(5) a) Consider the likelihood fn,

$$l = \begin{cases} \prod_{k=1}^{N} e^{-(y(k)-\theta)} & \text{if } \min(y_k) > \theta \\ 0 & \text{if } \min(y_k') < \theta \end{cases}$$

$$\Rightarrow l = \begin{cases} \exp\left(\sum_{k=1}^{N} \theta - y[k]\right) & \min(y(k)) > \theta \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} \exp(N\theta)\exp\left(\sum -y(k)\right) & \min(y(k)) > \theta \\ 0 & \text{otherwise} \end{cases}$$

~~Using the Heaviside step fn,~~

$$\cancel{H(x) = \begin{cases} 1 & x > 0 \\ 0 & x \end{cases}}$$

Consider the function,

$$H(x) = \begin{cases} 1 & x > 0 \\ 0 & x \leq 0 \end{cases}$$

$$\Rightarrow l = \left[\exp(N\theta)\left(\exp\left(\sum -y(k)\right)\right)\left(H(\min(y_N) - \theta)\right)\right]$$

$$= \left[H(\min(y_N) - \theta)\exp(N\theta)\right]\left[\exp\left(-\sum y(k)\right)\right]$$

consider ~~eq~~ term 1 as $\phi(T(y_N), \theta)$ $\left(\because T(\underline{y}_N) = \min(\underline{y}_N)\right)$

term 2 as $K(\underline{y}_N)$

∴ By the Neyman-Fisher factorisation theorem,

$T_N = \min(y_N)$ is a sufficient statistic.

Further we know that it is a complete statistic

By Rao-Blackwell theorem, we need to construct an unbiased estimator using T_N to get the MVUE

from Q1 b), we know an unbiased estimator of Θ as,

$$T_N' = \frac{T_N}{2} - \frac{1}{N}$$

$$\Rightarrow \text{MVUE of } \Theta = \frac{T_N}{2} - \frac{1}{N}$$

$$= \underline{\min(y_N)} - \frac{1}{N}$$

$$\Rightarrow \boxed{\hat{\Theta}_{MVUE} = \min(y_N) - \frac{1}{N}}$$