SIO 207A: Fundamentals of Digital Signal Processing
Class 5

Florian Meyer

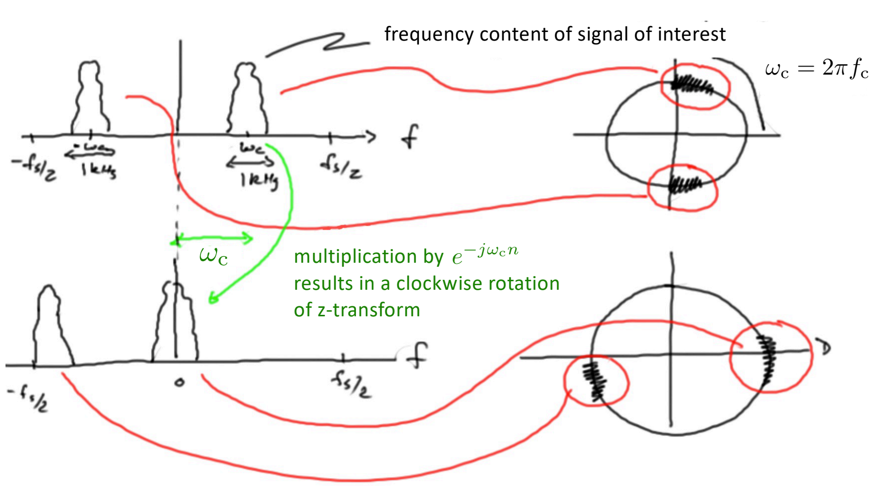
Scripps Institution of Oceanography
Electrical and Computer Engineering Department
University of California San Diego

SCRIPPS INSTITUTION OF OCEANOGRAPHY

UC San Diego
JACOBS SCHOOL OF ENGINEERING

0

Complex Basebanding



1

1

Z-Transform of Finite Length Sequences

$N = 4$
$a = 1$

- Assume sequence is of length N

$$X(z) = \sum_{n=0}^{N-1} x[n] z^{-n} \qquad x[n] = a^n \text{ for } n = 0, \ldots, N-1$$

$N = 4$
$a = 0.9$

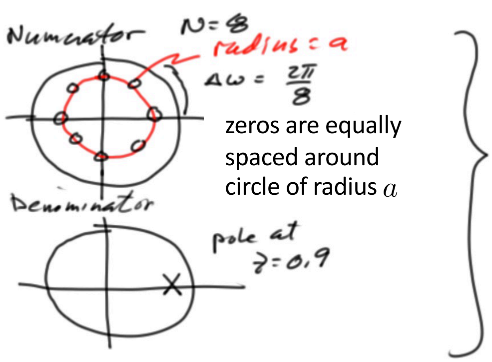
$$\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha} = \frac{1-(az^{-1})^N}{1-(az^{-1})} = \left(\frac{z}{z-a}\right)\left(\frac{z^N - a^N}{z^N}\right)$$

2

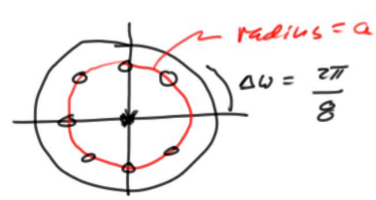
2

Z-Transform of Finite Length Sequences

$$X(z) = \left(\frac{z}{z-a}\right)\left(\frac{z^N - a^N}{z^N}\right) \qquad x[n] = a^n \text{ for } n = 0, \ldots, N-1$$



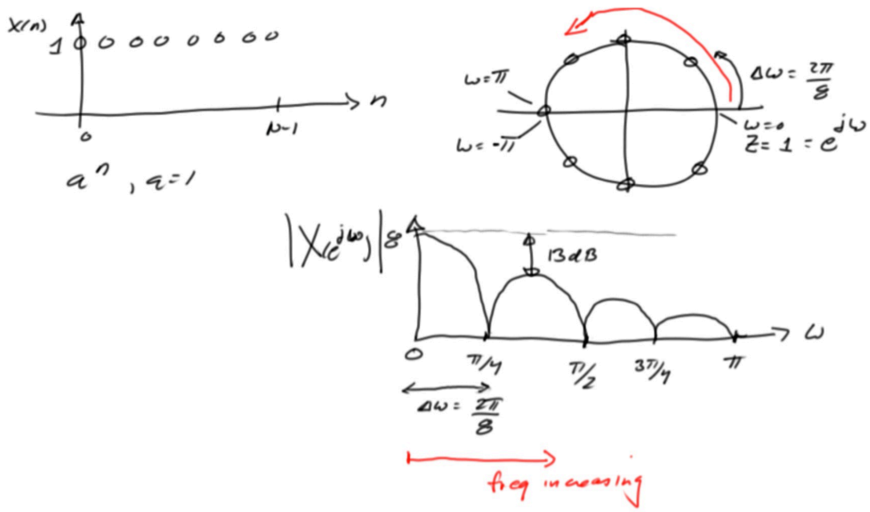
zeros are equally spaced around circle of radius a



3

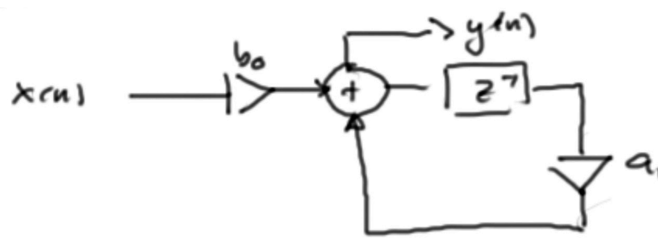
3

Z-Transform of Finite Length Sequences

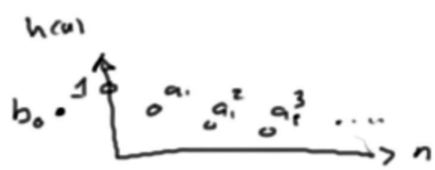


Computing the Output of a Linear System

$$y[n] = b_0 x[n] + a_1 y[n-1]$$
$$= \sum_{k=0}^{\infty} h[k] x[n-k]$$



- The impulse response is given as $h[n] = b_0 a_1^n u[n]$ where $u[n]$ is the unit step function



Computing the Output of a Linear System

- We want to express output $y[n]$ due to unit step input
- What is $y[n]$ for $x[n] = u[n]$

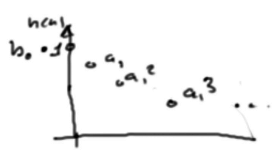
$$y[n] = \sum_{k=0}^{\infty} \left(b_0 a_1^k\right) x[n-k]$$
$$= \sum_{k=0}^{n} b_0 a_1^k$$
$$= b_0 \left(\frac{1 - a_1^{n+1}}{1 - a_1} \right)$$

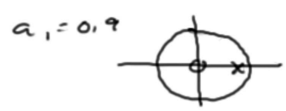
6

6

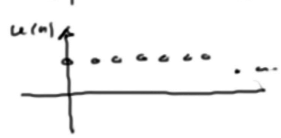
Computing the Output of a Linear System

$h(n) = b_0 a_1^n u(n)$



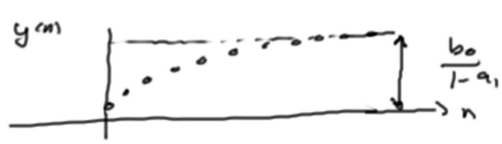


$x(n) = u(n)$



- **A:** Time domain output signal $y[n]$

$y(n) = b_0 \left(\frac{1 - a_1^{n+1}}{1 - a_1} \right)$



DC gain: $\frac{b_0}{1 - a_1}$

7

7

Computing the Output of a Linear System

- **B:** Z-domain output signal $Y(z)$: Recognize that a convolution in the time domain is a product in the z-domain

$$Y(z) = H(z)X(z)$$
$$= \left(\frac{b_0 z}{z - a_1}\right)\left(\frac{z}{z-1}\right)$$
$$\xrightarrow{\text{partial fraction expansion}} = C_0 + C_1 \frac{z}{z - a_1} + C_2 \frac{z}{z-1}$$

Determine constants C_0, C_1 **and** C_2

(a) let $z = 0 \longrightarrow C_0 = 0$

(b) multiply through by $(z - a_1)/z$

let $z = a_1 \longrightarrow C_1 = \dfrac{b_0 a_1}{a_1 - 1} = \dfrac{-b_0 a_1}{1 - a_1}$

(c) multiply through by $(z-1)/z$

let $z = 1 \longrightarrow C_2 = \dfrac{b_0}{1 - a_1}$

8

8

Computing the Output of a Linear System

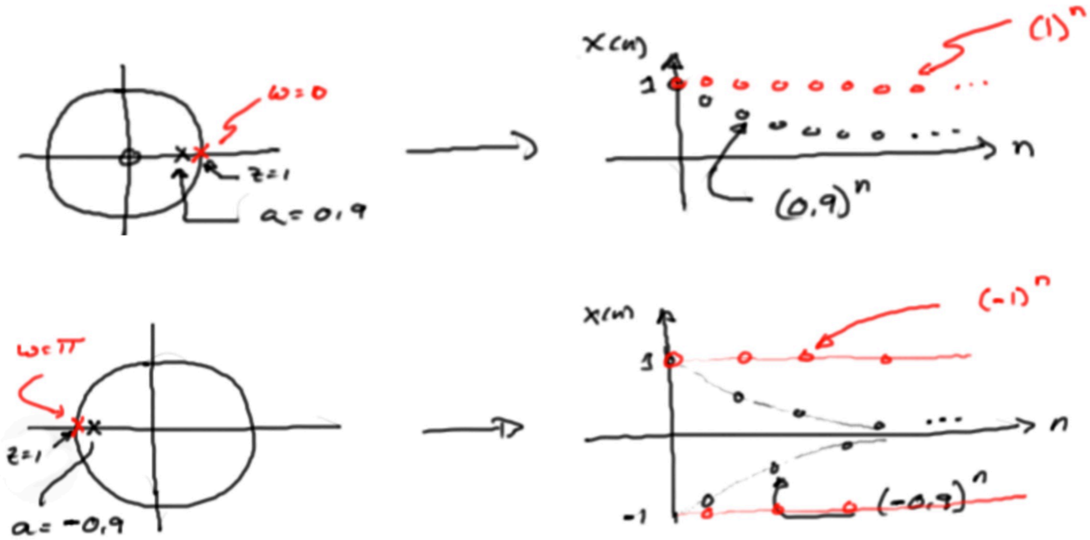
- **C:** Time-domain signal after partial fraction extension

$$y[n] = C_1(a_1)^n + C_2(1)^n$$
$$= \underbrace{\boxed{\left(\frac{b_0}{1 - a_1}\right)}}_{\text{DC gain}}[1 - a_1^{n+1}]$$

9

9

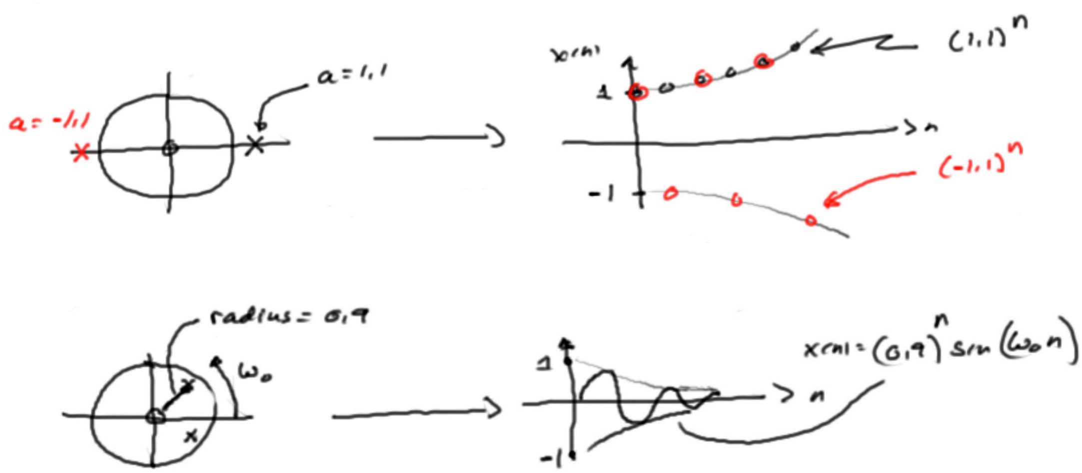
Z-Plane Pole Locations and Their Corresponding Time Series



10

10

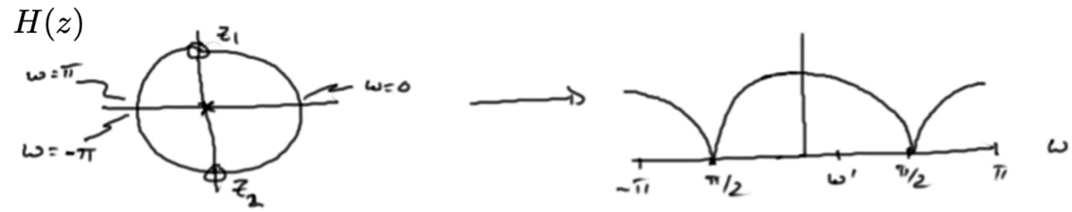
Z-Plane Pole Locations and Their Corresponding Time Series



11

11

Z-Domain Analysis



$$H(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2} = \frac{(z-z_1)(z-z_2)}{z^2}$$

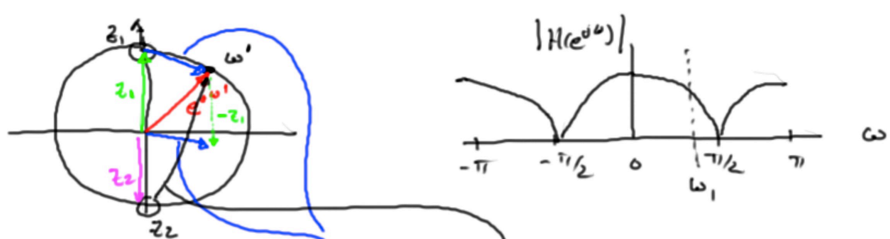
$$H(z)\Big|_{z=e^{j\omega'}} = \frac{(e^{j\omega'}-z_1)(e^{j\omega'}-z_2)}{\boxed{(e^{j\omega'})^2}}$$ magnitude equal to 1

$$\left|H\left(e^{j\omega'}\right)\right| = \left|e^{j\omega'}-z_1\right|\left|e^{j\omega'}-z_2\right|$$

12

12

Z-Domain Analysis

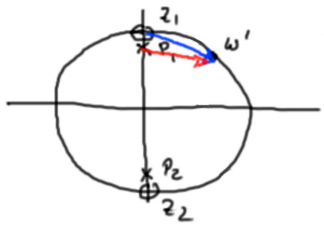


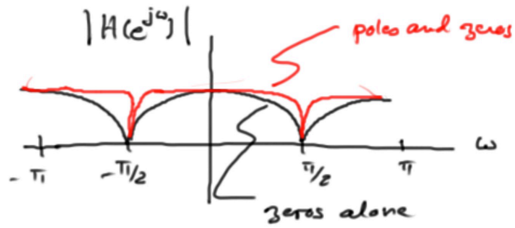
$$\left|H(e^{j\omega'})\right| = \left|e^{j\omega'} - z_1\right| \left|e^{j\omega'} - z_2\right|$$

13

13

Z-Domain Analysis





$$H(z) = \frac{(z - z_1)(z - z_2)}{(z - p_1)(z - p_2)}$$

$$\left|H(e^{j\omega'})\right| = \frac{|e^{j\omega'} - z_1||e^{j\omega'} - z_2|}{|e^{j\omega'} - p_1||e^{j\omega'} - p_2|}$$

14

14