KBTU KAZAKH-BRITISH TECHNICAL UNIVERSITY

School of Applied Mathematics
Discipline: Linear Algebra for Engineers
Spring Semester 2024. Final Examination
Duration: 120 minutes (2 hours)
Each task is graded by 5 points. The overall score is 40 points

Examination card № 0

1. Solve the system of linear equations by using Cramer's rule: $\begin{cases} 2x_1 + 3x_2 + 5x_3 = 10 \\ 3x_1 + 7x_2 + 4x_3 = 3 \\ x_1 + 2x_2 + 2x_3 = 3 \end{cases}$

2. Find the inverse of the given matrix, if it exists, $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$.

3. Let $f(x) = -5x^3 + 10x$ and $A = \begin{pmatrix} 1 & -2 & 3 \\ -7 & -4 & 6 \\ -1 & 3 & 2 \end{pmatrix}$. Calculate $f(A)$.

4. Let $U = \{(a, b, c, d) \mid a = c, b = 2d, a, b, c, d \in \mathbb{R}\}$. Is U a subspace of $\mathbb{R}^4$? Why? In case, U is a subspace, find its basis.

5. Let $F : \mathbb{R}^3 \to \mathbb{R}^3$ be a mapping defined by $F(x, y, z) = (2x + y, y, -z)$. Is F a linear? Why?

6. Let $S = \{(1, 1, 1), (1, -1, 0), (2, 0, 0)\}$ be a basis of $\mathbb{R}^3$. Find matrix representation of the linear mapping $F(x, y, z) = (2x - y, 3z, y)$ on the basis S.

7. Given the vectors $v_1 = (1, 1, 0), v_2 = (1, 0, 1)$ and $v_3 = (0, 1, 1)$. Transform the vectors into orthogonal vectors using Gram-Schmidt Orthogonalization Process.

8. Let $A = \begin{pmatrix} 0 & -1 & -1 \\ -3 & -1 & -2 \\ 7 & 5 & 6 \end{pmatrix}$. Is A diagonalizable? Why?

Bonus Task: Find all possible Jordan form matrices and corresponding geometric multiplicities of each eigenvalue if the characteristic and minimum polynomials of a linear mapping F are respectively $\Delta_A(\lambda) = (\lambda - 5)^3(\lambda - 2)^3$ and $m(\lambda) = (\lambda - 5)^2(\lambda - 2)^2$.

Good Luck !

Minutes #6 of School of Applied Mathematics meeting, 24.11.2023

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