Assignment 8

Due: Saturday, 14th November at 5 pm

- Please submit your work on MATLAB Grader. For hand calculation problems, please provide the final answer in the appropriate vector

Problem 1: Pole Placement (Hand Calculations) **(6 points)**

Consider the state-space model from the previous assignment:

$$x(k+1) = \begin{bmatrix} 0.1 * \mathbf{p} & -0.4 \\ 0.1 * \mathbf{q} & 0.25 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

where p and q are the last two digits of your roll number. Thus, if your roll number is CH01B2**34**, then $p = \mathbf{3}$, $q = \mathbf{4}$ and the matrix `A=[0.3,-0.4;0.4,0.25];`.

1. Compute the pole-placement design that gives deadbeat controller. Please return the controller gain as a matrix `L_db`.

2. Compute the pole-placement design that gives a controller with one pole at 0.25 and another at 0.4. Please return the controller gain as a matrix `L_pp`.

~~Problem 2~~ Demo Example: Simulations of closed-loop system **(0 points)**

This is now a demo problem and it will not be graded: Consider regulating the system from $x_0 = [1 \quad 0.5]^T$. We will do this for five discrete steps, to obtain $x(1),\ x(2),\ x(3),\ x(4),\ x(5)$. Prefer to do this with hand calculations, though MATLAB is also permitted.

- Using dead-beat controller, compute $x(1)$ to $x(5)$. Please return the results in a 2×5 matrix `Xdb`
- Repeat for the pole placement controller. Please return the results in a 2×5 matrix `Xpp`

Problem 2: Steady State LQR **(6 points)**

3. Using the algebraic Ricatti equation or discrete Lyapunov equations in MATLAB, compute the controller gain if $Q = I$ and $R = 1$. Please report the results as a matrix `L`.

4. What are the controller poles for this controller? Please return them in a 2×1 vector `LQRpoles`.

Problem 3: Effect of Q and R **(8 points)**

5. Aggressive control: Repeat Problem 3 with new weights: $Q = 100I,\ R_2 = 1$. Compute the poles

6. Sluggish control: Repeat again with $Q = I,\ R_2 = 100$. Compute the poles

7. Very aggressive control: Repeat again with $Q = 10^4 I,\ R_2 = 1$. Compute the poles

8. Very sluggish control: Repeat again with $Q = I,\ R_2 = 10^4$. Compute the poles

Problems for Practice (not graded)

In the previous assignments, you had considered the following systems. For practice, you may repeat the pole placement controller and LQR for the following problems. Note that as the number of states increase, it is not immediately clear where one would want to place the closed-loop poles.

- $G(s) = \dfrac{5}{\tau s + 1} e^{-0.15s}, \qquad \tau = \dfrac{1 + a}{2}$ a = Last digit of roll number, $\qquad \Delta t = 0.5$
- $G(s) = \begin{bmatrix} \dfrac{2}{40s^2 + 16s + 1} & \dfrac{0.5}{20s^2 + 7s + 1} \\ \dfrac{1.2}{10s^2 + 5s + 1} & \dfrac{1}{36s^2 + 12s + 1} \end{bmatrix}, \qquad \Delta t = 2$

What to expect in Assignment 9?

The closed loop simulation problem (Problem 2) is now a demo problem. However, similar closed-loop simulation examples will be taken up in Assignment 9.