

## FE 542- HW1

### Problem 1

An urn contains three type A coins and two type B coin. When a type A coin is ipped, it comes up heads with probability  $1/3$ , whereas a type B coin is ipped, it comes up heads with robability  $2/3$ . A coin is randomly chosen from the urn and ipped. Given that theip landed on heads, what is the probability that it was a type B coin?

Solution:

0.5714286

$$\text{Prob}(B|\text{heads}) = P(\text{heads}|B) \cdot P(B) / (P(\text{heads}))$$

$P(\text{heads}|B)$ =if the coin is B prob of getting head is  $2/3$  which is given in the question

$P(\text{heads}|A)$ =if the coin is A prob of getting head is  $1/3$  which is given in the question

$P(B)=2/5$  There are totally 5 coins and 2 of them are B coins so the probability of chosing B coin is  $2/5$

$P(A)=3/5$  There are totally 5 coins and 3 of them are A coins so the probability of chosing A coin is  $3/5$

$$P(\text{heads}) = P(\text{heads}|B) \cdot P(B) + P(\text{heads}|A) \cdot P(A)$$

$$\text{Prob}(B|\text{heads}) = P(\text{heads}|B) \cdot P(B) / (P(\text{heads}))$$

- Calculating denominator of the equation

```
denominator=2/3*2/5 + 1/3*3/5
denominator
```

```
## [1] 0.4666667
```

- Calculating numerator of the equation

```
numerator=2/3*2/5
numerator
```

```
## [1] 0.2666667
```

- Showing the result

```
result=numerator/denominator
```

```
result
```

```
## [1] 0.5714286
```

## Problem 2

(i) Generate 100 observations from a normal distribution with mean 2 and variance 7.

```
sample=rnorm(100,mean=2,sd=sqrt(7))
```

```
sample
```

```
## [1] -4.39319271 2.14485098 0.16782802 1.24999814 1.95085494
3.37695288
## [7] -0.09983850 3.51536906 0.25479209 -3.41737617 -2.83300008
1.26784929
## [13] 6.21828415 4.93559201 2.92908836 3.68103218 -0.70468085
5.52113811
## [19] -2.68118838 5.70004080 5.38311197 2.63408977 -0.54217118 -
0.02963278
## [25] 3.32986502 -1.71698019 6.18417602 7.74089906 -1.34983370
1.16928112
## [31] 6.36333014 6.09995627 7.82872342 -0.05117579 -0.99696227
1.70894251
## [37] 6.69355490 1.96133355 4.69851664 3.07492388 3.44747303
1.49423940
## [43] 1.16253392 4.86653623 2.84427852 0.24793108 -2.60061588
7.07477571
## [49] -0.09511843 0.64983504 3.45096968 -0.32563908 2.09012501
0.78329327
## [55] -0.73704266 1.82728500 -0.54922955 3.73543878 -1.14264523 -
0.04276004
## [61] -0.39335280 4.85599984 -1.40847510 -0.97624015 2.58707994
1.70742788
## [67] 2.24554188 1.45597011 -3.08437790 -0.61904290 1.97440317
9.51518446
## [73] -1.75449114 5.15955868 1.12404805 -0.97147011 1.43355924
3.22375405
## [79] 5.48369089 6.93110660 -2.29646795 -3.49811820 0.84573388
7.27476165
## [85] 0.08175841 3.85296690 -1.83233339 2.80884243 1.50156079 -
2.05071184
## [91] 1.05228487 5.93271220 1.94415859 -0.26729167 2.45244755
3.08377865
## [97] -2.05149199 4.90560814 2.68053165 2.97145635
```

(ii) Compute the sample mean, standard deviation, skewness and kurtosis (if excess kurtosis state that clearly).

```
mean(sample)
```

```
## [1] 1.850321
```

```
sd(sample)
```

```
## [1] 2.977175
```

- Measure of assymetry
- if skewness is positive, the data is skewed right
- if skewness is negative, the data is skewed left
- if skewness is between  $-1$  and  $-\frac{1}{2}$  or between  $+\frac{1}{2}$  and  $+1$ , the distribution is moderately skewed.
- If skewness is between  $-\frac{1}{2}$  and  $+\frac{1}{2}$ , the distribution is approximately symmetric.
- If skewness is less than  $-1$  or greater than  $+1$ , the distribution is highly skewed.

We can say that sample is approximately symmetric since the skewness value is between  $-\frac{1}{2}$  and  $+\frac{1}{2}$

```
library(fBasics)

## Loading required package: timeDate

## Loading required package: timeSeries

skewness(sample)

## [1] 0.2393729
## attr(,"method")
## [1] "moment"
```

- Measure of tail thickness
- Negative excess kurtosis has short tails Platykurtic
- Positive excess kurtosis have heavy tails Leptokurtic

We can say that sample is Platykurtic(tails are light) since excess kurtosis value is negative

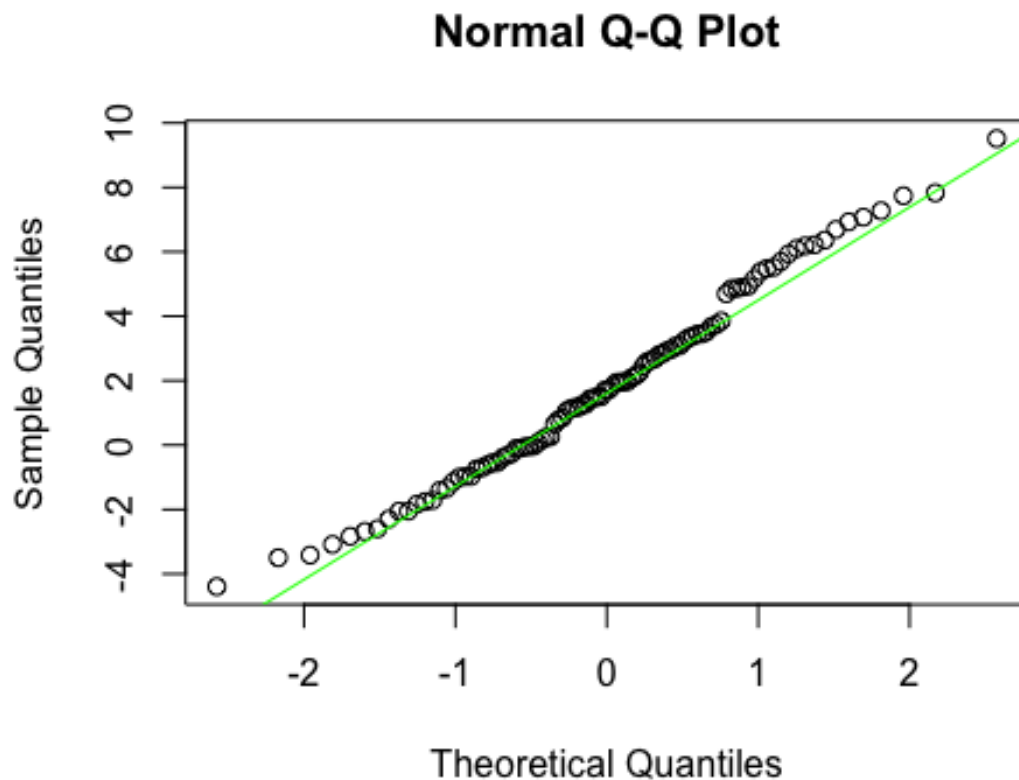
```
kurtosis(sample)

## [1] -0.5862402
## attr(,"method")
## [1] "excess"
```

(iii) Generate a Q-Q plot of the observations from (i) versus the standard normal distribution. Describe your plot in no more than 3 sentences.

- Data is fairly normal distributed since majority of the data points in line with the 45 degree normal line.

```
qqnorm(sample)
qqline(sample,col="green")
```



### Problem 3

- (i) Download daily price data for January 1, 2017 through December 31, 2020 of Microsoft stock from Yahoo Finance. You may use the quantmod package in R for this purpose.

```
library(quantmod)
```

```
## Loading required package: xts
```

```
## Loading required package: zoo
```

```
##
```

```
## Attaching package: 'zoo'
```

```
## The following object is masked from 'package:timeSeries':
```

```
##
```

```
##   time<-
```

```
## The following objects are masked from 'package:base':
```

```
##
```

```
##   as.Date, as.Date.numeric
```

```
## Loading required package: TTR
```

```
##
## Attaching package: 'TTR'

## The following object is masked from 'package:fBasics':
##
##      volatility

## Registered S3 method overwritten by 'quantmod':
##   method      from
## as.zoo.data.frame zoo

getSymbols('MSFT',src='yahoo',from='2017-01-01',to='2020-12-31')

## 'getSymbols' currently uses auto.assign=TRUE by default, but will
## use auto.assign=FALSE in 0.5-0. You will still be able to use
## 'loadSymbols' to automatically load data. getOption("getSymbols.env")
## and getOption("getSymbols.auto.assign") will still be checked for
## alternate defaults.
##
## This message is shown once per session and may be disabled by setting
## options("getSymbols.warning4.0"=FALSE). See ?getSymbols for details.

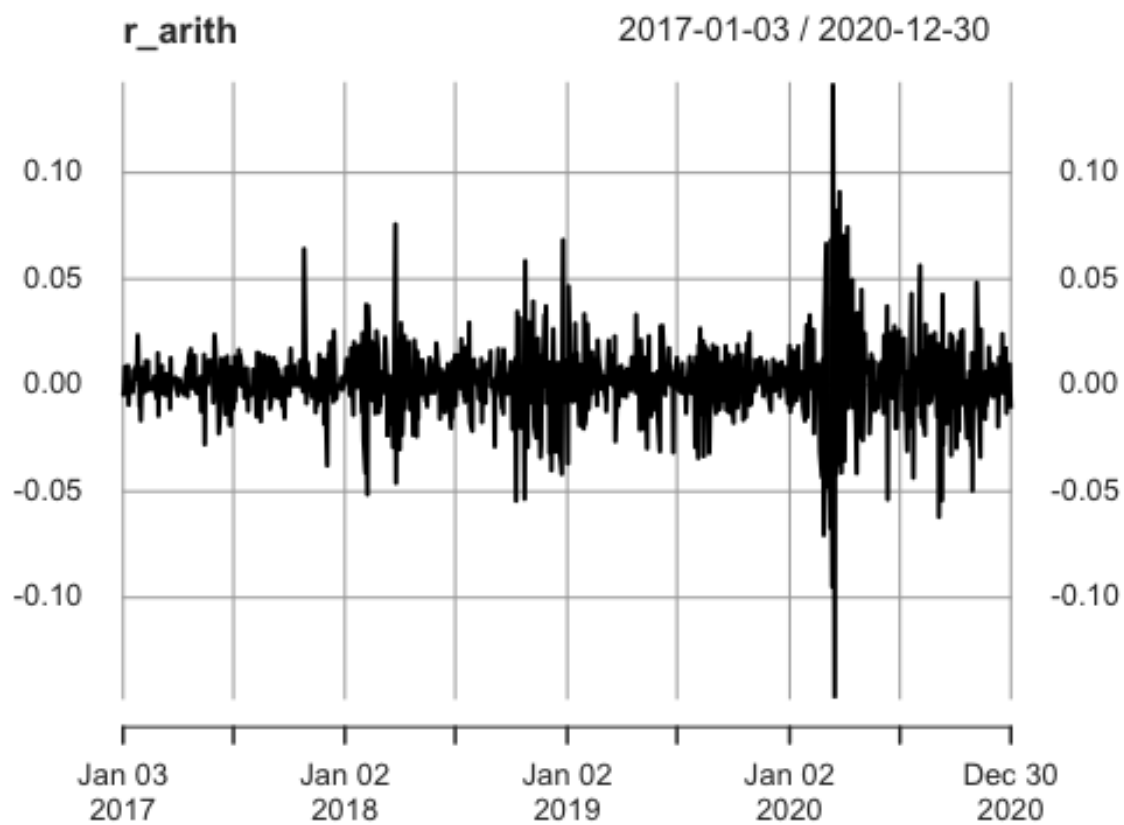
## [1] "MSFT"
```

(ii) Generate a plot of the closing prices and of daily log returns.

- plot of the daily return

```
r_arith=dailyReturn(MSFT)
```

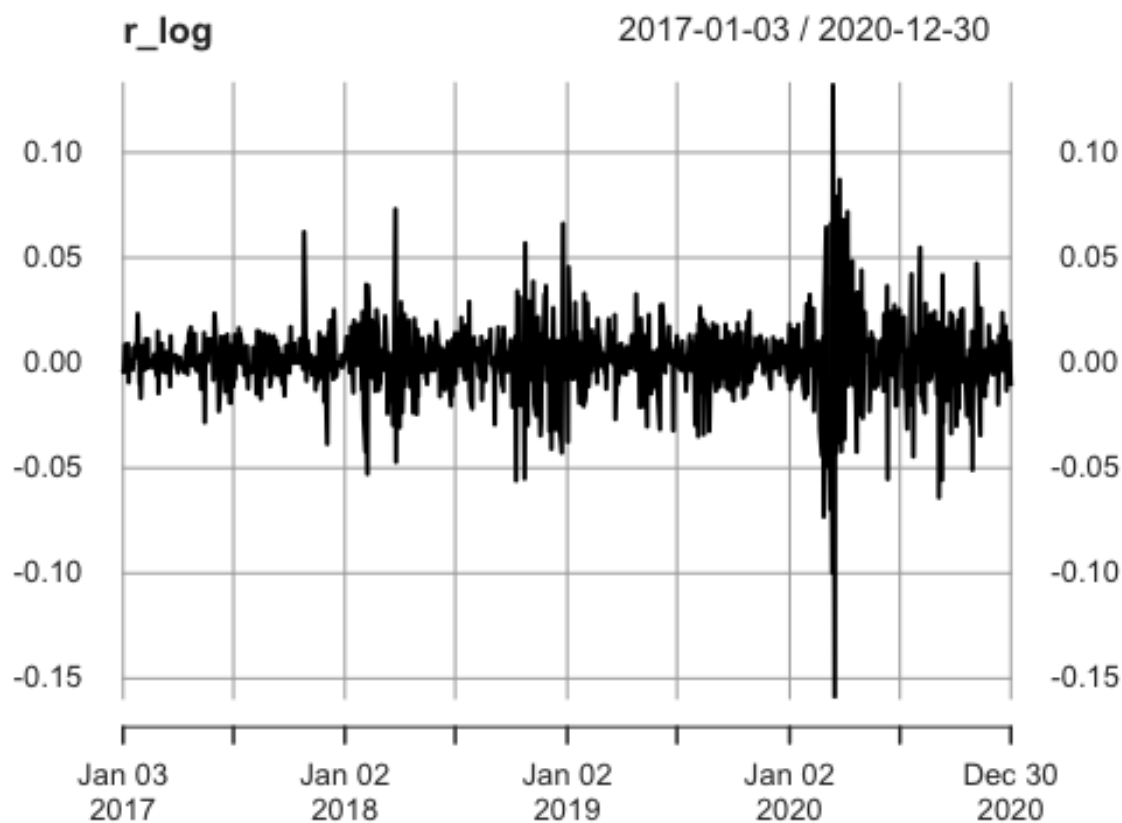
```
plot(r_arith)
```



- plot of daily log return

```
r_log = dailyReturn(MSFT,type='log')
```

```
plot(r_log)
```



(iii) Compute the sample mean, standard deviation, skewness and kurtosis (if excess kurtosis state that clearly) of the daily log returns.

```
mean(r_log)
## [1] 0.001253916

sd(r_log)
## [1] 0.01822633

skewness(r_log)
## [1] -0.3733297
## attr(,"method")
## [1] "moment"

kurtosis(r_log)
## [1] 11.62756
## attr(,"method")
## [1] "excess"
```

- (iv) Perform the Jarque-Bera Normality test (with 5% significance level) for daily log returns and interpret your results. Compare your result a visualization from a Q-Q plot.

Ho: (Normal Distribution) Ha: (Non-Normal Distribution)

- p value of Jarque - Bera Normality Test Normality Test is very small and smaller than significance level(0.05) so we can reject null hypothesis. Hence, the data is not normally distributed

```
normalTest(r_log,method="jb")

## Warning in if (class(x) == "fREG") x = residuals(x): the condition has
length >
## 1 and only the first element will be used

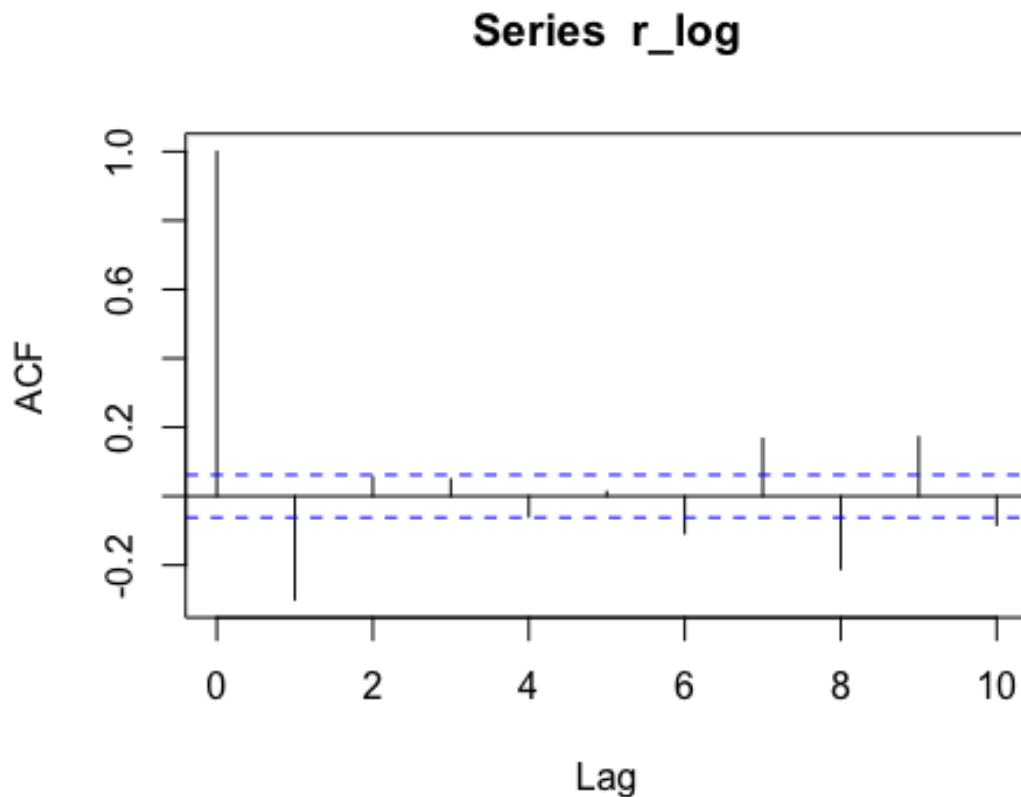
##
## Title:
##  Jarque - Bera Normalality Test
##
## Test Results:
##  STATISTIC:
##    X-squared: 5719.0022
##  P VALUE:
##    Asymptotic p Value: < 2.2e-16
##
## Description:
##  Sat Feb 20 06:30:24 2021 by user:
```

- (v) Compute and plot the lag-1 through lag-10 autocorrelation of the daily log returns.

- plot the autocorrelation
- calculate the autocorrelation

```
acf(r_log, lag.max=10, plot=TRUE)
```





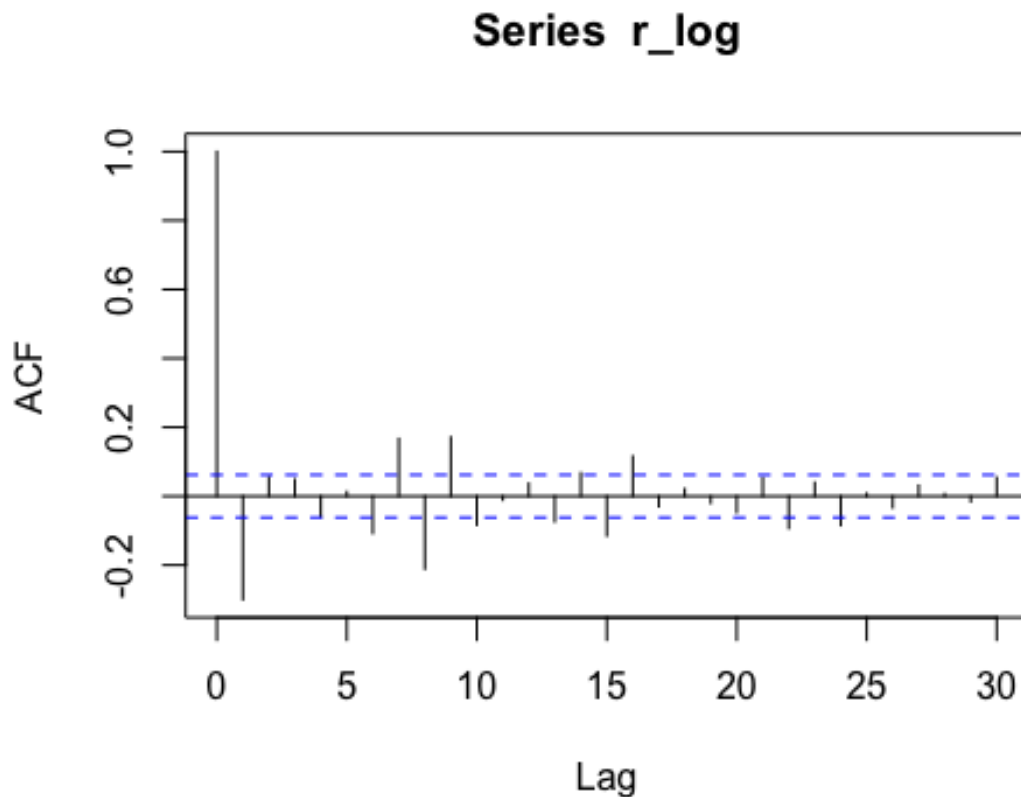
```
acf(r_log, lag.max=10,plot="False")
##
## Autocorrelations of series 'r_log', by lag
##
##      0      1      2      3      4      5      6      7      8      9
10
##  1.000 -0.300  0.055  0.049 -0.059  0.013 -0.107  0.167 -0.211  0.171 -
0.084
```

(vi) Test if the lag-5 autocorrelation is non-zero (with 5% significance level). Interpret your results.

$H_0 : \rho_l = 0$   $H_a : \rho_l \text{ not equal } 0$  if  $|t \text{ ratio}| > Z_{\alpha/2}$ , we reject null hypothesis

- According to individual ACF test, our t ratio is not bigger than qnorm value. Hence, we fail to reject null.

```
ac = acf(r_log,plot=TRUE)
```



```
T = length(r_log)
ratio = sqrt(T)*ac$acf[6]

significance = 0.05
abs(ratio) > qnorm(1-significance/2) #FALSE = FAIL TO REJECT NULL

## [1] FALSE
```

(vi) Test if the lag-5 autocorrelation is non-zero (with 5% significance level). Interpret your results.

(vii) Perform the Ljung-Box test to test if the first 2, first 5, and first 8 lagged autocorrelations are non-zero (with 5% significance level). Interpret your results.

H0: The data are independently distributed(no auto correlation) Ha: The data are not independently distributed; they exhibit serial correlation.

-First two autocorrelations are non zero because p- value of the Ljung-Box Tests is too small with given 5% significance level. Hence, we can reject null hypothesis(no auto correlation)

```
Box.test(r_log, lag=2, type="Ljung-Box")
```

```
##  
## Box-Ljung test  
##  
## data: r_log  
## X-squared = 93.936, df = 2, p-value < 2.2e-16
```

- First 5 autocorrelations are non zero because p- value of the Ljung-Box Tests is too small in comparison to given 5% significance level. Hence, we can reject null hypothesis( no auto correlation)

```
Box.test(r_log,lag=5,type="Ljung-Box")
```

```
##  
## Box-Ljung test  
##  
## data: r_log  
## X-squared = 100.04, df = 5, p-value < 2.2e-16
```

- First 8 autocorrelations are non zero because p- value of the Ljung-Box Tests is too small in comparison to given 5% significance level. Hence, we can reject null hypothesis( no auto correlation)

```
Box.test(r_log,lag=8,type="Ljung-Box")
```

```
##  
## Box-Ljung test  
##  
## data: r_log  
## X-squared = 185.33, df = 8, p-value < 2.2e-16
```