ECE 286: Bayesian Machine Perception

Class 10: Graph-Based Multiobject Tracking I

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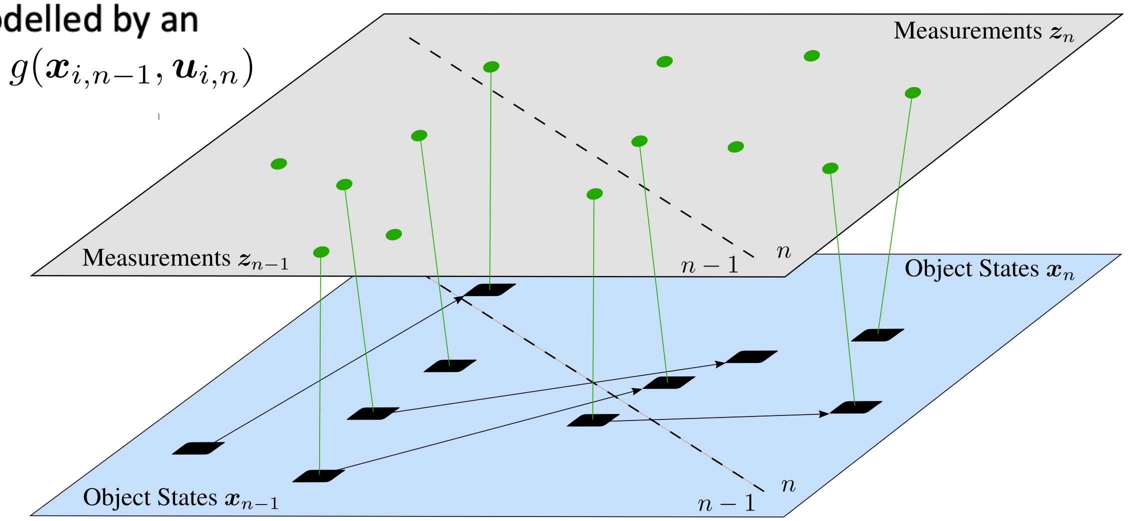
The Multiobject Tracking Problem

- At each time n: localize and track multiple objects $\boldsymbol{x}_n = [\boldsymbol{x}_{1,n}^{\mathrm{T}} \ldots \boldsymbol{x}_{I,n}^{\mathrm{T}}]^{\mathrm{T}}$ from measurements $\boldsymbol{z}_n = [\boldsymbol{z}_{1,n}^{\mathrm{T}} \ldots \boldsymbol{z}_{M_n,n}^{\mathrm{T}}]^{\mathrm{T}}$ with uncertain origin

A state $\boldsymbol{x}_{i,n}$ consists of the object's position and further parameters; its evolution is time modelled by an arbitrary model $\boldsymbol{x}_{i,n} = g(\boldsymbol{x}_{i,n-1}, \boldsymbol{u}_{i,n})$ with noise $\boldsymbol{u}_{i,n}$



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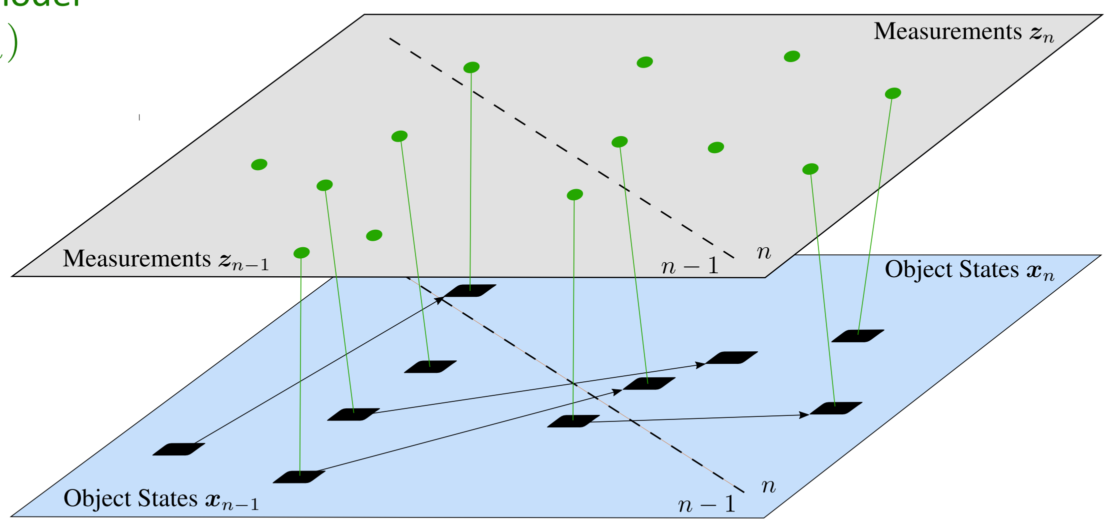
A measurement $\boldsymbol{z}_{m,n}$ is modelled by an arbitrary nonlinear model
$\boldsymbol{z}_{m,n} = h(\boldsymbol{x}_{k,n}, \boldsymbol{v}_{m,n})$
with noise $\boldsymbol{v}_{m,n}$



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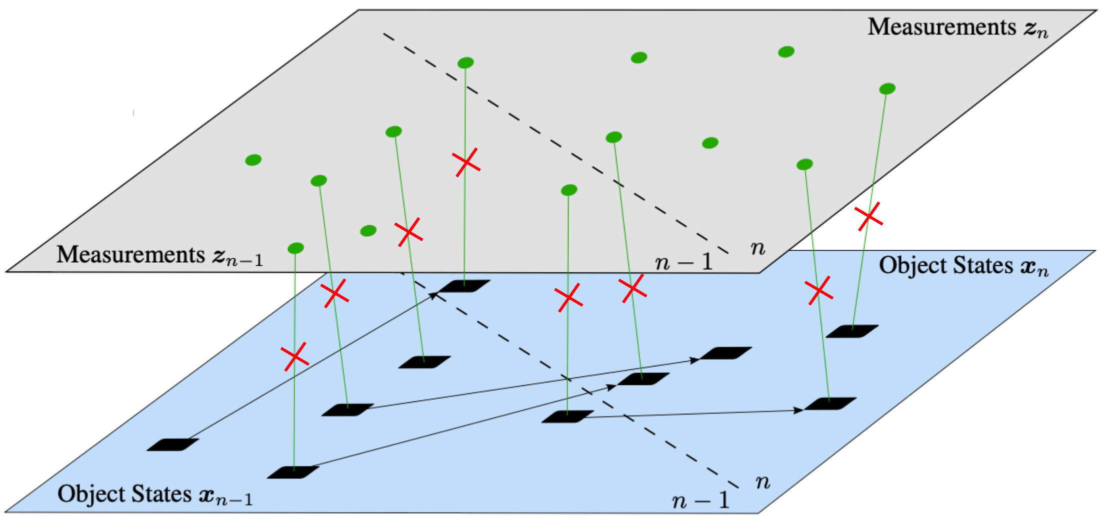

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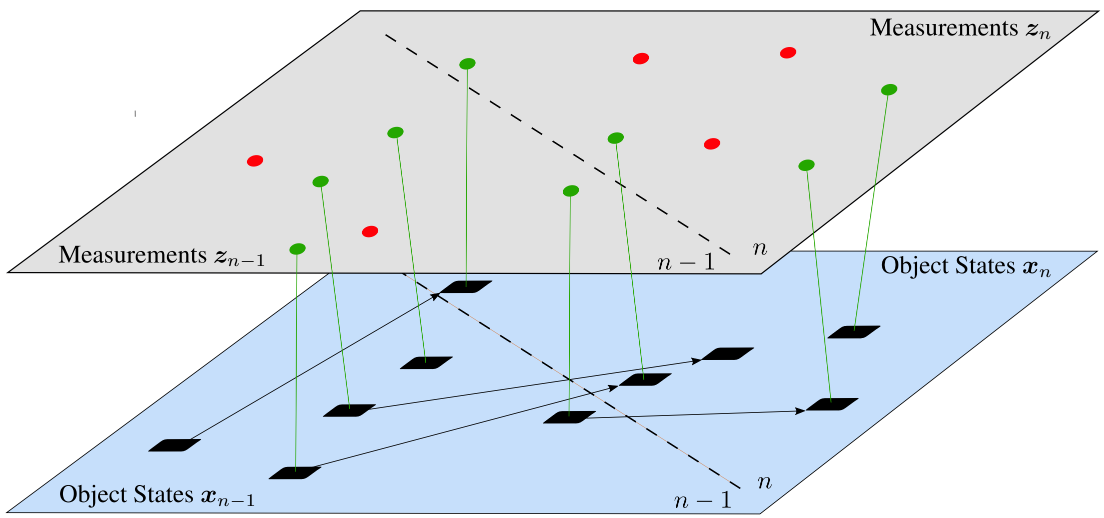
- Data association is challenging because of false clutter measurements and missing measurements



The Multiobject Tracking Problem

- At each time n: localize and track multiple objects $\boldsymbol{x}_n = [\boldsymbol{x}_{1,n}^{\mathrm{T}} \ldots \boldsymbol{x}_{I,n}^{\mathrm{T}}]^{\mathrm{T}}$ from measurements $\boldsymbol{z}_n = [\boldsymbol{z}_{1,n}^{\mathrm{T}} \ldots \boldsymbol{z}_{M_n,n}^{\mathrm{T}}]^{\mathrm{T}}$ with uncertain origin
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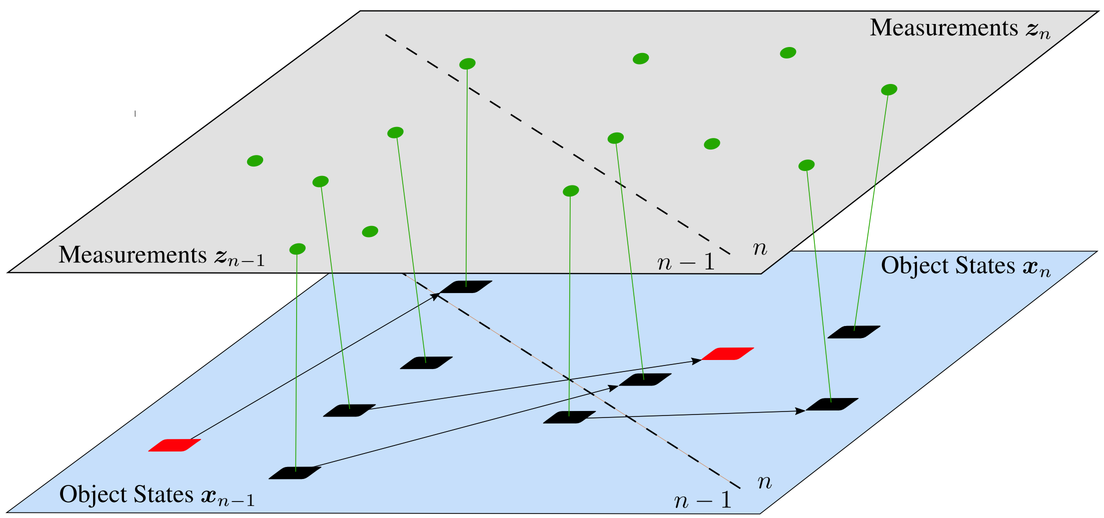
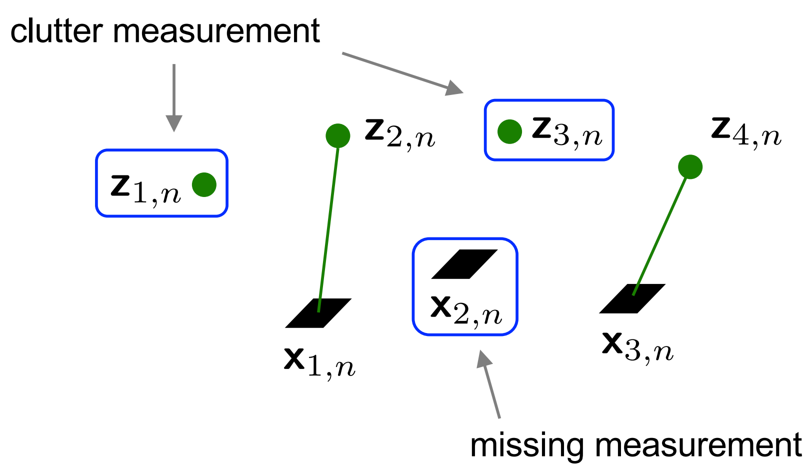


Association Vectors

- Recall measurement vector at time n, $\boldsymbol{z}_n = [\boldsymbol{z}_{1,n}^{\mathrm{T}}\ \boldsymbol{z}_{2,n}^{\mathrm{T}}\ \dots\ \boldsymbol{z}_{M_n,n}^{\mathrm{T}}]^{\mathrm{T}}$
- Object-oriented association vector $\boldsymbol{a}_n = [a_{1,n}\ a_{2,n}\ \dots\ a_{I,n}]^{\mathrm{T}}$
  - $a_{i,n} = m > 0$: at time n object i generates measurement with index m
  - $a_{i,n} = 0$: at time n object i did not generate a measurement



$M_n = 4$

$I = 3$

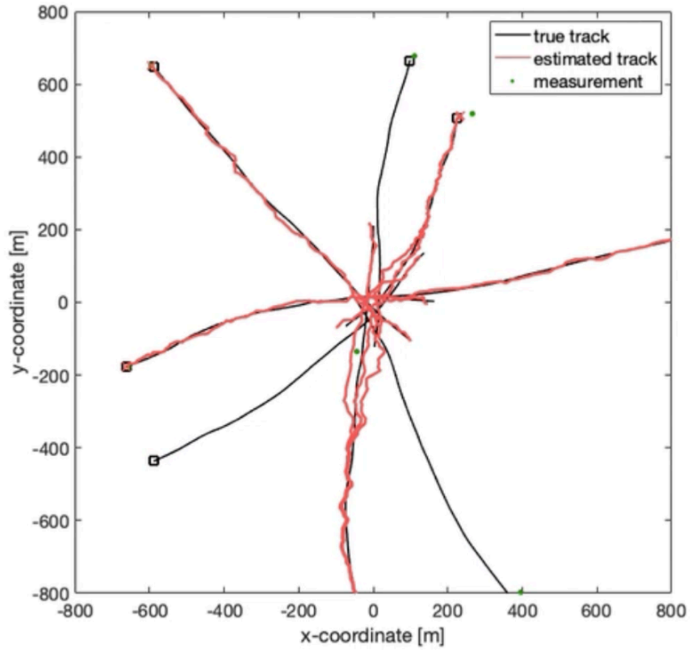
$a_{1,n} = 2$

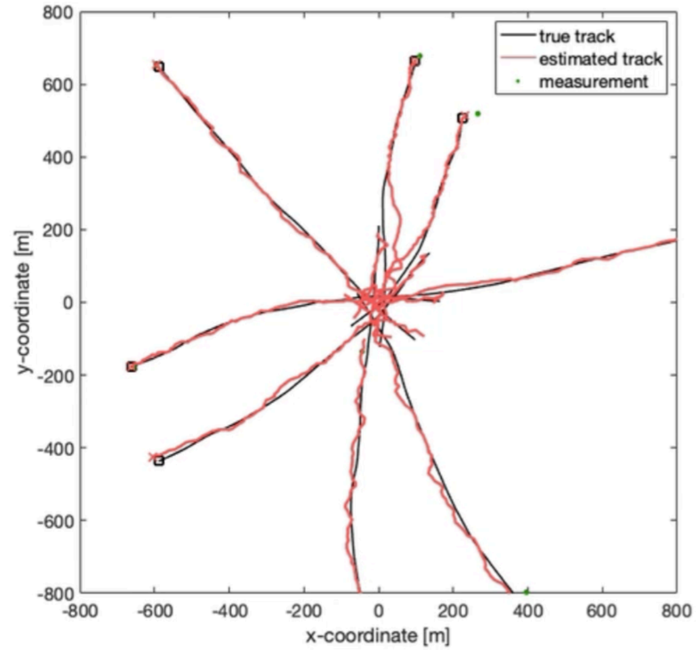
$a_{2,n} = 0$

$a_{3,n} = 4$

Why Multiobject Tracking?

- Separate single-object tracking (left) vs joint multiobject tracking (right)





- Only a joint multiobject tracking formulation works

Prior Distributions

- Assumptions:
  1. Object detections are independent Bernoulli trials with success probability $0 < p_{\mathrm{d}} \leqslant 1$
  2. The number of clutter measurements is Poisson distributed with mean μ_{c}
  3. At most one measurement is generated by each object
  4. A measurement can be generated from at most one object
- Assumptions 1-3 are parallel to the single object tracking case
- Every association event expressed by a vector $\boldsymbol{a}_n = [a_{1,n} \dots a_{I,n}]^{\mathrm{T}}$ automatically fulfills Assumption 3 (scalar association variable $a_{i,n}$ for each object)
- Assumption 4 can be enforced by the following check function

$$\varphi(\boldsymbol{a}_n) \triangleq \begin{cases} 0, & \exists i,j \in \{1,2,\dots,I\} \text{ such that } i \neq j \text{ and } a_{i,n} = a_{j,n} \neq 0 \\ 1, & \text{otherwise} \end{cases}$$

Prior Distributions

- Let us denote by $\mathcal{D}_{\boldsymbol{a}_n} = \big\{ i \in \{1, \ldots, I\} \,|\, a_{i,n} > 0 \big\}$ the set of detected object indexes corresponding to vector $\boldsymbol{a}_n$

- The prior pmf $p(\boldsymbol{a}_n, M_n)$ is given by

Check if every measurement is generated by at most one object

$$p(\boldsymbol{a}_n, M_n) = \boxed{\varphi(\boldsymbol{a}_n)} \bigg(\frac{p_\mathrm{d}}{\mu_\mathrm{c}(1 - p_\mathrm{d})} \bigg)^{|\mathcal{D}_{\boldsymbol{a}_n}|} \, \frac{e^{-\mu_\mathrm{c}} \mu_\mathrm{c}^{M_n}}{M_n!} \, (1 - p_\mathrm{d})^I$$

- $p(\boldsymbol{a}_n, M_n)$ is a valid pmf in the sense that it can be normalized as

$$\textstyle\sum_{M_n=0}^{\infty} \sum_{a_{1,n}=0}^{M_n} \cdots \sum_{a_{I,n}=0}^{M_n} p(\boldsymbol{a}_n, M_n) = 1$$

Y. Bar-Shalom, P. K. Willett, and X. Tian, *Tracking and Data Fusion: A Handbook of Algorithms*, YBS, 2011.

Prior Distributions - Examples

- Example 1: No detections, all clutter case

Probability that no object generates a measurement

$$p(\boldsymbol{a}_n, M_n) = \boxed{\frac{e^{-\mu_\mathrm{c}} \mu_\mathrm{c}^{M_n}}{M_n!}} \boxed{\big(1 - p_\mathrm{d}\big)^I}$$

Poisson pmf of the number of clutter measurements evaluated at M_n

Prior Distributions - Example

- Example 2: All detections, no clutter case ($\boldsymbol{a}^{\mathrm{d}}$ is any association vector that assigns exactly one measurement to each object, i.e., any permutation of $1, 2, \ldots, I$)

Poisson pmf of the number of clutter measurements evaluated at *0*

$$p(\boldsymbol{a}_n = \boldsymbol{a}_n^{\mathrm{d}}, M_n = I) = \frac{\boxed{e^{-\mu_{\mathrm{c}}}}}{\boxed{I!}} \boxed{p_{\mathrm{d}}^{I}}$$

Probability that every object generates a measurement

There are $I!$ different $\boldsymbol{a}^{\mathrm{d}}$

Prior Distributions

- Joint prior distribution of object states at time $n = 0$

$$f(\boldsymbol{x}_0) = \prod_{i=1}^{I} f(\boldsymbol{x}_{i,0})$$

- Joint state transition function (object states evolve independently)

Driving noise independent across objects

$$f(\boldsymbol{x}_n|\boldsymbol{x}_{n-1}) = \prod_{i=1}^{I} f(\boldsymbol{x}_{i,n}|\boldsymbol{x}_{i,n-1})$$

Driving noise independent across time n and independent of $\boldsymbol{x}_0$

- Joint prior distribution

$$f(\boldsymbol{x}_{0:n}) = f(\boldsymbol{x}_0) \prod_{n'=1}^{n} f(\boldsymbol{x}_{n'}|\boldsymbol{x}_{n'-1})$$

$$= \left(\prod_{j=1}^{I} f(\boldsymbol{x}_{j,0})\right) \prod_{n'=1}^{n} \prod_{i=1}^{I} f(\boldsymbol{x}_{i,n'}|\boldsymbol{x}_{i,n'-1})$$

Likelihood Function

- **Key Assumption II:**
  - Clutter measurements are independent and identically distributed (iid) according to $f_{\mathrm{c}}(\boldsymbol{z}_{m,n})$
  - Condition on $\boldsymbol{x}_{i,n}$, the object-generated measurement $\boldsymbol{z}_{a_{i,n},n}$ is conditionally independent of all the other measurements

- **Likelihood function:**
  - for $\boldsymbol{z}_n \in \mathbb{R}^{M_n d}$,

measurement model $\boldsymbol{z}_{m,n} = h_n(\boldsymbol{x}_n, \boldsymbol{v}_n)$ with noise $\boldsymbol{v}_n$

$$f(\boldsymbol{z}_n|\boldsymbol{x}_n, a_n, M_n) = \left(\prod_{i \in \mathcal{D}_{\boldsymbol{a}_n}} \frac{\boxed{f(\boldsymbol{z}_{a_{i,n},n}|\boldsymbol{x}_{i,n})}}{f_{\mathrm{c}}(\boldsymbol{z}_{a_{i,n},n})} \right) \prod_{m=1}^{M_n} f_{\mathrm{c}}(\boldsymbol{z}_{m,n})$$

  - For $\boldsymbol{z}_n \notin \mathbb{R}^{M_n d}$,

$$f(\boldsymbol{z}_n|\boldsymbol{x}_n, a_n, M_n) = 0$$

Y. Bar-Shalom, P. K. Willett, and X. Tian, *Tracking and Data Fusion: A Handbook of Algorithms*, YBS, 2011.

Joint Distributions

- Joint prior for $\boldsymbol{a}_{1:n}$ and $\boldsymbol{M}_{1:n}$

$$p(\boldsymbol{a}_{1:n}, \boldsymbol{M}_{1:n}) = \prod_{n'=1}^{n} p(a_{n'}, M_{n'})$$

Measurement generation independent across time n

- Joint likelihood function

$$f(\boldsymbol{z}_{1:n}|\boldsymbol{x}_{1:n}, \boldsymbol{a}_{1:n}, \boldsymbol{M}_{1:n}) = \prod_{n'=1}^{n} f(\boldsymbol{z}_{n'}|\boldsymbol{x}_{n'}, a_{n'}, M_{n'})$$

Measurement noise and clutter independent across time n

The Joint Posterior Distribution

- The joint posterior distribution ($M_{1:n}$ and $\boldsymbol{z}_{1:n}$ are observed and thus fixed)

$$f(\boldsymbol{x}_{0:n}, \boldsymbol{a}_{1:n} | \boldsymbol{z}_{1:n}) = f(\boldsymbol{x}_{0:n}, \boldsymbol{a}_{1:n}, \boldsymbol{M}_{1:n} | \boldsymbol{z}_{1:n}) \longleftarrow \boldsymbol{M}_{1:n} \text{ fixed}$$

Bayes rule $\longrightarrow$ $$\propto f(\boldsymbol{z}_{1:n} | \boldsymbol{x}_{1:n}, \boldsymbol{a}_{1:n}, \boldsymbol{M}_{1:n}) f(\boldsymbol{x}_{0:n}, \boldsymbol{a}_{1:n}, \boldsymbol{M}_{1:n})$$

$\boldsymbol{x}_{0:n} \perp\!\!\!\perp \boldsymbol{a}_{1:n}, \boldsymbol{M}_{1:n}$ $\longrightarrow$ $$= f(\boldsymbol{z}_{1:n} | \boldsymbol{x}_{1:n}, \boldsymbol{a}_{1:n}, \boldsymbol{M}_{1:n}) f(\boldsymbol{x}_{0:n}) p(\boldsymbol{a}_{1:n}, \boldsymbol{M}_{1:n})$$

Expressions for joint distributions $\longrightarrow$ $$= \left(\prod_{j=1}^{I} f(\boldsymbol{x}_{j,0}) \right) \prod_{n'=1}^{n} \left(\prod_{i=1}^{I} f(\boldsymbol{x}_{i,n'} | \boldsymbol{x}_{i,n'-1}) \right) f(\boldsymbol{z}_{n'} | \boldsymbol{x}_{n'}, \boldsymbol{a}_{n'}, M_{n'}) p(\boldsymbol{a}_{n'}, M_{n'})$$

Problem Formulation

- Input at time n:
  - All observations up to time $\boldsymbol{z}_{1:n}$
  - ``Markovian'' statistical model

$$f(\boldsymbol{x}_{0:n}, \boldsymbol{a}_{1:n} | \boldsymbol{z}_{1:n}) \propto \left(\prod_{j=1}^{I} f(\boldsymbol{x}_{j,0}) \right) \prod_{n'=1}^{n} \left(\prod_{i=1}^{I} f(\boldsymbol{x}_{i,n'} | \boldsymbol{x}_{i,n'-1}) \right) f(\boldsymbol{z}_{n'} | \boldsymbol{x}_{n'}, \boldsymbol{a}_{n'}, M_{n'}) p(\boldsymbol{a}_{n'}, M_{n'})$$

- Output at time n:
  - Estimates of all $\hat{\boldsymbol{x}}_{i,n}, i \in \{1, \ldots, I\}$

- Calculation of an estimates $\hat{\boldsymbol{x}}_{i,n}$ is based on the marginal posterior pdfs $f(\boldsymbol{x}_{i,n} | \boldsymbol{z}_{1:n})$

The Factor Graph

- Recall factorization of the joint posterior distribution:

$$f(\boldsymbol{x}_{0:n}, \boldsymbol{a}_{1:n} | \boxed{\boldsymbol{z}_{1:n}}) \propto \left(\prod_{j=1}^{I} f(\boldsymbol{x}_{j,0}) \right) \prod_{n'=1}^{n} \left(\prod_{i=1}^{I} f(\boldsymbol{x}_{i,n'} | \boldsymbol{x}_{i,n'-1}) \right) f(\boxed{\boldsymbol{z}_{n'}} | \boldsymbol{x}_{n'}, \boldsymbol{a}_{n'}, \boxed{M_{n'}}) \, p(\boldsymbol{a}_{n'}, \boxed{M_{n'}})$$

observed and fixed

The Factor Graph

- **Recall factorization of the joint posterior distribution:**

$$f(\boldsymbol{x}_{1:n}, \boldsymbol{a}_{1:n} | \boldsymbol{z}_{1:n}) \propto \Bigg(\prod_{j=1}^{I} f(\boldsymbol{x}_{j,0}) \Bigg) \prod_{n'=1}^{n} \Bigg(\prod_{i=1}^{I} f(\boldsymbol{x}_{i,n'} | \boldsymbol{x}_{i,n'-1}) \Bigg) f(\boldsymbol{z}_{n'} | \boldsymbol{x}_{n'}, \boldsymbol{a}_{n'}, M_{n'}) p(\boldsymbol{a}_{n'}, M_{n'})$$

$$\propto \Bigg(\prod_{j=1}^{I} f(\boldsymbol{x}_{j,0}) \Bigg) \prod_{n'=1}^{n} \Bigg(\prod_{i=1}^{I} f(\boldsymbol{x}_{i,n'} | \boldsymbol{x}_{i,n'-1}) g_1(\boldsymbol{x}_{i,n'}, a_{i,n'}) g_2(a_{i,n'}) \Bigg) \varphi(\boldsymbol{a}_{n'})$$

$$f(\boldsymbol{z}_n | \boldsymbol{x}_n, \boldsymbol{a}_n, M_n) = \Bigg(\prod_{i \in \mathcal{D}_{\boldsymbol{a}_n}} \frac{f(\boldsymbol{z}_{a_{i,n},n} | \boldsymbol{x}_{i,n})}{f_{\mathrm{c}}(\boldsymbol{z}_{a_{i,n},n})} \Bigg) \boxed{\prod_{m=1}^{M_n} f_{\mathrm{c}}(\boldsymbol{z}_{m,n})} \longleftarrow \text{constant}$$

$$g_1(\boldsymbol{x}_{i,n}, a_{i,n}) = \begin{cases} \dfrac{f(\boldsymbol{z}_{a_{i,n},n} | \boldsymbol{x}_n)}{f_{\mathrm{c}}(\boldsymbol{z}_{a_{i,n},n})} & a_{i,n} \in \{1, \ldots, M_n\} \\ 1 & a_{i,n} = 0 \end{cases}$$

The Factor Graph

- Recall factorization of the joint posterior distribution:

$$f(\boldsymbol{x}_{1:n}, \boldsymbol{a}_{1:n} | \boldsymbol{z}_{1:n}) \propto \left(\prod_{j=1}^{I} f(\boldsymbol{x}_{j,0}) \right) \prod_{n'=1}^{n} \left(\prod_{i=1}^{I} f(\boldsymbol{x}_{i,n'} | \boldsymbol{x}_{i,n'-1}) \right) f(\boldsymbol{z}_{n'} | \boldsymbol{x}_{n'}, \boldsymbol{a}_{n'}, M_{n'}) \, p(\boldsymbol{a}_{n'}, M_{n'})$$

$$\propto \left(\prod_{j=1}^{I} f(\boldsymbol{x}_{j,0}) \right) \prod_{n'=1}^{n} \left(\prod_{i=1}^{I} f(\boldsymbol{x}_{i,n'} | \boldsymbol{x}_{i,n'-1}) \, g_1(\boldsymbol{x}_{i,n'}, a_{i,n'}) \, g_2(a_{i,n'}) \right) \varphi(\boldsymbol{a}_{n'})$$

$$p(\boldsymbol{a}_n, M_n) = \varphi(\boldsymbol{a}_n) \boxed{\left(\frac{p_\mathrm{d}}{\mu_\mathrm{c}(1 - p_\mathrm{d})} \right)^{|\mathcal{D}_{\boldsymbol{a}_n}|}} \boxed{\frac{e^{-\mu_\mathrm{c}} \mu_\mathrm{c}^{M_n}}{M_n!} (1 - p_\mathrm{d})^I} \longleftarrow \text{constant}$$

$$g_2(a_{i,n}) = \begin{cases} \frac{p_\mathrm{d}}{\mu_\mathrm{c}(1-p_\mathrm{d})} & a_{i,n} \in \{1, \dots, M_n\} \\ 1 & a_{i,n} = 0 \end{cases}$$

The Factor Graph

- **Recall factorization of the joint posterior distribution:**

$$f(\boldsymbol{x}_{1:n}, \boldsymbol{a}_{1:n}|\boldsymbol{z}_{1:n}) \propto \left(\prod_{j=1}^{I} f(\boldsymbol{x}_{j,0})\right) \prod_{n'=1}^{n} \left(\prod_{i=1}^{I} f(\boldsymbol{x}_{i,n'}|\boldsymbol{x}_{i,n'-1})\right) f(\boldsymbol{z}_{n'}|\boldsymbol{x}_{n'}, \boldsymbol{a}_{n'}, M_{n'}) p(\boldsymbol{a}_{n'}, M_{n'})$$

$$\propto \left(\prod_{j=1}^{I} f(\boldsymbol{x}_{j,0})\right) \prod_{n'=1}^{n} \left(\prod_{i=1}^{I} f(\boldsymbol{x}_{i,n'}|\boldsymbol{x}_{i,n'-1}) g_1(\boldsymbol{x}_{i,n'}, a_{i,n'}) g_2(a_{i,n'})\right) \varphi(\boldsymbol{a}_{n'})$$

$$\propto \left(\prod_{j=1}^{I} f(\boldsymbol{x}_{j,0})\right) \prod_{n'=1}^{n} \left(\prod_{i=1}^{I} f(\boldsymbol{x}_{i,n'}|\boldsymbol{x}_{i,n'-1}) g_{\boldsymbol{z}_n}(\boldsymbol{x}_{i,n'}, a_{i,n'})\right) \boxed{\varphi(\boldsymbol{a}_{n'})}$$

Recall: Check if every measurement is generated by at most one object

$$g_{\boldsymbol{z}_n}(\boldsymbol{x}_{i,n}, a_{i,n}) = g_1(\boldsymbol{x}_{i,n}, a_{i,n}) g_2(a_{i,n}) = \begin{cases} \frac{p_{\mathrm{d}} f(\boldsymbol{z}_{a_{i,n},n}|\boldsymbol{x}_{i,n})}{\mu_{\mathrm{c}} f_{\mathrm{c}}(\boldsymbol{z}_{a_{i,n},n})} & a_{i,n} \in \{1, \ldots, M_n\} \\ (1 - p_{\mathrm{d}}) & a_{i,n} = 0 \end{cases}$$

The Factor Graph

- Recall factorization of the joint posterior distribution:

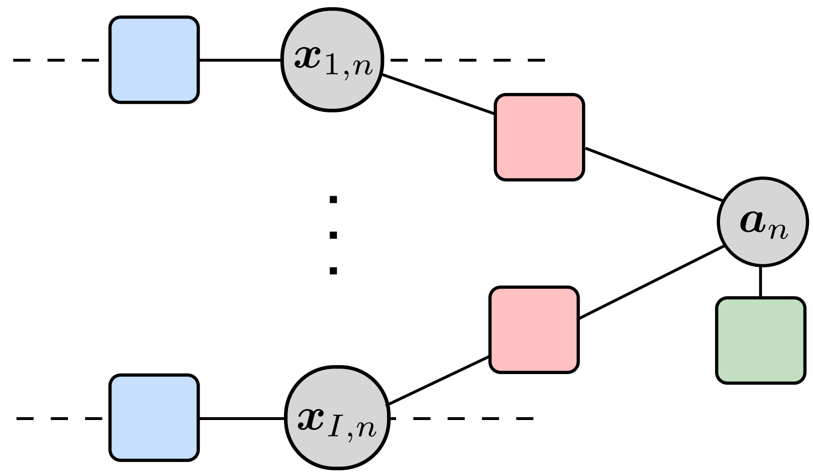
$$f(\boldsymbol{x}_{1:n}, \boldsymbol{a}_{1:n} | \boldsymbol{z}_{1:n}) \propto \left(\prod_{j=1}^{I} f(\boldsymbol{x}_{j,0}) \right) \prod_{n'=1}^{n} \left(\prod_{i=1}^{I} f(\boldsymbol{x}_{i,n'} | \boldsymbol{x}_{i,n'-1}) \, g_{\boldsymbol{z}_n}(\boldsymbol{x}_{i,n'}, a_{i,n'}) \right) \varphi(\boldsymbol{a}_{n'})$$

- Factor graph for time step n

$f(\boldsymbol{x}_{i,n'} | \boldsymbol{x}_{i,n'-1})$

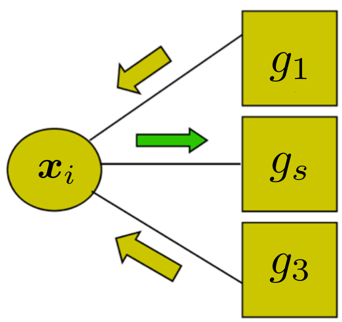
$g_{\boldsymbol{z}_n}(\boldsymbol{x}_{i,n'}, a_{i,n'})$

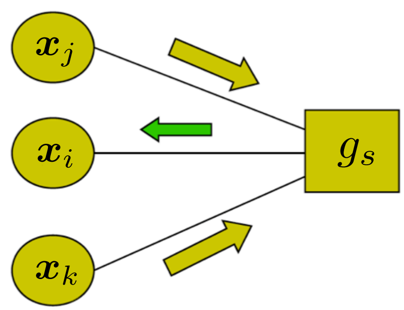
$\varphi(\boldsymbol{a}_n)$



Recall Message Passing Rules

- Message passing protocol: A message to a neighboring node can only be send when it has received messages from all its other neighbors
- Marginal distribution can be calculated as $b(\boldsymbol{x}_i) \propto \prod_{t \in \mathcal{N}(i)} \phi_{ti}(\boldsymbol{x}_i) = \phi_{ti}(\boldsymbol{x}_i)\nu_{it}(\boldsymbol{x}_i)$





$$\nu_{is}(\boldsymbol{x}_i) = \prod_{t \in \mathcal{N}(i)\backslash s} \phi_{ti}(\boldsymbol{x}_i)$$

$$\phi_{si}(\boldsymbol{x}_i) = \int \Big(g_s(\boldsymbol{x}_{\mathcal{N}(s)}) \prod_{j \in \mathcal{N}(s)\backslash i} \nu_{js}(\boldsymbol{x}_j) \Big) \mathrm{d}\boldsymbol{x}_{\mathcal{N}(s)\backslash i}$$

Prediction Step

- Prediction step:

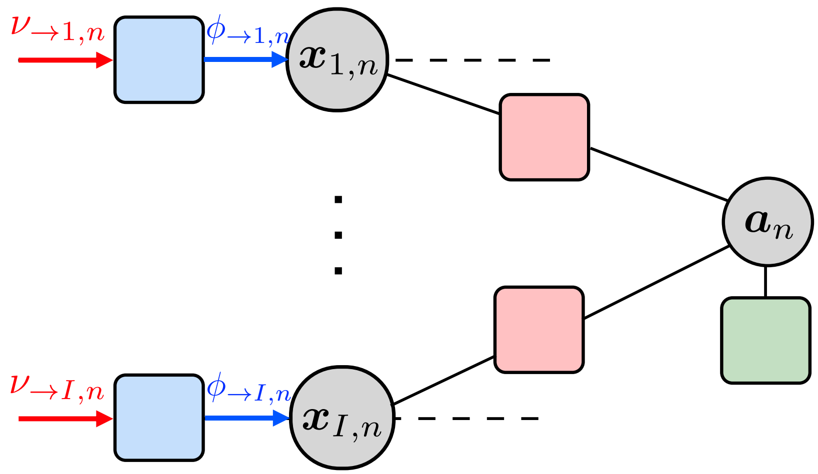
$$\phi_{\rightarrow i,n}(\boldsymbol{x}_{i,n}) = \int f(\boldsymbol{x}_{i,n}|\boldsymbol{x}_{i,n-1})\, \nu_{\rightarrow i,n}(\boldsymbol{x}_{i,n-1})\, \mathrm{d}\boldsymbol{x}_{i,n-1}$$

- Factor graph for time step n

$f(\boldsymbol{x}_{i,n'}|\boldsymbol{x}_{i,n'-1})$

$g_{\boldsymbol{z}_n}(\boldsymbol{x}_{i,n'}, a_{i,n'})$

$\varphi(\boldsymbol{a}_n)$



Measurement Evaluation

- Measurement evaluation:

$$\nu_{a_{i,n}}(\boldsymbol{x}_{i,n}) = \phi_{\rightarrow i,n}(\boldsymbol{x}_{i,n})$$

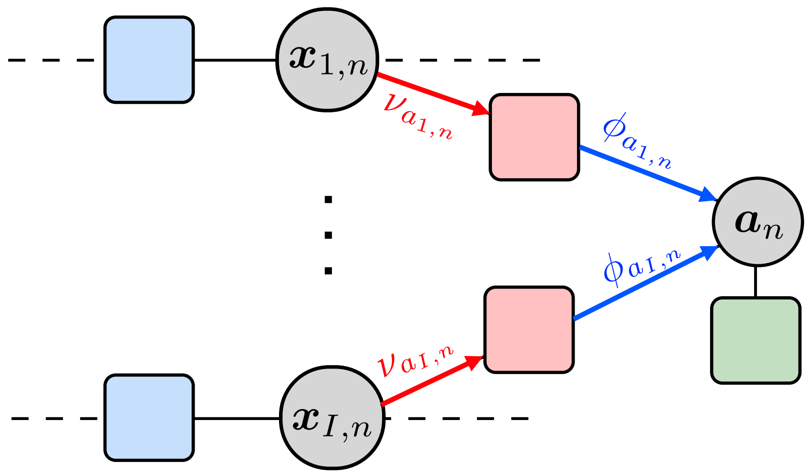
$$\phi_{a_{i,n}}(a_{i,n}) = \int g_{\boldsymbol{z}_n}(\boldsymbol{x}_{i,n}, a_{i,n})\, \nu_{a_{i,n}}(\boldsymbol{x}_{i,n})\, \mathrm{d}\boldsymbol{x}_{i,n}$$

- Factor graph for time step n

$f(\boldsymbol{x}_{i,n'} | \boldsymbol{x}_{i,n'-1})$

$g_{\boldsymbol{z}_n}(\boldsymbol{x}_{i,n'}, a_{i,n'})$

$\varphi(\boldsymbol{a}_n)$



Data Association

- Data association:

$$\nu_{\boldsymbol{x}_{i,n}}(\boldsymbol{a}_n) = \varphi(\boldsymbol{a}_n) \prod_{\substack{i=1 \\ i=i'}} \phi_{a_{i',n}}(a_{i',n})$$

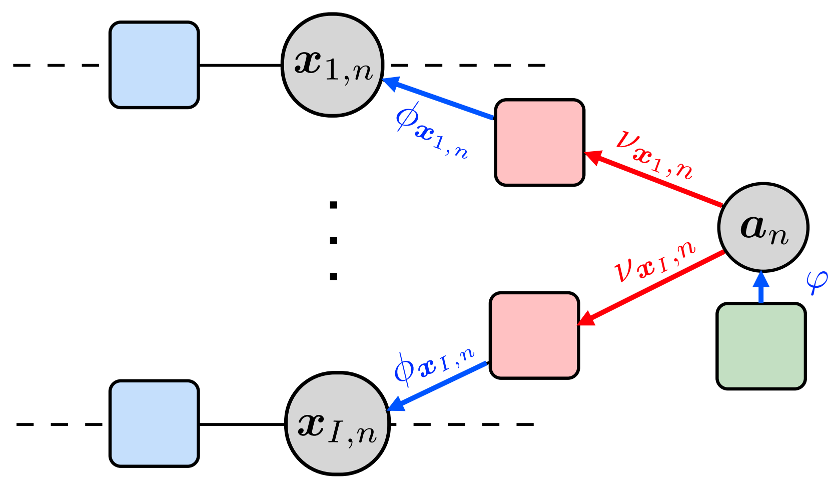
$$\phi_{\boldsymbol{x}_{i,n}}(\boldsymbol{x}_{i,n}) = \sum_{\boldsymbol{a}_n} g_{\boldsymbol{z}_n}(\boldsymbol{x}_{i,n}, a_{i,n})\, \nu_{\boldsymbol{x}_{i,n}}(\boldsymbol{a}_n)$$

- Factor graph for time step n

$f(\boldsymbol{x}_{i,n'}|\boldsymbol{x}_{i,n'-1})$

$g_{\boldsymbol{z}_n}(\boldsymbol{x}_{i,n'}, a_{i,n'})$

$\varphi(\boldsymbol{a}_n)$



Update Step

- Update step:

$$\tilde{f}(\boldsymbol{x}_{i,n}) \propto \phi_{\to i,n}(\boldsymbol{x}_{i,n})\,\phi_{\boldsymbol{x}_{i,n}}(\boldsymbol{x}_{i,n})$$

$$\nu_{\to i,n+1}(\boldsymbol{x}_{i,n}) = \phi_{i,n}(\boldsymbol{x}_{i,n})\,\phi_{\to i,n}(\boldsymbol{x}_{i,n})$$

$$\boxed{f(\boldsymbol{x}_{i,n}|\boldsymbol{z}_{1:n}) \approx \tilde{f}(\boldsymbol{x}_{i,n})}$$

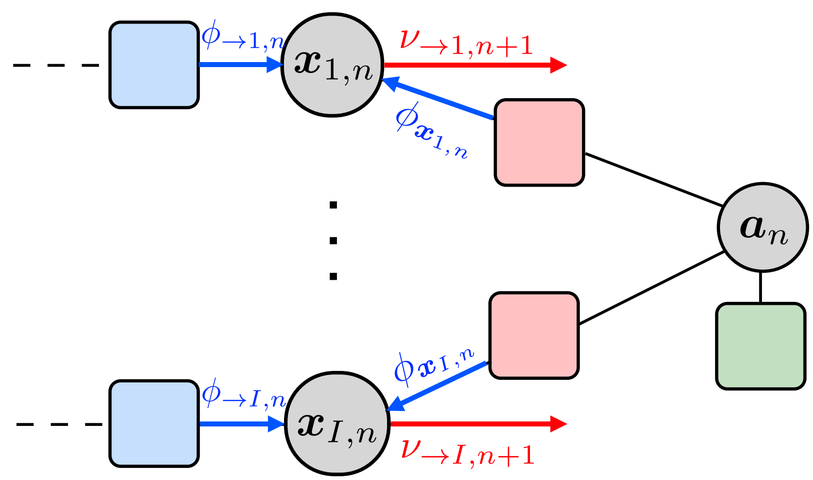
approx. since factor graph is not cycle-free

- Factor graph for time step n

$f(\boldsymbol{x}_{i,n'}|\boldsymbol{x}_{i,n'-1})$

$g_{\boldsymbol{z}_n}(\boldsymbol{x}_{i,n'}, a_{i,n'})$

$\varphi(\boldsymbol{a}_n)$



Summary

- Multiobject tracking
  - possible association events are modelled by a discrete random vector
  - measurement-origin uncertainty leads to a coupling of sequential estimation problems
  - the joint sequential estimation problem can be represented by a factor graph with cycles
  - approximate marginal posterior distributions can be calculated by passing messages on the factor graph