University of Liège

# Homework 2

## Applied digital signal processing

Quentin Graillet
Maxime Meurisse
Adrien Schoffeniels

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# 1 Noise elimination

An electrocardiogram signal (`hw2_electrocardiogram.mat`) is given with a sampling frequency of 250 Hz. This signal is noisy. The goal is to find the original signal, without the noise.

The Matlab code used is appended to this document.

### Question (a)

First, we plot the entire signal (figure 1).

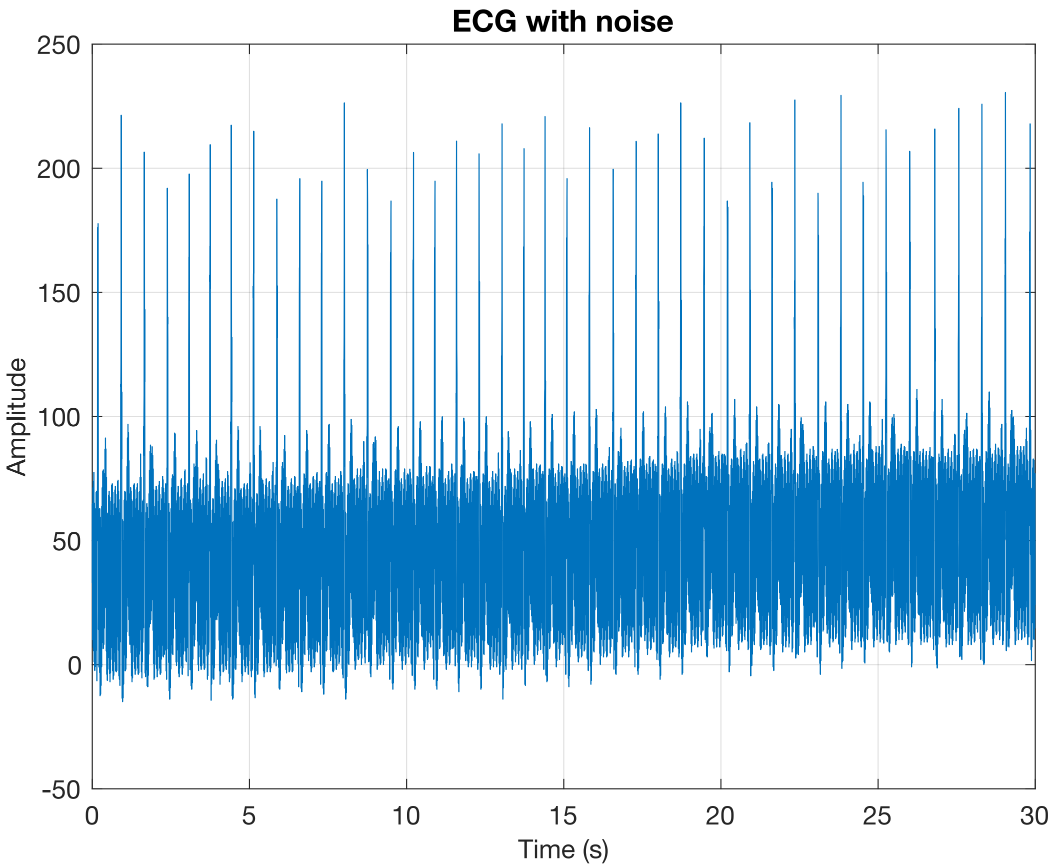


Figure 1 – Whole signal as a function of time (in seconds)

### Question (b)

Since the entire signal is not very visible, we only plot the signal between times 2 and 5 seconds (figure 2).

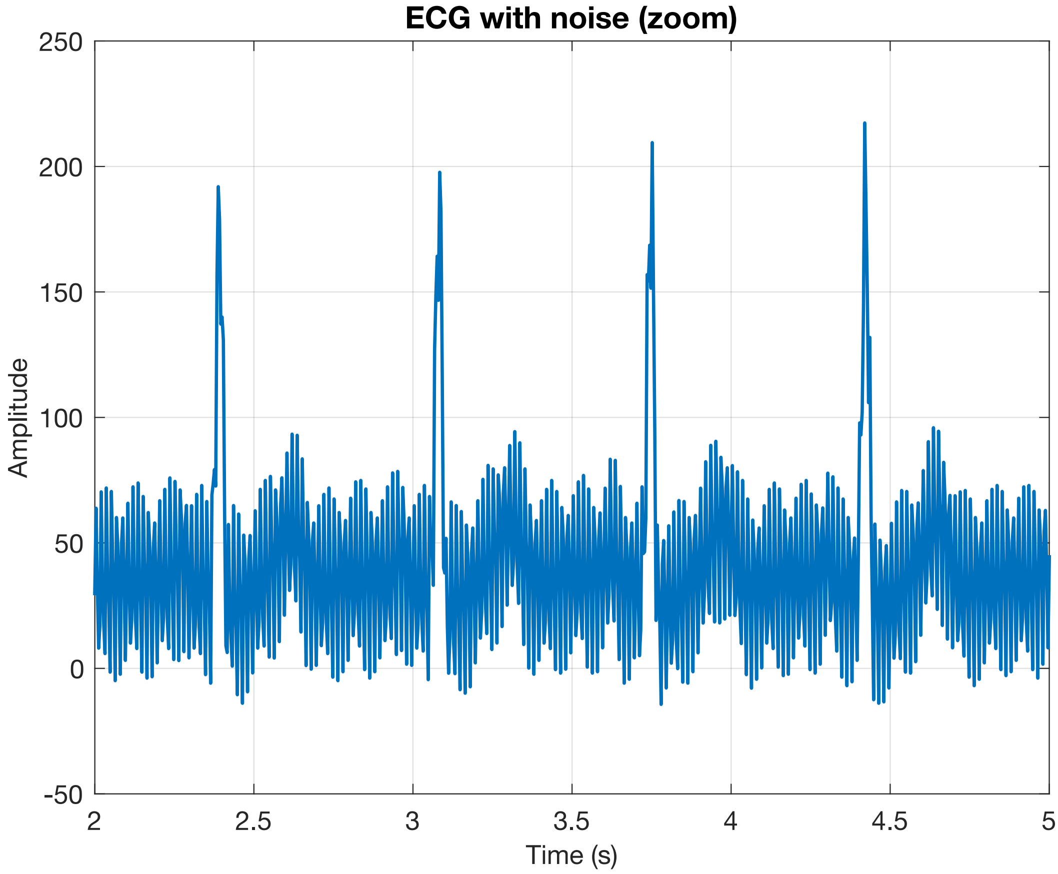


Figure 2 – Signal between times 2 and 5 seconds

### Question (c)

It is clearly observed that the signal is noisy. In order to identify the noisy frequencies, we plot the single-sided magnitude spectrum (thanks to the function `fft` of Matlab) of the signal (figure 3).

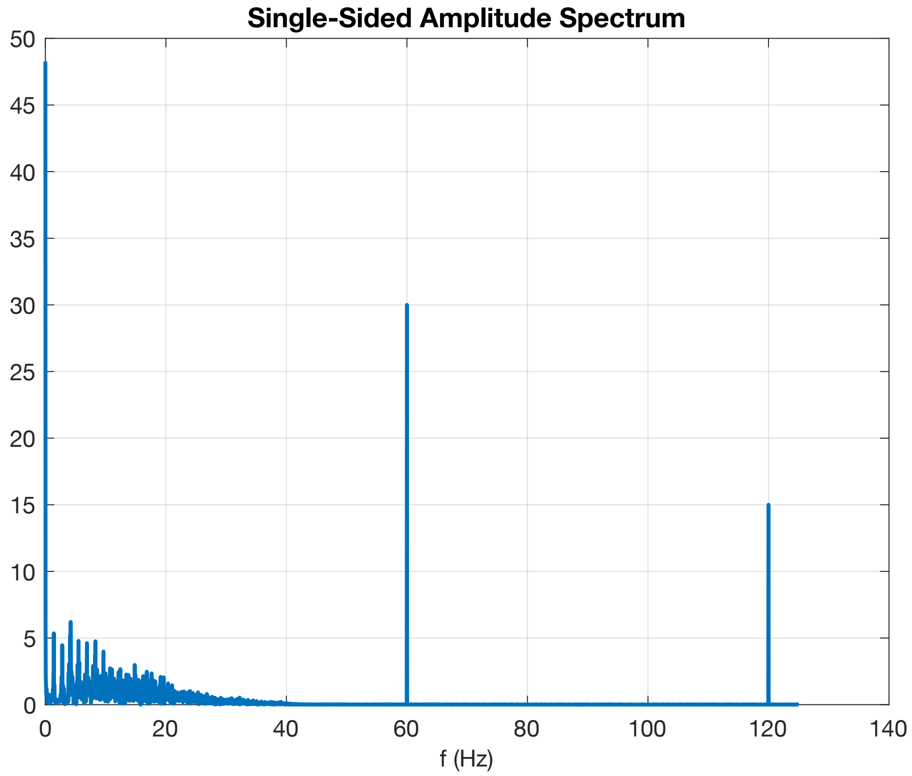


Figure 3 – Single-sided magnitude spectrum of the signal

### Question (d)

The two noisy frequencies are clearly identified by the two peaks: 60 Hz and 120 Hz.

### Question (e)

Both signals (noisy and non-noisy) are shown in figure 4.

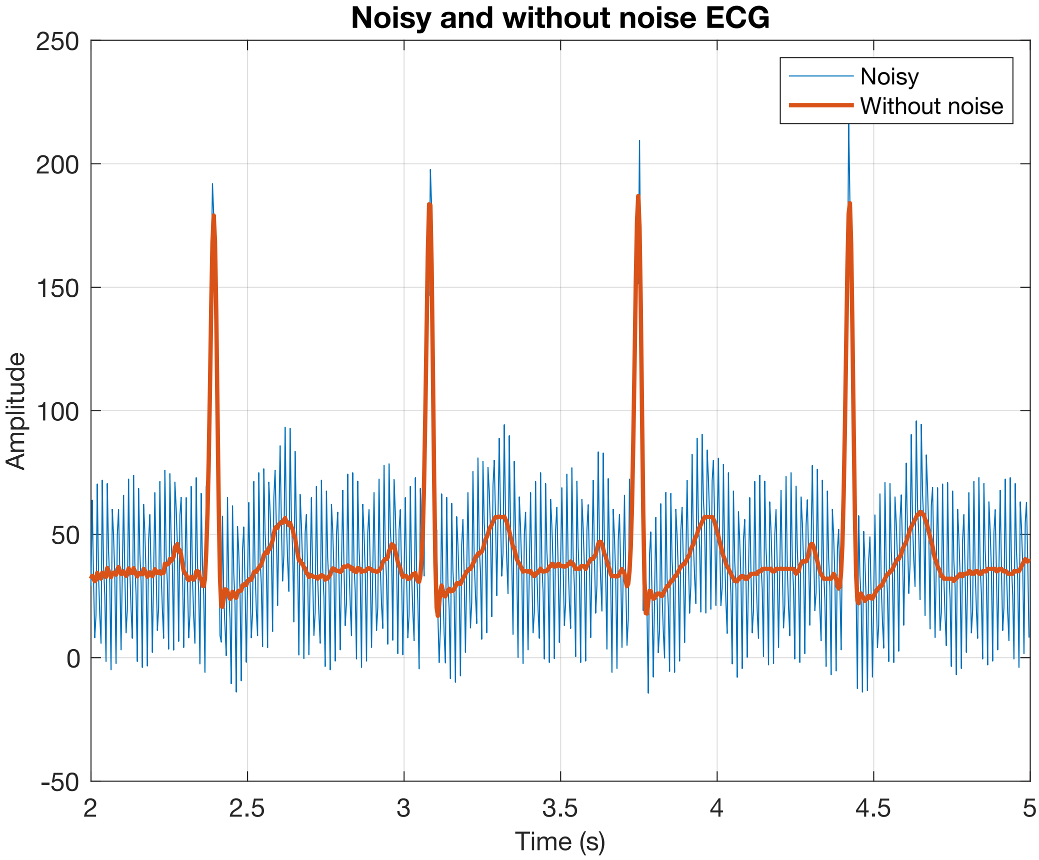


Figure 4 – Noisy and original signal (from the second 2 to second 5)

### Question (f)

We can create notch filters to filter the noisy frequencies. Indeed, a notch filter makes it possible to filter particular frequencies.

In our case, we create two filters: one to filter the noise at 60 Hz frequency and the other to filter the noise at 120 Hz frequency.

The filters were created with Matlab, with the procedure seen during the course.

The two filters are shown in figures 5 and 6. It can be seen that the magnitude response is effectively very low at 60 Hz and 120 Hz respectively (the filters therefore make it possible to remove the components at these frequencies).

The x-axis is expressed in $\pi$ rad/sample. However, it is very simple to do the conversion and to find that one falls well on 60 Hz and 120 Hz respectively :

$$\frac{60\,\text{Hz}}{F_{sample}/2} = \frac{60}{125} = 0.48 \quad \pi\ \text{rad/sample}$$
$$\frac{120\,\text{Hz}}{F_{sample}/2} = \frac{120}{125} = 0.96 \quad \pi\ \text{rad/sample}$$

The phases of the filters are linear, they do not deform the signal.

Applying these two filters in series, we can find the original signal, non-noisy.

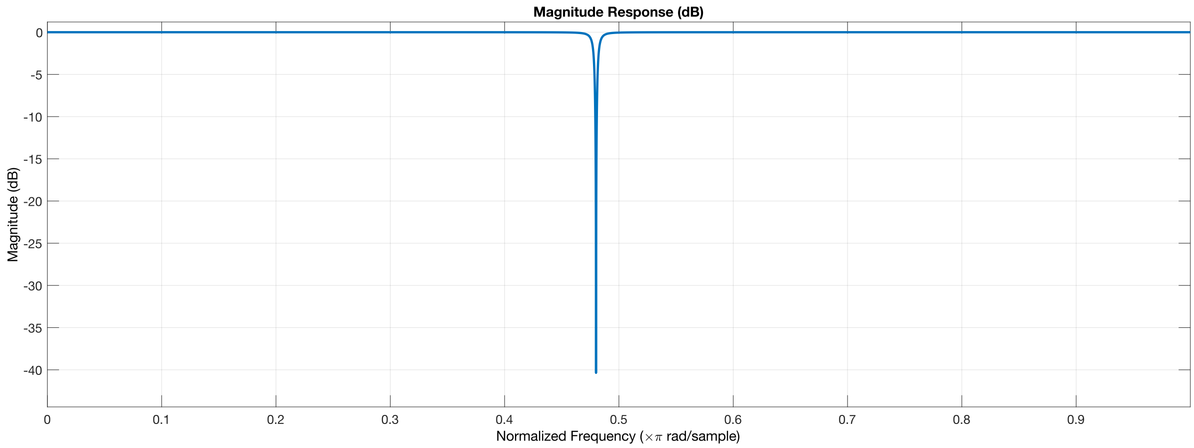


Figure 5 – Magnitude response of the notch filter (60 Hz)

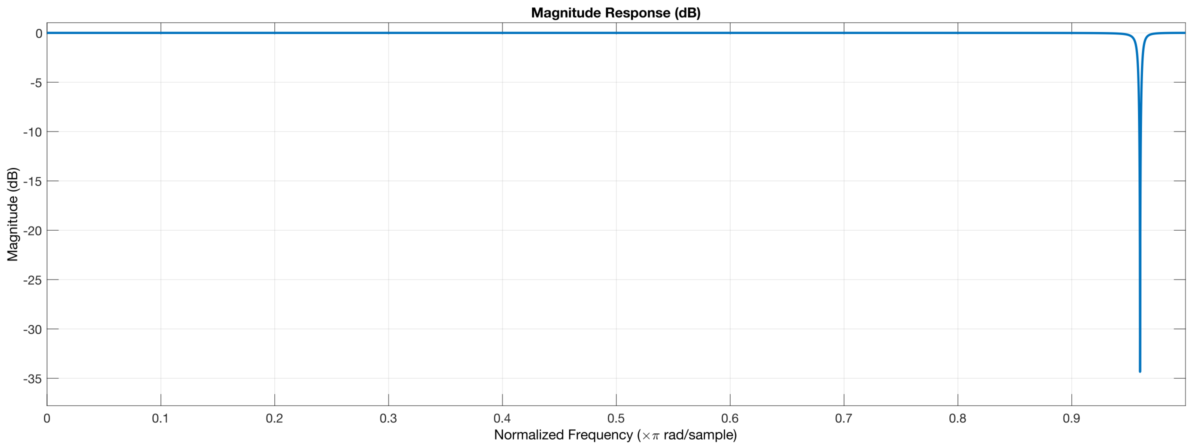


Figure 6 – Magnitude response of the notch filter (120 Hz)

# A Matlab code

```
%% Applied digital signal processing - Homework 2

% Question 1 - Noise elimination
%
% University of Liege
% Academic year 2018-2019
%
% Authors:
%    - Quentin Graillet
%    - Maxime Meurisse
%    - Adrien Schoffeniels

%% Figure properties

gca_fs = 12;
axis_fs = 12;
title_fs = 14;

%% Data

addpath('filters/')
load('hw2_electrocardiogram.mat', 'ecg')
Fs = 250; % sampling frequency

t_min = 2;
t_max = 5;

%% Subquestion a)

Ts = 1 / Fs; % sampling period
N = length(ecg);
Tmax = (N - 1) * Ts;
t = 0:Ts:Tmax; % time vector

figure
plot(t, ecg)
set(gca, 'FontSize', gca_fs)
xlabel('Time (s)', 'FontSize', axis_fs)
ylabel('Amplitude', 'FontSize', axis_fs)
title('ECG with noise', 'FontSize', title_fs)
grid on

%% Subquestion b)

figure
plot(t, ecg, 'LineWidth', 1.5)
set(gca, 'FontSize', gca_fs)
xlim([t_min t_max])
xlabel('Time (s)', 'FontSize', axis_fs)
ylabel('Amplitude', 'FontSize', axis_fs)
title('ECG with noise (zoom)', 'FontSize', title_fs)
grid on

```

```
%% Subquestion c)

ECG = fft(ecg); % Fast Fourier Transform
ECG2 = abs(ECG / N); % take abs and scale it
ECG1 = ECG2(1:(N / 2) + 1); % pick the first half
ECG1(2:end - 1) = 2 * ECG1(2:end - 1); % multiply by 2 (except on DC
    part) to compensate the removed side from the spectrum
F = Fs * (0:(N / 2)) / N; % frequency range

figure
plot(F, ECG1, 'LineWidth', 2)
set(gca, 'FontSize', gca_fs)
xlabel('f (Hz)', 'FontSize', axis_fs)
title('Single-Sided Amplitude Spectrum', 'FontSize', title_fs)
grid on

%% Subquestion e)

load('filters/filter60.mat', 'filter60')
load('filters/filter120.mat', 'filter120')

pure_ecg = ecg;
pure_ecg = filter(filter60, pure_ecg);
pure_ecg = filter(filter120, pure_ecg);

figure
plot(t(500:(length(t) / 6)), ecg(500:(length(t) / 6)))
set(gca, 'FontSize', gca_fs)
xlim([t_min t_max])
xlabel('Time (s)', 'FontSize', axis_fs)
ylabel('Amplitude', 'FontSize', axis_fs)
title('Noisy and without noise ECG', 'FontSize', title_fs)
hold on
plot(t(500:(length(t) / 6)), pure_ecg(500:(length(t) / 6)),
    'LineWidth', 2)
legend('Noisy', 'Without noise')
grid on

%% Filter analysis

fvtool(filter60)
fvtool(filter120)
```

Listing 1 – Elimination of noise on the signal `hw2_electrocardiogram.mat`.