EE6415: Nonlinear Systems Analysis

Jan-May, 2022

Tutorial 2

Instructions :

- **Submit on or before 11:59 PM, 13/02/2022**
- You have to turn in the well-documented code along with a detailed report of the results of the experiments. The code must be in a separate file(which can be directly run in matlab), and not a part of the report.
- Any kind of plagiarism will be dealt with severely. Acknowledge any and every resource used, including any coursemates you may have discussed with.
- Include any plots/images you deem necessary
- Your submission must be named "RollNo.pdf". For example, if your roll number is EE17B158, your submission must have the name "EE17B158.pdf".
- Your submissions must be made on moodle. Any emailed submissions will not be accepted.
- It is required that you use LaTeX for writing your report. A template has been provided along with this assignment for the same.

Identify the equilibrium point(s) for each of the following, and linearize the systems about the equilibrium point(s).

1.

$$\dot{x} = y + y^2$$
$$\dot{y} = -x + \frac{1}{5}y - xy + \frac{6}{5}y^2$$

2.

$$\dot{r} = r(1 - r^2)$$
$$\dot{\theta} = 1 - cos(\theta)$$

3. Find the equilibrium point(s), Explain how the system behaves locally around the equilibrium point(s).

$$\dot{x} = sin(y)$$
$$\dot{y} = x - x^3$$

4. (Leftists, rightists, centrists) Vasquez and Redner (2004, p. 8489) mention a highly simplified model of political opinion dynamics consisting of a population of leftists, rightists, and centrists. Let x, y, z represent the fraction of the population of leftists, rightists and centrists respectively. The leftists and rightists never talk to each other; they are too far apart politically to even begin a dialogue. But they do talk to the centrists, – this is how opinion change occurs. The population dynamics can be modelled as given below:

$$\begin{aligned}\dot{x} &= rxz \\ \dot{y} &= ryz \\ \dot{z} &= -rxz - ryz\end{aligned}$$

where $r \in \mathbb{R}\backslash\{0\}$.

Linearize around the fixed point(s) and explain how the population behaves for $r > 0$ and for $r < 0$.

5. A simple model of a satellite of unit mass moving in a plane can be described by the following equations of motion in polar coordinates:

$$\begin{aligned}\ddot{r}(t) &= r(t)\dot{\theta}^2(t) - \frac{\beta}{r^2(t)} + u_1(t) \\ \ddot{\theta}(t) &= -\frac{2\dot{r}(t)\dot{\theta}(t)}{r(t)} + \frac{u_2(t)}{r(t)}\end{aligned}$$

Linearize the system around $u^* = \begin{bmatrix} u_1^* \\ u_2^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and the trajectory $\begin{bmatrix} r^* \\ \dot{r}^* \\ \theta^* \\ \dot{\theta}^* \end{bmatrix} = \begin{bmatrix} r_0 \\ 0 \\ \omega_0 t + \theta_0 \\ \omega_0 \end{bmatrix}$ where $\omega_0 = \sqrt{\frac{\beta}{r_0^3}}$

6. Consider the nonlinear system

$$\begin{aligned}\dot{x} &= y + x(x^2 + y^2 - 1)\sin(\frac{1}{(x^2 + y^2 - 1)}) \\ \dot{y} &= -x + y(x^2 + y^2 - 1)\sin(\frac{1}{(x^2 + y^2 - 1)})\end{aligned}$$

Without solving the above equations explicitly, show that the system has infinite number of limit cycles.

7. The system

$$\begin{aligned}\dot{x}_1 &= -x_1 - \frac{x_2}{\ln\sqrt{x_1^2 + x_2^2}} \\ \dot{x}_2 &= -x_2 + \frac{x_1}{\ln\sqrt{x_1^2 + x_2^2}}\end{aligned}$$

has an equilibrium point at the origin

(a) Linearize the system about the origin, and show that the origin is a stable node.

(b) Plot the phase portrait of the system about the origin, and show that the origin is a stable focus

(c) Explain the discrepancy between the two results.

8. For the following systems, show that there exists a limit cycle

   (a) $\ddot{y} + y = \epsilon \dot{y}(1 - y^2 - \dot{y}^2)$

   (b) $\dot{x}_1 = x_2$, $\dot{x}_2 = -x_1 + x_2(2 - 3x_1^2 - 2x_2^2)$

   (c) $\dot{x}_1 = x_2$, $\dot{x}_2 = -x_1 + x_2 - 2(x_1 + 2x_2)x_2^2$

9. The following model is used to analyze the interaction between inhibitory and excitatory neurons in a biological system. In its simplest form, x_1 is the output of the excitatory neuron, and x_2 is the output of the inhibitory neurons.

$$\dot{x}_1 = -\frac{1}{\tau}x_1 + \tanh(\lambda x_1) - \tanh(\lambda x_2)$$
$$\dot{x}_2 = -\frac{1}{\tau}x_2 + \tanh(\lambda x_1) + \tanh(\lambda x_2)$$

   Show that, when $\lambda\tau > 1$, the system has a periodic orbit