ISYE 6669 - Project - Part A

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1 LP Formulation

Sets:
I: set of hours $= \{1, 2, 3, ..., 72\}$

Parameters:
s_i^E: Amount of electricity in MWh produced at solar farm for hour i
= rated output*% of rated output expected for hour i
p_i^E: Electricity price per MWh for hour i
p^H: Fixed hydrogen price per kg
d_i^E: Electricity demand of the plant in MWh for hour i
d_i^H: Hydrogen demand of the plant in kg for hour i

Decision Variables:
x_i^E: Amount of electricity in MWh purchased for hour i
x_i^H: Number of kgs of hydrogen purchased for hour i
n_i^H: Number of kgs of hydrogen stored at the end of hour i (inventory)

The objective is to minimise the cost of running the plan which is equal to the total cost of purchasing hydrogen and electricity over the plant running time.

$$\min \qquad \sum_{i \in I} (p_i^E x_i^E + p^H x_i^H)$$

s.t.

$$x_i^E + s_i^E \geq d_i^E \qquad \forall i \in I \tag{1}$$

$$x_1^H \geq d_1^H + n_1^H \tag{2}$$

$$n_{i-1}^H + x_i^H \geq d_i^H + n_i^H \qquad \forall i \in I/\{1\} \tag{3}$$

$$x_i^E, x_i^H, n_i^H \geq 0 \qquad \forall i \in I \tag{4}$$

Eq 1 ensures we have sufficient electricity (sum of purchased electricity and solar power) to meet the demand at each hour i. Eqns 2 and 3 ensure the total of hydrogen available in storage and the hydrogen purchased is greater or equal to the sum of hydrogen demand of the current hour and the hydrogen stored at the end of the hour. Eq 2 is a special case of eq 3 where the initial stored hydrogen is 0. These equations ensure that there is sufficient hydrogen to run the plant.

2 Python implementation and Output

Python code is provided in 'project_part-A_submission.py' file, and the hourly purchases of electricity and hydrogen are present in 'output.csv' file.

High-level summary:

- Total cost for running the plant (optimal objective value) = \$ 17916.013
- Total amount of hydrogen purchased = 1749.36 kg
- Total amount of electricity purchased = 18.01 MWh

3 Active Constraints and Time Horizon

3.1 Active Constraints

Yes, we observed active constraints. The core logic behind why the constraints are active is to ensure we do not purchase (electricity or hydrogen) more than what is required so as to minimise the cost.

Electricity purchase constraints (set of equations given by eq 1) are active whenever we have insufficient solar energy. We buy exactly the right amount of electricity to compensate the insufficiency of solar production in such hours. Buying extra electricity would increase our objective.

Hydrogen purchase (and storage) constraints (set of equations given by eqns 2 and 3) are always active. This is because, the amount of hydrogen purchased at the current time and amount stored till the current time will be exactly equal to the amount of hydrogen needed for the current hour and the amount of hydrogen to be stored for the future.

3.2 Time Horizon

- Reducing time horizon to t_H will not change the solution of electricity purchased till the time t_H (x_i^E, $i \leq t_H$), but since we are not considering beyond t_H, there will be no purchases after that. There is no change in solution since there is no inventory for electricity and we buy what is necessary to cover each hour independently.

- Reducing time horizon will affect the quantity of hydrogen purchased and stored(x_i^H and n_i^H). For instance, in the current solution, a massive amount of hydrogen is bought at the beginning, which had met the hydrogen demand until the final hour. However, if the problem is solved for 40 hours, a different amount of hydrogen will be purchased (and the purchase timing could also change). Since the x_i^H and n_i^H are dependent because of the active constraints in eqns 2 and 3, the solution for n_i^H would also change.

- Overall the objective function will be lower because, obviously, it takes lesser energy to run the plant for a lesser time

Task Allocation

All of us tried to solve the problem on our own and then sat together to come up with a formulation. Implemented the LP separately on our own and discussed the results (and ensured they aligned at the end).