Data Fusion for Multipath-Based SLAM: Combining Information from Multiple Propagation Paths: Supplementary Material

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This manuscript provides additional analysis for the publication "Data Fusion for Multipath-Based SLAM: Combining Information from Multiple Propagation Paths" by the same authors [1].

I. Geometrical Transformations

In this section, we derive the non-linear transformations [1, Eq. (3)] and [1, Eq. (4)] in Section [1, Sec. II].

A. *Derivation of Transformation from master virtual anchor (MVA) to virtual anchor (VA)*

To calculate [1, Eq. (3)], i.e., $\boldsymbol{p}_{ss,\mathrm{va}}^{(i)} = h_{\mathrm{va}}\big(\boldsymbol{p}_{s,\mathrm{mva}}, \boldsymbol{p}_{\mathrm{pa}}^{(i)}\big)$, we define the point $\boldsymbol{p}_{s,\mathrm{wp}}^{(i)}$ given by the intersection of reflective surface s and the line between $\boldsymbol{p}_{ss,\mathrm{va}}^{(i)}$ and $\boldsymbol{p}_{\mathrm{pa}}^{(i)}$. $\boldsymbol{p}_{s,\mathrm{wp}}^{(i)}$ can expressed as function of $\boldsymbol{p}_{s,\mathrm{mva}} = [p_{1,s,\mathrm{mva}}\ p_{2,s,\mathrm{mva}}]^{\mathrm{T}}$, i.e.,

$$\boldsymbol{p}_{s,\mathrm{wp}}^{(i)} = \gamma_1\ [-p_{2,s,\mathrm{mva}}\ p_{1,s,\mathrm{mva}}]^{\mathrm{T}} + \frac{\boldsymbol{p}_{s,\mathrm{mva}}}{2} \tag{1}$$

as well as function of $\boldsymbol{p}_{\mathrm{pa}}^{(i)}$ and $\boldsymbol{p}_{s,\mathrm{mva}}$

$$\boldsymbol{p}_{s,\mathrm{wp}}^{(i)} = \gamma_2\ \boldsymbol{p}_{s,\mathrm{mva}} + \boldsymbol{p}_{\mathrm{pa}}^{(i)} \tag{2}$$

where the constants γ_1 and γ_2 will be defined in what follows. Furthermore, we express the position of the VA $\boldsymbol{p}_{ss,\mathrm{va}}^{(i)} = [p_{1,ss,\mathrm{va}}^{(i)}\ p_{2,ss,\mathrm{va}}^{(i)}]^{\mathrm{T}}$ as function of $\boldsymbol{p}_{\mathrm{pa}}^{(i)} = [p_{1,\mathrm{pa}}^{(i)}\ p_{2,\mathrm{pa}}^{(i)}]^{\mathrm{T}}$ and $\boldsymbol{p}_{s,\mathrm{mva}}$, i.e.,

$$\boldsymbol{p}_{ss,\mathrm{va}}^{(i)} = 2\gamma_2\ \boldsymbol{p}_{s,\mathrm{mva}} + \boldsymbol{p}_{\mathrm{pa}}^{(i)}. \tag{3}$$

By combining (1) and (2), we obtain the following expression for γ_1 and γ_2

$$\gamma_1 = \frac{-(1/2+\gamma_2)\, p_{2,s,\mathrm{mva}} + p_{2,\mathrm{pa}}^{(i)}}{p_{1,s,\mathrm{mva}}} \tag{4}$$

and

$$\gamma_2 = -\frac{p_{1,\mathrm{pa}}^{(i)}\ p_{1,s,\mathrm{mva}} + p_{2,\mathrm{pa}}^{(i)}\ p_{2,s,\mathrm{mva}}}{p_{1,s,\mathrm{mva}}^2 + p_{2,s,\mathrm{mva}}^2} + \frac{1}{2}. \tag{5}$$

By plugging (5) into (3), the nonlinear transformation from MVA to VA is given by

$$\begin{aligned}\boldsymbol{p}_{ss,\mathrm{va}}^{(i)} &= h_{\mathrm{va}}\big(\boldsymbol{p}_{s,\mathrm{mva}}, \boldsymbol{p}_{\mathrm{pa}}^{(i)}\big)\\ &= -\left(\frac{2p_{1,\mathrm{pa}}^{(i)}\ p_{1,s,\mathrm{mva}} + 2p_{2,\mathrm{pa}}^{(i)}\ p_{2,s,\mathrm{mva}}}{p_{1,s,\mathrm{mva}}^2 + p_{2,s,\mathrm{mva}}^2} - 1\right)\boldsymbol{p}_{s,\mathrm{mva}} + \boldsymbol{p}_{\mathrm{pa}}^{(i)}\\ &= -\left(\frac{2\big\langle \boldsymbol{p}_{s,\mathrm{mva}}, \boldsymbol{p}_{\mathrm{pa}}^{(i)}\big\rangle}{\big\|\boldsymbol{p}_{s,\mathrm{mva}}\big\|^2} - 1\right)\boldsymbol{p}_{s,\mathrm{mva}} + \boldsymbol{p}_{\mathrm{pa}}^{(i)}\end{aligned} \tag{6}$$

where $\langle\cdot,\cdot\rangle$ denotes the inner-product between two vectors and $\|\cdot\|$ denotes the Euclidean norm of a vector.

B. Derivation of Transformation from VA to MVA

To calculate [1, Eq. (4)], i.e., $\boldsymbol{p}_{s,\mathrm{mva}} = h_{\mathrm{mva}}\big(\boldsymbol{p}_{ss,\mathrm{va}}, \boldsymbol{p}_{\mathrm{pa}}^{(i)}\big)$, we define the point $\boldsymbol{p}_{s,\mathrm{mwp}}$ given by an intersection of reflective surface s and the line between the origin $[0\,0]^{\mathrm{T}}$ and $\boldsymbol{p}_{s,\mathrm{mva}}$. $\boldsymbol{p}_{s,\mathrm{mwp}}$ can expressed as function of $\boldsymbol{p}_{ss,\mathrm{va}}^{(i)}$, $\boldsymbol{p}_{\mathrm{pa}}^{(i)}$, and $\boldsymbol{p}_{1}^{(i)} = [p_{1,1}^{(i)}\; p_{1,2}^{(i)}]^{\mathrm{T}} = \boldsymbol{p}_{\mathrm{pa}}^{(i)} - \boldsymbol{p}_{ss,\mathrm{va}}^{(i)}$, i.e.,

$$\boldsymbol{p}_{s,\mathrm{mwp}} = \gamma_3\; [-p_{2,1}\; p_{1,1}]^{\mathrm{T}} + \frac{\boldsymbol{p}_{\mathrm{pa}}^{(i)} + \boldsymbol{p}_{ss,\mathrm{va}}^{(i)}}{2} \tag{7}$$

as well as function of $\boldsymbol{p}_{1}^{(i)}$

$$\boldsymbol{p}_{s,\mathrm{mwp}} = \gamma_4\; \boldsymbol{p}_{1}^{(i)} \tag{8}$$

where the constants γ_3 and γ_4 will be defined in what follows. Furthermore, we express the position of the VA $\boldsymbol{p}_{s,\mathrm{mva}}$ as function of $\boldsymbol{p}_{1}^{(i)}$, i.e.,

$$\boldsymbol{p}_{s,\mathrm{mva}} = 2\gamma_4\; \boldsymbol{p}_{1}^{(i)}. \tag{9}$$

By combining (7) and (8), we obtain the following expression for γ_3 and γ_4

$$\gamma_3 = \frac{\gamma_4\; p_{2,1}^{(i)} - 1/2\,(p_{2,ss,\mathrm{va}}^{(i)} + p_{2,\mathrm{pa}}^{(i)})}{p_{1,1}^{(i)}} \tag{10}$$

and

$$\gamma_4 = \left(\frac{\big\langle \boldsymbol{p}_{1}^{(i)}, \boldsymbol{p}_{\mathrm{pa}}^{(i)}\big\rangle + \big\langle \boldsymbol{p}_{1}^{(i)}, \boldsymbol{p}_{ss,\mathrm{va}}^{(i)}\big\rangle}{\|\boldsymbol{p}_{1}^{(i)}\|^2}\right). \tag{11}$$

By plugging (11) into (9), the nonlinear transformation from VA to MVA is given by

$$\begin{aligned}\boldsymbol{p}_{s,\mathrm{mva}} &= h_{\mathrm{mva}}\big(\boldsymbol{p}_{s,\mathrm{mva}}, \boldsymbol{p}_{\mathrm{pa}}^{(i)}\big)\\ &= \left(\frac{\big\langle \boldsymbol{p}_{1}^{(i)}, \boldsymbol{p}_{\mathrm{pa}}^{(i)}\big\rangle + \big\langle \boldsymbol{p}_{1}^{(i)}, \boldsymbol{p}_{ss,\mathrm{va}}^{(i)}\big\rangle}{\big\|\boldsymbol{p}_{1}^{(i)}\big\|^2}\right)\boldsymbol{p}_{1}^{(i)}\\ &= \left(\frac{\big\|\boldsymbol{p}_{\mathrm{pa}}^{(i)}\big\|^2 - \big\|\boldsymbol{p}_{ss,\mathrm{va}}^{(i)}\big\|^2}{\big\|\boldsymbol{p}_{\mathrm{pa}}^{(i)} - \boldsymbol{p}_{ss,\mathrm{va}}^{(i)}\big\|^2}\right)\big(\boldsymbol{p}_{\mathrm{pa}}^{(i)} - \boldsymbol{p}_{ss,\mathrm{va}}^{(i)}\big).\end{aligned} \tag{12}$$

II. Statistical Model

In this section, we derive the expression of $f(\boldsymbol{y}_{0:n}, \boldsymbol{x}_{0:n}, \underline{\boldsymbol{a}}_{1:n}, \overline{\boldsymbol{a}}_{1:n}\,|\,\boldsymbol{z}_{1:n})$ in [1, Eq. (17)] which is represented by the factor graph in [1, Fig. 3] and provides the basis for the development of a sum-product algorithm (SPA) algorithm for data fusion multipath-based simultaneous localization and mapping (SLAM).

A. Joint Prior PDF

Before presenting derivations, we first define a few sets as follows: $\mathcal{D}_{\underline{\boldsymbol{a}}_n^{(i)}} \triangleq \{(s,s') \in \tilde{\mathcal{D}}_n^{(i)} : \underline{a}_{ss',n}^{(i)} \neq 0\}$ denotes the set of existing legacy potential MVAs (PMVAs), where $\tilde{\mathcal{D}}_n^{(i)} = (0,0) \cup \mathcal{D}_n^{(i)}$ with $\mathcal{D}_n^{(i)} \in \{(s,s') \in \mathcal{S}_n \times \mathcal{S}_n\} = \mathcal{D}_{\mathrm{S},n}^{(i)} \cup \mathcal{D}_{\mathrm{D},n}^{(i)}$, which is composed of the index sets for single-bounce propagation path $\mathcal{D}_{\mathrm{S},n}^{(i)}$ and double-bounce propagation paths $\mathcal{D}_{\mathrm{D},n}^{(i)}$, respectively (see [1, Sec. III]). $\mathcal{N}_{\overline{\boldsymbol{r}}_n^{(i)}} \triangleq \{m \in \{1,\dots,M_n^{(i)}\} : \overline{r}_{m,n}^{(i)} = 1, \overline{a}_{m,n}^{(i)} = 0\}$ denotes the set of existing new PMVAs.

The joint prior probability density function (PDF) of $\mathbf{y}_{0:n} = [\underline{\mathbf{y}}_{0:n}^{\mathrm{T}}, \overline{\mathbf{y}}_{1:n}^{\mathrm{T}}]^{\mathrm{T}}$, $\underline{\mathbf{a}}_{1:n}$, $\overline{\mathbf{a}}_{1:n}$, $\mathbf{x}_{1:n}$, and the number of the measurements $\mathbf{m}_{1:n} \triangleq \big[\mathsf{M}_1 \cdots \mathsf{M}_n\big]^{\mathrm{T}}$ factorizes as [2]–[4]

$$\begin{aligned} f(&\boldsymbol{x}_{0:n}, \boldsymbol{y}_{0:n}, \underline{\boldsymbol{a}}_{1:n}, \overline{\boldsymbol{a}}_{1:n}, \boldsymbol{m}_{1:n})\\ &= f(\boldsymbol{x}_{0:n}, \underline{\boldsymbol{y}}_{0:n}, \overline{\boldsymbol{y}}_{1:n}, \underline{\boldsymbol{a}}_{1:n}, \overline{\boldsymbol{a}}_{1:n}, \boldsymbol{m}_{1:n})\\ &= f(\boldsymbol{x}_0) \prod_{l=1}^{S_0} f(\boldsymbol{y}_{l,0}) \prod_{n'=1}^{n} f(\boldsymbol{x}_{n'}|\boldsymbol{x}_{n'-1}) \left(\prod_{s=1}^{S_{n'-1}} f(\underline{\boldsymbol{y}}_{s,n'}|\boldsymbol{y}_{s,n'-1}) \right) \end{aligned}$$

$$\times \Bigg(\prod_{j'=2}^{J} \prod_{s'=1}^{S_{n'}^{(j')}} f^{(i)}\big(\underline{\boldsymbol{y}}_{s',n'}^{(j')} \big| \underline{\boldsymbol{y}}_{s',n'}^{(j'-1)}\big) \Bigg) \Bigg(\prod_{j=1}^{J} f\big(\overline{\boldsymbol{p}}_{\mathrm{mva}}^{(i)} \big| \overline{\boldsymbol{r}}_{n'}^{(i)}, M_{n'}^{(i)}, \boldsymbol{x}_{n'}\big)\, p\big(\overline{\boldsymbol{r}}_{n'}^{(i)}, \underline{\boldsymbol{a}}_{n'}^{(i)}, \overline{\boldsymbol{a}}_{n'}^{(i)}, M_{n'}^{(i)} \big| \underline{\boldsymbol{y}}_{n'}^{(i)}, \boldsymbol{x}_{n'}\big) \Bigg). \tag{13}$$

We determine the prior PDF of new PMVAs $f(\overline{\boldsymbol{p}}_{\mathrm{mva}}^{(i)}|\overline{\boldsymbol{r}}_n^{(i)}, M_n^{(i)}, \boldsymbol{x}_n)$ and the joint conditional prior probability mass function (PMF) $p(\overline{\boldsymbol{r}}_n^{(i)}, \underline{\boldsymbol{a}}_n^{(i)}, \overline{\boldsymbol{a}}_n^{(i)}, M_n^{(i)}|\underline{\boldsymbol{y}}_n^{(i)}, \boldsymbol{x}_n)$ in what follows. Before the current measurements are observed, the number of measurements $\mathsf{M}_n^{(i)}$ is random. The Poisson PMF of the number of existing new PMVAs evaluated at $|\mathcal{N}_{\overline{\boldsymbol{r}}_n^{(i)}}|$ is given by

$$p\big(|\mathcal{N}_{\overline{\boldsymbol{r}}_n^{(i)}}|\big) = \mu_{\mathrm{n}}^{|\mathcal{N}_{\overline{\boldsymbol{r}}_n^{(i)}}|} / |\mathcal{N}_{\overline{\boldsymbol{r}}_n^{(i)}}|!\, \mathrm{e}^{\mu_{\mathrm{n}}}. \tag{14}$$

The prior PDF of the new PMVA state $\overline{\mathbf{x}}_n^{(i)}$ conditioned on $\overline{\mathbf{r}}_n^{(i)}$ and $\mathsf{M}_n^{(i)}$ is expressed as

$$f\big(\overline{\boldsymbol{p}}_{\mathrm{mva}}^{(i)} \big| \overline{\boldsymbol{r}}_n^{(i)}, M_n^{(i)}, \boldsymbol{x}_n\big) = \prod_{m \in \mathcal{N}_{\overline{\boldsymbol{r}}_n^{(i)}}} f_{\mathrm{n}}\big(\overline{\boldsymbol{p}}_{m,\mathrm{mva}}^{(i)} \big| \boldsymbol{x}_n\big) \prod_{m' \in \{1,\ldots,M_n^{(i)}\} \setminus \mathcal{N}_{\overline{\boldsymbol{r}}_n^{(i)}}} f_{\mathrm{d}}\big(\overline{\boldsymbol{p}}_{m',\mathrm{mva}}^{(i)}\big). \tag{15}$$

The joint conditional prior PMF of the binary existence variables of new PMVAs $\overline{\mathbf{r}}_n \triangleq [\overline{\mathsf{r}}_{1,n} \cdots \overline{\mathsf{r}}_{\mathsf{M}_n,n}]$, the association vectors $\underline{\mathbf{a}}_n$ and $\overline{\mathbf{a}}_n$ and the number of the measurements M_n conditioned on $\mathbf{x}_n$ and $\underline{\mathbf{y}}_n^{(i)}$ is obtained as [3]–[5]

$$p\big(\overline{\boldsymbol{r}}_n^{(i)}, \underline{\boldsymbol{a}}_n^{(i)}, \overline{\boldsymbol{a}}_n^{(i)}, M_n^{(i)} \big| \underline{\boldsymbol{y}}_n^{(i)}, \boldsymbol{x}_n\big) = \chi_{\overline{\boldsymbol{r}}_n^{(i)}, \underline{\boldsymbol{a}}_n^{(i)}, M_n^{(i)}} \Bigg(\prod_{m \in \mathcal{N}_{\overline{\boldsymbol{r}}_n^{(i)}}} \Gamma_{\underline{\boldsymbol{a}}_n^{(i)}}\big(\overline{r}_{m,n}^{(i)}\big) \Bigg) \Bigg(\prod_{(s,s') \in \mathcal{D}_{\underline{\boldsymbol{a}}_n}} p_{\mathrm{d}ss'}\big(\boldsymbol{p}_n, \underline{\boldsymbol{y}}_{s,n}^{(i)}, \underline{\boldsymbol{y}}_{s',n}^{(i)}\big) \Bigg)$$
$$\times \Psi\big(\underline{\boldsymbol{a}}_n^{(i)}, \overline{\boldsymbol{a}}_n^{(i)}\big) \Bigg(\prod_{(s,s') \in \tilde{\mathcal{D}}_n^{(i)} \setminus \mathcal{D}_{\underline{\boldsymbol{a}}_n^{(i)}}} \Big(1 - p_{\mathrm{d}ss'}\big(\boldsymbol{p}_n, \underline{\boldsymbol{y}}_{s,n}^{(i)}, \underline{\boldsymbol{y}}_{s',n}^{(i)}\big) \Big) \Bigg). \tag{16}$$

where binary indicator function $\Psi(\underline{\boldsymbol{a}}_n^{(i)}, \overline{\boldsymbol{a}}_n^{(i)})$ that check consistency for any pair $(\underline{\boldsymbol{a}}_n^{(i)}, \overline{\boldsymbol{a}}_n^{(i)})$ of PMVA-oriented and measurement-oriented association variables, read

$$\Psi\big(\underline{\boldsymbol{a}}_n^{(i)}, \overline{\boldsymbol{a}}_n^{(i)}\big) \triangleq \prod_{(s,s') \in \tilde{\mathcal{D}}_n^{(i)}} \prod_{m=1}^{M_n^{(i)}} \Psi\big(\underline{a}_{ss',n}^{(i)}, \overline{a}_{m,n}^{(i)}\big) \tag{17}$$

and

$$\Gamma_{\boldsymbol{a}_n^{(i)}}\big(\overline{r}_{m,n}^{(i)}\big) \triangleq \begin{cases} 0, & \overline{r}_{m,n}^{(i)} = 1 \text{ and } a_{ss',n}^{(i)} = m \\ \underline{r}_{s,n}\, p_{\mathrm{d}}\big(\boldsymbol{p}_n, \underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)}\big), & \text{otherwise} \end{cases}. \tag{18}$$

The function

$$p_{\mathrm{d}ss'}\big(\boldsymbol{p}_n, \underline{\boldsymbol{y}}_{s,n}^{(i)}, \underline{\boldsymbol{y}}_{s',n}^{(i)}\big) \triangleq \begin{cases} \underline{r}_{s,n}\, \underline{r}_{s',n}\, p_{\mathrm{d}}\big(\boldsymbol{p}_n, \underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)}, \underline{\boldsymbol{p}}_{s',\mathrm{mva}}^{(i)}\big), & s \neq s' \wedge (s,s') \neq (0,0) \\ \underline{r}_{s,n}\, p_{\mathrm{d}}\big(\boldsymbol{p}_n, \underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)}\big), & s = s' \wedge (s,s') \neq (0,0) \\ p_{\mathrm{d}}\big(\boldsymbol{p}_n\big), & (s,s') = (0,0) \end{cases} \tag{19}$$

provides the respective detection probability for the line-of-sight (LOS), single-bounce, and double-bounce VAs. The normalization constant $\chi_{\overline{\boldsymbol{r}}_n^{(i)}, \underline{\boldsymbol{a}}_n^{(i)}, M_n^{(i)}}$ is given as

$$\chi_{\overline{\boldsymbol{r}}_n^{(i)}, \underline{\boldsymbol{a}}_n^{(i)}, M_n^{(i)}} = \left(\frac{{\mu_{\mathrm{fp}}}^{M_n^{(i)}} \mathrm{e}^{-\mu_{\mathrm{n}} - \mu_{\mathrm{fp}}}}{M_n^{(i)}!} \right) \left(\left(\frac{\mu_{\mathrm{n}}}{\mu_{\mathrm{fp}}} \right)^{|\mathcal{N}_{\overline{\boldsymbol{r}}_n^{(i)}}|} {\mu_{\mathrm{fp}}}^{-|\mathcal{D}_{\underline{\boldsymbol{a}}_n^{(i)}}|} \right) \tag{20}$$

where the left-hand-side term (in brackets) is fixed after observing the current measurements given the assumption that the mean number of newly detected PMVAs μ_n and the mean number of false alarms μ_{fp} are known. The right-hand-side term can be merged with factors in the sets $\mathcal{N}_{\overline{\boldsymbol{r}}_n^{(i)}}$ and $\mathcal{D}_{\underline{\boldsymbol{a}}_n^{(i)}}$ respectively. The product of the prior PDF of new PMVAs (15) and the joint conditional prior PMF (16) can be written up to the normalization constant as

$$f\big(\overline{\boldsymbol{p}}_{\mathrm{mva}}^{(i)} \big| \overline{\boldsymbol{r}}_n^{(i)}, M_n^{(i)}, \boldsymbol{x}_n\big) p\big(\overline{\boldsymbol{r}}_n^{(i)}, \underline{\boldsymbol{a}}_n^{(i)}, \overline{\boldsymbol{a}}_n^{(i)}, M_n^{(i)} \big| \underline{\boldsymbol{y}}_n^{(i)}, \boldsymbol{x}_n\big)$$
$$\propto \Bigg(\psi\big(\underline{\boldsymbol{a}}_n^{(i)}, \overline{\boldsymbol{a}}_n^{(i)}\big) \prod_{(s,s') \in \mathcal{D}_{\underline{\boldsymbol{a}}_n}^{(i)}} \frac{p_{\mathrm{d}ss'}\big(\boldsymbol{p}_n, \underline{\boldsymbol{y}}_{s,n}^{(i)}, \underline{\boldsymbol{y}}_{s',n}^{(i)}\big)}{\mu_{\mathrm{fp}}} \prod_{(s'',s''') \in \tilde{\mathcal{D}}_n^{(i)} \setminus \mathcal{D}_{\underline{\boldsymbol{a}}_n^{(i)}}} \Big(1 - p_{\mathrm{d}ss'}\big(\boldsymbol{p}_n, \underline{\boldsymbol{y}}_{s'',n}^{(i)}, \underline{\boldsymbol{y}}_{s''',n}^{(i)}\big) \Big) \Bigg)$$

$$\times\Bigg(\prod_{m\in\mathcal{N}_{\overline{\boldsymbol{r}}_n^{(i)}}}\frac{\mu_n f_{\mathrm{n}}\big(\overline{\boldsymbol{p}}_{m',\mathrm{mva}}^{(i)}\big)}{\mu_{\mathrm{fp}}}\Gamma_{\underline{\boldsymbol{a}}_n}\big(\overline{r}_{m,n}^{(i)}\big)\prod_{m'\in\{1,\dots,M_n^{(i)}\}\backslash\mathcal{N}_{\overline{\boldsymbol{r}}_n^{(i)}}} f_{\mathrm{d}}\big(\overline{\boldsymbol{p}}_{m',\mathrm{mva}}^{(i)}\big)\Bigg). \tag{21}$$

With some simple manipulations using the definitions of exclusion functions $\Psi(\underline{\boldsymbol{a}}_n^{(i)},\overline{\boldsymbol{a}}_n^{(i)})$ and $\Gamma_{\underline{\boldsymbol{a}}_n}(\overline{r}_{m,n}^{(i)})$, Eq. (21) can be rewritten as the product of factors related to the legacy PMVAs and to the new PMVAs respectively, i.e.,

$$\begin{aligned}
&f(\overline{\boldsymbol{p}}_{\mathrm{mva}}^{(i)}|\overline{\boldsymbol{r}}_n^{(i)},M_n^{(i)},\boldsymbol{x}_n)p(\overline{\boldsymbol{r}}_n^{(i)},\underline{\boldsymbol{a}}_n^{(i)},\overline{\boldsymbol{a}}_n^{(i)},M_n^{(i)}|\underline{\boldsymbol{y}}_n^{(i)},\boldsymbol{x}_n)\\
&\quad\propto\Bigg(\prod_{j=1}^{J}\underline{q}_{\mathrm{P}1}\big(\boldsymbol{p}_n,\underline{a}_{00,n}^{(i)}\big)\prod_{m'=1}^{M_n^{(i)}}\Psi\big(\underline{a}_{00,n'}^{(i)},\overline{a}_{m',n'}^{(i)}\big)\Bigg)\Bigg(\prod_{s=1}^{S_n^{(i)}}\underline{q}_{\mathrm{S}1}\big(\underline{\boldsymbol{y}}_{s,n}^{(i)},\underline{a}_{ss,n}^{(i)},\boldsymbol{p}_n\big)\Bigg(\prod_{m'=1}^{M_n^{(i)}}\Psi\big(\underline{a}_{ss,n}^{(i)},\overline{a}_{m',n}^{(i)}\big)\Bigg)\\
&\quad\times\prod_{s'=1,s'\neq s}^{S_n^{(i)}}\underline{q}_{\mathrm{D}1}\big(\underline{\boldsymbol{y}}_{s,n}^{(i)},\underline{\boldsymbol{y}}_{s',n}^{(i)},\underline{a}_{ss',n}^{(i)},\boldsymbol{p}_n\big)\prod_{m'=1}^{M_n^{(i)}}\Psi\big(\underline{a}_{ss',n}^{(i)},\overline{a}_{m',n}^{(i)}\big)\Bigg)\Bigg(\prod_{m=1}^{M_n^{(i)}}\overline{q}_{\mathrm{S}1}\big(\overline{\boldsymbol{y}}_{m,n}^{(i)},\overline{a}_{m,n}^{(i)},\boldsymbol{p}_n\big)\Bigg).
\end{aligned}\tag{22}$$

We note that the factor $\prod_{m'=1}^{M_{n'}^{(i)}}\Psi(\underline{a}_{00,n'}^{(i)},\overline{a}_{m',n'}^{(i)})$ in (13) considers the joint data association with respect to the LOS component [3] that is assumed to always exist (but may not always be detected). The functions related to the physical anchor (PA) $\underline{q}_{\mathrm{P}1}\big(\boldsymbol{p}_n,\underline{a}_{00,n}^{(i)}\big)$ and to the legacy PMVA states $\underline{q}_{\mathrm{S}1}\big(\underline{\boldsymbol{y}}_{s,n}^{(i)},\underline{a}_{ss,n}^{(i)},\boldsymbol{p}_n\big)=\underline{q}_{\mathrm{S}1}\big(\underline{r}_{s,n}^{(i)},\underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)},\underline{a}_{ss,n}^{(i)},\boldsymbol{p}_n\big)$ and $\underline{q}_{\mathrm{D}1}\big(\underline{\boldsymbol{y}}_{s,n}^{(i)},\underline{\boldsymbol{y}}_{s',n}^{(i)},\underline{a}_{ss',n}^{(i)},\boldsymbol{p}_n\big)=\underline{q}_{\mathrm{D}1}\big(\underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)},\underline{r}_{s,n}^{(i)},\underline{\boldsymbol{p}}_{s',\mathrm{mva}}^{(i)},\underline{r}_{s',n}^{(i)},\underline{a}_{ss',n}^{(i)},\boldsymbol{p}_n\big)$ are respectively given by

$$\underline{q}_{\mathrm{P}1}\big(\boldsymbol{p}_n,\underline{a}_{00,n}^{(i)}\big)\triangleq\begin{cases}\dfrac{p_{\mathrm{d}}(\boldsymbol{p}_n)}{\mu_{\mathrm{fp}}}, & \underline{a}_{00,n}^{(i)}\in\mathcal{M}_n^{(i)}\\ 1-p_{\mathrm{d}}(\boldsymbol{p}_n), & \underline{a}_{00,n}^{(i)}=0\end{cases}, \tag{23}$$

$$\underline{q}_{\mathrm{S}1}\big(\underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)},\underline{r}_{s,n}^{(i)}=1,\underline{a}_{ss,n}^{(i)},\boldsymbol{p}_n\big)\triangleq\begin{cases}\dfrac{p_{\mathrm{d}}\big(\boldsymbol{p}_n,\underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)}\big)}{\mu_{\mathrm{fp}}}, & \underline{a}_{ss,n}^{(i)}\in\mathcal{M}_n^{(i)}\\ 1-p_{\mathrm{d}}\big(\boldsymbol{p}_n,\underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)}\big), & \underline{a}_{ss,n}^{(i)}=0\end{cases}, \tag{24}$$

$\underline{q}_{\mathrm{S}1}\big(\underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)},\underline{r}_{s,n}^{(i)}=0,\underline{a}_{ss,n}^{(i)},\boldsymbol{p}_n\big)\triangleq\delta\big(\underline{a}_{ss,n}^{(i)}\big)$,

$$\underline{q}_{\mathrm{D}1}\big(\underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)},\underline{r}_{s,n}^{(i)}=1,\underline{\boldsymbol{p}}_{s',\mathrm{mva}}^{(i)},\underline{r}_{s',n}^{(i)}=1,\underline{a}_{ss',n}^{(i)},\boldsymbol{p}_n\big)\triangleq\begin{cases}\dfrac{p_{\mathrm{d}}\big(\boldsymbol{p}_n,\underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)},\underline{\boldsymbol{p}}_{s',\mathrm{mva}}^{(i)}\big)}{\mu_{\mathrm{fp}}}, & a_{ss'}^{(i)}\in\mathcal{M}_n^{(i)}\\ 1-p_{\mathrm{d}}\big(\boldsymbol{p}_n,\underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)},\underline{\boldsymbol{p}}_{s',\mathrm{mva}}^{(i)}\big), & a_{ss'}^{(i)}=0\end{cases}, \tag{25}$$

and $\underline{q}_{\mathrm{D}1}\big(\underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)},\underline{r}_{s,n}^{(i)},\underline{\boldsymbol{p}}_{s',\mathrm{mva}}^{(i)},\underline{r}_{s',n}^{(i)},\underline{a}_{ss',n}^{(i)},\boldsymbol{p}_n\big)\triangleq\delta\big(\underline{a}_{ss',n}^{(i)}\big)$ if $\underline{r}_{s,n}^{(i)}=0$ or $\underline{r}_{s',n}^{(i)}=0$. The function $\overline{q}_{\mathrm{S}1}\big(\overline{\boldsymbol{y}}_{m,\mathrm{mva}}^{(i)},\overline{a}_{m,n}^{(i)},\boldsymbol{p}_n\big)=\overline{q}_{\mathrm{S}1}\big(\overline{\boldsymbol{p}}_{m,\mathrm{mva}}^{(i)},\overline{r}_{s,n}^{(i)},\overline{a}_{m,n}^{(i)},\boldsymbol{p}_n\big)$ related to new PMVA states reads

$$\overline{q}_{\mathrm{S}1}\big(\overline{\boldsymbol{p}}_{m,\mathrm{mva}}^{(i)},\overline{r}_{s,n}^{(i)}=1,\overline{a}_{m,n}^{(i)},\boldsymbol{p}_n\big)\triangleq\begin{cases}0, & \overline{a}_{m,n}^{(i)}\in\tilde{\mathcal{D}}_n^{(i)}\\ \frac{\mu_{\mathrm{n}}f_{\mathrm{n}}(\overline{\boldsymbol{p}}_{m,\mathrm{mva}}^{(i)}|\boldsymbol{p}_n)}{\mu_{\mathrm{fp}}}, & \overline{a}_{m,n}^{(i)}=0\end{cases} \tag{26}$$

and $\overline{q}_{\mathrm{S}1}\big(\overline{\boldsymbol{p}}_{m,\mathrm{mva}}^{(i)},\overline{r}_{s,n}^{(i)}=0,\overline{a}_{m,n}^{(i)},\boldsymbol{p}_n\big)\triangleq 1$.

Finally, by inserting (22) into (13), the joint prior PDF can be rewritten as

$$\begin{aligned}
&f(\boldsymbol{x}_{0:n},\boldsymbol{y}_{0:n},\underline{\boldsymbol{a}}_{1:n},\overline{\boldsymbol{a}}_{1:n},\boldsymbol{m}_{1:n})\\
&\quad\propto\Bigg(f(\boldsymbol{x}_0)\prod_{l=1}^{S_0}f(\boldsymbol{y}_{l,0})\Bigg)\prod_{n'=1}^{n}f\big(\boldsymbol{x}_{n'}|\boldsymbol{x}_{n'-1}\big)\Bigg(\prod_{j=1}^{J}\underline{q}_{\mathrm{P}1}\big(\boldsymbol{p}_{n'},\underline{a}_{00,n'}^{(i)}\big)\prod_{m'=1}^{M_{n'}^{(i)}}\Psi\big(\underline{a}_{00,n'}^{(i)},\overline{a}_{m',n'}^{(i)}\big)\Bigg)\Bigg(\prod_{s'=1}^{S_{n'-1}}f\big(\underline{\boldsymbol{y}}_{s',n'}|\boldsymbol{y}_{s',n'-1}\big)\Bigg)\\
&\quad\times\Bigg(\prod_{j'=2}^{J}\Bigg(\prod_{s'=1}^{S_{n'}^{(j')}}f^{(i)}\big(\underline{\boldsymbol{y}}_{s',n'}^{(j')}|\underline{\boldsymbol{y}}_{s',n'}^{(j'-1)}\big)\Bigg)\Bigg)\prod_{j=1}^{J}\Bigg(\prod_{s=1}^{S_{n'}^{(i)}}\underline{q}_{\mathrm{S}1}\big(\underline{\boldsymbol{y}}_{s,n'}^{(i)},\underline{a}_{ss,n'}^{(i)},\boldsymbol{p}_{n'}\big)\Bigg(\prod_{m'=1}^{M_{n'}^{(i)}}\Psi\big(\underline{a}_{ss,n'}^{(i)},\overline{a}_{m',n'}^{(i)}\big)\Bigg)
\end{aligned}$$

$$\times \prod_{s'=1,s'\neq s}^{S_{n'}^{(i)}} \underline{q}_{\mathrm{D1}}\big(\underline{\boldsymbol{y}}_{s,n'}^{(i)}, \underline{\boldsymbol{y}}_{s',n'}^{(i)}, \underline{a}_{ss',n'}^{(i)}, \boldsymbol{p}_{n'}\big) \prod_{m'=1}^{M_{n'}^{(i)}} \Psi\big(\underline{a}_{ss',n'}^{(i)}, \overline{a}_{m',n'}^{(i)}\big)\Bigg)\Bigg(\prod_{m=1}^{M_{n'}^{(i)}} \overline{q}_{\mathrm{S1}}\big(\overline{\boldsymbol{y}}_{m,n'}^{(i)}, \overline{a}_{m,n'}^{(i)}, \boldsymbol{p}_{n'}\big)\Bigg) \tag{27}$$

B. Joint Likelihood Function

Assume that the measurements $\mathbf{z}_n$ are independent across n, the conditional PDF of $\mathbf{z}_{1:n}$ given $\boldsymbol{x}_{1:n}$, $\underline{\mathbf{y}}_{1:n}$, $\overline{\mathbf{y}}_{1:n}$, $\underline{\mathbf{a}}_{1:n}$, $\overline{\mathbf{a}}_{1:n}$, and the number of measurements $\mathbf{m}_{1:n}$ is given by

$$f(\boldsymbol{z}_{1:n}|\boldsymbol{x}_{1:n}, \underline{\boldsymbol{y}}_{1:n}, \overline{\boldsymbol{y}}_{1:n}, \underline{\boldsymbol{a}}_{1:n}, \overline{\boldsymbol{a}}_{1:n}, \boldsymbol{m}_{1:n}) = \prod_{n'=1}^{n} f(\boldsymbol{z}_{n'}|\boldsymbol{x}_{n}, \underline{\boldsymbol{y}}_{n'}, \overline{\boldsymbol{y}}_{n'}, \underline{\boldsymbol{a}}_{n'}, \overline{\boldsymbol{a}}_{n'}, M_{n'}) \tag{28}$$

for $\boldsymbol{z}_n$ and $f(\boldsymbol{z}_n|\boldsymbol{x}_n, \underline{\boldsymbol{y}}_n, \overline{\boldsymbol{y}}_n, \underline{\boldsymbol{a}}_n, \overline{\boldsymbol{a}}_n, M_n) = 0$ otherwise. Assuming that the measurements $\mathbf{z}_{m,n}$ are conditionally independent across m given $\underline{\mathbf{y}}_{k,n}$, $\overline{\mathbf{y}}_{m,n}$, $\underline{\mathbf{a}}_{k,n}$, $\overline{\mathsf{a}}_{m,n}$, and M_n [3], [6], Eq. (28) factorizes as

$$\begin{aligned} &f(\boldsymbol{z}_{1:n}|\boldsymbol{x}_{1:n},\underline{\boldsymbol{y}}_{1:n}, \overline{\boldsymbol{y}}_{1:n}, \underline{\boldsymbol{a}}_{1:n}, \overline{\boldsymbol{a}}_{1:n}, \boldsymbol{m}_{1:n}) \\ &\quad = \prod_{n'=1}^{n} C(\boldsymbol{z}_{n'}) \left(\prod_{(s,s')\in\mathcal{D}_{\underline{\boldsymbol{a}}_n^{(i)},\underline{\boldsymbol{r}}_{ss',n}^{(i)}}} \frac{f_{ss'}(\boldsymbol{z}_{\underline{a}_{ss',n'},n'})|\boldsymbol{p}_n, \underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)}, \underline{\boldsymbol{p}}_{s',\mathrm{mva}}^{(i)})}{f_{\mathrm{fp}}(\boldsymbol{z}_{\underline{a}_{ss',n'},n'})} \right) \left(\prod_{m\in\mathcal{N}_{\overline{\boldsymbol{r}}_{n'}}} \frac{f(\boldsymbol{z}_{m,n}^{(i)}|\boldsymbol{p}_n, \overline{\boldsymbol{p}}_{m,\mathrm{mva}}^{(i)})}{f_{\mathrm{fp}}(\boldsymbol{z}_{m,n'})} \right) \end{aligned} \tag{29}$$

where $\mathcal{D}_{\underline{\boldsymbol{a}}_n^{(i)},\underline{\boldsymbol{r}}_{ss',n}^{(i)}} \triangleq \{(s,s') \in \mathcal{D}_{\underline{\boldsymbol{a}}_n^{(i)}} : \underline{r}_{s,n}^{(i)} \neq 0 \wedge \underline{r}_{s',n}^{(i)} \neq 0\}$ considers non existent PMVAs and

$$f_{ss'}(\boldsymbol{z}_{m,n}^{(i)}|\boldsymbol{p}_n, \underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)}, \underline{\boldsymbol{p}}_{s',\mathrm{mva}}^{(i)}) \triangleq \begin{cases} f(\boldsymbol{z}_{m,n}^{(i)}|\boldsymbol{p}_n, \underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)}, \underline{\boldsymbol{p}}_{s',\mathrm{mva}}^{(i)}), & s \neq s' \wedge (s,s') \neq (0,0) \\ f(\boldsymbol{z}_{m,n}^{(i)}|\boldsymbol{p}_n, \underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)}), & s = s' \wedge (s,s') \neq (0,0) \\ f(\boldsymbol{z}_{m,n}^{(i)}|\boldsymbol{p}_n), & (s,s') = (0,0) \end{cases} \tag{30}$$

provides the respective likelihood function for LOS, single-bounce and double-bounce VAs. Since the normalization factor $C(\boldsymbol{z}_n) = \prod_{m=1}^{M_n} f_{\mathrm{fa}}(\boldsymbol{z}_{m,n})$ depending on $\boldsymbol{z}_n$ and M_n is fixed after the current measurement $\boldsymbol{z}_n$ is observed and using $\mathbf{y}_{1:n} = [\underline{\mathbf{y}}_{1:n}^{\mathrm{T}}, \overline{\mathbf{y}}_{1:n}^{\mathrm{T}}]^{\mathrm{T}}$, the likelihood function in (29) can be rewritten up to the normalization constant as

$$\begin{aligned} &f(\boldsymbol{z}_{1:n}|\boldsymbol{x}_{1:n}, \boldsymbol{y}_{1:n}, \underline{\boldsymbol{a}}_{1:n}, \overline{\boldsymbol{a}}_{1:n}, \boldsymbol{m}_{1:n}) \\ &\quad\propto \prod_{n'=1}^{n}\prod_{j=1}^{J} \Bigg(\underline{q}_{\mathrm{P2}}\big(\boldsymbol{x}_{n'}, \underline{a}_{00,n'}^{(i)}; \boldsymbol{z}_{n'}^{(i)}\big) \prod_{s=1}^{S_{n'}^{(i)}} \underline{q}_{\mathrm{S2}}\big(\underline{\boldsymbol{y}}_{s,n'}^{(i)}, \underline{a}_{ss,n'}^{(i)}, \boldsymbol{x}_{n'}; \boldsymbol{z}_{n'}^{(i)}\big) \prod_{s'=1,s'\neq s}^{S_{n'}^{(i)}} \underline{q}_{\mathrm{D2}}\big(\underline{\boldsymbol{y}}_{s,n'}^{(i)}, \underline{\boldsymbol{y}}_{s',n'}^{(i)}, \underline{a}_{ss',n'}^{(i)}, \boldsymbol{x}_{n'}; \boldsymbol{z}_{n'}^{(i)}\big)\Bigg) \\ &\qquad\times \prod_{m=1}^{M_{n'}^{(i)}} \overline{q}_{\mathrm{S2}}\big(\overline{\boldsymbol{y}}_{m,n'}^{(i)}, \overline{a}_{m,n'}^{(i)}, \boldsymbol{x}_{n'}; \boldsymbol{z}_{n'}^{(i)}\big) \end{aligned} \tag{31}$$

where the factors related to the PA $\underline{q}_{\mathrm{P2}}\big(\boldsymbol{x}_n, \underline{a}_{00,n}^{(i)}; \boldsymbol{z}_n^{(i)}\big)$ is given by

$$\underline{q}_{\mathrm{P2}}\big(\boldsymbol{x}_n, \underline{a}_{00,n}^{(i)}; \boldsymbol{z}_n^{(i)}\big) \triangleq \begin{cases} \dfrac{f\big(\boldsymbol{z}_{m,n}^{(i)}\big|\boldsymbol{p}_n\big)}{f_{\mathrm{fp}}\big(\boldsymbol{z}_{m,n}^{(i)}\big)}, & \underline{a}_{00,n}^{(i)} = m \in \mathcal{M}_n^{(i)} \\ 1, & \underline{a}_{00,n}^{(i)} = 0 \end{cases} \tag{32}$$

and the factors related to legacy PMVA states $\underline{q}_{\mathrm{S2}}(\underline{\boldsymbol{y}}_{s,n}^{(i)}, \underline{a}_{ss,n}^{(i)}, \boldsymbol{x}_n; \boldsymbol{z}_n^{(i)}) = \underline{q}_{\mathrm{S2}}(\underline{r}_{s,n}^{(i)}, \underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)}, \underline{a}_{ss,n}^{(i)}, \boldsymbol{x}_n; \boldsymbol{z}_n^{(i)})$ and $\underline{q}_{\mathrm{D2}}\big(\underline{\boldsymbol{y}}_{s,n}^{(i)}, \underline{\boldsymbol{y}}_{s',n}^{(i)}, \underline{a}_{ss',n}^{(i)}, \boldsymbol{x}_n; \boldsymbol{z}_n^{(i)}\big) = \underline{q}_{\mathrm{D2}}\big(\underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)}, \underline{r}_{s,n}^{(i)}, \underline{\boldsymbol{p}}_{s',\mathrm{mva}}^{(i)}, \underline{r}_{s',n}^{(i)}, \underline{a}_{ss',n}^{(i)}, \boldsymbol{x}_n; \boldsymbol{z}_n^{(i)}\big)$ are given respectively by

$$\underline{q}_{\mathrm{S2}}(\underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)}, \underline{r}_{s,n}^{(i)} = 1, \underline{a}_{ss,n}^{(i)}, \boldsymbol{x}_n; \boldsymbol{z}_n^{(i)}) \triangleq \begin{cases} \dfrac{f\big(\boldsymbol{z}_{m,n}^{(i)}\big|\boldsymbol{p}_n, \underline{\boldsymbol{p}}_{s,\mathrm{mva}}^{(i)}\big)}{f_{\mathrm{fp}}\big(\boldsymbol{z}_{m,n}^{(i)}\big)}, & \underline{a}_{ss,n}^{(i)} = m \in \mathcal{M}_n^{(i)} \\ 1, & \underline{a}_{ss,n}^{(i)} = 0 \end{cases} \tag{33}$$

and $\underline{q}_{\mathrm{S2}}\big(\underline{\boldsymbol{p}}^{(i)}_{s,\mathrm{mva}}, \underline{r}^{(i)}_{s,n} = 0, \underline{a}^{(i)}_{ss,n}, \boldsymbol{x}_n; \boldsymbol{z}^{(i)}_n\big) \triangleq 1$ and by

$$\underline{q}_{\mathrm{D2}}\big(\underline{\boldsymbol{p}}^{(i)}_{s,\mathrm{mva}}, \underline{r}^{(i)}_{s,n} = 1, \underline{\boldsymbol{p}}^{(i)}_{s',\mathrm{mva}}, \underline{r}^{(i)}_{s',n} = 1, \underline{a}^{(i)}_{ss',n}, \boldsymbol{x}_n; \boldsymbol{z}^{(i)}_n\big) \triangleq \begin{cases} \dfrac{f\big(\boldsymbol{z}^{(i)}_{m,n}\big|\boldsymbol{p}_n, \underline{\boldsymbol{p}}^{(i)}_{s,\mathrm{mva}}, \underline{\boldsymbol{p}}^{(i)}_{s',\mathrm{mva}}\big)}{f_{\mathrm{fp}}\big(\boldsymbol{z}^{(i)}_{m,n}\big)}, & a^{(i)}_{ss'} = m \in \mathcal{M}^{(i)}_n \\ 1, & a^{(i)}_{ss'} = 0 \end{cases} \tag{34}$$

and $\underline{q}_{\mathrm{D2}}\big(\underline{\boldsymbol{p}}^{(i)}_{s,\mathrm{mva}}, \underline{r}^{(i)}_{s,n}, \underline{\boldsymbol{p}}^{(i)}_{s',\mathrm{mva}}, \underline{r}^{(i)}_{s',n}, \underline{a}^{(i)}_{ss',n}, \boldsymbol{x}_n; \boldsymbol{z}^{(i)}_n\big) \triangleq 1$ if $\underline{r}^{(i)}_{s,n} = 0$ or $\underline{r}^{(i)}_{s',n} = 0$. The factor related to new PMVA states $\overline{q}_{\mathrm{S2}}\big(\overline{\boldsymbol{y}}^{(i)}_{m,\mathrm{mva}}, \overline{a}^{(i)}_{m,n}, \boldsymbol{x}_n; \boldsymbol{z}^{(i)}_n\big) = \overline{q}_{\mathrm{S2}}\big(\overline{\boldsymbol{p}}^{(i)}_{m,\mathrm{mva}}, \overline{r}^{(i)}_{s,n}, \overline{a}^{(i)}_{m,n}, \boldsymbol{x}_n; \boldsymbol{z}^{(i)}_n\big)$ is given by

$$\overline{q}_{\mathrm{S2}}\big(\overline{\boldsymbol{p}}^{(i)}_{m,\mathrm{mva}}, \overline{r}^{(i)}_{s,n} = 1, \overline{a}^{(i)}_{m,n}, \boldsymbol{x}_n; \boldsymbol{z}^{(i)}_n\big) \triangleq \begin{cases} 0, & \overline{a}^{(i)}_{m,n} \in \tilde{\mathcal{D}}^{(i)}_n \\ \dfrac{f\big(\boldsymbol{z}^{(i)}_{m,n}\big|\boldsymbol{p}_n, \underline{\boldsymbol{p}}^{(i)}_{m,\mathrm{mva}}\big)}{f_{\mathrm{fp}}\big(\boldsymbol{z}^{(i)}_{m,n}\big)}, & \overline{a}^{(i)}_{m,n} = 0 \end{cases} \tag{35}$$

and $\overline{q}_{\mathrm{S2}}\big(\overline{\boldsymbol{p}}^{(i)}_{m,\mathrm{mva}}, \overline{r}^{(i)}_{s,n} = 0, \overline{a}^{(i)}_{m,n}, \boldsymbol{x}_n; \boldsymbol{z}^{(i)}_n\big) \triangleq 1$.

C. Joint Posterior PDF

We derive the factorization of $f(\boldsymbol{y}_{0:n}, \boldsymbol{x}_{0:n}, \underline{\boldsymbol{a}}_{1:n}, \overline{\boldsymbol{a}}_{1:n}|\boldsymbol{z}_{1:n})$ considering that the measurements $\boldsymbol{z}_{1:n}$ are observed and thus fixed (consequently M_n is fixed as well). By using Bayes'rule and by exploiting the fact that $\boldsymbol{z}_n$ implies M_n according to (29), we obtain [5], [7]

$$\begin{aligned} f(\boldsymbol{y}_{0:n}, \boldsymbol{x}_{0:n}, \underline{\boldsymbol{a}}_{1:n}, \overline{\boldsymbol{a}}_{1:n}|\boldsymbol{z}_{1:n}) &= \sum_{M'_1=0}^{\infty} \sum_{M'_2=0}^{\infty} \cdots \sum_{M'_n=0}^{\infty} f(\boldsymbol{y}_{0:n}, \boldsymbol{x}_{0:n}, \underline{\boldsymbol{a}}_{1:n}, \overline{\boldsymbol{a}}_{1:n}, \boldsymbol{m}'_{1:n}|\boldsymbol{z}_{1:n}) \\ &= \sum_{M'_1=0}^{\infty} \sum_{M'_2=0}^{\infty} \cdots \sum_{M'_n=0}^{\infty} f(\boldsymbol{z}_{1:n}|\boldsymbol{x}_{1:n}, \boldsymbol{y}_{1:n}, \underline{\boldsymbol{a}}_{1:n}, \overline{\boldsymbol{a}}_{1:n}, \boldsymbol{m}'_{1:n}) f(\boldsymbol{y}_{0:n}, \boldsymbol{x}_{0:n}, \underline{\boldsymbol{a}}_{1:n}, \overline{\boldsymbol{a}}_{1:n}, \boldsymbol{m}'_{1:n}) \end{aligned} \tag{36}$$

Using the factorized joint prior PDF (27) and the factorized joint likelihood function (31) the joint posterior PDF (36) can be rearranged as

$$\begin{aligned} &f(\boldsymbol{y}_{0:n}, \boldsymbol{x}_{0:n}, \underline{\boldsymbol{a}}_{1:n}, \overline{\boldsymbol{a}}_{1:n}|\boldsymbol{z}_{1:n}) \\ &\propto \bigg(f(\boldsymbol{x}_0) \prod_{l=1}^{S_0} f(\boldsymbol{y}_{l,0})\bigg) \prod_{n'=1}^{n} f\big(\boldsymbol{x}_{n'}|\boldsymbol{x}_{n'-1}\big) \bigg(\prod_{j=1}^{J} \underline{q}_{\mathrm{P1}}\big(\boldsymbol{p}_{n'}, \underline{a}^{(i)}_{00,n'}\big) \underline{q}_{\mathrm{P2}}\big(\boldsymbol{x}_{n'}, \underline{a}^{(i)}_{00,n'}; \boldsymbol{z}^{(i)}_{n'}\big) \prod_{m'=1}^{M^{(i)}_{n'}} \Psi\big(\underline{a}^{(i)}_{00,n'}, \overline{a}^{(i)}_{m',n'}\big) \bigg) \\ &\times \bigg(\prod_{s'=1}^{S_{n'-1}} f\big(\underline{\boldsymbol{y}}_{s',n'}|\boldsymbol{y}_{s',n'-1}\big)\bigg) \bigg(\prod_{j'=2}^{J} \bigg(\prod_{s'=1}^{S^{(j')}_{n'}} f^{(i)}\big(\underline{\boldsymbol{y}}^{(j')}_{s',n'}\big|\underline{\boldsymbol{y}}^{(j'-1)}_{s',n'}\big)\bigg)\bigg) \prod_{j=1}^{J} \bigg(\prod_{s=1}^{S^{(i)}_{n'}} \underline{q}_{\mathrm{S1}}\big(\underline{\boldsymbol{y}}^{(i)}_{s,n'}, \underline{a}^{(i)}_{ss,n'}, \boldsymbol{p}_{n'}\big) \underline{q}_{\mathrm{S2}}\big(\underline{\boldsymbol{y}}^{(i)}_{s,n'}, \underline{a}^{(i)}_{ss,n'}, \boldsymbol{x}_{n'}; \boldsymbol{z}^{(i)}_{n'}\big) \\ &\times \bigg(\prod_{m'=1}^{M^{(i)}_{n'}} \Psi\big(\underline{a}^{(i)}_{ss,n'}, \overline{a}^{(i)}_{m',n'}\big)\bigg) \prod_{s'=1, s' \neq s}^{S^{(i)}_{n'}} \underline{q}_{\mathrm{D1}}\big(\underline{\boldsymbol{y}}^{(i)}_{s,n'}, \underline{\boldsymbol{y}}^{(i)}_{s',n'}, \underline{a}^{(i)}_{ss',n'}, \boldsymbol{p}_{n'}\big) \underline{q}_{\mathrm{D2}}\big(\underline{\boldsymbol{y}}^{(i)}_{s,n'}, \underline{\boldsymbol{y}}^{(i)}_{s',n'}, \underline{a}^{(i)}_{ss',n'}, \boldsymbol{x}_{n'}; \boldsymbol{z}^{(i)}_{n'}\big)\bigg) \prod_{m'=1}^{M^{(i)}_{n'}} \Psi\big(\underline{a}^{(i)}_{ss',n'}, \overline{a}^{(i)}_{m',n'}\big)\bigg) \\ &\times \bigg(\prod_{m=1}^{M^{(i)}_{n'}} \overline{q}_{\mathrm{S1}}\big(\overline{\boldsymbol{y}}^{(i)}_{m,n'}, \overline{a}^{(i)}_{m,n'}, \boldsymbol{p}_{n'}\big) \overline{q}_{\mathrm{S2}}\big(\overline{\boldsymbol{y}}^{(i)}_{m,n'}, \overline{a}^{(i)}_{m,n'}, \boldsymbol{x}_{n'}; \boldsymbol{z}^{(i)}_{n'}\big)\bigg) \end{aligned} \tag{37}$$

The factors related to the legacy PMVAs and to the new PMVAs can be simplified as $\underline{q}_{\mathrm{P}}\big(\boldsymbol{x}_{n'}, \underline{a}^{(i)}_{00,n'}; \boldsymbol{z}^{(i)}_{n'}\big) \triangleq \underline{q}_{\mathrm{P1}}\big(\boldsymbol{p}_{n'}, \underline{a}^{(i)}_{00,n'}\big) \underline{q}_{\mathrm{P2}}\big(\boldsymbol{x}_{n'}, \underline{a}^{(i)}_{00,n'}; \boldsymbol{z}^{(i)}_{n'}\big)$, $\underline{q}_{\mathrm{S}}\big(\underline{\boldsymbol{y}}^{(i)}_{s,n'}, \underline{a}^{(i)}_{ss,n'}, \boldsymbol{x}_{n'}; \boldsymbol{z}^{(i)}_{n'}\big) \triangleq \underline{q}_{\mathrm{S1}}\big(\underline{\boldsymbol{y}}^{(i)}_{s,n'}, \underline{a}^{(i)}_{ss,n'}, \boldsymbol{p}_{n'}\big) \underline{q}_{\mathrm{S2}}\big(\underline{\boldsymbol{y}}^{(i)}_{s,n'}, \underline{a}^{(i)}_{ss,n'}, \boldsymbol{x}_{n'}; \boldsymbol{z}^{(i)}_{n'}\big)$, $\underline{q}_{\mathrm{D}}\big(\underline{\boldsymbol{y}}^{(i)}_{s,n'}, \underline{\boldsymbol{y}}^{(i)}_{s',n'}, \underline{a}^{(i)}_{ss',n'}, \boldsymbol{x}_{n'}; \boldsymbol{z}^{(i)}_{n'}\big) \triangleq \underline{q}_{\mathrm{D1}}\big(\underline{\boldsymbol{y}}^{(i)}_{s,n'}, \underline{\boldsymbol{y}}^{(i)}_{s',n'}, \underline{a}^{(i)}_{ss',n'}, \boldsymbol{p}_{n'}\big) \underline{q}_{\mathrm{D2}}\big(\underline{\boldsymbol{y}}^{(i)}_{s,n'}, \underline{\boldsymbol{y}}^{(i)}_{s',n'}, \underline{a}^{(i)}_{ss',n'}, \boldsymbol{x}_{n'}; \boldsymbol{z}^{(i)}_{n'}\big)$, and $\overline{q}_{\mathrm{S2}}\big(\overline{\boldsymbol{y}}^{(i)}_{m,n'}, \overline{a}^{(i)}_{m,n'}, \boldsymbol{x}_{n'}; \boldsymbol{z}^{(i)}_{n'}\big) \triangleq \overline{q}_{\mathrm{S1}}\big(\overline{\boldsymbol{y}}^{(i)}_{m,n'}, \overline{a}^{(i)}_{m,n'}, \boldsymbol{p}_{n'}\big) \overline{q}_{\mathrm{S2}}\big(\overline{\boldsymbol{y}}^{(i)}_{m,n'}, \overline{a}^{(i)}_{m,n'}, \boldsymbol{x}_{n'}; \boldsymbol{z}^{(i)}_{n'}\big)$ respectively, yielding [1, Eq. (17)].

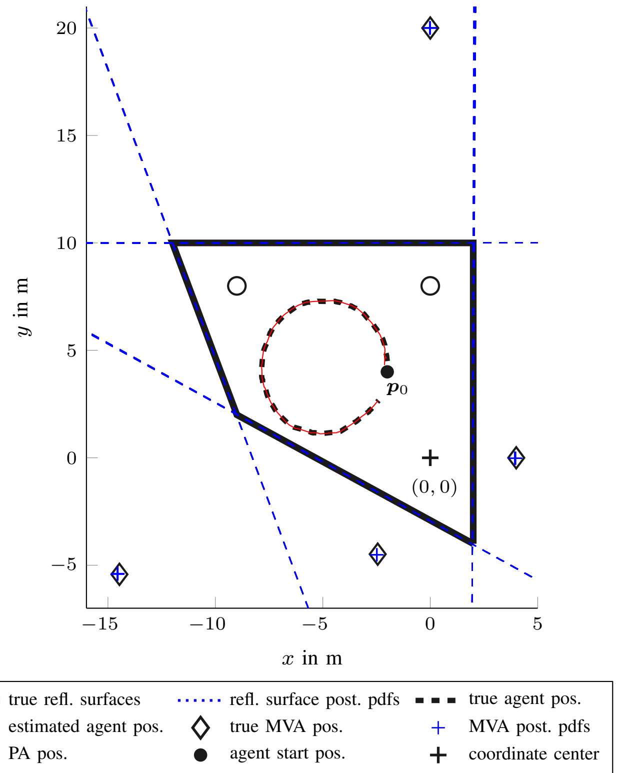

Fig. 1. Considered scenario for performance evaluation in non-rectangular room with two PAs, four reflective surfaces and corresponding MVAs, as well as agent trajectory.

III. Iterative Data Association

This section contains the detailed messages of Section [1, Sec. V-B6]. Using the messages $\beta\big(\underline{a}^{(i)}_{ss',n}\big)$ given [1, Sec. V-B4] as well as in [1, Sec. V-B3] and $\xi\big(\overline{a}^{(i)}_{m,n}\big)$ given in [1, Sec. V-B5], messages $\eta\big(\underline{a}^{(i)}_{ss',n}\big)$ and $\varsigma\big(\overline{a}^{(i)}_{m,n}\big)$ are obtained using loopy (iterative) BP. To keep the notation concise, we also define the sets $\mathcal{M}^{(i)}_{0,n} \triangleq \mathcal{M}^{(i)}_n \cup \{0\}$ and $\tilde{\mathcal{D}}^{(i)}_{0,n} \in \tilde{\mathcal{D}}^{(i)}_n \cup \{0\}$. For each measurement, $m \in \mathcal{M}^{(i)}_n \triangleq \{1, \ldots, M^{(i)}_n\}$, messages $\nu^{(p)}_{m\to s}\big(\underline{a}^{(i)}_{ss',n}\big)$ and $\zeta^{(p)}_{s\to m}\big(\overline{a}^{(i)}_{m,n}\big)$ are calculated iteratively according to [3], [8]

$$\nu^{(p)}_{m\to ss'}\big(\underline{a}^{(i)}_{ss',n}\big) = \sum_{\overline{a}^{(i)}_{m,n}\in\tilde{\mathcal{D}}^{(i)}_{0,n}} \xi\big(\overline{a}^{(i)}_{m,n}\big)\,\psi\big(\underline{a}^{(i)}_{ss',n}, \overline{a}^{(i)}_{m,n}\big) \prod_{(s'',s''')\in\tilde{\mathcal{D}}^{(i)}_n\setminus\{ss'\}} \zeta^{(p-1)}_{s''s'''\to m}\big(\overline{a}^{(i)}_{m,n}\big) \tag{38}$$

$$\zeta^{(p)}_{ss'\to m}\big(\overline{a}^{(i)}_{m,n}\big) = \sum_{\underline{a}^{(i)}_{ss',n}\in\mathcal{M}^{(i)}_{0,n}} \beta\big(\underline{a}^{(i)}_{ss',n}\big)\,\psi\big(\underline{a}^{(i)}_{ss',n}, \overline{a}^{(i)}_{m,n}\big) \prod_{m'\in\mathcal{M}^{(i)}_n\setminus\{m\}} \nu^{(p)}_{m'\to ss'}\big(\underline{a}^{(i)}_{ss',n}\big), \tag{39}$$

for $ss' \in \tilde{\mathcal{D}}^{(i)}_n$, $m \in \mathcal{M}^{(i)}_n$, and iteration index $p \in \{1, \ldots, P\}$. The recursion defined by (38) and (39) is initialized (for $p=0$) by $\zeta^{(0)}_{ss'\to m}\big(\overline{a}^{(i)}_{m,n}\big) = \sum_{\underline{a}^{(i)}_{ss',n}=0}^{M^{(i)}_n} \beta\big(\underline{a}^{(i)}_{ss',n}\big)\psi\big(\underline{a}^{(i)}_{ss',n}, \overline{a}^{(i)}_{m,n}\big)$. Then, after the last iteration $p = P$, the messages $\eta\big(\underline{a}^{(i)}_{ss',n}\big)$ and $\varsigma\big(\overline{a}^{(i)}_{m,n}\big)$ are calculated as

$$\eta\big(\underline{a}^{(i)}_{ss',n}\big) = \prod_{m\in\mathcal{M}^{(i)}_n} \nu^{(P)}_{m\to ss'}\big(\underline{a}^{(i)}_{ss',n}\big) \tag{40}$$

$$\varsigma\big(\overline{a}^{(i)}_{m,n}\big) = \prod_{ss'\in\mathcal{D}^{(i)}_n} \zeta^{(P)}_{ss'\to m}\big(\overline{a}^{(i)}_{m,n}\big). \tag{41}$$

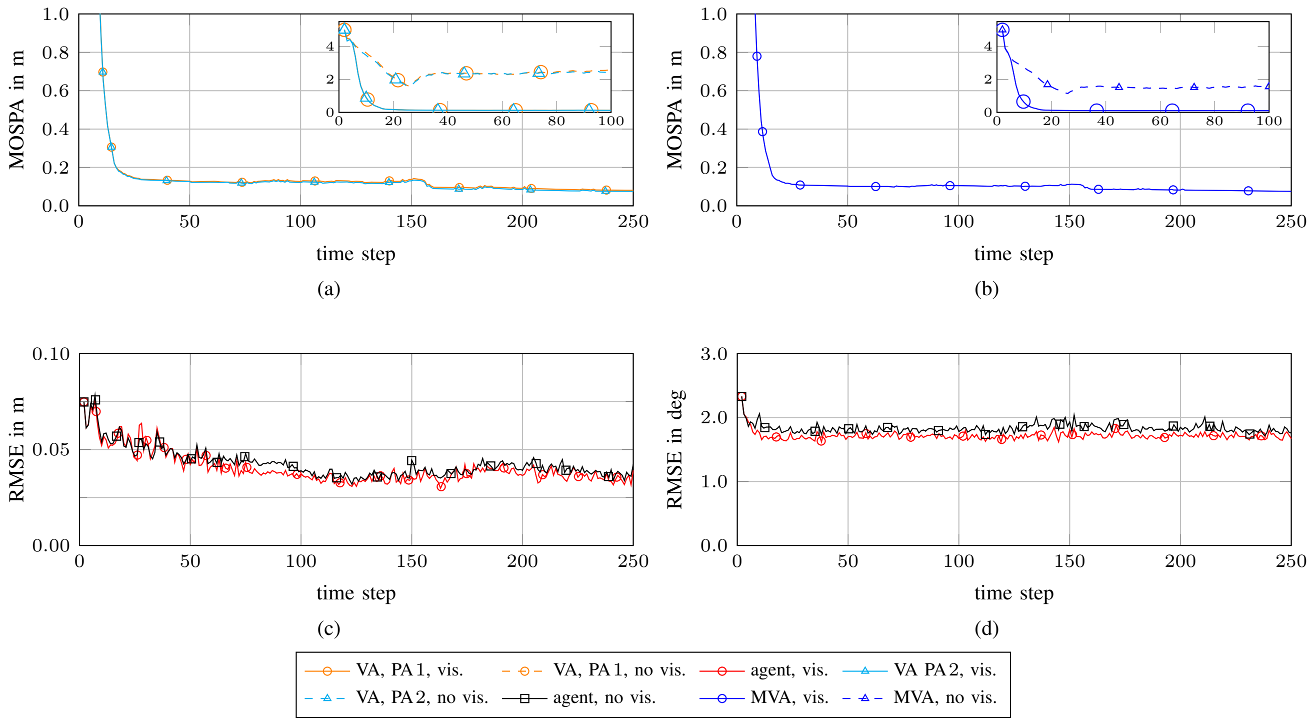

Fig. 2. Performance results: (a) MOSPA errors of the VAs of each PA, (b) MOSPA errors of the MVAs, (c) RMSEs of the mobile agent position, (d) RMSEs of the agent orientation.

IV. Additional Results: Performance in Non-Rectangular Room

In this section, we provide additional simulation results using synthetic measurements. In particular, this experiment demonstrates the need for an integrated ray-tracing (RT) to determine the available propagation paths. The setups and parameter are set according to Section [1, Sec. VI-A].

In this experiment, we compare the complete version of the proposed MVA-based SLAM algorithm (vis.), which includes the availability checks as introduced in Sec. [1, Fig. III-D] and Sec. [1, Fig. V-B2], with a reduced variant, where we deactivate availability checks (no vis.), i.e., $p_{\mathrm{d}}(\boldsymbol{p}_n, \boldsymbol{p}_{s,\mathrm{mva}}^{(i)}) = p_{\mathrm{d}}(\boldsymbol{p}_n, \boldsymbol{p}_{s,\mathrm{mva}}^{(i)}, \boldsymbol{p}_{s',\mathrm{mva}}^{(i)}) = p_{\mathrm{d}}$. We consider the indoor scenario shown in Figure 1. The scenario consists of four reflective surfaces, i.e., $K = 4$ MVAs, as well as two PAs at positions $\boldsymbol{p}_{\mathrm{pa}}^{(1)} = [-0.5 \;\; 6]^{\mathrm{T}}$, and $\boldsymbol{p}_{\mathrm{pa}}^{(2)} = [4.2 \;\; 1.3]^{\mathrm{T}}$. The acceleration noise standard deviation is $\sigma_w = 0.02\,\mathrm{m/s^2}$.

Fig. 2a shows the mean optimal subpattern assignment (MOSPA) errors for the two PAs all associated VAs, Fig. 2b shows the MOSPA errors for all MVAs, Fig. 2c shows the root mean-square error (RMSE) of the mobile agent's position, and Fig. 2d shows the RMSE of the mobile agent's orientation, all versus time n. As an example, Fig. 1 also depicts one simulation run of the complete version of the proposed method with availability checks. The posterior PDFs of the MVA positions represented by particles, the corresponding reflective surfaces, and the estimated agent tracks are also shown. The MOSPA errors in Fig. 2a and 2b of the algorithm variant with availability check converge faster and to a much smaller value than those of the variant without availability check. This is because in the scenario investigated, several VAs corresponding to the left as well as the lower walls are not available over large parts of the trajectory (this is the case especially for VAs of $\boldsymbol{p}_{\mathrm{pa}}^{(2)}$). See also Fig. [1, Fig. 2c], which provides a graphical explanation. Thus, the algorithm variant without availability check tends to deactivate the corresponding MVAs (i.e., strongly lower the probability of existence) as some of the corresponding VAs, which are expected to be detected with p_{d} are not observed for significant amounts of time. The RMSE of the agent position are not strongly influenced by this deactivation since still sufficient position-related information is provided by the two PAs and the remaining VAs.

V. Implementation of the Proposed MVA-based SLAM Method

Pseudocode for one time step of the proposed MVA-based SLAM method is provided in Algorithm 1. This pseudocode closely follows the presentation of the particle-based implementation discussed in [1, Section V]. Note that the existence probabilities are defined as $p^{\mathrm{e}}_{s,\mathrm{mva}} \triangleq p(r_{s,n}=1|\boldsymbol{z}_{1:n})$.

Algorithm 1: Proposed Particle-Based MVA SLAM Method — Single Time Step

$\Big[\{\boldsymbol{x}^{(i)}\}_{i=1}^{I}, \Big\{\{\boldsymbol{p}^{(i)}_{s,\mathrm{mva}}\}_{i=1}^{J}, p^{\mathrm{e}}_{s,\mathrm{mva}}\Big\}_{s=1}^{S^{(J)}+M^{(J)}}\Big] = \mathrm{DFSLAM}\Big[\{\boldsymbol{x}^{-(i)}\}_{i=1}^{I}, \Big\{\{\boldsymbol{p}^{-(i)}_{s,\mathrm{mva}}\}_{i=1}^{I}, p^{-\mathrm{e}}_{s,\mathrm{mva}}\Big\}_{k=1}^{S^-}, \{\boldsymbol{z}_m\}_{m=1}^{M}\Big]$

for $i = 1 : I$ **do**
  $\tilde{\boldsymbol{x}}^{(i)} \sim f(\boldsymbol{x}^{(i)}|\boldsymbol{x}^-)$ and $w^{(i)} = \frac{1}{I}$ `// prediction of agent`
end
for $s = 1 : S^-$ **do**
  Calculate $\underline{p}^{(0)\mathrm{e}}_{s,\mathrm{mva}} = p^{-\mathrm{e}}_{s,\mathrm{mva}}$ `// prediction legacy MVAs`
  for $i = 1 : I$ **do**
    $\underline{\boldsymbol{p}}^{(0,i)}_{s,\mathrm{mva}} \sim f_{\mathrm{reg}}(\boldsymbol{p}^{(i)}_{s,\mathrm{mva}})$ `// regularize MVA`
  end
end
$S^{(1)} = S^-$
$M^{(0)} = 0$
for $j = 1 : J$ **do**
  `// Loop PAs`
  for $i = 1 : I$ **do**
    for $m = 1 : M^{(j-1)}$ **do** `// stacking MVAs`
      $\tilde{\underline{\boldsymbol{p}}}^{(j,i)}_{S^{(j-1)}+m,\mathrm{mva}} = \overline{\boldsymbol{p}}^{(j-1,i)}_{m,\mathrm{mva}}$
      $\underline{p}^{(j)\mathrm{e}}_{S^{(j-1)}+m,\mathrm{mva}} = \overline{p}^{(j)\mathrm{e}}_{s,\mathrm{mva}}$
    end
  end
  $S^{(j)} = S^{(j-1)} + M^{(j-1)}$
  for $i = 1 : I$ **do**
    Draw $\boldsymbol{p}^{(j,i)}_{m,\mathrm{va}}$ from the inverse of (5) and (6)
    Calculate new PMVAs $\overline{\boldsymbol{p}}^{(i)}_{m,\mathrm{mva}}$ using $\boldsymbol{p}^{(j,i)}_{m,\mathrm{va}}$ and the transform in (4) `// draw samples new MVAs`
  end
  `visibilityCheck()` `// ray-tracing (RT)`
  `measEvalPAs()` `// MVAs messages to data association nodes`
  `measEvalLegacyMVAs()` `// legacy MVAs messages to data association nodes`
  `measEvalNewMVAs()` `// new MVAs messages to data association nodes`
  `dataAssociation()` `// data association`
  `measUpdateAgent()` `// measurement update agent`
  `measUpdateLegacyMVA()` `// measurement legacy MVAs`
  `measUpdateNewMVA()` `// measurement new MVAs`
  `resamplingMVA()` `// resampling of MVAs`
  `pruning()` `// remove unreliable MVAs`
end
`agentBelief()` `// agent belief update`
Output: $\{\boldsymbol{x}^{(i)}\}_{i=1}^{I}, \Big\{\{\boldsymbol{p}^{(i)}_{s,\mathrm{mva}}\}_{i=1}^{J}, p^{\mathrm{e}}_{s,\mathrm{mva}}\Big\}_{s=1}^{S^{(J)}+M^{(J)}}$

Sub-Routine 1: Visibility Check

Procedure `visibilityCheck()`

for $i = 1 : I$ **do**

  for $s = 1 : S^{(j)}$ **do**

    $\boldsymbol{p}^{(j,i)}_{ss,\mathrm{va}}$ is calculated according to (3) using $\underline{\tilde{\boldsymbol{p}}}^{(j,i)}_{s,\mathrm{mva}}$ and $\boldsymbol{p}^{(j)}_{\mathrm{pa}}$

    $\underline{v}^{(i)}_{ss} = f_{\mathrm{vis}}\big(\boldsymbol{x}, \boldsymbol{p}^{(j,i)}_{s,\mathrm{va}}\big)$ `// recursive visibility check [9], [10]`

    for $s' = 1 : S^{(j)}$ *and* $s \neq s'$ **do**

      $\boldsymbol{p}^{(j,i)}_{ss',\mathrm{va}}$ is calculated according to (3) using $\underline{\tilde{\boldsymbol{p}}}^{(j,i)}_{s,\mathrm{mva}}$ and $\boldsymbol{p}^{(j,i)}_{s,\mathrm{va}}$

      $\underline{v}^{(i)}_{ss'} = f_{\mathrm{vis}}\big(\boldsymbol{x}, \boldsymbol{p}^{(j,i)}_{ss',\mathrm{va}}\big)$ `// recursive visibility check [9], [10]`

    end

  end

end

Sub-Routine 2: measurement evaluation PAs

Procedure `measEvalPAs()`

for $m = 1 : M^{(j)}$ **do**

  Calculate $\beta^{(j)}_{00,m} = \frac{1}{\mu_{\mathrm{fp}}\, f_{\mathrm{fp}}\left(\boldsymbol{z}^{(j)}_m\right)} \sum_{i=1}^{I} w^{(i)} p^{(j)}_{\mathrm{d},00}\, f\left(\boldsymbol{z}^{(j)}_m | \tilde{\boldsymbol{x}}^{(i)}\right)$

end

Sub-Routine 3: measurement evaluation legacy MVAs

Procedure `measEvalLegacyMVAs()`

for $i = 1 : I$ **do**

  for $m = 1 : M^{(j)}$ **do**

    for $s = 1 : S^{(j)}$ **do**

      if $m == 0$ **then**

        $\beta^{(j)}_{ss,m} = \left(1 - \underline{p}^{(j)\,\mathrm{e}}_s\right) + \left(1 - \underline{v}^{(i)}_{ss}\; p^{(j)}_{\mathrm{d},ss}\right)$

      end

      Calculate $\beta^{(j)}_{ss,m} = \frac{\underline{p}^{(j)\,\mathrm{e}}_s}{\mu_{\mathrm{fp}}\, f_{\mathrm{fp}}\left(\boldsymbol{z}^{(j)}_m\right)} \sum_{i=1}^{I} w^{(i)}\; \underline{v}^{(i)}_{ss}\; p^{(j)}_{\mathrm{d},ss}\; f\left(\boldsymbol{z}^{(j)}_m | \tilde{\boldsymbol{x}}^{(i)}, \underline{\tilde{\boldsymbol{p}}}^{(j,i)}_{s,\mathrm{mva}}\right) + \underline{p}^{(j)\,\mathrm{e}}_s \left(1 - \underline{v}^{(i)}_{ss}\; p^{(j)}_{\mathrm{d},ss}\right)$

      for $s' = 1 : S^{(j)}$ *and* $s \neq s'$ **do**

        if $m == 0$ **then**

          Calculate $\beta^{(j)}_{ss',m} = \left(1 - \underline{p}^{(j)\,\mathrm{e}}_s\right)\left(1 - \underline{p}^{(j)\,\mathrm{e}}_{s'}\right) + \left(1 - \underline{v}^{(i)}_{ss'}\; p^{(j)}_{\mathrm{d},ss'}\right)$

        end

        Calculate $\beta^{(j)}_{ss,m} = \frac{\underline{p}^{(j)\,\mathrm{e}}_s\; \underline{p}^{(j)\,\mathrm{e}}_{s'}}{\mu_{\mathrm{fp}}\, f_{\mathrm{fp}}\left(\boldsymbol{z}^{(j)}_m\right)} \sum_{i=1}^{I} w^{(i)}\; \underline{v}^{(i)}_{ss'}\; p^{(j)}_{\mathrm{d},ss'}\; f\left(\boldsymbol{z}^{(j)}_m | \tilde{\boldsymbol{x}}^{(i)}, \underline{\tilde{\boldsymbol{p}}}^{(j,i)}_{s,\mathrm{mva}}, \underline{\tilde{\boldsymbol{p}}}^{(j,i)}_{s',\mathrm{mva}}\right)$

      end

    end

  end

end

Sub-Routine 4: measurement evaluation new MVAs

Procedure `measEvalNewMVAs()`

for $i = 1 : I$ **do**

  for $m = 1 : M^{(j)}$ **do**

    Calculate $\zeta^{(j)}_m = 1 + \frac{1}{\mu_{\mathrm{fp}}\, f_{\mathrm{fp}}\left(\boldsymbol{z}^{(j)}_m\right)} \sum_{i=1}^{I} w^{(i)}\; f\left(\boldsymbol{z}^{(j)}_m | \tilde{\boldsymbol{x}}^{(i)}, \underline{\tilde{\boldsymbol{p}}}^{(j,i)}_{m,\mathrm{mva}}\right)$

  end

end

Sub-Routine 5: data association

Procedure `dataAssociation()`

  Calculate $\eta^{(j)}_{ss',m}$ and $\varsigma^{(j)}_m$ from $\beta^{(j)}_{ss',m}$ and $\zeta^{(j)}_{m,ss'}$ with $(s, s') \in \tilde{\mathcal{D}}^{(j)}$ and $m \in \tilde{\mathcal{M}}^{(j)}_0$ (see Section III)

Sub-Routine 6: measurement update agent

Procedure `measUpdateAgent()`

Calculate $\gamma_{00}^{(j,i)} = \frac{1}{\mu_{\mathrm{fp}} f_{\mathrm{fp}}\left(\boldsymbol{z}_m^{(j)}\right)} \sum_{m=1}^{M^{(j)}} \eta_{00,m}^{(j)} w^{(i)} p_{\mathrm{d},00}^{(j)} f\left(\boldsymbol{z}_m^{(j)} | \underline{\tilde{\boldsymbol{p}}}^{(i)}\right) + \eta_{00,0}^{(j)}\left(1 - p_{\mathrm{d},00}^{(j)}\right)$

for $i = 1 : I$ **do**

for $s = 1 : S^{(j)}$ **do**

Calculate $\gamma_{ss}^{(j,i)} = \frac{\underline{p}_s^{(j)\,\mathrm{e}}}{\mu_{\mathrm{fp}} f_{\mathrm{fp}}\left(\boldsymbol{z}_m^{(j)}\right)} \sum_{m=1}^{M^{(j)}} \eta_{ss,m}^{(j)}\; w^{(i)}\; \underline{v}_{ss}^{(i)}\; p_{\mathrm{d},ss}^{(j)}\; f\left(\boldsymbol{z}_m^{(j)} | \tilde{\boldsymbol{x}}^{(i)}, \underline{\tilde{\boldsymbol{p}}}_{s,\mathrm{mva}}^{(i)}\right) + \eta_{ss,0}^{(j)}\; \underline{p}_s^{(j)\;\mathrm{e}}\left(1 - \underline{v}_{ss}^{(i)}\; p_{\mathrm{d},ss}^{(j)}\right) + \left(1 - \underline{p}_s^{(j)\,\mathrm{e}}\right)$

for $s' = 1 : S^{(j)}$ *and* $s \neq s'$ **do**

Calculate $\gamma_{ss'}^{(j,i)} = \frac{\underline{p}_s^{(j)\,\mathrm{e}}\; \underline{p}_{s'}^{(j)\,\mathrm{e}}}{\mu_{\mathrm{fp}} f_{\mathrm{fp}}\left(\boldsymbol{z}_m^{(j)}\right)} \sum_{m=1}^{M^{(j)}} \eta_{ss',m}^{(j)}\; w^{(i)}\; \underline{v}_{ss'}^{(i)}\; p_{\mathrm{d},ss'}^{(j)}\; f\left(\boldsymbol{z}_m^{(j)} | \tilde{\boldsymbol{x}}^{(i)}, \underline{\tilde{\boldsymbol{p}}}_{s,\mathrm{mva}}^{(i)}, \underline{\tilde{\boldsymbol{p}}}_{s',\mathrm{mva}}^{(i)}\right) + \eta_{ss',0}^{(j)}\; \underline{p}_s^{(j)\,\mathrm{e}}\; \underline{p}_{s'}^{(j)\,\mathrm{e}}\Big(1 -$
$\underline{v}_{ss'}^{(i)}\; p_{\mathrm{d},ss'}^{(j)}\Big) + \big(1 - \underline{p}_s^{(j)\,\mathrm{e}}\big)\big(1 - \underline{p}_{s'}^{(j)\,\mathrm{e}}\big)$

end

end

end

Sub-Routine 7: measurement update legacy MVAs

Procedure `measUpdateLegacyMVA()`

for $i = 1 : I$ **do**

for $s = 1 : S^{(j)}$ **do**

Calculate $\rho_{ss,1}^{(j,i)} = \frac{\underline{p}_s^{(j)\,\mathrm{e}}}{\mu_{\mathrm{fp}} f_{\mathrm{fp}}\left(\boldsymbol{z}_m^{(j)}\right)} \sum_{m=1}^{M^{(j)}} \eta_{ss,m}^{(j)}\; w^{(i)}\; \underline{v}_{ss}^{(i)}\; p_{\mathrm{d},ss}^{(j)}\; f\left(\boldsymbol{z}_m^{(j)} | \tilde{\boldsymbol{x}}^{(i)}, \boldsymbol{p}_{s,\mathrm{mva}}^{(i)}\right) + \eta_{ss,0}^{(j)}\; \underline{p}_s^{(j)\,\mathrm{e}}\left(1 - \underline{v}_{ss}^{(i)}\; p_{\mathrm{d},ss}^{(j)}\right)$

Calculate $\rho_{ss,0}^{(j)} = \eta_{ss,0}^{(j)}$

for $s' = 1 : S^{(j)}$ *and* $s \neq s'$ **do**

Calculate $\rho_{ss',1}^{(j,i)} = \frac{\underline{p}_s^{(j)\,\mathrm{e}} \underline{p}_{s'}^{(j)\,\mathrm{e}}}{\mu_{\mathrm{fp}} f_{\mathrm{fp}}\left(\boldsymbol{z}_m^{(j)}\right)} \sum_{m=0}^{M^{(j)}} \eta_{ss',m}^{(j)}\; w^{(i)}\; \underline{v}_{ss'}^{(i)}\; p_{\mathrm{d},ss'}^{(j)}\; f\left(\boldsymbol{z}_m^{(j)} | \tilde{\boldsymbol{x}}^{(i)}, \boldsymbol{p}_{s,\mathrm{mva}}^{(i)}, \boldsymbol{p}_{s',\mathrm{mva}}^{(i)}\right) + \eta_{ss',0}^{(j)}\; \underline{p}_s^{(j)\mathrm{e}}\; \underline{p}_{s'}^{(j)\,\mathrm{e}}$
$\times\left(1 - \underline{v}_{ss'}^{(i)}\; p_{\mathrm{d},ss'}^{(j)}\right)$

Calculate $\rho_{ss',0}^{(j)} = \eta_{ss',0}^{(j)}$

end

end

Calculate $\gamma_{s,1}^{(j,i)} = \rho_{ss,1}^{(j,i)} + \prod_{s'=1,s\neq s'}^{S_n^{(j)}} \rho_{ss',1}^{(j,i)}$

Calculate $\gamma_{s,0}^{(j)} = \rho_{ss,0}^{(j)} + \prod_{s'=1,s\neq s'}^{S_n^{(j)}} \rho_{ss',0}^{(j)}$

end

for $s = 1 : S^{(j)}$ **do**

$\underline{p}_s^{(j)\,\mathrm{e}} = \frac{\sum_{i=1}^{(I)} \gamma_{s,1}^{(j,i)}}{\gamma_{s,0}^{(j)} + \sum_{i=1}^{(I)} \gamma_{s,1}^{(j,i)}}$

$\gamma_s^{(j,i)} = \frac{\gamma_{s,1}^{(j,i)}}{\sum_{i=1}^{(I)} \gamma_{s,1}^{(j,i)}}$

end

Sub-Routine 8: measurement update new MVAs

Procedure `measUpdateNewMVA()`

  for $i = 1 : I$ **do**

    for $m = 1 : M^{(j)}$ **do**

      Calculate $\phi^{(j,i)}_{m,1} = \frac{1}{\mu_{\mathrm{fp}}\, f_{\mathrm{fp}}\left(\boldsymbol{z}^{(j)}_m\right)}\ \zeta^{(j)}_{m,0}\ w^{(i)}\ f\left(\boldsymbol{z}^{(j)}_m | \tilde{\boldsymbol{x}}^{(i)}, \overline{\boldsymbol{p}}^{(j,i)}_{m,\mathrm{mva}}\right)$

      Calculate $\phi^{(j)}_{m,0} = \sum_{\tilde{\mathcal{D}}^{(j)}_0} \varsigma^{(j)}_{m,ss'}$

      $\overline{p}^{(j)\,\mathrm{e}}_m = \frac{\sum_{i=1}^{I} \phi^{(j,i)}_{m,1}}{\phi^{(j)}_{m,0} + \sum_{i=1}^{I} \phi^{(j,i)}_{m,1}}$

      $\phi^{(j,i)}_m = \frac{\phi^{(j,i)}_{m,1}}{\sum_{i=1}^{I} \phi^{(j,i)}_{m,1}}$

    end

  end

Sub-Routine 9: resampling of MVAs

Procedure `resamplingMVA()`

  for $s = 1 : S^{(j)}$ **do**

    $\left[\left\{\frac{1}{I}, \underline{\boldsymbol{p}}^{(j,i)}_{s,\mathrm{mva}}\right\}_{i=1}^{I}\right] = \text{resampling}\left(\left\{\gamma^{(j,k)}_s, \tilde{\underline{\boldsymbol{p}}}^{(j,k)}_{s,\mathrm{mva}}\right\}_{k=1}^{I}\right)$ `// systematic resampling [11]`

  end

  for $m = 1 : M^{(j)}$ **do**

    $\left[\left\{\frac{1}{I}, \overline{\boldsymbol{p}}^{(j,i)}_{m,\mathrm{mva}}\right\}_{i=1}^{I}\right] = \text{resampling}\left(\left\{\phi^{(j,i)}_m, \tilde{\overline{\boldsymbol{p}}}^{(j,k)}_{m,\mathrm{mva}}\right\}_{k=1}^{I}\right)$ `// systematic resampling [11]`

  end

Sub-Routine 10: pruning of MVAs

Procedure `pruning()`

  $S_{\mathrm{pr}} = 0$

  for $s = 1 : S^{(j)}$ **do**

    if $\underline{p}^{(j)\,\mathrm{e}}_s < p_{pr}$ **then**

      $\underline{p}^{(j)\,\mathrm{e}}_s = []$

      $S_{\mathrm{pr}} = S_{\mathrm{pr}} + 1$

      for $i = 1 : I$ **do**

        $\underline{\boldsymbol{p}}^{(j,i)}_{s,\mathrm{mva}} = []$

      end

    end

  end

  $S^{(j)} = S^{(j)} - S_{\mathrm{pr}}$

  $M_{\mathrm{pr}} = 0$

  for $m = 1 : M^{(j)}$ **do**

    if $\overline{p}^{(j)\,\mathrm{e}}_m < p_{pr}$ **then**

      $\underline{p}^{(j)\,\mathrm{e}}_s = []$

      $M_{\mathrm{pr}} = M_{\mathrm{pr}} + 1$

      for $i = 1 : I$ **do**

        $\overline{\boldsymbol{p}}^{(j,i)}_{m,\mathrm{mva}} = []$

      end

    end

  end

  $M^{(j)} = M^{(j)} - M_{\mathrm{pr}}$

Sub-Routine 11: agent belief calculations MVAs

Procedure `agentBelief()`

for $i = 1 : I$ **do**

$\gamma^{(i)} = \prod_{ss' \in \tilde{\mathcal{D}}(J)} \gamma_{ss'}^{(J,i)}$

$\gamma^{(i)} = \frac{\gamma^{(i)}}{\sum_{i=1}^{(I)} \gamma^{(i)}}$

$\left[\left\{ \frac{1}{I}, \boldsymbol{x}^{(i)} \right\}_{i=1}^{I} \right] = \text{resampling}\left(\left\{ \gamma^{(i)}, \tilde{\boldsymbol{x}}^{(i)} \right\}_{k=1}^{I} \right)$ `// systematic resampling [11]`

end

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