INFS7450 SOCIAL MEDIA ANALYTICS
Tutorial Week 3

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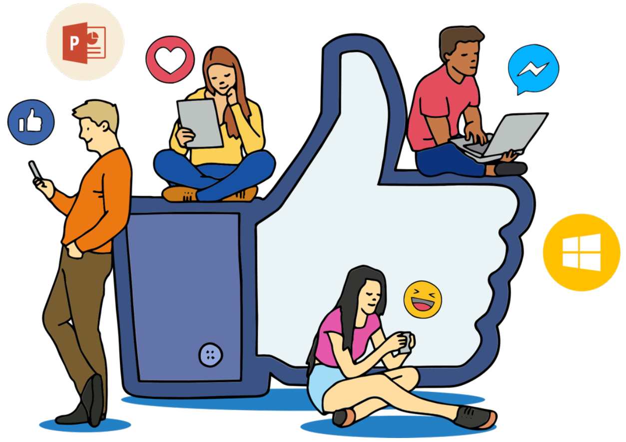
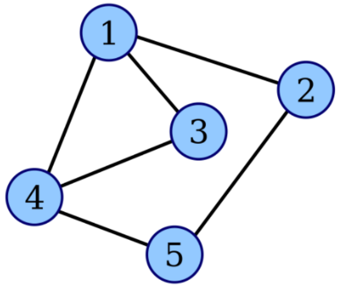


Outlines

- Quiz 1 Answers
- Lecture Knowledge Extension
- Coding Demo
  - Implementation of Node Centrality Measures
- Q&A

Section 1: Quiz 1 Answers

QUESTION 1

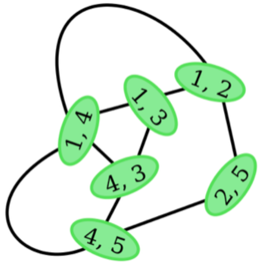


The above graph has 5 nodes connected by 6 edges, which of the following candidates is the correct line graph of the above graph?

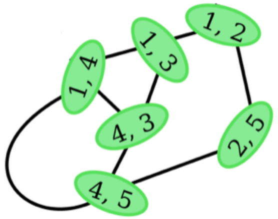
Line graph

- The **line graph** of an undirected graph *G* is another graph L(*G*) that represents the adjacencies between edges of *G*.
- L(G) is constructed in the following way:
  - for each edge in G, make a vertex in L(G);
  - for every two edges in G that have a vertex in common, make an edge between their corresponding vertices in L(G).

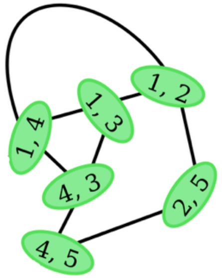
A.



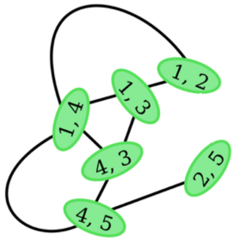
B.



C.



D.



- A. The first one (A)
- B. The second one (B)
- C. The third one (C)
- D. The last one (D)

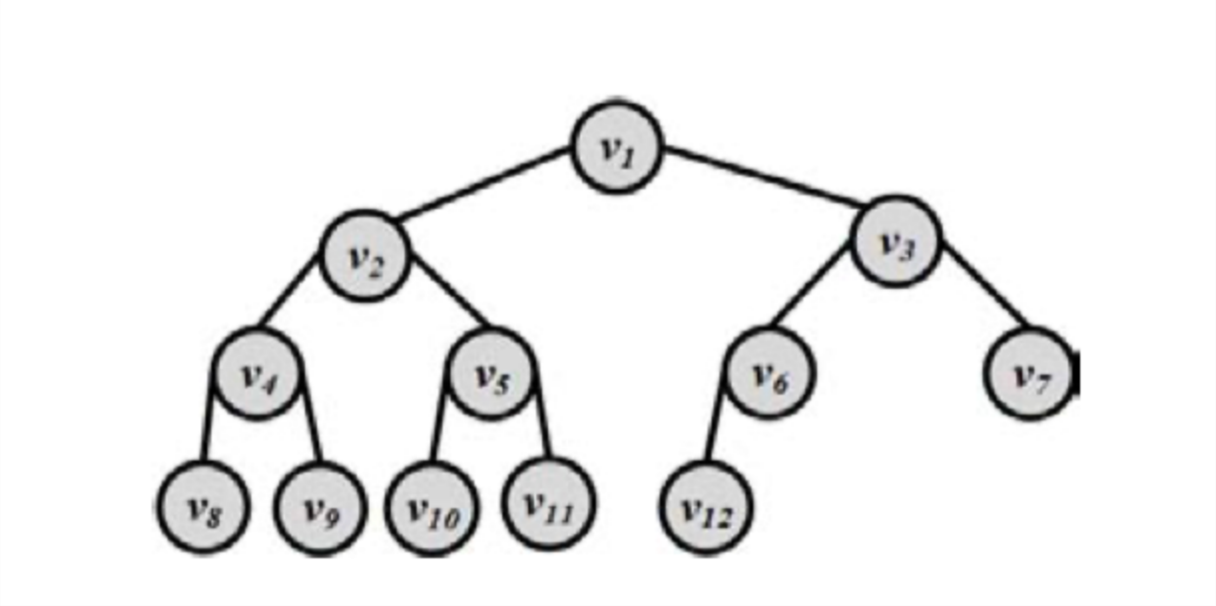
Question	Is it possible to have the following degrees in a graph with 7 nodes (let's assume the graph is an undirected graph) ? {4, 4, 4, 3, 5, 7, 2}.
Answer	Yes ✔ No
Correct Feedback	your are correct!
Incorrect Feedback	actually, the degree sum should be equal to double of the edges.

– Handshake Theorem: Let G=(V,E) be an undirected graph with m edges. Then

$$2m = \sum_{v \in V} \deg(v)$$

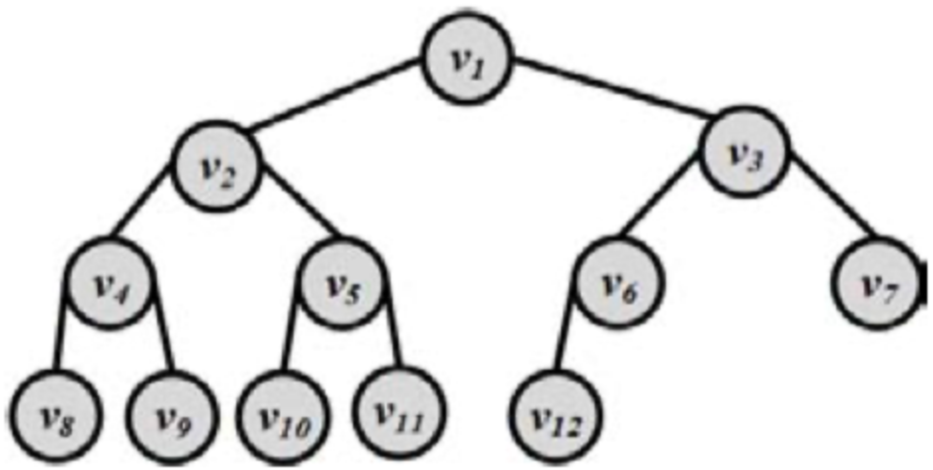
Proof: Each edge contributes twice to the total degree count of all vertices. Thus, both sides of the equation equal to twice the number of edges.

Question	Consider the tree shown in the following figure. Traverse the graph using BFS and list the order in which nodes are visited in each algorithm. Which of the following is correct?



Answer	✓ A. v1,v2,v3,v4,v5,v6,v7,v8,v9,v10,v11,v12
	B. v1,v2,v3,v4,v8,v9,v5,v10,v11,v6,v7,v12
	C. v1,v2,v4,v3,v6,v12,v5,v10,v11,v7,v8,v9
Correct Feedback	you are correct
Incorrect Feedback	A is correct

Question	Consider the tree shown in the following figure. Traverse the graph using DFS and list the order in which nodes are visited in each algorithm. Which of the following is correct?



Answer

A. v1,v2,v3,v4,v5,v6,v7,v8,v9,v10,v11,v12

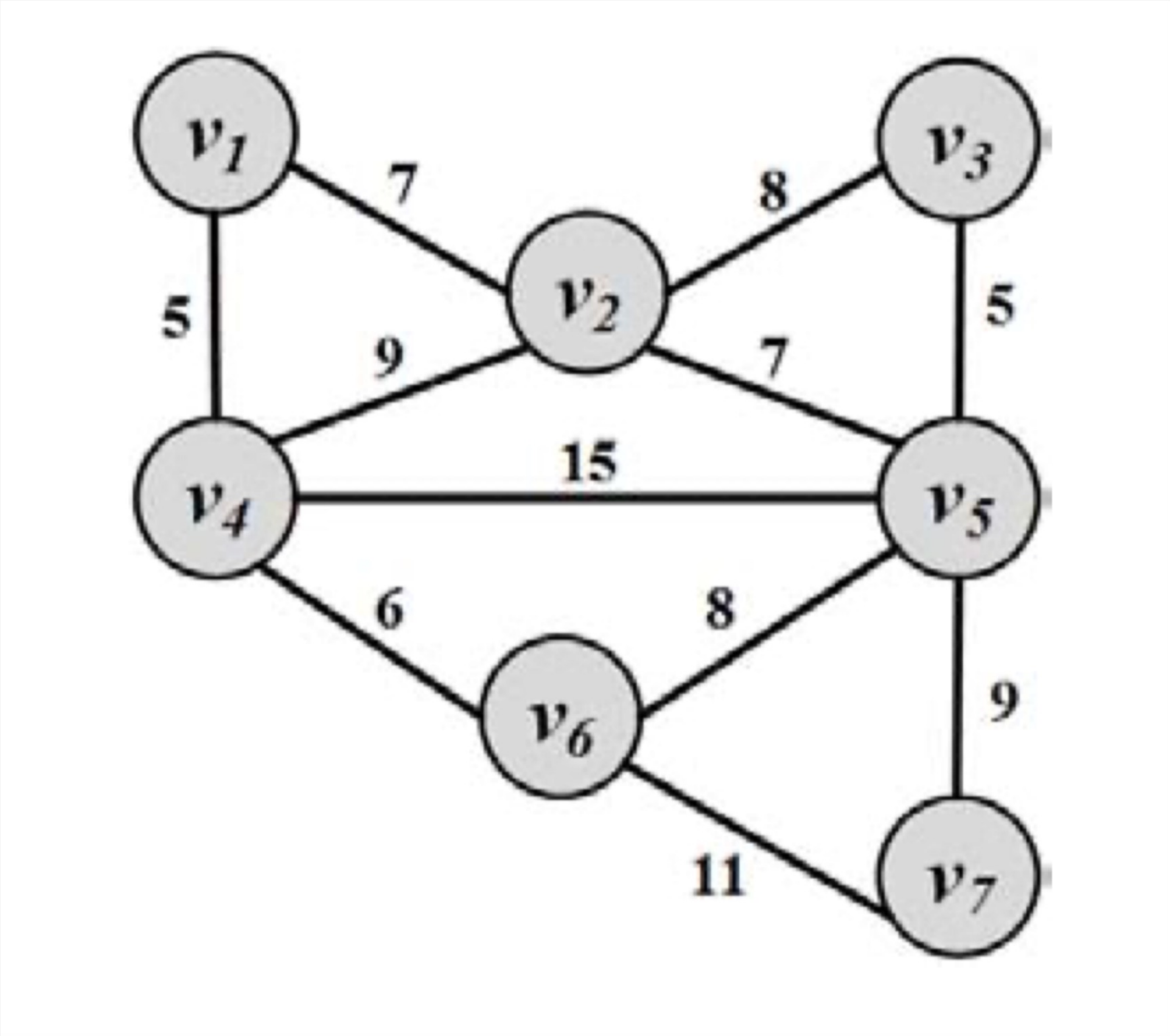
B. v1,v2,v4,v8,v9,v5,v10,v11,v3,v6,v12,v7

C. v1,v2,v4,v3,v6,v12,v5,v10,v11,v7,v8,v9

Correct Feedback	correct!
Incorrect Feedback	B is correct

Question

the shortest path length between v1 and v6 for the following graph is:



Answer

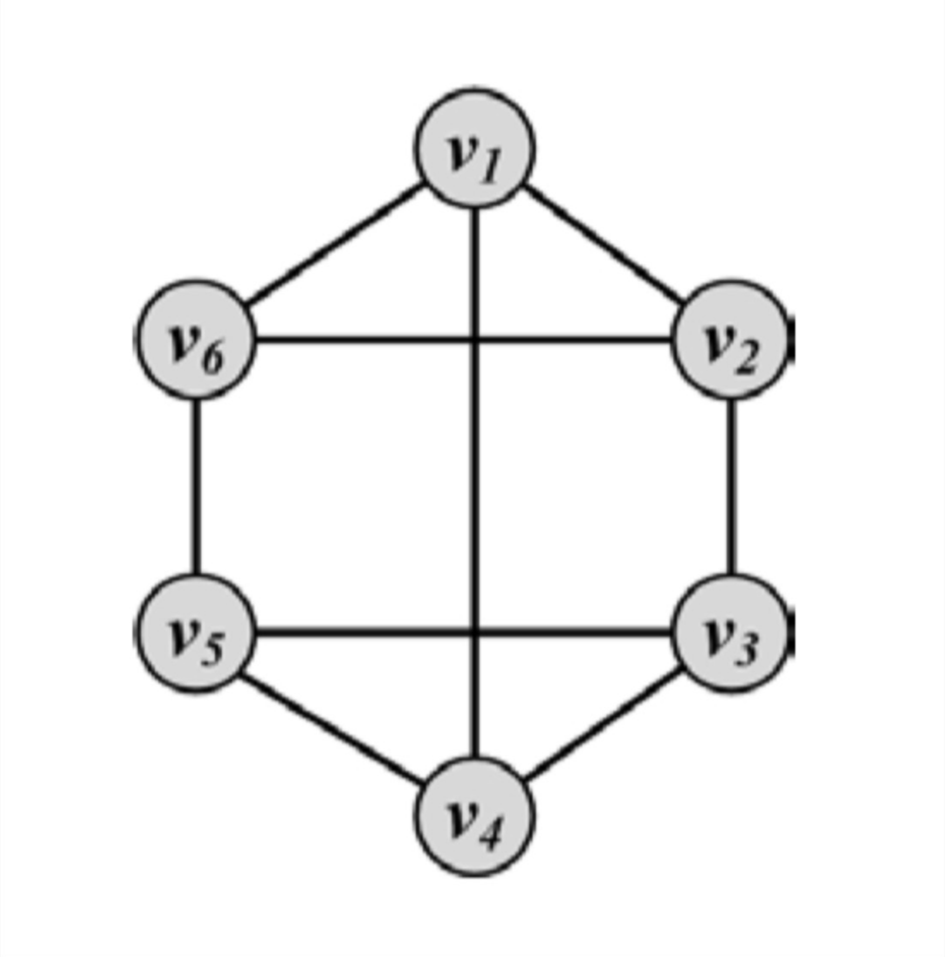
A. 5

B. 6

C. 11

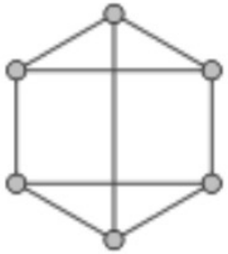
D. 2

Question	which kind of graph is correct according to the following Figure?

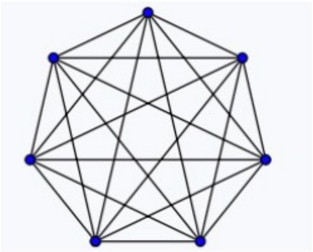


Answer	A. Complete graph
	B. directed graph
	✔ C. Regular graph
Correct Feedback	you are correct
Incorrect Feedback	C is correct

- Regular graph VS complete graph
  - A regular graph is a graph where each vertex has the same number of neighbors; i.e., every vertex has the same degree.
  - A complete graph is a graph in which each pair of graph vertices is connected by an edge.
  - A complete graph is always a regular graph.

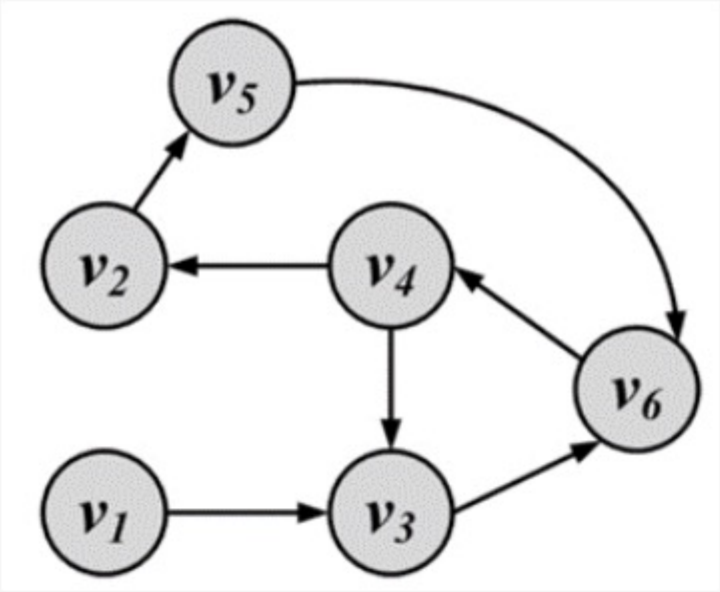


3-regular graph



Complete graph

Which of the following is correct?



A. v 4 , v 3 , v 6 , v 4 , v 2 is not a walk

B. v 4 , v 3 is not a path

C. v 4 , v 3 , v 6 , v 4 , v 2 is not a trail

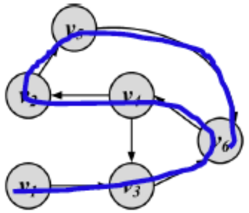
D. v 4 , v 3 , v 6 , v 4 is both a tour and a cycle.

Walk: A walk is a sequence of incident edges visited one after another

Path

- A walk where **nodes and edges are distinct** is called a **path**
- One special case – the starting node and end node can be the same one. In this case, it is called a **cycle**.

- A trail is a walk where **no edge is visited more than once** and all walk edges are distinct



- A closed trail (one that ends where it starts) is called a **tour** or **circuit**

Question	In an undirected graph, there are an/a _____ number of nodes having odd degree.
Answer	✓ even
	odd
Correct Feedback	you are correct
Incorrect Feedback	A is correct

– Theorem: An undirected graph has an even number of vertices of odd degree.

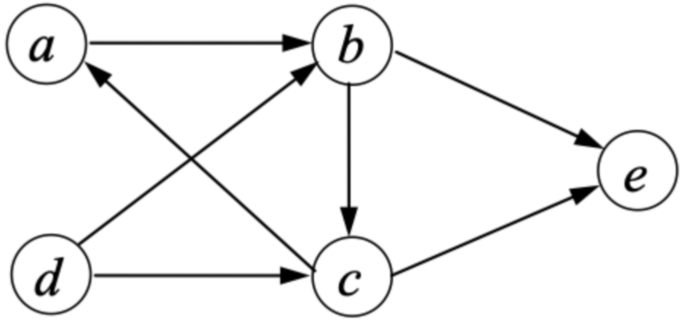
Proof: Let V_1 be the vertices of even degree and V_2 be the vertices of odd degree in an undirected graph G=(V,E) with m edges. Then

$$2m = \sum_{v \in V} \deg(v) = \sum_{v \in V_1} \deg(v) + \sum_{v \in V_2} \deg(v)$$

Must be even since deg(v) is even for each $v \in V_1$

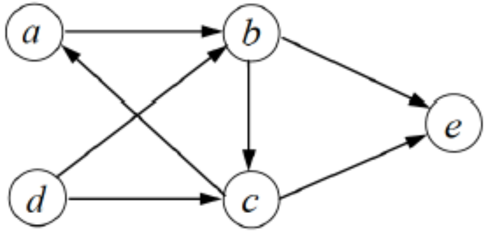
This sum must be even because 2m is even and the sum of the degrees of the vertices of even degrees is also even. Because this is the sum of the degrees of all vertices of odd degree in the graph, there must be an even number of such vertices.

Question	which of the following is a strongly connected component of this graph?



Answer	A. {a,b,c}
	B. {b,c}
	C. {a,b}
Correct Feedback	you are correct
Incorrect Feedback	A is correct

- What is a strongly connected component?
  - A strongly connected component is the portion of a directed graph in which there is a path from each vertex to another vertex.
  - It is applicable only on a directed graph.
  - Is {a,b,c} a strongly connected component?
  - Is {b,c,d} a strongly connected component?



Question	Consider the following adjacency matrix of a directed graph (let's say, node index starts from 0, and if A[i,j]=1, there exists an directed edge starting from node i, ending at node j.). Then the in-degree of node 0 is:

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0. \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Answer

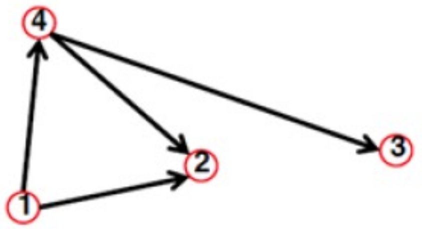
A. 0

B. 1 ✔

C. 2

D. 3

Directed



$$A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\boxed{d_i^{out} = \sum_{j=1}^{N} A_{ij}}$$

$$\boxed{d_j^{in} = \sum_{i=1}^{N} A_{ij}}$$

$$A_{ij} \neq A_{ji}$$
$$A_{ii} = 0$$

Correct Feedback	you are correct
Incorrect Feedback	correct is B

Section 2: Lecture Knowledge Extension

Section 2:

1. In-class Quiz about three geometric centrality measures
2. What are eigenvectors and eigenvalues?
3. How to calculate eigenvectors and eigenvalues?

In-Class Quiz (5 mins)

- Consider an undirected graph with 5 nodes (A,B,C,D,E) with the following edge list:

  (A,B)

  (A,C)

  (B,C)

  (B,D)

  (C,E)

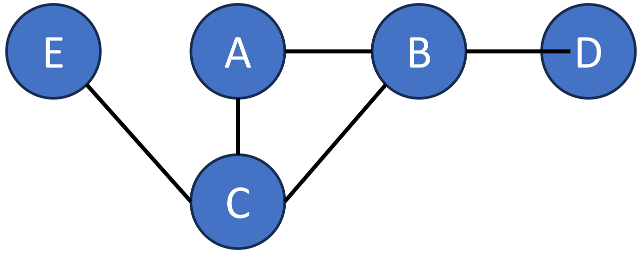
  Please calculate the centrality of nodes A and E using Degree Centrality, Closeness Centrality, and normalized Harmonic Centrality methods, respectively.

In-class quizzes will NOT be included in the final grade.

In-Class Quiz

Consider an undirected graph with 5 nodes (A,B,C,D,E) with the following edge list:

(A,B)
(A,C)
(B,C)
(B,D)
(C,E)



Please calculate the centrality of nodes A and E using Degree Centrality (DC), Closeness Centrality (CC), and normalized Harmonic Centrality (HC) methods, respectively.

(In-)Degree Centrality: The number of incoming links

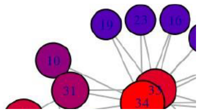
$$c_{\deg}(x) = d_{in}(x)$$

Closeness Centrality

Nodes that are more central have smaller distances

$$c_{\text{clos}}(x) = \frac{1}{\sum_y \boxed{d(y,x)}}$$

length of the shortest path from y to x



Rather than summing the distances of a node to all other nodes, the harmonic centrality algorithm sums the inverse of those distances. This enables it to deal with infinite values.

$$c_{\text{har}}(x) = \boxed{\sum_{y \neq x} \frac{1}{d(y,x)}} = \sum_{d(y,x)<\infty, y\neq x} \frac{1}{d(y,x)}$$

$$c_{\text{har}}(x) = \frac{1}{\text{n-1}} \boxed{\sum_{y \neq x} \frac{1}{d(y,x)}} = \frac{1}{\text{n-1}} \sum_{d(y,x)<\infty, y\neq x} \frac{1}{d(y,x)}$$

normalized version

	DC	CC	Normalized HC
Node A	2	$\frac{1}{1+1+2+2} = \frac{1}{6} = 0.17$	$\frac{1}{4}\left(\frac{1}{1}+\frac{1}{1}+\frac{1}{2}+\frac{1}{2}\right) = \frac{3}{4} = 0.75$
Node E	1	$\frac{1}{2+2+1+3} = \frac{1}{8} = 0.125$	$\frac{1}{4}\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{1}+\frac{1}{3}\right) = \frac{7}{12} = 0.58$

Section 2: Spectral Graph

Spectral Centrality Measures

- Eigenvector Centrality
- Kat's Index
- PageRank
- Hits

Why we call those as spectral methods? What's the word "spectral" means?

Section 2: Spectral Graph

- What is spectral?
  - "Spectral" can be understood that it simply means decomposing a signal/audio/image/graph into a combination (usually, a sum) of simple elements (wavelets, graphlets).
  - Such decomposing generally has some nice properties, for example, these simple elements are usually orthogonal, i.e., mutually linearly independent, and therefore form a basis.

- Spectral graph theory is the powerful and beautiful theory that arise from the following question:
  - What properties of a graph are exposed/revealed if we
    1) Represent the graph as a matrix
    2) Study the eigenvectors/eigenvalues of that matrix.

  - We will begin with the very basic observation that graphs can be represented as matrices, and then ask "what happens if we apply the linear algebraic tools to these matrices?"
    - How to calculate the eigenvalues and eigenvectors of a matrix?

Spectral Graph Theory:
https://web.stanford.edu/class/cs168/l/l11.pdf

Section 2: Eigenvectors and Eigenvalues

We start by considering the matrix A and vector $\vec{x}$ as given below.

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Multiplying $A\vec{x}$ gives:

$$\begin{aligned} A\vec{x} &= \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 5 \\ 5 \end{bmatrix} \\ &= 5 \begin{bmatrix} 1 \\ 1 \end{bmatrix} ! \end{aligned}$$

$$A\vec{x} = 5\vec{x}$$

Here, an interesting fact is that we multiply a **matrix** by a **vector** and get the same result as when we multiply a **scalar** (just a number) by that **vector**.

Section 2: Eigenvectors and Eigenvalues

Geometric interpretation of an eigenvector:

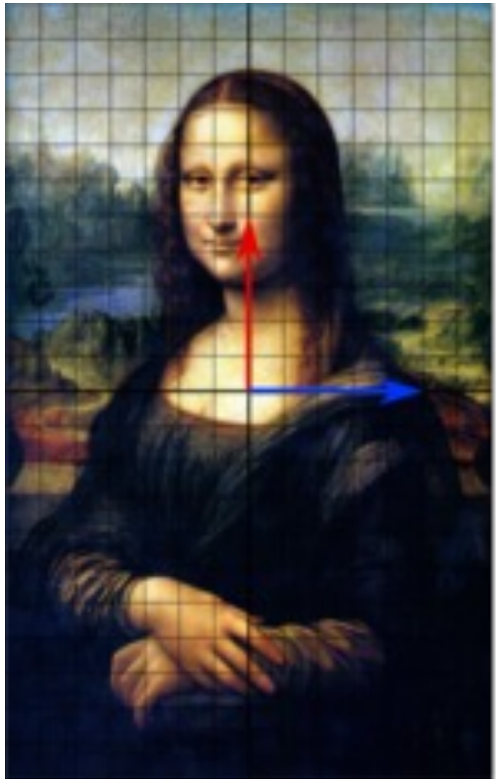
When we multiply matrix A by the eigenvector, the direction of the eigenvector remains unchanged.

Eigenvalues and Eigenvectors

Let A be an $n \times n$ matrix, $\vec{x}$ a nonzero $n \times 1$ column vector and λ a scalar. If

$$A\vec{x} = \lambda\vec{x},$$

then $\vec{x}$ is an *eigenvector* of A and λ is an *eigenvalue* of A.



Section 2: Eigenvectors and Eigenvalues

$$A\vec{x} = \lambda\vec{x} \quad \text{original equation}$$
$$A\vec{x} - \lambda\vec{x} = \vec{0} \quad \text{subtract } \lambda\vec{x} \text{ from both sides}$$
$$(A - \lambda I)\vec{x} = \vec{0} \quad \text{factor out } \vec{x}$$

not $A\vec{x} - \lambda\vec{x} = (A - \lambda)\vec{x}$

Recall $B\vec{x} = \vec{0}$

where $B = (A - \lambda I)$

Back then $\vec{x} = \vec{0}$

Now we're looking for an eigenvector that cannot be $\vec{0}$.

Thus we want a solutions to $(A - \lambda I)\vec{x} = \vec{0}$ other than $\vec{x} = \vec{0}$

Recall:

Theorem : any invertible matrix – $(A - \lambda I)$ – only has one solution.

Therefore, $(A - \lambda I)$ needs to not be invertible.

and

Theorem : noninvertible matrices all have a determinant of 0.

Thus det $(A - \lambda I)$ needs to equal 0.

Now we move forward by finding λ such that det $(A - \lambda I)$ = 0

Section 2: Eigenvectors and Eigenvalues

- Given a matrix, $\mathbf{A}$, $\mathbf{x}$ is the eigenvector and λ is the corresponding eigenvalue if $\mathbf{Ax} = \lambda\mathbf{x}$
  - $\mathbf{A}$ must be square and the determinant of $\mathbf{A} - \lambda\, \mathbf{I}$ must be equal to zero

$$\mathbf{Ax} - \lambda\mathbf{x} = 0 \text{ iff } (\mathbf{A} - \lambda\mathbf{I})\, \mathbf{x} = 0$$

    - Trivial solution is if $\mathbf{x} = 0$
    - The nontrivial solution occurs when $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$
- Are eigenvectors unique?
  - If $\mathbf{x}$ is an eigenvector, then $\beta\mathbf{x}$ is also an eigenvector and λ is an eigenvalue

$$\mathbf{A}(\beta\mathbf{x}) = \beta(\mathbf{Ax}) = \beta(\lambda\mathbf{x}) = \lambda(\beta\mathbf{x})$$

Section 2: Eigenvectors and Eigenvalues

Problem

Find the eigenvalues of A, that is, find λ such that $\det(A - \lambda I) = 0$,

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}.$$

Solution

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \\ &= \begin{bmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{bmatrix} \end{aligned}$$

Therefore,

$$\begin{aligned} \det(A - \lambda I) &= \begin{vmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{vmatrix} \\ &= (1-\lambda)(3-\lambda) - 8 \\ &= \lambda^2 - 4\lambda - 5 \end{aligned}$$

Since we want $\det(A - \lambda I) = 0$, we want $\lambda^2 - 4\lambda - 5 = 0$. This is a simple quadratic equation that is easy to factor:

$$\begin{aligned} \lambda^2 - 4\lambda - 5 &= 0 \\ (\lambda - 5)(\lambda + 1) &= 0 \\ \lambda &= -1,\ 5 \end{aligned}$$

Section 2: Eigenvectors and Eigenvalues

Problem

Find $\vec{x}$ such that $A\vec{x} = 5\vec{x}$, where

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}.$$

Solution

Recall that our algebra from before showed that if

$$A\vec{x} = \lambda\vec{x} \quad \text{then} \quad (A - \lambda I)\vec{x} = \vec{0}.$$

Therefore, we need to solve the equation $(A - \lambda I)\vec{x} = \vec{0}$ for $\vec{x}$ when $\lambda = 5$.

$$\begin{aligned} A - 5I &= \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -4 & 4 \\ 2 & -2 \end{bmatrix} \end{aligned}$$

To solve $(A - 5I)\vec{x} = \vec{0}$, we form the augmented matrix and put it into reduced row echelon form:

$$\begin{bmatrix} -4 & 4 & 0 \\ 2 & -2 & 0 \end{bmatrix} \quad \overrightarrow{\text{rref}} \quad \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Thus

$$\begin{aligned} x_1 &= x_2 \\ x_2 &\text{ is free} \end{aligned}$$

and

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

We have infinite solutions to the equation $A\vec{x} = 5\vec{x}$; any nonzero scalar multiple of the vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is a solution. We can do a few examples to confirm this:

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix} = 5\begin{bmatrix} 2 \\ 2 \end{bmatrix};$$

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ 7 \end{bmatrix} = \begin{bmatrix} 35 \\ 35 \end{bmatrix} = 5\begin{bmatrix} 7 \\ 7 \end{bmatrix};$$

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -3 \\ -3 \end{bmatrix} = \begin{bmatrix} -15 \\ -15 \end{bmatrix} = 5\begin{bmatrix} -3 \\ -3 \end{bmatrix}.$$

Section 2: Eigenvectors and Eigenvalues

In-class Quiz:

Find $\vec{x}$ such that $A\vec{x} = -1\vec{x}$, where

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}.$$

Problem

Solution

Section 2: Eigenvectors and Eigenvalues

Problem

Find $\vec{x}$ such that $A\vec{x} = -1\vec{x}$, where

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}.$$

Solution

We subtract $\lambda \times I$ from A, where $\lambda = -1$ and I is the 2×2 identity matrix. This gives us $(A - \lambda I) = \begin{bmatrix} 1-(-1) & 4 \\ 2 & 3-(-1) \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$.

We reduce this augmented matrix to row echelon form (ref), which is

$$\begin{bmatrix} 2 & 4 & 0 \\ 2 & 4 & 0 \end{bmatrix} \text{ rref } \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus $x_1 = -2x_2$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

x_2 could be any nonzero scalar

Q&A