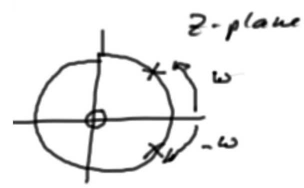
Z-Transform of Sinusoidal Sequence

- We consider the sinusoidal sequence $x[n] = (\sin \omega n)u[n]$; note that

$\sin \omega n = \left(e^{j\omega n} - e^{-j\omega n}\right)/2j$

$$
\begin{aligned}
X(z) &= \sum_{n=0}^{\infty} \sin[\omega n] z^{-n} \\
&= \sum_{n=0}^{\infty} \left(\frac{e^{jwn} - e^{-jwn}}{2j}\right) z^{-n} \\
&= \frac{1}{2j}\sum_{n=0}^{\infty} e^{j\omega n} z^{-n} - \frac{1}{2j}\sum_{n=0}^{\infty} e^{-j\omega n} z^{-n} \\
&= \frac{1}{2j}\left(\frac{z}{z-e^{j\omega}}\right) - \frac{1}{2j}\left(\frac{z}{z-e^{-j\omega}}\right) \\
&= \frac{z \sin \omega}{(z-e^{j\omega})(z-e^{-j\omega})} \qquad |z| > 1
\end{aligned}
$$



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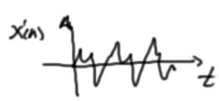
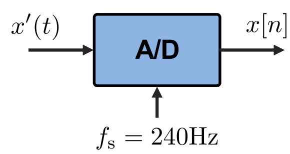
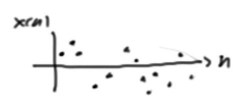
Interference Cancellation

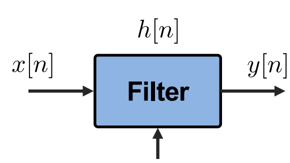




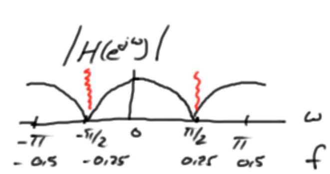


- Problem: $x'(t)$ is contaminated with $f_i = 60\text{Hz}$ interference
- For $f_s = 240\text{Hz}$, 60Hz interference is sampled 4 times per cycle, i.e., $f = 0.25\frac{\text{cycles}}{\text{samples}}$

$\omega_i = 2\pi \frac{f_i}{f_s} = 2\pi \frac{60}{240} = \frac{\pi}{2} \frac{\text{rad}}{\text{samples}}$



place zeros on interference



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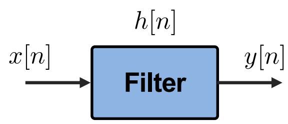
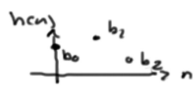
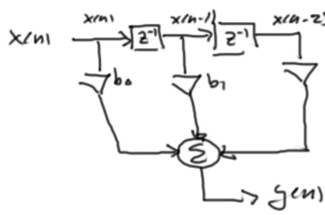
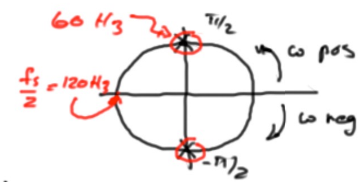
Frequency Domain Representation









O zero locations to cancel interference poles

$$y[n] = b_0x[n] + b_1x[n-1] + b_2x[n-2]$$

$$Y(z) = b_0X(z) + b_1z^{-1}X(z) + b_2z^{-2}X(z)$$
$$= (b_o + b_1z^{-1} + b_2z^{-2})X(z)$$

$$z_1, z_2 = \frac{-b_1 \pm \sqrt{b_1^2 - 4b_0b_2}}{2b_0}$$

$$H(z) = \frac{Y(z)}{X(z)} = b_0 + b_1z^{-1} + b_2z^{-2} = \frac{b_0z^2 + b_1z + b_2}{z^2} = \frac{(z-z_1)(z-z_2)}{z^2}$$

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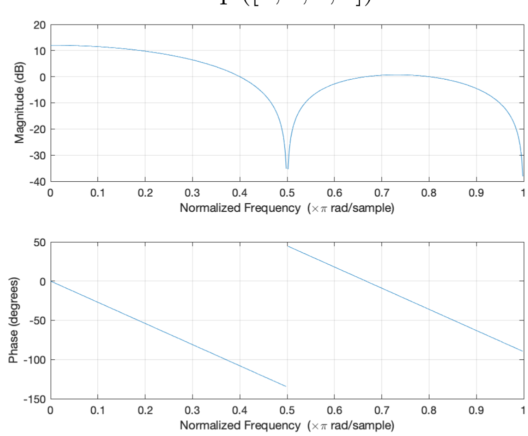
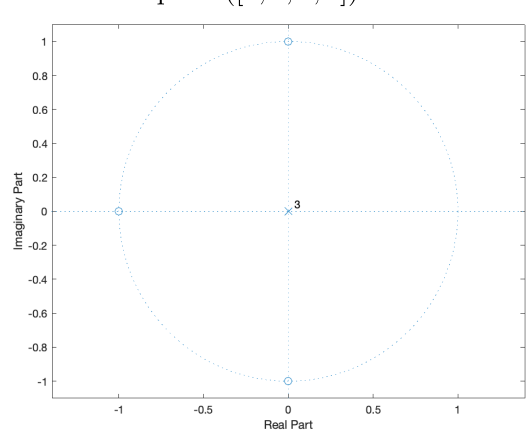
Z-Transform Examples

$$H(z) = \frac{z^3 + z^2 + z + 1}{z^3}$$

freqz([1, 1, 1, 1])



zplane([1, 1, 1, 1])



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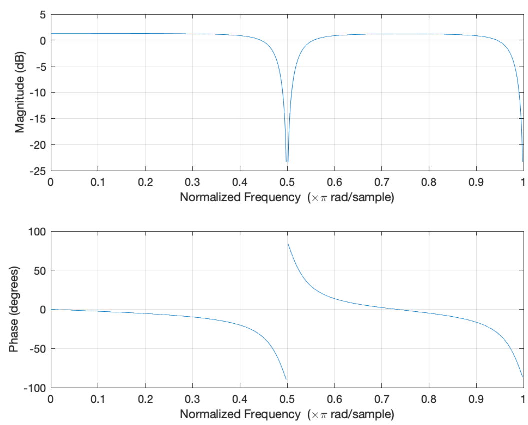
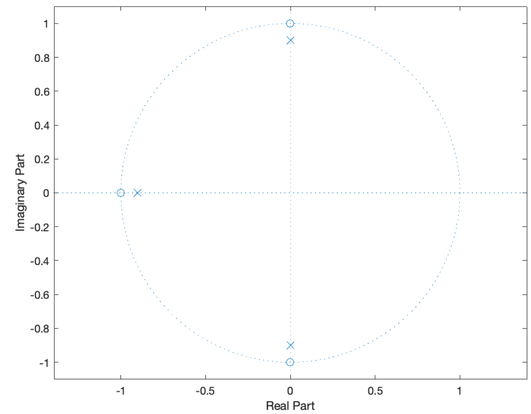
Z-Transform Examples

$$H(z) = \frac{z^3 + z^2 + z + 1}{z^3 + 0.9z^2 + 0.81z + 0.729}$$

$\text{freqz}([1,1,1,1],[1,0.9,0.81,0.729])$



$\text{zplane}([1,1,1,1],[1,0.9,0.81,0.729])$



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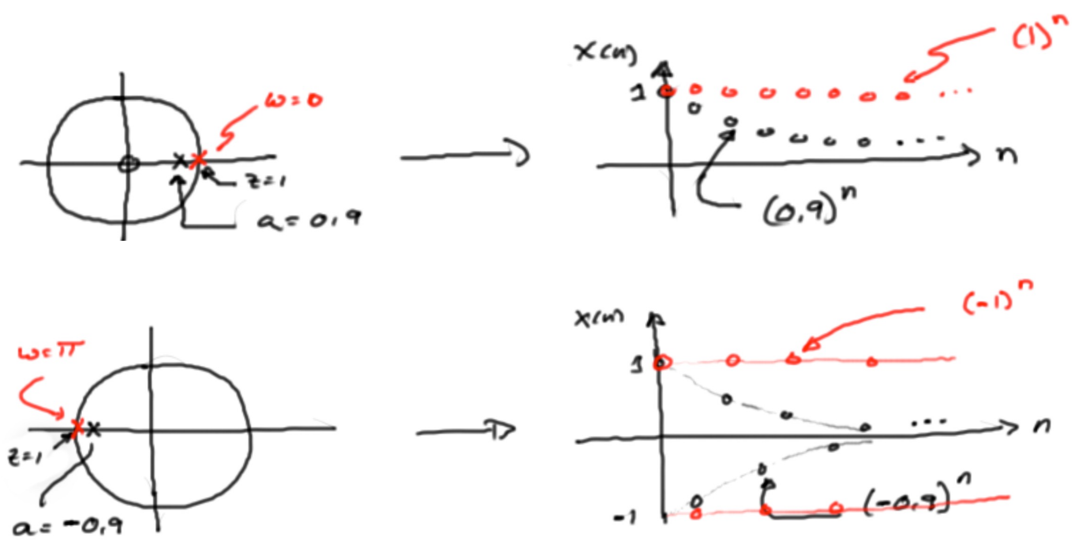
Z-Plane Pole Locations and Their Corresponding Time Series



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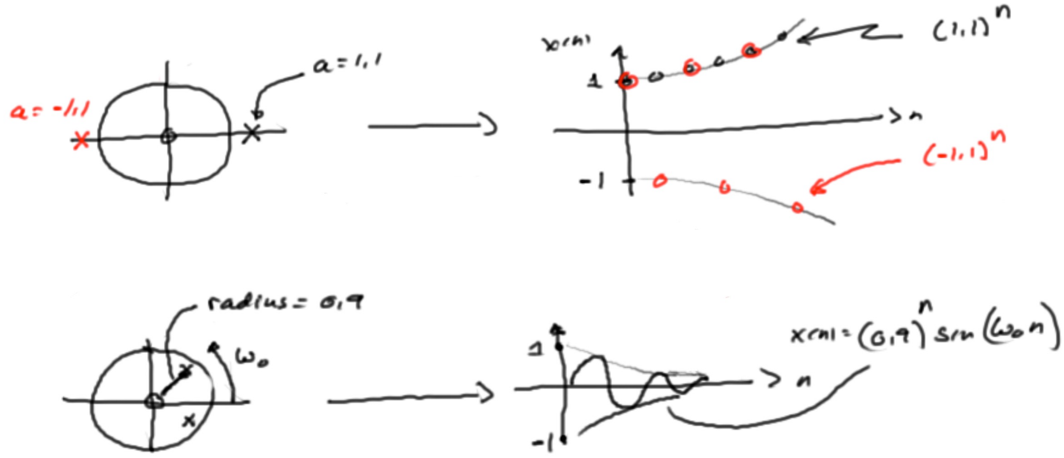
Z-Plane Pole Locations and Their Corresponding Time Series



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