EE6415: Nonlinear Systems Analysis

Jan-May, 2022

Tutorial 1

Instructions :

- **Submit on or before 11:59 PM, 29/01/2022**
- You have to turn in the well-documented code along with a detailed report of the results of the experiments. The code must be in a separate file(which can be directly run in matlab), and not a part of the report.
- Any kind of plagiarism will be dealt with severely. Acknowledge any and every resource used, including any coursemates you may have discussed with.
- Include any plots/images you deem necessary
- Your submission must be named "RollNo.zip". For example, if your roll number is EE17B158, your submission must have the name "EE17B158.zip".
- Your submissions must be made on moodle. Any emailed submissions will not be accepted.
- It is required that you use LaTeX for writing your report. A template has been provided along with this assignment for the same.

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1. Consider the following system :

$$\dot{x} = y$$
$$\dot{y} = -x - \beta y$$

   (a) Plot the Phase portrait for this system for
      i. $\beta = 2$
      ii. $\beta = 0$
      iii. $\beta = -2$

   (b) Provide inferences based on the difference in the graphs in terms of stability of equilibrium points.

2. Plot the phase portrait and comment on the stability of each of the equilibrium points and check whether there are any limit cycles in the systems described below.

   (a)

$$\dot{x} = x(5 - x - 2y)$$
$$\dot{y} = y(4 - x - y) \quad x, y \geq 0$$

(b)

$$\dot{x} = -x$$
$$\dot{y} = y^2$$

(c)

$$\dot{x} = y$$
$$\dot{y} = x - x^3$$

(d)

$$\dot{x} = \sin(y)$$
$$\dot{y} = \sin(x)$$

3. In the following system, plot phase portraits for $\mu = -1, 0$ and 1. Explain the differences in the phase portraits in terms of number and stability of equilibrium points, and existence of limit cycles.

$$\dot{x} = \mu x - y + xy^2$$
$$\dot{y} = x + \mu y + y^3$$

4. In the following system, plot phase portraits for $\mu = -2.5, -2$ and -1.5. Explain the differences in the phase portraits in terms of number and stability of equilibrium points, and existence of limit cycles.

$$\dot{x} = \mu x + y + \sin(x)$$
$$\dot{y} = x - y$$

5. A particle moves along a line joining two stationary masses, m_1 and m_2, and which are separated by a fixed distance a . Let x denote the distance of the particle from m_1.

   (a) Find a relationship between $\ddot{x}$ and x, using Newton's Law of Gravitation.

   (b) For $m_1 = 1, m_2 = 10, a = 10$, plot the phase portrait for the system, and identify the nature of the particles equilibrium.

6. In the following system, plot phase portraits for $\mu = -2, 0$ and 2. Explain the differences in the phase portraits in terms of number and stability of equilibrium points, and existence of limit cycles.

$$\dot{x} = \mu - x^2$$
$$\dot{y} = -y$$

7. Consider the equations of a normalized pendulum, given by

$$\ddot{q} + sin(q) + \dot{q} = 0$$

Plot the phase portrait for this system. Let us now assume that we modify the system to allow us to control the system as follows:

$$\ddot{q} + sin(q) + \dot{q} = u$$

where u is the input. Now, let us provide the input

$$u = sin(q) - q + 1$$

Plot the Phase portrait of the system after providing this feedback control.