SIO 209: Signal Processing for Ocean Sciences
Class 14

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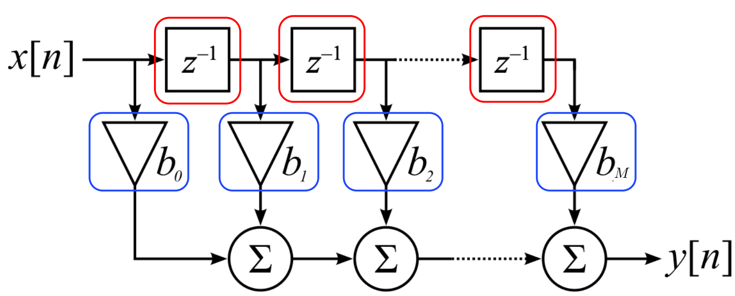
Review of FIR "All-Zeros" Filters

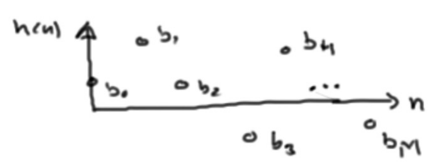
- Finite Impulse Response (FIR) Digital Filters



Unit Delay

Coefficients $b_k, k \in \{0, 1, \ldots, M\}$

Impulse Response:



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Review of FIR "All-Zeros" Filters

- **A: Unit sample response, i.e.,** $h[n] = b_n$
  (analogous to impulse response)
- **B: Generates only zeros**
  Example: low pass filter with all $b_k = 1$
  $x[n]$ is an unit alternating sequence
  Number of zeros: M
- **C: Filter description (input/output relationship)**

Difference Equations:

$$y[n] = b_0\,x[n] + b_1\,x[n-1] + \cdots + b_M\,x[n-M]$$

$$= \sum_{k=0}^{M} b_k\,x[n-k]$$

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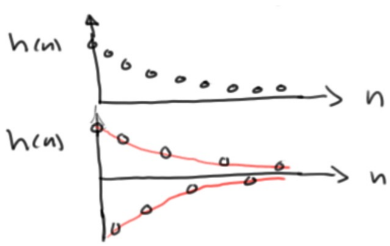
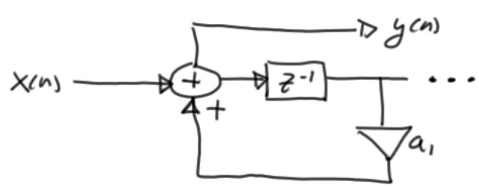
Review of IIR "All-Poles" Filters

- **B: Unit sample response** is a geometric sequence: $h[n] = (a_1)^n$
- Examples

$a_1 = 0.9$

$a_1 = -0.9$





- **C: Filter Description (input/output)**

Difference Equation:

$$y[n] = a_1\,y[n-1] + a_2\,y[n-2] + \cdots + a_N\,y[n-N] + x[n]$$

$$= \sum_{k=1}^{K} a_k\,y[n-k] + x[n]$$

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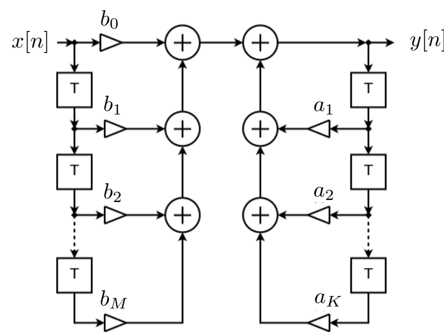
Review of General IIR "Zeros & Poles" Filters

- In general, IIR filters have both feedforward and feedback sections
- **A. Filter description (input/output)**
- Difference Equation: $y[n] = \sum_{k=1}^{K} a_k\, y[n-k] + \sum_{r=0}^{M} b_r\, x[n-r]$
- Convolution: $y[n] = \sum_{k=0}^{\infty} h[k] x[n-k]$



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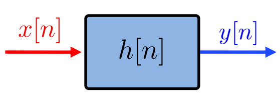
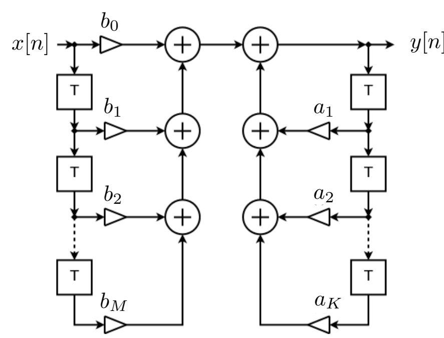
Review of General IIR "Zeros & Poles" Filters

- **B: Filter description (input/output) in z-Domain**





$$Y(z) = \sum_{k=1}^{K} a_k\, z^{-k}\, Y(z) + \sum_{r=0}^{M} b_r\, z^{-r} X(z)$$
$$= H(z) X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{r=0}^{M} b_r\, z^{-r}}{1 - \sum_{k=1}^{K} a_k\, z^{-k}}$$
$$= \frac{b_0 z^M + b_1 z^{M-1} + \cdots + b_M}{z^K + a_1 z^{K-1} + \cdots + a_K}$$

$$y[n] = \sum_{k=1}^{K} a_k y[n-k] + \sum_{r=0}^{M} b_r x[n-r]$$
$$= \sum_{k=-\infty}^{\infty} h[k] x[n-k] \quad \text{convolution summation}$$

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High Resolution Spectral Analysis

Process Generation

$w[n]$ → **All-Poles Filter** → $x[n]$

white noise

observed data

$H(z) = \dfrac{1}{A(z)}$

Auto-Regressive (AR) Model
(IIR filter with no zeros)

Analysis

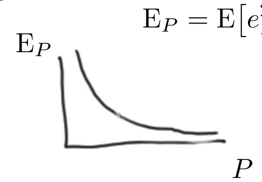
Goal: Estimate $P_{xx}(\omega) = |H(\omega)|^2$

→ **All-Zeros Filter** → $e_P(n) = \hat{w}(n)$

$\hat{A}(z)$

prediction error filter
"lpc" and "levinson"

$\mathrm{E}_P = \mathrm{E}[e_P^2[n]]$



Terms in $A(z)$ have the form

$$\frac{(z - z_k)(z - z_k^*)}{z^2}$$

$$k = 1 \ldots K/2$$

The polynomial coefficients of $A(z)$ can be obtained from "poly"

The all-pole filters can be implemented using "filter"

Note that there is also a ARMA (AR-moving-average) model which is based on a general IIR filter (poles & zeros)

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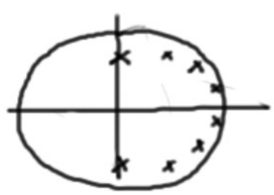
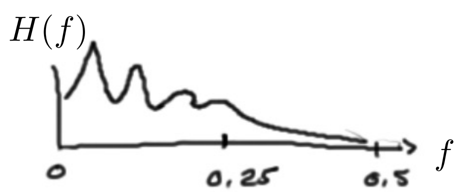
High Resolution Spectral Analysis

We aim to estimate the poles of $H(z)$ which can then be used for spectral analysis

Poles of $H(z)$





Handout (11.57) / Fig. 11.5

General IIR Structure: $$y[n] \pm \sum_{k=1}^{K} a_k\, y[n-k] = \sum_{m=0}^{M} b_m\, x[n-m]$$

Oppenheim et al. (6.26) / Fig. 6.14

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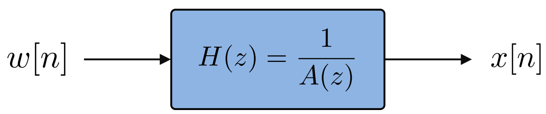
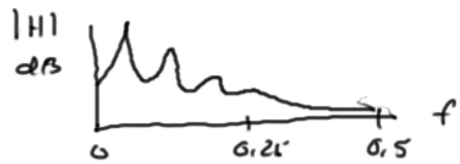
High Resolution Spectral Analysis

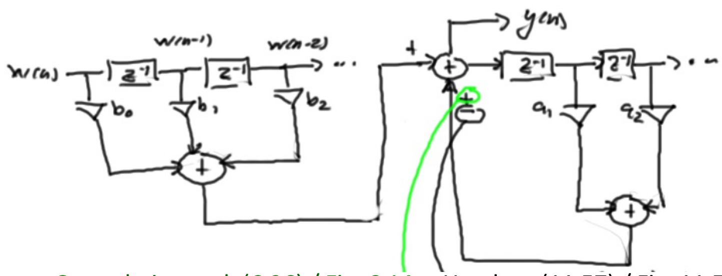
Autoregressive Process Generation





"filter" implements IIR structure using direct form (see Oppenheim et al. Ch 6,.3 and note sign difference)

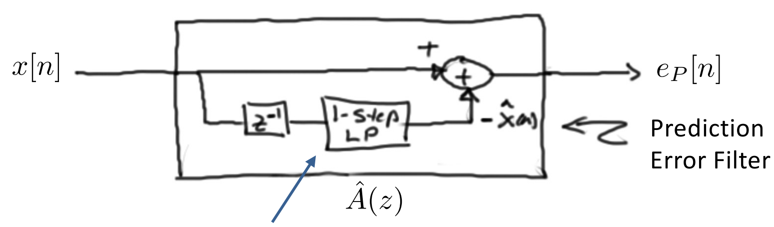
$z = e^{j\omega}$



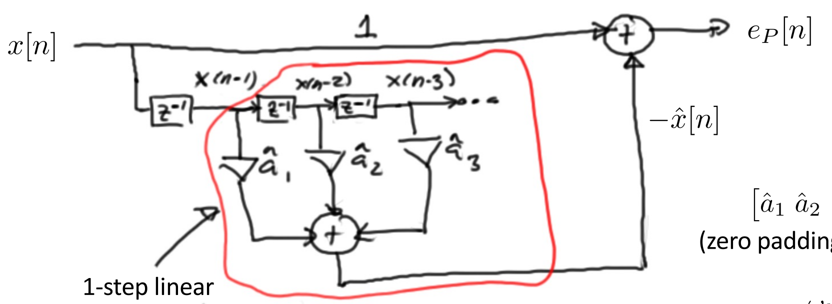
Oppenheim et al. (6.26) / Fig. 6.14   Handout (11.57) / Fig. 11.5

High Resolution Spectral Analysis

Analysis



linear predictor (LP) – window data going into "lpc"



1-step linear predictor

Analysis Result

$$10\log\frac{1}{|\hat{A}(k)|^2}$$

DFT of

$$[\hat{a}_1\ \hat{a}_2\ \cdots\ \hat{a}_P\ 0\ 0\ \cdots] \quad = 10\log|\hat{H}(\omega_k)|^2$$

(zero padding increases resolution)

$$\omega_k = \left(\frac{2\pi}{N}\right)k$$