LECTURE 9

Basic laws of distribution of discrete random variables

1. Binomial law of distribution

A discrete random variable X has *a binomial law of distribution* with parameters n and p if it takes on values 0, 1, 2, …, m, …, n with probabilities

$$\boxed{P(X=m)=C_n^m p^m q^{n-m}}$$

where $0 < p < 1$, $q = 1 - p$.

The binomial law of distribution represents the law of distribution of the number $X = m$ of occurrences of an event A in n independent trials in each of which it can occur with the same probability.

The series of distribution of a binomial law has the following form:

x_i	0	1	2	…	m	…	n
p_i	q^n	$C_n^1 p^1 q^{n-1}$	$C_n^2 p^2 q^{n-2}$	…	$C_n^m p^m q^{n-m}$	…	p^n

Obviously, $\sum_{i=0}^{n} p_i = 1$ because $\sum_{i=0}^{n} p_i$ is the sum of all members of decomposition of Newton binomial:

$$q^n + C_n^1 pq^{n-1} + C_n^2 p^2 q^{n-2} + \ldots + C_n^m p^m q^{n-m} + \ldots + p^n = (q+p)^n = 1^n = 1.$$

Therefore, the law is said to be binomial.

Theorem. The mathematical expectation of a random variable X distributed under a binomial law is $M(X) = np$, and its dispersion $D(X) = npq$.

The binomial law of distribution is widely used in the theory and practice of statistical control by a production quality, at the description of functioning of systems of mass service, at modeling the prices of actives, in the theory of shooting and in other areas.

<u>*Example*</u>. Footwear has arrived in a shop from two factories in the ratio 2:3. 4 pairs of footwear have been bought. Find the law of distribution of the number of the bought pairs of footwear made by the first factory. Find the mathematical expectation and the dispersion of this random variable.
Solution: The probability that a randomly chosen pair of footwear has been made by the first factory is $p = 2/(2 + 3) = 0,4$. The random variable X – the number of bought pairs of footwear made by the first factory among 4 selling pairs – has the binomial law of distribution with parameters $n = 4$, $p = 0,4$. The series of distribution of X has the following form:

x_i	0	1	2	3	4
p_i	0,1296	0,3456	0,3456	0,1536	0,0256

The values $p_i = P(X = m)$, where m = 0, 1, 2, 3, 4, are calculated by the formula:

$$P(X=m)=C_4^m \cdot 0{,}4^m \cdot 0{,}6^{4-m}$$

Find the mathematical expectation and the dispersion of the random variable X:

$$M(X) = np = 4 \cdot 0{,}4 = 1{,}6;\ D(X) = npq = 4 \cdot 0{,}4 \cdot 0{,}6 = 0{,}96.$$

2. The law of Poisson distribution

A discrete random variable X has *the law of Poisson distribution* with parameter $\lambda > 0$ if it takes on values 0, 1, 2, …, m, …(infinite countable set of values) with probabilities

$$\boxed{P(X = m) = \frac{\lambda^m e^{-\lambda}}{m!}}$$

The series of distribution of the Poisson law has the following form:

x_i	0	1	2	…	m	…
p_i	$e^{-\lambda}$	$\lambda e^{-\lambda}$	$\lambda^2 e^{-\lambda}/2!$	…	$\lambda^m e^{-\lambda}/m!$	…

Since the sum of the series

$$\sum_{i=0}^{\infty} p_i = e^{-\lambda} + \lambda e^{-\lambda} + \frac{\lambda^2 e^{-\lambda}}{2!} + \ldots + \frac{\lambda^m e^{-\lambda}}{m!} + \ldots =$$

$$= e^{-\lambda}\left(1 + \lambda + \frac{\lambda^2}{2!} + \ldots + \frac{\lambda^m}{m!} + \ldots\right) = e^{-\lambda} \cdot e^{\lambda} = 1,$$

the basic property of distribution series $\sum_{i=0}^{n} p_i = 1$ holds, and consequently the Poisson law is well-defined.

The Poisson probability distribution was introduced by S.D. Poisson in a book he wrote regarding the application of probability theory to lawsuits, criminal trials, and the like.

Theorem. The mathematical expectation and the dispersion of a random variable distributed under a Poisson law coincide and are equal to the parameter λ of the law, i.e. $M(X) = \lambda$, $D(X) = \lambda$.

The Poisson random variable has a tremendous range of applications in diverse areas because it may be used as an approximation for a binomial random variable with parameters (n, p) when n is large and p is small enough so that np is a moderate size. In other words, if n independent trials, each of which results in a success with probability p, are performed, then, when n is large and p small enough to make np moderate, the number of successes occurring is approximately a Poisson random variable with parameter $\lambda = np$.

Some examples of random variables that usually obey the Poisson probability law follow:

1. The number of misprints on a page (or a group of pages) of a book.
2. The number of people in a community living to 100 years of age.
3. The number of wrong telephone numbers that are dialed in a day.
4. The number of packages of dog biscuits sold in a particular store each day.
5. The number of customers entering a post office on a given day.
6. The number of vacancies occurring during a year in the federal judicial system.
7. The number of α-particles discharged in a fixed period of time from some radioactive material.

Each of the preceding, and numerous other random variables, are approximately Poisson for the same reason – namely, because of the Poisson approximation to the binomial. For instance, we can suppose that there is a small probability p that each letter typed on a page will be misprinted. Hence the number of misprints on a page will be approximately Poisson with $\lambda = np$, where n is the number of letters on a page.

Example. Suppose that the number of typographical errors on a single page of this book has a Poisson distribution with parameter $\lambda = 1/2$. Calculate the probability that there is at least one error on this page.
Solution: Letting X denote the number of errors on this page, we have

$$P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-1/2} = 0{,}393.$$

Example. Suppose that the probability that an item produced by a certain machine will be defective is 0,1. Find the probability that a sample of 10 items will contain at most 1 defective item.
Solution: The desired probability is

$$P(X = 0) + P(X = 1) = C_{10}^{0}(0{,}1)^{0}(0{,}9)^{10} + C_{10}^{1}(0{,}1)^{1}(0{,}9)^{9} = 0{,}7361,$$

whereas the Poisson approximation yields the value

$$P(X = 0) + P(X = 1) = e^{-1} + e^{-1} \approx 0{,}7358.$$

3. Geometric distribution

A discrete random variable X has *a geometric distribution* with the parameter p if it takes on values 1, 2, …, m, … (infinite countable set of values) with probabilities

$$\boxed{P(X = m) = pq^{m-1}}$$

where $0 < p < 1$, $q = 1 - p$.

The series of a geometric distribution has the following form:

x_i	1	2	3	…	m	…
p_i	p	pq	pq^2	…	pq^{m-1}	…

It is easy to see that the probabilities p_i form the geometric progression with the first member p and denominator q (therefore, the law is said to be geometric).

Since $\sum p_i = p + pq + \ldots + pq^{m-1} + \ldots = p\left(1 + q + \ldots + q^{m-1} + \ldots\right) = p \cdot \dfrac{1}{1-q} = 1,$ the geometric distribution is well-defined.

A random variable X having a geometric distribution represents the number m of trials which have been carried out under Bernoulli circuit with probability p of occurrence of the event in each trial till the first positive outcome.

Theorem. The mathematical expectation of a random variable X having the geometrical distribution with parameter p is $M(X) = 1/p$, and its dispersion $D(X) = q/p^2$ where $q = 1 - p$.

Example. Testing a big batch of items up to detection of a rejected item (without restriction of the number of tested items) is carried out. Compose the law of distribution of the number of tested items. Find its mathematical expectation and dispersion if it is known that the probability of reject for each item is equal to 0,1.
Solution: The random variable X – the number of tested items up to detection of rejected item – has geometrical distribution with the parameter $p = 0{,}1$. Therefore, the series of distribution has the following form:

x_i	1	2	3	4	…	m	…
p_i	0,1	0,09	0,081	0,0729	…	$0{,}9^{m-1} \cdot 0{,}1$	…

$M(X) = 1/p = 1/0{,}1 = 10$; $D(X) = q/p^2 = 0{,}9/(0{,}1)^2 = 90$.

4. Hypergeometric distribution

Hypergeometric distribution is widely used in practice of statistical acceptance control by quality of industrial production, in the problems connected to the organization of sampling inspections, and other areas.

A discrete random variable X has *the hypergeometric distribution* with parameters n, M, N if it takes on values $0, 1, 2, \ldots, m, \ldots, \min(n, M)$ with the probabilities

$$\boxed{P(X=m)=\frac{C_M^m C_{N-M}^{n-m}}{C_N^n}}$$

where $M \le N$, $n \le N$; n, M, N are natural numbers.

Let there be M standard items in a batch of N items. n items are randomly selected from the batch (each item can be extracted with the same probability), and the selected item is not replaced in the batch before selecting the next item (therefore the Bernoulli formula here is not applicable). Then the random variable X which is the number m of standard items among n selected items has hypergeometric distribution.

Theorem. The mathematical expectation of a random variable X having a hypergeometric distribution with parameters n, M, N is $M(X)=n\dfrac{M}{N}$, and its dispersion $D(X)=n\dfrac{M}{N-1}\left(1-\dfrac{M}{N}\right)\left(1-\dfrac{n}{N}\right)$.

<u>*Example*</u>. In a lottery «Sportloto 6 of 45» monetary prizes are received by participants who have guessed 3, 4, 5 and 6 kinds of sports from randomly selected 6 kinds of 45 (the size of a prize increases with an increasing the number of guessed kinds of sports). Find the law of distribution of a random variable X – the number of guessed kinds of sports among randomly selected 6 kinds. What is the probability of receiving a monetary prize? Find the mathematical expectation and the dispersion of the random variable X.

Solution: Obviously, the number of guessed kinds of sports in the lottery "6 of 45" is a random variable having hypergeometric distribution with the parameters n = 6, M = 6, N = 45. The series of its distribution has the following form:

x_i	0	1	2	3	4	5	6
p_i	0,40056	0,42413	0,15147	0,02244	0,00137	0,00003	0,0000001

The probability of receiving a monetary prize

$$P(3\le X\le 6)=\sum_{i=3}^{6}P(X=i)=0{,}02244+0{,}00137+0{,}00003+0{,}0000001=$$
$$=0{,}02384\approx 0{,}024.$$

$M(X) = n \cdot M/N = 6 \cdot 6/45 = 0{,}8$; $D(X) = 6 \cdot 39/44\,(1 - 39/45)(1 - 6/45) = 0{,}6145$.

Glossary

binomial – биномиальный; **Poisson** – Пуассон
lawsuit – судебный процесс; **tremendous** – огромный
moderate – небольшой, доступный; **diverse areas** – разнообразные области
particle – частица; **to discharge** – разряжать
circuit – схема; **hypergeometric** – гипергеометрический

Exercises for Seminar 9

9.1. A die is tossed three times. Write the law of distribution of the number of appearance of 6.

9.2. Find an average number (mathematical expectation) of typing errors on page of the manuscript if the probability that the page of the manuscript contains at least one typing error is 0,95. It is supposed that the number of typing errors is distributed under the Poisson law (typing error – опечатка; an average number – среднее число).

The answer: 3.

9.3. The switchboard of an enterprise serves 100 subscribers. The probability that a subscriber will call on the switchboard within 1 minute is equal 0,02. Which of two events is more probable: 3 subscribers will call or 4 subscribers will call within 1 minute? (Subscriber – абонент, switchboard – коммутатор).

9.4. A die is tossed before the first landing «six» aces. Find the probability that the first appearance of «six» will take place:

(a) at the second tossing the die;
(b) at the third tossing the die;
(c) at the fourth tossing the die.

The answer: *(a)* 5/36.

9.5. Suppose that a batch of 100 items contains 6 that are defective and 94 that are non-defective. If X is the number of defective items in a randomly drawn sample of 10 items from the batch, find (a) $P(X = 0)$ and (b) $P(X > 2)$.

9.6. There are 7 standard items in a set of 10 items. 4 items are randomly taken from the set. Find the law of distribution of the random variable X equal to the number of standard items among the taken items.

9.7. An urn contains 5 white and 20 black balls. 3 balls are randomly taken from the urn. Compose the law of distribution of the random variable X equal to the number of taken out white balls.

9.8. At horse-racing competitions it is necessary to overcome four obstacles with the probabilities equal 0,9; 0,8; 0,7; 0,6 respectively. At the first failure the sportsman in the further competitions does not participate. Compose the law of distribution of a random variable X – the number of taken obstacles. Find the mathematical expectation of the random variable X (obstacle – препятствие).

The answer: $M(X) = 2{,}4264$.

9.9. Two shooters make on one shot in a target. The probability of hit by the first shooter at one shot is 0,5, and by the second shooter – 0,4.

(a) Find the law of distribution of the random variable X – the number of hits in the target;
(b) Find the probability of the event $X \geq 1$.

The answer: b) 0,7.

9.10. A set of families has the following distribution on number of children:

x_i	x_1	x_2	2	3
p_i	0,1	p_2	0,4	0,35

Determine x_1, x_2, p_2, if it is known that $M(X) = 2$, $D(X) = 0{,}9$.

L E C T U R E 10

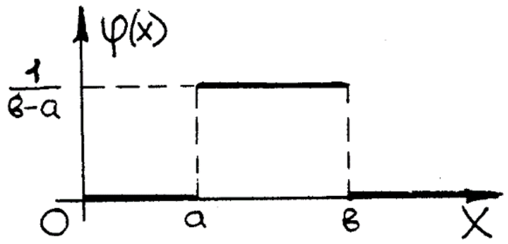
Basic laws of distribution of continuous random variables

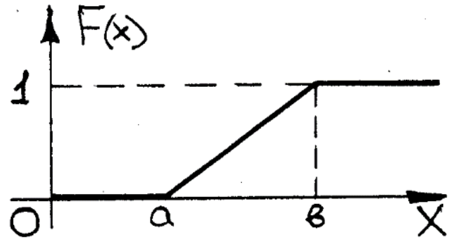
1. The uniform law of distribution

A continuous random variable X has *a uniform law of distribution* on the segment $[a;\ b]$ if its probability density $\varphi(x)$ is a constant on the segment and equals 0 outside of the segment, i.e.

$$\varphi(x)=\begin{cases}\dfrac{1}{b-a} & if \quad a\le x\le b,\\ 0 \quad if & x<a \ or \ x>b.\end{cases}$$

The distribution curve $\varphi(x)$ and the graph of the distribution function $F(x)$ of the random variable X are the following:





Theorem. The distribution function of a random variable X distributed under a uniform law is

$$F(x)=\begin{cases}0 & if \quad x\le a\\ (x-a)/(b-a) & if \quad a<x\le b\\ 1 & if \quad x>b\end{cases}$$

Its mathematical expectation and its dispersion are the following:

$$M(X)=\frac{a+b}{2}, \qquad D(X)=\frac{(b-a)^2}{12}$$

The uniform law of distribution is used at an analysis of mistakes of a rounding off at carrying out of numerical calculations (for example, the mistake of rounding a number to an integer is uniformly distributed in the interval [– 0,5; + 0,5], in a series of problems of mass service, at statistical modeling the observations subordinated to the given distribution.

<u>*Example*</u>. Trains of underground go on a regular basis with an interval of 2 minutes. A passenger leaves on a platform at a random moment of time. What is the probability that the passenger has to wait no more than half of minute? Find the mathematical expectation and the mean square deviation of a random variable X – the time of waiting a train.

Solution: The random variable X – the time of waiting a train on temporal (in minutes) segment [0; 2] has the uniform law of distribution with $\varphi(x)$ = 1/2. Therefore, the probability that the passenger has to wait no more than half of minute is equal to 1/4 of the area of the rectangle, i.e.

$$P(X\le 0{,}5)=\int_0^{0{,}5}\frac{1}{2}dx=\frac{1}{2}x\Big|_0^{0{,}5}=\frac{1}{4};\ M(X)=\frac{0+2}{2}=1$$

$$D(X)=\frac{(2-0)^2}{12}=\frac{1}{3},\ \sigma(X)=\sqrt{D(X)}\approx 0{,}58\ \text{min}.$$

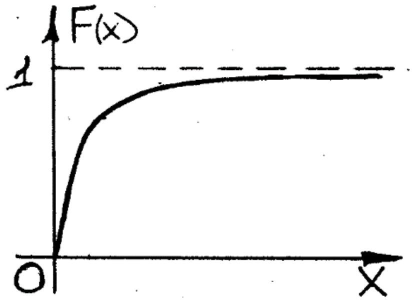
2. Exponential law of distribution

A continuous random variable X has *an exponential law of distribution* with the parameter $\lambda > 0$ if its probability density has the following form:

$$\varphi(x) = \begin{cases} \lambda e^{-\lambda x} & for\ x \geq 0, \\ 0 & for\ \ x < 0. \end{cases}$$

The exponential distribution often arises, in practice, as being the distribution of the amount of time until some specific event occurs. For instance, the amount of time (starting from now) until an earthquake occurs, or until a new war breaks out, or until a telephone call you receive turns out to be a wrong number are all random variables that tend in practice to have exponential distributions.

The distribution curve $\varphi(x)$ and the graph of function of distribution $F(x)$ of the random variable X are the following:



Theorem. The function of distribution of a random variable X distributed under an exponential law is $F(x) = \begin{cases} 0 & for\ x < 0, \\ 1 - e^{-\lambda x} & for\ x \geq 0. \end{cases}$

Its mathematical expectation $M(X) = \dfrac{1}{\lambda}$, and its dispersion $D(X) = \dfrac{1}{\lambda^2}$.

From the theorem follows that for a random variable distributed under an exponential law, the mathematical expectation is equal to the mean square deviation, i.e. $M(X) = \sigma(X) = 1/\lambda$.

The exponential law of distribution plays the big role in the theory of mass service and the theory of reliability.

Example. It has been established that the time of repairing a TV is a random variable X distributed under an exponential law. Determine the probability that it is required no less than 20 days for repairing a TV if the average time of repairing TVs makes 15 days. Find the probability density, the distribution function and the mean square deviation of the random variable X.

Solution: By the hypothesis the mathematical expectation $M(X) = 1/\lambda = 15$, and consequently $\lambda = 1/15$, and the probability density and the distribution function have the following form:

$$\varphi(x) = \frac{1}{15} e^{-\frac{1}{15}x}; \ F(x) = 1 - e^{-\frac{1}{15}x} \ \ (x \geq 0).$$

The required probability $P(X \geq 20)$ can be found by integrating the probability density:

$$P(X \geq 20) = P(20 \leq X < +\infty) = \int_{20}^{+\infty} \varphi(x)dx = \int_{20}^{+\infty} \frac{1}{15} e^{-\frac{1}{15}x} dx$$

But it is simply to do this by using the distribution function:

$$P(X \geq 20) = 1 - P(X < 20) = 1 - F(20) = 1 - (1 - e^{\frac{-20}{15}}) = e^{\frac{-20}{15}} = 0{,}264.$$

Find the mean square deviation: $\sigma(X) = M(X) = 15$ days.

3. Normal law of distribution

The normal law of distribution is most frequently met in practice. The main feature allocating it among other laws is that it is the limiting law to which other laws of distribution are approximated at rather frequently meeting typical conditions.

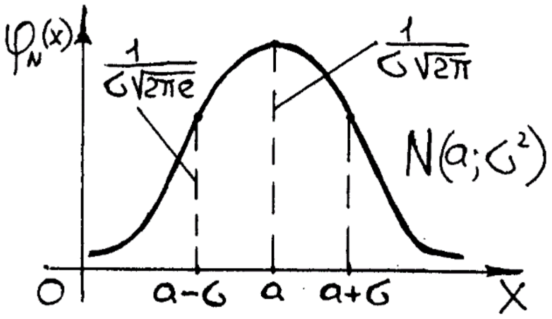
The normal distribution was introduced by the French mathematician Abraham DeMoivre in 1733 and was used by him to approximate probabilities associated with binomial random variables when the binomial parameter n is large. This result was later extended by Laplace and others and is now encompassed in a probability theory known as the central limit theorem, which is discussed later. The central limit theorem, one of the two most important results in probability theory, gives a theoretical base to the often noted empirical observation that, in practice, many random phenomena obey, at least approximately, a normal probability distribution. Some examples of this behavior are the height of a man, the velocity in any direction of a molecule in gas, and the error made in measuring a physical quantity.

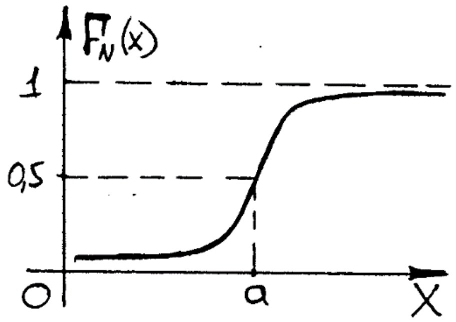
A continuous random variable *X* has *a normal law of distribution (Gauss law)* with the parameters *a* and σ^2 if its probability density has the following form:

$$\boxed{\varphi_N(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-a)^2}{2\sigma^2}}}$$

Theorem. The mathematical expectation of a random variable *X* distributed under a normal law is $M(X) = a$, and its dispersion $D(X) = \sigma^2$.

The normal law of distribution of a random variable with the parameters $a = 0$, $\sigma^2 = 1$, i.e. $N(0; 1)$, is said to be *standard* or *normalized*.





A curve of the normal law of distribution is said to be *normal* or *Gauss curve*.

Theorem. The distribution function of a random variable *X* distributed under a normal law is expressed by the Laplace function $\Phi(x)$:

$$\boxed{F_N(x) = \frac{1}{2} + \Phi\left(\frac{x-a}{\sigma}\right)}$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-t^2/2} dt$.

Property 1. The probability of hit of a random variable *X* distributed under a normal law in the interval $[x_1; x_2]$ is

$$\boxed{P(x_1 \le X \le x_2) = \Phi(t_2) - \Phi(t_1)}$$

where $t_1 = \frac{x_1 - a}{\sigma}$, $t_2 = \frac{x_2 - a}{\sigma}$.

Property 2. The probability that deviation of a random variable X distributed under a normal law from the mathematical expectation a doesn't exceed the quantity $\Delta > 0$ (on absolute value) is equal to

$$\boxed{P(|X - a| \le \Delta) = 2\Phi(t)}$$

where $t = \Delta/\sigma$.

The rule of three sigmas: *If a random variable X has the normal distribution with the parameters a and σ^2, i.e. N(a; σ^2), then it is practically reliable that its values are in the interval (a – 3σ; a + 3σ), i.e.*

$$P(|X - a| < 3\sigma) = 2\Phi\left(\frac{3\sigma}{\sigma}\right) = 2\Phi(3) = 2 \cdot 0{,}4965 = 0{,}9973$$

The violation of «the rule of three sigmas», i.e. the deviation of normally distributed random variable X is more than on 3σ (by absolute value), is an event which is practically impossible since its probability is rather small:

$$P(|X - a| > 3\sigma) = 1 - P(|X - a| \le 3\sigma) = 1 - 0{,}9973 = 0{,}0027.$$

Example. Assuming that the height of men of a certain age group is a normally distributed random variable X with the parameters $a = 173$ and $\sigma^2 = 36$, find:

1. a) the probability density and the distribution function of the random variable X; b) parts of suits of the 4-th height (176-182) and the 3-rd height (170-176) which need to be provided in total amount of a factory for a given age group.
2. Formulate «the rule of three sigmas» for the random variable X.

Solution: 1. a) We have $\varphi_N(x) = \dfrac{1}{6\sqrt{2\pi}} e^{-\frac{(x-173)^2}{2 \cdot 36}}$;

$$F_N(x) = \frac{1}{6\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(x-173)^2}{2\cdot 36}} dx = \frac{1}{2} + \Phi\left(\frac{x-173}{6}\right).$$

b) The part of suits of the 4-th height (176 – 182 cm) in total amount of the factory is determined as probability:

$$P(176 \le X \le 182) = \Phi(t_2) - \Phi(t_1) = \Phi(1{,}50) - \Phi(0{,}50) =$$
$$= 0{,}4332 - 0{,}1915 = 0{,}2417,$$

since $t_1 = \dfrac{176-173}{6} = 0{,}50;\quad t_2 = \dfrac{182-173}{6} = 1{,}50.$

The part of suits of the 3rd height (170 – 176 cm) can be determined analogously, but it is simply to make by the following way:

$$P(170 \le X \le 176) = P(|X - 173| \le 3) = 2\Phi\left(\frac{3}{6}\right) = 2 \cdot 0{,}1915 = 0{,}383.$$

2. It is practically reliable that the height of men of the given age group is enclosed in boundaries from $a - 3\sigma = 173 - 3 \cdot 6 = 155$ to $a + 3\sigma = 173 + 3 \cdot 6 = 191$, i.e. $155 \le X \le 191$ (cm).

Remark. DeMoivre, who used the normal distribution to approximate probabilities connected with coin tossing, called it the exponential bell-shaped curve. Its usefulness, however, became truly apparent only in 1809, when the famous German mathematician K.F. Gauss used it as an integral part of his approach to predicting the location of astronomic entities. As a result, it became common after this time to call the *Gaussian distribution*.

During the mid to late nineteenth century, however, most statisticians started to believe that the majority of data sets would have histograms conforming to the Gaussian bell-shaped form.

Indeed, it came to be accepted that it was “normal” for any well-behaved data set to follow this curve. As a result, following the lead of the British statistician Karl Pearson, people began referring to the Gaussian curve by calling it simply the *normal* curve.

Glossary

uniform – равномерный; **exponential** – показательный; **sigma** – сигма
to round off – округлить; **reliability** – надежность;
violation – нарушение; **part** – доля; **temporal** – временной
bell-shaped – конусообразный; **to encompass** – заключать
to conform – подчиняться; **lead** – первенство

Exercises for Seminar 10

10.1. A random variable X is uniformly distributed in the interval (-2; N). Find:
(a) the differential function of the random variable X;
(b) the integral function;
(c) the probability of hit of the random variable into the interval (-1; N/2);
(d) the mathematical expectation, the dispersion and the mean square deviation of the random variable X. *The answer*: c) 0,5.

10.2. Buses arrive at a specified stop at 15-minute intervals starting at 7 a.m. That is, they arrive at 7, 7:15, 7:30, 7:45, and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 7 and 7:30, find the probability that he waits:
(a) less than 5 minutes for a bus; (b) more than 10 minutes for a bus. *The answer:* a) 1/3; b) 1/3.

10.3. You arrive at a bus stop at 10 o’clock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30.
(a) What is the probability that you will have to wait longer than 10 minutes?
(b) If at 10:15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes? *The answer:* a) 2/3; b) 1/3.

10.4. A random variable X is distributed under an exponential law with parameter $\lambda = 0{,}5$. Find:
(a) the probability density and the distribution function of X;
(b) the probability of hit of the random variable X into the interval (2; 4);
(c) the mathematical expectation, the dispersion and the mean square deviation of X.
The answer: b) 0,233.

10.5. The time (in hours) required to repair a machine is an exponentially distributed random variable with parameter $\lambda = 0{,}5$. What is
(a) the probability that a repair time exceeds 2 hours;
(b) the conditional probability that a repair takes at least 10 hours, given that its duration exceeds 9 hours? *The answer:* a) 0,3679; b) 0,9956.

10.6. Jones figures that the total number of thousands of miles that an auto can be driven before it would need to be junked is an exponential random variable with parameter 1/20. Smith has a used car that he claims has been driven only 10,000 miles. If Jones purchases the car, what is the probability that she would get at least 10,000 additional miles out of it? Repeat under the assumption that the lifetime mileage of the car is not exponentially distributed but rather is (in thousands of miles) uniformly distributed over (0, 40) (to figure – считать; to junk – утилизировать). *The answer:* 0,7614; 0,75.

10.7. A normally distributed random variable X is given by the differential function: $f(x) = \frac{1}{4\sqrt{2\pi}} \cdot e^{-\frac{(x-5)^2}{32}}$. Determine:
(a) the mathematical expectation and the dispersion of the random variable X;
(b) the probability of hit of the random variable X into the interval (3; 9). *The answer*: b) 0,5328.