EE6415: Nonlinear Systems Analysis

Jan-May, 2022

End-Semester Examination

Instructions :

- **Submit on moodle on or before 02/05/22, 11:59 pm**
- All questions carry **10 marks** each.
- It is required that you use LaTeX for writing your report.
- Include any plots/images you deem necessary. (A good plot is worth a thousand words). You can learn how to make good MATLAB plots here.
- Your report("RollNo.pdf"), along with the codes must be submitted as a single .zip file.
- Your submission must be named "RollNo.zip". For example, if your roll number is EE17B158, your submission must have the name "EE17B158.zip".
- Any kind of malpractice will be dealt with severely. You have enough time to figure things out on your own..!

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1. Write a MATLAB code to compute the matrix P as defined in the the (KYP-lemma)PR-lemma, and to check whether a system is (strictly) positive-real.
   Consider the transfer functions given below:

$$G_0(s) = \frac{10}{s+4}$$

$$G_1(s) = \frac{4}{(s+1)(0.5s+1)(\frac{s}{3}+1)}$$

$$G_2(s) = \frac{s+2}{s^2+2s+1}$$

$$G_3(s) = \frac{s^2+8s+15}{s^2+6s+8}$$

   (a) Analyse the passivity of G_1, G_2, G_3 using the code you have written.

   (b) Using your code, check which combination of these systems in a negative feedback interconnection give (i) a passive system, (ii) a strictly passive system. (Check all the 6 combinations with G_1, G_2, G_3)

   (c) Check if the system G_iG_j (cascaded) is passive, for $i, j \in \{1, 2, 3\}, i \neq j$.

   (d) How can you validate whether the code that you have written is correct?
   Hint: YALMIP, Passivity.

2. Consider the model of a single-link manipulator with flexible joints:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{-MgL}{J_1} sin(x_1) - \frac{k}{J_1}(x_1 - x_3) \\ x_4 \\ \frac{k}{J_2}(x_1 - x_3) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_2} \end{bmatrix} u$$

where J_1, J_2 are the inertia constants, M the mass of the arm, k the torsional spring constant, and L the length of the arm. x_1 and x_3 are the states corresponding to the flexible link angles. Take all constants M, L, J_1, J_2, k as unity and $g = 10\ m/s^2$

(a) Design an output function such that the zero dynamics of the system is of the order 0,1,2 and 3.

(b) Is the system fully feedback-linearizable(with appropriate output)? If yes, design a full-state feedback-linearization control law such that the output tracks a reference $r(t) = \sin(t)$. If no, then do the partial-feedback linearization of the system, with the output tracking the reference $r(t) = \sin(t)$. Simulate the system using MATLAB and plot the state trajectories w.r.t. time, and also the zero-dynamics(if present).

(c) Assume the output measured is the angle x_3. Is the system minimum phase? Justify your answers by appropriate analysis(Lyapunov) and plots(state trajectories vs time, or phase portraits).

3. Consider the second-order system below:

$$\begin{aligned}
\dot{x}_1 &= x_1 + (1-a)x_2 \\
\dot{x}_2 &= bx_2^2 + x_1 + u \\
y &= x_1
\end{aligned}$$

It is known that the value of $a = 0.5$ and $1 \le b \le 2$. Use a sliding-mode controller to stabilize the system. Plot the trajectory on the phase plane $x_2(t)$ vs $x_1(t)$ and also the control signal $u(t)$ vs time. Is there chattering in the control signal? If yes introduce a boundary layer to eliminate chattering and plot the control signal as well as the trajectories in the phase plane $x_2(t)$ vs $x_1(t)$.

Hint: Diffeomorphism(Change of coordinates).

4. Consider the nonlinear affine system

$$\begin{aligned}
\dot{x} &= f(x) + g(x)u \\
y &= h(x)
\end{aligned}$$

Does the relative degree and zero dynamics remain invariant under a feedback transformation of the form $u = \alpha(x) + \beta(x)v$. Based on this result, comment whether the below systems can be made passive via feedback. Justify your answers

(a)

$$\begin{aligned}
\dot{x} &= \begin{bmatrix} x_3 - x_2^2 \\ -x_2 \\ x_1^2 - x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} u \\
y &= x_1
\end{aligned}$$

(b)

$$\begin{aligned}
\dot{x} &= \begin{bmatrix} x_1^3 \\ \cos x_1 \cos x_2 \\ x_2 \end{bmatrix} x + \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \\
y &= x_2
\end{aligned}$$

(c) Based on the above two observations, check if one can design an appropriate passivity based control law that asymptotically stabilizes the system to its equilibrium point

$$\dot{x}_1 = x_1^2 x_2$$
$$\dot{x}_2 = u$$

Hint: Choose a quadratic storage function and identify an output function $y = h(x)$ that makes the system passive via appropriate state feedback

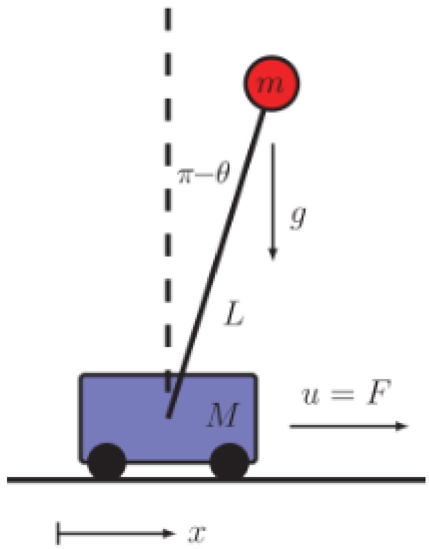

Figure 1: Cart-pendulum

5. Consider the cart-pendulum system shown in figure 1. The dynamics of the system is given by:

$$\dot{x} = v$$
$$\dot{v} = \frac{-m^2L^2gcos(\theta)sin(\theta) + mL^2(mL\omega^2sin(\theta) - \delta v) + mL^2u(t)}{mL^2(M + m(1 - cos^2(\theta)))}$$
$$\dot{\theta} = \omega$$
$$\dot{\omega} = \frac{(m + M)mgLsin(\theta) - mLcos(\theta)(mL\omega^2sin(\theta) - \delta v) - mLcos(\theta)u(t)}{mL^2(M + m(1 - cos^2(\theta)))}$$

Where $x(t)$ is the position of the cart, $v(t)$ the cart velocity, $\theta(t)$ the pendulum angle measured as shown in the figure, and $\omega(t)$ the pendulum angular velocity. M, m, L, g, δ are the cart mass, pendulum mass, pendulum length, acceleration due to gravity, and the damping constant respectively. Assume we have sensor that measures the pendulum angle θ. Take $m = 1\ Kg,\ M = 5\ Kg,\ L = 2\ m,\ g = -10m/s^2,\ \delta = 1$;

(a) What is the relative degree of the system?

(b) Feedback-linearize the system (partially or fully) and find the linearizing control-law u(t).

(c) Does the system have any zero-dynamics? Is the system minimum-phase?

(d) Simulate the system dynamics with the control law that you have found, and plot the trajectories of $x(t), v(t), \theta(t), \omega(t)$ vs time and report your inferences.

6. Consider the second order nonlinear system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 + x_2(1 - x_1^2 - x_2^2)$$

(a) Discuss the stability of the origin.

(b) What happens to the trajectories that do not start from the origin?
Hint: Use Lyapunov stability theory and LaSalles principle for the analysis