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Problem 1 (a) $S=\mathbb{R}$. $\Omega = \{\theta \mid \theta \in \mathbb{R}\}$, $D = \{\hat\theta \mid \hat\theta \in \mathbb{R}\}$

$$L(d,\theta) = \frac{(\theta-d)^2}{1+\theta^2}$$

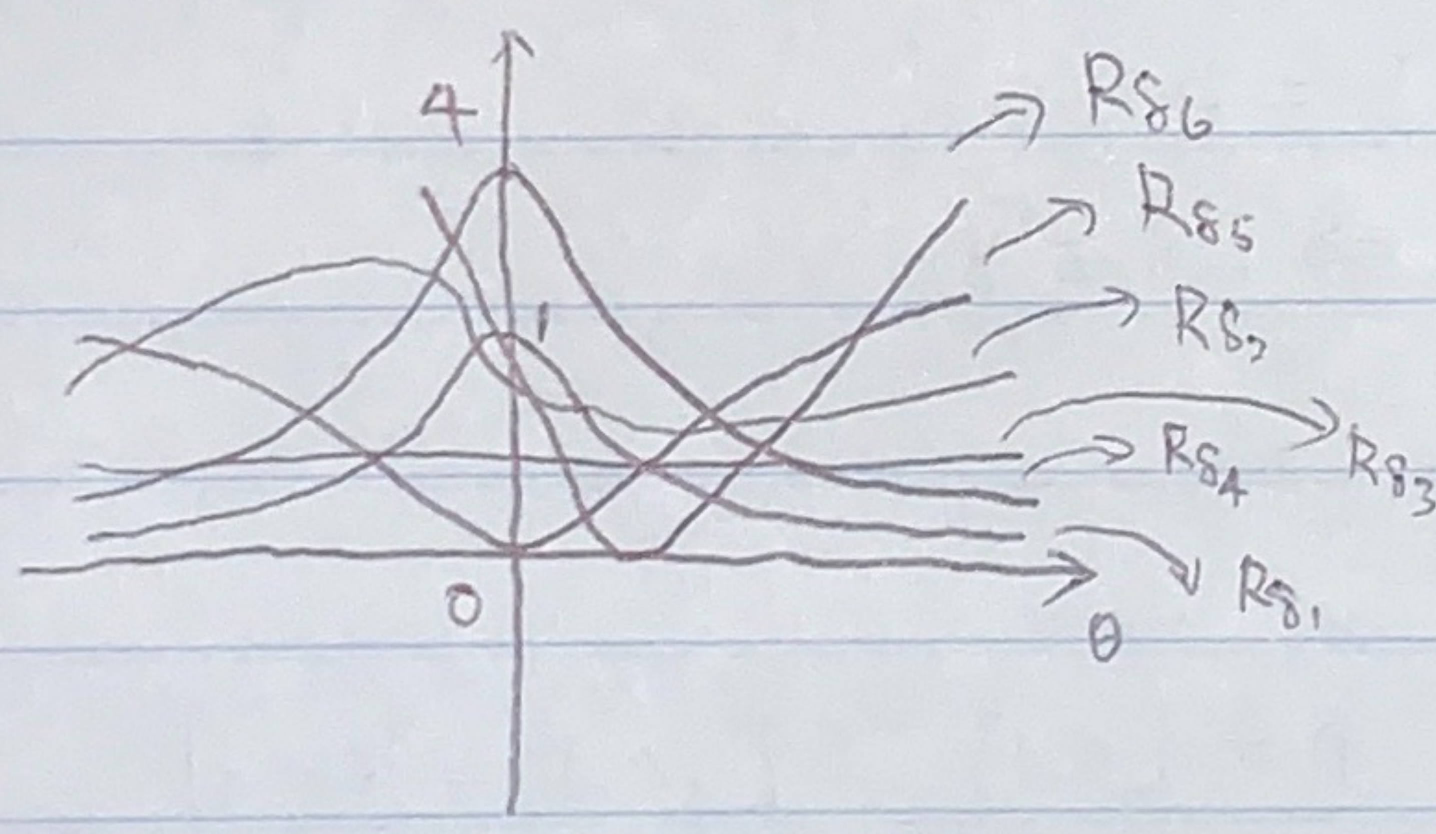
(b) for $\delta(x) = a+bx$, $R_\delta(\theta) = E\left[\frac{(\theta-\delta)^2}{1+\theta^2}\right]$

$$= \frac{1}{1+\theta^2}\left[(E_\delta(d)-\theta)^2 + Var(d)\right]$$

considering $E_\delta(d) = a+b\theta$, $Var(d) = b^2$

$$\Rightarrow R_{\delta_1}(\theta) = \frac{1}{1+\theta^2}, \quad R_{\delta_2}(\theta) = \frac{\theta^2-2\theta+2}{4\theta^2+4}, \quad R_{\delta_3}(\theta) = \frac{1}{4}$$

$$R_{\delta_4}(\theta) = \frac{\theta^2+4}{1+\theta^2}, \quad R_{\delta_5}(\theta) = \frac{\theta^2}{1+\theta^2}, \quad R_{\delta_6}(\theta) = \frac{(1-\theta)^2}{1+\theta^2}$$



(c) δ_4 is inadmissible, it's always larger than δ_1 and δ_5

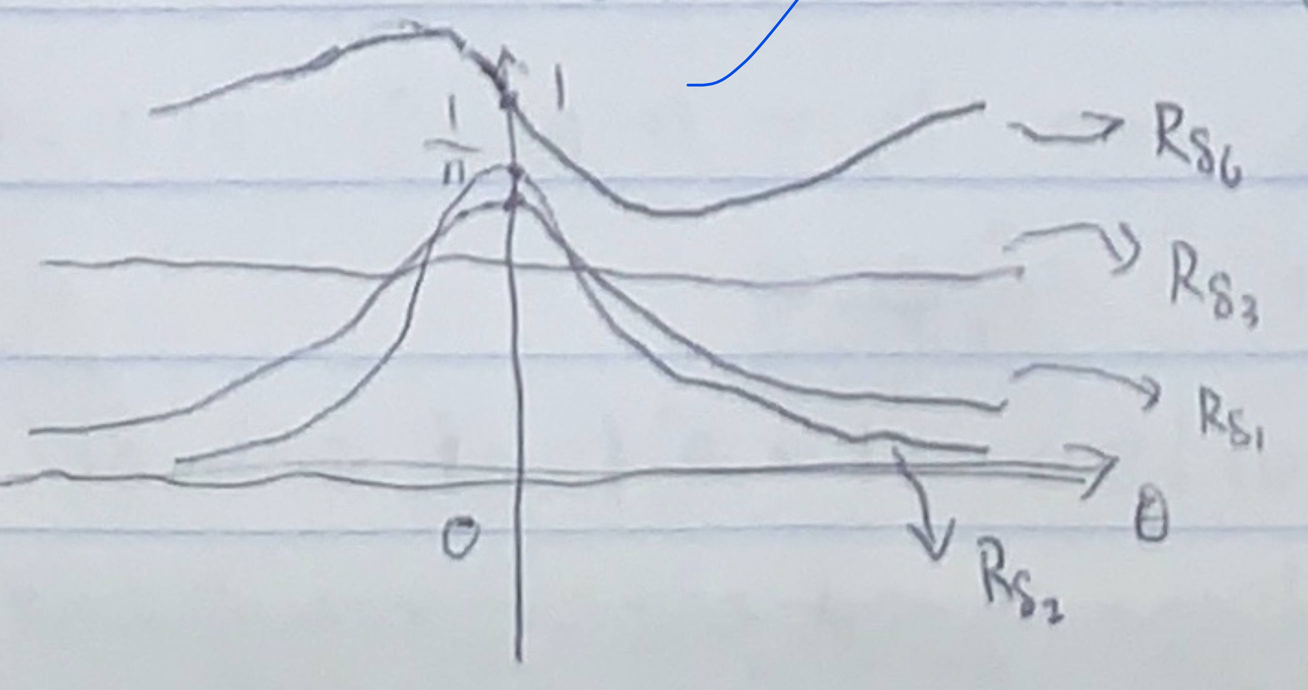
(d) I would use δ_3 since it's stable

(e) for general $\delta_n(x_1, \cdots, x_n) = a + b\bar{x}_n$, we have

$$R_{\delta_n}(\theta) = (1+\theta^2)^{-1} E(a+b\bar{x}_n-\theta)^2$$

$$= \frac{1}{1+\theta^2}\left[(a+(b-1)\theta)^2 + \frac{b^2}{n}\right]$$

$$\Rightarrow R_{\delta_1}(\theta) = \frac{1}{n(1+\theta^2)}, \quad R_{\delta_2}(\theta) = \frac{(\theta-1)^2+n}{(1+n^2)(1+\theta^2)}, \quad R_{\delta_3}(\theta) = \frac{1}{(1+\sqrt{n})^2}, \quad R_{\delta_6} = \frac{(\theta-1)^2}{1+\theta^2}$$



(f) δ_1 and δ_2, their overall risk is lower

(g) $R_{\delta_n}(\theta) = \frac{1}{1+\theta^2}\left[(a+(b-1)\theta)^2 + \frac{b^2}{n}\right] \to 0$ holds if and only if the order of numerator is smaller than the order of denominator in θ

which means it's smaller than 2

this holds if and only if $b=1$

proof for a = 0?

(h) $R_{\delta_6}(\theta) = \frac{(\theta-1)^2}{1+\theta^2}$, if there's another δ s.t. $R_\delta \le R_{\delta_6}$

then $R_\delta(1) \le R_{\delta_6}(1) = 0 \Rightarrow R_\delta(1) = 0$

$\Rightarrow \left[(a+b-1)^2 + \frac{b^2}{n}\right] = 0$

$\Rightarrow b=0,\ a=1 \Rightarrow \delta \equiv \delta_6$

so δ_6 is admissible

The proof is incorrect because to prove admissibility one must show that NONE of the procedures are better than the given procedure.
However the given proof shows that only procedures of the form a + bX are not better than the given procedure

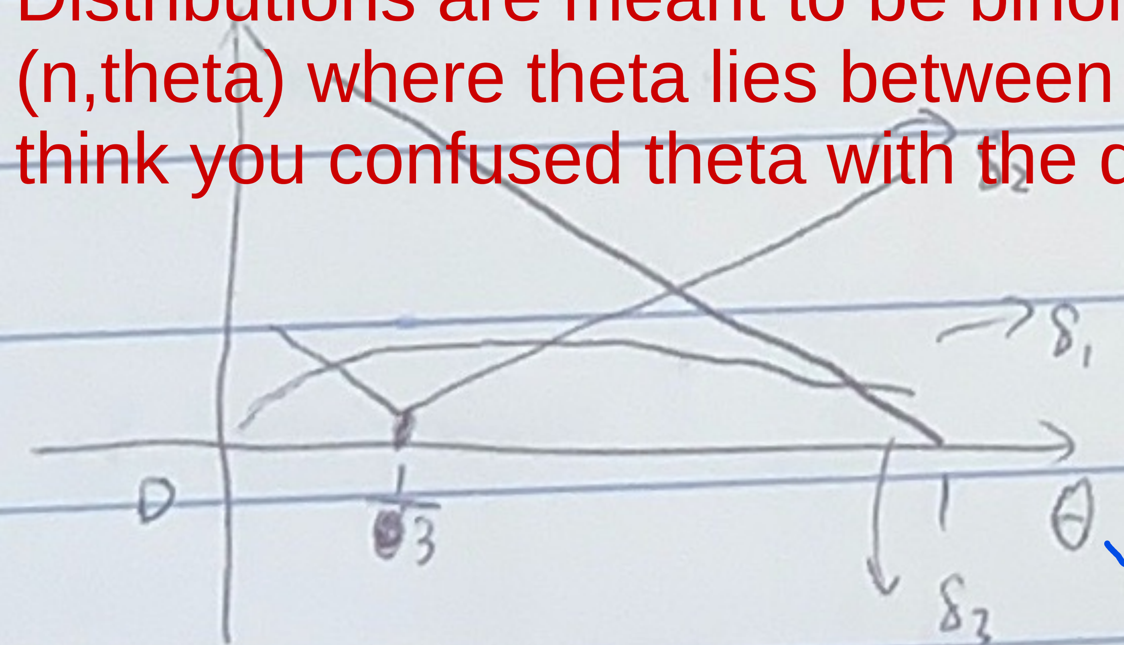
Problem 2 (a) $S = [n]$, $\Omega = [0,1]$, $D = [0,1]$

S = {0,...,n} (i am not familiar with the notation you used, so apologies if you meant the same thing)
Distributions are meant to be binomial (Bin (n,theta) where theta lies between 0 and 1). I think you confused theta with the distribution

(b) $R_{\delta_1}(\theta) = \sum_{i=1}^{n} \binom{n}{i} \theta^i (1-\theta)^{n-i} \left|\frac{i}{n} - \theta\right|$

$R_{\delta_2}(\theta) = \left|\frac{1}{3} - \theta\right|$

$R_{\delta_3}(\theta) = |1-\theta|$



(c) if another procedure has $R_\delta(\frac{1}{3}) = 0$

$\Rightarrow P(\delta = \frac{1}{3}) = 1$, otherwise $E|\delta - \frac{1}{3}| > 0$

$\Rightarrow \delta \equiv \frac{1}{3}$ almost everywhere

$\Rightarrow \delta = \delta_2$

incomplete. Questions expects you to show admissibility of delta_2

(d) if $\exists\, \delta$ s.t. $R_\delta \le R_{\delta_3} \Rightarrow R_\delta = (1-\theta)^2|\delta(0)-\theta| + 2\theta(1-\theta)|\delta(1)-\theta| + \theta^2|\delta(2)-\theta|$

let $\theta = 1 \Rightarrow \delta(2) = 1$

invalid to divide by 1-theta and subst theta= 1 because 1-theta becomes zero

$\Rightarrow (1-\theta)^2|\delta(0)-\theta| + 2\theta(1-\theta)|\delta(1)-\theta| + \theta^2|1-\theta| \le |1-\theta|$

$\Rightarrow (1-\theta)|\delta(0)-\theta| + 2\theta|\delta(1)-\theta| \le 1-\theta^2$, let $\theta=1 \Rightarrow \delta(1)=1$, $\delta(0)=1$ if do again

$\Rightarrow (1-\theta)|\delta(0)-\theta| + 2\theta|\delta(1)-\theta|$