Module 1: Foundations
Scientific Measurement Theory

Fundamentals of Chemistry Open Course

Learning Objectives | Module 1

1. Extract useful quantitative information from problems; generate a list of known and unknown quantities from the text of a problem.
2. Solve equations for a single unknown variable using standard algebraic operations.
3. Draw and interpret graphs relating physical variables with relevance to chemistry.
4. Recognize the essential components of a measurement.
5. Apply dimensional analysis with knowns and unknowns to solve equations involving measured quantities.
6. Calculate measures of accuracy and precision to assess the quality of a set of measurements.
7. Express quantities calculated from measurements at the appropriate level of precision by applying the rules for significant digits.
8. Recognize and distinguish between physical and chemical properties.
9. Classify different types of matter as pure substances or mixtures; compare and contrast homogeneous and heterogeneous mixtures.
10. Describe and apply the scientific method.

Essential Components of a Measurement

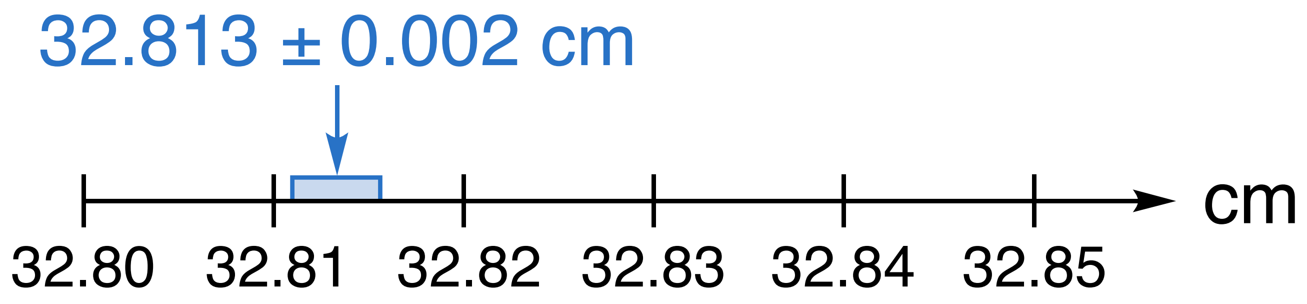
- A **measurement** is a quantitative observation of a physical phenomenon with respect to a given scale.

- Every measurement includes two components:
    - **Units** indicate the phenomenon and the size of an increment on the scale.
    - **Value** positions the measurement on the scale. Value is never an infinitely small point—it is always a range.

- All measurements have some degree of uncertainty; this may be indicated with a standard deviation.

$$32.813(2)\ \mathrm{cm}$$

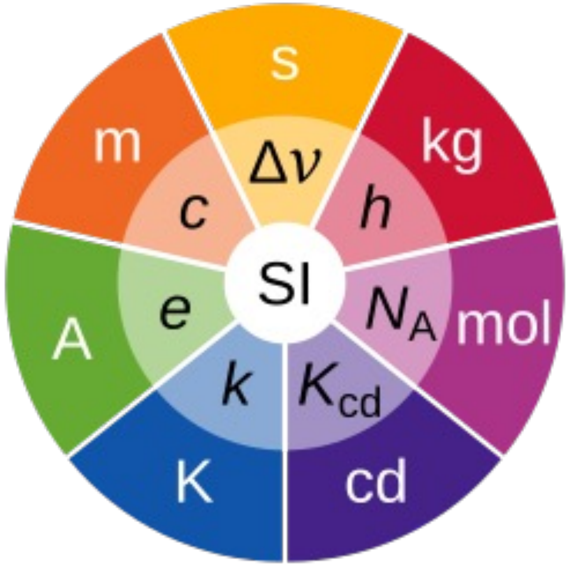
$$32.813 \pm 0.002\ \mathrm{cm}$$



The International System of Units (SI)

- The **SI system** of units includes the **metric system** and a set of base and derived units for measurement.

- The seven **base units** are defined using physical constants such as the speed of light and elementary charge.



Symbol	Name	Quantity
s	second	time
m	metre	length
kg	kilogram	mass
A	ampere	electric current
K	kelvin	thermodynamic temperature
mol	mole	amount of substance
cd	candela	luminous intensity

- Units for other quantities are constructed by multiplying or dividing the base units.

- Examples of derived units include the Newton ($kg \cdot m \cdot s^{-2}$), Coulomb ($A \cdot s$), and Pascal ($kg \cdot m^{-1} \cdot s^{-2}$ or $N\ m^{-2}$).

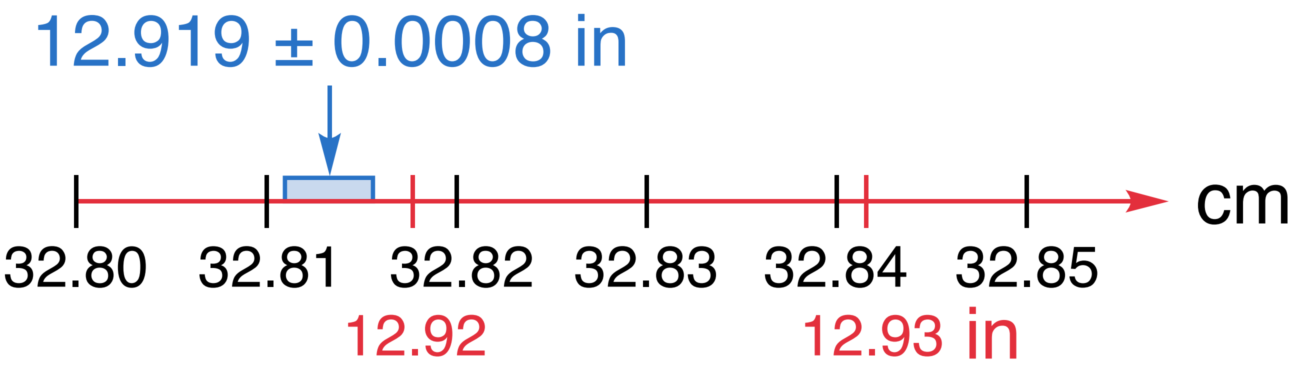
Metric Prefixes

- The **metric system** includes a set of unit prefixes to represent powers of ten. These prefixes enable scaling of the increment on a measurement scale to a “human-friendly” level.

Prefix	Symbol	Factor	Power
tera-	T	1000000000000	10^{12}
giga-	G	1000000000	10^{9}
mega-	M	1000000	10^{6}
kilo-	k	1000	10^{3}
hecto-	h	100	10^{2}
deca-	da	10	10^{1}
—	—	1	10^{0}
deci-	d	0.1	10^{-1}
centi-	c	0.01	10^{-2}
milli-	m	0.001	10^{-3}
micro-	µ	0.000001	10^{-6}
nano-	n	0.000000001	10^{-9}
pico-	p	0.000000000001	10^{-12}

Unit Conversions

- Measurements of a given physical phenomenon can be expressed in an infinite variety of units. Conceptually, **unit conversion** is just a change in the increment on the scale of the measurement.



- **Conversion factor:** the ratio of the size of one unit to the size of another unit associated with the same phenomenon

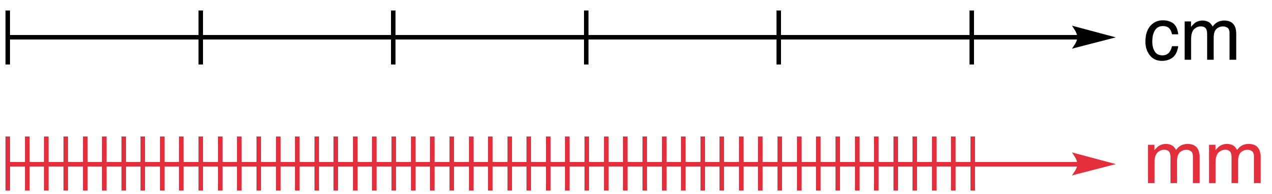
- Conversion factors can be raised to a power to create new factors.

Example. Convert 3.26 cubic meters (m^3) to cubic feet (ft^3). One meter is equivalent to 3.28 feet.

Unit Conversions

- Conversion between units differing only in a metric prefix can be done with a conversion factor built from equivalent quantities in the two units.

- This is usually done to bring the value of a measurement into “human-friendly” territory, between 1 and 1000 or so.

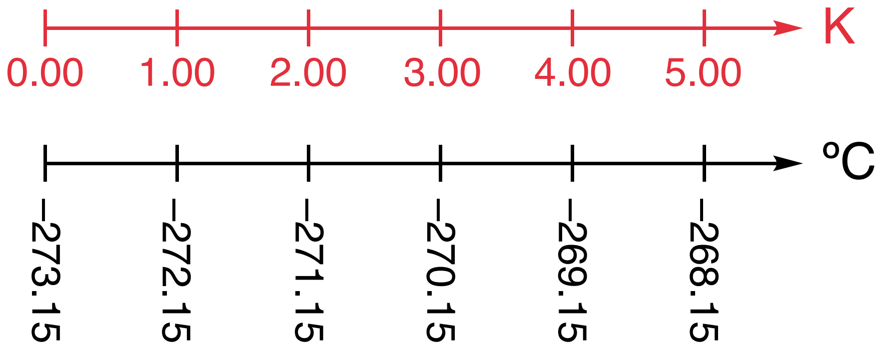


Example. The mole is, in general, a very large unit. For example, 1 gram of the drug rapamycin corresponds to only 0.00109 moles of the substance. Express this amount in millimoles (mmol) and micromoles (μmol).

Converting Between Temperature Units

- The Celsius and Kelvin temperature scales have the same increment; they are just shifted or “translated” relative to one another.

$$T(^\circ\mathrm{C}) = T(\mathrm{K}) + 273.15$$



- The Fahrenheit scale is shifted *and* uses a different increment than the Celsius and Kelvin scales.

$$T(^\circ\mathrm{C}) = \frac{5}{9}[T(^\circ\mathrm{F}) - 32]$$

$$T(^\circ\mathrm{F}) = \frac{9}{5}T(^\circ\mathrm{C}) + 32$$

Scientific Notation

- In **scientific notation**, a measured value is expressed as the product of a number between 1 and 10 (the **significand**) and a power of ten (10 to the **exponent**).
  - This notation clearly shows the level of precision of a measurement or result derived therefrom.
  - It is commonly used for very large or very small values, although this is not a requirement.

$$\overset{\text{significand}}{2.99792458} \times 10^{\overset{\text{exponent}}{8}}\ \mathrm{m/s}$$

- Note that values in scientific notation are easily multiplied (divided): the significands are multiplied (divided) and the exponents are added (subtracted).

- The significand may be negative.

Example. The moon is 394000 km from earth. Express this distance in scientific notation.

Scientific Notation

- To express a given measurement in scientific notation,
    1. Place a decimal point to the right of the leftmost non-zero digit in the value.
    2. Count the number of "steps" it takes to get from the new decimal point to the standard location of the decimal point between the ones and tenths place. This is the exponent.
    3. Discard any leading zeros (if the given value is less than 1) to generate the significand.

Example. The moon is 394000 km from earth. Express this distance in scientific notation.

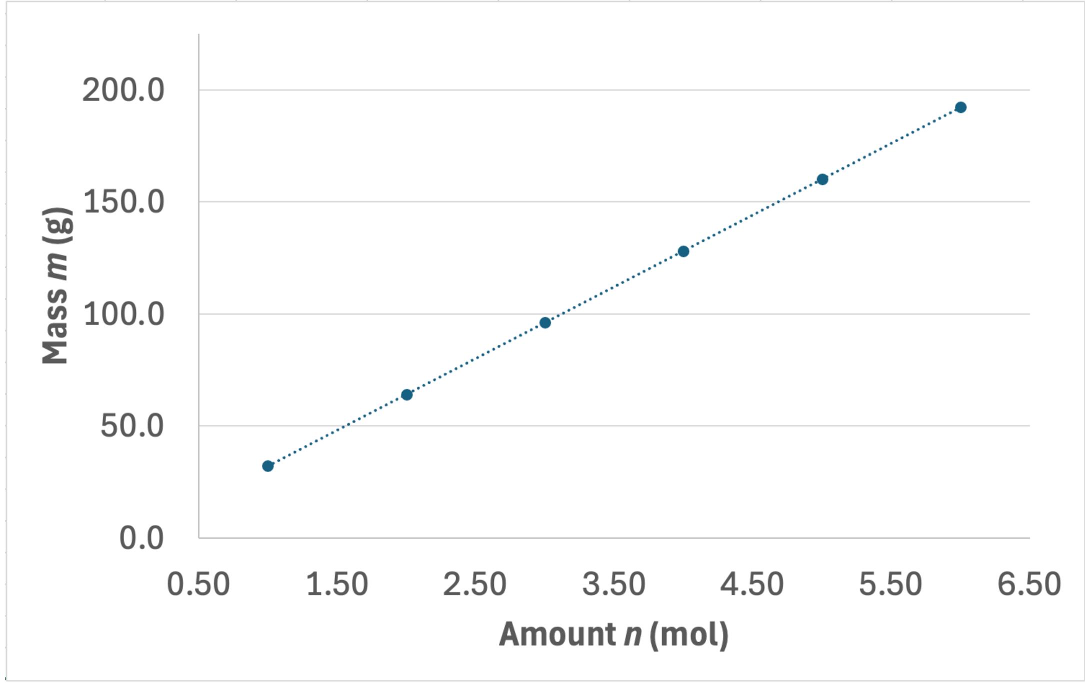
Example. Red light has a wavelength of about 0.00000062 m. Express this length in scientific notation.

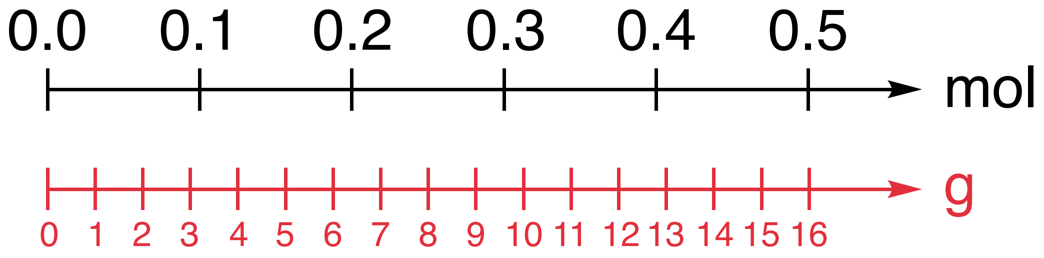
Dimensional Analysis and Proportional Reasoning

- Derived units involving a division of two or more units can be written as a ratio of two measured quantities, with “1 [unit]” in the denominator. For example,

$$32.0\ \mathrm{g/mol} = 32.0\ \mathrm{g\,mol^{-1}} = \frac{32.0\ \mathrm{g}}{1.00\ \mathrm{mol}}$$

- These ratios of measurements can be used to calculate one quantity if the other is known. The ratio expresses a **proportionality** between the two unit scales, which holds at any point along either scale.





Dimensional Analysis and Proportional Reasoning

- The reciprocal measurement expresses the same proportionality and may need to be set up if the unknown quantity is in the denominator of the ratio as given. For example,

$$(32.0\ \mathrm{g/mol})^{-1} = \frac{1.00\ \mathrm{mol}}{32.0\ \mathrm{g}} = 0.031\ \mathrm{mol\,g^{-1}}$$

- Recognizing that ratios of units express linear relations or proportionalities is the basis of **proportional reasoning**. Multiplying a known measured quantity by an appropriate ratio can tell us the value of a related quantity.

- When the exact same unit appears in the numerator and denominator of a calculation, it divides out.

- **Dimensional analysis** is an application of proportional reasoning that takes us from values on known dimensions (scales) to a value on an unknown dimension, using proportionalities as described above.

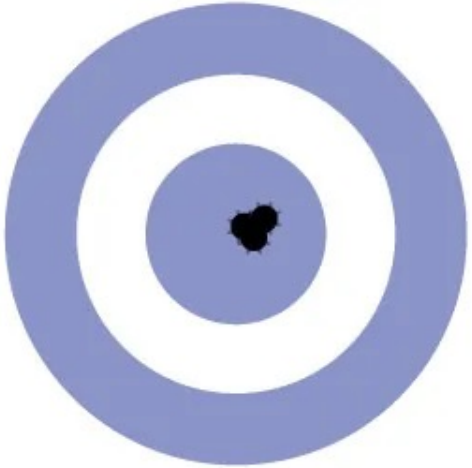
Example. The molar mass of dioxygen (O_2) is 32.0 g/mol. Determine the amount in moles of a sample of dioxygen with a mass of 0.19 g.

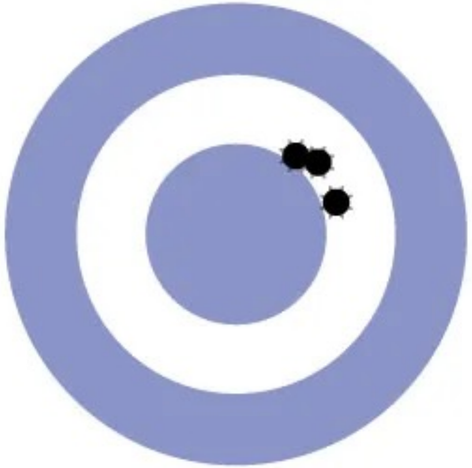
Evaluating the Quality of Measurements

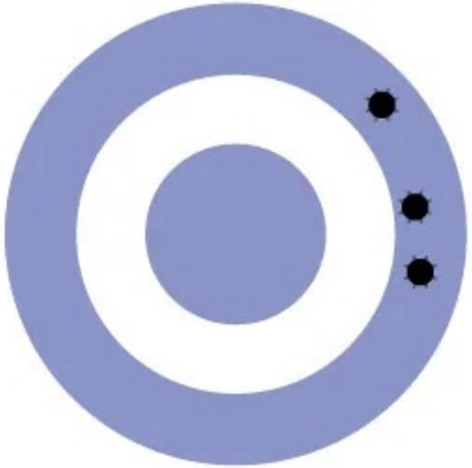
- Equivalent measurements are usually made multiple times (in **replicate**).
  - The average of replicate measurements is our “best guess” of the true value.
  - The **standard deviation** of replicate measurements captures the “spread” or uncertainty of the set.

- There are two ways to evaluate the quality of a set of replicate measurements…
  - **Accuracy** is the closeness of the average to a previously measured value or reference value.
  - **Precision** is the closeness of the measurements *to each other*, without reference to an outside value.

Example. Evaluate the accuracy and precision of the three sets of dart throws below, assuming the center of each target is the “reference location.”







Evaluating the Quality of Measurements

- Accuracy can be expressed quantitatively as the deviation of the average (**mean**) from the reference value x_{ref} divided by the reference value times 100%, a measure known as **percent error**.

- We divide by the reference value so that the deviation is expressed as a *percentage* of this value—this makes the accuracy measure independent of the units of the measurements.

- Percent error may be signed but is usually expressed as a positive quantity regardless of the sign of the deviation.

$$PE = \frac{\bar{x} - x_{ref}}{x_{ref}} \times 100\%$$

Evaluating the Quality of Measurements

- Precision can be expressed quantitatively as the **standard deviation (σ)** of the set of measurements. This is the square root of the average of the squared deviation of each measurement from the mean. For N measurements x_i,

$$\sigma = \sqrt{\frac{\sum_i^N (x_i - \bar{x})^2}{N}}$$

- The standard deviation has the same units as the measurements themselves.
- The larger the standard deviation, the greater the “spread” in the data.
- Standard deviation is frequently included with reported means either as “± σ” or in parentheses at the end of the value. Digits inside the parentheses correspond to the rightmost digits of the mean.

Significant Digits

- **Significant digits (figures)** are non-placeholding digits that communicate the value of a measurement as opposed to its power of ten.

- Equivalently, significant digits are those that appear in the significand when the number is written in scientific notation (n in $n \times 10^m$).

- The greater the number of significant digits, the greater the certainty of the measurement.

$$\mathbf{1.0555} \times 10^2 \text{ g}$$

Significant Digits

- **Rules for determining significant digits:**
  1. All nonzero digits are significant.
  2. Zeros between two significant figures are themselves significant.
  3. Zeros at the beginning of a number (leading zeros) are never significant.
  4. Zeros at the end of a number and after the decimal point are always significant.
  5. Zeros at the end of a number and before the decimal point are not significant if a decimal point is not present.

- Note that **exact numbers**, including counting numbers and mathematical constants (π, e), do not limit significant digits in calculations involving these numbers.

Significant Digits

1. **For addition and subtraction.** The result has the same level of precision as the least precise measurement. In other words, the number of decimal places in the result is the same as the measurement with the smallest number of decimal places.

   Example: Adding or subtracting two volumes

$$\begin{array}{r} 865.\mathbf{9}\ \text{mL} \\ -\ 2.8121\ \text{mL} \\ \hline 863.0879\ \text{mL} = \mathbf{863.1}\ \text{mL} \end{array}$$

$$\begin{array}{r} 83.\mathbf{5}\ \text{mL} \\ +\ 23.28\ \text{mL} \\ \hline 106.78\ \text{mL} = \mathbf{106.8}\ \text{mL} \end{array}$$

2. **For multiplication and division.** The result has the same number of significant digits as the least precise measurement. In other words, the least precise measurement limits the precision of the result.

   Example: Calculating a volume

$$\mathbf{9.2}\ \text{cm} \times 6.8\ \text{cm} \times 0.3744\ \text{cm} = 23.4225\ \text{cm}^3 = \mathbf{23}\ \text{cm}^3$$

Significant Digits

1. **Rounding with the first digit removed *greater than 5.*** The least significant digit in the result is increased by 1.

   Example: 5.379 rounds to 5.38 or 5.4.

2. **Rounding with the first digit removed *less than 5.*** The least significant digit in the result is unchanged.

   Example: 0.2413 rounds to 0.241 or 0.24.

3. **Rounding with the *only* digit removed *equal to 5.*** Round up if the preceding digit is odd and round down if the preceding digit is even.

   Example: 0.415 rounds to 0.42 but 0.445 rounds to 0.44.

4. **Rounding with the first digit removed *equal to 5.***
   Round up (apply rule 1) unless all subsequent digits are zero (apply rule 3).

   Example: 8.6252 rounds to 8.63; 8.625 and 8.62500 round to 8.62.

As a rule, don't worry about rounding or significant digits until the very end of a calculation.