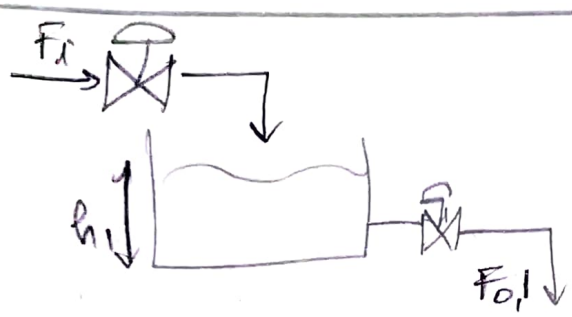
③

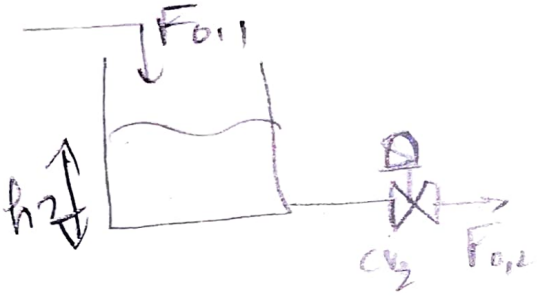


Mass balance over tank 1:

$$\frac{A_1 dh_1}{dt} = F_i - F_{o,1}$$

$$= F_i - C_{v_1} h_1$$

$$\Rightarrow \frac{A_1 dh_1}{dt} = \frac{F_i}{A_1} - \frac{C_{v_1} h_1}{A_1} \quad \text{—①}$$



Mass balance over tank 2:

$$\frac{A_2 dh_2}{dt} = F_{o,1} - F_{o,2}$$

$$= C_{v_1} h_1 - C_{v_2} h_2$$

$$\Rightarrow \frac{dh_2}{dt} = \frac{C_{v_1}}{A_2} h_1 - \frac{C_{v_2}}{A_2} h_2$$

$$\Rightarrow \begin{bmatrix} \frac{dh_1}{dt} \\ \frac{dh_2}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-C_{v_1}}{A_1} & 0 \\ \frac{C_{v_1}}{A_2} & \frac{-C_{v_2}}{A_2} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} F_i$$

$$\Rightarrow \dot{\underline{x}} = A_c \underline{x} + B_c u$$

$$\underline{x} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} ; \; u = F_i$$

$$A_c = \begin{bmatrix} \frac{-C_{v_1}}{A_1} & 0 \\ \frac{C_{v_1}}{A_2} & \frac{-C_{v_2}}{A_2} \end{bmatrix} \quad \text{and} \quad B_c = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

If only $F_{o,2}$ is measured

$$F_{o,2} = C_{v_2} h_2 \Rightarrow y = \begin{bmatrix} 0 & C_{v_2} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

$$\Rightarrow C_C = \begin{bmatrix} 0 & C_{V_2} \end{bmatrix}$$