1 AMT TUTORIAL - 8

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CH18B020

1 Method 1: Right angled triangle method.

a)
- Using the given data, plot the raffinate and extract curves. Plot $F(0, 0.35)$, $S(1,0)$, $R_N(0.005, 0.1)$
- y-axis will indicate acetone concentration and x-axis will indicate concentration of tce.
- Plot the tie lines: Upon plotting tie lines we can see that the tie line that intersects $R_N S$ the farthest is the topmost tie line (δ_{min})

  [R_N is plot using the acetone composition as y coordinate & finding the point with that y coordinate on the raffinate curve]

- Join F to the δ_{min} point. Extend the line till the extract curve. The intersection point of $F\delta_{min}$ on extract is E_1

  $E_1 \equiv (0.485, 0.479)$

- Intersection of FS and $R_N E_1$ gives M.

$\Rightarrow$ Solve $y - y_F = \frac{-y_F}{1}(x)$ (FS)

$y - y_{RN} = \frac{y_{E1} - y_{RN}}{x_{E1} - x_{RN}}(x - x_{RN})$ ($R_N E$)

$$\Rightarrow x = \frac{y_F + \left(\frac{y_{E1} - y_{RN}}{x_{E1} - x_{RN}}\right) - y_{RN}}{\left(\frac{y_{E1} - y_{RN}}{x_{E1} - x_{RN}}\right) + y_F}$$

$\Rightarrow (x_M, y_M) = (0.02293, 0.272)$

a) $\frac{S_{min}}{F} = \frac{(x_A)_F - (x_A)_M}{(x_A)_M - (x_A)_S}$ (derived in class using $F+S=M$ and $(x_A)_F F + (x_A)_S S = x_M M$)

$F = 1300$ kg/h, $(x_A)_F = 0.35$, $(x_A)_M = 0.272$, $(x_A)_S = 0$

$\Rightarrow S_{min} = 372.9289$

$\Rightarrow \boxed{S_{min} = 372.93 \text{ kg/h}}$

b) $S = 1.5\, S_{min} = 559.39$ kg/h.

- $(x_A)_M = \frac{F \times (x_A)_F}{F+S}$ ($\because (x_A)_S = 0$)
  $= 0.2447$

$(x_{TCE})_M = \frac{S}{F+S} = 0.3008$

Join RM. Extend it to the extract curve to get E1.

- Join FE_1 and join R_NS. The intersection of FE_1 and R_NS gives the new delta δ point
- Determine the conjugate curve by drawing lines ‖el to y axis from extract end of tie line & line ‖el to x-axis from raffinate end.
  Intersection of these 2 gives a point on conjugate curve.

- From the 3 available tie lines we get 3 points on conjugate curve which are fit using a cubic spline
- From E_1 drop to tie line. From the point on the tie line draw a horizontal to ~~y-cont~~ raffinate
- Join E_1 with the point on raffinate curve. This is the tie line passing through E_1 & the pt. on raffinate is R_1.
- Join ~~R_1 to~~ δ to R_1. Extend it to Extract curve. The intersection of δR_1 on extract curve is E_2.
- Repeat the previous 2 steps till R_N is crossed

b) From the graph we can see that there are $\boxed{\text{3 stages}}$ (Actually more than 3 but the end of 3rd stage is very close to R_N compared to end of the 4th stage).

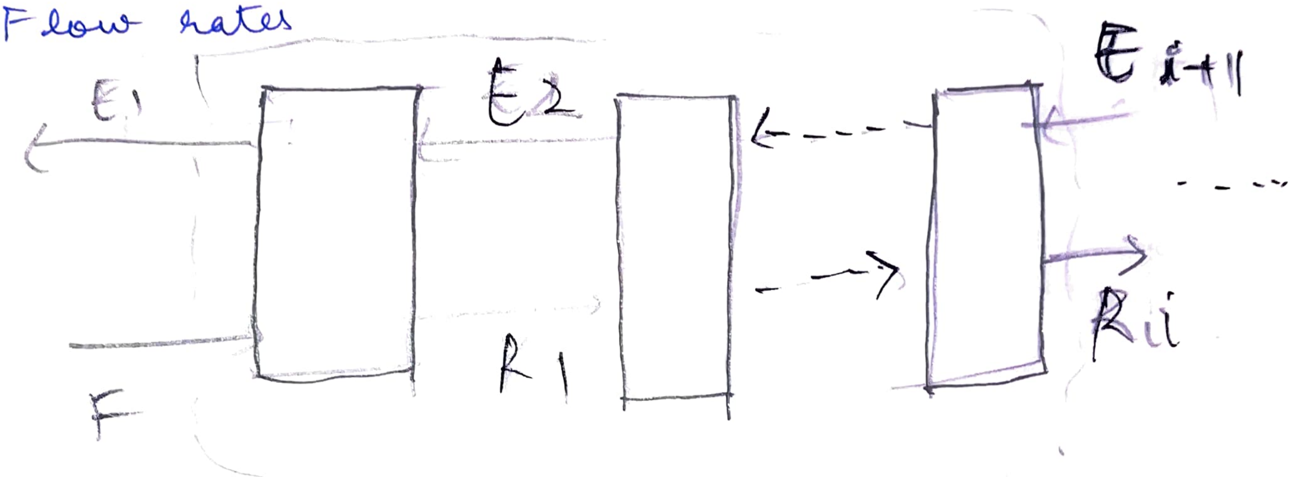
c) Compositions:

	E_1	E_2	E_3
TCE → $(x_{TCE})_E$	0.587	0.695	0.818
Aceton → $(x_{A})_E$	0.385	0.286	0.171
$(x_{water})_E$	0.029	0.019	0.014

4

	R_1	R_2	R_3
$(x_A)_R$	0.278	0.2	0.113
$(x_{TCE})_R$	0.018	0.01	0.004
$(x_{water})_R$	0.704	0.790	0.882

o Flow rates



Total material balance:

$$E_1 + \cancel{E_{i+1}} + R_i = F + E_{i+1} \quad \text{①}$$

Acetone Balance:
(assuming x_α indicates ~~aceton~~ composition in stream α).

$$x_{E_1} E_1 + x_{R_i} R_i = x_F F + x_{E_{i+1}} E_{i+1} \quad \text{②}$$

Solving ① & ②,

$$E_{i+1} = \frac{(x_{E_1} - x_{Ri}) E_1 - (x_F - x_{Ri}) F}{(x_{Ei+1} - x_{Ri})}$$

$$R_i = \frac{(x_{E_1} - x_{Ei+1}) E_1 - (x_F - x_{Ei+1}) E_1}{(x_{Ri} - x_{Ei+1})}$$

R_N, E_1 can be found using overall mass balance for the LLE system.

$$E_1 + R_N = F + S$$

$$E_1 y_{E1} + y_{RN} R_N = F y_F + S \cancel{x_S}^{\,0}$$

$$\Rightarrow E_1 = \frac{F \times (y_F - y_{RN}) - S \times y_{RN}}{(y_{E1} - y_{RN})} = \cancel{1142\ kg/h} = 942.43\ kg/h$$

$$R_3 = F + S - E_1$$
$$= 913.96\ kg/h$$

mass flow rate of E_1 = $\boxed{942.43\ kg/h}$

E_2 = $\boxed{829.82\ kg/h}$

E_3 = 693.23 kg/h.

R_1 = 1184.4 kg/h.

R_2 = 1047.8 kg/h.

R_3 = ~~913.9~~ 913.96 kg/h.

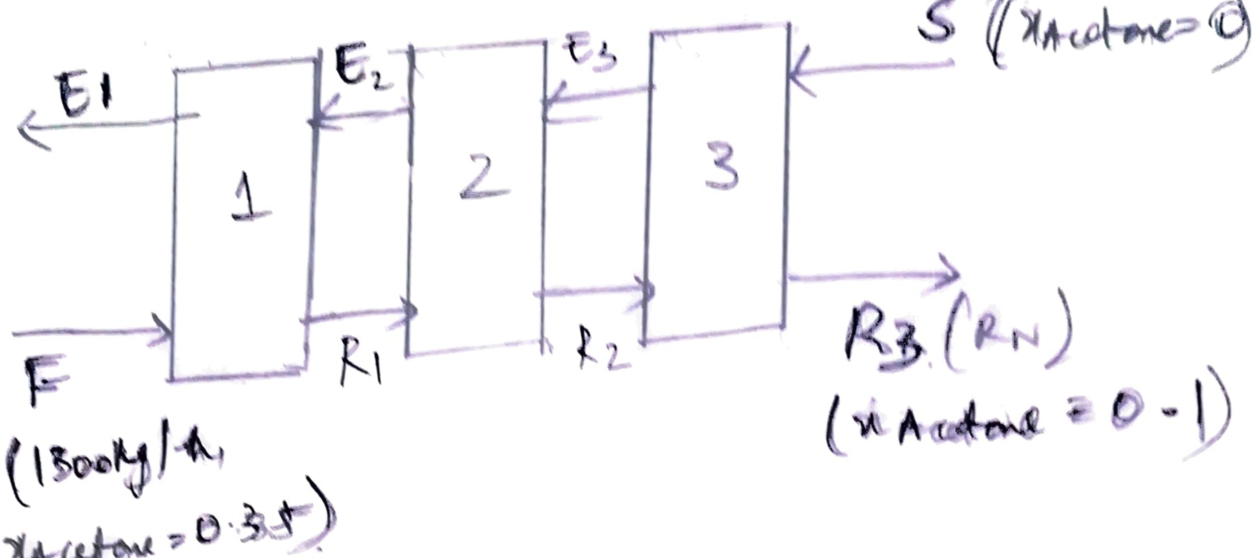
→ For this case all eqns have been solved in MATLAB to determine coordinates (rather than just getting coordinate from graph)

→ In the code 'y' represent composition of acetone and 'x' represents TCE composition.

This naming convention was used because

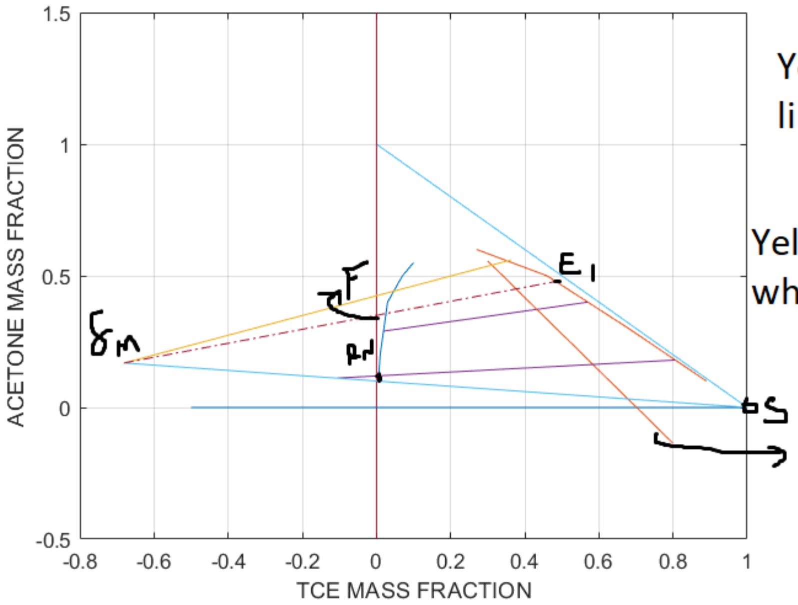
y-axis represents acetone concentration

x- axis " TCE composition.


E1
E2
E3
S ((x_Acetone = 0)
1
2
3
F
R1
R2
R3 (RN)
(x Acetone = 0.1)
(1800kg/h,
x_Acetone = 0.35)

Question 1 – RIGHT TRIANGLE METHOD

a) Delta point for minimum solvent flow rate

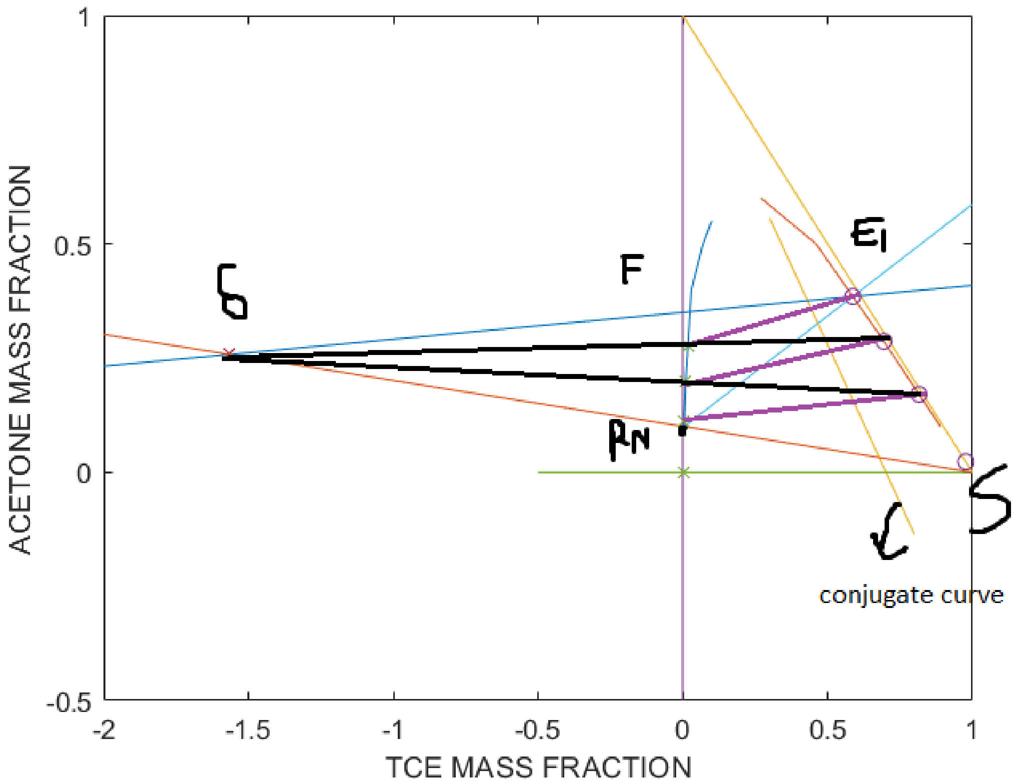


Yellow and purple lines are tie lines.

Yellow is the tie line which gives delmin.

conjugate curve

b) And c) Number of stages and composition of each stream at the exit of each stage



Code for evaluating the points and plotting:

```
close all; clear;
%% Equilibrium Data
r_tce = [0.1,0.07,0.03,0.02,0.01,0.005];
r_a = [0.55,0.5,0.4,0.3,0.2,0.1];
r_p = spline(r_a,r_tce);
e_a = [0.6,0.5,0.4,0.3,0.2,0.1];
e_tce = [0.27,0.46,0.57,0.68,0.785,0.89];
e_p = spline(e_a,e_tce);
%% Given Feed Data
yF =0.35;
yRN = 0.1;
F = 1300;
%% Tie Line data
ac_raff = [0.44,0.29,0.12];
ac_ext = [0.56,0.4,0.18];
tce_r = ppval(r_p,ac_raff);
tce_e = ppval(e_p,ac_ext);
xRN = ppval(r_p,yRN);
%% Plotting the data
plot(r_tce,r_a,e_tce,e_a);
hold on; grid on; grid minor;
for i = 1:3
   plot([tce_r(i),tce_e(i)],[ac_raff(i),ac_ext(i)]);
end
%Completing the triangle
plot(linspace(0,1,5),1-linspace(0,1,5));
%Plotting the axes
plot(zeros(1,2),linspace(-0.5,1.5,2));
plot(linspace(-0.5,1,2),zeros(1,2));
%% Conjugate curve
curve = spline(ac_raff,tce_e);
plot(linspace(0.3,0.8,5),ppval(curve,linspace(0.3,0.8,5)));
%% Tie line intersection with RS
% From the plot we can infer topmost tie line gives farthest S
line1 = polyfit([tce_r(1),tce_e(1)],[ac_raff(1),ac_ext(1)],1);
line3 = polyfit([tce_r(3),tce_e(3)],[ac_raff(3),ac_ext(3)],1);
RS = @(x)(yRN + (x-xRN)*(-yRN/(1-xRN)));
int1 = @(x)(RS(x)-polyval(line1,x));
xint1 = fsolve(int1,0);
int2 = @(x)(RS(x)-polyval(line3,x));
xint2 = fsolve(int2,0);
plot([xint1,tce_r(1),tce_e(1)],[polyval(line1,xint1),ac_raff(1),ac_ext(1)]);
plot([xint2,tce_r(3),tce_e(3)],[polyval(line3,xint2),ac_raff(3),ac_ext(3)]);
plot([xint1,1],[polyval(line1,xint1),0]);
%% Connecting F and delmin
xint = xint1;
yint = polyval(line1,xint1);
Fdel = polyfit([xint,0],[yint,yF],1);
int = @(x)(polyval(Fdel,x)-ppval(e_p,x));
xE1 = fsolve(int,0);
```

```
yE1 = ppval(e_p,xE1);
%hold off;
%figure();
hold on; grid on; grid minor;
plot([xint1,1],[polyval(line1,xint1),0]);
plot([xint,xE1],polyval(Fdel,[xint,xE1]),'-.');
xlabel('TCE MASS FRACTION');
ylabel('ACETONE MASS FRACTION');
hold off;
%% Getting Smin
%Find intersection of EF and RS
mER = (yE1-yRN)/(xE1-xRN);
xM = (yF-yRN+mER*xRN)/(mER+yF);
yM = -yF*xM+yF;
Smin = F*(yF-yM)/(yM);
%% Stages
figure();
plot(r_tce,r_a,e_tce,e_a);
hold on;
%Conjugate Curve
plot(linspace(0.3,0.8,5),ppval(curve,linspace(0.3,0.8,5)));
%Plotting the axes
plot(zeros(1,2),linspace(-0.5,1,2));
plot(linspace(-0.5,1,2),zeros(1,2));
S = 1.5*Smin;
yMnew = F*yF/(F+S);
RM = polyfit([xRN,S/(F+S)],[yRN,yMnew],1);
fun1 = @(x)(polyval(RM,x)-spline(e_tce,e_a,x));
xE1 = fsolve(fun1,1);
yE1 = polyval(RM,xE1);
plot([0,1],polyval(RM,[0,1]));
FE = polyfit([xE1,0],[yE1,yF],1);
RS = polyfit([xRN,1],[yRN,0],1);
fun = @(x)(polyval(FE,x)-polyval(RS,x));
delx = fsolve(fun,-1.48);
dely = polyval(FE,delx);
plot(delx,dely,'x');
x = [-2,1];
plot(x,polyval(FE,x),x,polyval(RS,x));
% Stepping process
% Conjugate curve
cc = spline(tce_e,ac_raff);
i = 1;
yp = yE1;
xp = xE1;
EC = spline(e_tce,e_a);
ycoords= zeros(1,4);
xcoords= zeros(1,4);
ycoords(1)= yp;
xcoords(1) = xp;
ycoords2= zeros(1,4);
```

```
xcoords2= zeros(1,4);
while yp >= yRN
   yp = ppval(cc,xp);
   xp = ppval(r_p,yp);
   ycoords2(i) = yp;
   xcoords2(i) = xp;
   OL = polyfit([xp,delx],[yp,dely],1);
   f = @(x)(ppval(EC,x)-polyval(OL,x));
   xp = fsolve(f,0.5);
   yp = polyval(OL,xp);
   ycoords(i+1)= yp;
   xcoords(i+1) = xp;
   i = i + 1;
   if i > 25
      break;
   end
end
plot(linspace(0,1,5),1-linspace(0,1,5));
plot(xcoords,ycoords,'o',xcoords2,ycoords2,'x');
xlabel('TCE MASS FRACTION');
ylabel('ACETONE MASS FRACTION');
%% Mass
E1  = (F*(yF-yRN)-S*yRN)/(yE1-yRN);
E = ((yE1-ycoords2(1:2))*E1-(yF-ycoords2(1:2))*F)./(ycoords(2:3)-ycoords2(1:2));
R = ((yE1-ycoords(2:3))*E1-(yF-ycoords(2:3))*F)./(ycoords(2:3)-ycoords2(1:2));
```

Method 2: Equilateral triangle method.

a). • Given S, F, R_N and equilibrium and tie line data are plot on the equilateral

• I used the ternary plot generator to plot the points on ternary diagram.
& paint 3D to draw the lines.

• From the previous method, we know that the topmost tie line intersects $R_N S$ at δ_{min}.

• Join F & δ_{min}. Extend line till E_1 (intersection of raffinate & δF).

• Find intersection of $R_N E_1$ & SF → gives M.

• The acetone composition @ M, $x_M = 0.275$

(The least count of my plot is 0.05; in case the point lies between 2 lines, I took that to be 0.025 or 0.075 as appropriate)

• $$\frac{S_{min}}{F} = \frac{(x_A)_F - (x_A)_M}{(x_A)_M - (x_A)_S}$$

$F = 1300$ kg/h ; $(x_A)_F = 0.35$, $(x_A)_M = 0.275$, $(x_A)_S = 0$

→ $\boxed{S_{min} = 354.5453 \text{ kg/h}}$

b) $S = 1.5\, S_{min}$ ⇒ $S = 531.818$

Now $(x_A)_M = \dfrac{F \times (x_A)_F}{F+S} = 0.248$

and $(x_{TCE})_M = \dfrac{S}{F+S} = 0.29$

} ⇒ Plot M on ternary diagram

- Join RM. Extend it to the extract curve to get E_1
- Join FE_1 and R_NS.

Intersection of FE_1 and R_NS is the new δ point

- Plot the conjugate curve:
  - → draw lines parallel to the Water and acetone line from ~~extract and~~ raffinate and extract points of a tie line respectively.
  - → ~~Join the~~. The intersection of such a pair of lines lies on conjugate curve
  - → connect the 3 points (from 3 tie lines) to get the conjugate curve.
  - → Done using 3 point curve in paint 3D
- Draw a tie line through E_1. The end point of the tie line on the raffinate curve gives R_1
  - → The procedure to draw new tie lines is same as the one described earlier.
- Draw a line from δ to R_1 ie extend it till extract curve. The intersection of δR_1 with extract curve gives ~~R_2~~ E_2.
- Repeat the previous 2 steps till you get close enough to R_N

No. of stages ≈ $\boxed{3.}$ (actually more, but 3rd stage is closer to R_n than end of 4th stage)

c) Compositions
(rounded off to nearest 0.025)

	E_1	E_2	E_3
$(x_A)_E$	0.375	0.3	0.175
$(x_{TCE})_E$	0.6 ~~0.675~~	0.7 ~~0.8~~	0.825 ~~0.111~~
$(x_{water})_E$	0.025	0	0
Raffinate	R_1	R_2	R_3
$(x_A)_R$	0.275	0.2	0.115
$(x_{TCE})_R$	0.025	0	0.01
$(x_{water})_R$	~~0.025~~ 0.70	0.8	0.875

Balance eqns same as previous case.

$$E_1 = \frac{F \times ((x_A)_F - (x_A)_{RN}) - S(x_A)_{RN}}{(x_A)_{E_1} - (x_A)_{RN}}$$

$$= \frac{F \times ((x_A)_F - (x_A)_{RN}) - S(x_A)_{RN}}{(x_A)_{E_1} - (x_A)_{RN}}$$

$$= \cancel{950} \; \underline{955.12 \text{ kg/h}}$$

(9)

$R_N = F + S - E_1 = \underline{877\ kg/h}.$

- We can use the same equations derived for ~~equilateral Δ^{le}~~ class. right Δ^{le} case.

$$E_{i+1} = \frac{\left((x_A)_{E_1} - (x_A)_{R_i}\right)E_1 - \left((x_A)_F - (x_A)_{R_i}\right)F}{\left((x_A)_{E_{i+1}} - (x_A)_{R_i}\right)}$$

$$R_i = \frac{\left((x_A)_{E_1} - (x_A)_{E_{i+1}}\right)E_1 - \left((x_A)_F - (x_A)_{E_{i+1}}\right)E_1}{\left((x_A)_{R_i} - (x_A)_{E_{i+1}}\right)}$$

- Mass flow rates are found to be

$E_1 = \underline{955\ kg/h}.$

$E_2 = \underline{889.7\ kg/h}.$ *

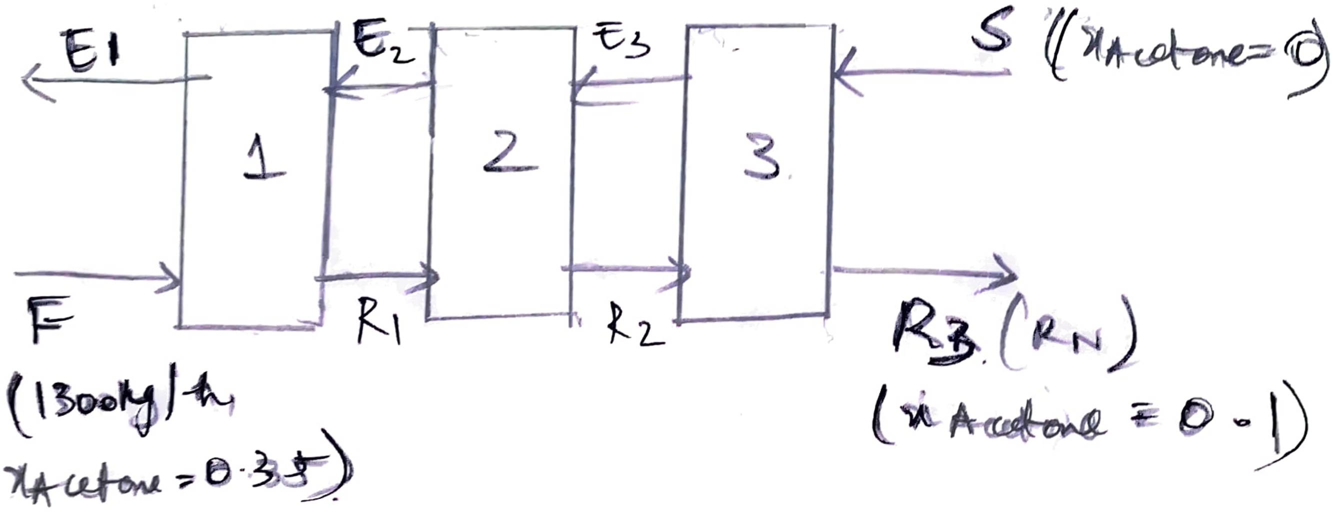
$E_3 = \underline{747.74\ kg/h}.$

$R_1 = \underline{1234.6\ kg/h}.$ *

$R_2 = \underline{1092.0\ kg/h}$

$R_3 = \underline{877\ kg/h}.$

* Instead of using 0.3 and 0.275 as extract & raffinate compositions, I used a more accurate values of 0.29 & 0.28. This is done to reduce is round-off errors which is high in this case.


E1
E2
E3
S (xAcetone = 0)
1
2
3
F
R1
R2
R3 (RN)
(1300kg/h
xAcetone = 0.35)
(xAcetone = 0.1)

Legend to the ~~provided~~ 2nd drawing attached:

i) purple lines represent stages (tie lines)

ii) red lines represent the original tie lines given as data (note that the final stage almost coincides with one of the given tie lines)

iii) Δ in centre of the graph denotes mixing point.

iv) FE, RNS, F and all operating lines are black

v) Light yellow and light blue are lines parallel to sides of the Δle used in new tie line determination by conjugate curve approach.

vi) Dark blue is the conjugate curve.

vii) Thin light blue on left - raffinate curve

Thin light orange on right - extract curve.

Legend to first diagram:

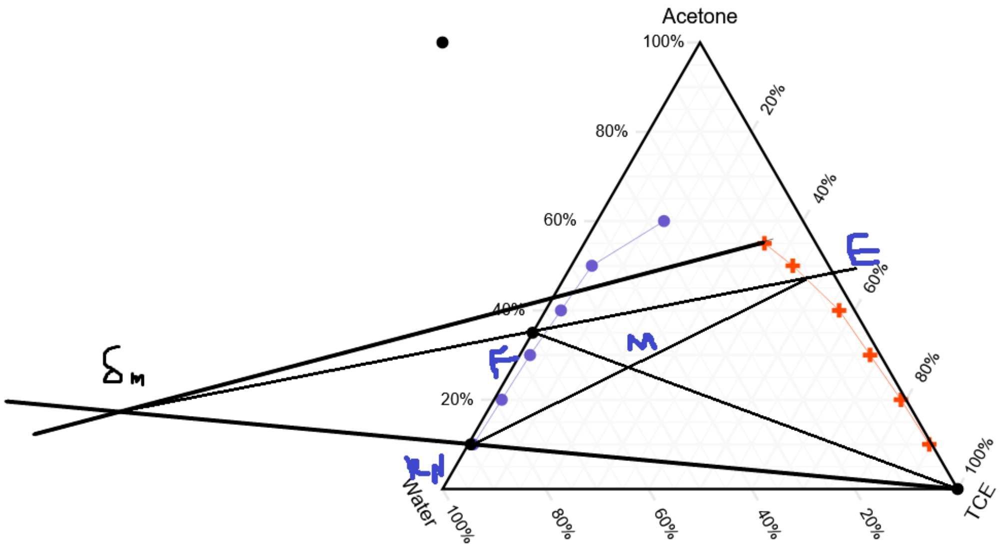
→ light blue curve - raffinate

→ light orange " - extract.

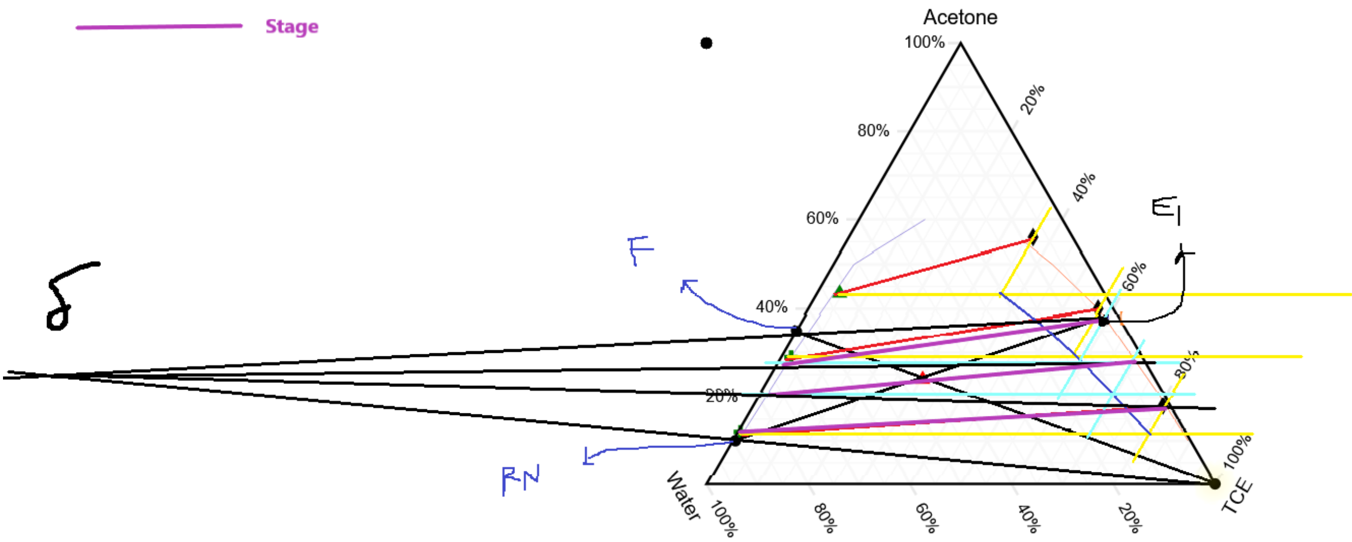
Question 1- EQUILATERAL TRIANGLE METHOD

Diagram source: www.ternaryplot.com

a) Smin determination **(Graph Lines not visible when I upload it in word)**



b) And c) Stages and composition determination**(Graph Lines not visible when I upload it in word)**



Legend for each of these graphs is in the handwritten part

MATLAB Code for solving the Mass balance equations

```
xcoords = [0.60,0.70,0.825];
ycoords = [0.375   0.29    0.175];
ycoords2 = [0.28,0.20,0.115];
xcoords2 = 1-ycoords2-[0.70 0.80 0.875];
Snew = 1.5*354.5455;
E1  = (F*(yF-yRN)-Snew*yRN)/(yE1-yRN);
```

```
E = ((yE1-ycoords2(1:2))*E1-(yF-
ycoords2(1:2))*F)./(ycoords(2:3)-ycoords2(1:2));
R = ((yE1-ycoords(2:3))*E1-(yF-
ycoords(2:3))*F)./(ycoords(2:3)-ycoords2(1:2));
RNew = F + Snew- E1;
```