EE6415: Nonlinear Systems Analysis

Jan-May, 2022

Tutorial 4

Instructions :

- **Submit on or before 11:59 PM, 07/03/2022**
- Any kind of plagiarism will be dealt with severely. Acknowledge any and every resource used, including any coursemates you may have discussed with.
- Include any plots/images you deem necessary
- Your submission must be named "RollNo.pdf". For example, if your roll number is EE17B158, your submission must have the name "EE17B158.pdf".
- Your submissions must be made on moodle. Any emailed submissions will not be accepted.
- It is required that you use LaTeX for writing your report. A template had been provided along with the first assignment for the same.

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1. Let X be the set of all ordered triples of zeros and ones. Show that X consists of eight elements and a metric d on X is defined by $d(x, y) =$ number of places where x and y have different entries. For example, $d(010, 111) = 2$. (This metric is called Hamming distance).

2. Show that the function d on the set X defined by

$$d(x, y) = \int_a^b |x(t) - y(t)| dt$$

is a metric, where X is the set of all real-valued functions $x, y, \cdots$ which are functions of an independent real variable t and are defined and continuous on a given closed interval $J = [a, b]$.

3. Consider the space of all sequences $x = (\zeta_i)$, $y = (\eta_i)$. Prove that

$$d(x, y) = \sum_{j=1}^{\infty} \frac{1}{2^j} \frac{|\zeta_j - \eta_j|}{1 + |\zeta_j - \eta_j|}$$

is a metric. Further, show that

$$d_2(x, y) = \sum_{j=1}^{\infty} r_j \frac{|\zeta_j - \eta_j|}{1 + |\zeta_j - \eta_j|}$$

is a metric for any sequence (r_j) for which every element is positive, and $\sum r_j$ converges.

4. Show that $d(x, y) = \sqrt{|x - y|}$ is a metric on the set of Real Numbers.

5. Show that a Cauchy sequence is bounded. Is boundedness of a sequence in a metric space sufficient for the sequence to be Cauchy? Convergent?

6. Let d be a metric on X. Determine all constants k such that

   (a) kd

(b) $d + k$

is a metric on X

7. Does $d(x, y) = (x - y)^2$ define a metric on the set of all real numbers?

8. The triangle inequality has several useful consequences. Show that the following inequalities are true for any metric d

   (a) $|d(x, y) - d(z, w)| \leq d(x, z) + d(y, w)$

   (b) $|d(x, z) - d(y, z)| \leq d(x, y)$

9. Consider the normed linear vector space of rational numbers Q with norm $\|x\| = |x|$. For each of the sequences an given next, find whether a_n is (a) convergent in Q (b) a Cauchy sequence.

   (a) $a_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n}$

   (b) $a_n = \frac{F_{n+1}}{F_n}$ where F_n is the n^{th} Fibonacci Number.