Assignment 4

Due: Thursday, 15th October at 5 pm

- Due to the non-instructional break (1st to 5th October), we had split Module-3 in two parts. This week's assignment will be MATLAB-based Assignment.
- Please submit this assignment on MATLAB Grader

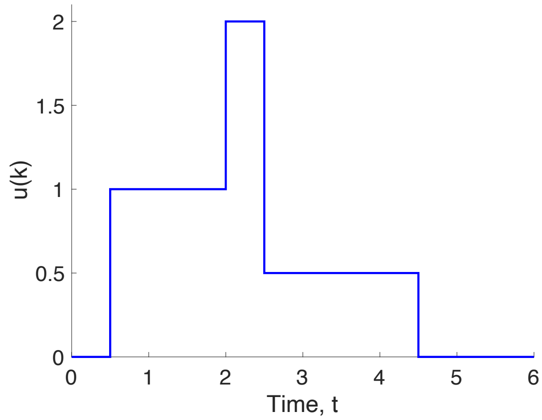
Problem 1: Comparison of State-Space and Step-Response Models **(2 + 2 points)**

In the previous assignments, you developed step-response and state space models for the following first-order system:

$$G(s) = \frac{5}{\tau s + 1} e^{-0.15s}, \qquad \tau = 4.2, \qquad \Delta t = 0.5$$

Here, we made a slight change from the problem by choosing a different value of τ compared to previous assignments.

The MATLAB code to obtain step-response and state space models will already be provided to you. You will run the *discrete-time models* to obtain the output response to the input moves shown in the adjoining figure.



Please run the models until $t = 10$. The results must be returned in vector `Y_###`, as described below. This must be a 1×20 row vector, such that the k^{th} element $Y(k)$ represents $[y]_{(t=0.5k)}$.

1. Use the step-response model given in matrix `S` to obtain $Y(k)$. Store this in a vector `Y_fsr`.
2. Use the state-space model ($x_{k+1} = Ax_k + Bu_k$, $y_k = Cx_k$, with the matrices `A, B, C` provided to you on MATLAB Grader). Please store the results in vector `Y_ss`.

Note: In both problems, we start at origin. In other words: $x(0) = 0$; and $y(i) = 0$, $u(i) = 0$ for $0 < i$.

Problem 1b: Repeat with arbitrary input moves **(2 + 2 points)**

The following lines of code generates input moves, $u(k)$, $k = 0, 1, \ldots, 20$. Note that $k = 20$ represents time $t = 10$. The generated input is stored in vector U (which is a 1×21 vector). The code that generates random input sequence (in the range $-1.5 \le u(k) \le 1.5$) is:

```
U=randi([-3,3],1,21)/2;
```

This line will generate different values of inputs every time you run the code in MATLAB.

3. Use the step-response model and report the results in vector `Y_fsr`.
4. Use the state-space model and report the results in `Y_ss`.

Note: If your MATLAB code is written in a flexible manner, then you will be able to use the exact same code for Problem 1a and 1b

Problem 2: Nonlinear and Linearized Model Simulations **(4 + 2 points)**

The nonlinear reactor model from the previous assignment is given by:

$$\frac{d}{dt}\begin{bmatrix} h \\ C \end{bmatrix} = \begin{bmatrix} \frac{u}{A} - \frac{\kappa}{A}\sqrt{h} \\ \frac{u}{Ah}(C_{in} - C) - kC^2 \end{bmatrix}$$

where, $A = 0.2;\ \kappa = 0.5,\ k = 1.5$. In the above model, u is the manipulated variable, F_{in}. The nominal operating point is $F_{in,ss} = 0.5$. We will simulate the system for a 10% step up change in the MV to $F_{in} = 0.55$ starting from the steady state. We will take $C_{in} = 4$ in this problem.

5. Please simulate the original nonlinear model using MATLAB solver `ode15s`. We will consider the sampling time to be $\Delta t = 0.2$. Please simulate the response of the nonlinear model to 10% step change until $t = 5$. Please report the concentration values in vector `C_nonlin`, which is a 1×25 array (note that value of y_0 is not required).

6. Repeat the above case of 10% step change (i.e., $u = 0.05$), using linearized model. For the state-space model $x_{k+1} = Ax_k + Bu_k,\ y_k = x_k$, the values of A, B, C are provided to you. Note that the concentration values are in deviation variables. Please return them in 1×25 array `Cdash`.

Problem 3: Simulation of MIMO system to series of step inputs **(6 points)**

We will develop a step-response model for the above system, with $n_u = 1, n_y = 2$, and $n = 25$ steps in the step-response model. The model parameters will be provided as matrix `Smodel`, which will be pre-loaded for you. Please use this $(n_y.n) \times 1$ matrix for your computations.

Please simulate the step-response model for a series of step input moves generated randomly. The following code will be used to generate the input moves in the range $-0.05 \leq u(k) \leq 0.05$:

```
U=(0.5-rand(1,41))/10;
```

Please simulate the step-response model with the above input moves. This represents simulation unti time $t = 40\Delta t = 8$.

Hint: The step-response simulations for MIMO system are: $\tilde{Y}_{k+1} = \tilde{Y}_k^{\text{shift}} + S\Delta u_k$, where, $\Delta u_k = u_k - u_{k-1}$ (the initial input, $\Delta u_0 = u_0$) and $\tilde{Y}^{\text{shift}}$ represents shift operation:

$$\begin{bmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \\ a_3 \\ b_3 \\ \vdots \\ a_{24} \\ b_{24} \\ a_{25} \\ b_{25} \end{bmatrix} \xrightarrow{\text{shift}} \begin{bmatrix} a_2 \\ b_2 \\ a_3 \\ b_3 \\ a_4 \\ b_4 \\ \vdots \\ a_{25} \\ b_{25} \\ \boldsymbol{a_{25}} \\ \boldsymbol{b_{25}} \end{bmatrix}$$