KAZAKH-BRITISH TECHNICAL UNIVERSITY
RESEARCH AND EDUCATIONAL CENTER OF MATHEMATICS AND CYBERNETICS

Approved by
Dean of School of Applied Mathematics
A.V. Sinitsa
On 11.01.2024 protocol № 10

Syllabus
Discipline: **Calculus II**
Semester: Spring 2024
2023/2024 Academic Year
4 credits (2/0/2)

COURSE DURATION: 4 credits, 15 weeks, 60 class hours

COURSE DESCRIPTION
The goals of the course are to familiarize students with the important branches of Calculus.

Course Objectives:
Calculus II is a mandatory component of all Bachelor programs at KBTU. This course is usually delivered at the 2nd semester of the undergraduate education.
Calculus II is one of the important parts of the mathematical background required by mathematicians, engineers, computer scientists, physicists, economists, statisticians and etc.
This course is mostly considered as an extension of the Calculus I for the case of functions of severable variables. Main topics of the course are:

- Numerical Integration
- Sequences and Series; Power Series; Taylor Series and Convergence;
- Parametrizations of Plane Curves; Calculus with Parametric Curves;
- Vector-valued Functions;
- Differential Calculus of Functions of Many Variables;

Upon successful completion of this course,
Students will be able to:

- operate with sequences and series
- operate with parametrizations of plane curves
- operate with vector-valued functions
- evaluate the derivatives of functions of many variables

Competencies (learning outcomes):
At the end of the course, students are expected to:

- evaluate the limits of sequences;
- determine convergence of series and find sums of series;
- evaluate the derivatives of functions of many variables;
- apply the derivatives to extreme problems.

Prerequisites: Calculus 1
Post-requisitions: Algorithms and programming languages, Data analysis

REFERENCES

Main:
1. George B. Thomas Jr., Maurice D. Weir, Joel Hass. Thomas' Calculus (early transcendentals), 13th edition, Pearson, 2014, 1205 pages.
2. Vladimir A. Zorich, Mathematical Analysis 1. Springer-Verlag, 2003, 597 pages.

Supplementary:
3. B.P. Demidovich, Collection of Problems and Exercises in Mathematical Analysis, Moscow: Moscow University, 13th edition, 1997, 624 pages (in Russian).

COURSE CALENDAR

Week	Class work			
	Topic	Lectures	Seminars	Chapters for reading
1	**Numerical Integration.** Trapezoidal Approximations. Simpson's Rule: Approximations Using Parabolas.	2	2	[1], Ch. 8.7, pp. 494-501. Problem Set: pp. 501-504.
2	**Infinite Sequences and Series.** Convergence and Divergence. Calculating Limits of Sequences. Bounded Monotonic Sequences. The Monotonic Sequence Theorem.	2	2	[1], Ch. 10.1, pp. 572-581. Problem Set: pp. 581-584.
3	**Infinite Series.** Definitions. Geometric Series. The nth-Term Test for a Divergent Series. Combining Series. Adding or Deleting Terms. **The Integral Test**. Non-decreasing Partial Sums. Bounds for the Remainder in the Integral Test. Comparison Tests.	2	2	[1], Ch. 10.2, 10.3, pp. 584-591, 593-598. Problem Set: pp. 591-593, 598-599.
4	**The Ratio and Root Tests.** Comparison Tests. Alternating Series, Absolute and Conditional Convergence.	2	2	[1], Ch. 10.5, 10.6, pp. 604-609, 610-614. Problem Set: pp. 609-610, 615-616.
5	**Power Series** and Convergence. Definitions. Examples. The Convergence Theorem for Power Series. The Radius of Convergence of a Power Series. Operations on Power Series.	2	2	[1], Ch. 10.7, pp. 616-624. Problem Set: pp. 624-626.
6	**Taylor and Maclaurin Series**: Definitions. Taylor Polynomials: Definition and Examples. Convergence of Taylor Series.	2	2	[1], Ch. 10.8, 10.9, pp. 626-630, 631-637. Problem Set: pp. 630-631, 637-638.
7	**Parametrizations of Plane Curves.** Calculus with Parametric Curves.Tangents and Areas. Parametric Formulas for the 1st and 2nd Derivatives. Length of a Parametrically Defined Curve. Length of a Curve.	2	2	[1], Ch. 11.1, 11.2, pp. 1653-658, 661-669. Problem Set: pp. 659-661, 669-671.

8	**Vector-valued Functions**. Curves in Space and Their Tangents. Limits and Continuity. Derivatives and Motion. Differentiation Rules. Integrals of Vector Functions. Arc length in Space.	2	2	[1], Ch. 13.1, 13.2, 13.3, pp.751-757, 759-764, 768-771. Problem Set: pp. 757-759, 765-768, 771-772.
9	**Functions of several variables**. Domains and Ranges. Functions of Two Variables. Graphs, Level Curves, and Contours of Functions of Two Variables. Functions of Three Variables. Limits and Continuity in Higher Dimensions. Limits for Functions of Two Variables.	2	2	[1], Ch.14.1, 14.2, pp. 793-798, 801-807. Problem Set: pp. 799-801, 807-810.
10	**Partial Derivatives of a Function of Two Variables**: Definitions and Calculations. Continuity. Second-Order Partial Derivatives. The Mixed Derivative Theorem. Differentiability. **The Chain Rule**. Chain Rule for Functions of Two Intermediate Variables. Chain Rule for Functions of Three Intermediate Variables. Functions Defined on Surfaces. Chain Rule for Two Independent Variables and Three Intermediate Variables. A Formula for Implicit Differentiation.	2	2	[1], Ch. 14.3, 14.4, pp. 810-819. 821-828. Problem Set: pp. 819-821, 828-830.
11	**Directional Derivatives and Gradient Vectors**. Interpretation of the Directional Derivative. Calculation and Gradients: The Directional Derivative Is a Dot Product. Properties of the Directional Derivative. Gradients and Tangents to Level Curves.	2	2	[1], Ch. 14.5, pp. 830-837. Problem Set: p. 838.
12	**Tangent Planes and Differentials**. Tangent Planes and Normal Lines. How to Linearize a Function of Two Variables. Differentials. Total Differential. Extreme Values and Saddle Points.	2	2	[1], Ch. 14.6, 14.7, pp. 839-845, 848-854. Problem Set: pp. 845-848, 855-857.
13	**Lagrange Multipliers.** Constrained Maxima and Minima. The Method of Lagrange Multipliers. The Orthogonal Gradient Theorem. Lagrange Multipliers with Two Constraints. Taylor's Formula for Two Variables. Derivation of the Second Derivative Test. Partial Derivatives with Constrained Variables.	2	2	[1], Ch. 14.8, 14.9, 14.10, pp. 857-864, 866-870, 870-874. Problem Set: pp. 864-866, 870, 874-875.
14	**Multiple integrals.** Double Integrals. Double Integrals as Volumes. Fubini's Theorem. Double Integrals over Bounded, Nonrectangular Regions. Finding Limits of Integration. Properties of Double Integrals. Area by Double Integration. Areas of Bounded Regions in the Plane. Average Value	2	2	[1], Ch. 15.1, 15.2, 15.3, pp. 882-886, 887-893, 896-899. Problem Set: pp. 886-887, 894-896, 899.
15	**Multiple integrals.** Triple Integrals. Volume of a Region in Space. Finding Limits of Integration in the Order dz dy dx. Average Value of a Function in Space. Properties of Triple Integrals. Moments and Centers of Mass. Masses and First Moments. Moments of Inertia.	2	2	[1], Ch. 15.5, 15.6, pp. 906-912, 915-920. Problem Set: pp. 912-915, 920-922.

COURSE ASSESSMENT PARAMETERS

Attendance	0 %
Activity on lessons	8 %
Home works	14 %
Quizes and mid/end-terms	38 %
Final exam	40 %
Total	**100 %**

№	Assessment criteria	Weeks															Total
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16 17	
1.	Attendance	has to be more than 70%															
2.	Activity on lessons	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1		8
3.	Home works		1	1	1	1	1	1	1	1	1	1	1	1	1	1		14
4.	Quizes and mid/end-terms				8			10				10			10			38
5.	Final examination																40	40
	Total															60	40	100

Grading policy:

Intermediate attestations (on 7th and 14th week) join topics of all lectures, laboratories, homework, quiz and materials for reading discussed to the time of attestation. Maximum number of points within attendance, activity, homework, quiz and laboratories for each attestation is 30 points.

Final exam joins and generalizes all course materials, is conducted in the complex form with questions and problems. Final exam duration is 120 min. Maximum number of points is 40. At the end of the semester you receive overall total grade (summarized index of your work during semester) according to conventional KBTU grade scale.

ACADEMIC POLICY

Students are required:

- to be respectful to the teacher and other students;
- to switch off mobile phones during classes;
- DO NOT cheat. Plagiarized papers shall be graded with zero points;
- to come to classes prepared and actively participate in classroom work; to meet the deadlines;
- to enter the room before the teacher starts the lesson;
- to attend all classes. No make-up tests or quiz are allowed unless there is a valid reason for missing it;
- to follow KBTU academic policy regarding **W, AW, I, F** grades;
- When students are absent for 30% of the lessons or more, then their grade is F;
- When students have a score of 29 or less for attestation 1 added to attestation 2, then their grade is F;
- When students have a score of 19 or less (less than 50%) for their final exam, then their grade is F;

- When students do not come for their final exam, then their grade is F.

Students are encouraged to

- consult the teacher on any issues related to the course;
- make up within a week's time for the works undone for a valid reason without any grade deductions;

Lecturer _____ Yeldos Zhandaulet