Assignment 7

Due: Saturday, 7th November at 5 pm

Problem 1: Multiple Choice Questions with Explanation **(5 points)**

1. Consider the system: $\frac{dx}{dt} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$

Compute the eigenvalues of the `Ac` matrix and comment whether it is stable, unstable or marginally stable system. (Although this was not covered in the class, please refer either to your UG material or any online reference to answer this question.)

2. Convert the above system into discrete-time model $x_{k+1} = Ax_k + Bu_k$ with sampling interval $h = 0.2$. From the eigenvalues of `A`, comment if the system is stable, unstable or marginally stable.

3. Compute the controllability Grammian and comment whether the system is controllable.

4. Is the system stabilizable? Why or why not?

5. In the previous assignment, you considered the transfer function model:

$$G(s) = \frac{5}{\tau s + 1} e^{-0.15s}, \qquad \tau = \frac{1 + a}{2} \quad \text{a} = \text{Last digit of roll number}, \qquad \Delta t = 0.5$$

You considered unit step response of the model in the previous assignment. Based on the step response of the system, explain whether the system is stable, unstable or marginally stable.

Problem 2: Stability and System Response **(1+1+3 points)**

Consider the following system

$$x(k+1) = \begin{bmatrix} 0.1 * \mathbf{p} & -0.4 \\ 0.1 * \mathbf{q} & 0.25 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

where p and q are the last two digits of your roll number. Thus, if your roll number is CH01B2**34**, then $p = \mathbf{3}$, $q = \mathbf{4}$ and the matrix `A=[0.3,-0.4;0.4,0.25];`.

6. Please comment whether the system is asymptotically stable or not

7. Is the system controllable?

8. Starting with $x_0 = [1 \quad 1]^T$, compute $x(10)$ for autonomous system (i.e., $u = 0$).

Problem 3: Controllability, Observability and Subspaces **(10 points)**

Consider the following state space system

$$x_{k+1} = \begin{bmatrix} 0.65 & 0.1 & 0 & -0.45 \\ 0.35 & 0.4 & 0.3 & -0.15 \\ 0 & 0.25 & 0.35 & -0.2 \\ -0.2 & -0.05 & -0.15 & 0.4 \end{bmatrix} x_k + \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \end{bmatrix} u_k$$

$$y_k = \begin{bmatrix} 2 & 0 & 0 & -2 \end{bmatrix} x_k$$

9. Compute the controllability matrix $\mathcal{W}^c$. Compute its rank. Is the system controllable?

10. Likewise, compute the observability matrix, $\mathcal{W}^o$. Compute its rank. Is the system observable?

11. Apply the Hautus condition for controllability. Is the system controllable? Is it stabilizable?

12. Apply the Hautus condition for observability. Is the system observable?

13. Coordinate Transform: One of the useful transformations is to split the system into controllable and uncontrollable subspaces.

For this purpose, perform *singular value decomposition* on the $\mathcal{W}^c = U\Sigma V^T$. Let U be the transformation matrix. Use the coordinate transformation, $x = Uz$, and compute the new matrices $\hat{A}$ and $\hat{B}$. Based on the matrices $\hat{A}$ and $\hat{B}$ obtained after the coordinate transform, please comment about what controllability / stabilizability physically means for this system?