Assignment 1

Due: Thursday, 24th September at 5 pm

Problem 1: Change of basis **(2 + 2 points)**

Part-1.1: Change of basis for a vector

Let $\mathbf{x} = \begin{bmatrix} a \\ b \end{bmatrix}$, where a and b are the last two digits of your roll number. Thus, if this is CH17B987, then $\mathbf{x} = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$. (If your roll number ends in 00, then use $a = 1$ and $b = 1$).

1. Express this vector $\mathbf{x}$ in terms of new basis, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Part-1.2: Change of bases for linear transformation

Recall that the problem of blending of two streams was a three-input-two-output problem. The inputs were flowrates F_1, F_2, F_{out} and the outputs were h, x_B. The gain matrix is given by:

$$K = \begin{bmatrix} 4 & 2 & 4 \\ 0.5 & 1 & 0 \end{bmatrix}$$

2. How will this matrix change if the domain space is expressed in terms of the following bases:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \qquad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

and the co-domain space is expressed in terms of

$$\mathbf{w}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad \mathbf{w}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Problem 2: Linearly Dependence **(1 + 3 points)**

Let $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \in \mathbb{R}^n$ be linearly independent vectors in n-dimensional space.

3. If the above three vectors are linearly independent, what is/are the possible value(s) of n?

(a) $n = 1$ (b) $n = 2$ (c) $n = 3$ (d) $n = 4$ (e) $n = 5$

4. Consider the three vectors $\mathbf{u} = \mathbf{x}_1 + \mathbf{x}_2$, $\mathbf{v} = \mathbf{x}_1 + \mathbf{x}_3$, $\mathbf{w} = \mathbf{x}_2 + \mathbf{x}_3$. Are the vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ linearly independent? **Prove this**.

Problem 3: Null and Image Spaces **(4 points)**

Consider a matrix $L = \begin{bmatrix} a & b & 4 \\ 0.5 & 1 & 0 \end{bmatrix}$, where a and b are the last two digits of your roll number. If your roll number ends in 00, use $a = 1$ and $b = 1$.

5. Using definition, determine null space and image space of L.

Using MATLAB: Not Graded, For Practice Only

Also confirm the same using SVD (please use MATLAB for SVD).

Problem 4: Eigenvalue Decomposition and Matrix Exponent **(1 + 1 + 1 + 1 points)**

Consider a matrix: $B = \begin{bmatrix} 1 & 0 \\ a & b \end{bmatrix}$, where a and b are the last two digits of your roll number.

Thus, if this is `CH17B987`, then $B = \begin{bmatrix} 1 & 0 \\ 8 & 7 \end{bmatrix}$. (If your roll number ends in 00, use $a = 1$ and $b = 1$).

6. Obtain the characteristic equation and hence compute the eigenvalues of B.

7. Substitute B in its characteristic equation and thus verify Cayley Hamilton Theorem.

8. Perform eigenvalue decomposition for the matrix B

9. Using eigenvalue decomposition, compute matrix exponent e^B

Using MATLAB: Not Graded, For Practice Only

Use MATLAB and compute the matrix exponent of B.

Problem 5: Jordan Decomposition **(2 + 2 points)**

10. Find the eigenvalues of the matrix, $C = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$. Since eigenvalues are repeated, compute eigenvector and generalized eigenvector

11. Representing $C = X\Lambda X^{-1}$ in Jordan canonical form, compute the matrix exponent

Hints for Problems 4 and 5

- Please see the discussion about effect of similarity transform on exponent
- Consider the following rules

$$\exp\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = \begin{bmatrix} e^{\lambda} & 0 \\ 0 & e^{\lambda} \end{bmatrix}, \qquad \exp\left(\begin{bmatrix} \lambda & a \\ 0 & \lambda \end{bmatrix}\right) = \begin{bmatrix} e^{\lambda} & ae^{\lambda} \\ 0 & e^{\lambda} \end{bmatrix}$$