EE6415: Nonlinear Systems Analysis

Jan-May, 2022

Tutorial 3

Instructions :

- **Submit on or before 11:59 PM, 21/02/2022**
- You have to turn in the well-documented code along with a detailed report of the results of the experiments. The code must be in a separate file(which can be directly run in matlab), and not a part of the report.
- Any kind of plagiarism will be dealt with severely. Acknowledge any and every resource used, including any coursemates you may have discussed with.
- Include any plots/images you deem necessary
- Your submission must be named "RollNo.zip". For example, if your roll number is EE17B158, your submission must have the name "EE17B158.zip".
- Your submissions must be made on moodle. Any emailed submissions will not be accepted.
- It is required that you use LaTeX for writing your report. A template has been provided along with this assignment for the same.

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1. Find and classify the bifurcation for the system:

$$\dot{x} = y - 2x$$
$$\dot{y} = \mu + x^2 - y$$

2. Classify the bifurcation for the system below, and draw its bifurcation diagram:

$$\dot{x} = \mu x + y + sin(x)$$
$$\dot{y} = \mu + x - y$$

3. Classify the bifurcations in each of the following systems as μ varies

   (a) $\dot{x}_1 = x_2; \quad \dot{x}_2 = \mu(x_1 + x_2) - x_2 - x_1^3 - 3x_1^2 x_2$
   (b) $\dot{x}_1 = x_2; \quad \dot{x}_2 = \mu - x_2 - x_1^2 - 2x_1 x_2$
   (c) $\dot{x}_1 = x_2; \quad \dot{x}_2 = \mu(x_1 + x_2) - x_2 - x_1^3 + 3x_1^2 x_2$

4. The following system is a model for a genetic control system.

$$\dot{x} = -ax + y$$
$$\dot{y} = \frac{x^2}{1 + x^2} - by$$

   where $a, b > 0$. Analyze the bifurcations that occur as the parameter "a" is varied and find the critical value of "a" in terms of the parameter b.

5. Consider the system

$$\dot{x}_1 = ax_1 - x_1x_2$$
$$\dot{x}_2 = bx_1^2 - cx_2$$

where a, b, c are positive constants, and $c > a$. Let $D = \{x \in \mathbb{R}^2 | x_2 \geq 0\}$.

(a) Show that every trajectory starting in D stays in D for all future time.

(b) Show that there are no periodic orbits in D.

6. For each of the following systems, show that the system has no limit cycles

(a) $\dot{x}_1 = -x_1 + x_2; \quad \dot{x}_2 = g(x_1) + ax_2 \quad a \neq 1$

(b) $\dot{x}_1 = -x_1 + x_1^3 + x_1x_2^2; \quad \dot{x}_2 = -x_2 + x_2^3 + x_1^2x_2$

(c) $\dot{x}_1 = 1 - x_1x_2^2; \quad \dot{x}_2 = x_1$

7. A model that is used to analyze chemical oscillators is given by

$$\dot{x}_1 = a - x_1 - \frac{4x_1x_2}{1 + x_1^2}, \quad \dot{x}_2 = bx_1\left(1 - \frac{x_2}{1 + x_1^2}\right)$$

where x_1, x_2 are dimensionless concentrations of certain chemicals, and a, b are positive constants.

(a) Prove that the system has a periodic orbit, when $b < 3a/5 - 25/a$

(b) Find and classify the bifurcations that occur as b varies, with a fixed a.

8. Consider the system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -(2b - g(x_1))ax_2 - a^2x_1$$

where a, b are positive constants, and

$$g(x) = \begin{cases} 0, & |x| > 1 \\ k, & |x| \leq 1 \end{cases}$$

(a) Show that there are no periodic orbits for $k < 2b$.

(b) Show that there exists a periodic orbit for $k > 2b$.

9. Using a programming language of your choice(Matlab would be the easiest), create a video showing the evolution of the phase portrait with μ for any system of your choice that displays

(a) Saddle Node Bifurcation

(b) Transcritical Bifurcation

(c) Supercritical Pitchfork Bifurcation

(d) Subcritical Pitchfork Bifurcation

(e) Supercritical Hopf Bifurcation

Write the system equations as a part of the report.