Homework 2

FE-542

Due: March 12, 2021 at 5PM

Problem 1 (30pt)
Suppose that the daily log return of a security follows the AR(2) model:

$$r_t = 0.03 + 0.2r_{t-2} + a_t$$

where a_t is a Gaussian white noise series with mean zeros and variance 0.1.

(i) What are the mean and variance of the return series r_t?

(ii) Compute the lag-1 and lag-2 autocorrelations of r_t.

(iii) Assume that $r_{100} = -0.02$ and $r_{99} = 0.01$. Compute the 1- and 2-step ahead forecasts of the return series at the forecast origin $t = 100$. What are the associated standard deviations of the forecast errors?

(iv) In R create a report in pdf format using RMarkdown (or, if you choose to use Python instead, create a Jupyter notebook) to:

  (a) Simulate 1000 terms of this time series with $r_0 = -0.02$ and $r_{-1} = 0.01$.

  (b) Using the generated time series, find the sample mean and variance. How do these values compare with those computed analytically?

  (c) Using the generated time series, find the sample lag-1 and lag-2 autocorrelations. How do these values compare with those computed analytically?

  (d) Consider how you might use repeated simulations to forecast this time series. Use your method with 1000 repeated simulations of the time series to forecast the 1- and 2-step ahead returns with $r_t = -0.01$ and $r_{t-1} = 0.02$. What is the sample standard deviation? How do these values compare with those computed analytically?

Problem 2 (30pt)
Suppose that the simple return of a monthly bond index follows the MA(1) model:

$$R_t = a_t + 0.2a_{t-1}$$

where a_t is a Gaussian white noise series with mean zero and variance 0.001.

(i) What are the mean and variance of the return series R_t?

(ii) Compute the lag-1 and lag-2 autocorrelations of R_t.

(iii) Assume that $a_{100} = -0.01$. Compute the 1- and 2-step ahead forecasts of the return series at the forecast origin $t = 100$. What are the associated standard deviations of the forecast errors?

(iv) In R create a report in pdf format using RMarkdown (or, if you choose to use Python instead, create a Jupyter notebook) to:

  (a) Simulate 1000 terms of this time series.

  (b) Using the generated time series, find the sample mean and variance. How do these values compare with those computed analytically?

  (c) Using the generated time series, find the sample lag-1 and lag-2 autocorrelations. How do these values compare with those computed analytically?

  (d) Consider how you might use repeated simulations to forecast this time series. Use your method with 1000 repeated simulations of the time series to forecast the 1- and 2-step ahead returns with $a_t = 0.01$. What is the sample standard deviation? How do these values compare with those computed analytically?

Problem 3 (40pt)
In R create a report in pdf format using RMarkdown (or, if you choose to use Python instead, create a Jupyter notebook) to:

(i) Import the monthly yields of Moody's Aaa seasoned bonds from January 1, 1962 to December 31, 2020 from **homework02.csv** provided on Canvas. The data are obtained from the Federal Reserve Bank of St. Louis. Monthly yields are averages of daily yields.

(ii) Obtain the summary statistics (sample mean, standard deviation, skewness, *excess* kurtosis) of this yield series.

(iii) Build a time series model for this data. Evaluate its performance. Justify your choices.