Assignment 9

Due: Saturday, 21st November at 5 pm

- Please submit your work on MATLAB Grader. For hand calculation problems, please provide the final answer in the appropriate vector

Problem 1: Pole Placement (Hand Calculations) **(4 points)**

Consider the state-space model from the previous assignment:

$$x(k+1) = \begin{bmatrix} 0.1 * \mathbf{p} & -0.4 \\ 0.1 * \mathbf{q} & 0.25 \end{bmatrix} x(k) + \begin{bmatrix} 0.2 \\ 1.2 \end{bmatrix} \varepsilon(k)$$

$$y(k) = [1 \quad 0] x(k) + \nu(k)$$

where p and q are the last two digits of your roll number. Thus, if your roll number is CH01B2**34**, then $p = \mathbf{3}$, $q = \mathbf{4}$ and the matrix `A=[0.3,-0.4;0.4,0.25];`.

Let $\text{cov}(\varepsilon) = 1$ and $\text{cov}(\nu) = 0.25$.

1. Compute the pole-placement design that gives deadbeat observer, with the result provided in the estimator gain matrix, `K_db`.

2. Compute the pole-placement design that gives an observer with one pole at 0.25 and another at 0.4. Please return the observer gain as a matrix `K_pp`.

Problem 2: Steady State LQR **(6 points)**

3. Compute the steady-state Kalman *filter* gain, K^{∞} and report it matrix `K_kf`.

4. Compute the poles of the filter equation (i.e., eigenvalues of $A - KC$) in vector `Lam_Kf`.

5. Also compute the error covariances P^{∞} and $\bar{P}^{\infty}$ and return them in `P_inf` and `Pbar_inf`, respectively.

Problem 3: Effect of R_1 and R_2 **(4 points)**

6. <u>Low measurement noise</u>: Repeat Problem 2 with the same state noise covariance, but with lower measurement noise, $\text{cov}(\nu) = 2.5 \times 10^{-5}$. Report the estimator gain and poles, `K_kf1` and `Lam_1`, respectively.

7. <u>High confidence on the model</u>: Repeat Problem 2 with lower covariance of state noise, $\text{cov}(\varepsilon) = 0.0001$. Report the results in matrices `K_kf2` and `Lam_2`.

Problem 4: Estimator Simulations

For the example considered in this assignment, we will run some simulation cases and compare the estimator performance. One can generate a realization of $\varepsilon(k)$ and $\nu(k)$ using the

MATLAB command `randn(200,1)`, which yields Gaussian white noise with unit covariance. We will assume the *true state* to be $x(0) = [1 \quad 1]^T$, whereas the initial estimates to be $\hat{x}(0|0) = [0 \quad 0]^T$

Simulations of a dead-beat observer (No marks: Setting up for the next problem)

You had developed a dead-beat observer in Problem 1.

I have written the code for dead-beat observer, where the only task required from you is to enter your observer gain. We will use this observer to compute $\hat{x}(k|k)$. We will also compute error $e(k) = [x(k) - \hat{x}(k|k)]$ and hence the SSE = $\sum_{i=1}^{200}(e^T e)$. Your code will return 2×200 array `Xhat` (i^{th} column will contain $\hat{x}(i|i)$) and scalar `SSE`.

Part-B: Kalman Filter Simulations

You had also computed the steady-state Kalman filter gains, K_∞. Use the Kalman gain and compute $\hat{x}(k|k)$. Plot $x(k)$ and $\hat{x}(k|k)$ for the entire sequence.

8. Return the stored values of $\hat{x}(k|k)$ in a 2×200 array `Xhat`.

9. Compute the error $e(k) = [x(k) - \hat{x}(k|k)]$ and hence compute the SSE = $\sum_{i=1}^{200}(e^T e)$. Return the result in scalar `SSE`.

10. Compare the results of dead-beat observer and Kalman filter

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Related to a Question asked on Nov 11th Live Session: Error Covariances (NOT GRADED)

In Problem-1, you computed two different filters. In Problem-2 also, you computed the filter. The overall one-step prediction error is given by:

$$x_e(k+1) = (A - KC)x_e(k) + \varepsilon(k) + K\nu(k)$$

For each of the estimator designs, use the definition $\bar{P} = E\{x_e x_e^T\}$ to compute the steady state error covariance. Note that as $k \to \infty$, $\bar{P}(k+1) \to \bar{P}(k)$ since all the three estimators are stable.

- Hint: Discrete Lyapunov equation is given by $\Phi X \Phi^T + X + \Gamma = 0$. Check MATLAB help for the function `dlyap`.