Direct Multipath-Based SLAM: Supporting Documents

Mingchao Liang, Erik Leitinger, and Florian Meyer

This manuscript provides derivations for the publication, "Direct Multipath-Based SLAM" by the same authors [1].

1 The Joint Posterior PDF $f(\mathbf{x}_{0:k}, \mathbf{y}_{0:k}, \mathbf{C}_{\boldsymbol{\epsilon},0:k}|\mathbf{z}_{1:k})$

In this section, we present the derivation of the factorization of the joint posterior probability density function (PDF) [1], which lays the foundation of the factor graph representation in [1] and the starting point of the proposed belief propagation (BP) algorithm.

1.1 The Joint Likelihood PDF $f(\mathbf{z}_{1:k}|\mathbf{x}_{1:k}, \mathbf{y}_{1:k}, \mathbf{C}_{\boldsymbol{\epsilon},1:k})$

At each time step k, each physical anchor (PA) j receives a measurement $\mathbf{z}_k^{(j)} \in \mathbb{C}^M$ is received. The measurements $\mathbf{z}_k^{(j)}$ are independent conditioned on the $\mathbf{x}_k$, $\mathbf{y}_k^{(j)}$ and $\mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)}$ as discussed in [1, Sec. II-B]. As a result, the joint likelihood function can be represented as

$$f(\mathbf{z}_{1:k}|\mathbf{x}_{1:k}, \mathbf{y}_{1:k}, \mathbf{C}_{\boldsymbol{\epsilon},1:k}) = \prod_{k'=1}^{k} \prod_{j=1}^{J} f(\mathbf{z}_{k'}^{(j)}|\mathbf{x}_{k'}, \mathbf{y}_{k'}^{(j)}, \mathbf{C}_{\boldsymbol{\epsilon},k'}^{(j)}) \tag{1}$$

where $f(\mathbf{z}_{k'}|\mathbf{x}_{k'}, \mathbf{y}_{k'}, \mathbf{C}_{\boldsymbol{\epsilon},k'}^{(j)})$ is introduced in [1, eq. (5)].

1.2 The Joint Prior PDF $f(\mathbf{x}_{0:k}, \mathbf{y}_{0:k}, \mathbf{C}_{\boldsymbol{\epsilon},0:k})$

We start from applying the product rule on the joint prior PDF $f(\mathbf{x}_{0:k}, \mathbf{y}_{0:k}, \mathbf{C}_{\boldsymbol{\epsilon},0:k})$, from which we have

$$f(\mathbf{x}_{0:k}, \mathbf{y}_{0:k}, \mathbf{C}_{\boldsymbol{\epsilon},0:k}) = f(\mathbf{x}_0, \mathbf{y}_0, \mathbf{C}_{\boldsymbol{\epsilon},0}) \prod_{k'=1}^{k} f(\mathbf{x}_{k'}, \mathbf{y}_{k'}, \mathbf{C}_{\boldsymbol{\epsilon},k'}|\mathbf{x}_{0:k'-1}, \mathbf{y}_{0:k'-1}, \mathbf{C}_{\boldsymbol{\epsilon},0:k'-1}) \tag{2}$$

We then apply the product rule to $f(\mathbf{x}_k, \mathbf{y}_k, \mathbf{C}_{\boldsymbol{\epsilon},k}|\mathbf{x}_{0:k-1}, \mathbf{y}_{0:k-1}, \mathbf{C}_{\boldsymbol{\epsilon},0:k-1})$

$$\begin{aligned} f(\mathbf{x}_k, \mathbf{y}_k, \mathbf{C}_{\boldsymbol{\epsilon},k}|\mathbf{x}_{0:k-1}, \mathbf{y}_{0:k-1}, \mathbf{C}_{\boldsymbol{\epsilon},0:k-1}) = {} & f(\mathbf{x}_k|\mathbf{x}_{0:k-1}, \mathbf{y}_{0:k-1}, \mathbf{C}_{\boldsymbol{\epsilon},0:k-1}) \\ & \times f(\mathbf{y}_k|\mathbf{x}_{0:k}, \mathbf{y}_{0:k-1}, \mathbf{C}_{\boldsymbol{\epsilon},0:k-1}) \\ & \times f(\mathbf{C}_{\boldsymbol{\epsilon},k}|\mathbf{x}_{0:k}, \mathbf{y}_{0:k}, \mathbf{C}_{\boldsymbol{\epsilon},0:k-1}). \end{aligned}$$

Following the conditional independence assumption in [1, Sec. II-C], we obtain

$$\begin{aligned} f(\mathbf{x}_k|\mathbf{x}_{0:k-1}, \mathbf{y}_{0:k-1}, \mathbf{C}_{\boldsymbol{\epsilon},0:k-1}) &= f(\mathbf{x}_k|\mathbf{x}_{k-1}) \\ f(\mathbf{y}_k|\mathbf{x}_{0:k}, \mathbf{y}_{0:k-1}, \mathbf{C}_{\boldsymbol{\epsilon},0:k-1}) &= \prod_{j=1}^{J} \prod_{n=1}^{N_{k-1}^{(j)}} f(\mathbf{y}_{k,n}^{(j)}|\mathbf{y}_{k-1,n}^{(j)}) \prod_{n=N_{k-1}^{(j)}+1}^{N_k^{(j)}} f(\mathbf{y}_{k,n}^{(j)}|\mathbf{x}_k) \\ f(\mathbf{C}_{\boldsymbol{\epsilon},k}|\mathbf{x}_{0:k}, \mathbf{y}_{0:k}, \mathbf{C}_{\boldsymbol{\epsilon},0:k-1}) &= \prod_{j=1}^{J} f(\mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)}|\mathbf{C}_{\boldsymbol{\epsilon},k-1}^{(j)}) \end{aligned}$$

The the variables $\mathbf{x}_0$, $\mathbf{C}_{\boldsymbol{\epsilon},0}^{(j)}$, $j \in \{1,\ldots,J\}$, and $\mathbf{y}_{0,n}^{(j)}$, $n \in \{1,\ldots,N_0^{(j)}\}$, $j \in \{1,\ldots,J\}$ are also assumed to be independent, i.e.,

$$f(\mathbf{x}_0, \mathbf{y}_0, \mathbf{C}_{\boldsymbol{\epsilon},0}) = f(\mathbf{x}_0) \prod_{j=1}^{J} f(\mathbf{C}_{\boldsymbol{\epsilon},0}^{(j)}) \bigg(\prod_{n=1}^{N_0^{(j)}} f(\mathbf{y}_{0,n}^{(j)}) \bigg).$$

Finally, we insert the results to (2) and have

$$\begin{aligned} f(\mathbf{x}_{0:k}, \mathbf{y}_{0:k}, \mathbf{C}_{\boldsymbol{\epsilon},0:k}) = f(\mathbf{x}_0) \prod_{j=1}^{J} f(\mathbf{C}_{\boldsymbol{\epsilon},0}^{(j)}) \bigg(\prod_{n=1}^{N_0^{(j)}} f(\mathbf{y}_{0,n}^{(j)}) \bigg) \prod_{k'=1}^{k} f(\mathbf{x}_{k'}|\mathbf{x}_{k'-1}) \\ \times \prod_{j=1}^{J} f(\mathbf{C}_{\boldsymbol{\epsilon},k'}^{(j)}|\mathbf{C}_{\boldsymbol{\epsilon},k'-1}^{(j)}) \bigg(\prod_{n=1}^{N_{k'-1}^{(j)}} f(\mathbf{y}_{k',n}^{(j)}|\mathbf{y}_{k'-1,n}^{(j)}) \bigg) \bigg(\prod_{n=N_{k'-1}^{(j)}+1}^{N_{k'}^{(j)}} f(\mathbf{y}_{k',n}^{(j)}|\mathbf{x}_{k'}) \bigg). \end{aligned} \quad (3)$$

1.3 Final Expression for the Joint Posterior PDF $f(\mathbf{x}_{0:k}, \mathbf{y}_{0:k}, \mathbf{C}_{\boldsymbol{\epsilon},0:k}|\mathbf{z}_{1:k})$

Using Bayes' rule and inserting (1) and (3), the joint posterior PDF $f(\mathbf{x}_{0:k}, \mathbf{y}_{0:k}, \mathbf{C}_{\boldsymbol{\epsilon},0:k}|\mathbf{z}_{1:k})$ can be represented as in [1, eq. (13)]

$$\begin{aligned} f(\mathbf{x}_{0:k}, \mathbf{y}_{0:k}, \mathbf{C}_{\boldsymbol{\epsilon},0:k}|\mathbf{z}_{1:k}) &\propto f(\mathbf{z}_{1:k}|\mathbf{x}_{0:k}, \mathbf{y}_{0:k}, \mathbf{C}_{\boldsymbol{\epsilon},0:k}) f(\mathbf{x}_{0:k}, \mathbf{y}_{0:k}, \mathbf{C}_{\boldsymbol{\epsilon},0:k}) \\ &= f(\mathbf{z}_{1:k}|\mathbf{x}_{1:k}, \mathbf{y}_{1:k}, \mathbf{C}_{\boldsymbol{\epsilon},1:k}) f(\mathbf{x}_{0:k}, \mathbf{y}_{0:k}, \mathbf{C}_{\boldsymbol{\epsilon},0:k}) \\ &= f(\mathbf{x}_0) \prod_{j=1}^{J} f(\mathbf{C}_{\boldsymbol{\epsilon},0}^{(j)}) \bigg(\prod_{n=1}^{N_0^{(j)}} f(\mathbf{y}_{0,n}^{(j)}) \bigg) \prod_{k'=1}^{k} f(\mathbf{x}_{k'}|\mathbf{x}_{k'-1}) \\ &\quad \times \prod_{j=1}^{J} \bigg(\prod_{n=1}^{N_{k'-1}^{(j)}} f(\mathbf{y}_{k',n}^{(j)}|\mathbf{y}_{k'-1,n}^{(j)}) \bigg) \bigg(\prod_{n=N_{k'-1}^{(j)}+1}^{N_{k'}^{(j)}} f(\mathbf{y}_{k',n}^{(j)}|\mathbf{x}_{k'}) \bigg) \\ &\quad \times f(\mathbf{C}_{\boldsymbol{\epsilon},k'}^{(j)}|\mathbf{C}_{\boldsymbol{\epsilon},k'-1}^{(j)}) f(\mathbf{z}_{k'}^{(j)}|\mathbf{x}_{k'}, \mathbf{y}_{k'}^{(j)}, \mathbf{C}_{\boldsymbol{\epsilon},k'}^{(j)}). \end{aligned} \quad (4)$$

2 Computation of Matrices $\mathbf{C}_{\iota,k}^{(j)}(\mathbf{x}_k)$, $\mathbf{C}_{\kappa,k,n}^{(j)}(\boldsymbol{\phi}_{k,n}^{(j)})$, and $\mathbf{C}_{\nu,k}^{(j)}$

In this section, we present the derivation of the covariance matrices in [1, (21)-(23)]. By definition, $\mathbf{C}_{\kappa,k,n}^{(j)}$ is computed as

$$\begin{aligned} \mathbf{C}_{\kappa,k,n}^{(j)}(\boldsymbol{\phi}_{k,n}^{(j)}) &= \int \mathbf{z}_k^{(j)} \mathbf{z}_k^{(j)\mathrm{H}} \kappa(\mathbf{y}_{k,n}^{(j)}; \mathbf{z}_k^{(j)}) \, \mathrm{d}\mathbf{z}_k^{(j)} \\ &= \sum_{r_{k,1}^{(j)} \in \{0,1\}} \cdots \sum_{r_{k,n-1}^{(j)} \in \{0,1\}} \sum_{r_{k,n+1}^{(j)} \in \{0,1\}} \cdots \sum_{r_{k,N_k^{(j)}}^{(j)} \in \{0,1\}} \int \cdots \int \bigg(\int \mathbf{z}_k^{(j)} \mathbf{z}_k^{(j)\mathrm{H}} f(\mathbf{z}_k^{(j)}|\mathbf{x}_k, \mathbf{y}_k^{(j)}, \mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)}) \, \mathrm{d}\mathbf{z}_k^{(j)} \bigg) \\ &\quad \times \xi(\mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)}) \beta(\mathbf{x}_k) \prod_{\substack{n'=1 \\ n' \neq n}}^{N_k^{(j)}} \alpha(\mathbf{y}_{k,n'}^{(j)}) \mathrm{d}\boldsymbol{\phi}_{k,1}^{(j)} \cdots \mathrm{d}\boldsymbol{\phi}_{k,n-1}^{(j)} \, \mathrm{d}\boldsymbol{\phi}_{k,n+1}^{(j)} \cdots \mathrm{d}\boldsymbol{\phi}_{k,N_k^{(j)}}^{(j)} \, \mathrm{d}\mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)} \, \mathrm{d}\mathbf{x}_k \end{aligned} \quad (5)$$

where $\int \mathbf{z}_k^{(j)}\mathbf{z}_k^{(j)\mathrm{H}} f(\mathbf{z}_k^{(j)}|\mathbf{x}_k, \mathbf{y}_k^{(j)}, \mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)}) \,\mathrm{d}\mathbf{z}_k^{(j)} = \mathbf{C}_{k,n}^{(j)} = \mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)} + \sum_{n=1}^{N_k^{(j)}} r_{k,n}^{(j)}\gamma_{k,n}^{(j)}\mathbf{h}_{k,n}^{(j)}\mathbf{h}_{k,n}^{(j)\mathrm{H}}$ as discussed in [1, Sec. II-B]. Plugging in the result we obtain

$$
\begin{aligned}
\mathbf{C}_{\kappa,k,n}^{(j)}(\boldsymbol{\phi}_{k,n}^{(j)}) &= \sum_{r_{k,1}^{(j)}\in\{0,1\}} \cdots \sum_{r_{k,n-1}^{(j)}\in\{0,1\}} \sum_{r_{k,n+1}^{(j)}\in\{0,1\}} \cdots \sum_{r_{k,N_k^{(j)}}^{(j)}\in\{0,1\}} \int \cdots \int \left(\mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)} + \sum_{n=1}^{N_k^{(j)}} r_{k,n}^{(j)}\gamma_{k,n}^{(j)}\mathbf{h}_{k,n}^{(j)}\mathbf{h}_{k,n}^{(j)\mathrm{H}} \right) \\
&\quad \times \xi(\mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)})\beta(\mathbf{x}_k) \prod_{\substack{n'=1 \\ n'\neq n}}^{N_k^{(j)}} \alpha(\mathbf{y}_{k,n'}^{(j)}) \mathrm{d}\boldsymbol{\phi}_{k,1}^{(j)} \cdots \mathrm{d}\boldsymbol{\phi}_{k,n-1}^{(j)} \,\mathrm{d}\boldsymbol{\phi}_{k,n+1}^{(j)} \cdots \mathrm{d}\boldsymbol{\phi}_{k,N_k^{(j)}}^{(j)} \,\mathrm{d}\mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)} \,\mathrm{d}\mathbf{x}_k \\
&= \sum_{r_{k,1}^{(j)}\in\{0,1\}} \cdots \sum_{r_{k,n-1}^{(j)}\in\{0,1\}} \sum_{r_{k,n+1}^{(j)}\in\{0,1\}} \cdots \sum_{r_{k,N_k^{(j)}}^{(j)}\in\{0,1\}} \int \cdots \int \left(\sum_{n=1}^{N_k^{(j)}} r_{k,n}^{(j)}\gamma_{k,n}^{(j)}\mathbf{h}_{k,n}^{(j)}\mathbf{h}_{k,n}^{(j)\mathrm{H}} \right) \\
&\quad \times \beta(\mathbf{x}_k) \prod_{\substack{n'=1 \\ n'\neq n}}^{N_k^{(j)}} \alpha(\mathbf{y}_{k,n'}^{(j)}) \mathrm{d}\boldsymbol{\phi}_{k,1}^{(j)} \cdots \mathrm{d}\boldsymbol{\phi}_{k,n-1}^{(j)} \,\mathrm{d}\boldsymbol{\phi}_{k,n+1}^{(j)} \cdots \mathrm{d}\boldsymbol{\phi}_{k,N_k^{(j)}}^{(j)} + \int \mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)} \xi(\mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)}) \,\mathrm{d}\mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)} \\
&= r_{k,n}^{(j)}\gamma_{k,n}^{(j)} \int \mathbf{h}_{k,n}^{(j)}\mathbf{h}_{k,n}^{(j)\mathrm{H}} \beta(\mathbf{x}_k) \,\mathrm{d}\mathbf{x}_k + \sum_{\substack{n'=1 \\ n'\neq n}}^{N_k^{(j)}} \sum_{r_{k,n'}^{(j)}\in\{0,1\}} \int r_{k,n'}^{(j)}\gamma_{k,n'}^{(j)}\mathbf{h}_{k,n'}^{(j)}\mathbf{h}_{k,n'}^{(j)\mathrm{H}} \alpha(\mathbf{y}_{k,n'}^{(j)})\beta(\mathbf{x}_k) \,\mathrm{d}\boldsymbol{\phi}_{k,n'}^{(j)} \,\mathrm{d}\mathbf{x}_k \\
&\quad + \int \mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)} \xi(\mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)}) \,\mathrm{d}\mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)} \\
&= r_{k,n}^{(j)}\gamma_{k,n}^{(j)} \int \mathbf{h}_{k,n}^{(j)}\mathbf{h}_{k,n}^{(j)\mathrm{H}} \beta(\mathbf{x}_k) \,\mathrm{d}\mathbf{x}_k + \sum_{\substack{n'=1 \\ n'\neq n}}^{N_k^{(j)}} \int \gamma_{k,n'}^{(j)}\mathbf{h}_{k,n'}^{(j)}\mathbf{h}_{k,n'}^{(j)\mathrm{H}} \alpha(\boldsymbol{\phi}_{k,n'}^{(j)}, 1)\beta(\mathbf{x}_k) \,\mathrm{d}\boldsymbol{\phi}_{k,n'}^{(j)} \,\mathrm{d}\mathbf{x}_k \\
&\quad + \int \mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)} \xi(\mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)}) \,\mathrm{d}\mathbf{C}_{\boldsymbol{\epsilon},k}^{(j)} \\
&= r_{k,n}^{(j)} \mathbf{C}_{2,k,n}^{(j)}(\boldsymbol{\phi}_{k,n}^{(j)}) + \sum_{\substack{n'=1 \\ n'\neq n}}^{N_k^{(j)}} \mathbf{C}_{3,k,n'}^{(j)} + \mathbf{C}_{\xi,k}^{(j)}.
\end{aligned}
\tag{6}
$$

which is the same as [1, (21)-(23)]. The matrices $\mathbf{C}_{\iota,k}^{(j)}(\mathbf{x}_k)$ and $\mathbf{C}_{\nu,k}^{(j)}$ can be derived in a similar way.

3 Monte Carlo Integration of $\mathbf{C}_{1,k,n}^{(j)}(\mathbf{x}_k)$, $\mathbf{C}_{2,k,n}^{(j)}(\boldsymbol{\phi}_{k,n}^{(j)})$, and $\mathbf{C}_{3,k,n}^{(j)}$

In this section how “stacking technique” in [1, (32)-(34)] is achieved. We start from showing the “non-stacked” Monte Carlo integration of $\mathbf{C}_{3,k,n}^{(j)}$

$$
\mathbf{C}_{3,k,n}^{(j)} \approx \sum_{p=1}^{P} \sum_{p'=1}^{P} w_{\beta,k}^{(p)} w_{\alpha,k,n}^{(j,p')} \mathbf{H}_{k,n}(\mathbf{x}_k^{(p)}, \boldsymbol{\phi}_{k,n}^{(j,p')}).
\tag{7}
$$

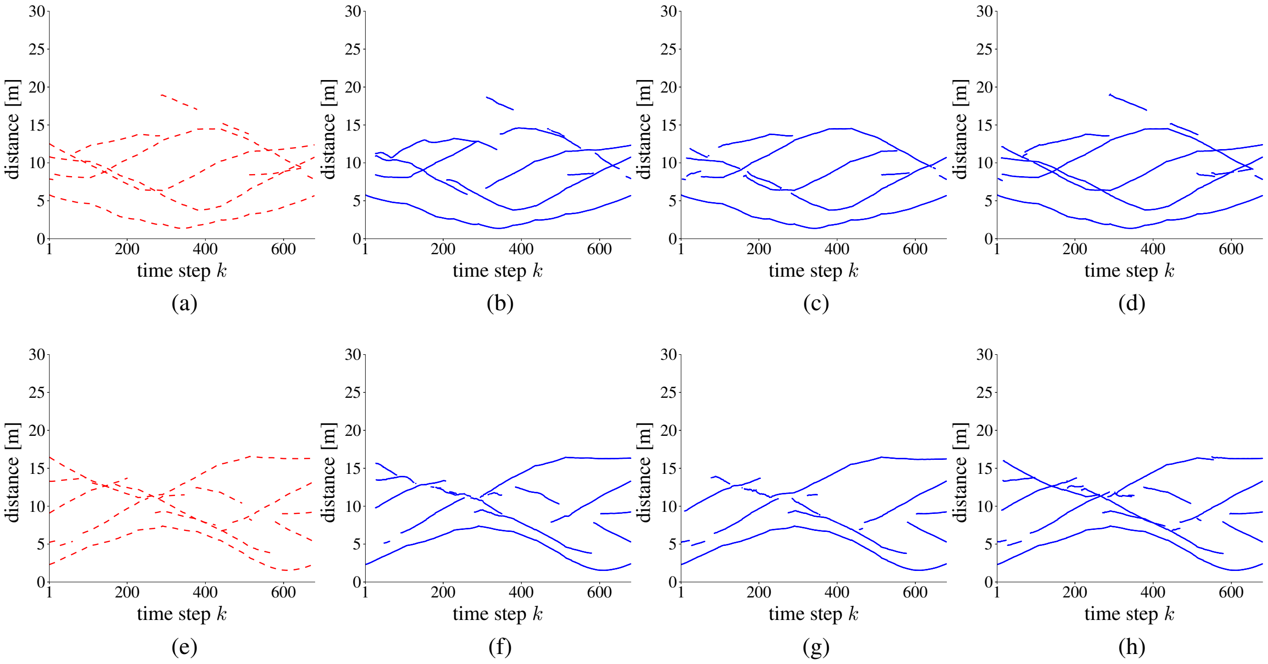

Figure 1: Visualization of the distance between the agent position and the true/estimated feature position. The first column with (a)/(e) shows the true distances for PA 1 and PA 2, respectively. The following columns show the averaged distance between the true agent position and the particles of PF positions obtained using (b)/(f) BP-SLAM, (c)/(g) BP-SLAM-AI, and (d)/(h) Direct SLAM with 400MHz bandwidth and for PA 1 and PA 2, respectively.

This "non-stacked" version has complexity $\mathcal{O}(P^2)$. To avoid the square complexity, we stack the particles and only evaluate pairs $(p, p'), p = p'$. As a result,

$$\begin{aligned} \mathbf{C}_{3,k,n}^{(j)} &\approx \sum_{p=1}^{P} w_{\beta,k}^{(p)} \Big(\sum_{p'=1}^{P} w_{\alpha,k,n}^{(j,p')} \Big) \mathbf{H}_{k,n}(\mathbf{x}_k^{(p)}, \boldsymbol{\phi}_{k,n}^{(j,p)}) \\ &= \sum_{p=1}^{P} w_{\beta,k}^{(p)} \Big(\sum_{p'=1}^{P} w_{\alpha,k,n}^{(j,p')} \Big) \tilde{\mathbf{H}}_{k,n}^{(j,p)} = \tilde{\mathbf{C}}_{3,k,n}^{(j)}. \end{aligned} \tag{8}$$

Similarly, $\mathbf{C}_{1,k,n}^{(j)}(\mathbf{x}_k)$ and $\mathbf{C}_{2,k,n}^{(j)}(\boldsymbol{\phi}_{k,n}^{(j)})$ are approximated as

$$\begin{aligned} \mathbf{C}_{1,k,n}^{(j)}(\mathbf{x}_n^{(p)}) &\approx \sum_{p'=1}^{P} w_{\alpha,k,n}^{(j,p')} \mathbf{H}_{k,n}(\mathbf{x}_k^{(p)}, \boldsymbol{\phi}_{k,n}^{(j,p')}) \\ &\approx \Big(\sum_{p'=1}^{P} w_{\alpha,k,n}^{(j,p')} \Big) \mathbf{H}_{k,n}(\mathbf{x}_k^{(p)}, \boldsymbol{\phi}_{k,n}^{(j,p)}) \\ &= \Big(\sum_{p'=1}^{P} w_{\alpha,k,n}^{(j,p')} \Big) \tilde{\mathbf{H}}_{k,n}^{(j,p)} = \tilde{\mathbf{C}}_{1,k,n}^{(j)}(\mathbf{x}_n^{(p)}) \end{aligned} \tag{9}$$

$$\mathbf{C}_{2,k,n}^{(j)}(\boldsymbol{\phi}_{k,n}^{(j,p)}) \approx \sum_{p'=1}^{P} w_{\beta,k}^{(j,p')} \mathbf{H}_{k,n}(\mathbf{x}_k^{(p')}, \boldsymbol{\phi}_{k,n}^{(j,p)})$$

$$
\approx \Big(\sum_{p'=1}^{P} w_{\beta,k}^{(j,p')} \Big) \mathbf{H}_{k,n}(\mathbf{x}_k^{(p)}, \boldsymbol{\phi}_{k,n}^{(j,p)})
$$

$$
= \mathbf{H}_{k,n}(\mathbf{x}_k^{(p)}, \boldsymbol{\phi}_{k,n}^{(j,p)}) = \tilde{\mathbf{H}}_{k,n}^{(j,p)} = \tilde{\mathbf{C}}_{2,k,n}^{(j)}\big(\boldsymbol{\phi}_{k,n}^{(j,p)}\big). \tag{10}
$$

4 Further Simulation Results

In this section, we plot the average distance between the particles of the potential features (PF) positions and the true agent positions for one exemplary simulation run in Fig. 1b-1d and 1f-1h. Since BP-SLAM and BP-SLAM-AI rely on a channel estimator that cannot resolve multipath components (MPCs) having similar path lengths, both BP-SLAM and BP-SLAM-AI have problems when features “cross” or have similar distances with respect to the agent position. Conversely, Direct-SLAM directly operates on the radio signal and does consider the superposition of MPCs in the statistical model. For this reason, Direct-SLAM alleviates performance issues in situations with complicated propagation environments.

References

[1] M. Liang, E. Leitinger, and F. Meyer, “Direct multipath-based SLAM,” *IEEE Trans. Signal Process.*, 2024, to appear.