Dynamic Portfolio Allocation by Regime Detection

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Abstract

The main objective of this paper is to develop dynamic portfolio allocation strategies by regime detection using deep learning models. In this study, a data set covers the period of January 2000-December 2021 of major US and global financial indices. The market regime is defined as volatile and non-volatile, in other words; abnormal and normal. Since supervised learning methods used to model the market regime, two indices VIX and Z3 used as threshold values for data labeling purpose. The deep learning models: Recurrent Neural Networks and Long-Short Term Memory is used to predict the market regime. We implemented four different portfolio allocation strategies including both static and dynamic ones along with predicted market regimes then compared and evaluated results. The study presents that the dynamic strategies can outperform benchmark static strategies under variety of scenarios and conditions.

Keywords: Regime Modeling, Deep Learning, Factor Modeling, Portfolio Optimization

1 Introduction

Asset allocation is a question of distributing limited resources optimally over various investment instruments .When it comes to long term investing, such as pension funds, the portfolio composition do not change frequently, but in general, it is necessary to adjust the portfolio composition according to changes in economic conditions.

The Modern Portfolio Theory is introduced by Harry Markowitz in 1952[1]. The Markowitz's Mean Variance theory aims to maximize the expected return of assets with given risk level or minimize the risk with given level of expected return by changing the weight composition of portfolio. The theory is based on a single period utility analysis, and it requires estimation of expected returns and covariance matrix. Non-stationary and stochastic nature of financial assets causes estimation errors in asset parameters which leads to poor out of sample performance[2].

To reduce the effect of estimation error, the ideal situation would require high frequency rebalancing which is in practice not useful due to potential adverse impact of transaction cost. On the other hand, very low rebalancing frequency usually leads to significant ex-ante ex-post distance in terms of efficient frontiers. That leads to the need to study optimal rebalancing strategies to account for dynamics of the risk factors associated to a given covariance matrix.

Another approach to deal with estimation error and frequent rebalancing is considering market regime and adjusting the portfolio allocation accordingly. The economic state of the market is called regime. The Regime-based asset allocation will be studied to determine optimal re-balancing strategy and portfolios. The Regime-based asset allocation requires adjusting portfolio holdings in accordance with the change in market conditions. Deep Learning methods are being used in many fields such as computer vision, speech recognition, and language translation[3]. The non-linearity, non-stationary, and noisiness characteristics of financial markets leads to use of Machine Learning and Deep Learning algorithms for complex problems in finance field[4][5]. Supervised Deep Learning Models will be used to predict market regime.

This paper aims to find empirical evidence whether the Regime-based asset allocation model can produce better results than traditional asset allocation strategies.

2 Methodology

In this section, we first introduce the dataset used throughout the research. Then we present the labeling construction, which serves as threshold value for classifying market regime as volatile and non-volatile. Further, we show deep learning models used for market regime peridiction. Finally, we introduce both static and dynamic portfolio allocation strategies.

2.1 Data

This section presents the dataset used. The dataset covers the period starting from 03 January 2000 to 31 December 2021 and includes twenty-three features grouped under five main categories namely: Equity, Credit, Commodity, Fixed Income, and Sector ETFs as shown in Table 1. Data labeling conducted to classify output, current market regime, as non-volatile and volatile, since supervised methods used in the research. The output data is binary: 0 and 1 representing respectively non-volatile and volatile market regime. For labeling purpose, variety of indicators used such as CBOE Volatility Index (VIX) and Z3. Data labeling explained further in the next section.

Ticker	Description
XLB	Materials Select Sector SPDR Fund
XLE	Energy Select Sector SPDR Fund
XLF	Financial Select Sector SPDR Fund
XLI	Industrial Select Sector SPDR Fund
XLK	Technology Select Sector SPDR Fund
XLP	Consumer Staples Select Sector SPDR Fund
XLU	Utilities Select Sector SPDR Fund
XLV	Health Care Select Sector SPDR Fund
XLY	Consumer Discretionary Select Sector
SPY	SPDR S&P 500 ETF Trust
SPY_V	SPDR S&P 500 ETF Trust Volume
SPY_HL	SPDR S&P 500 ETF Trust High Minus Low
QQQ	Invesco QQQ Trust- Nasdaq
VIX	CBOE Volatility Index
EWJ	iShares MSCI Japan ETF
DAA	Moody's Seasoned Aaa Corporate Bond Yield
DBAA	Moody's Seasoned Baa Corporate Bond Yield
DPRIME	Bank Prime Loan Rate
DGS3MO	Market Yield on U.S. Treasury Securities at 3-Month
DGS2	Market Yield on U.S. Treasury Securities at 2-Year
DGS10	Market Yield on U.S. Treasury Securities at 10-Year
WTICO	Crude Oil Prices: West Texas Intermediate
BCOM	Bloomberg Commodity Index

Table 1: Features

2.2 Labeling

The first indicator used for data labeling purpose is CBOE Volatility Index (VIX) and the idea of using an index to label the data is inspired by paper by Cuneyt Sevim(2014)[6]. The VIX is calculated based on options of S&P500 Index by CBOE Global Markets, and widely used in finance industry to gauge market's expectation of future volatility. To label the market condition as volatile and non-volatile, past crisis and their corresponding VIX values observed. During Subprime Mortgage crisis, on 04 September 2008, S&P500 index dropped 3.4% and corresponding VIX value reached to 25.47. Similarly, amid Covid crisis, on 24 February 2020, S&P500 went down 3.351% where VIX hit to 25.03 and kept soaring. After evaluating both crisis and associated VIX values in detail, VIX 27 level is chosen as a threshold value to label the data as volatile and non-volatile market.

$$R = \begin{cases} 1 & \text{VIX} \geq 27 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Where:

- R: is the current market regime, VIX above 27 indicating volatile regime, and below non-volatile regime

The second indicator used for market classification is so called Z3 indicator, which is defined based on the empirical properties of stock market. The empirical study conducted by Rama Cont(2000)[7] states that volatility of stock returns show positive auto-correlation, that is, high-volatility tends to cluster over time. Furthermore, volatility is negatively correlated with returns meaning that high volatility often accompanied by bearish market. In fact, the observed market conditions are mostly divided into two main categories : a market where high volatility persistence along with low returns and low volatility along with high returns. The opposite rarely observed. Considering these two categories, indicator Z3 constructed to reflect market's empirical properties better. Z3 consists of two other indicators respectively, Z1 and Z2 where Z1 is 10-day absolute drawdown of S&P500, and Z2 is 15-day S&P500 volatility. Hence, Z3 is sum of the normalized values of Z1 and Z2.

$$Z1 = \left| \frac{P_t - max(P_{(t-9,t)})}{max(P_{(t-9,t)})} \right| \tag{2}$$

$$Z2 = \frac{1}{15} \sum_{i=0}^{14} \sqrt{(R_{t-i} - \mu)^2} \tag{3}$$

Z3= Z1+Z2 (4)

Where:

- P_t: is the price at time t
- $P_{(t-9,t)}$: is the last 10 days S&P500 price
- R_{t-i}: is the return of S&P500 on (t-i)th day
- μ: mean of the S&P 500 return for 15 days

Briefly, rise in Z3 value indicates that, market is getting shifted to unfavorable state which is high volatility along with low returns, on the other hand drop in Z3 implies cooling down in market and moving towards non-volatile regime. During Covid crisis on 24 February 2020, Z3 reached over 1.5 and kept soaring after that. Therefore, 1.5 Z3 level chosen as threshold value to classify the market as volatile and non-volatile.

$$R = \begin{cases} 1 & \text{Z3} \geq 1.5 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

2.3 Deep Learning

Many studies conducted on market regime detection using Hidden Markov Models, Gaussian Mixture Models[8][9], this research utilizes deep learning models for regime identification. Using Deep Learning models does not require feature engineering, and underlying features can be learned directly from the data, furthermore, lack of studies on regime classification using deep learning are the reasons behind the usage of deep learning in the research. Two models used for regime detection namely Recurrent Neural Networks (RNN) and Long Short-Term Memory (LSTM).

Both RNN and LSTM models uses twenty-three features as presented in Table 1 to predict the binary output 0 and 1 representing non-volatile and volatile regime respectively. Output is labeled using VIX and Z3 indices separately.The models use last five days to predict to next day's regime and use sigmoid activation function and binary cross entropy as loss function since the it's a multi label classification problem. As shown in Table 2, RNN models outperformed LSTM models in terms of prediction accuracy for both indicators, therefore RNN model chosen for market regime prediction. Although, LSTM algorithm includes more advance properties such as using both long-term and short-term memory,the reason we think RNN outperforms LSTM in this specific circumstance: volatile market regime is rare and data set is consisted of mostly non-volatile regime, when there is a shift in regime from non-volatile to volatile LSTM's long-term memory feature does not let the model adjust the prediction immediately.

$$sig(x) = \frac{1}{1 + e^{-x}} \tag{sigmoid activation function}$$

$$L = -1/N \sum_{i=1}^{N} y_i \times \log \hat{y_i} + (1 - y_i) \times \log (1 - \hat{y_i}) \tag{binary cross entropy loss}$$

Accuracy	VIX	Z3
RNN	0.9918	0.9299
LSTM	0.9609	0.9197

Table 2: Model Accuracy

2.4 Portfolio Strategy

We implemented two main allocations strategies: Static Asset Allocation and Dynamic Asset Allocation. The static strategy is not considering regime whereas dynamic one implemented using predicted market regimes. The static strategy used as a benchmark for comparison purpose of different strategies. Dynamic strategies namely: Stop Investing, GLD&TLT, and Beta Neutral will be further explained next.

The portfolio investment universe is 11 sector ETFs shown in Table 3, and the reason behind of choosing sector ETFs, because sector ETFs rotate according to economic cycle and the associated risk could be reduced using different sectors.

Ticker	Description
XLB	Materials Select Sector SPDR Fund
XLE	Energy Select Sector SPDR Fund
XLF	Financial Select Sector SPDR Fund
XLI	Industrial Select Sector SPDR Fund
XLK	Technology Select Sector SPDR Fund
XLP	Consumer Staples Select Sector SPDR Fund
XLU	Utilities Select Sector SPDR Fund
XLV	Health Care Select Sector SPDR Fund
XLY	Consumer Discretionary Select Sector
XLRE	Real Estate Select Sector
XLC	Communication Services Select Sector

Table 3: Investment Universe

2.4.1 Static Asset Allocation

The static asset allocation strategy uses Markowitz Mean-Variance theory by maximizing the objective function along with target beta and weight constraints represented below. The beta is single factor market risk measure set to one which is same as the market portfolio S&P500. The weight constraint of each

ETFs ranges between zero to one third and sum up to one. The portfolio is reallocated every five days.

$$Max\ \rho^T w - w^T \Sigma w$$
$$\sum_{i=1}^{11} \beta_i w \ = 1\ ,\ \sum_{i=1}^{11} w_i \ = \ 1\ ,\ 0 \leq \ w_i \leq \ \frac{1}{3}$$

where Σ is the cov-var matrix of the security returns, w_p is the prior-portfolio weights while rebalancing portfolio (the first allocation equally weighted (1/11) , $\beta = \frac{cov(r_i, r_M)}{\sigma^2(r_M)}$ is the Beta of security, as defined in the CAPM. [10]

Fama and French Three- Factor Model (Momentum, Value and Size) used to estimate the return and cov-var matrix of the portfolio[11]. Momentum factor is the expected excess return of the market compared to risk-free return. Size factor is the difference between expected returns of large cap and small cap companies. Value factor is the difference between expected return of value stocks (High value) and growth stocks (Low value).

$$r_i = r_f + \beta_i^m (r_M - r_f) + \beta_i^s \ r_{SMB} + \beta_i^v \ r_{HML} + \alpha_i + \epsilon_i$$

The factor model assumes that the expectation of ϵ_i is 0 and the the different asset's returns are uncorrelated. Cov(r_i, r_j)=0 ,where i $\neq j$.

$$\widehat{r_i} = r_f + \beta_i^m (r_M - r_f) + \beta_i^s \ r_{SMB} + \beta_i^v \ r_{HML} + \alpha_i,$$
$$\widehat{\Sigma} = w^T \Sigma_F w + diag(\sigma_i^2 (r_i - \widehat{r_i}))$$

where Σ_F is the covariance matrix of factors, $\sigma_i^2(r_i - \widehat{r_i})$ is the idiosyncratic risk in CAPM.

The return and covariance matrix is estimated by the returns of the factors for 60 days and 90 days' respectively.

2.4.2 Dynamic Asset Allocation

Dynamic Asset Allocation strategy takes proactive approach and adjust the portfolio according to market regime. As stated previously, we consider two market regimes: volatile and non-volatile. When regime shifts from non-volatile to volatile, the new strategies should be adopted.During non-volatile regime, portfolio should be rebalanced effectively to minimize the transaction cost while maximizing the return. Hence, it is vital to consider prior portfolio weights in the objection function below for each rebalancing.

$$Max\ \rho^T w - (w - w_p)\Sigma(w - w_p)$$

$$\sum_{i=1}^{11} \beta_i w \ = 1\ ,\ \sum_{i=1}^{11} w_i \ = \ 1\ ,\ 0 \leq \ w_i \leq \ \frac{1}{3}$$

On the other hand, during high volatility and low return regime, portfolio could be altered in different ways. The first and the simplest strategy we implemented is so called Stop Investing. As the names states this strategy would stop investing and hold cash till market regime shifts back to normal regime

$$\sum_{i=1}^{11} w_i \ = \ 0\ ,\ w_i = 0\ for\ all\ i$$

The second strategy is GLD&TLT which invests in gold and treasury bond equally since these are safe assets for volatile markets. Moreover, monetary policy tools by central banks such as Quantitative Easing and Forward Guidance is likely to affect these two assets in favor.

$$w_{GLD} = w_{TLT} = 0.5$$

Third strategy sets the target beta of the portfolio to zero to make the portfolio to move in an independent direction of the market by allowing short selling. The objective function is changed to consider the overall volatility

$$Max\ \rho^T w - w^T \Sigma w$$

$$\sum_{i=1}^{11} \beta_i w \ = 0\ ,\ \sum_{i=1}^{11} w_i \ = \ 1\ ,\ -\frac{2}{3} \leq \ w_i \leq \ \frac{2}{3}$$

3 Results

Among all dynamic asset allocation strategies, the Stop Investing and GLD+TLT strategies are ways to reduce volatility while keeping returns stable during volatile regime. The Beta Neutral strategy pursues absolute returns by long-short strategy; however, volatility could raise more than benchmark static asset allocation. Hence, it's wise to look at Sharpe Ratio when evaluating, and comparing different dynamic and static allocation strategies.

We have conducted back testing for two different periods: 2019-2021 including Covid crisis and 2007- 2010 including Subprime Mortage crisis, by using two different predicted indicators separately Z3 and VIX. Looking at the results as shown in Table 4, back testing using Z3 indicator during 2019-2021 period, Stop

Investing and GLD&TLT strategies managed lowering volatility from 0.237 to 0.1331 and 0.1562 respectively, similarly cumulative return went down for both strategies, resulted in lower Sharpe Ratio 0.5902 and 0.5284 whereas benchmark (Static AA) ratio is 0.6232. In the case of Beta Neutral strategy, Sharpe Ratio increased to 0.7242, due to dramatic increase in cumulative return from 0.5109 to 0.8441.

The Figure 1 shows the results of Static AA and Dynamic AA strategies during 2019-2021. The vertical axis represents the value of portfolio starting with $100 investment, and horizontal axis represents the investment period. The shaded areas on the graph is showing predicted crisis periods using Z3 indicator. Moreover, the dotted black line is the benchmark, Static AA, orange line and grey line are Stop investing and GLD&TLT strategy, respectively, lastly blue line representing the beta neutral strategy.

Strategy	Cumulative Return	Volatility	Sharpe Ratio	Max DrawDown
Predicted Z3 / 2019-2021				
Static AA	0.5109	0.237	0.6232	0.3774
Stop Investing	0.2542	0.1331	0.5902	0.1503
GLD+TLT	0.2683	0.1562	0.5284	0.14
Beta Neutral	0.8441	0.3129	0.7242	0.2853
Predicted Z3 / 2007-2010				
Static AA	0.0984	0.2725	0.0874	0.5085
Stop Investing	0.22	0.1677	0.3048	0.2963
GLD+TLT	0.2355	0.1908	0.2855	0.3467
Beta Neutral	-0.3680	0.3499	-0.3107	0.6961
Predicted VIX / 2019-2021				
Static AA	0.5109	0.237	0.6232	0.3774
Stop Investing	0.2592	0.1243	0.6431	0.1033
GLD+TLT	0.3794	0.15	0.7556	0.1491
Beta Neutral	1.0646	0.3149	0.8692	0.2853
(d) Predicted VIX / 2007-2010				
Static AA	0.0984	0.2725	0.0874	0.5085
Stop Investing	0.1005	0.1535	0.1583	0.2542
GLD+TLT	0.1932	0.1825	0.2482	0.3234
Beta Neutral	-0.4471	0.3599	-0.3836	0.6888

Table 4: Results: Dynamic AA and Static AA

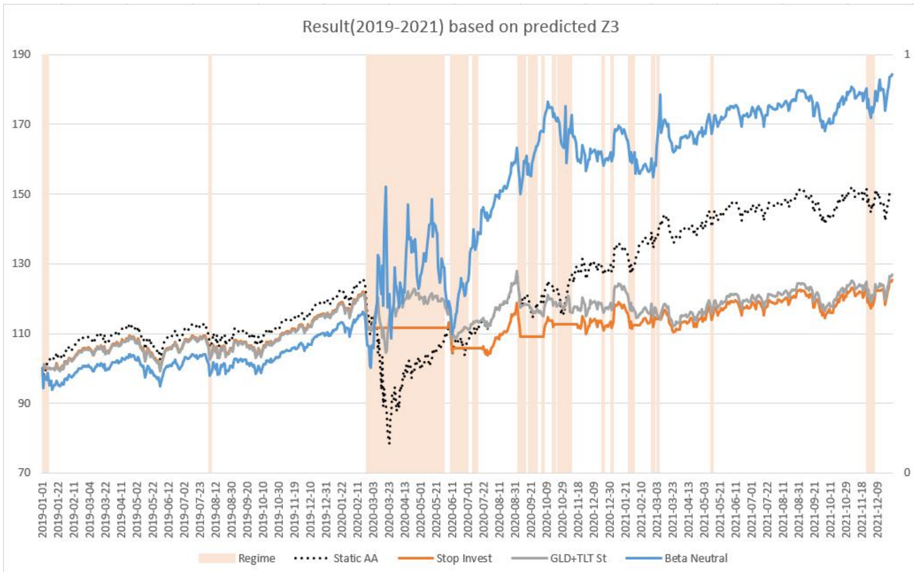

Figure 1: The Result based on predicted Z3 (2019-2021)

Throughout 2007-2010 period, Stop Investing and GLD&TLT strategies outperformed the benchmark in terms of volatility, cumulative return, and Sharpe Ratio which are 0.3048 and 0.2855 respectively, whereas benchmark Sharpe Ration is 0.0874. On the other hand, Beta Neutral strategy performed worse than benchmark in all metrics, indeed max draw down hit to 0.7, which caused -0.368 cumulative return.

Looking at back testing results using VIX indicator for regime detection during 2019-2021period, Stop Investing and GLD&TLT strategies showed better performance in terms of volatility and max draw down, however also showed lower cumulative returns, and resulted in higher Sharpe ratio of 0.6431 and 0.7556 respectively comparing to benchmark of 0.6232 value. The Beta Neutral strategy beat all strategies with a Sharpe ratio of 0.8692, due to high cumulative return of 1.0646.

In the period 2007-2010, although the Stop Investing and GLD+TLT strategies showed worse performance than those using Z3, but still showed higher performance than Benchmark's Sharpe ratio. The Sharpe ratio of these two strategies was 0.1583 and 0.2482, respectively, exceeding benchmark 0.0874.

The Beta Neutral strategy performed worst with a cumulative return of -0.4471 and max draw down of 0.6888. However, it cannot be said that the regime-based asset allocation is not effective for this strategy, since it involves short selling, leverage inevitably used, causing increase in volatility. Moreover, it is difficult to recover once a large loss occurs.

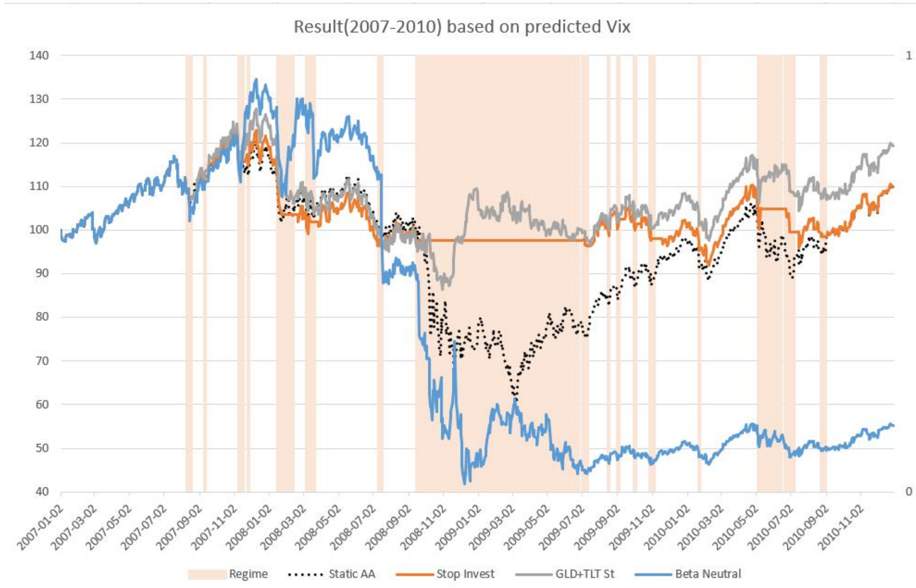

Figure 2: The Result based on predicted VIX (2007-2010)

The Figure 2 shows the results of Static AA and Dynamic AA during 2007-2010 period. The initial amount of money assumed is \$100, and the orange shaded period is VIX predicted volatile regime. The dotted black line is the benchmark, Static AA whereas orange and grey line are Stop Investing and GLD&TLT strategy and the blue line representing the beta neutral strategy

4 Conclusion

To sum up, Dynamic Asset Allocation strategies could perform better in some scenarios as shown in prior section. Depending on the characteristics of the fund, investment profile and risk appetite, it could be adopted accordingly. For instance, open-end funds aim to reduce volatility and max drawdown, in this case, it's wise to use Stop Investing or GLD&TLT strategy when regime shifts to volatile market.

Although predicted VIX using RNN model performed quite well, findings using predicted Z3 values using RNN were not as good as expected, especially during the periods when regime changed frequently, it showed poor prediction during transition from nonvolatile to volatile. To illustrate, in May 2020 the stock market rebounded fairly from Covid Crisis, however RNN model couldn't capture the rebound in high accuracy, roughly fifty 50 percent accuracy. On the other hand, using original Z3 values as a threshold value performed superior as shown in Figure 3 below.

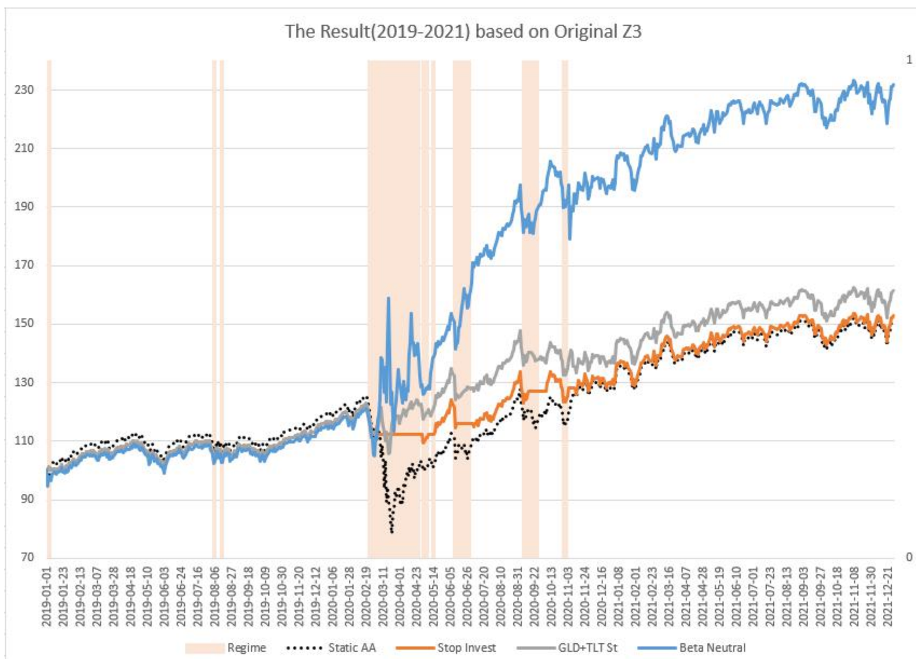

Figure 3: The Result based on original z3(2019-2021)

It seems that some changes can be made to improve the performance of the portfolio strategies. To improve the results, additional explanatory variables could be added. Some economic data could not be used since they are either weekly or monthly data, adjusting and adding them could give us better prediction results. Moreover, throughout the research constant Z3 and VIX values used as threshold values, instead moving average values of these two indicators could be used. Also, market regime only divided into two regimes, using three or four regimes could be used (Maybe citation) to reflect the market regimes in detail. Finally, some unsupervised learning techniques could be used since there is no need for threshold value for data labeling purpose.

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A Appendix

This section provides all the graphs constructed throughout the research by using original indicator values and predicted values. There are total 8 graphs, first two graphs, Figure 4 and Figure 5, showing the results of strategies using original Z3 and predicted Z3 during 2019-2021. Figure 6 and Figure 7 presenting the results of strategies using original VIX and predicted VIX during 2019-2021. The remaining figures follow the same format for period 2007-2010

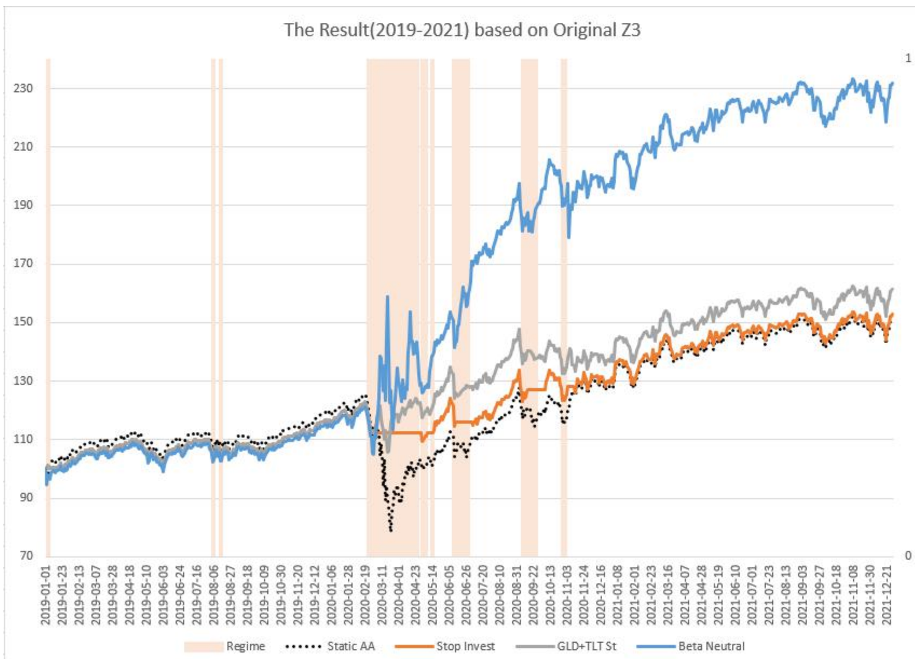

Figure 4: The Result based on original Z3(2019-2021)

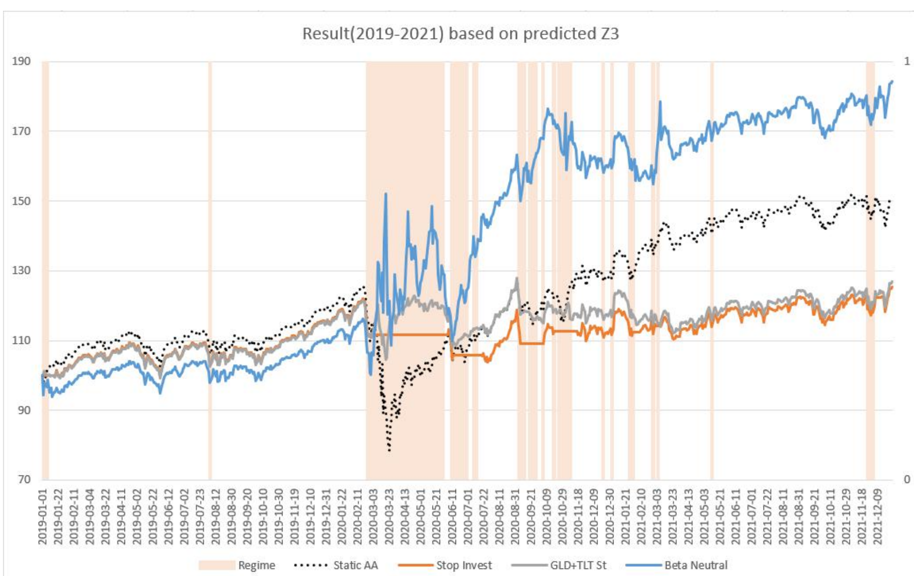

Figure 5: The Result based on predicted Z3(2019-2021)

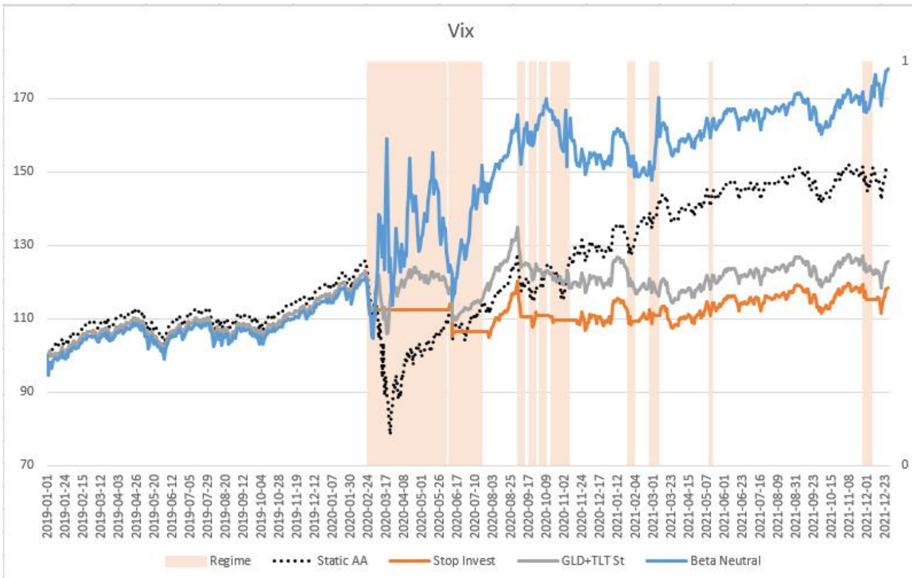

Figure 6: The Result based on original VIX (2019-2021)

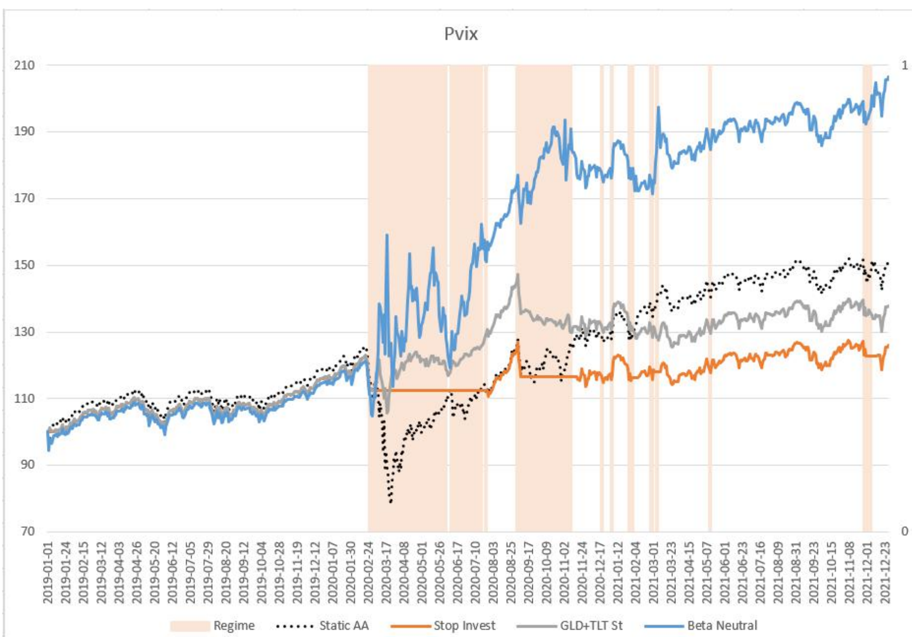

Figure 7: The Result based on predicted VIX (2019-2021)

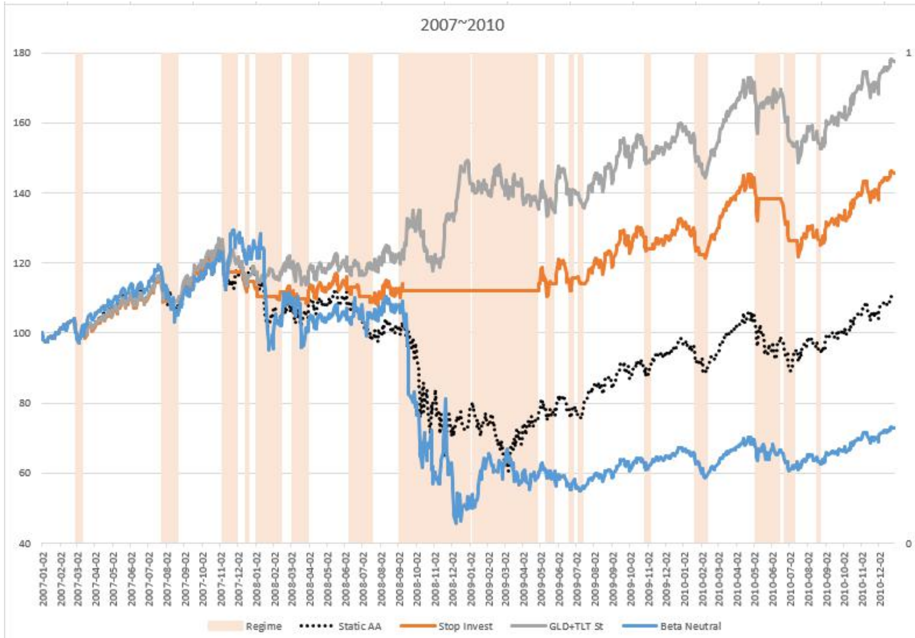

Figure 8: The Result based on original Z3 (2007-2010)

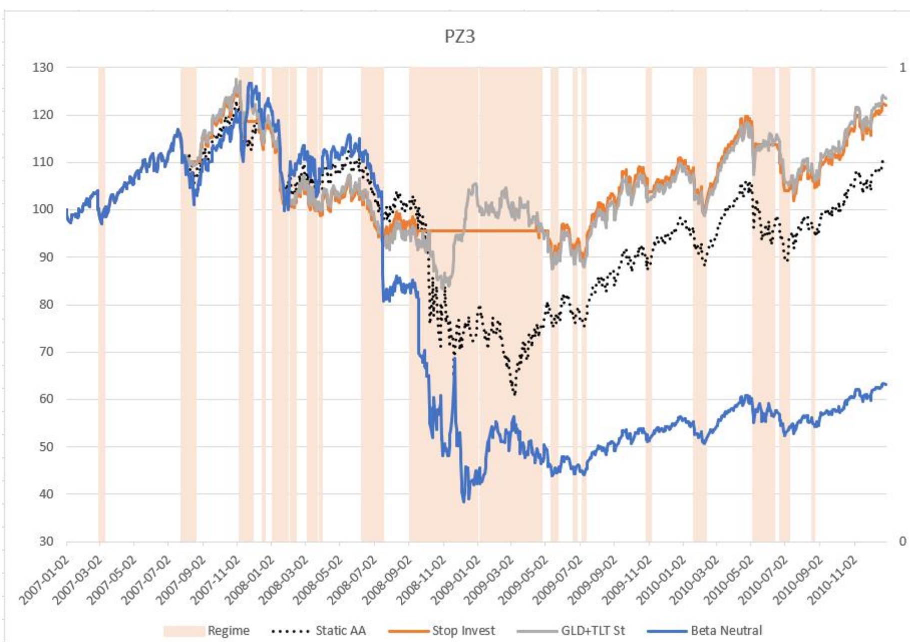

Figure 9: The Result based on predicted Z3 (2007-2010)

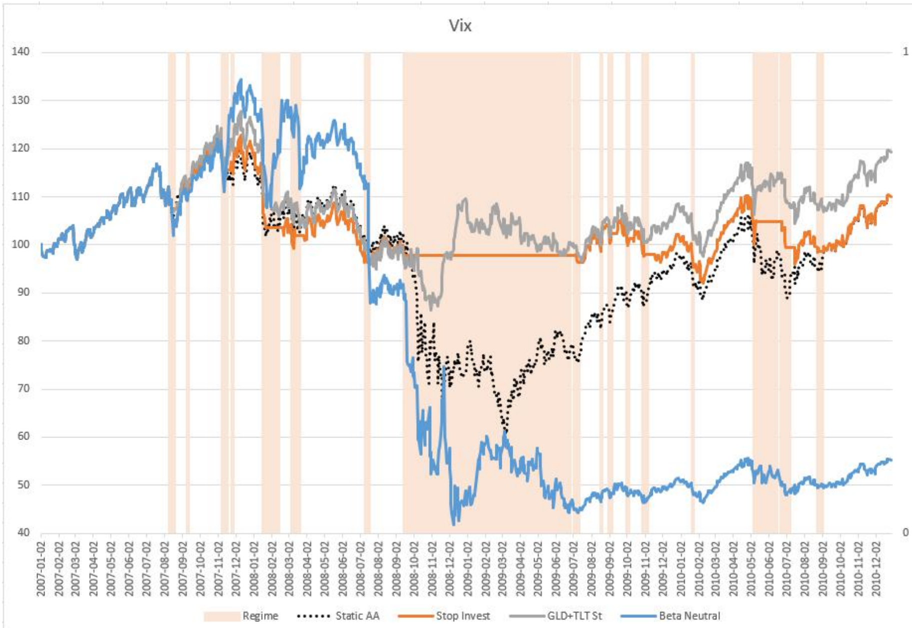

Figure 10: The Result based on VIX (2007-2010)

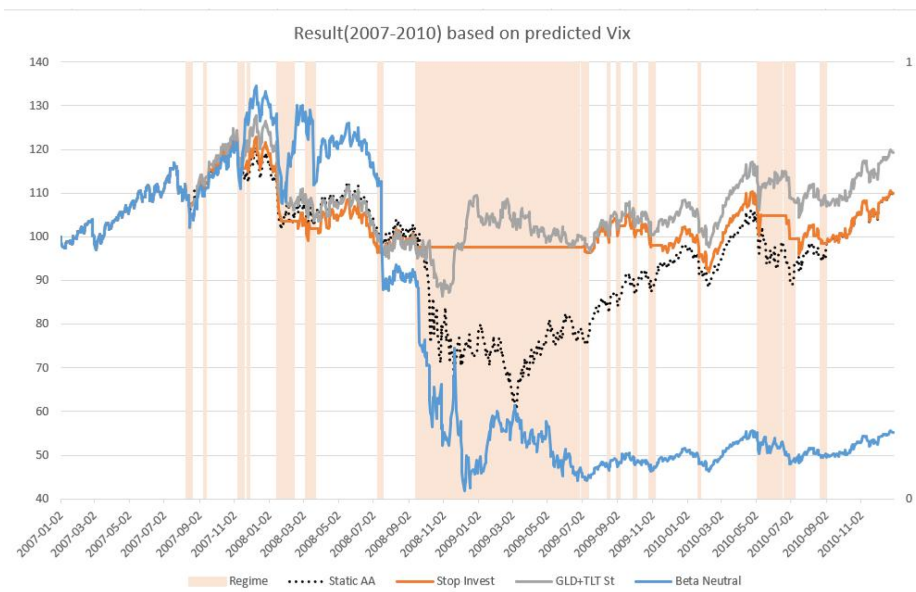

Figure 11: The Result based on predicted VIX (2007-2010)