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CH5115 - QUIZ - 2

(1) $T_1 = \sum_{i=1}^{n} X_i$

Joint pdf is given by :

$$f(\underline{y_N};\theta) = \begin{cases} \prod_{i=1}^{N} \theta^{y_i}(1-\theta)^{(1-y_i)} & \text{if all } y_i = 0,1 \\ 0 & \text{if otherwise.} \end{cases}$$

Let $K(\underline{y_N})$ be an function such that,

$$K(\underline{y_n}) = \begin{cases} 1 & \text{if all } y_i \in \{0,1\} \\ 0 & \text{if any } y_i \notin \{0,1\} \end{cases}$$

$$\Rightarrow f(\underline{y_N};\theta) = \prod_{i=1}^{N} \theta^{y_i}(1-\theta)^{(1-y_i)} K(\underline{y_N})$$

$$= \left(\frac{\theta}{1-\theta}\right)^{\sum_{i=1}^{N} y_i} (1-\theta)^N K(y_N)$$

$$= \theta^{\sum y_i}(1-\theta)^{N-\sum y_i} K(y_N)$$

$\theta^{\sum y_i}(1-\theta)^{N-\sum y_i}$ is a function of y_i only through $T_1 = \sum_{i=1}^{N} y_i$

consider $\theta^{\sum y_i}(1-\theta)^{N-\sum y_i} = \phi(T_1;\theta)$

$$\Rightarrow \phi(T_1;\theta) = \theta^{T_1}(1-\theta)^{N-T_1}$$

$\Rightarrow f(\underline{y_N}, \theta) = \phi(T_1, \theta)\, K(y_N)$

By the Neyman-Fisher factorisation theorem, T_1 is a sufficient statistic for θ.

Now, $E(T) = E\left(\sum Y_i\right) = \sum_{i=1}^{N} E(Y_i)$

$\cancel{E\, y_i}$ $f(y;\theta) = \begin{cases} \theta & y=1 \\ 1-\theta & y=0 \\ 0. & \text{otherwise} \end{cases}$

$\Rightarrow E(y_i) = \theta \times 1 + (1-\theta) \times 0$

$= \theta$

$\therefore E(T) = N E(y_i)$

$= N\theta$.

Since T is a complete sufficient statistic for θ, we just need to do bias correction to make it MVUE.

$\Rightarrow T_{new} = \frac{1}{N} T_1$ $\left(\Rightarrow E(T_{new}) = \frac{1}{N} E(T_1) = \frac{1}{N} \times \theta N = \theta\right)$

$\Rightarrow T_{new}$ is unbiased

$\therefore$ the MVUE for θ is

$$\frac{1}{N} \sum_{i=1}^{N} Y_i.$$