# Assignment 6

Due: Thursday, 29th October at 5 pm

**DMC Codes Uploaded on Moodle**

In the class, we will develop step-response MPC (i.e., classical DMC algorithm) for control of a SISO system. The system was given by the following transfer function

$$y(s) = \frac{2.5e^{-7s}}{20s+1}u(s)$$

with sampling-time of $h = 5$. Recall that we chose $n = 24$ number of steps and developed DMC algorithm to control the system at the set-point of $r = 1$. Controller tuning parameters: $m = 4$, $p = 10$, $Q = 1$, $R = 0.04$. The following constraints are implemented: $-0.5 \le u(k) \le 0.5, \ \ |\Delta u(k)| < 0.05$.

You will repeat the simulations for the following SISO models: (i) Case with no disturbance; (ii) extension to measured disturbance case; (iii) extension to plant-model mismatch case.

**Problem 1: SISO MPC** **(6 points)**

In the previous assignments, you developed step-response model, simulated open-loop response and developed matrices for using with DMC algorithm for the first-order system:

$$G(s) = \frac{5}{\tau s + 1}e^{-0.15s}, \qquad \tau = \frac{1+a}{2} \ \ \mathrm{a = Last\ digit\ of\ roll\ number}, \qquad \Delta t = 0.5$$

In this assignment, please modify the uploaded DMC code for controlling your system

Choose $n$ long enough for the system to reach steady state. Typically, $n = (4 \text{ to } 5) \times \tau$. Choose $n$ in the range $16 \le n \le 50$. Controller tuning parameters: $m = 4$, $p = 10$, $Q = 1$, $R = 0.1$. The following constraints are implemented: $-0.4 \le u(k) \le 0.4, \ \ |\Delta u(k)| < 0.025$.

**Problem 2: Extension to Measured Disturbance Case** **(8 points)**

Modify the uploaded SISO system code to simulate the case of measured disturbance:

$$y(s) = \frac{2.5e^{-7s}}{20s+1}u(s) + \frac{0.4e^{-4s}}{10s+1}d(s)$$

Modify the uploaded code to handle the effect of measured disturbance, with

- Case-1: A step-change of 0.5 in the disturbance
- Case-2: A series of step changes, with $d = 0.5$, 1.0 and –0.2 made at $k = 0$, 12, 20

Note that there is no model plant mismatch for this problem.

**Hint: Model with measured disturbances**

Consider a model: $y = G_p u + G_d d$, where $y \in \mathbb{R}^{n_y}$ are process outputs, $u \in \mathbb{R}^{n_u}$ are manipulated inputs and $d \in \mathbb{R}^{n_d}$ are measured disturbances. As we have seen in previous assignments, we can obtain step response coefficient matrices $S$ and $S_d$ from $G_p$ and $G_d$.

The model formulation, thus, becomes:

$$\tilde{Y}(k+1) = M\tilde{Y}(k) + S\,\Delta u(k) + \boldsymbol{S_d\,\Delta d(k)}$$

whereas, the $p$-step prediction equation becomes:

$$\mathcal{Y}_p(k+1) = \mathcal{M}\tilde{Y}(k) + \mathcal{S}^U \Delta\mathcal{U}_m(k) + \mathcal{S}^d \Delta d(k)$$

Following the discussion in the course videos, the Hessian remains same, whereas gradient becomes:

$$g^T = [\mathcal{S}^u]^T \Gamma^y \{\mathcal{M}\tilde{Y}(k) + \boldsymbol{\mathcal{S}^d \Delta d(k)} - \mathcal{R}\}$$

You only need to focus on the two highlighted equations. Other equations remain unchanged. The uploaded code does not have the bolded term involving $\Delta d(k)$. You need to edit the code at only the appropriate locations to: (i) Obtain $S_d$ and $\mathcal{S}^d$ matrices of size $(n.n_y) \times n_d$ and $(p.n_y) \times n_d$;
(ii) edit plant behavior and model predictions to include the effect of $d(k)$; and
(iii) edit the gradient calculation required for obtaining the input moves.

These are the main changes required in the code. Other than these, the idea remains the same.

**Problem 3: Extension to Model-Plant Mismatch Case** **(6 points)**

Now consider the case of Model-Plant Mismatch. Let us assume that the true *Plant* is:

$$y(s) = \frac{2.75e^{-6.2s}}{18.5s + 1} u(s)$$

Modify the uploaded code to handle the case of MPM. Note that the code needs to be changed to include load disturbances so as to ensure steady-state bias correction.

**<u>Hint</u>: Handling unmeasured disturbances or Model-Plant Mismatch**

When there are *unmeasured* disturbances or model-plant mismatch, the model predictions $\tilde{Y}(k)$ differ from the actual response of the system being controlled, $y(k)$. Hence, we need to introduce bias correction, based on the error: $e(k) = y(k) - \tilde{y}(k)$.

Recall that $\tilde{y}(k)$ is nothing but the first $n_y$ elements of $\tilde{Y}(k)$. In this assignment, $n_y = 1$.

As in the previous problem, the Hessian remains same, whereas gradient becomes:

$$g^T = [\mathcal{S}^u]^T \Gamma^y \{\mathcal{M}\tilde{Y}(k) + \mathcal{I}_p \boldsymbol{e(k)} - \mathcal{R}\}$$

where, $\mathcal{I}_p = \begin{bmatrix} I_{n_y \times n_y} \\ \vdots \\ I_{n_y \times n_y} \end{bmatrix} \updownarrow p$ times.

Note that you will need to run two separate step-response models: One for the actual plant (in cyan highlight above) and another for the DMC-model.