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CH5120 - Modern Control Theory
Assignment - 8 - Solutions

P1: Pole Placement (Hand Calculations)

CH16B004 $\longrightarrow$ $A = \begin{bmatrix} 0 & -0.4 \\ 0.4 & 0.25 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$u(k) = -Lx(k) \Rightarrow x(k+1) = (A - BL)x(k)$

Let $L = [l_1 \ \ l_2] \Rightarrow A - BL = \begin{bmatrix} 0 & -0.4 \\ 0.4 & 0.25 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} [l_1 \ \ l_2]$

$= \begin{bmatrix} -l_1 & -0.4 - l_2 \\ 0.4 & 0.25 \end{bmatrix}$

1. Deadbeat controller : $\Rightarrow$ eigenvalues of $(A - BL) = 0, 0$

Char eqn of $(A - BL) = \begin{vmatrix} -l_1 - \lambda & -0.4 - l_2 \\ 0.4 & 0.25 - \lambda \end{vmatrix} = 0$

$= (\lambda - 0.25)(\lambda + l_1) + (0.4 + l_2)(0.4) = 0.$

$= \lambda^2 + (-0.25 + l_1)\lambda + (0.4 l_2 - 0.25 l_1 + 0.16) = 0$

Since poles are $0, 0 \Rightarrow \lambda^2 = 0$ is the char eqn of $A - BL$.

Comparing coeffs. $\Rightarrow$ $-0.25 + l_1 = 0$

& $0.4 l_2 - 0.25 l_1 + 0.16 = 0$

$\Rightarrow l_1 = -0.25$ & $l_2 = -0.24375$

$$\therefore \boxed{L_{db} = [-0.25 \quad -0.24375]}$$

2) Pole placement controller : Poles $\Rightarrow$ 0.25 & 0.4

Char eqn of $(A - BL) = \lambda^2 + (-0.25 + l_1)\lambda + (0.4 l_2 - 0.25 l_1 + 0.16) = 0$

Given the poles, Char eqn
$$= \lambda^2 - (0.25 + 0.4) + 0.25 \times 0.4 = 0$$
$$= \lambda^2 - 0.65\lambda + 0.1 = 0$$

Comparing coeffs : $-0.25 + l_1 = -0.65$
$\quad$ & $0.4 L_2 - 0.25 L_1 + 0.16 = 0.1$

$$\Rightarrow l_1 = -0.4 \quad \& \quad l_2 = -0.4$$

$$\Rightarrow \boxed{L_{pp} = [-0.4 \quad -0.4]}$$

P2 Steady state LQR : L_∞ & Controller poles were
$[Q = I_2, \; R = 1]$ calculated using dare() & dlqr() in MATLAB

In this case,
$$\boxed{\begin{array}{l} L_\infty = [0.1318, \; -0.2985] \\ \text{\& poles are : } 0.0591 \pm 0.355i \end{array}}$$

Date: __/__/____

3

P3) Effect of Q & R: Sol'n in given MATLAB code (Using dlqr())

(NOT GRADED) Recursive LQR:

$$S(p-1) = A^T S(p) A + Q - A^T S(p) B \left(B^T S(p) B + R \right)^{-1} B^T S(p) A$$

This recursive formula may be used till convergence to get Steadystate ~~S~~ values or S_∞.