• CH5120 Assignment - 2 solutions :-

→ 1.) Roll number :- CH16B001

⇒ $a = 1$

⇒ $\tau = 0.5(a+1) = 1.$

⇒ $\boxed{G_1(s) = \frac{5}{s+1}}$

⇒ $Y(s) = G_1(s)\,U(s)$

⇒ For unit step input.

⇒ $U(s) = \frac{1}{s}$ ⇒ $Y(s) = \frac{5}{s(s+1)}$

⇒ $Y(s) = 5\left[\frac{1}{s} - \frac{1}{s+1}\right]$

⇒ $Y(s) = \frac{5}{s} - \frac{5}{(s+1)}$

⇒ $\boxed{y(t) = \mathcal{L}^{-1}(Y(s)) = 5(1 - e^{-t})}$

Sampling time, $\Delta t = 0.5$

⇒ $s_0 = y(t=0) = 0.$

⇒ $s_1 = y(t = 0.5) = 5(1 - e^{-0.5}) = \underline{1.9673}$

⇒ $s_2 = y(t = 2 \times 0.5) = y(t=1) = \underline{3.1606}$

20 $s_n = y(t = n \times T_s) = y(t = nT_s)$

T_s = sampling time = 0.5

· Following this procedure, we get :-

Index (i)	Step response model parameter (s_i)	Value
1	s_1	1.9673
2	s_2	3.1606
3	s_3	3.8843
4	s_4	4.3233
5	s_5	4.5896
6	s_6	4.7511
7	s_7	4.849
8	s_8	4.9084
9	s_9	4.9444
10	s_{10}	4.9663
11	s_{11}	4.9796
12	s_{12}	4.9876
13	s_{13}	4.9925

→ 2.) Now θ = delay = 0.15

$$\Rightarrow G(s) = \frac{5e^{-0.15s}}{(s+1)}$$

We use the property of Laplace transforms:

$$\boxed{\mathcal{L}[f(t-t_0)] = e^{-st_0}\bar{f}(s)}$$

where $\boxed{\bar{f}(s) = \mathcal{L}(f(t))}$

Using this property :

$$y(t) = \mathcal{L}^{-1}(Y(s)) = \mathcal{L}^{-1}\left(\frac{5e^{-0.15s}}{s(s+1)}\right)$$

$$\boxed{y(t) = 5\left(1 - e^{-(t-0.15)}\right), \quad t \geq 0.15}$$

$$\Rightarrow s_1 = y(t = 0.5) = 1.4765$$

$$\Rightarrow s_2 = y(t = 2 \times 0.5) = y(t = 1) = 2.8629$$

$$\Rightarrow s_3 = y(t = 3 \times 0.5) = y(t = 1.5) = 3.7038$$

$$\Rightarrow s_4 = y(t = 4 \times 0.5) = y(t = 2) = 4.2138$$

$$\Rightarrow \boxed{\begin{aligned} s_n &= y(t = nT_s) && \text{if } nT_s > 0 \\ &= 0 && \text{if } nT_s < 0 \end{aligned}}$$

* Following this procedure :

Index (i)	Step response parameter (s_i)	Value
1	s_1	1.4765
2	s_2	2.8629
3	s_3	3.7038
4	s_4	4.2138
5	s_5	4.5231
6	s_6	4.7108
7	s_7	4.8246
8	s_8	4.8936
9	s_9	4.9355
10	s_{10}	4.9608
11	s_{11}	4.9762
12	s_{12}	4.9856
13	s_{13}	4.9913
14	s_{14}	4.9947
15	s_{15}	4.9968

→ 3) Now $\theta = 1.5 = T_3$

$$G(s) = \frac{5e^{-1.5s}}{(s+1)}$$

$$\Rightarrow \quad y(t) = 5\left(1 - e^{-(t-1.5)}\right), \quad t > 1.5$$

$$\Rightarrow \quad s_n = \begin{cases} 0 & \text{if} \quad nT_3 \leq \theta \\ y(t = nT_3) & \text{if} \quad nT_3 > \theta \end{cases}$$

$$\Rightarrow \quad \boxed{s_1 = s_2 = s_3 = 0}$$

$$\Rightarrow \quad \underline{s_4} = y(t = 0.5 \times 4) = y(t = 2) = \underline{1.9763}$$

$$\Rightarrow \quad \underline{s_5} = y(t = 0.5 \times 5) = y(t = 2.5) = \underline{3.1606}$$

Following this procedure

⇒

Index (i)	Step response parameter (s_i)	Value
4	s_4	1.9673
5	s_5	3.1606
6	s_6	3.8843
7	s_7	4.3233
8	s_8	4.5896
9	s_9	4.7511
10	s_{10}	4.849

• Problem 2 : Impulse Response Model :

⇒ $G(s) = \dfrac{5}{(s+1)}$, $\Delta t = 0.5$

• Unit pulse input : $u(s) = \dfrac{1}{s}(1 - e^{-0.5s})$

⇒ $Y(s) = G(s)\,U(s)$

⇒ $Y(s) = \dfrac{5}{(s+1)} \cdot \dfrac{1}{s}(1 - e^{-0.5s})$

$$Y(s) = \frac{5}{s(s+1)} - \frac{5}{s(s+1)} e^{-0.5s}$$

$$y(t) = \mathcal{L}^{-1}(Y(s)) = 5(1 - e^{-t}) - 5(1 - e^{-(t-0.5)})$$

$\hookrightarrow t \geq 0.5$

⇒ $h_1 = y(t = 0.5) = 5(1 - e^{-0.5}) = 1.9673$

⇒ $h_2 = y(t = 2 \times 0.5) = y(t = 1) = 1.1932$

⇒ $h_3 = y(t = 3 \times 0.5) = y(t = 1.5) = 0.7237.$

⇒ $h_n = y(t = n\Delta t)$

Index (i)	FIR model coefficient (h_i)	Value
1	h_1	1.9673
2	h_2	1.1932
3	h_3	0.7237
4	h_4	0.44
5	h_5	0.2662
6	h_6	0.1615
7	h_7	0.0979
8	h_8	0.0594
9	h_9	0.036
10	h_{10}	0.0218
11	h_{11}	0.0132
12	h_{12}	8.04×10^{-3}
13	h_{13}	4.876×10^{-3}

- Relation between step and FIR coefficients

$$h_n = s_n - s_{n-1}$$

⇒ from problem - 1 → FIR coefficients

$s_1 = 1.9673$ ⇒ $h_1 = s_1 = 1.9673$

$s_2 = 3.1606$ ⇒ $h_2 = s_2 - s_1 = 1.1933$

$s_3 = 3.8843$ ⇒ $h_3 = s_3 - s_2 = 0.7237$

$s_4 = 4.3233$ ⇒ $h_4 = s_4 - s_3 = 0.439$

$s_5 = 4.5896$ ⇒ $h_5 = s_5 - s_4 = 0.2663$

$s_6 = 4.7511$ ⇒ $h_6 = s_6 - s_5 = 0.1615$

Script

Reset MATLAB Documentation

```
% Transfer functions
G11=tf(2,[40,16,1]);
G12=tf(0.5,[20,7,1]);
G21=tf(1.2,[10,5,1]);
G22=tf(1,[36,12,1]);
G=[G11, G12; G21, G22];

% Parameters
ts=2; n=25;

% --- Start typing your code below this line ---
y_11 = step(G11,[0:ts:n*ts]);
y_12 = step(G12,[0:ts:n*ts]);
y_21 = step(G21,[0:ts:n*ts]);
y_22 = step(G22,[0:ts:n*ts]);
S = [];
for i = 1:n
    newmat = [y_11(i+1) y_12(i+1);y_21(i+1) y_22(i+1)];
    S = [S;newmat];
end
disp(S)
```