Exercises for practice lesson 8

(1) Assume in the vector space R^2 given the mapping

$$\langle u, v\rangle = u_1v_1 - u_1v_2 - u_2v_1 + 3u_2v_2$$

(a) Show that this mapping is dot product.

(b) For $u = \begin{pmatrix}1\\5\end{pmatrix}$ and $v = \begin{pmatrix}3\\4\end{pmatrix}$ find $\langle u, v\rangle$, $\|u\|$ and $\|v\|$.

(2) Let V be the vector space of polynomials over R of degree ≤ 2 with dot product defined by

$$\langle f, g\rangle = \int_0^1 f(t)g(t)dt$$

Find a basis of the subspace W orthogonal to $h(t) = 2t + 1$.

(3) Let $w = \begin{pmatrix}1\\-2\\-1\\3\end{pmatrix}$ be a vector in Euclidean space R^4. Find

(a) an orthogonal basis for $w^\perp$;

(b) an orthonormal basis for $w^\perp$.

(4) Let U be the subspace of Euclidean space R^4 spanned by

$$u_1 = \begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \quad u_2 = \begin{pmatrix}1\\-1\\2\\2\end{pmatrix}, \quad u_3 = \begin{pmatrix}1\\2\\-3\\-4\end{pmatrix}$$

(a) Apply the Gram–Schmidt algorithm to find an orthogonal and an orthonormal basis for U;

(b) Find the projection of $v = \begin{pmatrix}1\\2\\-3\\4\end{pmatrix}$ onto U.

(5) Consider $P(t)$ with dot product

$$\langle f, g\rangle = \int_{-1}^1 f(t)g(t)dt$$

and the subspace $W = P_3(t)$:

(a) Find an orthogonal basis for W by applying the Gram–Schmidt algorithm to $\{1, t, t^2, t^3\}$;

(b) Find the projection of $f(t) = t^5$ onto W.

Home work 8

(1) Show that each of the following is not a dot product on R^3
 (a) $\langle u, v\rangle = u_1v_1 + u_2v_2$;
 (b) $\langle u, v\rangle = u_1v_2u_3 + v_1u_2v_3$.

(2) Let $M = M_{2\times 2}$ with dot product $\langle A, B\rangle = tr(B^TA)$. Find an orthogonal basis for the orthogonal complement of
 (a) diagonal matrices;
 (b) symmetric matrices.

(3) Consider the subspace $W = P_2(t)$ of $P(t)$ with dot product

$$\langle f, g\rangle = \int_0^1 f(t)g(t)dt$$

 (a) Find an angle between vectors $f(t) = t + 1$ and $g(t) = t^2 - 1$;
 (b) Find an orthogonal basis for W with integer coefficients;
 (c) Find the projection of $f(t) = t^3$ onto W.

(4) Find an orthonormal basis for this subspace of Euclidean space R^4.

$$W = \{\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} : \ x - y - z + w = 0 \text{ and } x + z = 0\}.$$