INDIAN INSTITUTE OF TECHNOLOGY MADRAS
Department of Chemical Engineering

CH5115: Parameter and State Estimation

Assignment 1

Due: Thursday, October 01, 2020

---

1. (a) If two random variables have joint density

$$f(x,y) = \begin{cases} K\dfrac{e^{-x/y}e^{-y}}{y} & x > 0,\ y > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find (i) the value of K (ii) marginal density of Y, (iii) the probability Pr$(0 < X < 1,\ 0.2 < Y < 0.4)$ (iv) conditional expectation $E(X|Y)$. Use numerical integration routines (`integral` or `integral2` in MATLAB) if necessary.

(b) Show that for two RVs X and Y that have a joint Gaussian distribution, the conditional expectation $E(Y|X)$ is a linear function of X.

2. The covariance between two RVs is estimated from their samples $x[k]$ and $y[k]$ as

$$\hat{\sigma}_{yx} = \frac{1}{N}\sum_{k=1}^{N}(y[k]-\bar{y})(x[k]-\bar{x}) \tag{1}$$

where $\bar{x}$ and $\bar{y}$ are the sample means of X and Y, respectively and N is the sample size. Write a **function** in MATLAB to calculate this **sample covariance matrix** given samples of two random variables. Test your code on the case $X \sim \mathcal{N}(1,2)$ and $Y = 3X^2 + 5X$ by comparing the resulting covariance matrix with the values obtained from `cov` command in MATLAB. Finally, show by means of simulation that the estimate $\hat{\sigma}_{yx}$ tends to the theoretical value as $N \to \infty$.

3. Given the variance-covariance matrix of three random variables X_1, X_2 and X_3, $\Sigma = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{bmatrix}$,

(a) Find the correlation **matrix** ρ.

(b) Find the correlation between X_1 and $\frac{1}{2}X_2 + \frac{1}{2}X_3$.

4. (a) Determine the optimal MAE predictor of a random variable $X \sim \chi^2(10)$, numerically using MATLAB. Find the average absolute error at the optimum value $X^\star$.

(b) Determine Pr$(0.9X^\star < X < 1.1X^\star)$. Is this lower than Pr$(0.9\mu_X < X < 1.1\mu_X)$? Justify your observation.