Adaptive Algorithms for Automatic Link Selection in Multiple Access with Link Failures

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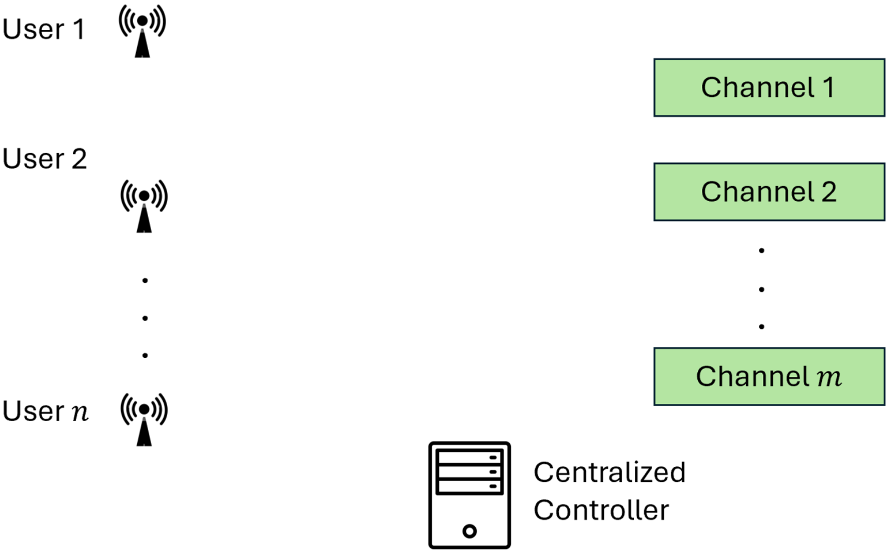
Outline

System Model



Link Selection

In each slot, the controller assigns the users to channels

- Each user is assigned to at most one channel
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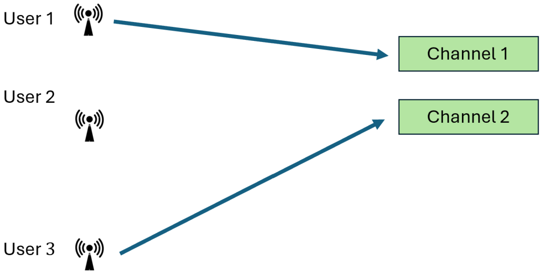
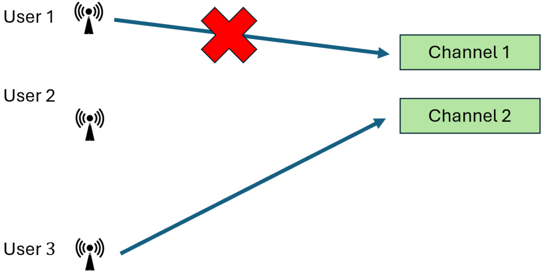


Assignment matrix

$$\boldsymbol{Y}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Link Failures

1. After assignment, link (i, j) fails with probability $1 - q_{i,j}$
2. $q_{i,j}$ have to be estimated using bandit feedback

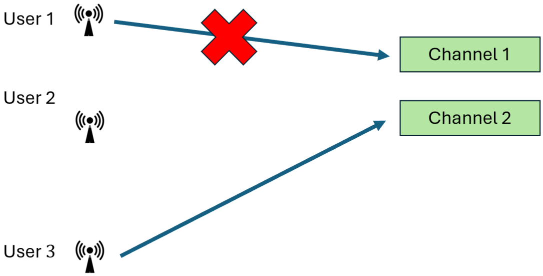


Link Failure Matrix

$$\boldsymbol{S}(t) = \begin{bmatrix} \boxed{0} & 1 \\ 0 & 0 \\ 1 & \boxed{1} \end{bmatrix}$$

Throughput Vector

1. Throughput vector $\boldsymbol{X}(t) \in \{0, 1\}^n$.



$$\boldsymbol{X}(t) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Maximizing System Throughput

Time horizon - T

$$\text{maximize} \sum_{i=1}^{n} \frac{1}{T} \sum_{t=1}^{T} X_i(t)$$

Pros: Widely analyzed
Cons: Unfair for some users

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$$\begin{array}{cc} & \begin{array}{cc} \text{Channel 1} & \text{Channel 2} \end{array} \\ \begin{array}{c} \text{User 1} \\ \text{User 2} \\ \text{User 3} \end{array} & \begin{bmatrix} 0.7 & \boxed{0.7} \\ 0.3 & 0.2 \\ \boxed{0.6} & 0.4 \end{bmatrix} \end{array}$$

Figure: Example $q_{i,j}$ values

Network Utility Maximization

$$\text{Maximize } \phi\left(\underbrace{\boxed{\frac{1}{T}\sum_{t=1}^{T} X_1(t)}, \frac{1}{T}\sum_{t=1}^{T} X_2(t), \ldots, \frac{1}{T}\sum_{t=1}^{T} X_n(t)}_{\text{Time-average throughput vector}}\right)$$

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Assumptions

1. Concave
2. Entrywise nondecreasing
3. Bounded subgradients $|\phi_i'(\boldsymbol{x})| \leqslant B$.

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3. $\phi(\boldsymbol{x}) = \sum_{i=1}^{n} \log(1 + \beta x_i)$.
4. $\phi(\boldsymbol{x}) = w_1 \min\{x_1, x_2, \ldots, x_n\} + w_2 \sum_{i=1}^{n} \log(1 + \beta x_i)$.

Known $q_{i,j}$ Values

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3. Optimal value ϕ^{opt}

Our Goal

- Can we get close to ϕ^{opt} when $q_{i,j}$ are **unknown.**

Outline

Finite Time Bounds

$$\phi^{\text{opt}} - \phi\left(\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{\boldsymbol{X}(t)\}\right) \leqslant g(T),$$

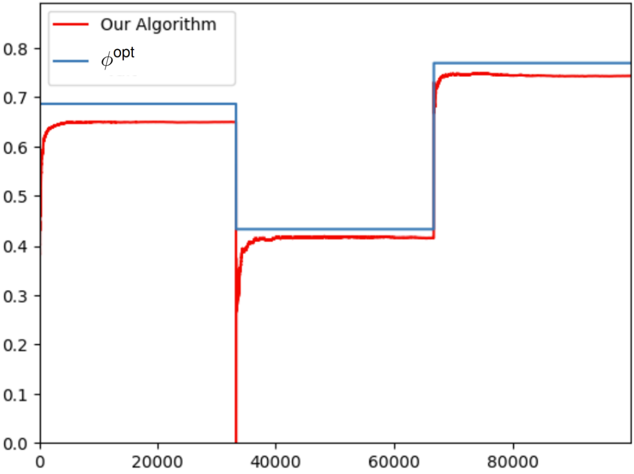
such that $g(x) \to 0$ as $x \to \infty$.

Prior Work on Single-Channel Case

- Can be solved using[1].
- **Issues:** Above UCB based algorithms are not **adaptive**.

[1] Shipra Agrawal and Nikhil R. Devanur. "Bandits with concave rewards and convex knapsacks". In: *Proceedings of the Fifteenth ACM Conference on Economics and Computation*. Palo Alto, California, USA, June 2014, pp. 989–1006.

Adaptiveness



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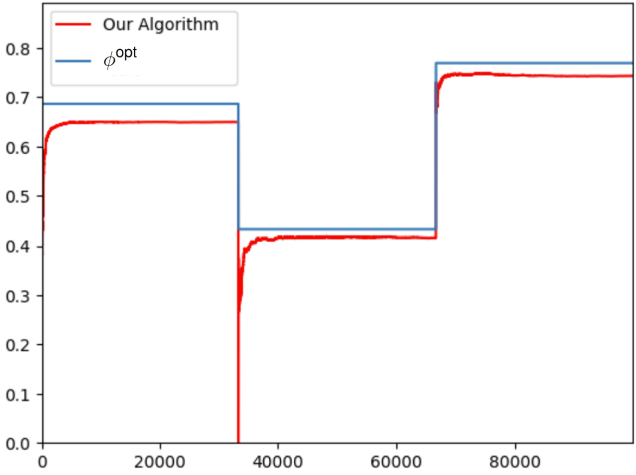

- **Idea:** Combining Lyapunov Optimization with Importance Sampling Based Estimation (EXP3 style)

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- **Estimate** $S(t)$ ($q_{i,j}$ values) using feedback
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- **Update** $\boldsymbol{P}(t+1)$

Virtual Queues and Auxiliary Variables

Auxiliary variables $\gamma(t) \in [0,1]^n$, Virtual queue $\boldsymbol{Q}(t) \in [0,\infty)^n$

1. $\gamma(t+1) = \arg\min_{\gamma \in [0,1]^n} \left[-V\phi(\gamma) + \sum_{i=1}^{n} Q_i(t)\gamma_i \right]$
2. $\boldsymbol{Q}(t+1) = \max\{\boldsymbol{Q}(t) + \gamma(t+1) - \boldsymbol{X}(t), \boldsymbol{0}\}$

Update for $\boldsymbol{P}(t)$

$$\boldsymbol{P}(t+1) = \arg \min_{\substack{\boldsymbol{P} \text{ is doubly stochastic} \\ \text{with each entry} \geqslant \varepsilon}} \Big[-\sum_{i=1}^{n} \sum_{j=1}^{m} Q_i(t) \hat{S}_{i,j}(t) P_{i,j} + \frac{1}{\eta} D(\boldsymbol{P} \| \tilde{\boldsymbol{P}}(t)) \Big]$$

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1. No closed form solution!!
2. **Possible approach :** Sinkhorn's iterations.

Even/Odd Iterations

1. $\tilde{\boldsymbol{P}}(t+1) = \arg\min_{\boldsymbol{P}\in\mathcal{Q}} \left[-\sum_{i=1}^{n}\sum_{j=1}^{m} Q_i(t)\hat{S}_{i,j}(t)P_{i,j} + \frac{1}{\eta} D(\boldsymbol{P}\|\tilde{\boldsymbol{P}}(t)) \right],$
   - $\mathcal{Q}$ = column stochastic matrices with each entry $\geqslant \varepsilon$, when t is **even**
   - $\mathcal{Q}$ = row stochastic matrices with each entry $\geqslant \varepsilon$, when t is **odd**

[2] Jason Altschuler, Jonathan Niles-Weed, and Philippe Rigollet. "Near-linear time approximation algorithms for optimal transport via Sinkhorn iteration". In: *Advances in Neural Information Processing Systems*. Dec. 2017.

Even/Odd Iterations

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2. $\boldsymbol{P}(t+1) = \text{ROUND}(\tilde{\boldsymbol{P}}(t+1))$

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2. $\boldsymbol{P}(t+1) = \text{ROUND}(\tilde{\boldsymbol{P}}(t+1))$
3. Used for transport polytopes in[2]

[2] Altschuler, Niles-Weed, and Rigollet, "Near-linear time approximation algorithms for optimal transport via Sinkhorn iteration".

Why Do We Need to Sample Alternatively

1. **Idea :** In the long run, $\tilde{\boldsymbol{P}}(t)$ will get closer and closer to a doubly stochastic matrix (close to $\boldsymbol{P}(t)$).

Outline

Performance Bound

Theorem

For any $T, T_0 \in \mathbb{N}$,

$$\phi^{opt} - \phi\left(\frac{1}{T}\sum_{t=T_0}^{T+T_0-1}\mathbb{E}\{\boldsymbol{X}(t)\}\right) = O\left(\frac{nm\max^2\{n,m\}\log(T)}{T^{1/3}}\right)$$

Proof: `https://arxiv.org/abs/2501.14971`

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Adaptiveness: This holds independent of the behavior outside of $[T_0 : T + T_0 - 1]$.

Bound for the Single Channel Case

Recall for our multi-channel algorithm

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2. If we enforce each user to transmit more that θ fraction of the time,

$$\phi^{\text{opt}} - \phi\left(\frac{1}{T}\sum_{t=T_0}^{T+T_0-1}\mathbb{E}\{\boldsymbol{X}(t)\}\right) = O\left(\frac{n}{T^{1/2}}\right)$$

Outline

Conclusions

1. We developed an **adaptive** algorithm for the problem of automatic link selection in multi-channel multiple access with link failures.
2. We provided theoretical guarantees for the convergence.
3. The algorithm simplifies in the single channel case with faster convergence.

Future Work

1. Time-correlated scenarios, where channel successes are modulated by an unknown Markov process.

Thank You