Assignment 3

Due: Thursday, 8th October at 5 pm

- Due to the non-instructional break (1st to 5th October), we will split Module-3 in two parts. This week's assignment will be a pen-and-paper assignment. Next week's Assignment-4 will be a MATLAB Only Assignment.
- Please submit this assignment on Moodle

Problem 1: Model Conversion **(3 + 3 points)**

Consider the first-order system from previous assignment: $G(s) = \frac{5}{\tau s+1}$ which can be rewritten in the convenient form:

$$G(s) = \frac{5\alpha}{s+\alpha}, \qquad \alpha = 1/\tau$$

This can be converted into observable canonical form:

$$\frac{dx}{dt} = (-\alpha)x + (5\alpha)u, \qquad y = (1)x$$

The terms in red-color font are the terms A^c, B^c, C^c in the state-space model.

Recall that the value of τ was based on the last digit of your roll number. If the last digit of your roll number is a, then $\tau = 0.5(1+a)$ and $\alpha = 1/\tau$.

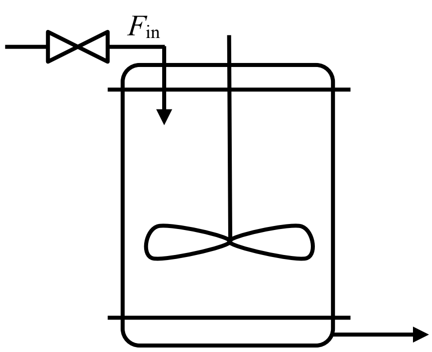
Thus, if your roll number is `CH20D000`, the $a = 0$, $\tau = 0.5$ and $\alpha = 2$.

Thus, if your roll number is `CH20D999`, the $a = 9$, $\tau = 5$ and $\alpha = 0.2$.

Question 1: Discretize the above model (highlighted in yellow) and obtain a discrete-time state space model. Use sampling time of $\Delta t = 0.5$. Please do hand-calculations and avoid using MATLAB.

Question 2: As in Assignment-2, repeat the above for an input delay of 1.5, i.e., $G(s) = \frac{5e^{-1.5\,s}}{\tau s+1}$

Problem 2: Model Linearization **(4 + 4 points)**



Consider a reactor as shown in the adjoining figure. The CVs are liquid level and concentration whereas inlet flowrate, F_{in} is the MV. The model of the form $\dot{x} = f(x,u)$ is given by:

$$\frac{dh}{dt} = \frac{1}{A}\left(F_{in} - \kappa\sqrt{h}\right), \qquad A = 0.2;\ \kappa = 0.5$$

$$\frac{dC}{dt} = \frac{F_{in}}{Ah}(C_{in} - C) - kC^2, \qquad k = 1.5.$$

As we have been doing in earlier assignments, we will choose the inlet concentration based on the last digit of your roll number "a" as: $C_{in} = 5 + 0.2a$. Thus, if your roll number is `CH20D999`, then `Cin = 5+0.2*9 = 6.8`.

Question 3

The system is to be linearized around the steady state. The first step is to compute the steady state values of *h* and *C*. Consider the operating point at $F_{in} = 0.5$ and compute the steady state.

Question 4

Linearize the model around the steady state values, (x_{ss}, u_{ss}) computed in the above problem. You can linearize using Taylor's series expansion:

$$f(x,u) = f(x_{ss}, u_{ss}) + \left.\frac{\partial f}{\partial x}\right|_{(x_{ss},u_{ss})} (x - x_{ss}) + \left.\frac{\partial f}{\partial u}\right|_{(x_{ss},u_{ss})} (u - u_{ss})$$

Defining deviation variables, $x' = x - x_{ss},\ u' = u - u_{ss},\ y' = y - y_{ss}$, obtain the linear state space model of the form:

$$\frac{dx'}{dt} = A^c x' + B^c u', \qquad y' = Ix'$$

Problem 3: MIMO System **(6 points)**

Consider the MIMO system from Assignment-2:

$$G(s) = \begin{bmatrix} \dfrac{2}{40s^2 + 16s + 1} & \dfrac{0.5}{20s^2 + 7s + 1} \\ \dfrac{1.2}{10s^2 + 5s + 1} & \dfrac{1}{36s^2 + 12s + 1} \end{bmatrix}$$

The following steps were performed in MATLAB (you will do something similar next week):

- We used the `tf` function to obtain the above four transfer functions: `Gp11`, `Gp12`, `Gp21`, `Gp22`.
- We then used the `ss` function to obtain the four state-space models: `Gc11`, `Gc12`, `Gc21`, `Gc22`.
- We used `c2d` with $\Delta t = 2$ to obtain discrete-time state space models: `G11`, `G12`, `G21`, `G22`.

$$G_{11} = \frac{2}{40s^2 + 16s + 1} \quad \Rightarrow \quad A_{11} = \begin{bmatrix} 0.42 & -0.27 \\ 0.17 & 0.96 \end{bmatrix}, \qquad B_{11} = \begin{bmatrix} 0.68 \\ 0.097 \end{bmatrix}, \qquad C_{11} = [0 \quad 0.8]$$

$$G_{12} = \frac{0.5}{20s^2 + 7s + 1} \quad \Rightarrow \quad A_{12} = \begin{bmatrix} 0.43 & -0.28 \\ 0.35 & 0.92 \end{bmatrix}, \qquad B_{12} = \begin{bmatrix} 0.35 \\ 0.1 \end{bmatrix}, \qquad C_{12} = [0 \quad 0.4]$$

$$G_{21} = \frac{1.2}{10s^2 + 5s + 1} \quad \Rightarrow \quad A_{21} = \begin{bmatrix} 0.27 & -0.47 \\ 0.30 & 0.86 \end{bmatrix}, \qquad B_{21} = \begin{bmatrix} 0.59 \\ 0.18 \end{bmatrix}, \qquad C_{21} = [0 \quad 0.96]$$

$$G_{22} = \frac{1}{36s^2 + 12s + 1} \quad \Rightarrow \quad A_{22} = \begin{bmatrix} 0.48 & -0.32 \\ 0.18 & 0.96 \end{bmatrix}, \qquad B_{22} = \begin{bmatrix} 0.72 \\ 0.1 \end{bmatrix}, \qquad C_{22} = [0 \quad 0.44]$$

Question 5

Combine the above four SISO state-space models to obtain a single MIMO model of the form

$$x(k+1) = Ax(k) + Bu(k), \qquad y(k) = Cx(k)$$

Hint: Since you have 2 CVs and 2 MVs, $y \in \mathbb{R}^2, u \in \mathbb{R}^2$. When you combine the four models, you will get eight states in the model, i.e., $x \in \mathbb{R}^8$.