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CH18B020

CH5120 - MCT - ASSIGNMENT - 5 PROBLEM (1)

(1) Prediction horizon, $p = 5$

Control horizon, $m = 2$

$$\tau = \frac{1+a}{2} = \frac{1+0}{2} = 0.5 \quad (\because \text{Roll no: CH18B020})$$

Step responses for the given system $G(s) = \frac{5}{\tau s+1} e^{-0.15s}$

is obtained as,

$$S = \begin{bmatrix} 2.52 \\ 4.09 \\ 4.66 \\ 4.88 \\ 4.95 \\ 4.99 \\ \vdots \end{bmatrix}$$

S^u will be
$$\begin{bmatrix} S_1 & 0 \\ S_2 & S_1 \\ S_3 & S_2 \\ S_4 & S_3 \\ S_5 & S_4 \end{bmatrix}$$ $p \times m$ matrix.
(5×2 matrix

$$\Rightarrow \quad S^u = \begin{bmatrix} 2.52 & 0 \\ 4.09 & 2.52 \\ 4.66 & 4.09 \\ 4.88 & 4.66 \\ 4.95 & 4.88 \end{bmatrix}$$

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(2) Hessian, $\mathcal{H} = S^{uT} \Gamma^y S^u + \Gamma^u$.

S^u is determined in the previous subpart

$$\Gamma^y = \begin{bmatrix} Q & & 0 \\ & \ddots & \\ 0 & & Q \end{bmatrix}$$

Since it is a SISO system, Q is a scalar
ie $Q = 1$.

$$\Rightarrow \Gamma^y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$ (p×p matrix)

$$\Gamma^u = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$ (m×m, diagonal matrix)

($\because R = 0.5$).

Substituting these values,

$$H = S^{u,T} \Gamma^y S^u + \Gamma^u$$

$\Gamma^y = I \Rightarrow H = S^{uT} S^u + \Gamma^u$.

Doing the matrix multiplication in MATLAB,

H is obtained as

$$\boxed{H = \begin{bmatrix} 93.611 & 86.7606 \\ 86.7606 & 87.2606 \end{bmatrix}}$$

3

a) Input constraints

$u(k) \leq 1$

$\Rightarrow u(k-1) + \Delta u(k) \leq 1$

$\Rightarrow \Delta u(k) \leq 1 - u(k-1)$ — 1

$u(k+1) \leq 1$

$\Rightarrow \Delta u(k+1) + \Delta u(k) \leq 1 - u(k-1)$ — 2

Similarly $u(k) \geq 0$

$\Rightarrow \Delta u(k) \geq -u(k-1) \Rightarrow -\Delta u(k) \leq u(k-1)$ — 3

$u(k+1) \geq 0 \Rightarrow \Delta u(k) + \Delta u(k+1) \geq 0 - u(k-1)$

$\Rightarrow -(\Delta u(k) + \Delta u(k+1)) \leq +u(k-1)$ — 4

~~Similarly~~ b) the ~~output~~ rate constraints can be written

as $\Delta u(k) \leq 0.1$; $\Delta u(k+1) \leq 0.1$

$\cancel{-\Delta u(k) \geq 0.1}$ $\cancel{-\Delta u(k}$

$-\Delta u(k) \leq 0.1$; $-\Delta u(k+1) \leq 0.1$

$\Rightarrow (-)(\Delta u(k) + \Delta u(k+1)) \leq 0.2$

$C \, \Delta U(k)$

$$U(k) = \begin{bmatrix} u(k) \\ u(k+1) \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ -1 & 0 \\ -1 & -1 \end{bmatrix}$$

(4) We can compare the rate & the input (converted to rate) constraints and find their intersection.

~~Rate~~ Input : $\Delta u(k) \le 1 - \cancel{u(k-1)}^{0}$

$\Rightarrow \Delta u(k) \le 1$

and $\Delta u(k) + \Delta u(k+1) \le 1 - \cancel{u(k-1)}^{0}$ — (1)

$\Rightarrow \Delta u(k) + \Delta u(k+1) \le 1$

Rate : $\Delta u(k) \le 0.1$ & $\Delta u(k+1) \le 0.1$

~~⇒ Imp. Sol.~~ $\Rightarrow \Delta u(k) + \Delta u(k+1) \le 0.2$ — (2)

⇒ common area (intersection) of (1) & (2) :

$$\Delta u(k) \le 0.1$$

$$\Delta u(k) + \Delta u(k+1) \le 0.2.$$

Input : $-\Delta u(k) \le u(k-1) \Rightarrow -\Delta u(k) \le 0$

$-(\Delta u(k+1) + \Delta u(k)) \le 0$ — (3)

Rate : $-\Delta u(k) \le +0.1$ & $-\Delta u(k+1) \le 0.1$

$\Rightarrow -(\Delta u(k) + \Delta u(k+1)) \le 0.2$ — (4)

common region of (3) & (4) :

$$-\Delta u(k) \le 0$$

& $-(\Delta u(k) + \Delta u(k+1)) \le 0$

$$C\,U(k) \le \begin{bmatrix} 0.1 \\ 0.2 \\ 0 \\ 0 \end{bmatrix}$$

where $\boxed{CRHS = \begin{bmatrix} 0.1 \\ 0.2 \\ 0 \\ 0 \end{bmatrix}}$

C & U same as in previous part.