Assignment 2

Due: Wednesday, 30th September at 10 pm

- Note that this will be a shorter assignment, due on Wednesday, since 1st to 5th October are non-instructional days. The due date is on Wednesday at 10 pm
- Please submit Problems 1 and 2 on Moodle and Problem 3 on MATLAB Grader

Problem 1: Step Response Model **(2 + 2 + 2 points)**

Consider the following first-order system:

$$G(s) = \frac{5}{\tau s + 1}$$

Hint: The step response of this system is $y(t) = 5\left(1 - e^{-\frac{t}{\tau}}\right)$

1. Compute the step-response model parameters with sampling time of $\Delta t = 0.5$.
2. Please compute the step response parameters if the above system has a delay of $\theta = 0.15$. In other words, $G(s) = \frac{5e^{-0.15s}}{\tau s+1}$.
3. Repeat the above when the delay is 1.5, i.e., $G(s) = \frac{5e^{-1.5\,s}}{\tau s+1}$

   (Note: You are expected to know Laplace transform and its inverse)

Important: The value of τ is based on the last digit of your roll number. If the last digit of your roll number is a, then $\tau = 0.5(1 + a)$. Thus, if your roll number is `CH20D000`, the $a = 0$ and $\tau = 0.5$. If your roll number is `CH20D999`, the $a = 9$ and $\tau = 5$

Problem 2: Impulse Response Model **(2 + 2 points)**

We will now compute the impulse response coefficients for the same system as above, i.e., $G(s) = \frac{5}{\tau s+1}$, in two different ways. Recall that $\Delta t = 0.5$

4. Transfer function of a unit pulse input is $u(s) = \frac{1}{s}\left(1 - e^{-\Delta t.s}\right)$. Please compute the FIR model coefficients as a response of the system to the unit pulse input.
5. In the lectures, we had discussed the relationship between step and impulse response coefficients. Use this to compute the impulse response coefficients from the step-response coefficients computed in Question 1 of Problem 1 above.

----- Continued on Next Page -----

Problem 3: Step Response Model in MATLAB for a MIMO System **(10 points)**

Please submit this problem directly on MATLAB Grader.

Consider the following two-input two-output system

$$G(s) = \begin{bmatrix} \dfrac{2}{40s^2 + 16s + 1} & \dfrac{0.5}{20s^2 + 7s + 1} \\ \dfrac{1.2}{10s^2 + 5s + 1} & \dfrac{1}{36s^2 + 12s + 1} \end{bmatrix}$$

Let the sampling interval $\Delta t = 2$ and number of step-response coefficients be $n = 25$.

In this problem, we will compute the step response coefficients for each of the four transfer functions, $G_{ij}(s)$ with sampling interval of $\Delta t = 2$. Let $S_{i,j}(k)$ represent the step response of the i^{th} output in response to the j^{th} input at the time instance k. For example, step response coefficients for the transfer function G_{21} are $\{S_{2,1}(1), S_{2,1}(2), \cdots, S_{2,1}(n)\}$. Choose $\Delta t = 2$ and $n = 25$ for this problem.

Compute the step response coefficients and build the step response matrix as follows:

$$S = \begin{bmatrix} \begin{bmatrix} S_{1,1}(1) & S_{1,2}(1) \\ S_{2,1}(1) & S_{2,2}(1) \end{bmatrix} \\ \vdots \\ \vdots \\ \begin{bmatrix} S_{1,1}(n) & S_{1,2}(n) \\ S_{2,1}(n) & S_{2,2}(n) \end{bmatrix} \end{bmatrix}$$

Write a MATLAB code to compute the S matrix given above. Please ensure that the sampling interval is `ts=2` and number of steps is `n=25`. Report the result in 50×2 matrix `S`.