

1. Determine which of the following statements is true after one step of evaluation using Small-Step (Transition) Semantics.

$$(a) \langle \text{while } x > 5 \text{ do } x := x - 1 \text{ od}, \{x := 10\} \rangle \\ \longrightarrow \langle \text{while } x > 5 \text{ do } x := x-1 \text{ od}, \{x := 9\} \rangle$$

False

$$(b) \langle \text{while } x > 5 \text{ do } x := x - 1 \text{ od}, \{x := 10\} \rangle \\ \longrightarrow \langle x := x - 1; \text{ while } x > 5 \text{ do } x := x - 1 \text{ od}, \{x := 10\} \rangle$$

True

$$(c) \langle \text{while } x > 5 \text{ do } x := x - 1 \text{ od}, \{x := 10\} \rangle \\ \longrightarrow \langle x := x - 1, \{x := 10\} \rangle$$

False

$$(d) \langle \text{while } x > 5 \text{ do } x := x - 1 \text{ od}, \{x := 10\} \rangle \\ \longrightarrow \langle \text{while } x > 5 \text{ do } x := x - 1 \text{ od}, \{x := 10\} \rangle$$

False

2. Determine which of the following statements is true after one step of evaluation using Small-Step (Transition) Semantics.

$$(a) \langle \text{if } x > 10 \text{ then } x := 10 \text{ else skip}, \{x \mapsto 11\} \rangle \\ \longrightarrow \langle x > 10, \{x \mapsto 11\} \rangle$$

False

$$(b) \langle \text{if } x > 10 \text{ then } x := 10 \text{ else skip}, \{x \mapsto 11\} \rangle \\ \longrightarrow \langle \text{skip}, \{\} \rangle$$

True

$$(c) \langle \text{while } x < 5 \text{ do } x := x - 1 \text{ od}, \{x \mapsto 10\} \rangle \\ \longrightarrow \langle E, \{x \mapsto 10\} \rangle$$

True

$$(d) \langle \text{while true do } x := 5 \text{ od}, \{\} \rangle \\ \longrightarrow \langle x := 5; \text{ while true do } x := 5 \text{ od}, \{\} \rangle$$

True

3. Complete the proof below.

$$\begin{array}{c}
 \frac{}{(x, \{x \mapsto 4\}) \Downarrow 4} \text{Var} \quad \frac{}{(5, \{x \mapsto 4\}) \Downarrow 5} \text{Int} \quad \frac{}{(x, \{x \mapsto 4\}) \Downarrow 4} \text{Var} \quad \frac{}{(3, \{x \mapsto 4\}) \Downarrow 3} \text{Int} \\
 \hline
 \frac{}{(x > 5, \{x \mapsto 4\}) \Downarrow \text{false}} \text{RelOp} \quad \frac{}{(x * 3, \{x \mapsto 4\}) \Downarrow 12} \text{BinOp} \\
 \hline
 \frac{}{(y := x * 3, \{x \mapsto 4\}) \Downarrow \{x \mapsto 4, y \mapsto 12\}} \text{Asgn} \\
 \hline
 \frac{}{(\text{if } x > 5 \text{ then } y := x * 5 \text{ else } y := x * 3 \text{ fi}, \{x \mapsto 4\}) \Downarrow \{x \mapsto 4, y \mapsto 12\}} \text{If}
 \end{array}$$

4. Complete the proof below.

$$\begin{array}{c}
 \frac{}{(x, \{x \mapsto 4\}) \Downarrow 4} \text{Var} \quad \frac{}{(1, \{x \mapsto 4\}) \Downarrow 1} \text{Int} \\
 \hline
 \frac{}{(x + 1, \{x \mapsto 4\}) \Downarrow 5} \text{BinOp} \quad \frac{}{(x, \{x \mapsto 5\}) \Downarrow 5} \text{Var} \\
 \hline
 \frac{}{(x := x + 1, \{x \mapsto 4\}) \Downarrow \{x \mapsto 5\}} \text{Asgn} \quad \frac{}{(y := x, \{x \mapsto 5\}) \Downarrow \{x \mapsto 5, y \mapsto 5\}} \text{Asgn} \\
 \hline
 \frac{}{(x < 5, \{x \mapsto 4\}) \Downarrow \text{true}} \text{RelOp} \quad \frac{}{(x := x + 1; y := x, \{x \mapsto 4\}) \Downarrow \{x \mapsto 5, y \mapsto 5\}} \text{Seq} \quad \frac{}{(x < 5, \{x \mapsto 5; y \mapsto 5\}) \Downarrow \text{false}} \text{RelOp} \\
 \hline
 \frac{}{(\text{while } x < 5 \text{ do } x := x + 1; y := x \text{ od}, \{x \mapsto 4\}) \Downarrow \{x \mapsto 5, y \mapsto 5\}} \text{While}
 \end{array}$$

5. Given the following Hoare Logic statement, fill in the blanks appropriately.

$$\{ \boxed{A} \} x := x * 2; \{ \boxed{B} \} y := y * 2 \{x = 10; y = 20\}$$

- (a) The box for A should be filled in with: $x := 5; y := 10$
 - (b) The box for B should be filled in with: $x := 10; y := 10$
6. Given the following Hoare Logic statement, fill in the blanks appropriately.

$$\{ \boxed{\dots} \} \text{ if } x > 0 \text{ then } \{ \boxed{A} \} x := x * 2 \{x > 10\} \\ \text{ else } \{ \boxed{B} \} x := -x \{x > 10\} \text{ fi } \{ \boxed{\dots} \}$$

- (a) The box for A should be filled in with: $x > 5$
- (b) The box for B should be filled in with: $x < -10$