ec3: Elliptic Curve Cryptography Compiler

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Diffie-Hellman

Diffie-Hellman Key Exchange

Alice and Bob want to *establish a shared secret*.

Use integers modulo a *prime p*, and choose a multiplicative *generator g*.

Protocol

Alice Bob

$$\xrightarrow{g^x \pmod{p}}$$

$$\xleftarrow{g^y \pmod{p}}$$

Protocol

Alice Bob

$$g^x \pmod{p}$$
$$\xrightarrow{\hspace{6cm}}$$

$$g^y \pmod{p}$$
$$\xleftarrow{\hspace{6cm}}$$

Alice chooses secret x.

Sends $g^x \pmod{p}$ to Bob.

Protocol

Alice $\qquad\qquad\qquad\qquad$ Bob

$$\xrightarrow{\qquad g^x \pmod{p} \qquad}$$

$$\xleftarrow{\qquad g^y \pmod{p} \qquad}$$

Bob chooses secret y.

Sends $g^y \pmod{p}$ to Alice.

Both Alice and Bob can compute

$$g^{xy}$$

Alice knows x and received g^y so computes

$$(g^y)^x \pmod{p}$$

Bob knows y and received g^x so computes

$$(g^x)^y \pmod{p}$$

You could break this if you could *find x given* g^x.

This is the discrete log problem.

$$\mathcal{O}\left(e^{c\sqrt[3]{(\log p)(\log\log p)^2}}\right)$$

Discrete log for integers modulo p
is sub-exponential.

Requirements

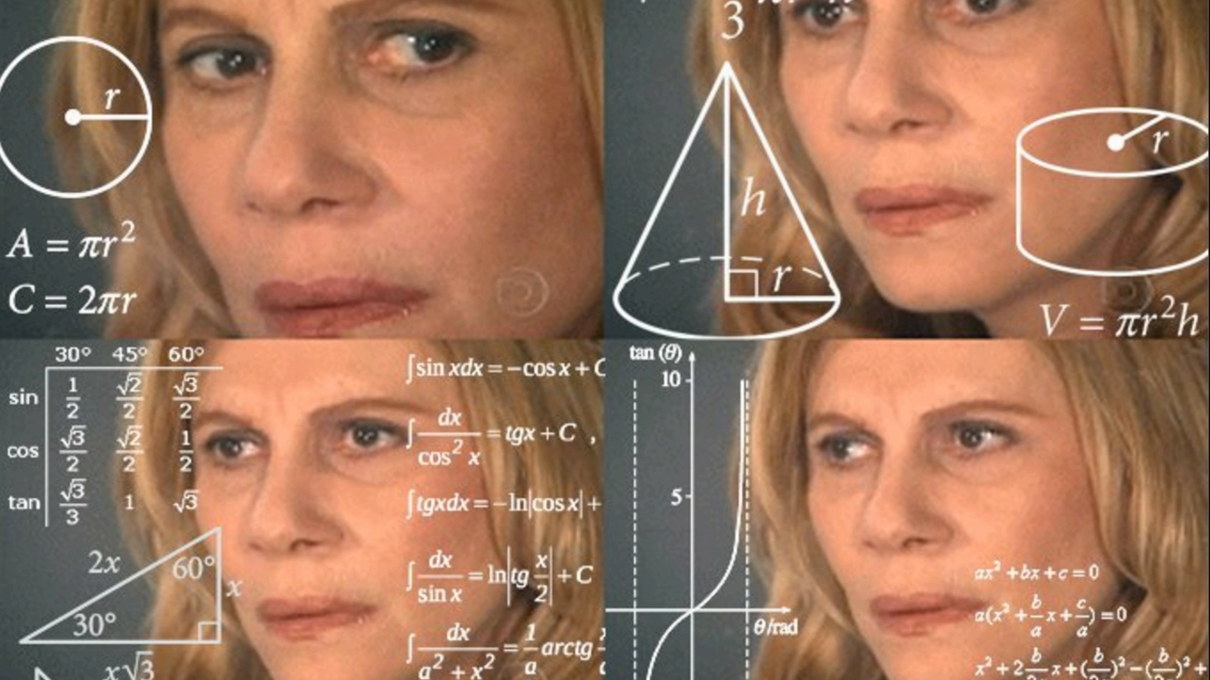
- **Set:** set of elements G to manipulate
- **Operation:** definition of g^x
- **Security:** discrete log problem is *hard*

We can perform Diffie-Hellman on any mathematical group.

Elliptic Curves

Not to be confused with Ellipse.

Smooth, projective, algebraic curve of genus one.

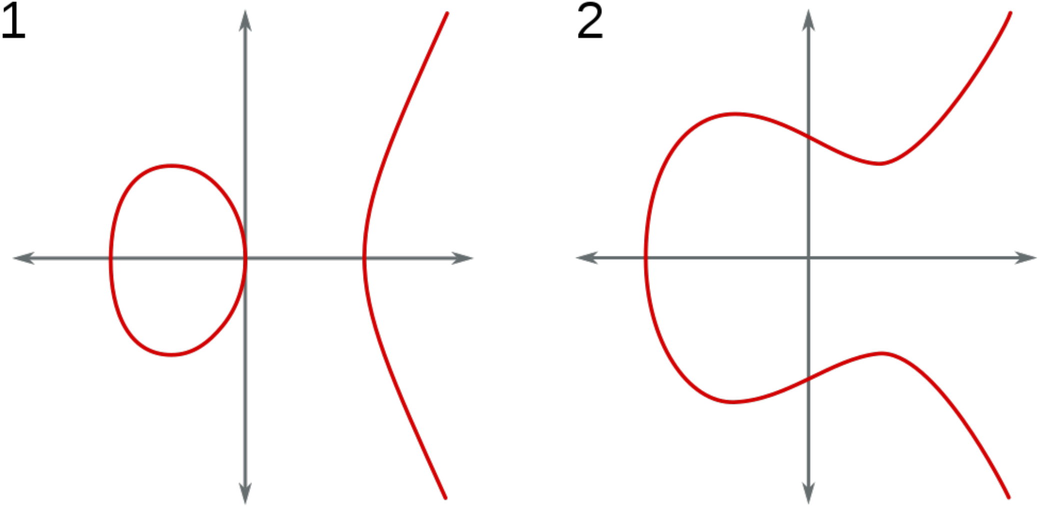


r
$A = \pi r^2$
$C = 2\pi r$
3
h
r
r
$V = \pi r^2 h$
30° 45° 60°
sin $\frac{1}{2}$ $\frac{\sqrt{2}}{2}$ $\frac{\sqrt{3}}{2}$
cos $\frac{\sqrt{3}}{2}$ $\frac{\sqrt{2}}{2}$ $\frac{1}{2}$
tan $\frac{\sqrt{3}}{3}$ 1 $\sqrt{3}$
$2x$
60°
x
30°
$x\sqrt{3}$
$\int \sin x dx = -\cos x +$
$\int \frac{dx}{\cos^2 x} = tgx + C$,
$\int tgxdx = -\ln|\cos x| +$
$\int \frac{dx}{\sin x} = \ln\left|tg\frac{x}{2}\right| + C$
$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arctg$
tan (θ)
10
5
θ/rad
$ax^2 + bx + c = 0$
$a(x^2 + \frac{b}{a}x + \frac{c}{a}) = 0$

$$y^2 = x^3 + ax + b$$

1

2

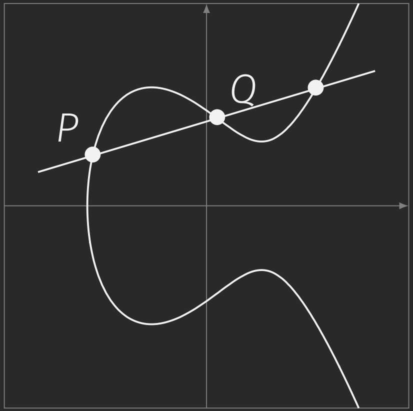


$y^2 = x^3 - x$

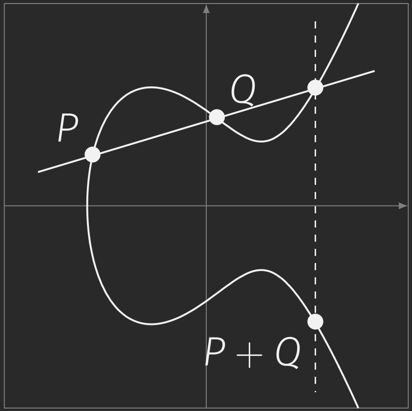
$y^2 = x^3 - x + 1$

Elliptic Curve Diffie-Hellman

- **Set:** points (x, y) on the elliptic curve
- **Operation:** repeated point addition kP
- **Security:** discrete log on elliptic curves has *exponential complexity*



Addition $P + Q$ “Chord rule”



Addition $P+Q$ “Chord rule”

In *Elliptic Curve Diffie-Hellman* we replace exponentiation g^x with repeated addition of points kP.

Elliptic Curve Diffie-Hellman

- **Set:** points (x, y) on the elliptic curve
- **Operation:** repeated point addition kP
- **Security:** discrete log on elliptic curves has *exponential complexity*

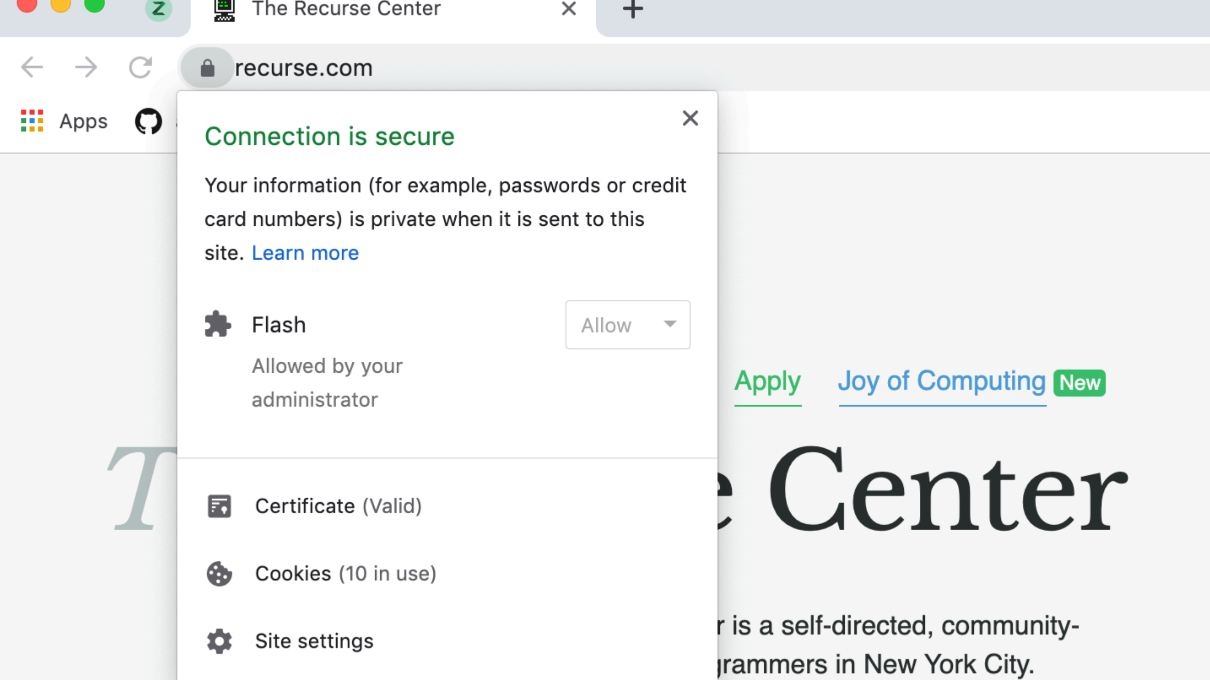
$$\mathcal{O}\left(\sqrt{p}\right)$$

Discrete log for Elliptic Curves is exponential.

Elliptic Curve Diffie-Hellman

- **Set:** points (x, y) on the elliptic curve
- **Operation:** repeated point addition kP
- **Security:** discrete log on elliptic curves has *exponential complexity*

Elliptic Curves are smaller and faster for the same security level.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Elliptic Curve Cryptography (ECC) is very widely used in TLS.

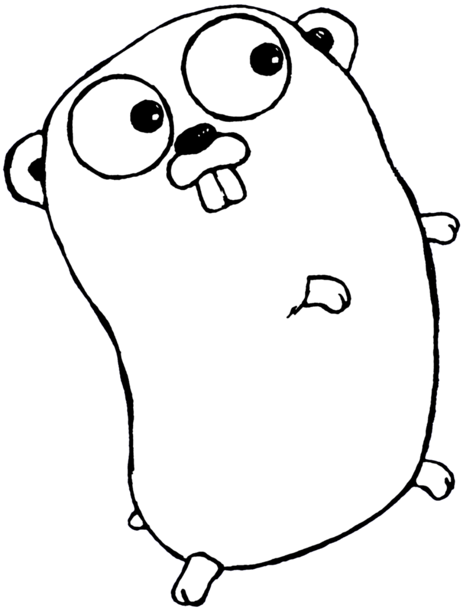
Implementation

Implementations must be secure.

Error-free and constant-time.

Implementations must be fast.

Low-level arithmetic almost always implemented in assembly.



crypto/elliptic/p256_asm_amd64.s

```
	MOVOU X3, (16*3)(DX)

	RET
/* ---------------------------------------*/
// func p256OrdMul(res, in1, in2 []uint64)
TEXT ·p256OrdMul(SB),NOSPLIT,$0
	MOVQ res+0(FP), res_ptr
	MOVQ in1+24(FP), x_ptr
	MOVQ in2+48(FP), y_ptr
	// x * y[0]
	MOVQ (8*0)(y_ptr), t0

	MOVQ (8*0)(x_ptr), AX
	MULQ t0
	MOVQ AX, acc0
	MOVQ DX, acc1
```

```
	MOVQ acc5, acc3
	MOVQ acc0, t0
	MOVQ acc1, t1
	// Subtract p256
	SUBQ p256ord<>+0x00(SB), acc4
	SBBQ p256ord<>+0x08(SB) ,acc5
	SBBQ p256ord<>+0x10(SB), acc0
	SBBQ p256ord<>+0x18(SB), acc1
	SBBQ $0, acc2

	CMOVQCS x_ptr, acc4
	CMOVQCS acc3, acc5
	CMOVQCS t0, acc0
	CMOVQCS t1, acc1

	MOVQ acc4, (8*0)(res_ptr)
```

```
/* ---------------------------------------*/
TEXT p256SqrInternal(SB),NOSPLIT,$0

	MOVQ acc4, mul0
	MULQ acc5
	MOVQ mul0, acc1
	MOVQ mul1, acc2

	MOVQ acc4, mul0
	MULQ acc6
	ADDQ mul0, acc2
	ADCQ $0, mul1
	MOVQ mul1, acc3

	MOVQ acc4, mul0
```

```
MOVL t1, sel_save
MOVL t2, zero_save
// Negate y2in based on sign
MOVQ (16*2 + 8*0)(CX), acc4
MOVQ (16*2 + 8*1)(CX), acc5
MOVQ (16*2 + 8*2)(CX), acc6
MOVQ (16*2 + 8*3)(CX), acc7
MOVQ $-1, acc0
MOVQ p256const0<>(SB), acc1
MOVQ $0, acc2
MOVQ p256const1<>(SB), acc3
XORQ mul0, mul0
// Speculatively subtract
SUBQ acc4, acc0
SBBQ acc5, acc1
SBBQ acc6, acc2
```

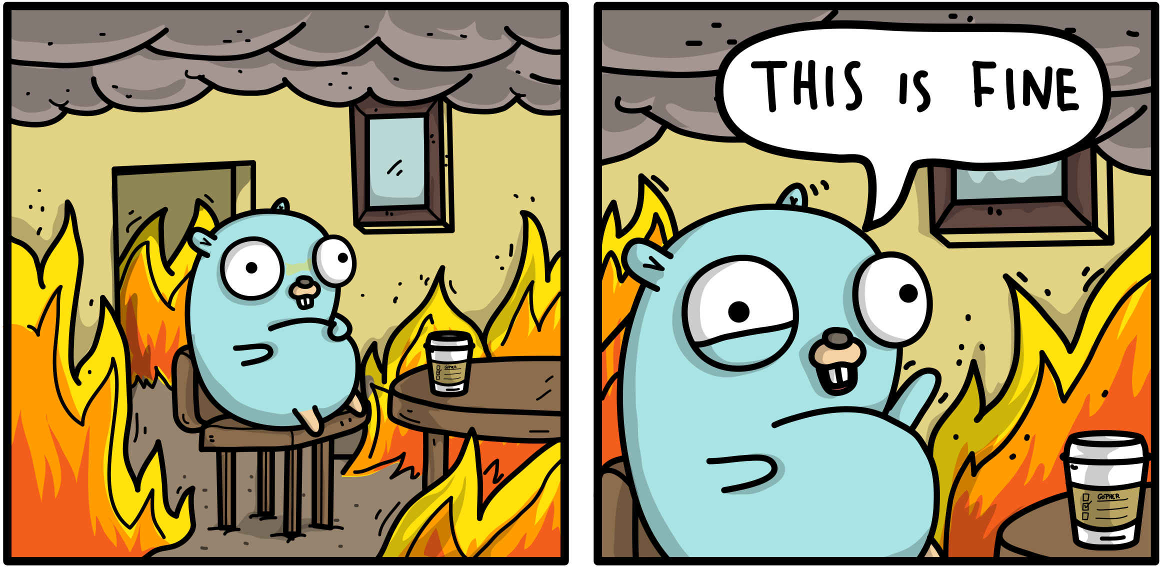
```
	LDt (y)
	CALL p256SubInternal(SB)
	MOVQ rptr, AX
	// Store y
	MOVQ acc4, (16*2 + 8*0)(AX)
	MOVQ acc5, (16*2 + 8*1)(AX)
	MOVQ acc6, (16*2 + 8*2)(AX)
	MOVQ acc7, (16*2 + 8*3)(AX)
	///////////////////////
	MOVQ $0, rptr

	RET
/* ---------------------------------------*/

```

```
        RET
/* ----------
```

Is this fine?

crypto/elliptic: carry bug in x86-64 P-256 #20040

Closed agl opened this issue on Apr 19, 2017 · 12 comments


THIS IS FINE
GOPHER

Elliptic Curve Cryptography Compiler

Most elliptic curve implementations are hand rolled.

It’s just f**king arithmetic!

Can we automate code-generation for elliptic curves?

ec3: Elliptic Curve Cryptography Compiler

ec3

Inputs:

- Curve type and equation
- Coordinate system
- Finite field implementation

Outputs assembly and Go code to implement the curve.

Status

Generates correct elliptic curve code for P-256.

```
DATA p<>+0(SB)/8, $0xffffffffffffffff
DATA p<>+8(SB)/8, $0x00000000ffffffff
DATA p<>+16(SB)/8, $0x0000000000000000
DATA p<>+24(SB)/8, $0xffffffff00000001
GLOBL p<>(SB), RODATA|NOPTR, $32

// func double(X1_ *Elt, X3_ *Elt, Y1_ *Elt, Y3_ *Elt, Z1_ *E
TEXT ·double(SB), $800-48
	MOVQ X1_+0(FP), BX
	MOVQ (BX), AX
	MOVQ 8(BX), CX
	MOVQ 16(BX), DX
	MOVQ 24(BX), BX
	MOVQ AX, 160(SP)
	MOVQ CX, 168(SP)
```

10106 lines (9230 sloc) | 170 KB

```
// Code generated by ec3. DO NOT EDIT.

#include "textflag.h"

// func lookup(p *Jacobian, tbl []Jacobian, idx int)
TEXT ·lookup(SB), $0-40
        MOVQ p+0(FP), AX
        MOVQ tbl_base+8(FP), CX
        MOVQ tbl_len+16(FP), DX
        MOVQ idx+32(FP), BX

        // Initialize a 1 register.
        PXOR    X0, X0
```

Performance

```
pkg: github.com/mmcloughlin/ec3/examples/p256
BenchmarkScalarMult-4       136772   89073 ns/op
BenchmarkStdScalarMult-4    213002   56462 ns/op
```

Thank you!

@mbmcloughlin