

**KAZAKH-BRITISH TECHNICAL UNIVERSITY  
RESEARCH AND EDUCATIONAL SCHOOL OF APPLIED MATHEMATICS**



Approved by  
**Dean of the School of Applied  
Mathematics**  
**Artem V. Sinita**  
On 11.01.2024, protocol No. 10

**Syllabus**

Discipline: **Linear Algebra for Engineers**

Semester: Spring 2024

2023/2024 Academic Year

3 credits (2/0/1)

**Lecturer and seminar instructors:**

Candidate of Physical and Mathematical Sciences, Assistant Professor, Tulenbayev Kaisar Maratovich.  
Doctor PhD, Associate Professor, Kalmurzaev Birzhan Seilkhanovich.

Assistant-professor, Bazarbayeva Larissa Ermurzaevna.

PhD candidate, Senior-Lecturer, Rakymzhankyzy Fariza.

Master of sciences in engineering and technologies, Senior-Lecturer, Mukash Nazgul Kanyshkyzy.

Master of Engineering, Lecturer, Yerimbet Meruyert Tastanbekkyzy.

PhD candidate, Lecturer, Askarbekkyzy Aknur.

PhD candidate, Tutor, Nurlanbek Dias.

PhD candidate, Tutor, Alish Daryn.

MSc candidate, Tutor, Iskakov Alibek Masgutovich.

MSc candidate, Tutor, Giniyatov Igluk Bolatovich.

MSc candidate, Tutor, Zhakulov Alshyn Sauletuly.

Personal Information about the Instructor	Time and place of classes		Contact information
	Lessons	Office Hours	e-mail
Kaisar M. Tulenbayev	According to the schedule, wsp.kbtu.kz	Will be announced via M-Teams and wsp.kbtu.kz	<a href="mailto:k.tulenbayev@kbtu.kz">k.tulenbayev@kbtu.kz</a> teacher's room # 313
Birzhan S. Kalmurzaev			<a href="mailto:b.kalmurzaev@kbtu.kz">b.kalmurzaev@kbtu.kz</a> teacher's room # 311
Larissa E. Bazarbayeva			<a href="mailto:l.bazarbaeva@kbtu.kz">l.bazarbaeva@kbtu.kz</a> <a href="mailto:ermurza.larisa@mail.ru">ermurza.larisa@mail.ru</a> teacher's room # 315
Fariza Rakymzhankyzy			<a href="mailto:f.rakymzhankyzy@kbtu.kz">f.rakymzhankyzy@kbtu.kz</a>
Nazgul K. Mukash			<a href="mailto:n.mukash@kbtu.kz">n.mukash@kbtu.kz</a> teacher's room # 311
Meruyert T. Yerimbet			<a href="mailto:m.yerimbet@kbtu.kz">m.yerimbet@kbtu.kz</a>
Aknur Askarbekkyzy			<a href="mailto:a.askarbekkyzy@kbtu.kz">a.askarbekkyzy@kbtu.kz</a>
Dias Nurlanbek			<a href="mailto:d.nurlanbek@kbtu.kz">d.nurlanbek@kbtu.kz</a> teacher's room # 311
Daryn Alish			<a href="mailto:d.alish@kbtu.kz">d.alish@kbtu.kz</a>
Igluk B. Giniyatov			<a href="mailto:i.giniyatov@kbtu.kz">i.giniyatov@kbtu.kz</a> teacher's room # 315

Alshyn S. Zhakulov			<a href="mailto:a.zhakulov@kbtu.kz">a.zhakulov@kbtu.kz</a> teacher's room # 315
Alibek M. Iskakov			<a href="mailto:a.iskakov@kbtu.kz">a.iskakov@kbtu.kz</a> teacher's room # 313

**COURSE DURATION:** 3 credits, 15 weeks, 45 class hours

**COURSE PRE-REQUISITES:** Good knowledge of school mathematics is enough.

## COURSE DESCRIPTION

The goals of the course are to familiarize students with the important branches of Linear Algebra.

### Course objectives

This course is designed to meet the following program objectives:

- To use mathematically correct language and notation for Linear Algebra.
- To become computational proficiency involving procedures in Linear Algebra.
- To understand the axiomatic structure of a modern mathematical subject and learn to construct simple proofs.
- To solve problems that apply Linear Algebra to Chemistry, Economics and Engineering.

The topics that will enable this course to meet its objectives are:

- the basic arithmetic operations on vectors and matrices, including inversion and determinants, using technology where appropriate;
- solving systems of linear equations, using technology to facilitate row reduction;
- the basic terminology of linear algebra in Euclidean spaces, including linear independence, spanning, basis, rank, nullity, subspace, and linear transformation;
- the abstract notions of vector space and inner product space;
- finding eigenvalues and eigenvectors of a matrix or a linear transformation, and using them to diagonalize a matrix;
- projections and orthogonality among Euclidean vectors, including the Gram-Schmidt orthonormalization process and orthogonal matrices;
- the common applications of Linear Algebra, possibly including Markov chains, areas and volumes, Cramer's rule, the adjoint, and the method of least squares;
- the nature of a modern mathematics course: how abstract definitions are motivated by concrete examples, how results follow from the axiomatic definitions and are specialized back to the concrete examples, and how applications are woven in throughout. This course will present various "characterization" theorems (e.g. characterizing isomorphic finite-dimensional vector spaces by their dimension and characterizing invertible matrices by various criteria);
- basic proof and disproof techniques, including mathematical induction, verifying that axioms are satisfied, standard "uniqueness" proofs, proof by contradiction, and disproof by counterexample.

### Course outcomes

Upon successful completion of this course, students will:

- Solve systems of linear equations using multiple methods, including Gaussian elimination and matrix inversion.
- Carry out matrix operations, including inverses and determinants.
- Demonstrate understanding of the concepts of vector space and subspace.
- Demonstrate understanding of linear independence, span, and basis.
- Determine eigenvalues and eigenvectors and solve eigenvalue problems.
- Apply principles of matrix algebra to linear transformations.
- Demonstrate understanding of inner products and associated norms.

**Knowledge:** during the study of this course, students must obtain knowledge about how to explain with examples the basic terminology of linear systems, vector spaces, maps between spaces, isomorphism and homomorphism, determinants, similarity, quadratic forms and polynomials.

**Skills:** As a result of studying this course, students must be able to

- Solve systems of linear equations using various methods including Gaussian and Gauss-Jordan elimination and inverse matrices.
- Perform matrix algebra, invertibility, and the transpose and understand vector algebra in  $R^n$
- Determine relationship between coefficient matrix invertibility and solutions to a system of linear equations and the inverse matrices.
- Define special matrices: diagonal, triangular, and symmetric.
- Understand determinants and their properties.
- Understand real vector spaces and subspaces and apply their properties.
- Understand linear independence and dependence.
- Find basis and dimension of a vector space and understand change of basis.
- Find a basis for the row space, column space and null space of a matrix and find the rank and nullity of a matrix.
- Compute linear transformations, kernel and range, and inverse linear transformations, and find matrices of general linear transformations.
- Find the dimension of spaces such as those associated with matrices and linear transformations.
- Find eigenvalues and eigenvectors and use them in applications.
- Diagonalize, and orthogonally diagonalize symmetric matrices.
- Evaluate the dot product, norm, angle between vectors, and orthogonality of two vectors in  $R^n$ .
- Compute inner products on a real vector space and compute angle and orthogonality in inner product spaces.
- Create orthogonal and orthonormal bases: Gram-Schmidt process and use bases and orthonormal bases to solve application problems.
- Prove basic results in linear algebra using appropriate proof-writing techniques such as linear independence of vectors; properties of subspaces; linearity, injectivity and surjectivity of functions; and properties of eigenvectors and eigenvalues.

## REFERENCES

### Main:

1. Jim Hefferon, Linear Algebra. – Virginia Commonwealth University Mathematics (June 1, 2009), ISBN-10: 0982406215, 450 pages. <https://www.amazon.com/Linear-Algebra-Jim-Hefferon/dp/0982406215>
2. Seymour Lipschutz, Marc Lars Lipson, Linear Algebra, Fourth Edition. – Schaum's Outline Series, The McGraw-Hill Companies, Inc., 2009. eBook also appears in the print version of this title: ISBN: 978-0-07-154352-1, MHID: 0-07-154352-X.

### Supplementary

3. David C. Lay, Steven R. Lay, Judi J. McDonald. Linear algebra and its applications, 5th edition. Pearson Education Limited, 2016, 579 pages.
4. Badaev S. A., Issakhov As. A., Mukash N. K., Ostemirova U. B., Rakhymzhankyzy F. – Analytic Geometry. Exercises, ISBN 978-601-269-090-3, 150 pages.

## COURSE CALENDAR

Week №	General Information				SIS and assessments
	Lessons' content	Lecture classes	Seminar classes	Topics and materials to study	
1	Linear Systems: solving linear systems.	2	1	Lecture notes and [1, Chapter 1]: 1.1, 1.2, 1.3.	SIS 1
2	Linear Systems: linear geometry; reduced echelon form.	2	1	Lecture notes and [1, Chapter 1]: 2.1, 3.1, 3.2.	SIS 2
3	Vector Spaces: definition of vector space; linear independence.	2	1	Lecture notes and [1, Chapter 2]: 1.1, 1.2, 2.1.	SIS 3
4	Vector Spaces: basis and dimension.	2	1	Lecture notes and [1, Chapter 2]: 3.1, 3.2, 3.3.	<b>Quiz-1</b>
5	Maps Between Spaces: isomorphism; homomorphism.	2	1	Lecture notes and [1, Chapter 3]: 1.1, 1.2, 2.1, 2.2,	SIS 4
6	Maps Between Spaces: computing linear maps; matrix operations.	2	1	Lecture notes and [1, Chapter]: 3: 3.1, 3.2, 4.1, 4.2, 4.4.	SIS 5
7	Maps Between Spaces: change of basis.	2	1	Lecture notes and [1, Chapter 3]: 5.1, 5.2,	<b>Midterm assessment</b>

8	Maps Between Spaces: projection, Gram-Schmidt Or- thogonalization, pro- jection into a subspace	2	1	Lecture notes and [1, Chapter 3]: 6.1, 6.2, 6.3	SIS 6
9	Determinants: defini- tion, properties.	2	1	Lecture notes and [1, Chapter 4]: 1.1, 1.2, 1.3.	SIS 7
10	Determinants: geome- try of determinants; Laplace's expansion, Cramer's rule.	2	1	Lecture notes and [1, Chapter 4]: 2.1, 3.1.	SIS 8
11	Similarity: complex vector space.	2	1	Lecture notes and [1, Chapter 5]: 1.1, 1.2.	<b>Quiz-2</b>
12	Similarity: diagonali- zability, eigenvalues and eigenvectors.	2	1	Lecture notes and [1, Chapter 5]: 2.1, 2.2, 2.3.	SIS 9
13	Similarity: nilpotence.	2	1	Lecture notes and [1, Chapter 5]: 3.1, 3.2.	SIS 10
14	Similarity: Jordan form.	2	1	Lecture notes and [1, Chapter 5]: 4.1, 4.2	<b>Endterm assessment</b>
15	Quadratic forms and polynomials.	2	1	Lecture notes	SIS 11



## COURSE ASSESSMENT PARAMETERS

The tentative timetable of exams and tasks:

№	Assessment Criteria	Week №														Total
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	
1	Activity on practice classes	DW <sup>1</sup>	0.5	0.5	Q-1	0.5	0.5	MT	0.5	0.5	Q-2	0.5	0.5	ET	0.5	5 <sup>2</sup>
2	Homeworks and IWS <sup>2</sup>	DW <sup>1</sup>	1	1	*	1	1	*	1	1	*	1	1	*	1	10
3	Midterm	*	*	*	*	*	*	15	*	*	*	*	*	*	*	15
4	Endterm	*	*	*	*	*	*	*	*	*	*	*	*	15	*	15
5	Quizzes	*	*	*	10	*	*	*	*	*	10	*	*	*	*	20
6	Final examination	*	*	*	*	*	*	*	*	*	*	*	*	*	*	40
5	Total	*	*	*	*	*	*	*	*	*	*	*	*	*	*	100(5) <sup>4</sup>

<sup>1</sup>DW – Drop Week

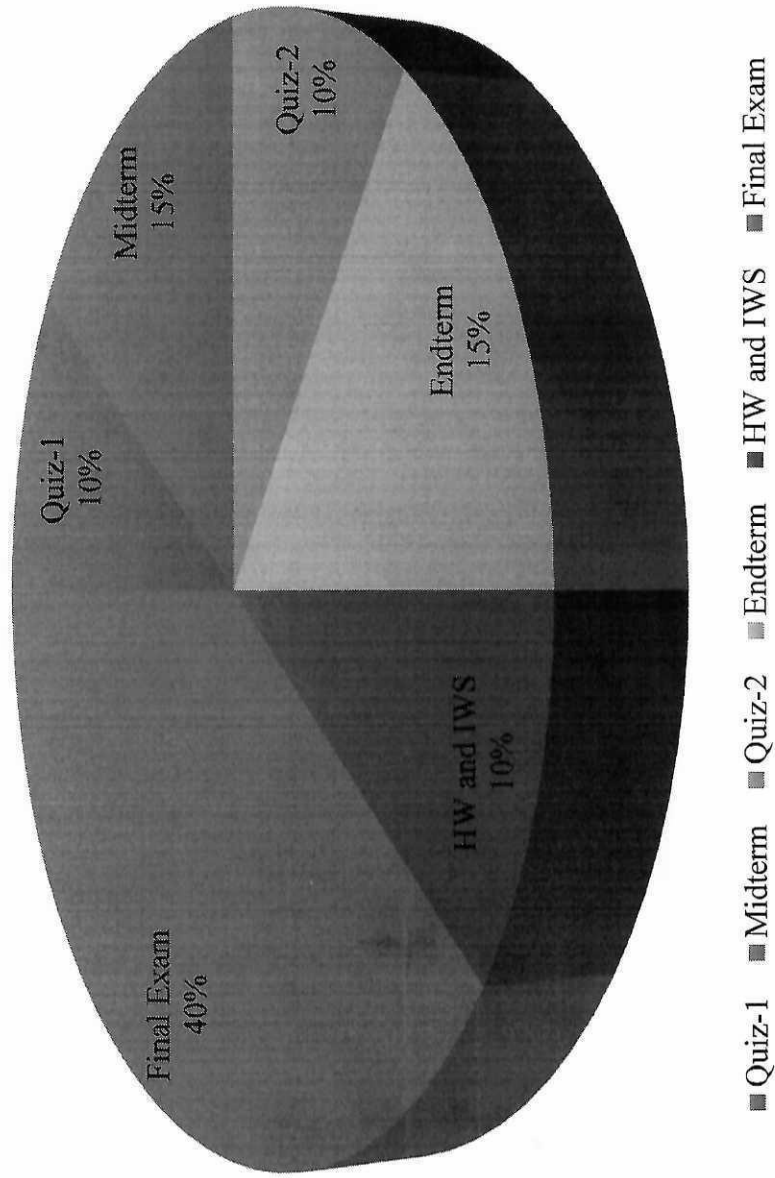
<sup>2</sup>Seminar activity bonus points

<sup>3</sup>IWS – Individual Work of Student

<sup>4</sup>Total bonus points

Grading criteria:

**Percentage**



**Lectures** are conducted in the form of explaining the theory given in the course, that is why students are supplied with handouts uploaded onto the intranet. Activity and attendance in lessons is mandatory. Mandatory requirement is preparation for each lesson.

**Grading policy:**

Intermediate attestations (on 7<sup>th</sup> and 14<sup>th</sup> week) join topics of all lectures, laboratories, homework, quiz and materials for reading discussed to the time of attestation. The maximum number of points within attendance, activity, homework, quiz and laboratories for each attestation is 30 points. Every intermediate attestation and quiz are held during practice hours in the university.

Final exam joins and generalizes all course materials, is conducted in the complex form with questions and problems. Final exam duration is 120 min. Maximum number of points is 40. At the end of the semester, you receive an overall total grade (summarized index of your work during semester) according to conventional KBTU grade scale.

**ACADEMIC POLICY**

**Students are required:**

- to be respectful to the teacher and other students;
- to switch off mobile phones during classes;
- DO NOT cheat. Plagiarized papers shall be graded with zero points!
- to come to classes prepared and actively participate in classroom work; to meet the deadlines;
- to enter the room before the teacher starts the lesson;
- to attend all classes. No make-up tests or quiz are allowed unless there is a valid reason for missing it;
- to follow KBTU academic policy regarding **W, AW, I, FX, F** grades.
- When students are absent for 30% of the lessons or more, then their grade is **F**.
- When students have a score of 29 or less for attestation 1 added to attestation 2, then their grade is **F**.
- When students have a score between 9,5 and 19,4 for their final exam, then their grade is **FX**.
- When students have a score of 9,4 or less (less for their final exam, then their grade is **F**.
- When students do not come for their final exam, then their grade is **F**.

**Students are encouraged to**

- consult the teacher on any issues related to the course;
- make up within a week's time for the works undone for a valid reason without any grade deductions;

Candidate of Phys.-Math. Sciences, Assistant Professor, Kaisar M. Tulenbayev \_\_\_\_\_

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Master of Engineering, Lecturer, Meruyert T. Yerimbet \_\_\_\_\_

PhD candidate, Lecturer, Aknur Askarbekkyzy \_\_\_\_\_

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