

① Freundlich isotherm:

$$q = k C^{1/n}; \text{parameters: } q \text{ and } n.$$

Model is fit using nlinfit function in MATLAB.

$$k = \underline{5.655} \quad n = \underline{2.176}$$

$$\Rightarrow q = 5.655 C^{0.4596}.$$

$$\text{Mean Squared error} = \underline{3.467}$$

Langmuir Isotherm:

$$q = \frac{q_m K C}{1 + K C}; \text{parameters: } q_m \text{ and } K.$$

Model is fit using nlinfit function in MATLAB.

$$K = \underline{0.0433}, \quad q_m = \underline{52.002}$$

$$\Rightarrow q = \frac{2.252 C}{1 + 0.0433 C}$$

$$\text{Mean squared error} = \underline{0.801}.$$

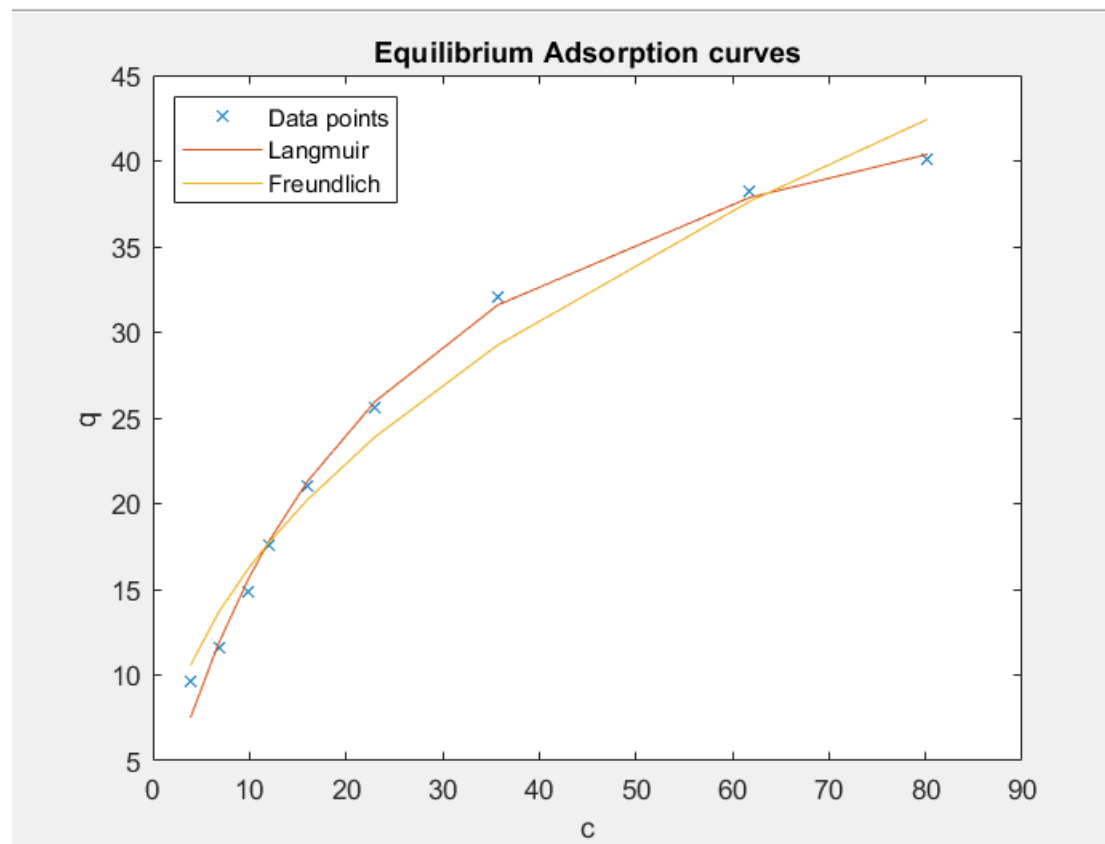
By comparing the mean squared errors as well as the visual examination of the adsorption isotherm plots, we find that

Langmuir isotherm best fits the data.

$$(k = \underline{5.655}, n = \underline{2.176}) \rightarrow q_m = 52.002)$$

Question 1 MATLAB CODE AND PLOT

```
clear;close all; clc;
c = [3.9;6.8;9.9;12;16;23;35.7;61.8;80.2];
q= [9.6;11.6;14.9;17.6;21.0;25.6;32.1;38.2;40.1];
options = statset('MaxIter',500);
[beta,R,J,COVB,MSE] = nlinfit(c,q,@Freund,[1,1]);
[beta1,R1,J1,COVB1,MSE1] =
nlinfit(c,q,@Lang,[0.00001,2*10000],options);
qlang = Lang(beta1,c);
qfreund = Freund(beta,c);
plot(c,q,'x',c,qlang,c,qfreund);
title('Equilibrium Adsorption curves');
legend('Data points','Langmuir','Freundlich');
xlabel('c');
ylabel('q');
function y = Freund(parms,c)
    k = parms(1);
    n = parms(2);
    y = k.*(c.^(1./n));
end
function y= Lang(parms,c)
    k = parms(1);
    qm = parms(2);
    y = qm.*(k.*c./(1+k.*c));
end
```



② a) At minimum Adsorbent condition,

$$\frac{W_{\min}}{L} = \frac{Y_0 - Y_1}{X_1 - X_0}$$

$$Y_0 = 3.5 \text{ mg/L} \quad Y_1 = 0.1 \text{ mg/L}$$

$X_0 = 0$ and X_1 is in eqbm with Y_1

$$\Rightarrow X_1 = 50(0.1)^{0.32}$$

$$= 23.932 \text{ mg compound / g charcoal}$$

$$L = 10 \text{ Litres}$$

$$\Rightarrow W_{\min} = 10 \times \left(\frac{3.5 - 0.1}{23.932} \right)$$

$$\Rightarrow W_{\min} = 1.4207 \text{ g charcoal}$$

b) $Sh = \frac{k_L \times (\text{diameter of particle})}{D_{AB}}$

$$D_{AB} = 1.5 \times 10^{-5} \text{ cm}^2/\text{s} = 1.5 \times 10^{-9} \text{ m}^2/\text{s}$$

$$Sh = 30 \therefore \text{particle diameter} = 2 \times 10^{-3} \text{ m.}$$

$$\therefore k_L = \frac{1.5 \times 10^{-9} \times 30}{2 \times 10^{-3}} = 2.25 \times 10^{-5} \text{ m/s.}$$

$$W = 2 W_{\min} = 2.841 \text{ g}$$

Adsorbate Balance:

$$-\frac{dc}{dt} = k_L a (c - c^*)$$

c is the concentration of adsorbate in the solution at any time t .

a is the surface area of charcoal per unit volume of solution.

$$\Rightarrow a = \frac{5 \text{ m}^2/\text{kg charcoal} \times (2.841 \times 10^{-3} \text{ kg charcoal})}{10.1 \times 10^{-3} \text{ m}^3 \text{ solution}} \\ = 1.421 \text{ m}^2/\text{m}^3 \text{ solution.}$$

c^* is the eqbm concentration of adsorbate in solution

$$\Rightarrow c^* = \left(\frac{(c_i - c)L}{wK} \right)^n \text{ mg/L}$$

From the given isotherm,

$$n = \frac{1}{0.32} = 3.125, K = 50.$$

Substituting all the values in the eqn,

$$\frac{-dc}{dt} = k_L a \left(c - \left(\frac{(c_i - c)L}{wK} \right)^n \right)$$

$$\Rightarrow \int_0^t dt = \int_{0.1}^{0.1} \frac{-dc}{k_L a \left(c - \left(\frac{(c_i - c)L}{wK} \right)^n \right)} \\ = \int_{0.1}^{0.1} \frac{dc}{k_L a \left(c - \left(\frac{(c_i - c)L}{wK} \right)^n \right)}$$

Integrating this expression numerically in MATLAB,

$$t = 1.1428 \times 10^5 \text{ s} \\ = \boxed{31.743 \text{ hours}}$$

Question 2 MATLAB CODE

```
clear; close all;
kl = 2.25*10^-5;%m/s
L = 10;%Litres
wmin = (3.5-0.1)/(50*0.1^(0.32))*L;%g
w = 2*wmin;%g
ci = 3.5;%mg/L
K = 50;%q should be in mg/g, c in mg/L
a = 5;%m^2/kg
time = integral(@dtbydc,3.5,0.1);
t_inhours = time/3600;
function val = dtbydc(c)
    kl = 2.25*10^-5;%m/s
    L = 10;%Litres
    wmin = (3.5-0.1)/(50*0.1^(0.32))*L;%g
    w = 2*wmin;%g
    ci = 3.5;%mg/L
    K = 50;%q should be in mg/g, c in mg/L
    a = 5;%m^2/kg
    %units of val
    %c is the conc in solution, second term is the ideal
    c in soln
    val = -1./(kl*(a*w/L).*(c-((ci-
c)*L/(w*K)).^(1/0.32))));
end
```

③ a) $L_{\text{empt}} = 0.2 \text{ m.}$

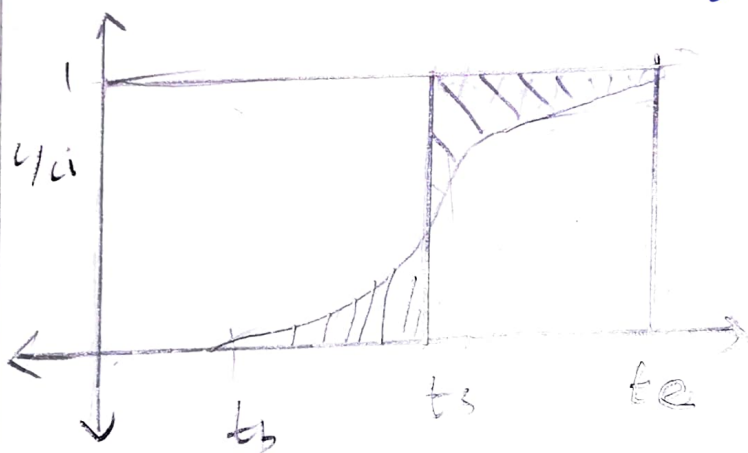
$\mu_{\text{superficial}} = 0.29 \text{ m/s}$

$c_i = 0.11 \text{ gmol/lm}^3.$

Breakthrough time : time at which $\frac{c}{c_i} = 0.03.$

$\Rightarrow t_b = 20.626 \text{ minutes}$

(Obtained using Spline interpolation in MATLAB)



At stoichiometric time Area above curve = Area below the curve

$$\Rightarrow \int_{t_b}^{t_s} \frac{c}{c_i} dt = (t_e - t_s) - \int_{t_s}^{t_e} \frac{c}{c_i} dt$$

$$\Rightarrow \int_{t_b}^{t_s} \frac{c}{c_i} dt + \int_{t_s}^{t_e} \frac{c}{c_i} dt = (t_e - t_s)$$

$$\Rightarrow (t_e - t_s) - \int_{t_b}^{t_s} \frac{c}{c_i} dt$$

t_e is the saturation time - the bed is fully used up. ($c/c_i = 1$)

\Rightarrow from the data $t_e = 108.6 \text{ minutes}$

Performing the integration numerically,

$$t_s = 45.18 \text{ minutes}$$

$$u_s = \frac{L}{t_s} = \frac{0.2}{45.18} = 4.427 \times 10^{-3} \text{ m/min}$$

$$\Rightarrow u_s = 0.2656 \text{ m/h.}$$

$$LUB = L \times \left(1 - \frac{t_b}{t_s} \right)$$

$$= 0.2 \times \left(1 - \frac{20.63}{45.18} \right)$$

$$\Rightarrow \boxed{LUB = 0.1087 \text{ m.} = 0.109 \text{ m.}}$$

Since the curve is not symmetric ($\frac{c}{c_i}$ vs t curve)

we can't use $MTZ = 2 \times LUB$.

Instead we use the general expression,

$$MTZ \text{ Length of mass transfer zone} = L \times \left(1 - \frac{t_b}{t_e} \right)$$

$$\Rightarrow \text{Length of mass transfer zone} = \boxed{0.162 \text{ m}}$$

(Note: If ^{the curve} the curve was symmetric,

$$* MTZ = 2 \times LUB)$$

$$b) \dot{V} = 3000 \text{ m}^3/\text{h}.$$

$$= \frac{3000}{3600} \text{ m}^3/\text{s}$$

$$\Rightarrow \dot{V} = 0.833 \text{ m}^3/\text{s} \quad ; \quad u_{\text{superficial}} = 0.29 \text{ m/s}$$

$$\text{Area of Bed} = \frac{\dot{V}}{u_{\text{superficial}}}$$

$$\Rightarrow \text{Diameter of bed} = \sqrt{\frac{4 \dot{V}}{\pi u_{\text{superficial}}}}$$

$$= \underline{1.913 \text{ m.}}$$

Adsorption cycle, $t = 8 \text{ hours}$

$$\Rightarrow \text{length of used bed} = u_s \times t$$

$$= \underline{2.125 \text{ m}}$$

LVB and length of mass transfer zone will remain the same before and after scale up because the wavefront is the same.

$$\Rightarrow \text{Length of bed} = \text{Length used} + \text{Length unused}$$

↓
(LVB)

$$\Rightarrow L = 2.125 + 0.1087$$

$$\Rightarrow L_{\text{total}} = \underline{2.233 \text{ m.}}$$

Weight of adsorbate adsorbed till break through, $w = V \times C_i \times t_b$.
(\because Cunit 40)

$$\Rightarrow w = \frac{2640 \text{ g mol}}{190.37 \text{ kg}} \quad (mw = 72.11 \text{ g mol}^{-1})$$

$$\text{Average loading} = \frac{\text{Mass of adsorbate}}{\text{Mass of bed}}$$

$$= \frac{\text{mass of adsorbate}}{S_{\text{bed}} \frac{\pi D^2}{4} L_{\text{total}}}$$

$$= 0.5876 \text{ g mol / kg bed}$$

$$= 0.0424 \text{ kg adsorbate / kg bed.}$$

$$\text{Maximum loading} = \frac{\text{mass of adsorbate}}{\text{mass of used part of bed}}$$

$$= \frac{w}{S_{\text{bed}} \frac{\pi D^2}{4} L_{\text{used}} \quad \text{MEK}}$$

$$\Rightarrow \text{Maximum loading} = 0.6177 \text{ g mol / kg bed}$$

$$= 0.0445 \text{ kg MEK / kg Bed.}$$

Question 3 MATLAB CODE AND PLOT

```
clear; close all;
%% Given data
u_superficial = 0.29;
t =
[9.5,19,21,25.7,34.3,39,42,46.7,51.4,56.2,64.7,68.6,72.4,
77.1,84.8,97.1,104.7,108.6];
te = 108.6;
c =
[0,0.018,0.037,0.083,0.287,0.435,0.491,0.62,0.713,0.768,0
.852,0.935,0.952,0.963,0.97,0.987,0.991,1];
L = 0.2;
ci = 0.11;
plot(t,c);
title('c/ci vs t');
ylabel('c/ci');
xlabel('time');

%% Part a
t_break = spline(c,t,0.03);
pp = spline(t,c);
n = 40;
time = linspace(9.5,108.6,n*(10));
ts = te - trapz(time,ppval(pp,time));
% Following code evaluates both the integrals numerically
and minimises it.
% No need to run this part because we have already
simplified it into a simple linear eqn in ts.
% min = trapz(time,ppval(pp,time));
% ts = 230;
% ar_diff = zeros(n*10-1,1);
% l = length(time);
% for i = 2:l-1
%     ar_diff(i) = trapz(time(1:l),ppval(pp,time(1:l)))-
(108.6-time(i));
%     if min > abs(ar_diff(i))
%         min = abs(ar_diff(i));
%         ts = time(i);
%     end
% end
us = L/ts*60; %m/h
LUB = L*(1-t_break/ts);
MTZ = (te-t_break)/te*L;%Since curve is NOT symmetric

%% Part b
V = 3000/60/60;%m^3/s
D = sqrt(4*V/(u_superficial*pi));
```

```
t_cycle = 8;  
bed_density = 700;  
L_used = 8*us;  
L_scaleup = LUB + L_used;  
sol_adsorbed = V*ci*t_cycle*3600;  
avg_loading =  
sol_adsorbed/(L_scaleup*bed_density*pi*D^2/4);  
max_loading = sol_adsorbed/(L_used*bed_density*pi*D^2/4);
```

