ISYE 6669 - Project - Part B

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Correction of the Previous Model

First of all, we would like to start with the correction of our LP model which we used for our Part A submission. Based on the provided feedback, we revised the model, which will serve as a basis for the models developed in Part B. Following is the revised model:

Sets:
I: set of hours

Parameters:
s_i^E: Percentage of the rated solar power output for hour i
p_i^E: Electricity price per MWh for hour i
p^H: Fixed hydrogen price per kg
d_i^E: Electricity demand of the plant in MWh for hour i
d_i^H: Hydrogen demand of the plant in kg for hour i

Decision Variables:
x_i^E: Amount of electricity in MWh purchased for hour i
x_i^H: Number of kgs of hydrogen purchased for hour i

$$\min \quad \sum_{i \in I} (p_i^E x_i^E + p^H x_i^H)$$

s.t.

$$x_i^E + 2s_i^E \geq d_i^E \qquad \forall i \in I \tag{1}$$

$$\sum_{k=0}^{i} x_k^H - \sum_{k=0}^{i-1} d_k^H \geq d_i^H \qquad \forall i \in I \tag{2}$$

$$x_i^E, x_i^H \geq 0 \qquad \forall i \in I \tag{3}$$

In the following we answer the questions for Part B.

1 Electrolyser

a

To represent the electrolyser option in our LP model, we should introduce a new decision variable and update constraints (1) and (2).

The new decision variable is:

a_i: 1, if electrolyser is on for hour i; 0, otherwise

We add a constraint:

$$a_i \in \{0, 1\} \qquad \forall i \in I \tag{4}$$

After the update, constraints (1) and (2) become:

$$x_i^E + 2s_i^E - 0.5a_i \geq d_i^E \qquad \forall i \in I \tag{5}$$

$$\sum_{k=1}^{i}(x_k^H + 9a_k) - \sum_{k=1}^{i-1} d_k^H \geq d_i^H \qquad \forall i \in I \tag{6}$$

b

Related files are provided in the submission file.

c

Looking at the results, the benefit of electrolyser due to its high output/input ratio lead to using it in all hours, which caused changes in the solution and the objective value. For example, the amount of electricity purchased in each hour, x_i^E, increased by 0.5 MW because this is the cost of using electrolyser in terms of electricity consumption. Moreover, we can observe that the the total of hydrogen purchased, $\sum_i x_i^H$, decreased by 648 kg because 9 kgs of hydrogen produced in each hour for 72 hours by the electrolyser, which diminished the need for hydrogen purchase. The benefit of decreasing hydrogen purchase cost exceeded the increase in the electricity purchase cost, as a result of which our objective value became \$11,919.92, which means a savings of \$5996.098 compared to the base case.

2 Solar Panel

a

Adding the two 0.2MW solar panels, constraint (1) becomes:

$$x_i^E + (2 + 0.2 * 2)s_i^E \geq d_i^E \qquad \forall i \in I \tag{7}$$

$$\implies x_i^E + 2.4s_i^E \geq d_i^E \qquad \forall i \in I \tag{8}$$

With adding the additional 2 0.2 MW solar panels (total capacity increased from 2 MW to 2.4 MW), we see a decrease in the objective function from \$17916.013 to \$17906.249 (a savings of \$9.764).
This small decrease in cost over this 3 day period can be explained by the fact that whenever we have non-zero solar forecasted, the current setup has more capacity than our requirements for most time steps (barring 2-3 cases out of the 72 time steps) making the extra panels redundant. Further, if we look at the expected solar output and the electricity prices, we see that electricity prices are the lowest/highest when the solar output is the highest/lowest.
These reasons show that the relative increase in solar output we get from purchasing multiple panels will lead to a relatively small decrease in the cost we pay for electricity overall.

b

To represent the solar panel options in our LP model, we should introduce a new model and update constraint (1). We took the price of a new 0.2 MW block to be $\$900,0000 * \frac{1}{3.5 \text{ yrs}} * \frac{1 \text{ yrs}}{365 \text{ days}} \approx \$704.5$ per day, bringing the total cost to \$2,113.5 per 3 days.

The new decision variable is:

b: Number of additional solar panels purchased

The new objective function becomes:

$$\min \qquad \sum_{i \in I} (p_i^E x_i^E + p^H x_i^H + 2,113.5 * b)$$

We add a constraint:

$$b \in \{0, \mathbb{N}\} \tag{9}$$

After the update, constraint (1) becomes:

$$x_i^E + (2 + 0.2b)s_i^E \geq d_i^E \qquad \forall i \in I \tag{10}$$

Note that we assumed that solar panels can only be bought in blocks of 0.2 MW, for instance, we can't purchase a 0.1 MW block at $0.45m because we are only offered 0.2 MW blocks at $0.9m.

c

The optimal number of new solar installations is 0. The new operational cost over the three days is $17916.013.

3 Battery

a

We introduce a new decision variable:

c_i: Charge stored in the battery at the end of hour i

We make the following modifications to the constraints:
Firstly we add a constraint that the charge stored can't exceed 0.5MWh

$$c_i <= 0.5 \tag{11}$$

We update the electricity balance constraint (assuming zero charge to begin with) equation (1) becomes:

$$x_i^E + 2s_i^E + c_{i-1} \geq d_i^E + c_i \qquad \forall i \in I \setminus \{1\} \tag{12}$$

$$x_1^E + 2s_1^E \geq d_1^E + c_1 \tag{13}$$

Sign constraint:

$$c_i \geq 0 \qquad \forall i \in I \tag{14}$$

b

Code implemented in the Python File, output stored in output_part-B_Q3.csv.

c

As expected adding a battery to store the excess electricity generated/bought improves our objective. This is because, as we see in the solution, it enables us to buy electricity at cheaper prices (or at 0 price if solar is in excess) and use it when it is costlier. The objective falls to $17807.03 from $17916.01. And specifically looking at reduction in energy costs (since hydrogen costs are higher and dominate the objective and battery plays a role only in energy costs) we observe a considerable percentage reduction from $422.41 to $313.42. This results in a savings of $108.98 every 3 days.

4 Comparing Investments

In the following, we calculate the buy-back time for each investment:

- **Electrolyser:** Our findings in Question 1 suggest that using a 0.5 MW Electrolyser results in a savings of \$5996.1 in total of three days, which can be restated as it enables a savings of \$1,998.7 in a single day. Since the investment costs \$500,000, its buy-back time can be found by $\frac{500{,}000}{1{,}998.7}$ which is $\approx$ **250.16 days**.

- **Solar Panel:** Our findings in Question 2 suggest that purchasing 2 0.2 MW solar panels results in a savings of \$9.764 every three days. Recalculating for 1 0.2 MW solar panel gives us a savings of \$4.882 every 3 days. This can be transformed into a savings of $\approx$ \$1.6273 per day. Because the investment cost is \$900,000, its buy-back time can be found by $\frac{900{,}000}{1.6273}$ which is $\approx$ **553,052.14 days**.

- **Battery:** Our findings in Question 2 suggest that purchasing a 0.5 MWh battery results in a savings of \$108.98 every 3 days. This can be transformed into a savings of $\approx$ \$32.3267 per day. Because the investment cost is \$75,000, its buy-back time can be found by $\frac{75000}{32.3267}$ which is $\approx$ **2320.07 days**.

Based on the calculated buy-back time for each investment, we recommend using the electrolyser, because it has the shortest buy-back time.

Task Allocation

Came up with the formulations for each of the problems together. Each person implemented one of the three sections individually. And we discussed and finalised the results together.