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CH5LS15 QUIZ - I

① $f(y;\theta) = \begin{cases} \cancel{y\theta^{y+1}}\ \theta y^{\theta-1}, & 0 < y < 1 \\ 0, & \text{else where.} \end{cases}$

$$L = \log \prod_{i=1}^{N} \theta\, y_i^{\theta-1}$$

$$\cancel{= \sum \log \theta_i + \sum (\theta_i - 1) \log y}$$

$$\cancel{\frac{\partial L}{\partial \theta} = \sum \frac{1}{\theta_i} + \sum \log y}$$

$$\Rightarrow L = N \log \theta + \sum_{i=1}^{N} (\theta - 1) \log y_i .$$

$$\Rightarrow \frac{\partial L}{\partial \theta} = \frac{N}{\theta} + \sum_{i=1}^{N} \log y_i .$$

$$\text{So } \frac{\partial L}{\partial \theta} = 0$$

$$\Rightarrow \frac{N}{\theta} = -\sum \log y_i$$

$$\Rightarrow \boxed{\hat{\theta} = \frac{-\sum \log y_i}{N}}$$

(⇒ $\hat{\theta} > 0$, so makes sense)

$$\frac{\partial^2 L}{\partial \theta^2} = \frac{-N}{\theta^2} .$$

$$FI(\theta) = -E\left(\frac{\partial^2 L}{\partial \theta^2}\right)$$

$$\Rightarrow \boxed{I(\theta) = \frac{N}{\theta^2}}$$

② $\hat{v}[k] = \phi_{11} v[k-1] + \phi_{12} v[k-2]$

$\hookrightarrow$ AR(2) model.

$J = E((v[k] - \hat{v}[k])^2)$ (MSE)

$= E((v[k] - \phi_{11} v[k-1] - \phi_{12} v[k-2])^2)$

$$\frac{\partial J}{\partial \phi_{11}} = -2E(v[k]v[k-1]) + 2\phi_{11} E(v^2[k-1]) + 2\phi_{12} E(v[k-1]v[k-2]) = 0. \quad -①$$

$$\frac{\partial J}{\partial \phi_{12}} = -2E(v[k]v[k-2]) + 2\phi_{12} E(v^2[k-2]) + 2\phi_{11} E(v[k-1]v[k-2]) = 0 \quad -②$$

$① \Rightarrow -\sigma_{vv}[1] + \phi_{11}\sigma_{vv}[0] + \phi_{12}\sigma_{vv}[1] = 0$

$\Rightarrow \phi_{11}\sigma_{vv}[0] + \phi_{12}\sigma_{vv}[1] = \sigma_{vv}[1]$

$② \Rightarrow \phi_{11}\sigma_{vv}[1] + \phi_{12}\sigma_{vv}[0] = \sigma_{vv}[2]$

'Yule-walker eqns for AR(2)'

What is known: $\rho[\pm 1], \rho[\pm 2]$

$$\rho[l] = \frac{\sigma_{vv}[l]}{\sigma^2_v} = \frac{\sigma_{vv}[l]}{\sigma_{vv}[0]}$$

Dividing the eqns by $\sigma_{vv}[0]$,

$$\phi_{11} + \phi_{12}\, \rho[1] = \rho[1] \Rightarrow \cancel{\phi_{11} + \phi_{12}(0) = 0} \qquad (3)$$

$$\phi_{11}\, \rho[1] + \phi_{12} = \rho[2] \qquad (4)$$

(3) $\Rightarrow \phi_{11} + 0(\phi_{12}) = 0 \Rightarrow \phi_{11} = 0$

(4) $\Rightarrow 0(\phi_{11}) + \phi_{12} = 0.6$

$\Rightarrow \phi_{12} = 0.6.$

$\therefore$ The required AR(2) model is

$$\boxed{\hat{v}[k] = 0.6\, v[k-2]}$$

(It is the MMSE AR(2) model)