Note: Correctness of the answers (if shown) has not been verified.

2 Papers with 55 minutes for each given.

Paper-1

Question **1**
Complete
Marked out of 1.00

Consider a $n \times n$ matrix A.
The determinant of A is zero and its largest singular value is 5.0.
Please report its smallest singular value.

Question **2**
Complete
Marked out of 1.00
Flag question

The step-response based Model Predictive Control algorithm is known as DMC. The full-form of DMC is: Dynamic Matrix Control

Question **3**
Complete
Marked out of 1.00
Flag question

tf is the MATLAB command to obtain a **Transfer Function** model.

Question **4**
Complete
Marked out of 1.00
Flag question

What is the MATLAB command for solving the quadratic programming problem:

```
deltaU= quadprog (hessian,gradient,C_lhs,c_rhs);
```

Question **5**
Complete
Marked out of 0.80
Flag question

You developed impulse response model in your assignment from a continuous-time model of a system. In this problem, you will do the same, but using a discrete time model. Consider the following system:

$x(k+1) = 0.5x(k) + 0.2u(k)$
$y(k) = x(k)$

The impulse response matrix for this system is given by:

$H = [h_1 \quad h_2 \quad h_3 \quad h_4 \quad h_5 \quad \cdots]^T$

Please give the values of h_1 to h_5 in various questions below.

If you are unable to obtain impulse response coefficients, then report step-response coefficients for half-credit.

Question **10**

Complete

Marked out of 4.00

Flag question

Consider the following continuous time system, but with an input delay of $\theta = 1$:

$\frac{dx}{dt} = -0.3x + 0.5u, \qquad y = x$

Discretize the above with sampling time of $\tau = 1$. The resulting discrete-time model is of the form:

$z(k+1) = Az(k) + Bu(k)$

$y(k) = [1 \quad 0]\, z(k)$

In the above,

A =

0.7408	0.4320
0	0

B =

0
1

Question **11**

Complete

Marked out of 4.00

Flag question

Consider a **two-input-one-output** system. As there are two manipulated inputs, the following two step-tests were run.

First, a unit step was given in the first input u_1 and the following response of y was recorded:

```
Time, k: 1      2      3

  y(k): 0.45   0.54   0.56
```

Next, a unit step was given in the second input u_2 and the following response of y was recorded:

```
Time, k: 1      2      3      4      5

  y(k): 0.55   0.83   0.96   1.03   1.07
```

Note that in this problem, $n_y = 1, n_u = 2, n = 5$. Build the step-response matrix (S) and report it below.

Question **12**

Complete

Marked out of 4.00

Flag question

We wish to design DMC controller for a SISO system of interest, with $p = 4$ as prediction horizon and $m = 1$ as control horizon. I have pre-computed the matrix $\mathcal{S}^U$ (i.e., `bigSu` matrix) as given below:

$$\mathcal{S}^U = \begin{bmatrix} 0.35 \\ 0.5 \\ 0.65 \\ 0.75 \end{bmatrix}$$

Let the weights used in DMC be $\Gamma^y = 9$ and $\Gamma^u = 4$. With the information given above, please compute and report the value of Hessian:

Paper-2

Question **1**
Complete
Marked out of 1.00
Flag question

Consider the following state-space model

$$x(k+1) = \begin{bmatrix} 0.8 & 1 \\ 0.25 & 0.8 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ -1 \end{bmatrix} u(k)$$

Please answer the following questions about stability and controllability of this system.

Compute both the eigenvalues of A. The **larger** of the two eigenvalues is 1.3 .

Question **2**
Complete
Marked out of 1.00
Flag question

Based on these eigenvalues, the system is Unstable

Question **3**
Complete
Marked out of 1.00
Flag question

For the above system,

$$x(k+1) = \begin{bmatrix} 0.8 & 1 \\ 0.25 & 0.8 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ -1 \end{bmatrix} u(k),$$

calculate the rank of the Hautus matrix, $[A - \lambda I, B]$ for both eigenvalues.

The rank of Hautus matrix is 2 (i.e., full rank) for λ = 0.3

Question **4**

Complete

Marked out of 1.00

Flag question

The Hautus matrix is rank-deficient (i.e., rank = 1) for λ =

Question **5**

Complete

Marked out of 4.00

Remove flag

Designing LQG Control

Consider the following first order system

$$x(k+1) = 0.7x(k) + 0.4u(k) + 0.8\varepsilon(k)$$

$$y(k) = x(k) + \nu(k)$$

According to the separation principle, we can design the LQ controller and KF independently, as considered below.

LQ Control: Let's design an aggressive controller by choosing $Q = 1$ and $R = 0$ as the output and input weights, respectively.

1. The infinite-horizong LQ Controller gain is L_∞ = 1.75

2. Please report the closed-loop LQR pole, i.e., λ_{LQR} = 0

Kalman filter

ε and ν Gaussian white noise with covariances **1** and **0.25**, respectively.

3. The steady-state covariance of one-step prediction error, $\bar{P}_\infty$ = 1.09132

4. The steady-state Kalman one-step predictor gain, $\bar{K}_\infty$ = 0.5695

[Hint (optional)]

For infinite-horizon LQR:

$$S_\infty = A^T S_\infty A + Q - A^T S_\infty B(B^T S_\infty B + R)^{-1} B^T S_\infty A$$

For steady-state KF:

$$\bar{P}_\infty = A\bar{P}_\infty A^T + R_1 - A\bar{P}_\infty C^T (C\bar{P}_\infty C^T + R_2)^{-1} C\bar{P}_\infty A^T$$

Question **6**

Complete

Marked out of 1.00

Remove flag

Let $w(k)$ be a stochastic signal, which is given by:

$$w(k+1) = 0.4w(k) + 0.8\varepsilon(k)$$

where $\varepsilon(k)$ is a zero-mean Gaussian white noise sequence with covariance 1.

Please answer the following sub-questions.

3.a. What is the expected value, $E\{w(k)\}$ for large values of k?

Question **7**

Complete

Marked out of 2.00

Remove flag

What is the covaraince of $w(k)$ for large values of k, where

$$w(k+1) = 0.4w(k) + 0.8\varepsilon(k)$$

3.b. In other words, please report

$$E\{w(k)w^T(k)\}, \quad k \to \infty$$

Question **8**
Complete
Marked out of 1.00
Flag question

3.c. Which of the following statements is true about the above signal $w(k)$:

Select one or more:

- [ ] A. $w(k)$ is a **white noise** sequence
- [x] B. $w(k)$ is a **random-walk** noise
- [ ] C. $w(k)$ is a **stationary** noise
- [x] D. $w(k)$ is an **integrating white** noise

Question **9**
Complete
Marked out of 4.00
Flag question

Consider the following state-space model:

$$x(k+1) = Ax(k) + Bu(k-2) + \eta(k)$$
$$y(k) = Cx(k)$$

where, η is an integrating white noise sequence such that $\Delta\eta(k) = \varepsilon(k)$.

We discussed the rate form of MPC/LQG, where augmented state is defined as:

$$z(k) = \begin{bmatrix} \Delta x(k) \\ \Delta u(k-1) \\ \Delta u(k-2) \\ y(k) \end{bmatrix}$$

With this definition: $z(k+1) = \Phi z(k) + \Gamma\Delta u(k) + \Psi\varepsilon(k)$

For output equation: $y(k) = \Xi z(k)$, we can write: `Xi = [0, 0, 0, I];`

Similarly, please provide expressions for: `Phi` (Φ), `Gamma` (Γ) and `Psi` (Ψ).

Note: We aren't looking for exact code, but an understanding of how augmentation works.

Question **10**
Complete
Marked out of 1.00
Flag question

Consider the following state space model

$$x(k+1) = \begin{bmatrix} 1 & -0.5 \\ 0.1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u(k)$$
$$y(k) = \begin{bmatrix} 1 & 0.5 \end{bmatrix} x(k)$$

We designed the following pole-placement controller:

$$L = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

One of the closed-loop pole is $\lambda_1 = 0$. What is the value of the other closed-loop pole?

Question **11**
Complete
Marked out of 1.00
Flag question

Let the initial state be

$$x(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Please report the value of $y(0)$:

Question **12**

Complete

Marked out of 1.00

Flag question

Stating with this initial state:

$$x(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix},$$

we run simulations and obtained outputs $y(k)$ vs. time.

Please report the value of $y(1)$ obtained:

Question **13**

Complete

Marked out of 0.50

Flag question

Please continue the closed loop simulations and report the value of $y(2)$ obtained:

Answer: 0.825

Question **14**

Complete

Marked out of 0.50

Flag question

Please continue the closed loop simulations and report the value of $y(3)$ obtained:

Answer: 0.4125