Question **1**

Not yet answered

Marked out of 45.00

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| **Question 1** | **[45 M]** |
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1. GaNithanAyaka (GN) knows that two RVs $Y^\star$ and $X^\star$ (with mean $\mu_{X^\star}$ and variance $\sigma^2_{X^\star}$) share a perfect linear relationship $Y^\star = \alpha_0 X^\star$. He has access to, however, only corrupted observations of $Y^\star$ and $X^\star$, essentially, $y[k] = y^\star[k] + \varepsilon_y[k]$ and $x[k] = x^\star[k] + \varepsilon_x[k]$, where $\varepsilon_x[k]$ and $\varepsilon_y[k]$ are zero-mean i.i.d. sequences of variances $\lambda_X$ and $\lambda_Y$, respectively.

   (a) Provide MoM estimates of the unknowns $\alpha_0$, $\mu^\star_X$, $\sigma^2_{X^\star}$, $\lambda_X$ and $\lambda_Y$ using the first and second-order moments only. Examine the consistency $\lim_{N\to\infty} \hat{\alpha}_{\text{MoM}}$. How does the solution change when $\mu_{X^\star} = 0$?

   (b) GN intends to estimate $\alpha_0$ using the LS approach, i.e., that which minimizes $\sum_{k=1}^{N}(y[k] - \alpha x[k])^2$. Examine the consistency $\lim_{N\to\infty} \hat{\alpha}_{\text{LS}}$. Can you explain the result?

Question **2**

Not yet answered

Marked out of 55.00

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| **Question 2** | **[55 M]** |
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i. Determine the MLE of $\lambda$ given $N$ observations of a RV $Y \sim \text{Poisson}(\lambda)$ with the constraint that $\lambda \geq b$, where $b > 0$ is a known constant.

ii. A random sample of $N$ observations of $Y \sim \Gamma(1, \theta)$ is available. Parignya chooses a prior as $\pi(\theta) \propto 1/\theta^2$.

   (a) Is Parignya's choice in the class of Jeffreys' priors? If not, correct Parignya's choice.

   (b) Find $f(\theta|\mathbf{y}_N)$ with Parignya's / corrected prior up to a proportionality constant.

   (c) If $\tau = 1/\theta$, determine the Bayesian (MMSE) estimator of $\tau$.



# Quiz 3: Probability estimation and Linear Regression

0 solutions submitted (max: 4)

Observations of a process variable $y[k]$ over a duration of time are available. Answer / do the following:

1. Find the probability $\Pr(a \leq y[k] \leq b)$, where $a$ and $b$ are known constants. Report your answer in `prob_est` variable.
2. Heera and Panna wish to fit an MA(3) model $y[k] = e[k] + c_1 e[k-1] + c_2 e[k-2] + c_3 e[k-3]$ to the given data. Realizing that the MLE method is quite complicated, H-P propose the following idea so as to use linear LS. Fit a high-order AR model such that the residuals are white. Use these residuals to fit an LR model for $y[k] - e[k]$ with appropriate regressors. Obtain estimates of $c_1$, $c_2$ and $c_3$ using this algorithm (report in MATLAB **vector** variable `Chat` = [c1hat c2hat c3hat])
3. Determine the significant coefficients (at $\alpha = 0.05$) and report them in `Cflags` = [c1flag c2flag c3flag] (1 for significant and 0 for otherwise).
4. Do you expect $\Pr(a \leq y[k] | y[k-1] \leq b)$ to be **significantly** different from $\Pr(a \leq y[k] \leq b)$? Report your answer in `prob_ans` (1 for 'Yes' and 0 for 'No').

**Note:** The data set $y[k]$ $k = 1, 2, \cdots N$ and the values of $a$, $b$ are auto-generated by the script datagen_ts.m supplied with this question.