FE542-HW5

M.Furkan Isik

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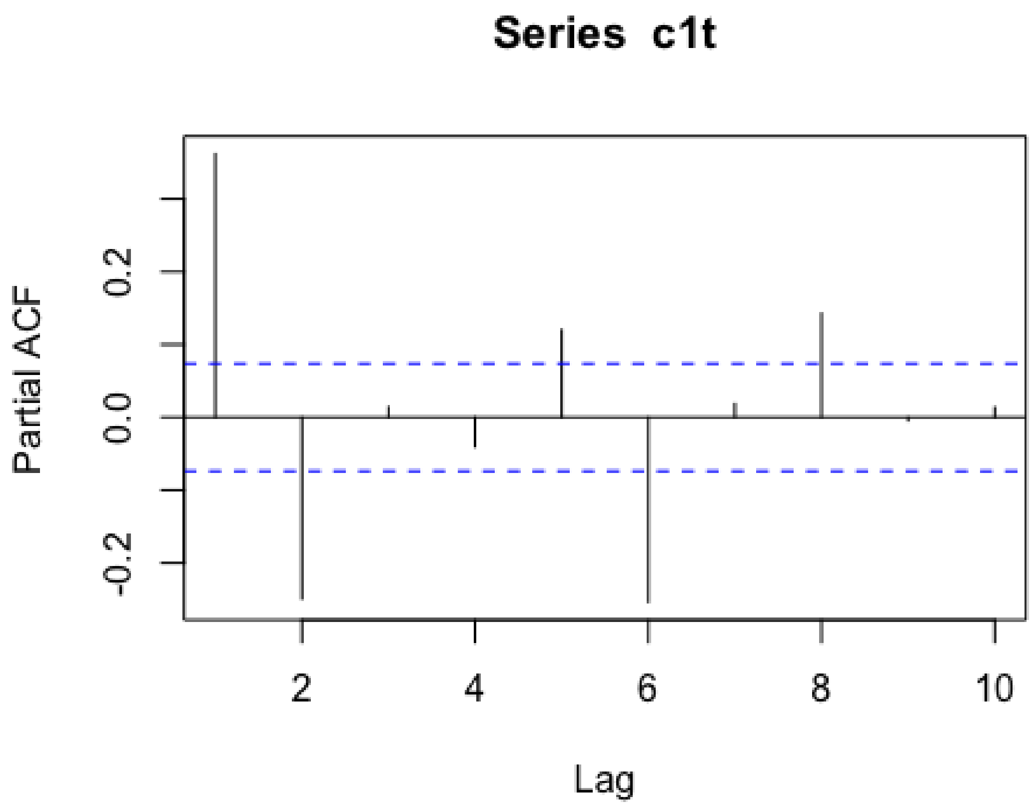
Problem 1

(i) Construct single time series autoregressive models for the single time series c1t and c10t.

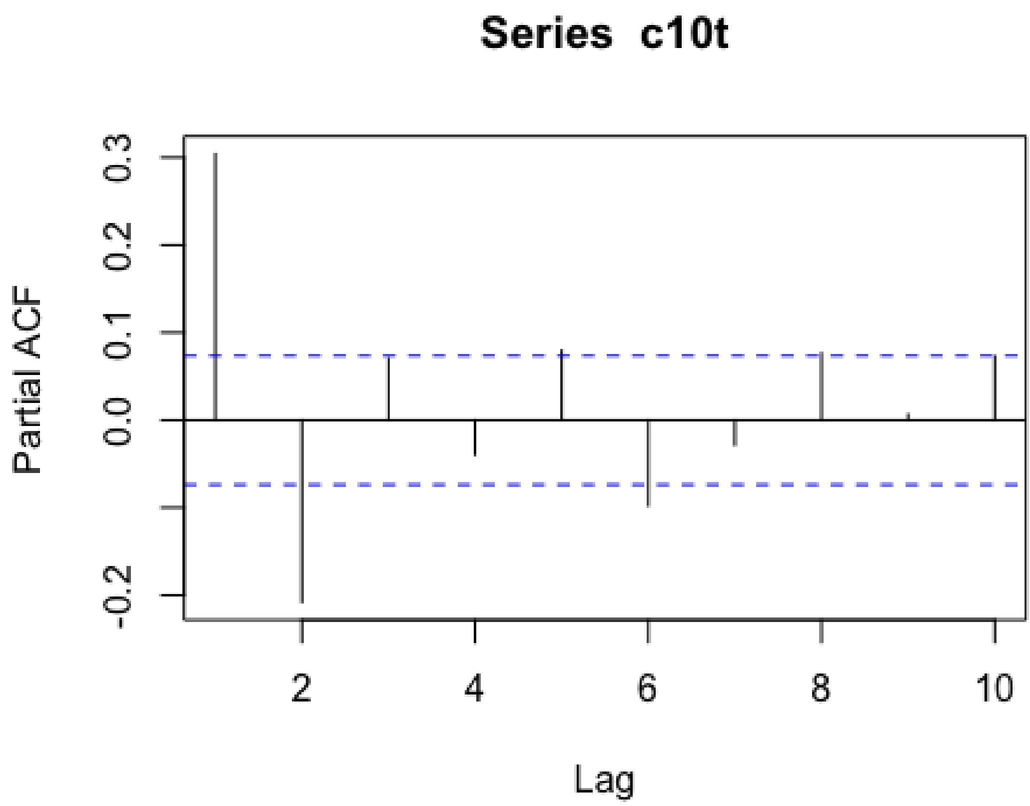
- To decide order of the model, pacf values with different lags observed
- For c1t we have lowest pacf value at lag 3 and lag 7, therefore lag 2 was chosen for order of the model
- For c10t we have lowest pacf value at lag 4 and lag 7, therefore lag 3 was chosen for order model

```
df <- read.csv("homework05.csv")
c1t=df$DGS1[2:708]-df$DGS1[1:707]
c10t= df$DGS10[2:708]-df$DGS10[1:707]

ct <- data.frame(c1t,c10t)
c1t.pacf <- pacf(x = c1t, lag.max = 10)
```




```
c10t.pacf <- pacf(c10t, lag.max = 10)
```



- Constructing ar models with order 2 and order 3

```
ar1 <- arima(c1t, order = c(2,0,0))
ar10 <- arima(c10t, order = c(3,0,0))
ar1

##
## Call:
## arima(x = c1t, order = c(2, 0, 0))
##
## Coefficients:
##          ar1      ar2  intercept
##       0.4509  -0.2485    -0.0044
## s.e.  0.0364   0.0364     0.0178
##
## sigma^2 estimated as 0.1427:  log likelihood = -315.09,  aic = 638.18

ar10

##
## Call:
## arima(x = c10t, order = c(3, 0, 0))
##
## Coefficients:
```

```
##          ar1      ar2     ar3  intercept
##       0.3812  -0.2331  0.0703    -0.0044
## s.e.  0.0375   0.0391  0.0374     0.0125
##
## sigma^2 estimated as 0.06744:  log likelihood = -50.06,  aic = 110.12
```

ii) Build a bivariateautoregressive model for the two change series.

- Used Varselect function to find the lowest AIC value
- Lag 2 has the huge drop in AIC value, after that although values are smaller, there is not a huge drop, so lag 2 is chosen

```
library(vars)

## Loading required package: MASS

## Loading required package: strucchange

## Loading required package: zoo

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

## Loading required package: sandwich

## Loading required package: urca

## Loading required package: lmtest

ct.var <- VARselect(ct, lag.max = 5)
ct.var$criteria

##                    1            2            3            4            5
## AIC(n) -5.407818909 -5.486275729 -5.496909863 -5.507872436 -5.509269022
## HQ(n)  -5.392775056 -5.461202641 -5.461807540 -5.462740877 -5.454108228
## SC(n)  -5.368896401 -5.421404882 -5.406090678 -5.391104912 -5.366553160
## FPE(n)  0.004481404  0.004143248  0.004099425  0.004054736  0.004049087

VAR_est <- VAR(ct,2)
summary(VAR_est)

##
## VAR Estimation Results:
## =========================
## Endogenous variables: c1t, c10t
## Deterministic variables: const
## Sample size: 705
## Log Likelihood: -53.869
## Roots of the characteristic polynomial:
## 0.4443 0.4443 0.4392 0.4392
```

```
## Call:
## VAR(y = ct, p = 2)
##
##
## Estimation results for equation c1t:
## ====================================
## c1t = c1t.l1 + c10t.l1 + c1t.l2 + c10t.l2 + const
##
##          Estimate Std. Error t value Pr(>|t|)
## c1t.l1   0.332956   0.057122   5.829 8.52e-09 ***
## c10t.l1  0.239154   0.083213   2.874  0.00418 **
## c1t.l2  -0.138684   0.056136  -2.470  0.01373 *
## c10t.l2 -0.195586   0.083230  -2.350  0.01905 *
## const   -0.003159   0.014183  -0.223  0.82379
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.3765 on 700 degrees of freedom
## Multiple R-Squared: 0.1978,  Adjusted R-squared: 0.1932
## F-statistic: 43.15 on 4 and 700 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation c10t:
## =====================================
## c10t = c1t.l1 + c10t.l1 + c1t.l2 + c10t.l2 + const
##
##          Estimate Std. Error t value Pr(>|t|)
## c1t.l1  -0.006801   0.039677  -0.171    0.864
## c10t.l1  0.375589   0.057800   6.498 1.54e-10 ***
## c1t.l2   0.026274   0.038992   0.674    0.501
## c10t.l2 -0.237496   0.057812  -4.108 4.46e-05 ***
## const   -0.003586   0.009852  -0.364    0.716
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.2615 on 700 degrees of freedom
## Multiple R-Squared: 0.1321,  Adjusted R-squared: 0.1272
## F-statistic: 26.64 on 4 and 700 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##          c1t    c10t
## c1t  0.14176 0.07513
## c10t 0.07513 0.06839
##
## Correlation matrix of residuals:
##        c1t  c10t
```

```
## c1t  1.000 0.763
## c10t 0.763 1.000
```

- The reduced form of VAR(1) model is similar to this r(t)=phi(0)+phi(1)*r(t-1)+phi(2)*r(t-1)+a(t)

```
phi0 =
c(VAR_est$varresult$c1t$coefficients[5],VAR_est$varresult$c10t$coefficients[5
])
phi1 = matrix(c(VAR_est$varresult$c1t$coefficients[1:2],
VAR_est$varresult$c10t$coefficients[1:2]), nrow = 2, byrow = T)
phi2 = matrix(c(VAR_est$varresult$c1t$coefficients[3:4],
VAR_est$varresult$c10t$coefficients[3:4]),nrow = 2, byrow = T)
phi0

##        const        const
## -0.003159349 -0.003585675

phi1

##              [,1]      [,2]
## [1,]  0.332956265 0.2391538
## [2,] -0.006801116 0.3755892

phi2

##             [,1]       [,2]
## [1,] -0.13868370 -0.1955858
## [2,]  0.02627412 -0.2374957
```

iii) Transform the constructed bivariate model into a structural form.

- The bi-variate model will be transfered into structural form
- In order to do that, covariance matrix of residuals created and cholesky decompostion applied
- The VAR(2) structural form would be, Ar(t)=phi0.star + phi1.star*r(t-1) + phi2.star*r(t-2) + b(t)

```
amat = diag(2)
amat[2,1] = NA
svar1 = SVAR(VAR_est, Amat = amat)
A = svar1$A
B = svar1$B
phi0.star = A%*%phi0
phi1.star = A%*%phi1
phi2.star = A%*%phi2
phi0.star

##              [,1]
## c1t  -0.003159349
## c10t -0.001911321

phi1.star
```

```
##             [,1]      [,2]
## c1t   0.3329563 0.2391538
## c10t -0.1832573 0.2488453
```

```
phi2.star
```

```
##              [,1]       [,2]
## c1t  -0.13868370 -0.1955858
## c10t  0.09977205 -0.1338415
```

- phi1 value is -0.006801116
- phi 2 value is 0.02627412 and both phi1 and phi2 is very close to 0.
- it could be said that lag 1 and lag 2 don't have crucial impact on c10t and c1t prediction

```
phi1
```

```
##               [,1]      [,2]
## [1,]  0.332956265 0.2391538
## [2,] -0.006801116 0.3755892
```

```
phi2
```

```
##              [,1]       [,2]
## [1,] -0.13868370 -0.1955858
## [2,]  0.02627412 -0.2374957
```

(iv) Briefly discuss the implications of the vector autoregressive model and compare with the single time series models.

- Both models were compared in terms of Rsquared values
- R squared value of AR model c1t=0.1846312 and c10t=0.1359105
- R squared value of VAR model c1t= 0.1978 c10t=0.1321
- Therefore, it could be said that VAR model is slighly better in comparison to sigle time series model in terms of explaning the variability

```
SST1 = c1t-mean(c1t)
SST1 = SST1^2
SST1 = sum(SST1)
RST1 = (ar1$residuals)^2
RST1 = sum(RST1)
Rsqrd1 <- 1 - (RST1/SST1)
Rsqrd1
```

```
## [1] 0.1846312
```

```
SST2 = c10t-mean(c10t)
SST2 = SST2^2
SST2 = sum(SST2)
RST2 = (ar10$residuals)^2
RST2 = sum(RST2)
```

```
Rsqrd2 <- 1 - (RST2/SST2)
Rsqrd2

## [1] 0.1359105
```