EE6415: Nonlinear Systems Analysis

Jan-May, 2022

Tutorial 5

Instructions :

- **Submit on or before 11:59 PM, 15/03/2022**
- Any kind of plagiarism will be dealt with severely. Acknowledge any and every resource used, including any coursemates you may have discussed with.
- Include any plots/images you deem necessary
- Your submission must be named "RollNo.pdf". For example, if your roll number is EE17B158, your submission must have the name "EE17B158.pdf".
- Your submissions must be made on moodle. Any emailed submissions will not be accepted.

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1. Consider the discrete time system $x[k+1] = Ax[k]$ with

$$A = \begin{bmatrix} 1.3 & 0.2 & 0.2 \\ -1 & 0.4 & -0.4 \\ -0.4 & -0.2 & 0.7 \end{bmatrix}$$

Given

$$Q = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

for the discrete time Lyapunov equation $A^T PA - P = -Q$

(a) Find P
(b) Find eigenvalues of P
(c) Comment on the stability of the system

2. Consider the system

$$\dot{x}(t) = \begin{bmatrix} 2.8 & 9.6 \\ 9.6 & -2.8 \end{bmatrix} x(t)$$

Given

$$Q = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix},$$

find P for the Lyapunov equation $A^T P + PA = -Q$.

(a) Write the matrix P, is it unique?
(b) Find the expression (or value) for the eigenvalues of P.

(c) Comment on the stability of x = 0 of the system.

3. Consider a second order system given as

$$\ddot{\omega} + g(\omega)\dot{\omega} + \omega = 0$$

with equilibrium point $\omega = \dot{\omega} = 0$. Find the conditions on the g(0) such that convergence to the origin can be guaranteed ?

4. Comment on the stability of the origin for the following system:

(a)

$$\begin{aligned}
\dot{x}_1 &= x_2(1 - x_1^2) \\
\dot{x}_2 &= -(x_1 + x_2)(1 - x_1^2)
\end{aligned}$$

(b)

$$\begin{aligned}
\dot{x}_1 &= x_1^3 + x_1^2 x_2 \\
\dot{x}_2 &= -x_2 + x_2^2 + x_1 x_2 - x_1^3
\end{aligned}$$

(c)

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -x_1^3 - x_2^3
\end{aligned}$$

5. Consider the system

$$\begin{aligned}
\dot{x}_1 &= x_1 - x_1^3 + x_2 \\
\dot{x}_2 &= 3x_1 - x_2
\end{aligned}$$

(a) Find all equilibrium point of the system.

(b) Using linearization, study the stability of each equilibrium point.

(c) Using quadratic Lyapunov functions, estimate the region of attraction of each asymptotically stable equilibrium point. Try to make your estimate as large as possible.

(d) Construct the phase portrait of the system and show on it the exact regions as attraction as well as your estimates

6. Consider the system

$$\begin{aligned}
\dot{x}_1 &= h(t)x_2 - g(t)x_1^3 \\
\dot{x}_2 &= -h(t)x_1 - g(t)x_2^3
\end{aligned}$$

where $h(t)$ and $g(t)$ are bounded, continuously differentiable functions and $g(t) > 0$, for all $t \geq 0$.

(a) Is the equilibrium point $x = 0$ uniformly asymptotically stable?

(b) Is it exponentially stable?

(c) Is it globally uniformly asymptotically stable?

(d) Is it globally exponentially stable?

7. Suppose the set M in LaSalle's theorem consists of a finite number of isolated points. Show that $\lim_{x\to\infty} x(t)$ exists and equals one of these points.

8. Consider the function given below:

$$V(x) = \frac{(x_1 + x_2)^2}{1 + (x_1 + x_2)^2} + (x_1 - x_2)^2$$

Is this a valid Lyapunov candidate function to deduce the global asymptotic stability of an equilibrium point? Justify your answer by:

(a) Proving/Disproving that it is radially unbounded.

(b) Plotting the level sets, and inferring from the plot.