Homework 4

FE-542

Due: April 16, 2021 at 5PM

Problem 1 (50pt)
Suppose that the monthly log returns, in percentages, of a stock follow the following Markov switching model:

$$r_t = 1.25 + a_t, \qquad a_t = \sigma_t \epsilon_t,$$
$$\sigma_t^2 = \begin{cases} 0.2a_{t-1}^2 + 0.9\sigma_{t-1}^2 & \text{if } s_t = 1 \\ 4.25 + 0.1a_{t-1}^2 + 0.75\sigma_{t-1}^2 & \text{if } s_t = 2 \end{cases}$$

where the transition probability are

$$\mathbb{P}(s_t = 2 \mid s_{t-1} = 1) = 0.2, \qquad \mathbb{P}(s_t = 1 \mid s_{t-1} = 2) = 0.1.$$

Suppose that $a_{100} = 6$, $\sigma_{100}^2 = 50$, and $s_{100} = 2$ with probability 1.

(i) What is the 1-step-ahead volatility forecast at the forecast origin $t = 100$?

(ii) If the probability of $s_{100} = 2$ is reduced to 0.75, what is the 1-step-ahead volatility forecast origin $t = 100$.

Bonus In R create a report in pdf format using RMarkdown (or, if you choose to use Python instead, create a Jupyter notebook) to implement this Markov switching model and compare the forecasts you computed to simulated results.

Problem 2 (50pt)
In R create a report in pdf format using RMarkdown (or, if you choose to use Python instead, create a Jupyter notebook) to:

(i) Download daily price data for January 1, 2000 through December 31, 2020 of Amazon stock from Yahoo Finance. Compute the *weekly* logarithmic returns r_t. You may use the quantmod package in R for this purpose.

(ii) Using lagged returns $r_{t-1}, r_{t-2}, r_{t-3}$ as input, build a 3-2-1 feed-forward neural network to forecast 1-step-ahead returns. Use data up to December 31, 2018 as the training data set and the remainder as the testing data. Calculate the mean squared error on the test data.

(iii) Using lagged returns $r_{t-1}, r_{t-2}, r_{t-3}$ and their signs (directions) to build a 6-5-1 feed-forward neural network to forecast the 1-step-ahead *direction* of Microsoft stock price movement (with 1 denoting upward movement and 0 downward movement). Use data up to December 31, 2018 as the training data set and the remainder as the testing data. Calculate the mean squared error on the test data.
Note: Let `rtn` denote a time series in R. To create a direction variable for `rtn`, use the command

```
drtn = ifelse(rtn > 0, 1, 0)
```