University of Liège

Homework 3

Applied digital signal processing

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Noise filtering

We have a noisy signal

$$x_{\text{ns}}[n] = x[n] + v[n]$$

where

$$x[n] = \cos(20\pi t) + 0.5\cos(40\pi t + 1.4) + 0.8\cos(120\pi t + 0.7)$$

and $v[n]$ is an arbitrary noise.

Signals $x_{\text{ns}}[n]$ and $x[n]$ are sampled at 1000 Hz.

The goal is to design a filter to remove the noise from $x_{\text{ns}}[n]$ without distortion.

Question a

We plot signals $x[n]$ and $x_{\text{ns}}[n]$ in the same axis (figure 1).

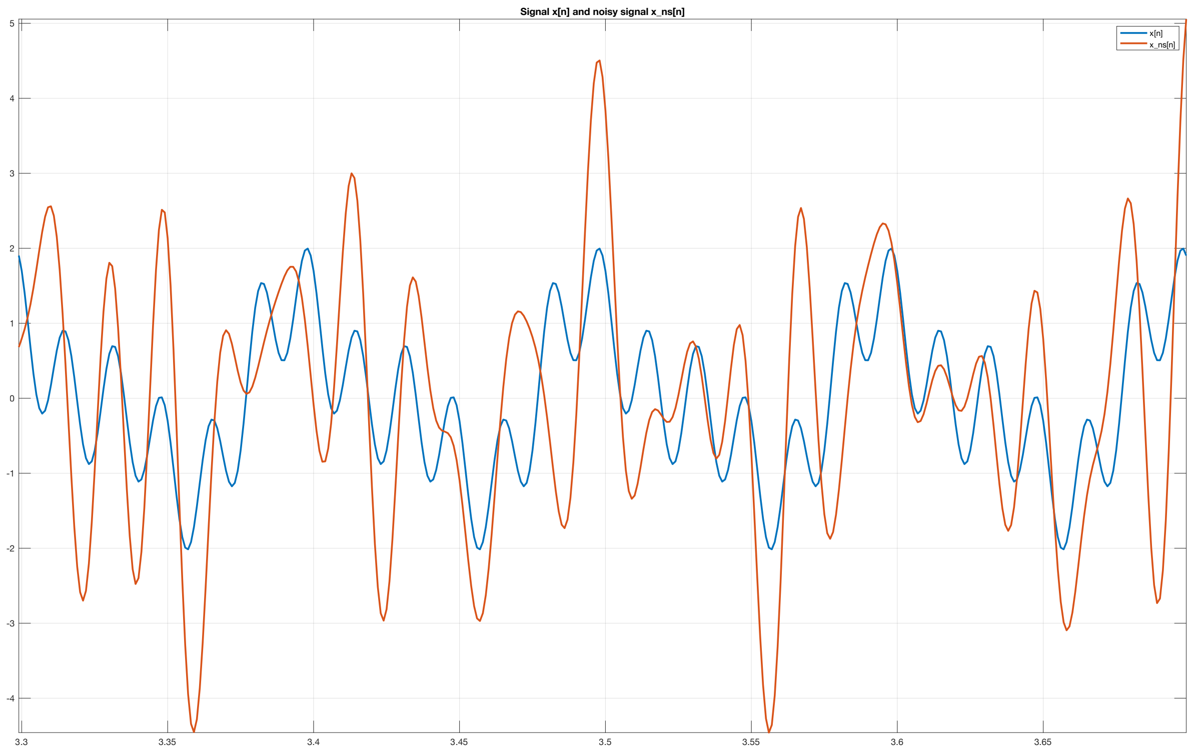

Figure 1 – Signals $x[n]$ and $x_{\text{ns}}[n]$ in the same axis.

Question b

We plot the single-sided amplitude spectrum of the noisy signal $x_{\text{ns}}[n]$ (figure 2).

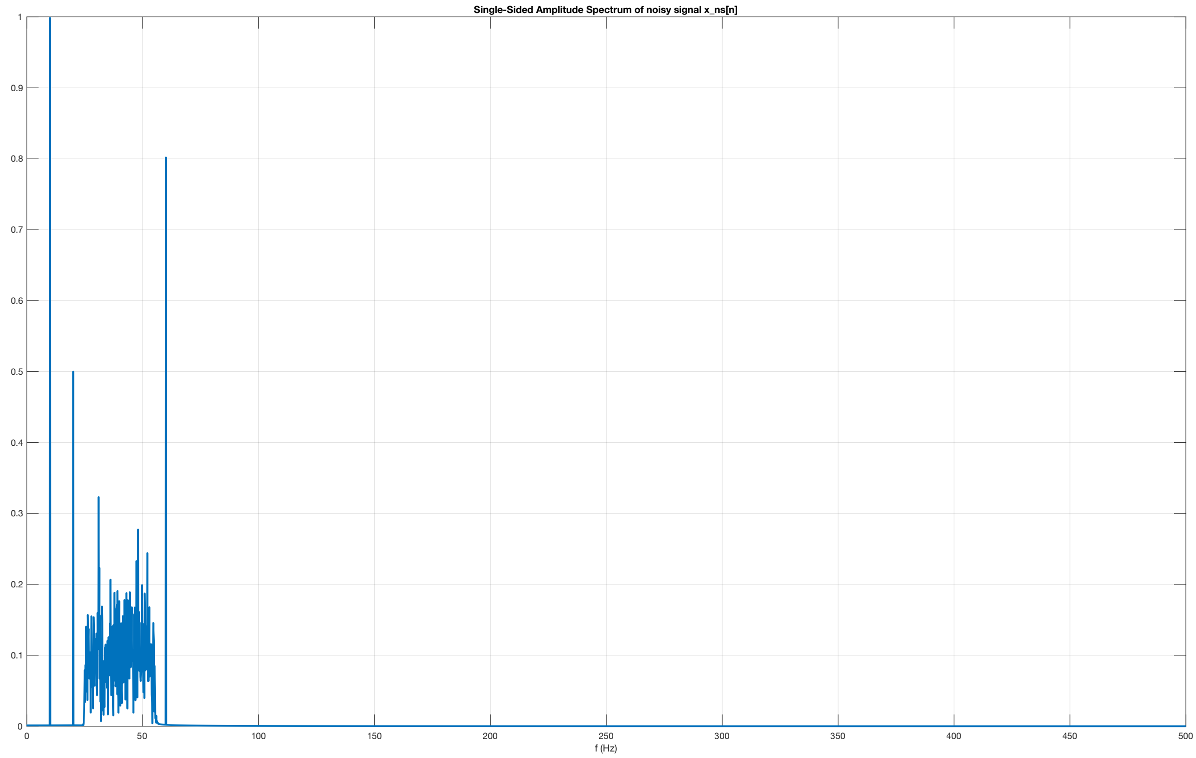

Figure 2 – Single-sided amplitude spectrum of the noisy signal $x_{\mathrm{ns}}[n]$.

Question c

Thanks to the figure 2 and a piece of code, we determined that the approximate frequency range of the noise $v[n]$ is $[24, 56]$ [Hz].

Question d

As we want to preserve the shape of the signal, we have to design an *FIR* filter.

Thanks to Matlab, we can create such a filter easily. Properties of the created filter are presented at figure 3.

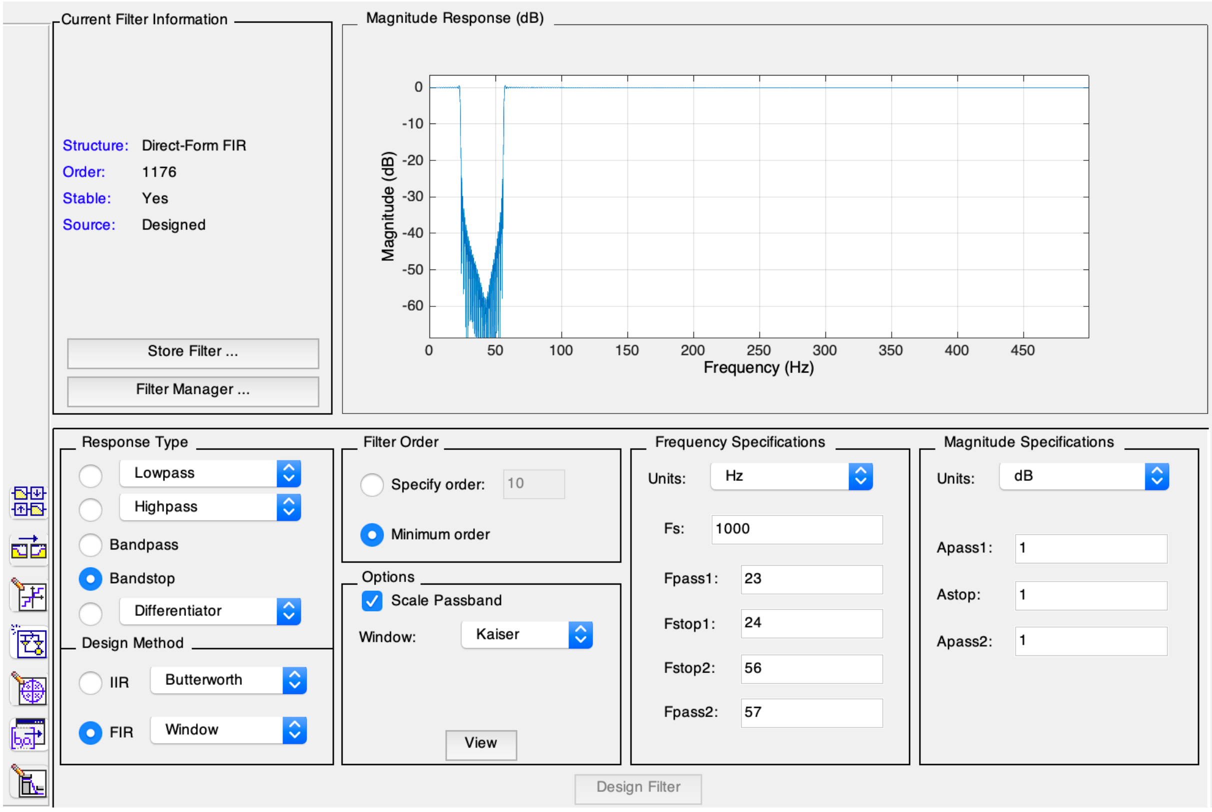

Figure 3 – Design of the *FIR* filter.

So, we choose a *FIR* filter in the options. We set F_s to 1000 Hz, and F_{stop1} and F_{stop2} respectively to 24 and 56 Hz. We also set A_{stop} to 1 dB.

Question e

After applying the filter on the noisy signal, we plot the single-sided amplitude spectrum of the filtered signal $x_{\text{filt}}[n]$ (figure 4).

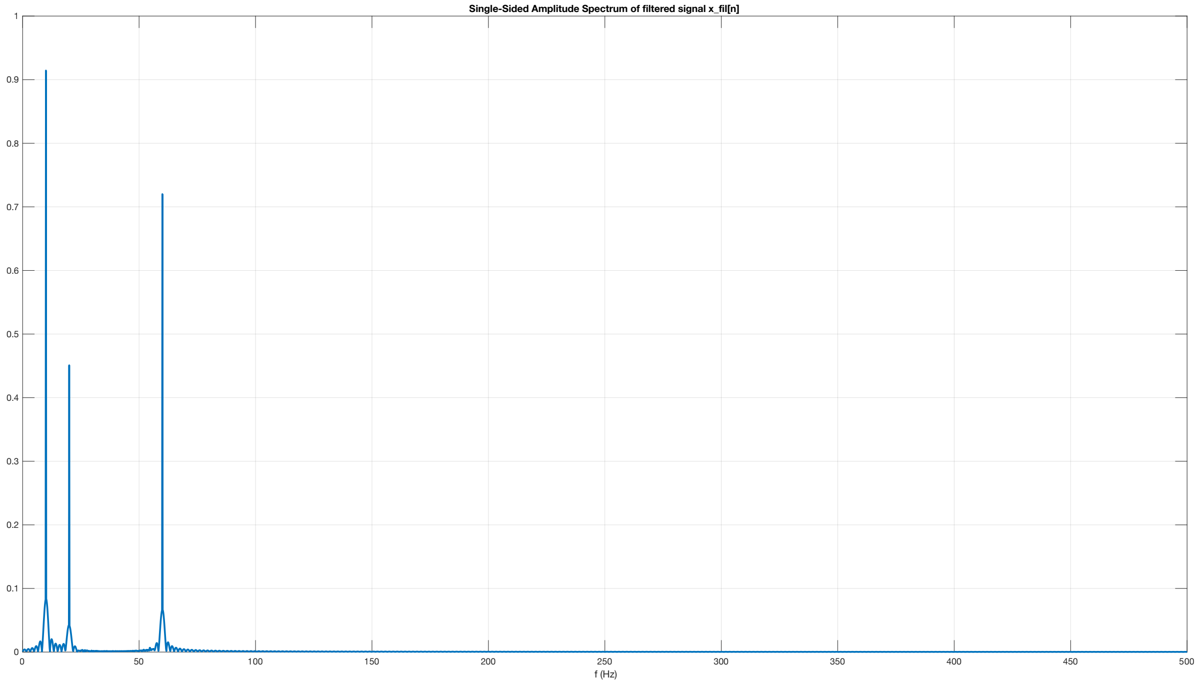

Figure 4 – Single-sided amplitude spectrum of the filtered signal $x_{\text{filt}}[n]$.

We clearly see that the noise in the frequency range $[24, 56]$ [Hz] has been attenuated.

Question f

Finally, we plot signals $x[n]$ and $x_{\text{fil}}[n]$ in the same axis to check the result of our *FIR* filter (figure 5).

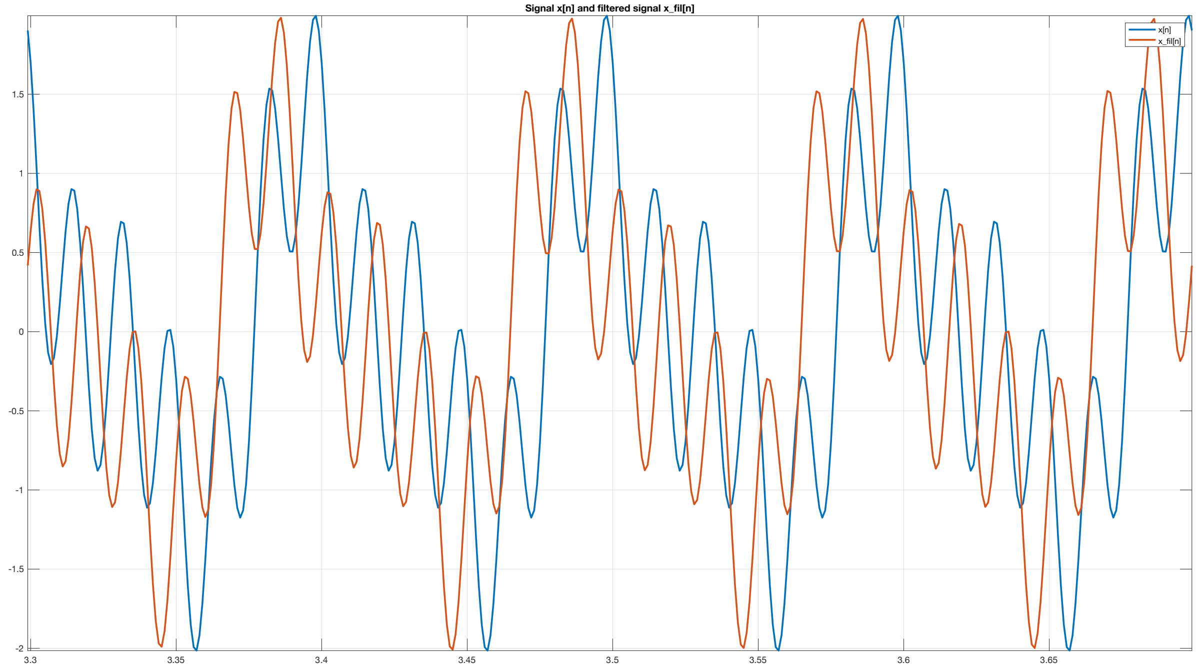

Figure 5 – Signals $x[n]$ and $x_{\text{fil}}[n]$ in the same axis.

We can observe that the signal $x_{\text{fil}}[n]$ is exactly the same than the original signal $x[n]$ (no distortion occurred because *FIR* filter has linear phase response).

Matlab code

```
%% Applied digital signal processing — Homework 3

% Question 1 — Noise filtering
%
% University of Liege
% Academic year 2018—2019
%
% Authors:
%    — Quentin Graillet
%    — Maxime Meurisse
%    — Adrien Schoffeniels

%% Load

load('hw3_noisy_signal.mat');

%% Data

Fs = 1000; % sampling period
Ts = 1 / Fs; % sampling interval
N = length(x); % length of the signal
Tmax = (N — 1) * Ts; % maximum time
t = 0:Ts:Tmax; % time vector

%% Subquestion a)

plot_range = ((N / 2 — 200):(N / 2 + 200));

figure
plot(t(plot_range), x(plot_range), 'LineWidth', 2);
hold on
plot(t(plot_range), x_ns(plot_range), 'LineWidth', 2);
axis tight
title('Signal x[n] and noisy signal x\_ns[n]');
legend('x[n]', 'x\_ns[n]');
grid on

%% Subquestion b)

X = fft(x_ns);
X2 = abs(X / N);
X1 = X2(1:N/2 + 1);
X1(2:end — 1) = 2 * X1(2:end — 1);

F = Fs * (0:(N / 2)) / N;

figure
plot(F, X1, 'LineWidth', 2);
title('Single—Sided Amplitude Spectrum of noisy signal x\_ns[n]');
xlabel('f (Hz)');
grid on

%% Subquestion c)
```

```

fr = find((X1 >= 0.1) & (X1 <= 0.4));

fprintf('Approximate frequency range of the noise v[n] : [%f - %f] Hz\n', F(fr(1)), F(fr(end)));

%% Subquestion e)

load('fir.mat');
fvtool(fir);

x_clean_FIR = filter(fir, x_ns);

X = fft(x_clean_FIR);
X2 = abs(X / N);
X1 = X2(1:N/2 + 1);
X1(2:end - 1) = 2 * X1(2:end - 1);
F = Fs * (0:(N / 2)) / N;

figure
plot(F, X1, 'LineWidth', 2);
title('Single-Sided Amplitude Spectrum of filtered signal x\_fil[n]');
xlabel('f (Hz)');
grid on

%% Subquestion f)

figure
plot(t(plot_range), x(plot_range), 'LineWidth', 2);
hold on
plot(t(plot_range), x_clean_FIR(plot_range), 'LineWidth', 2);
axis tight
title('Signal x[n] and filtered signal x\_fil[n]');
legend('x[n]', 'x\_fil[n]');
grid on

%% Cleaning workspace

clearvars
```