ISyE 6650: Probabilistic Models and Their Applications **Handout 7**
Sigrún Andradóttir **September 18, 2023**

Assignment 4

Due September 26, 2023, 11 PM

Remember to attach a fully completed Honor Code Statement to your homework

Problem 1 (8 points)

a. (2 points) Do Problem 4.20, page 278 in the text.

b. (2 points) Do Problem 4.29, page 280 in the text.

c. (2 points) In part **b**, suppose that the time an employee spends in a job in each of the three job classifications is random with mean 2 years, 5 years, and 3 years, respectively. Suppose furthermore that the yearly salary for an employee in each of the three job classifications is random with means \$50,000, \$70,000, and \$90,000, respectively. Finally, suppose that whenever a person changes job classifications, a training cost of \$5,000 is incurred (no training cost is incurred when a person changes jobs if the new job has the same classification as the previous job). Determine the average yearly (combined) salary and training cost per employee for the company. Explain.

d. (2 points) Do Problem 4.42, page 282 in the text.

Problem 2 (10 points)

This problem continues Problem 3 on Assignment 2.

a. (2 points) Determine the steady-state probabilities.

b. (3 points) The cost of lost profit and loss of customer satisfaction associated with losing a customer to Quality Cameras is \$40. Placing an order for m cameras costs $\$50 + 10 \times m$ (\$50 is a fixed cost regardless of the size of the order, and \$10 is the cost per ordered camera). Finally, the cost of storing Canon Rebel cameras overnight is \$5 per night per camera. This includes both the cost of the storage space and interest on the investment. Determine the long-run average cost per unit time. Explain.

c. (5 points) Would it be better for Lenox Photo to use an (s, S) inventory policy with $s = 2$ and $S = 3$? Explain.

Comment: When the objective of the analysis is to compare different inventory policies, then it is not necessary to incorporate revenue from sales because this can always be modeled as the revenue from sales if no demand is lost (which is the same for all inventory policies) minus the revenue lost due to the lost demand (which can be incorporated into the cost of losing a customer).

Problem 3 (9 points)

This problem continues Problem 4 on Assignment 2.

a. (2 points) Determine the steady-state probabilities.

b. (2 points) Determine the long-run average hourly production rate of the machining center (i.e., the long-run average number of produced products per hour). Explain.

c. (3 points) The raw material needed for producing one GT100 costs \$10. This raw material cost cannot be recovered for scrapped products. The selling price per unit of GT100 is set at \$20, and it is safe to assume that all the products manufactured can be sold in the market at this price. The hourly cost of operating the machining center is \$50. Determine the long-run average hourly profit from the sale of GT100s. Explain.

d. (2 points) The situation described in this problem involves several (simplifying) assumptions that may not always be reasonable. Identify at least five such assumptions

Problem 4 (16 points)

a. (4 points) Do Problem 4.45, page 283 in the text.

b. (3 points) Do Problem 4.59, page 286 in the text (it is not necessary to solve for $M_0, \ldots, M_N$ in the case when $p \neq 0.5$).

c. (4 points) Consider a discrete time Markov chain with state space $S = \{0, 1, 2, 3\}$ and transition probability matrix

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0.3 & 0.4 & 0.1 & 0.2 \\ 0 & 0 & 1 & 0 \\ 0.2 & 0.3 & 0.4 & 0.1 \end{array} \right] \end{array} .$$

Note that this Markov chain has two absorbing states (0 and 2) and two transient states (1 and 3). For all $i \in S$, let P_i denote the probability that the Markov chain gets absorbed in state 0 given that it started in state i, and let M_i denote the expected number of transitions before the Markov chain gets absorbed in either state 0 or state 2 given that it started in state i. Determine P_i and M_i for all $i \in S$. Explain.

d. (5 points) Consider a discrete time Markov chain with state space $S = \{1, 2, 3, 4\}$ and transition probability matrix

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 0.2 & 0 & 0 & 0.8 \\ 0 & 1 & 0 & 0 \\ 0.1 & 0.4 & 0.2 & 0.3 \\ 0.7 & 0 & 0 & 0.3 \end{array} \right] \end{array} .$$

For all $i, j \in S$, let $\pi_{i,j}$ denote the expected long-run average fraction of time the Markov chain spends in state j given that it started in state i. Determine the value of $\pi_{i,j}$ for all $i, j \in S$. Explain.

Reading Assignment

Read Sections 4.4, 4.5, 4.6, and 7.6 of the text.