Solution: Homework 1 CS 421: Homework 1

It's worth noting that white space does not matter in lambda calculus; the body of lambda abstractions extend as far right as possible regardless of spacing. For example, the lambda abstraction $(\lambda x.x\ y\ z\ a)$ has a body from x to a.

1.
$$(\lambda x.x) \ y \xrightarrow{\beta} y$$

2.
$$(\lambda x.y) \ x \xrightarrow{\beta} y$$

3.
$$(\lambda x.x \ y) \ (\lambda y.y \ z) \xrightarrow{\beta} (\lambda y.y \ z)y \xrightarrow{\beta} y \ z$$

4.
$$(\lambda x.x\ y)\ (\lambda a.a\ b)\ p \xrightarrow{\beta} ((\lambda a.a\ b)y)p \xrightarrow{\beta} (y\ b)\ p = y\ b\ p$$

5.
$$(\lambda x.x \ y) \ (\lambda a.b \ a) \ p \xrightarrow{\beta} ((\lambda a.b \ a)y)p \xrightarrow{\beta} (b \ y) \ p = b \ y \ p$$

6.
$$(\lambda x.(\lambda y.x\ y))\ y \xrightarrow{\alpha} (\lambda x.(\lambda a.x\ a))\ y \xrightarrow{\beta} (\lambda a.y\ a)$$

7.
$$(\lambda x.y \ x) \ y \xrightarrow{\beta} y \ y$$

8.
$$(\lambda x.\lambda y.x\ y\ z)\ (\lambda x.x\ y)\ z\xrightarrow{\alpha} (\lambda x.\lambda a.x\ a\ z)\ (\lambda x.x\ y)\ z\xrightarrow{\beta} (\lambda a.(\lambda x.x\ y)\ a\ z)\ z$$

$$\xrightarrow{\beta} ((\lambda x.x\ y)\ z\ z)\xrightarrow{\beta} (z\ y)\ z=z\ y\ z$$

9.
$$(\lambda x.y \ x) \ x \xrightarrow{\beta} y \ x$$

10.
$$(\lambda y.y \ x) \ (\lambda z.z \ y) \xrightarrow{\beta} (\lambda z.z \ y)x \xrightarrow{\beta} x \ y$$