Assignment-5

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March 16, 2022

Question-1

Part a)

$$
\begin{aligned}
d(T(f),T(g)) &= max_{x1[0,1]}|T(f)-T(g)| && (1)\\
&= max_{t1[0,1]}|\int_0^t (f(s)-g(s))\,ds| && (2)\\
&\leq max_{t1[0,1]}\int_0^t |f(s)-g(s)|\,ds \text{took mod inside the integral} && (3)
\end{aligned}
$$

Let $d(f,g) = max_{x1[0,1]}|f(x)-g(x)| = |f(x1)-g(x1)|$ and since the term inside the integral is always positive, we can do the integration upto its upper limit.

$$
\begin{aligned}
&\implies d(T(f),T(g)) \leq \int_0^1 |f(x1)-g(x1)|\,ds && (4)\\
&\implies d(T(f),T(g)) \leq d(f,g) * 1 && (5)\\
&\implies d(T(f),T(g)) \leq d(f,g) && (6)
\end{aligned}
$$

So a simple example that will hold the equality is f = 1 and g = 0. LHS and RHS both will be equal to 1.

Part b)

In equation 4, if we replace 1 with 0.5,

$$
\begin{aligned}
&\implies d(T(f),T(g)) \leq \int_0^0 .5|f(x1)-g(x1)|\,ds && (7)\\
&\implies d(T(f),T(g)) \leq 0.5 * d(f,g) && (8)\\
& && (9)
\end{aligned}
$$

So, T is a contraction with $\rho = 0.5$

Question-2

Part a)

$$
\begin{aligned}
|f(x)-f(y)| &= |\frac{1}{1+x} - \frac{1}{1+y}| && (10)\\
\implies |f(x)-f(y)| &\leq |\frac{y-x}{(1+x)(1+y)}| && (11)\\
\implies |f(x)-f(y)| &\leq d(x,y)\frac{1}{(1+x)(1+y)} (\because x,y \geq 0) && (12)\\
\implies \rho &= max_{x,y}\frac{1}{(1+x)(1+y)} && (13)
\end{aligned}
$$

We note that we get $\rho = 1$ as the solution. So f is not a contraction.

Part b)

$$
\begin{aligned}
f(x^*) &= x^* \quad (14) \\
\implies \frac{1}{1+x^*} &= x^* \quad (15) \\
\implies (x^*)^2 + x^* - 1 &= 0 \quad (16) \\
\implies x^* &= \frac{-1 \pm \sqrt{5}}{2} \quad (17)
\end{aligned}
$$

Since, $x^* \in [0, 1]$, we have a unique fixed point, $x^* = \frac{-1+\sqrt{5}}{2}$

Question-3

Part a)

Part b)

Question-4

Part a)

Part b)

Part c)

Question-5

Question-6

Part a)

Part b)

Question-7

Question-8

Question-9

References

- Students discussed with:
  1. Arvind Ragghav ME18B086
  2. Karthik Srinivasan ME18B149
- Course notes used:
  1. Class notes
- Hassan Khalil