Problem 1

i) $E(r_t) = \frac{\phi_0}{1-\phi_1-\phi_2}$ where $r_t = \underbrace{0.03}_{\phi_0} + \underbrace{0}_{\phi_1} r_{t-1} + \underbrace{0.2}_{\phi_2} r_{t-2} + a_t$

$E(r_t) = \frac{0.03}{1-0-0.2} = \underline{\underline{0.0375}}$

$Var(r_t) = \frac{\sigma_a^2}{1-\phi_1^2-\phi_2^2} = \frac{0.1}{1-0-(0.20)^2}$
$= \underline{\underline{0.104166}}$

ii) $\rho_1 = \frac{\phi_1}{1-\phi_2}$ lag one auto correlation $\rightarrow \frac{0}{1-0.2} = \underline{\underline{0}}$

$\rho_\ell = \phi_1 \rho_{\ell-1} + \phi_2 \rho_{\ell-2}$, $\ell \geq 2 \rightarrow \rho_2 = \phi_1 \rho_1 + \phi_2 \rho_0 = 0 \times 0 + 0.2 \times 1 = \underline{\underline{0.2}}$

* For stationary AR(2) series r_t, we have $\rho_0 = 1$

iii)

$\hat{r}_{100}(1) = \phi_0 + \phi_1 r_{100} + \phi_2 r_{99}$ where $\phi_0 = 0.03$ $\phi_1 = 0$ $\phi_2 = 0.2$

$\hat{r}_{100}(1) = 0.03 + 0 \times (-0.02) + 0.2(0.01) = \underline{\underline{0.032}}$

$Var(e_{100}(1)) = \sigma_a^2 = \underline{\underline{0.1}}$

$\hat{r}_{100}(2) = \phi_0 + \phi_1 \hat{r}_{100}(1) + \phi_2 r_{100} = 0.03 + 0 \times (0.032) + 0.20(-0.02) = \underline{\underline{0.026}}$

$Var(e_h(2)) = (1+\phi_1^2)\sigma_a^2 \rightarrow (1+0) . 0.1 = \underline{\underline{0.1}}$

Problem 2

i) $E[R_t] = E[a_t] + 0.2E[a_{t-1}]$ Mean of white noise equals 0

Then $= E[R_t] = \underline{\underline{0}}$

$Var[R_t] = (1+\theta_1^2)\sigma_a^2$ where $\theta_1 = 0{,}2$

$Var(R_t) = (1+0{,}2^2) \times 0{,}001 = \underline{\underline{0{,}00104}}$

ii)

$\rho_1 = \dfrac{\theta_1}{1+\theta_1^2}$ where $\theta_1 = 0{,}2 \rightarrow \rho_1 = \dfrac{0{,}2}{1+(0{,}4)^2} = \underline{\underline{0{,}1923}}$

$\rho_l = 0$ for $l > 1$ then $\rho_2 = 0$

iii) $\hat{R}_{100}(1) = \mu + \theta_1 a_{100}$ where $\mu = 0$ and $\theta_1 = 0{,}20$, $a_{100} = -0{,}01$

$\hat{R}_{100}(1) = 0 + 0{,}20 \times (-0{,}01) = \underline{\underline{-0{,}002}}$

$Var(e_{100}(1)) = \sigma_a^2 = 0{,}001$

$\hat{R}_{100}(2) = 0$ $\quad Var(e_{100}(2)) = (1+\theta_1^2)\sigma_a^2 = (1+(0{,}2)^2) \times 0{,}001$

$= \underline{\underline{0{,}00104}}$

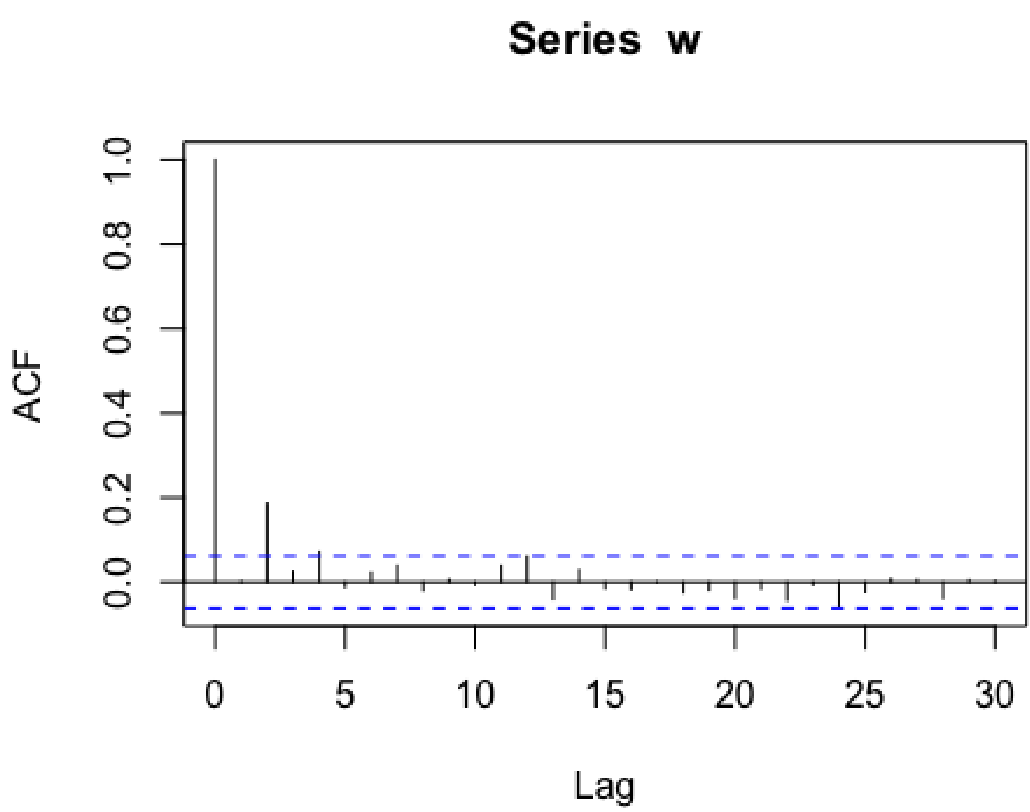
FE 542-HW2

```
library(stats)
```

Problem 1

(a) Simulate 1000 terms of this time series withr0=−0.02 andr−1= 0.01.

```
w=arima.sim(n=1000,model=list(ar=c(0,0.20)), innov =
rnorm(n=1000,mean=0,sd=sqrt(0.1)))
head(w)

## [1] -0.08716183  0.68384449 -0.02036859  0.04187873 -0.01063129
0.46784668

acf(w)
```




```
#ts.plot(w)
```

(b) Using the generated time series, find the sample mean and variance. How do thesevalues compare with those computed analytically?

- The mean is close to 0 which is the analytical value for the mean

- variance is almost 0.1 which is the analytical value for the variance

```
mean(w)

## [1] 0.02084584

var(w)

## [1] 0.1062774
```

(c) Using the generated time series, find the sample lag-1 and lag-2 autocorrelations.How do these values compare with those computed analytically?

- Lag one auto correlation is 0.055 which is very close to analytical value of 0
- Lag two auto correlation is 0.165 which is also close to analytical value of 0.2

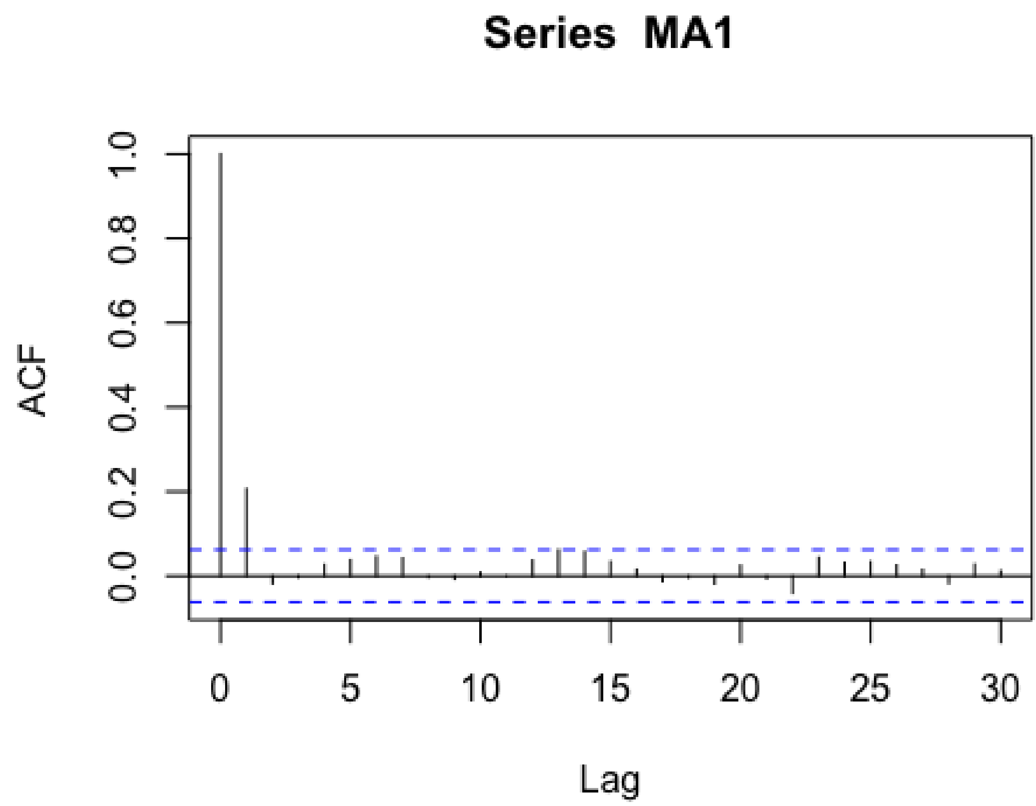
```
acf(w,lag.max=2,plot=FALSE)

##
## Autocorrelations of series 'w', by lag
##
##     0     1     2
## 1.000 0.001 0.186
```

Problem 2

iv) In R create a report in pdf format using RMarkdown (or, if you choose to use Pythoninstead, create a Jupyter notebook) to:

(a) Simulate 1000 terms of this time series.

```
MA1=arima.sim(n=1000, model=list(ma=c(0.20)),innov =
rnorm(1000,mean=0,sd=sqrt(0.001)))

head(MA1)

## [1]  0.24867696 -0.02556188  0.04355564  0.03901960 -0.05015004 -
0.01207619

acf(MA1)
```



(b) Using the generated time series, find the sample mean and variance. How do thesevalues compare with those computed analytically?

- The mean is close -0.003000503 which is very close to the analytical value of 0

- The variance is 0.001246287 which is very close to the analytical value of 0.00104

```
mean(MA1)

## [1] -0.0007889318

var(MA1)

## [1] 0.001124226
```

(c) Using the generated time series, find the sample lag-1 and lag-2 autocorrelations.How do these values compare with those computed analytically?

```
acf(MA1,lag.max = 2,plot=FALSE)

##
## Autocorrelations of series 'MA1', by lag
##
##      0      1      2
##  1.000  0.207 -0.018
```

- Lag one auto correlation is 0.191 whereas analytical value is 0.1923, they are pretty close to each other

- Lag two auto correlation is 0.019 whereas analytical value is 0, they are close to each other, it could be better to increase the number of terms

Problem 3

(i) Import the monthly yields of Moody's Aaa seasoned bonds from January 1, 1962 toDecember 31, 2020 fromhomework02.csvprovided on Canvas. The data are obtainedfrom the Federal Reserve Bank of St. Louis. Monthly yields are averages of daily yields

```
Moodys=read.csv("/Users/metuhead/Desktop/FE 542/homework02.csv")

head(Moodys)

##         DATE  AAA
## 1 1962-01-01 4.42
## 2 1962-02-01 4.42
## 3 1962-03-01 4.39
## 4 1962-04-01 4.33
## 5 1962-05-01 4.28
## 6 1962-06-01 4.28
```

(ii) Obtain the summary statistics (sample mean, standard deviation, skewness,excesskurtosis) of this yield series.

- Sample Mean is 7.063729

- Standard Deviation is 2.677317

- Skewness is 0.724596 the data is moderately skewed right

- Kurtosis is 0.286346 the tails of the distribution is almost behaves the same as normal distribution

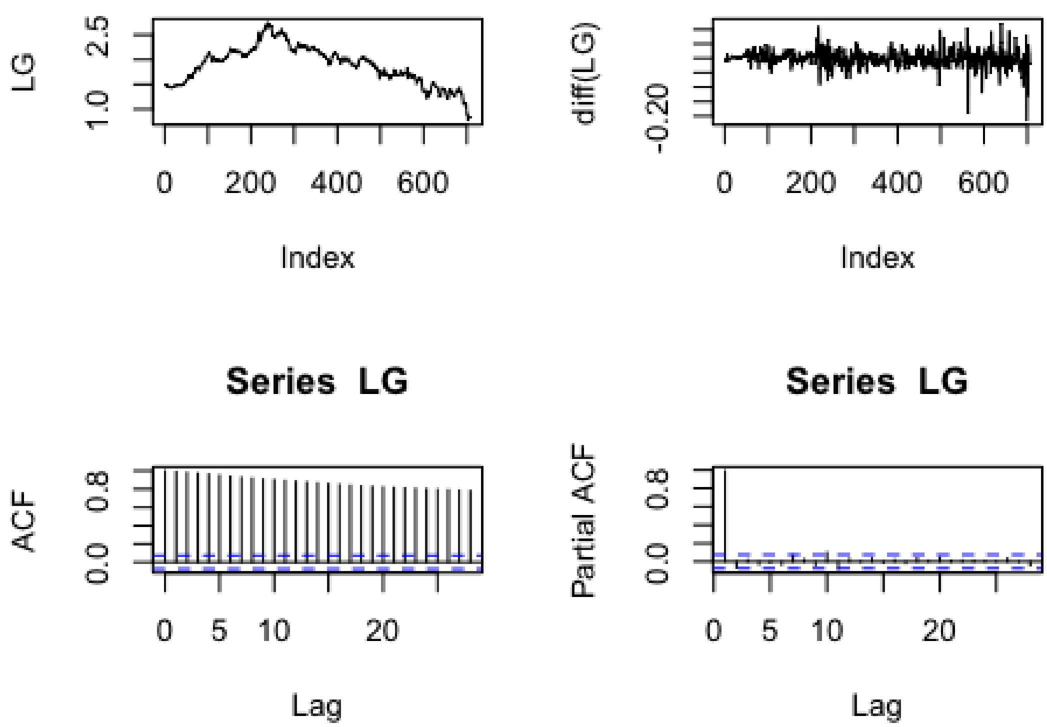
```
library(fBasics)

## Loading required package: timeDate

## Loading required package: timeSeries

basicStats(Moodys["AAA"])

##                    AAA
## nobs        708.000000
## NAs           0.000000
## Minimum       2.140000
## Maximum      15.490000
## 1. Quartile   4.987500
## 3. Quartile   8.602500
## Mean          7.063729
```

```
## Median         7.055000
## Sum         5001.120000
## SE Mean        0.100620
## LCL Mean       6.866180
## UCL Mean       7.261278
## Variance       7.168026
## Stdev          2.677317
## Skewness       0.724596
## Kurtosis       0.286346
```

(iii) Build a time series model for this data. Evaluate its performance. Justify your choices.

- taking log of the yields
- From the graph we see a exponential decay in a ACF, then for now AR model would be wise to use

```r
LG=log(Moodys$AAA)

par(mfrow=c(2,2))

# Plot of log of the yields
plot(LG,type="l")

# Plot of differencing of log of the yields

plot(diff(LG),type="l")

# Auto correlation plot of acf
acf(LG)
pacf(LG)
```



- From the AIC we see that AIC is lowest (0) at lag 8, so using order of 8 would be wise

```
m1=ar(diff(LG),method="mle")

m1$aic

##          0          1          2          3          4          5
6
## 32.8299572  3.0987734  1.1630814  0.9988615  2.2114128  3.9877824
5.9628130
##          7          8          9         10         11         12
##  4.1295669  0.0000000  1.7843503  3.2513379  4.9900488  6.9316511
```

- Unit Root Test : perform the test H0 : β = 1 versus Ha : β < 1.
- Null hypothesis is there is a unit root, Alternative hypothesis there is no unit root
- When the process has no unit root, it is stationary and vice versa

The p value of our Augmented Dickey-Fuller Test is 0.4128, then we fail to reject null hypothesis. Hence the data is non stationary, it could be a good idea using ARIMA model

```
library(fUnitRoots)

adfTest(LG,lags=8)
```

```
##
## Title:
##  Augmented Dickey-Fuller Test
##
## Test Results:
##   PARAMETER:
##     Lag Order: 8
##   STATISTIC:
##     Dickey-Fuller: -0.6384
##   P VALUE:
##     0.4128
##
## Description:
##  Fri Mar 12 12:38:17 2021 by user:
```

- Qutoe from book: "In some studies, interest rates, foreign exchange rates, or the price series of an asset are of interest.These series tend to be nonstationary.In this case, the log price series is unit-root nonstationary and hence can be treated as an ARIMA process"
- We use difference method to transform nonstationary process to stationary process,
- I used the arima model for the data

```
m3=arima(diff(LG), order=c(8,0,0))
m3
```

```
##
## Call:
## arima(x = diff(LG), order = c(8, 0, 0))
##
## Coefficients:
##          ar1      ar2     ar3      ar4     ar5     ar6      ar7     ar8
##       0.2388  -0.0918  0.0602  -0.0280  0.0034  0.0321  -0.0974  0.0944
## s.e.  0.0374   0.0383  0.0384   0.0385  0.0386  0.0392   0.0390  0.0380
##       intercept
##         -0.0009
## s.e.     0.0014
##
## sigma^2 estimated as 0.0008608:  log likelihood = 1491.61,  aic = -2963.22
```