K. H. Olsen
B-063

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To: E. W. Bivans

From: K. E. Perry and E. J. Aho

Subject: A Fourier Synthesis Character Generator

ABSTRACT

An analog device has been developed for displaying ~~alphabetic~~ or numeric characters on a cathode-ray tube face by deflecting the spot so that it traces out the character smoothly and ~~contin-~~uously. The necessary "x" and "y" deflection voltages are obtained by a Fourier synthesis technique that involves combining various harmonic frequencies of a fundamental frequency. A single character is displayed in about thirty microseconds.

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Approved Paul B. Aebring

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A FOURIER SYNTHESIS CHARACTER GENERATOR

A number of schemes has been devised in the past for scribing numeric and alphabetic characters on a scope face by spot deflection. Digital as well as analog deflection has been employed. A new analog circuit recently developed in Group 24 for this purpose has some advantages in both simplicity and versatility.

Consider the Arabic numerals zero through seven. Each of these may be represented as a segment of a continuous closed curve given in Cartesian coordinates by the equation $y = f(x)$. In general, y is a multi-valued function of x, but we can also represent the curve by two parametric equations:

$$y = f_1(t) \qquad\qquad t_0 \leq t \leq t_1$$
$$x = f_2(t)$$

where f_1 and f_2 are single-valued functions of t. If t is the time, then these functions define the continuous motion of a point along the curve. They must clearly be single-valued functions, since the spot cannot be in two different positions at the same time. If the tangential speed of the point is known at all times (specifically if it is constant), then the parametric equations are defined by the equation $y = f(x)$. Thus, if $f_1(t)$ and $f_2(t)$ represent the voltage waveforms that are applied to the y and x deflection amplifiers, the desired curve will be traced on the scope face. Since most of the symbols are not closed curves, an unblanking function must be provided to intensify the desired segment.

The functions f_1 and f_2 are defined in the interval $t_0 \leq t \leq t_1$ where $(t_1 - t_0)$ is the time required for the spot to trace the entire closed curve. A function of this type can be expanded, according to Fourier's theorem, into a series of terms of sines and cosines, thus:

$$f_1(t) = A_0 + A_1 \sin \omega t + B_1 \cos \omega t + A_2 \sin 2\omega t + \ldots.$$

where $\omega \doteq 2\pi/t_1 - t_0$ and $t_0 = 0$. The procedure for finding the coefficients A_n, B_n is as follows: The desired character is drawn on graph paper, including a retrace segment which closes the curve.[+] Twenty-four

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[+]In order that all characters can use the same unblanking function, closed figures like "zero" and "eight" have redundant retrace segments tacked on as an appendix.

points are laid off along the curve at roughly equal intervals. (The number of points used is arbitrary.) These points divide the time $(t_1 - t_0)$ into twenty-four equal intervals. The x and y coordinates of each point are tabulated with t_0 taken as the center of the retrace segment (Fig. 11). These tabulated values represent the two functions $f_1(t)$ and $f_2(t)$. These functions may be analyzed by any one of several graphical and numerical integration methods. The method we now use is a purely graphical one where each x or y value is laid off as a vector at an angle equal to $(n\omega t)$. These vectors are added head to tail and the projections of the resultant vector give the coefficients A_n, B_n. (See Bibliography, Reference 1.)

A circuit has been developed for synthesizing the desired voltage waveforms $f_1(t)$ and $f_2(t)$ from artificially generated sine and cosine waveforms. Five harmonics are used with fundamental frequency of 30 kc. Ten tuned circuits (5 sine and 5 cosine) are simultaneously shock-excited into oscillation by a gate 33 µs wide. Thus are generated one cycle of 30 kc, two cycles of 60 kc, three of 90 kc, four of 120 kc and five of 150 kc. These ten signals are fed through emitter-follower buffers to the primaries of ten toroidal transformers. Secondaries are wound on these toroids with direction of winding and number of turns determined by the sign and magnitude of the Fourier coefficients determined above. When these secondaries are connected in series and one end of the series circuit grounded, the desired voltage waveforms appear at the other end.

Fig. 1 is a complete block diagram of the prototype system. The circuit as depicted here will display the numerals 0 through 7, four rows deep (32 characters). This can be displayed on any oscilloscope which is provided with "x" and "y" inputs and an external unblanking connection.

A 120-kc sine wave is fed into a clock generator (Fig. 3) which shapes the signal into a square wave. The prime side of the clock generator output is commutatively coupled to Flip Flop C_1, the first of a chain of eight serial counters.+ The unblanking function is generated in the intensity flip flop, controlled by C_1, C_2, C_3 and the clock generator. Referring to the master timing diagram (Fig. 2), we see that the intensity pulse (waveform No. 12) starts one-half a clock cycle (about 4 µs) after the prime side of Flip Flop C_3 (pin No. 10) goes up, and ends 4 µs

+For a description of the type of flip flop circuitry used, see Bibliography Reference 2.

before the same point goes down. This, it will be seen, unblanks that segment of the lissajous pattern which forms the desired character. One fourth of this continuous closed curve is blanked out.

Flip Flop C_3 which shock-excites the ringing circuits in the harmonic generator (Fig. 8) is operating at exactly one-half the rate of the fundamental frequency used in the synthesis. The ringing period of the shock-excited oscillators occurs during the time the prime side of Flip Flop C_3 is high. Since the fundamental frequency (30 kc) is twice the frequency of Flip Flop C_3, we get one complete cycle into the slot before the ringing is terminated by a change of state in C_3.

In like manner, we have two cycles of the second harmonic, three of the third, four of the fourth and five of the fifth, all initiated and terminated at the same time (Fig. 12). The sine waves and the cosine waves are generated in parallel-resonant and series-resonant circuits respectively. Input A (Fig. 8) which is connected to five sine-wave ringing circuits is controlled by Flip Flop C_3'. When C_3' goes up, the five input transistors connected to point A are cut off and the parallel resonant circuits composed of L_1, C_1 and C_2 ring at their respective frequencies (30, 60, 90, 120 and 150 kc). The output is a positive sine wave. Damping of oscillations is small due to the high "Q" powdered iron cores used for L_1 and L_2. Input B which is connected to five cosine-wave series ringing circuits is controlled by Flip Flop output C_3. These circuits oscillate at their resonant frequencies when the input transistor is on (point "B" low). The output is a negative cosine wave. See Fig. 12.

It is interesting to note that since these ringing circuits are cut off at a point in the cycle exactly corresponding to the turn-on point, there is no damping transient and the operation is not duty-cycle sensitive. In other words, at the instant of turn-off the voltage on the capacitor and the current through the inductor are very near to the quiescent values. This would be exactly true except for the losses during ringing. It is only necessary to leave the circuit turned off long enough for this small amount of lost energy to be replaced.

The values of L and C are determined by setting $\sqrt{L/C} = R$ where R is the critical damping resistor (the R value arbitrarily chosen as 1 k) and L and C are unknown. Then solving first for L in terms of C and substituting this result in the equation $\sqrt{LC} = 1/2\pi f$ and solving for C, L can then be found from either equation. The trimmer condenser C_2 has a range of from 100 μμf to 500 μμf and is adequate for adjusting the ringing circuit for any LC inconsistencies.

Each ringing circuit is followed by an emitter-follower amplifier buffer which also serves to drive the base of a power transistor in an emitter-follower amplifier configuration. The output of the power transistor is coupled through a 1-μf capacitor to the primary of a toroidal transformer. Referring to both Fig. 1 and Fig. 8, we see that X_0 and X_0', X_1 and X_1', etc., or Y_0 and Y_0' or Y_1 and Y_1', etc., on the harmonic generator block (Fig. 1) are the terminals to the series secondary windings on the toroidal transformers. Every time that Flip Flop C_3 cycles, these circuits have $f_1(t)$ and $f_2(t)$ waveforms on them. These secondary waveforms will not, however, be passed through the "or" diodes to the scope unless the X and Y inputs are high. The d-c level of the unprimed ends of the secondary windings (X_N and Y_N -- Fig. 1) are controlled by the state of their associated switches (Figs. 6 and 7). When a switch output is high, the corresponding "or" diode (Fig. 1) is forward-biased and the signal on that particular secondary winding is transferred to the scope.

The switches (Figs. 6 and 7) are PNP transistors in a grounded-emitter configuration. The collector controls the d-c level of the associated secondary winding in the harmonic generator. The base inputs have two states. When the base is high the collector is at -6.5 volts and its associated secondary winding sees an open diode in the "or" circuit preceding the scope (Fig. 1). When the base is low, the collector will be at ground or some small negative voltage above -6.5 volts, determined by the fixed resistor at the emitter. The purpose of this resistor is to adjust the level of the synthesized waveform $f_1(t)$ and $f_2(t)$. In the original graphical analysis for $f_1(t)$ and $f_2(t)$, no attempt was made to compute the d-c Fourier coefficient A_0 since the zero frequency cannot be accommodated in the transformers. Therefore, some of the numerals would be displaced from their proper relative positions on the scope face. It is this descrepancy in d-c level that is adjusted by the resistors.

The diode matrix (Fig. 5) selects the number to be displayed under control of Flip Flops C_4, C_5 and C_6. A different number will be displayed during each unblanking pulse. Only one output is low at any time. This voltage turns on a pair of switching transistors in the selection-switch package. (Figs. 6 and 7.)

The four resistors on the X input of the scope (Fig. 1) are used to generate an eight-step ladder of voltages at the same rate as the unblanking function, thus displacing each numeral consecutively.

The three resistors on the Y input in conjunction with the slower running Flip Flops C_7 and C_8 displace the whole row of eight numbers vertically four times.

A simulation device was built to try the effect of various combinations of coefficients in generating various characters. The circuit is shown in Fig. 10. The toroid primaries are substituted for the toroids in the harmonic generator, and the 250-ohm potentiometers are adjusted to the proper coefficient values. The resulting character can then be observed.

Fig. 9 is a tabulation of the coefficients (number of turns) for the numerals "zero" through "seven" and Figs. 12 and 13 show the circuit waveforms. Fig. 14 shows the type of octal tabular display obtainable. Fig. 15 shows a miscellaneous assortment of characters generated with the simulator and demonstrates the versatility of this device. In Fig. 16 the harmonic generator is shown in the center with the control circuitry beneath. The simulator is on the left.

BIBLIOGRAPHY

1. Blow, Thomas C., "Graphical Fourier Analysis," Electronics, p. 194, December 1947.

2. Baker, R. H., "Boosting Transistor Switching Speed," Electronics, p. 190, March 1, 1957.

LIST OF FIGURES

Gray Fig - AHO B-6903-11

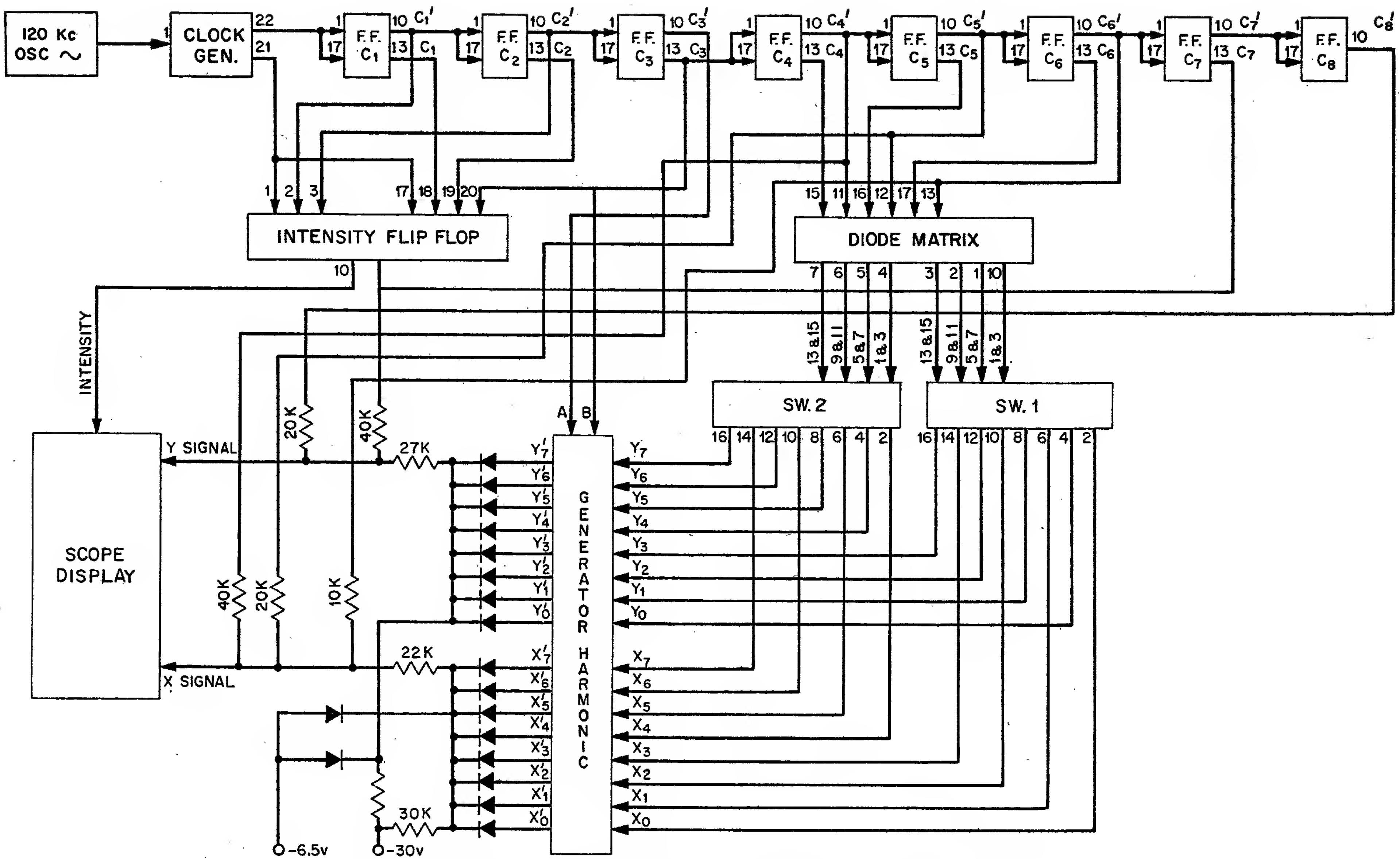


FIG. 1. FOURIER SYNTHESIS CHARACTER GENERATOR BLOCK DIAGRAM

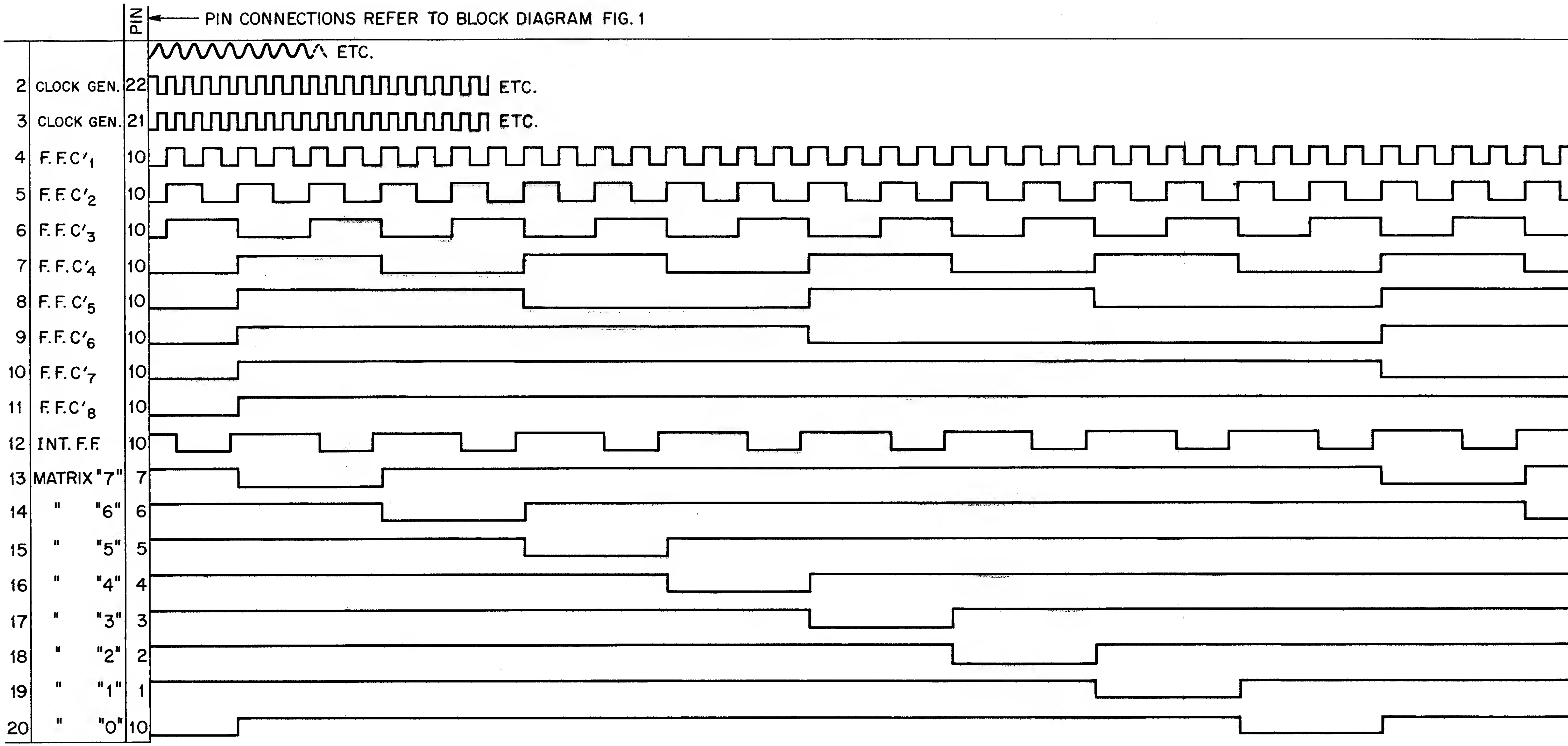
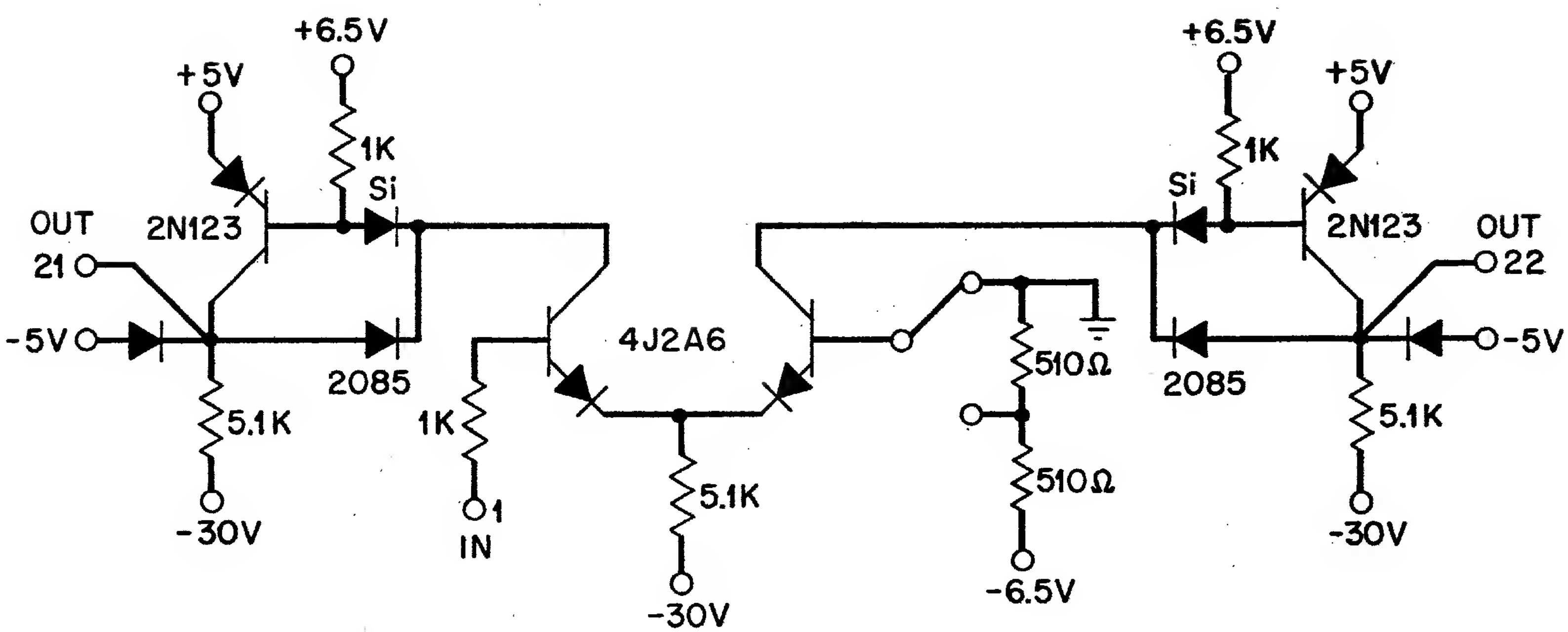


FIG. 2. F.S.C.G. MASTER TIMING DIAGRAM

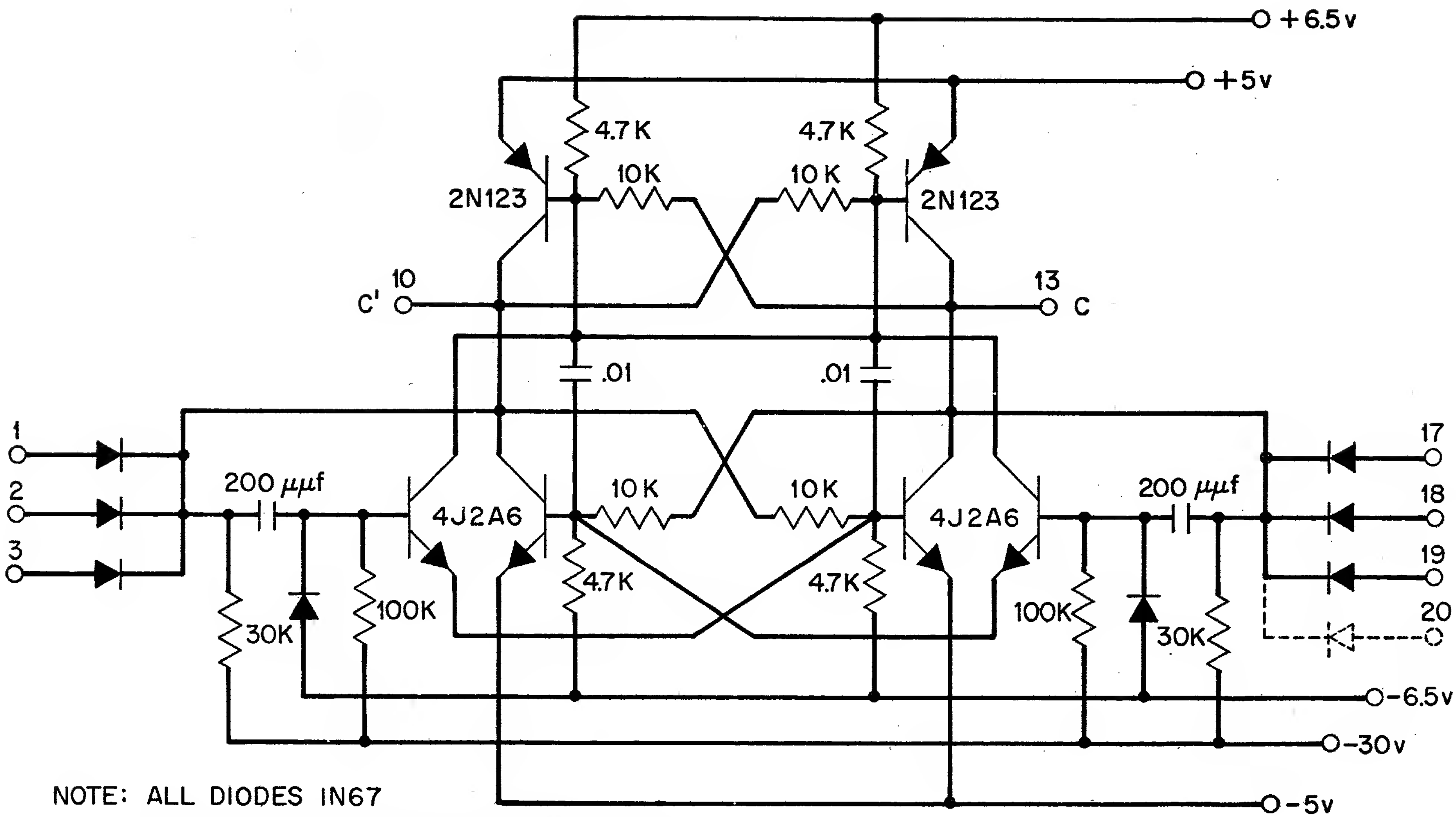
A-6903-9 Spofford Fig. AHO



NPN = 4JD2A6
PNP = 2N123 OR 4JD1A23
Si = SG22
DIODES = 1N67

FIG. 3. CLOCK GENERATOR.

Gray Fig. - AHO A-6903-6



NOTE: ALL DIODES IN67
PIN #20 USED ONLY IN INTENSITY F.F.

FIG. 4. FLIP FLOP 100 Kc NEGATIVE LOGIC

Gray Fig. - AHO

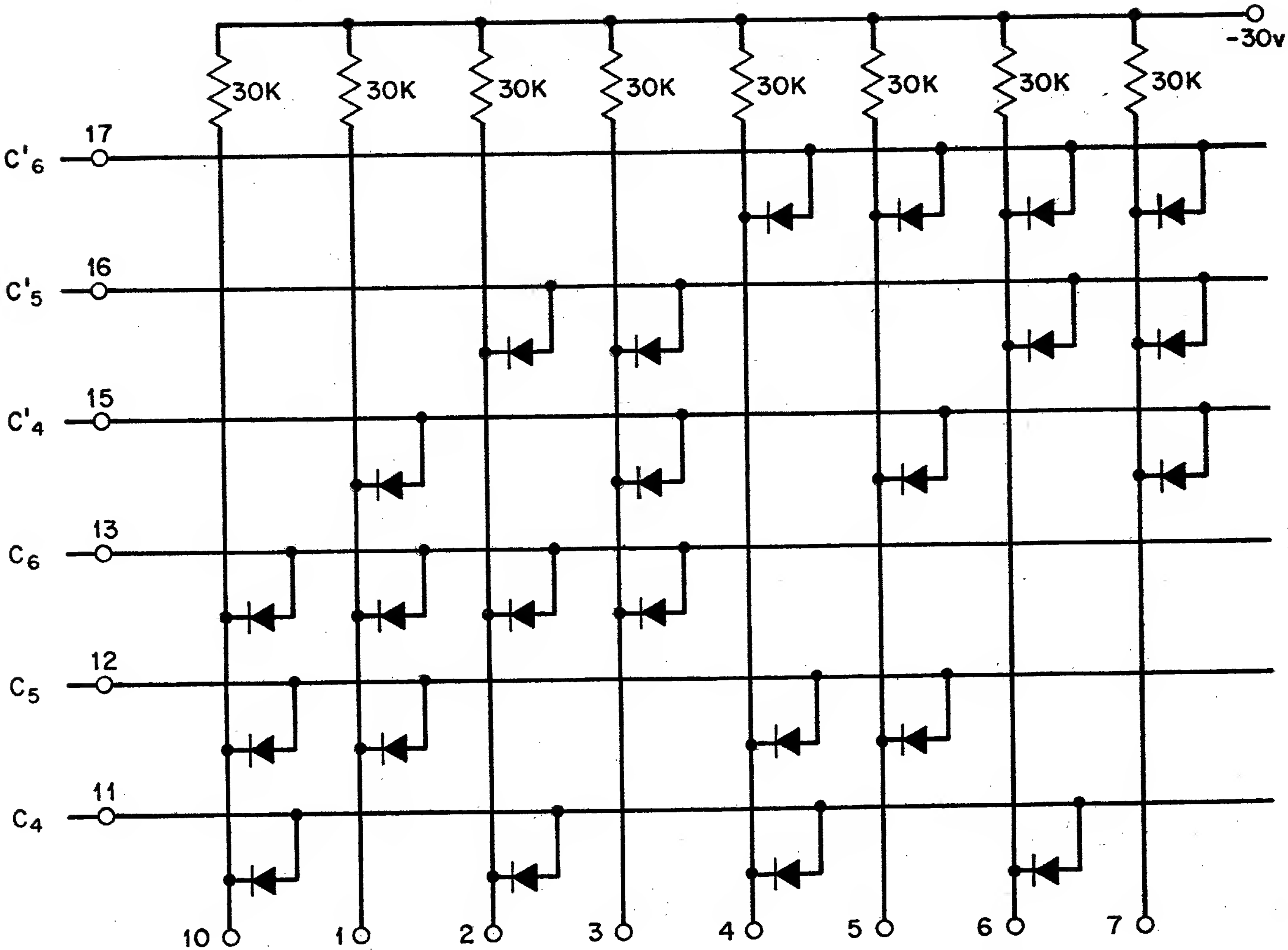


FIG. 5. F. S.C.G. MATRIX

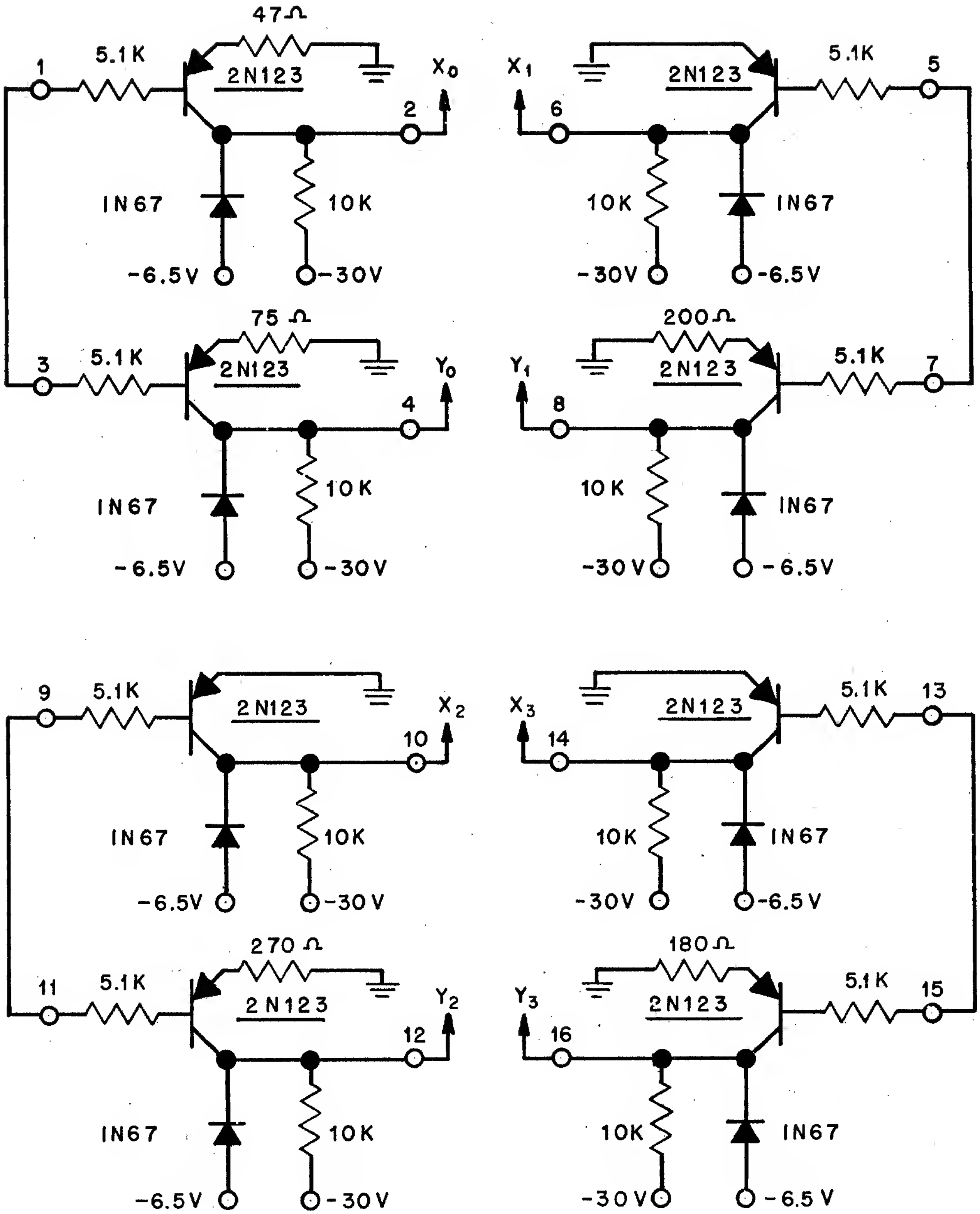


FIG. 6. F.S.C.G. CHARACTER SELECTION SWITCH NO. 1

Lapierre Fig. AHO A-6903

A-6903-2

Lapierre Fig. 4HO

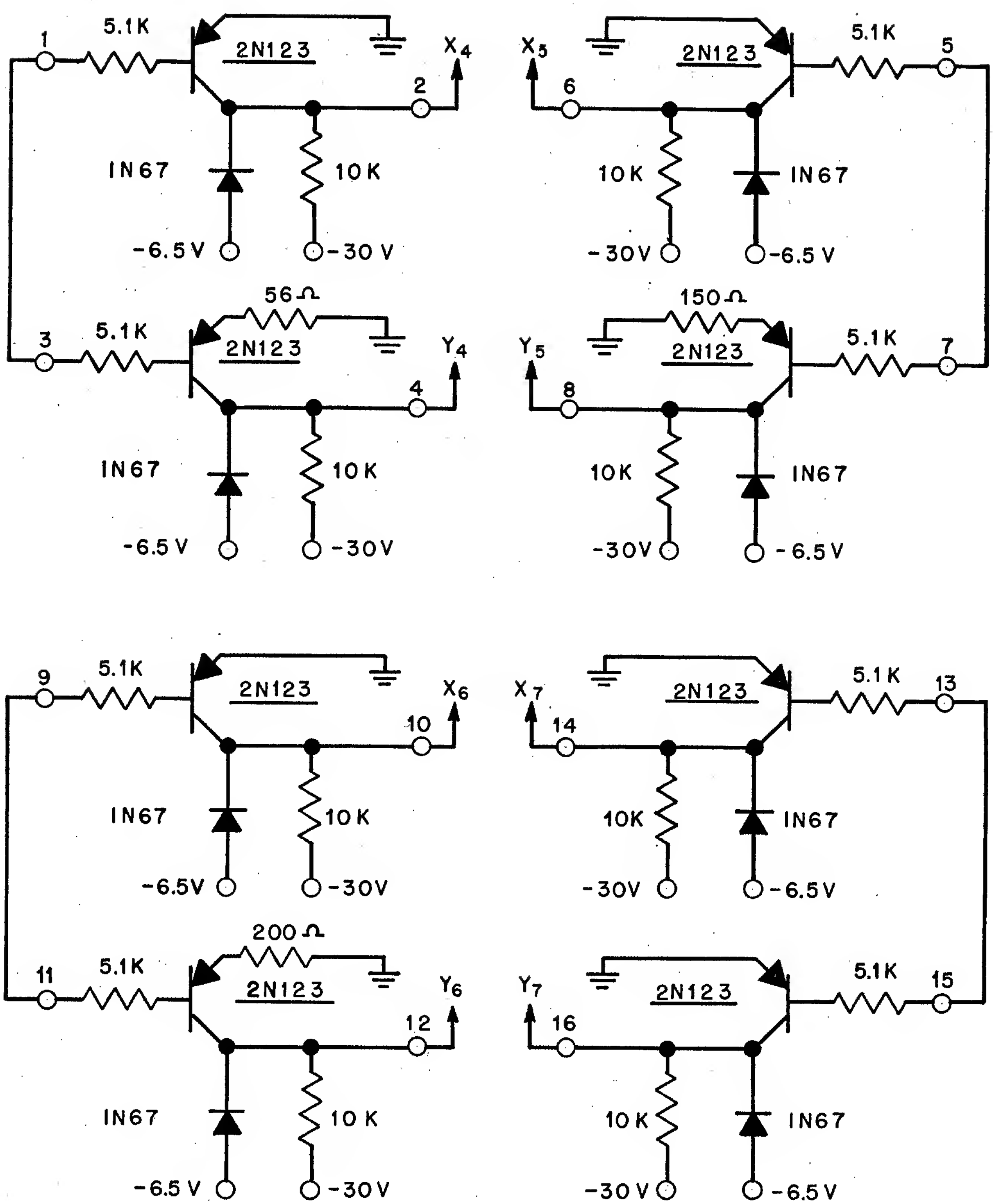
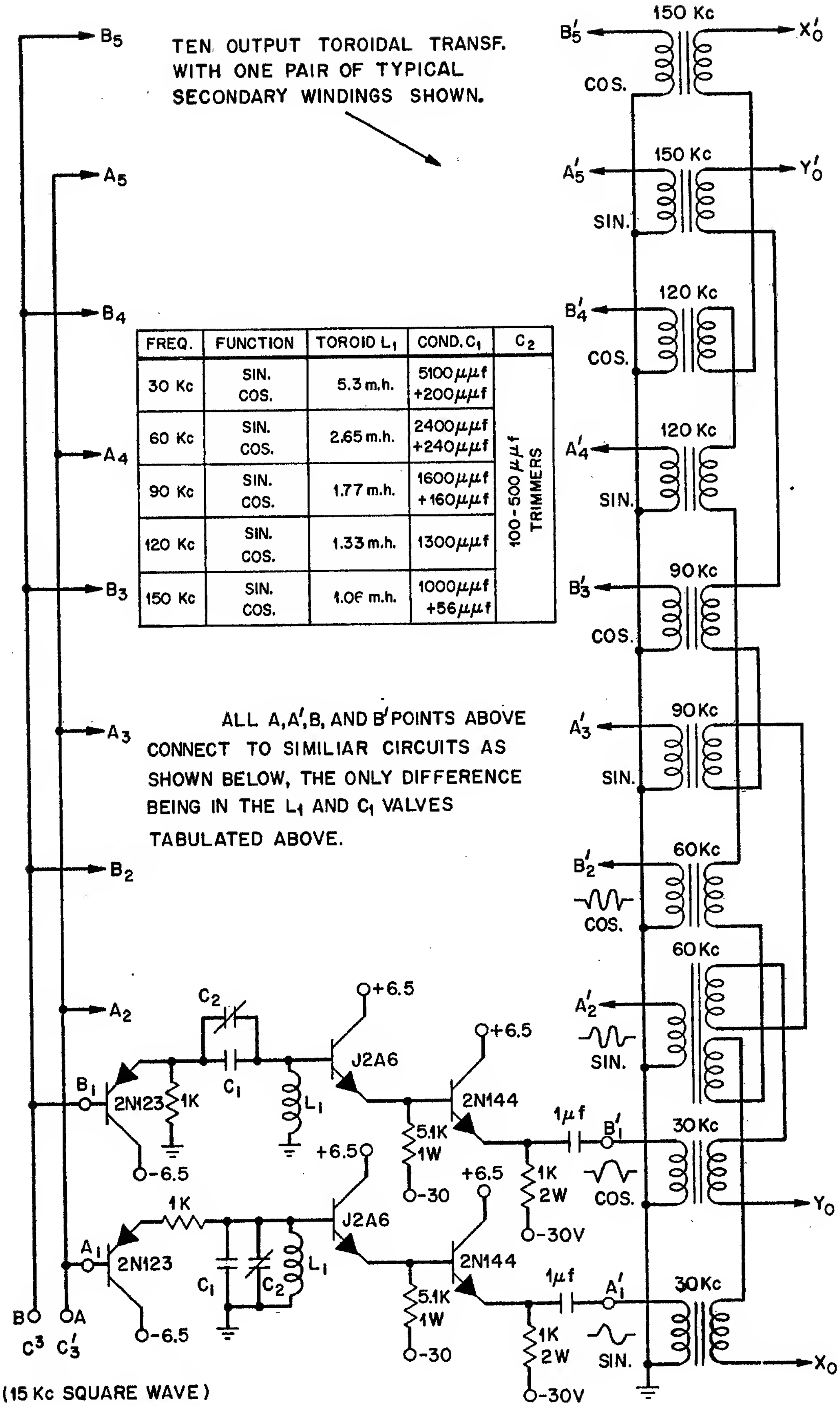


FIG. 7. F.S.C.G. CHARACTER SELECTION SWITCH NO. 2.



TEN OUTPUT TOROIDAL TRANSF. WITH ONE PAIR OF TYPICAL SECONDARY WINDINGS SHOWN.

FREQ.	FUNCTION	TOROID L_1	COND. C_1	C_2
30 Kc	SIN. COS.	5.3 m.h.	5100μμf +200μμf	100-500 μμf TRIMMERS
60 Kc	SIN. COS.	2.65 m.h.	2400μμf +240μμf	
90 Kc	SIN. COS.	1.77 m.h.	1600μμf +160μμf	
120 Kc	SIN. COS.	1.33 m.h.	1300μμf	
150 Kc	SIN. COS.	1.06 m.h.	1000μμf +56μμf	

ALL A, A', B, AND B' POINTS ABOVE CONNECT TO SIMILIAR CIRCUITS AS SHOWN BELOW, THE ONLY DIFFERENCE BEING IN THE L_1 AND C_1 VALVES TABULATED ABOVE.

FIG. 8. HARMONIC GENERATOR.

Spofford Fig. AHO

B-6903-8

A-6903-7

FIG. 9. A TABULATION OF THE NUMBER OF ACTUAL TURNS TAKEN ON THE TOROIDAL TRANSFORMER SECONDARIES TO GENERATE THE REQUIRED WAVEFORMS ON X AND Y FOR EACH NUMERAL.

X (0)		Y (0)	
$a_1 + 28$	$b_1 - 8$	a_1 0	$b_1 + 40$
$a_2 - 20$	b_2 0	a_2 0	$b_2 - 20$
a_3 0	b_3 0	a_3 0	b_3 0
a_4 0	b_4 0	a_4 0	b_4 0
a_5 0	b_5 0	a_5 0	b_5 0

X (1)		Y (1)	
a_1 0	b_1 0	a_1 0	b_1 0
a_2 0	b_2 0	a_2 0	b_2 0
a_3 0	b_3 0	a_3 0	b_3 0
a_4 0	b_4 0	a_4 0	b_4 0
a_5 0	b_5 0	a_5 0	$b_5 + 47$

X (2)		Y (2)	
$a_1 - 3$	b_1 0	$a_1 + 43$	$b_1 + 9$
$a_2 - 39$	b_2 0	a_2 0	b_2 0
a_3 0	b_3 0	$a_3 - 2$	$b_3 + 6$
a_4 0	b_4 0	a_4 0	$b_4 + 6$
a_5 0	b_5 0	a_5 0	b_5 0

X (3)		Y (3)	
$a_1 + 6$	$b_1 - 22$	$a_1 - 37$	$b_1 - 7$
$a_2 - 8$	$b_2 - 5$	$a_2 - 6$	b_2 0
$a_3 - 6$	$b_3 + 22$	$a_3 + 7$	b_3 0
$a_4 - 9$	b_4 0	a_4 0	b_4 0
a_5 0	b_5 0	a_5 0	b_5 0

X (4)		Y (4)	
$a_1 - 15$	$b_1 + 21$	$a_1 + 4$	$b_1 - 37$
$a_2 + 18$	$b_2 + 7$	$a_2 + 24$	$b_2 - 3$
a_3 0	$b_3 - 5$	$a_3 + 2$	$b_3 - 1$
a_4 0	$b_4 + 1$	$a_4 + 3$	b_4 0
$a_5 - 1$	b_5 0	$a_5 + 1$	b_5 0

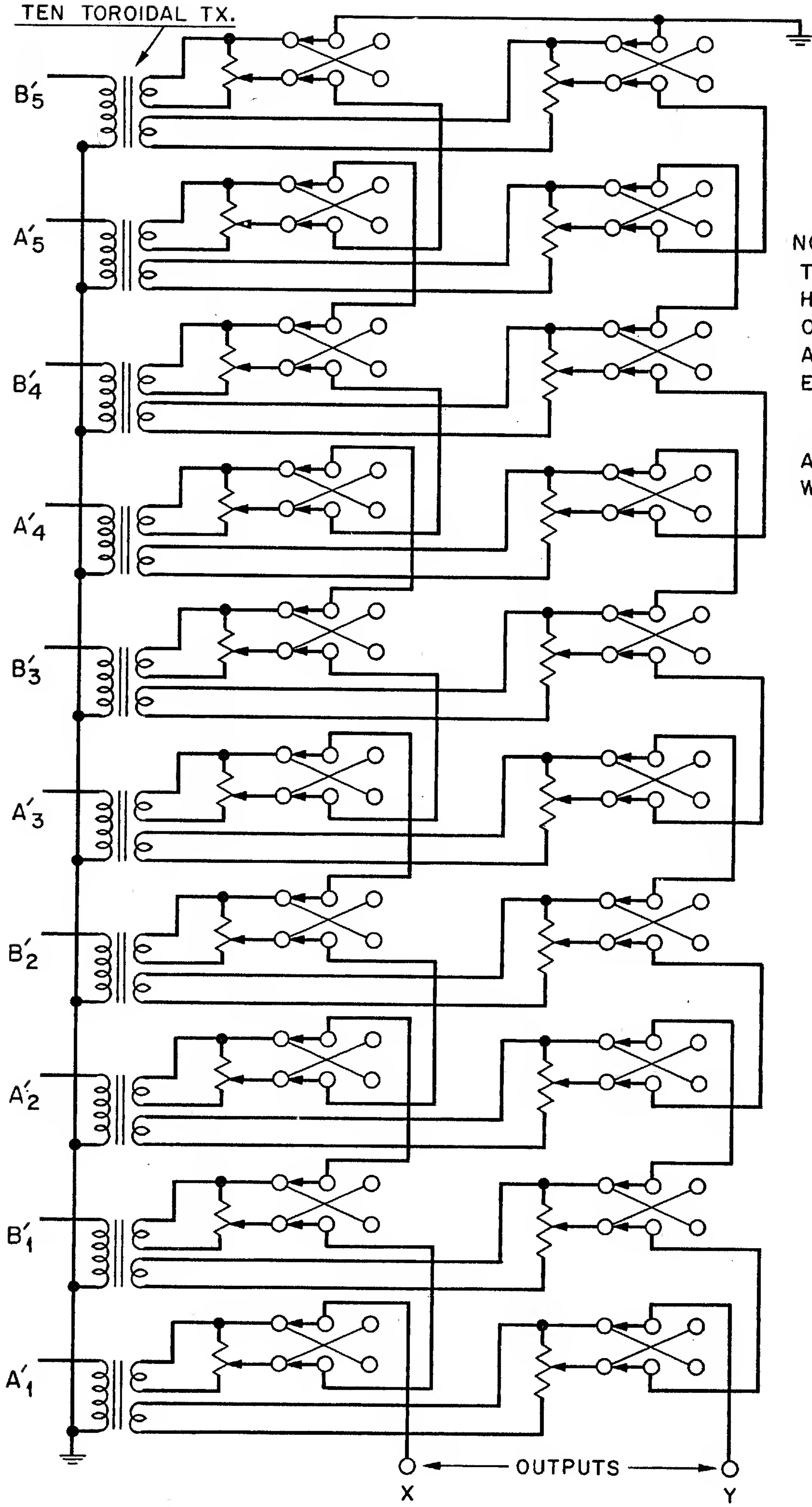
X (5)		Y (5)	
$a_1 - 9$	$b_1 + 4$	$a_1 + 41$	$b_1 + 1$
$a_2 + 37$	$b_2 + 2$	$a_2 + 6$	$b_2 + 8$
$a_3 + 4$	$b_3 + 11$	$a_3 - 6$	b_3 0
$a_4 + 3$	$b_4 - 1$	$a_4 + 3$	b_4 0
$a_5 - 1$	$b_5 + 6$	$a_5 - 3$	$b_5 - 2$

X (6)		Y (6)	
$a_1 - 4$	$b_1 - 16$	a_1 0	$b_1 + 28$
$a_2 + 15$	$b_2 + 32$	$a_2 + 21$	$b_2 + 7$
$a_3 + 6$	$b_3 + 6$	a_3 0	b_3 0
a_4 0	b_4 0	a_4 0	b_4 0
a_5 0	b_5 0	a_5 0	b_5 0

X (7)		Y (7)	
a_1 0	$b_1 - 21$	$a_1 + 1$	$b_1 + 24$
a_2 0	$b_2 - 19$	a_2 0	$b_2 - 32$
$a_3 + 1$	$b_3 + 20$	$a_3 - 1$	$b_3 + 9$
a_4 0	$b_4 + 13$	$a_4 - 1$	$b_4 - 10$
a_5 0	b_5 0	a_5 0	$b_5 - 4$

Lapière Fig. AHO

A - 6903 - 10



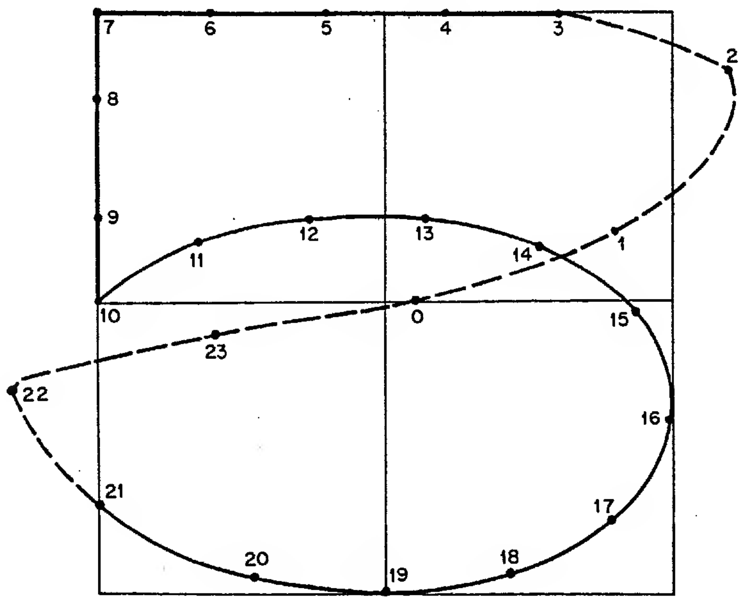
NOTE: THE TEN TOROIDAL TRANSF. HAVE 100 TURNS ON THE PRIMARY AND 30 TURNS ON EACH SECONDARY.

ALL POTS. ARE 250 Ω WIRE WOUND.

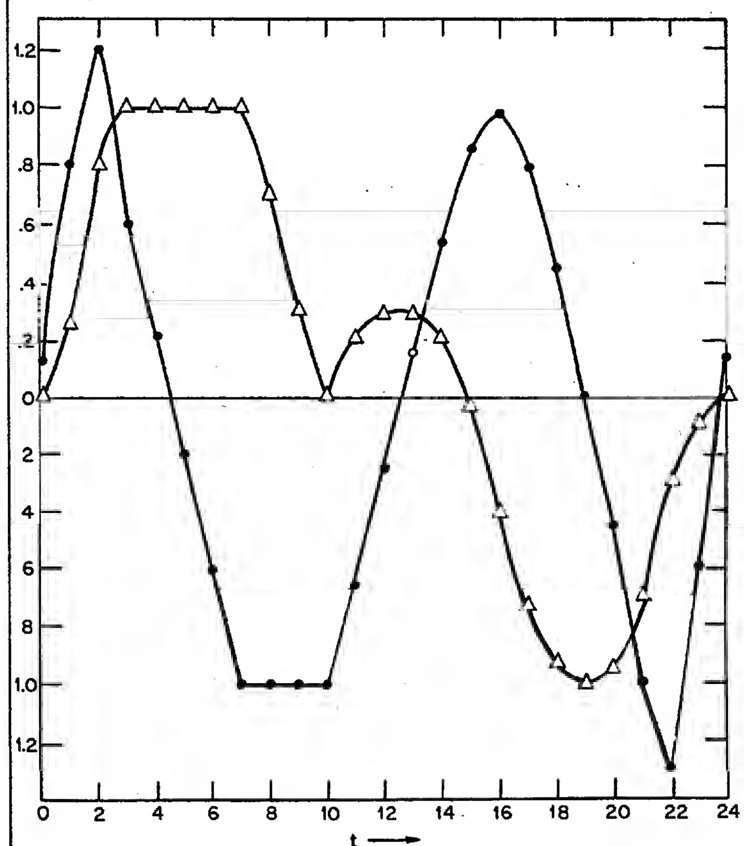
FIG. 10
F.S.C.G.
SIMULATOR

Compton - Fig. - AHO -

Lapierre - Fig. AHO-C-6903-5



COORDINATES		
t	X	Y
0	+ .11	0
1	+ .80	+ .25
2	+1.20	+ .80
3	+ .60	+1.00
4	+ .20	+1.00
5	− .20	+1.00
6	− .60	+1.00
7	−1.00	−1.00
8	−1.00	+ .70
9	−1.00	+ .30
10	−1.00	0
11	− .65	+ .20
12	− .25	+ .29
13	+ .15	+ .29
14	+ .53	+ .20
15	+ .87	− .03
16	+ .99	− .40
17	+ .79	− .73
18	+ .43	− .93
19	0	−1.00
20	−. 46	− .95
21	−1.00	− .70
22	−1.30	− .30
23	− .60	− .10



X AND Y WAVEFORMS REQUIRED

TO GENERATE THE NUMERAL 5

$(\bullet) = X = f_1(t)$

$(\Delta) = Y = f_2(t)$

FIG. 11. ANALYSIS OF NO. 5

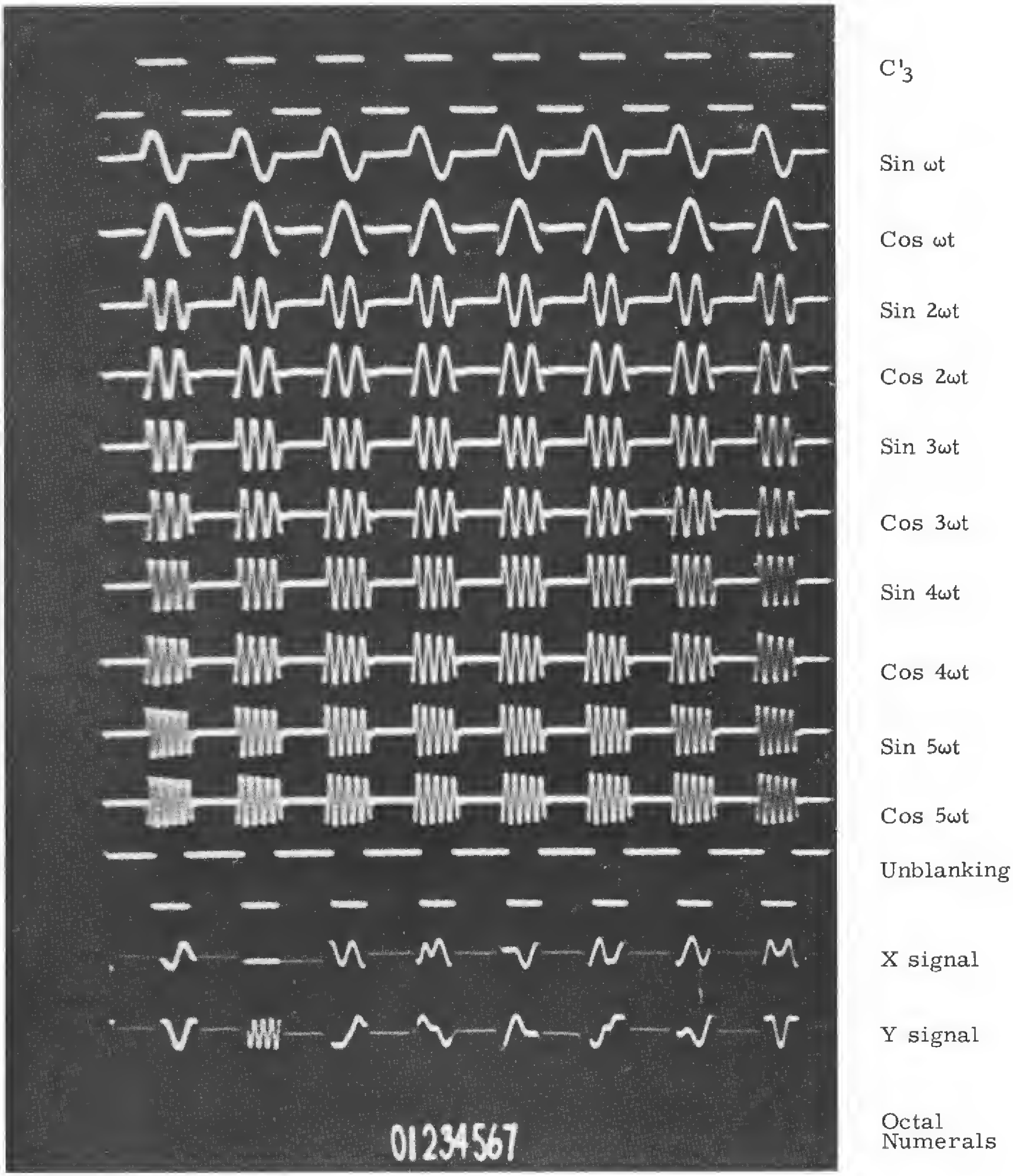


Figure 12. Waveforms

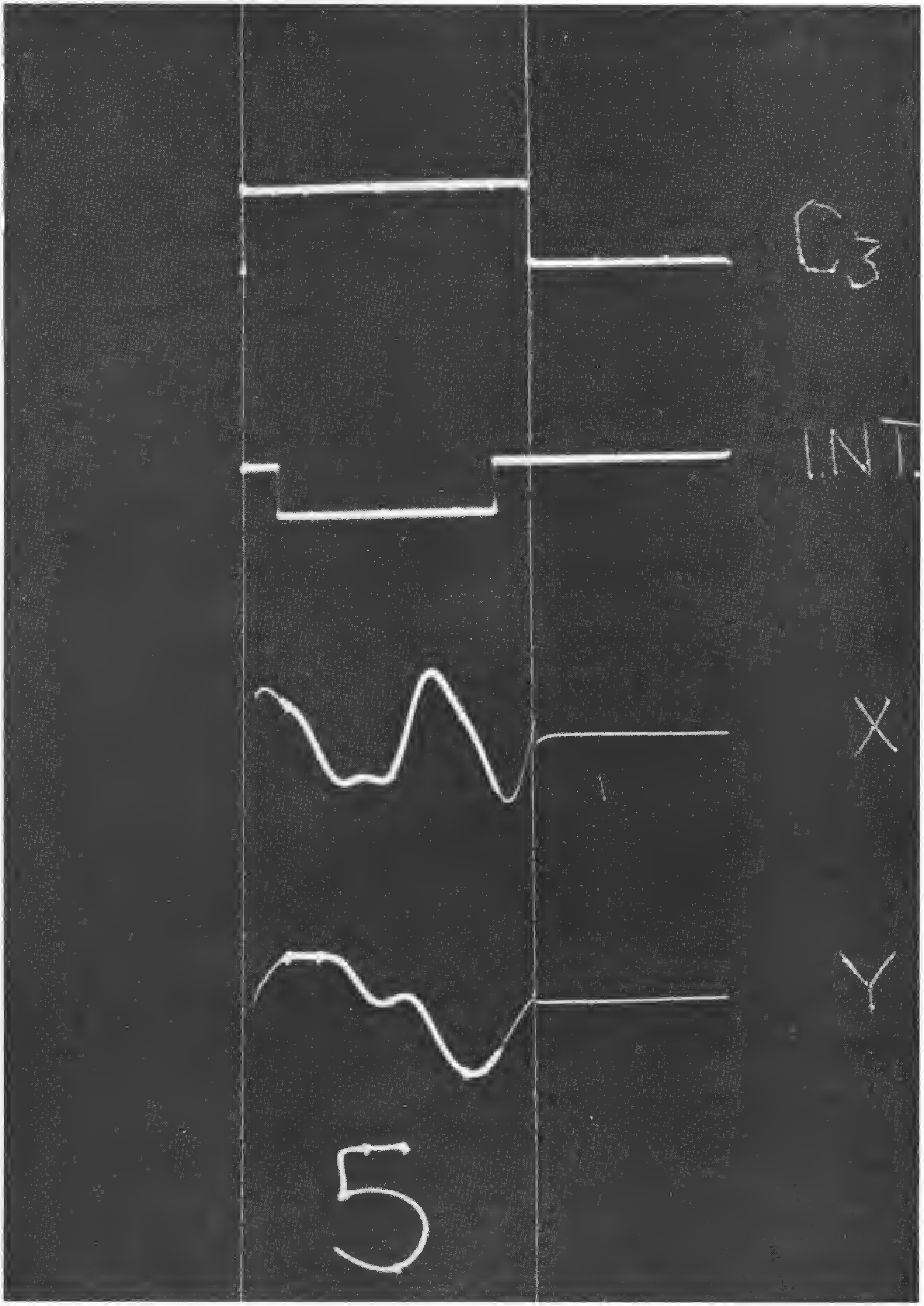


Figure 13. Enlarged Waveforms for No. 5.

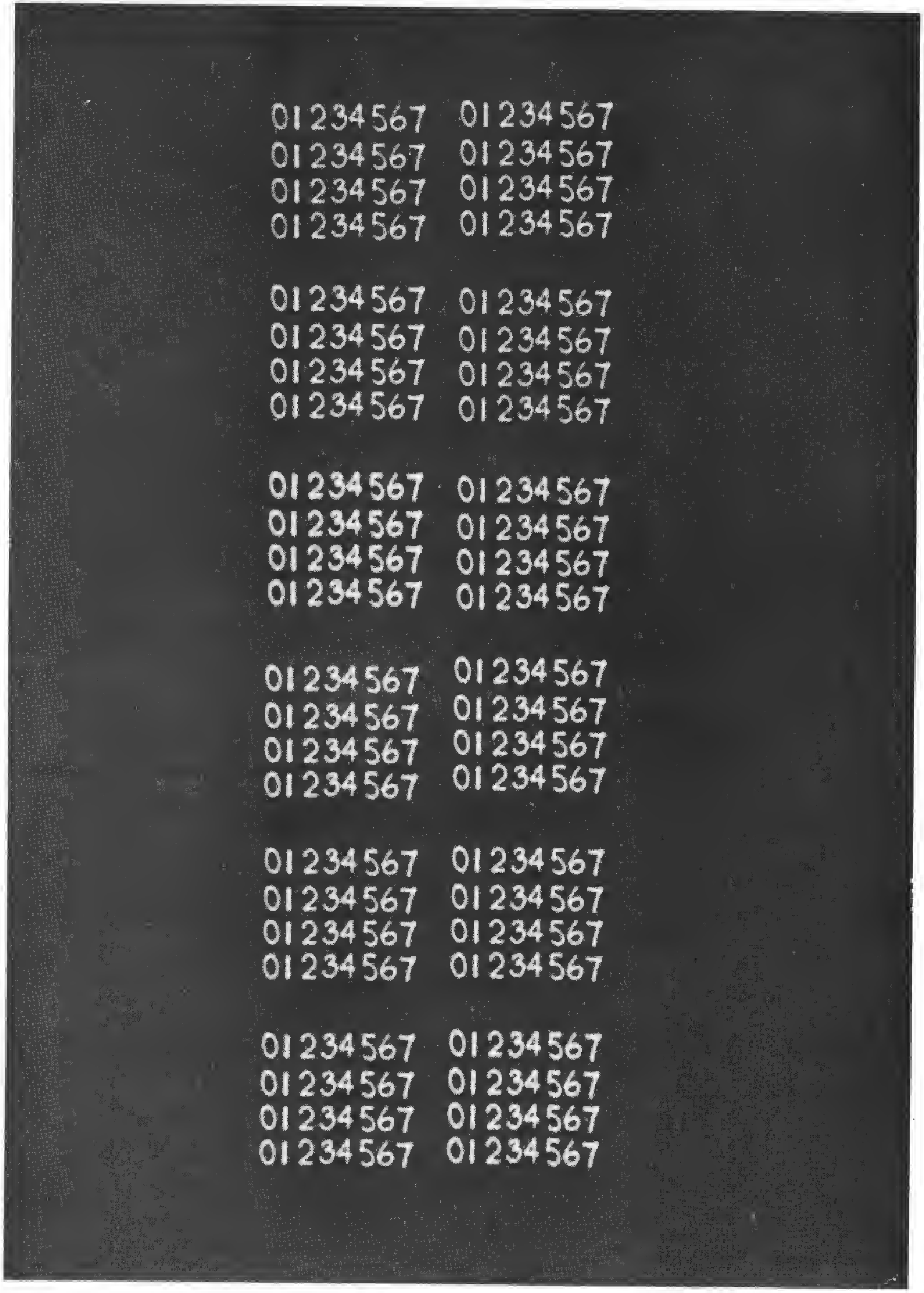


Figure 14. Octal Numerals Tabular Display.

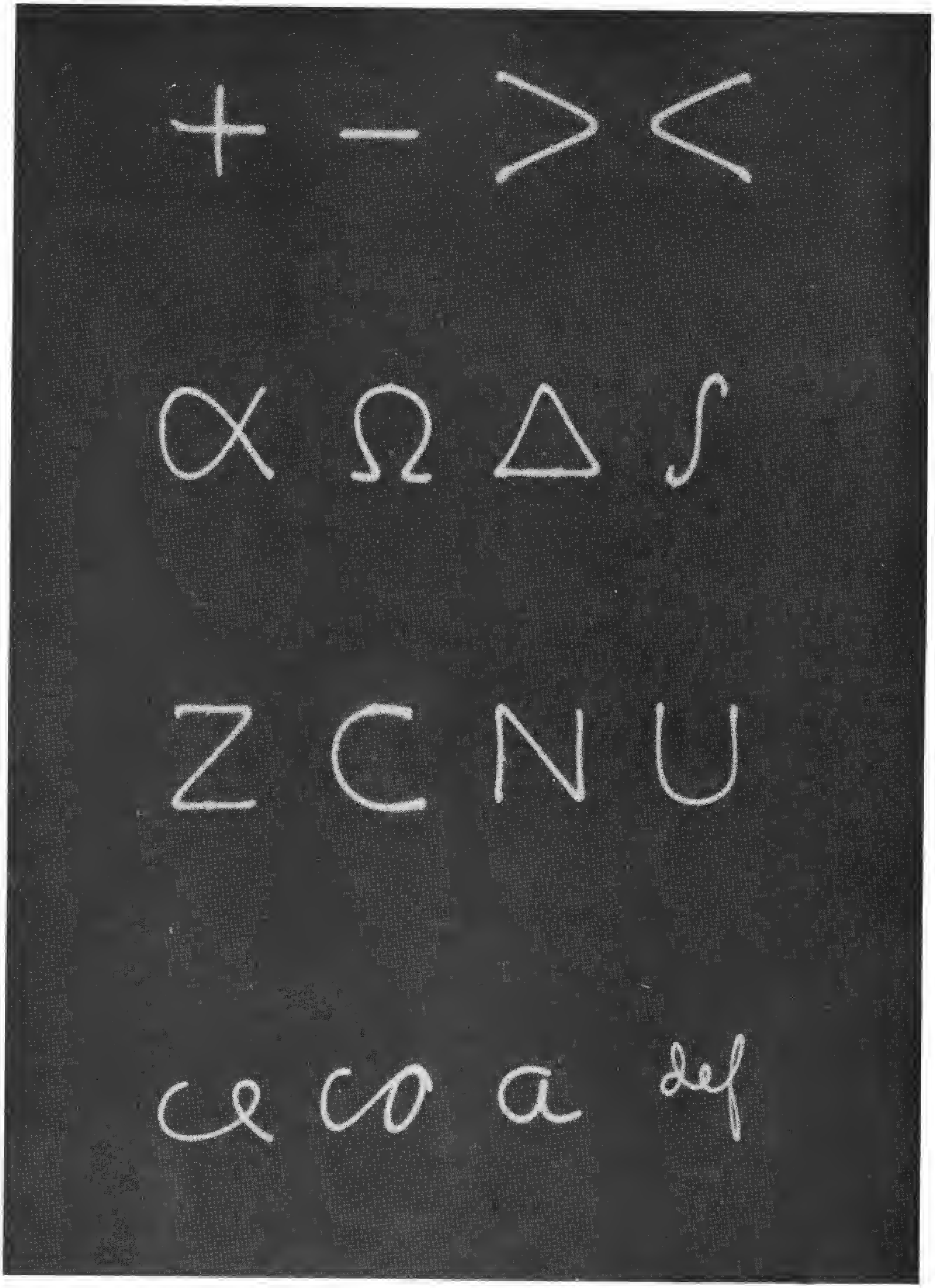


Figure 15. Miscellaneous Characters.

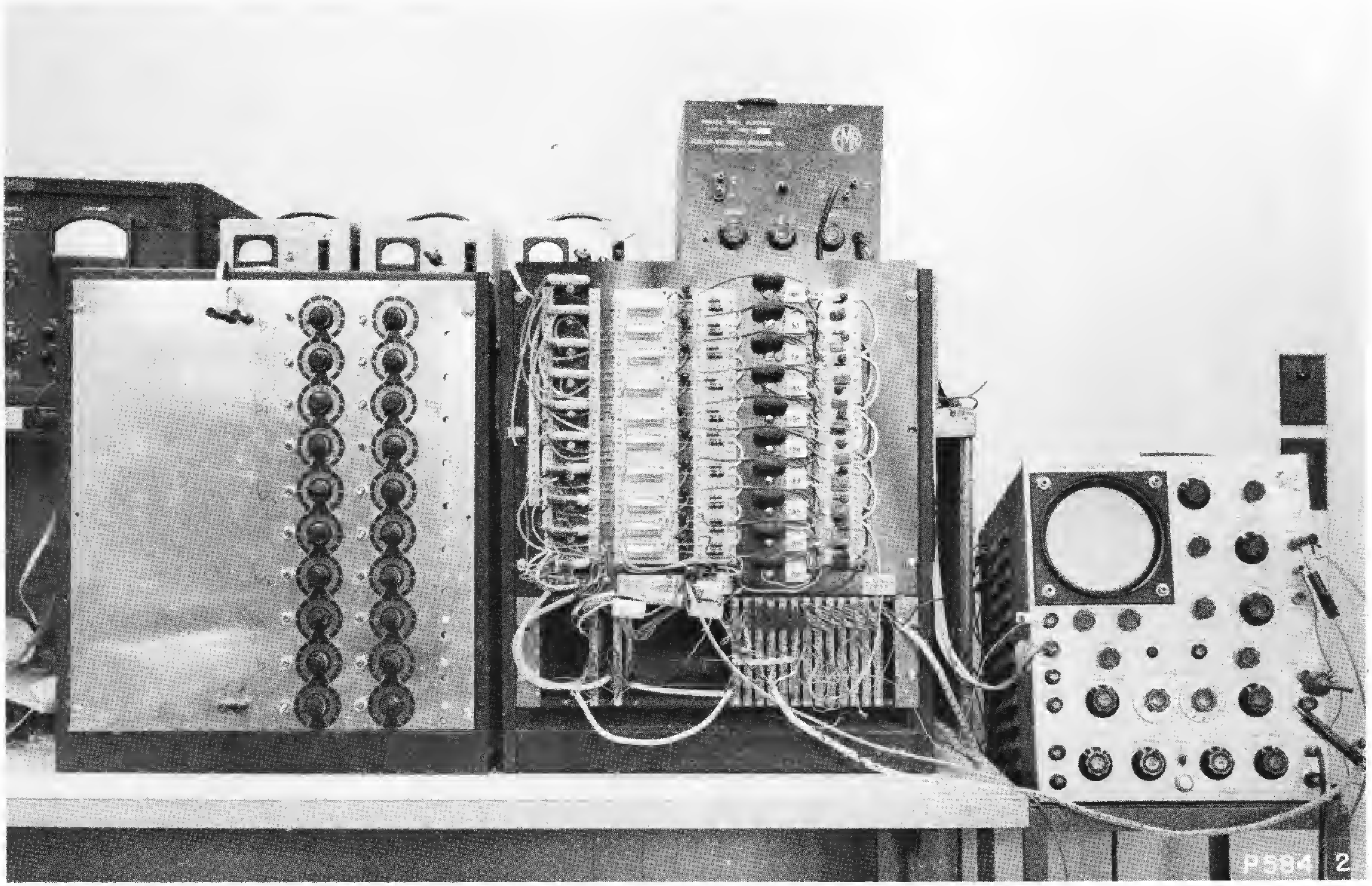


Figure 16. Photograph of Equipment.

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