0,0]=lnS

CT=np.zeros((M,1))

for j in range(0,M-1):


    #lnSt1[0,0]=lnS
    #lnSt2[0,0]=lnS


    e=np.random.normal(0,1,1)

    lnSt1[j,0]= lnS+ nudt+ sigsdt*e

    lnSt2[j,0]= lnS+ nudt+ sigsdt*(-e)




    St1[j,0]= np.exp(lnSt1[j,0])

    St2[j,0]= np.exp(lnSt2[j,0])
```

```
        if optype== "c":

            CT[j,0]= 0.5* ( max(0,St1[j,0]-K) + max(0, St2[j,0]-K) )

        elif optype== "p":

            CT[j,0]= 0.5* ( max(K-St1[j,0],0) + max(K-St2[j,0],0) )


        sum_CT= sum_CT+ CT[j,0]

        sum_CT2= sum_CT2+ CT[j,0]*CT[j,0]


    #return lnSt2

    #return St2

    #return CT

    #return sum_CT


    call_value= sum_CT/M*np.exp(-r*T)

    SD= np.sqrt( ( sum_CT2 - sum_CT*sum_CT/M )* np.exp(-2*r*T)/ (M-1) )

    SE= SD/np.sqrt(M)


    end=time.time()

    duration= end - start


    return call_value, SE, duration
```

[19]:
```
a=MCAV(K=100,T=1,S=100,sig=0.2,r=0.06,div=0.03,N=1,M=100,optype="p")
a
#a.shape
```

[19]: (5.7619804070276714, 0.44208439876288697, 0.0025610923767089844)

1.3.1 Delta based Contral Variate

Blac_Scholes_delta

[30]:
```
def Blac_Scholes_delta(K,T,t,S,sig,r,div,optype):

    d1 = 1 / (sig * np.sqrt(T)) * ( np.log(S/K) + (r- div+ sig**2/2) * T)

    if optype == 'c':
        return np.exp(-div*(T-t))*norm.cdf(d1)
    if optype == 'p':
        return np.exp(-div*(T-t)*(norm.cdf(d1)-1))
```

[31]:
```
Blac_Scholes_delta(K=100,T=1,t=0.1,S=100,sig=0.2,r=0.06,div=0.03,optype="c")
```

[31]: 0.5827575324752046

[32]:
```
def MCDCV(K,T,S,sig,r,div,N,M,optype="c"):

    start=time.time()

    dt=T/N

    nudt= (r-div-0.5*sig**2)*dt

    sigsdt= sig*np.sqrt(dt)

    erddt= np.exp(( r-div)*dt)

    beta1= -1

    sum_CT=0

    sum_CT2=0



    for j in range(1,M+1):

        St= S

        cv= 0

        for i in range(1,N+1):


            t= (i-1)*dt


            delta=Blac_Scholes_delta(K,T,t,St,sig,r,div,optype)
```

```
            # return delta

            e= np.random.normal(0,1,1)

            Stn= St*np.exp(nudt +sigsdt*e)

            cv= cv + delta*(Stn-St*erddt)

            St=Stn

        if optype== "c":

            CT= max(0,St-K) + beta1*cv

        elif optype== "p":

            CT= max(0,K-St) + beta1*cv

        sum_CT= sum_CT+ CT

        sum_CT2= sum_CT2+ CT*CT

    #return delta

    call_value= float(sum_CT/ M*np.exp(-r*T))

    SD= np.sqrt(( sum_CT2- sum_CT*sum_CT/M )*np.exp(-2*r*T)/ (M-1))

    SE= float(SD/np.sqrt(M))

    end=time.time()

    duration= end-start

    return call_value,SE, duration
```

[33]:
```
MCDCV(K=100,T=1,S=100,sig=0.2,r=0.06,div=0.03,N=10,M=100,optype="c")
```

[33]: (8.999277918921342, 0.2719212359540458, 0.1628100872039795)

1.4 Antithetic and Delta- based contral variates

[34]:
```
def MCADCV(K,T,S,sig,r,div,N,M,optype):


    start=time.time()

    dt= T/N

    nudt= (r-div-0.5*sig**2)*dt

    sigsdt= sig*np.sqrt(dt)

    erddt= np.exp(( r-div)*dt)

    beta1= -1

    sum_CT= 0

    sum_CT2= 0


    for j in range(1,M+1):

        St1=S

        St2=S

        cv1=0

        cv2=0

        for i in range(1,N+1):


            t= (i-1)*dt

            delta1= Blac_Scholes_delta(K,T,t,St1,sig,r,div,optype)

            delta2= Blac_Scholes_delta(K,T,t,St2,sig,r,div,optype)

            e= np.random.normal(0,1,1)

            Stn1= St1*np.exp( nudt+ sigsdt*e)

            Stn2= St2*np.exp( nudt+ sigsdt*(-e) )
```

```
            cv1= cv1+ delta1*(Stn1-St1*erddt)

            cv2= cv2+ delta2*(Stn2-St2*erddt)


            St1= Stn1

            St2= Stn2


        if optype=="c":

            CT= 0.5* ( max(0,St1-K)+ beta1*cv1+  max(0,St2-K)+ beta1*cv2)


        elif optype=="p":


            CT= 0.5* ( max(K-St1,0)+ beta1*cv1+  max(K-St2,0)+ beta1*cv2)


        sum_CT= sum_CT + CT

        sum_CT2= sum_CT2+ CT*CT


    call_value= float(sum_CT/M *np.exp(-r*T))

    SD= np.sqrt(( sum_CT2- sum_CT*sum_CT/M)* np.exp(-2*r*T)/ (M-1) )


    SE= float(SD/np.sqrt(M))


    end= time.time()


    duration= end- start

    return call_value, SE, duration
```

[35]:

```
MCADCV(K=100,T=1,S=100,sig=0.2,r=0.06,div=0.03,N=10,M=100,optype="c")
```

[35]: (9.132385646104707, 0.23575012321495664, 0.26267004013061523)

1.5 Comparison of the results

1.5.1 European Call

[40]:
```
mcc1=MCEU(K=100,T=1,S=100,sig=0.2,r=0.06,div=0.03,N=10,M=100,optype="c")
mcc2=MCAV(K=100,T=1,S=100,sig=0.2,r=0.06,div=0.03,N=1,M=100,optype="c")
mcc3=MCDCV(K=100,T=1,S=100,sig=0.2,r=0.06,div=0.03,N=10,M=100,optype="c")
mcc4=MCADCV(K=100,T=1,S=100,sig=0.2,r=0.06,div=0.03,N=10,M=100,optype="c")
```

[41]:
```
pd.DataFrame({"MC":mcc1,"Antithetic Variates":mcc2,"Delta-based Control␣
↪Variate":mcc3,"Antithetic&Delta-based":mcc4},index=["call␣
↪value","SE","Duration"])
```

[41]:
```
                   MC  Antithetic Variates  Delta-based Control Variate  \
call value   8.186609             8.664609                     8.829642
SE           1.631225             0.697869                     0.251916
Duration     0.016041             0.003621                     0.136241

             Antithetic&Delta-based
call value                 9.215379
SE                         0.239224
Duration                   0.220909
```

1.5.2 European Put

[42]:
```
mcp1=MCEU(K=100,T=1,S=100,sig=0.2,r=0.06,div=0.03,N=10,M=100,optype="p")
mcp2=MCAV(K=100,T=1,S=100,sig=0.2,r=0.06,div=0.03,N=1,M=100,optype="p")
mcp3=MCDCV(K=100,T=1,S=100,sig=0.2,r=0.06,div=0.03,N=10,M=100,optype="p")
mcp4=MCADCV(K=100,T=1,S=100,sig=0.2,r=0.06,div=0.03,N=10,M=100,optype="p")
```

[43]:
```
pd.DataFrame({"MC":mcp1,"Antithetic Variates":mcp2,"Delta-based Control␣
↪Variate":mcp3,"Antithetic&Delta-based":mcp4},index=["call␣
↪value","SE","Duration"])
```

[43]:
```
                   MC  Antithetic Variates  Delta-based Control Variate  \
call value   5.393831             6.060820                    -0.917028
SE           0.957845             0.467409                     2.598999
Duration     0.014960             0.003132                     0.139879

             Antithetic&Delta-based
call value                 6.131404
SE                         0.247954
Duration                   0.221451
```

[44]:
```
def Blac_scholes(K,T,S,sig,r,div,optype):

    d1 = (np.log(S / K) + (r - div + 0.5 * sig ** 2) * T) / (sig * np.sqrt(T))
    d2 = (np.log(S / K) + (r - div - 0.5 * sig ** 2) * T) / (sig * np.sqrt(T))

    C = (S * np.exp(-div * T) * norm.cdf(d1, 0.0, 1.0) - K * np.exp(-r * T) *␣
 ↪norm.cdf(d2, 0.0, 1.0))
    P = (K * np.exp(-r * T) * norm.cdf(-d2, 0.0, 1.0) - S * np.exp(-div*T) *␣
 ↪norm.cdf(-d1, 0.0, 1.0))

    if optype == "c":
        return C

    elif optype =="p":
        return P
```

[45]:
```
Blac_scholes(100,1,100,0.2,0.06,0.03,"c")
```

[45]: 9.135195269350568

[46]:
```
Blac_scholes(100,1,100,0.2,0.06,0.03,"p")
```

[46]: 6.267095272924621

Among all the Monte Carlo methods, we could say that Antithetic Variates and Delta-based Control Variate method is more close to Blackscholes value. Moreover it has the lowest SE value.

Monte carlo without any variance reduction method works the fastest comparing to other methods, however, it gives poor results, very far away from black scholes value

2 Problem 2

2.1 (a) Arithmetic Asian call option

[54]:
```
def MCAS(K,T,S,sig,r,div,N,M,optype):

    dt=T/N
    nudt= (r- div- 0.5*sig**2)*dt
    sigsdt= sig*np.sqrt(dt)

    sum_CT= 0
    sum_CT2= 0

    for j in range(0,M):

        St=S
```

```
        sumSt=0

        for i in range(0,N):


            e= np.random.normal(0,1,1)

            St= St* np.exp(nudt+ sigsdt*e)

            sumSt= sumSt+ St


        A= sumSt/N


        if optype=="c":

          CT= max(0,A-K)

        elif optype=="p":


            CT= max(0,K-A)


        sum_CT= sum_CT +CT

        sum_CT2= sum_CT2+ CT*CT


    option_value= sum_CT/M *np.exp(-r*T)
    SD= np.sqrt(( sum_CT2- sum_CT*sum_CT/M)* np.exp(-2*r*T)/ (M-1))
    SE= SD/np.sqrt(M)


    return option_value
```

[55]:
```
MCAS(K=100,T=1,S=100,sig=0.2,r=0.06,div=0.03,N=12,M=100,optype="c")
```

[55]: array([5.43640907])

2.2 (b) Up and Out Barrier Call Option

[56]:
```
def MCUOB(K,T,S,sig,r,div,H,N,M,optype):

    dt=T/N
    nudt= (r-div- 0.5*sig**2)*dt
    sigsdt= sig*np.sqrt(dt)

    sum_CT= 0

    sum_CT2= 0


    for j in range(0,M):

        St=S


        for i in range(0,N):


            e= np.random.normal(0,1,1)

            St= St* np.exp(nudt+ sigsdt*e)

            if St>H:

                break



        if optype=="c":

            CT= max(0,St-K)

        elif optype=="p":

            CT= max(0,K-St)



        sum_CT= sum_CT+ CT

        sum_CT2= sum_CT2+ CT*CT
```

```
    call_value= sum_CT/M*np.exp(-r*T)

    SD= np.sqrt(( sum_CT2-sum_CT*sum_CT/M  ))* np.exp(-2*r*T)/(M-1)

    SE= SD/np.sqrt(M)



    return call_value
```

[57]:
```
MCUOB(K=100,T=1,S=100,sig=0.2,r=0.06,div=0.03,H=110,N=12,M=100,optype="c")
```

[57]: array([6.32154556])

3 Problem 3

3.1 (a) Cholesky decomposition

[58]:
```
def cholesky(a):

    a= np.array(a,float)

    L=np.zeros_like(a)

    n,b=np.shape(a)

    for j in range(n):

        for i in range(j,n):

            if i== j:

                sumk=0

                for k in range(j):

                    sumk+= L[i,k]**2

                L[i,j]= np.sqrt(a[i,j]- sumk)

            else:
```

```python
                sumk=0
                for k in range(j):
                    sumk+= L[i,k] * L[j,k]
                L[i,j]= (a[i,j]- sumk) / L[j,j]
    return L
```

[59]:
```python
A=[[1,0.5,0.2],[0.5,1,-0.4],[0.2,-0.4,1]]
A
```

[59]:
```
[[1, 0.5, 0.2], [0.5, 1, -0.4], [0.2, -0.4, 1]]
```

[60]:
```python
B=cholesky(A)
B
```

[60]:
```
array([[ 1.        ,  0.        ,  0.        ],
       [ 0.5       ,  0.8660254 ,  0.        ],
       [ 0.2       , -0.57735027,  0.79162281]])
```

[61]:
```python
np.dot(B,3)
```

[61]:
```
array([[ 3.        ,  0.        ,  0.        ],
       [ 1.5       ,  2.59807621,  0.        ],
       [ 0.6       , -1.73205081,  2.37486842]])
```

[62]:
```python
np.dot(B,np.transpose(B))
```

[62]:
```
array([[ 1. ,  0.5,  0.2],
       [ 0.5,  1. , -0.4],
       [ 0.2, -0.4,  1. ]])
```

[63]:
```python
W1=np.random.normal(0,1,1)
W2=np.random.normal(0,1,1)
W3=np.random.normal(0,1,1)
brown_mot=[[W1,W2,W3]]
#brown_mot
np.dot(B,brown_mot)
```

[63]:
```
array([[[0.27621182]],

       [[0.00357075]],
```

```
        [[0.40664941]]])
```

[64]:
```
np.dot(B,brown_mot)
```

[64]:
```
array([[[0.27621182]],

        [[0.00357075]],

        [[0.40664941]]])
```

3.2 (b) Correlated GBM simulation

[65]:
```
def gbm(S0,T,mu,sig,A,N,M):


    dt= T/N

    value=[]

    L=cholesky(A)

    for i in range(M):

        S1=S0[0]
        S2=S0[1]
        S3=S0[2]

        res1=[]


        for j in range(N):

            row_z= np.random.normal(0,1,3)

            z= np.dot(L,row_z)

            z1= z[0]
            z2= z[1]
            z3= z[2]


            S1= S1+ mu[0]*S1*dt+ sig[0]*S1*z1*np.sqrt(dt)

            S2= S2+ mu[1]*S2*dt+ sig[1]*S2*z2*np.sqrt(dt)

            S3= S3+ mu[2]*S3*dt+ sig[2]*S3*z3*np.sqrt(dt)
```

```
            res2=[S1,S2,S3]

            res1.append(res2)
        value.append(res1)
    return value
```

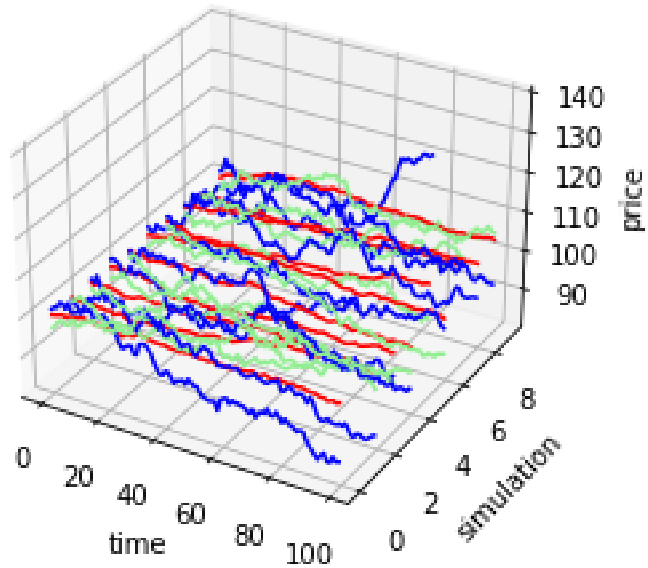
[66]:
```
a= np.matrix([[1,0.5,0.2],
                [0.5,1,-0.4],
                [0.2,-0.4,1]])
a
```

[66]:
```
matrix([[ 1. ,  0.5,  0.2],
        [ 0.5,  1. , -0.4],
        [ 0.2, -0.4,  1. ]])
```

[67]:
```
res=gbm(S0=[100,101,98],T=100/365,mu=[0.03,0.06,0.02],sig=[0.05,0.2,0.
 ↪15],A=a,N=100,M=1000)
```

[68]:
```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
for mm in range(10):
    xt = []
    yt = []
    zt = []
    for nn in range(len(res[mm])):
        xt.append(res[mm][nn][0])
        yt.append(res[mm][nn][1])
        zt.append(res[mm][nn][2])
    ax.plot(range(100),np.asarray([mm]*100),xt,color = "red")
    ax.plot(range(100),np.asarray([mm]*100),yt,color = "blue")
    ax.plot(range(100),np.asarray([mm]*100),zt,color = "lightgreen")
ax.set_xlabel('time')
ax.set_ylabel('simulation')
ax.set_zlabel('price')
```

[68]:
```
Text(0.5, 0, 'price')
```



3.3 (c) Basket options simulation

[74]:
```
def gbm_basket(S0,K,r,T,mu,sig,a,optype,A,N,M):

    dt= T/N

    L= cholesky(A)

    disc= np.exp(-r*T)

    sum_C= 0

    for i in range(M):

        S1=S0[0]
        S2=S0[1]
        S3=S0[2]



        for j in range(N):

            row_z= np.random.normal(0,1,3)

            z= np.dot(L,row_z)
```

```
            S1= S1+ mu[0]*S1*dt+ sig[0]*S1*z[0]*np.sqrt(dt)

            S2= S2+ mu[1]*S2*dt+ sig[1]*S2*z[1]*np.sqrt(dt)

            S3= S3+ mu[2]*S3*dt+ sig[2]*S3*z[2]*np.sqrt(dt)


        Ut= a[0]*S1+ a[1]*S2+ a[2]*S3


        if optype== "c":

            C= disc*max(Ut-K,0)

        elif optype== "p":

            C= disc*max(-Ut+K,0)

        sum_C+= C


    value= sum_C/M

    return value
```

[75]:
```
S0=[100,101,98]
sig=[0.05,0.2,0.15]
mu=[0.03,0.06,0.02]
a=[1/3,1/3,1/3]
A= np.matrix([[1,0.5,0.2],
                [0.5,1,-0.4],
                [0.2,-0.4,1]])
```

[76]:
```
gbm_basket(S0=S0,K=100,r=0.06,T=1,mu=mu,sig=sig,a=a,optype="c",A=A,N=100,M=100)
```

[76]: 4.259055339249731

[77]:
```
gbm_basket(S0=S0,K=100,r=0.06,T=1,mu=mu,sig=sig,a=a,optype="p",A=A,N=100,M=100)
```

[77]: 1.5867162526345826

3.4 (d) Exotic basket option

3.4.1 (i)

[78]:
```
def exgbm_basket1(S0,K,r,T,mu,sig,a,optype,B,A,N,M):


    dt=T/N

    L=cholesky(A)

    disc= np.exp(-r*T)

    sum_C=0

    #nudt= (r+0.5*sig**2)*dt
    #sigsdt= sig*np.sqrt(dt)

    for i in range(M):


        S1= S0[0]

        S2= S0[1]

        S3= S0[2]

        indic=0

        for j in range(N):


            #nudt= (r-div- 0.5*sig**2)*dt
            #sigsdt= sig*np.sqrt(dt)


            row_z=np.random.normal(0,1,3)

            z= np.dot(L,row_z)

            S1= S1*np.exp( (mu[0]+0.5*sig[0]**2)*dt+ sig[0]*np.sqrt(dt)*z[0])

            S2= S2*np.exp( (mu[1]+0.5*sig[1]**2)*dt+ sig[1]*np.sqrt(dt)*z[1])

            S3=S3*np.exp( (mu[2]+0.5*sig[2]**2)*dt+ sig[2]*np.sqrt(dt)*z[2])
```

```
            if S2> B:

                indic=1
            else:

                indic= indic


        Ut= a[0]*S1+ a[1]*S2+ a[2]*S3

        if indic==1:

            payoff= disc*max(S2-K,0)


        else:

            payoff= disc*max(Ut-K,0)


        sum_C+= payoff



    value= sum_C/M

    return value
```

[79]:
```
S0=[100,101,98]
sig=[0.05,0.2,0.15]
mu=[0.03,0.06,0.02]
a=[1/3,1/3,1/3]
A= np.matrix([[1,0.5,0.2],
                [0.5,1,-0.4],
                [0.2,-0.4,1]])
```

[80]:
```
exgbm_basket1(S0=S0,K=100,r=0.
 ↪06,T=1,mu=mu,sig=sig,a=a,optype="c",B=104,A=A,N=100,M=100)
```

[80]: 16.47735805824951

- using different technique

[81]:
```
def exgbm_basket1(S0,K,r,T,mu,sig,a,optype,B,A,N,M):


    dt=T/N

    L=cholesky(A)

    disc= np.exp(-r*T)

    sum_C=0

    #nudt= (r+0.5*sig**2)*dt
    #sigsdt= sig*np.sqrt(dt)

    for i in range(M):


        S1= S0[0]

        S2= S0[1]

        S3= S0[2]

        indic=0

        for j in range(N):


            #nudt= (r-div- 0.5*sig**2)*dt
            #sigsdt= sig*np.sqrt(dt)


            row_z=np.random.normal(0,1,3)

            z= np.dot(L,row_z)

            S1= S1+ mu[0]*S1*dt+ sig[0]*S1*z[0]*np.sqrt(dt)

            S2= S2+ mu[1]*S2*dt+ sig[1]*S2*z[1]*np.sqrt(dt)

            S3= S3+ mu[2]*S3*dt+ sig[2]*S3*z[2]*np.sqrt(dt)



            if S2> B:

                indic=1
```

```
            else:

                indic= indic


        Ut= a[0]*S1+ a[1]*S2+ a[2]*S3

        if indic==1:

            payoff= disc*max(S2-K,0)


        else:

            payoff= disc*max(Ut-K,0)


        sum_C+= payoff



    value= sum_C/M

    return value
```

[82]:
```
exgbm_basket1(S0=S0,K=100,r=0.
↪06,T=1,mu=mu,sig=sig,a=a,optype="c",B=104,A=A,N=100,M=100)
```

[82]: 13.534326385243855

3.4.2 (ii)

[84]:
```
def exgbm_basket1(S0,K,r,T,mu,sig,a,optype,B,A,N,M):


    dt=T/N

    L=cholesky(A)

    disc= np.exp(-r*T)

    sum_C=0
```

```
#nudt= (r+0.5*sig**2)*dt
#sigsdt= sig*np.sqrt(dt)

for i in range(M):


    S1= S0[0]

    S2= S0[1]

    S3= S0[2]

    indic=0

    max2=S2
    max3=S3

    for j in range(N):


        #nudt= (r-div- 0.5*sig**2)*dt
        #sigsdt= sig*np.sqrt(dt)




        row_z=np.random.normal(0,1,3)

        z= np.dot(L,row_z)

        S1= S1*np.exp( (mu[0]+0.5*sig[0]**2)*dt+ sig[0]*np.sqrt(dt)*z[0])

        S2= S2*np.exp( (mu[1]+0.5*sig[1]**2)*dt+ sig[1]*np.sqrt(dt)*z[1])

        S3=S3*np.exp( (mu[2]+0.5*sig[2]**2)*dt+ sig[2]*np.sqrt(dt)*z[2])


        if S2>max2:

            max2=S2
        else:

            max2=max2


        if S3>max3:
```

```
            max3=S3

        else:

            max3=max3





    if max2> max3:

        indic=1
    else:

        indic=0



    Ut= a[0]*S1+ a[1]*S2+ a[2]*S3


    if indic==1:


        payoff= disc*max(S2**2-K,0)

    elif indic==0 :


        payoff= disc*max(Ut-K,0)


    sum_C+= payoff



value= sum_C/M

return value
```

```
[85]: exgbm_basket1(S0=S0,K=100,r=0.
       ↪06,T=1,mu=mu,sig=sig,a=a,optype="c",B=104,A=A,N=100,M=100)
```

```
[85]: 9131.30610829641
```

3.4.3 (iii) & (iv)

```
[86]: def exgbm_basket2(S0,K,r,T,mu,sig,a,optype,B,A,N,M):


          dt=T/N

          L=cholesky(A)

          disc= np.exp(-r*T)

          sum_C=0

          #nudt= (r+0.5*sig**2)*dt
          #sigsdt= sig*np.sqrt(dt)

          for i in range(M):


              S1= S0[0]

              S2= S0[1]

              S3= S0[2]

              indic=0

              sumS2=0
              sumS3=0


              for j in range(N):


                  #nudt= (r-div- 0.5*sig**2)*dt
                  #sigsdt= sig*np.sqrt(dt)
```

```
        row_z=np.random.normal(0,1,3)

        z= np.dot(L,row_z)

        S1= S1*np.exp( (mu[0]+0.5*sig[0]**2)*dt+ sig[0]*np.sqrt(dt)*z[0])

        S2= S2*np.exp( (mu[1]+0.5*sig[1]**2)*dt+ sig[1]*np.sqrt(dt)*z[1])

        S3=S3*np.exp( (mu[2]+0.5*sig[2]**2)*dt+ sig[2]*np.sqrt(dt)*z[2])



        sumS2+=S2

        sumS3+=S3


    A2= sumS2/N

    A3= sumS3/N

    if A2>A3:

        indic=1

    else:

        indic=0



    Ut= a[0]*S1+ a[1]*S2+ a[2]*S3


    if indic==1:


        payoff= disc*max(A2-K,0)

    elif indic==0 :


        payoff= disc*max(Ut-K,0)
```

```
        sum_C+= payoff



    value= sum_C/M

    return value
```

[87]:
```
exgbm_basket2(S0=S0,K=100,r=0.
 ↪06,T=1,mu=mu,sig=sig,a=a,optype="c",B=104,A=A,N=100,M=100)
```

[87]: 8.7660945324578

4 Problem 4

4.1 (a)

[88]:
```
def Monte_Carlo8(n_,m_,T,S0,V0,k,theta,sig,rho,r,K,f1,f2,f3):
    start = time.time()
    dt = T/n_
    dis = np.exp(-r*T)
    sum_C = 0
    sum_C2 = 0
    for i in range(int(m_)):
        lnSt = np.log(S0)
        Vtt = V0
        Vt = V0
        for j in range(int(n_)):
            z1 = np.random.randn()
            z2 = np.random.randn()
            w1 = z1
            w2 = rho*z1+np.sqrt(1-rho**2)*z2
            Vtt = f1(Vtt)-k*dt*(f2(Vtt)-theta)+sig*f3(Vtt)**0.5*w1*np.sqrt(dt)
            lnSt += (r-0.5*Vt)*dt+np.sqrt(Vt)*w2*np.sqrt(dt)
            Vt = f3(Vtt)
        St = np.exp(lnSt)
        C = dis*max(St-K,0)
        sum_C += C
        sum_C2+= C**2
    mean_C = sum_C/m_
    bias = abs(6.8061-mean_C)
```

```python
    se = np.sqrt((sum_C2-m_*mean_C**2)/(m_-1)/m_)
    end = time.time()
    return mean_C, bias, se, (end-start)

def fa(x):
    return max(x,0)
def fb(x):
    return abs(x)
def fc(x):
    return x
```

[89]:
```python
Monte_Carlo8(n_=1000,m_=100,T=1,S0=100,V0=0.010201,
             k=6.21,theta=0.019,sig=0.61,rho=-0.7,r=0.0319,
             K=100,f1=fa,f2=fa,f3=fa)
```

[89]:
```
(6.1044497330434115,
 0.7016502669565883,
 0.7587362593289545,
 0.8396680355072021)
```

[90]:
```python
Monte_Carlo8(n_=100,m_=500,T=1,S0=100,V0=0.010201,
             k=6.21,theta=0.019,sig=0.61,rho=-0.7,r=0.0319,
             K=100,f1=fb,f2=fb,f3=fb)
```

[90]:
```
(7.3252737683098825,
 0.5191737683098827,
 0.3551039754424081,
 0.3923211097717285)
```

[91]:
```python
Monte_Carlo8(n_=100,m_=500,T=1,S0=100,V0=0.010201,
             k=6.21,theta=0.019,sig=0.61,rho=-0.7,r=0.0319,
             K=100,f1=fc,f2=fc,f3=fb)
```

[91]:
```
(7.02392340535475,
 0.21782340535474987,
 0.3529166508380937,
 0.3840038776397705)
```

[92]:
```python
Monte_Carlo8(n_=1000,m_=100,T=1,S0=100,V0=0.010201,
             k=6.21,theta=0.019,sig=0.61,rho=-0.7,r=0.0319,
             K=100,f1=fc,f2=fc,f3=fa)
```

[92]:
```
(7.035748224270574, 0.22964822427057374, 0.8174595162274783, 0.784987211227417)
```

[93]:
```python
Monte_Carlo8(n_=1000,m_=100,T=1,S0=100,V0=0.010201,
             k=6.21,theta=0.019,sig=0.61,rho=-0.7,r=0.0319,
             K=100,f1=fc,f2=fa,f3=fa)
```

[93]: (7.269970379439593, 0.4638703794395935, 0.7677858645423148, 0.8132710456848145)

4.2 (b)

[94]:
```
def simpson_int(func,a,b,tol):
    n=10000
    delta = (b-a)/n
    x = np.linspace(a,b,n+1)
    f_x = np.asarray([func(i) for i in x])
    res0 = 0
    res1 = delta/3*(f_x[0]+f_x[-1]+4*f_x[1:-1][::2].sum()+2*f_x[1:-1][1::2].
↪sum())
    while abs(res1-res0)>tol:
        n= n + 10000
        x = np.linspace(a,b,n+1)
        f_x = np.asarray([func(i) for i in x])
        delta = (b-a)/n
        res0 = res1
        res1 = delta/3*(f_x[0]+f_x[-1]+4*f_x[1:-1][::2].sum()+2*f_x[1:-1][1::2].
↪sum())
    return res1
```

[95]:
```
simpson_int(func=(lambda x: x**2),a=0,b=100,tol=0.00001)
```

[95]: 333333.3333333334

[96]:
```
import scipy.integrate as integrate
def C_integral(tau,S0,V0,k,theta,sig,rho,r,K):
    u1 = 0.5
    u2 = -0.5
    a = k*theta
    b1 = k-rho*sig
    b2 = k

    def f1(u):
        com = np.complex(b1,-rho*sig*u)
        d1 = np.sqrt((-com)**2-sig**2*(np.complex(0,2*u1*u)-u**2))
        g1 = (com+d1)/(com-d1)
        C1 = np.complex(0,r*u*tau)+a/sig**2*((com+d1)*tau-2*np.log((1-g1*np.
↪exp(d1*tau))/(1-g1)))
        D1 = (com+d1)/sig**2*((1-np.exp(d1*tau))/(1-g1*np.exp(d1*tau)))
        phi1 = np.exp(C1+D1*V0+np.complex(0,u*np.log(S0)))
        res = ((np.exp(np.complex(0,-np.log(K)*u))*phi1)/(np.complex(0,u))).real
        return res
    def f2(u):
        com = np.complex(b2,-rho*sig*u)
```

```python
        d2 = np.sqrt((-com)**2-sig**2*(np.complex(0,2*u2*u)-u**2))
        g2 = (com+d2)/(com-d2)
        C2 = np.complex(0,r*u*tau)+a/sig**2*((com+d2)*tau-2*np.log((1-g2*np.
↪exp(d2*tau))/(1-g2)))
        D2 = (com+d2)/sig**2*((1-np.exp(d2*tau))/(1-g2*np.exp(d2*tau)))
        phi2 = np.exp(C2+D2*V0+np.complex(0,u*np.log(S0)))
        res = ((np.exp(np.complex(0,-np.log(K)*u))*phi2)/(np.complex(0,u))).real
        return res
    P1 = 0.5+integrate.quad(f1,0.0001,1500)[0]/np.pi
    P2 = 0.5+integrate.quad(f2,0.0001,1500)[0]/np.pi
    result = S0*P1-K*np.exp(-r*tau)*P2
    return result
```

[97]:
```python
C_integral(tau=1,S0=100,V0=0.010201,
             k=6.21,theta=0.019,sig=0.61,rho=-0.7,r=0.0319,
             K=100)
```

[97]: 8.472087268998855

- Comparing to values in a and b, the results are similar with given paramaters: tau=1 S0=100 V0=0.010201 k=6.21 theta=0.019 sig=0.61 rho=-0.7 r=0.0319 K=100

[ ]: