Multisensor Multiobject Tracking with Improved Sampling Efficiency: Supplementary Material

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This manuscript provides supplementary material for the paper, "Multisensor Multiobject Tracking with Improved Sampling Efficiency" by the same authors [1].

1 Review of Graph-Based Multiobject Tracking Using Multiple Sensors

In what follows, we will review the system model, and the corresponding statistical formulation of graph-based multiobject tracking (MOT) using multiple sensors as presented in [2, 3]. We will first review the concept of potential object (PO) states, which is used to model an unknown number of objects. Then we review the state-transition model and the measurement-origin uncertainty (MOU) likelihood function. Finally, we derive the posterior probability density function (pdf) and the corresponding factor graph used for statistical inference.

1.1 PO States and State-Transition Model

We denote by $\underline{\mathbf{y}}_k \triangleq \big[\underline{\mathbf{y}}_k^{(1)\mathrm{T}} \cdots \underline{\mathbf{y}}_k^{(J_{k-1})\mathrm{T}}\big]^{\mathrm{T}}$ and by $\overline{\mathbf{y}}_k \triangleq \big[\overline{\mathbf{y}}_k^{(1)\mathrm{T}} \cdots \, \overline{\mathbf{y}}_k^{(\mathsf{M}_k)\mathrm{T}}\big]^{\mathrm{T}}$ as the joint vector of all legacy PO and all new PO states, respectively. In addition, the vector of all the PO states at time k is denoted as $\mathbf{y}_k \triangleq \big[\underline{\mathbf{y}}_k^{\mathrm{T}} \, \overline{\mathbf{y}}_k^{\mathrm{T}}\big]^{\mathrm{T}} \triangleq \big[\mathbf{y}_k^{(1)\mathrm{T}} \cdots \mathbf{y}_k^{(J_k)\mathrm{T}}\big]^{\mathrm{T}}$. For each augmented PO state $\mathbf{y}_{k-1}^{(j)}$, $j \in \{1, \dots, J_{k-1}\}$ at time $k-1$, there is one "legacy" PO state $\underline{\mathbf{y}}_{k,1}^{(j)}$, $j \in \{1, \dots, J_{k,0}\}$ at time k when measurements of sensor $s = 1$ are considered. The evolution of the augmented PO state between consecutive steps is modeled by a single-object state transition pdf $f\big(\underline{\boldsymbol{y}}_{k,1}^{(j)}|\boldsymbol{y}_{k-1}^{(j)}\big)$. It is assumed that objects evolve independently in time, i.e., $f\big(\underline{\boldsymbol{y}}_{k,1}|\boldsymbol{y}_{k-1}\big) = \prod_{j=1}^{J_{k-1}} f\big(\underline{\boldsymbol{y}}_{k,1}^{(j)}|\boldsymbol{y}_{k-1}^{(j)}\big)$. The state-transition pdf of the augmented PO state$f\big(\underline{\boldsymbol{y}}_{k,1}^{(j)}|\boldsymbol{y}_{k-1}^{(j)}\big)$ is a function of the probability of object survival p_{su} and the state-transition pdf $f\big(\underline{\boldsymbol{x}}_{k,1}^{(j)}|\boldsymbol{x}_{k-1}^{(j)}\big)$.

The functional form of the single-object state transition model and how it models the motion and disappearance of objects is presented in [3, Section VIII-C]. At time $k = 0$, the prior augmented states $\boldsymbol{y}_0^{(j)}$ are statistically independent across POs j. Often no prior information is available, i.e., $J_0 = 0$. Object birth at time k is modeled by a Poisson point process with mean μ_{b} and arbitrary pdf $f_{\mathrm{b}}\big(\overline{\boldsymbol{x}}_k\big)$.

1.2 MOU Measurement Model for Multiple Objects

At time k, a random number, $\mathsf{M}_{k,s}$, of measurements are generated at the sensor s. The measurements are subject to MOU, i.e., each measurement $\mathbf{z}_{k,s}^{(m)}$, $m \in \{1, \dots, \mathsf{M}_{k,s}\}$ can originate from one of the following three sources, i.e., it is either (i) originated from an object (represented by a legacy PO) that has already generated a measurement in the past; (ii) originated from an object (represented by a new PO) that has never generated a measurement in the past; or (iii) a false positive (FP). The object represented by (new or legacy) PO $j \in \{1, \dots, \mathsf{J}_{k,s}\}$ generates a measurement $\mathbf{z}_{k,s}^{(m)}$ with probability of detection p_{d}. In case PO with index j generates a measurement with index m, the statistical relationship of a measurement $\mathbf{z}_{k,s}^{(m)}$ and a PO state $\mathbf{x}_{k,s}^{(j)}$ is described by the conditional pdf $f\big(\boldsymbol{z}_{k,s}^{(m)}|\boldsymbol{x}_{k,s}^{(j)}\big)$, which can be derived based on the measurement model of the sensor. FP measurements are independent of the object states and modeled by a Poisson point process with mean μ_{fp} and pdf $f_{\text{fp}}\big(\boldsymbol{z}_{k,s}^{(m)}\big)$.

We make use of the well-established data association assumption [4–6], i.e., it is assumed that at any time k and sensor s, an object can generate at most one measurement and a measurement can originate from at most one object. The association between the $\mathsf{M}_{k,s}$ measurements and the $\mathsf{J}_{k,s-1}$ legacy POs at time k and sensor s is represented by an "object-oriented" data association (DA) vector $\mathbf{a}_{k,s} = \big[\mathsf{a}_{k,s}^{(1)} \cdots \mathsf{a}_{k,s}^{(\mathsf{J}_{k,s-1})}\big]^{\mathrm{T}}$ with object-oriented association variables $\mathsf{a}_{k,s}^{(j)} \in \big\{0, 1, \ldots, \mathsf{M}_{k,s}\big\}$. Every association vector represents an association event that explains the origin of each measurement. In particular, $\mathsf{a}_{k,s}^{(j)} = m \in \{1, \ldots, \mathsf{M}_{k,s}\}$ if legacy PO j generates measurement m at sensor s and $\mathsf{a}_{k,s}^{(j)} = 0$ if legacy PO j is missed by the sensor [3,4]. Note that only existing legacy POs can generate measurements, i.e., $\underline{\mathsf{r}}_{k,s}^{(j)} = 0$ implies $\mathsf{a}_{k,s}^{(j)} = 0$. MOU for multiple objects leads to a combinatorial number of association events $\mathbf{a}_{k,s}$. To obtain a scalable and efficient SPA (see [2,3,7] for details), we also introduce the "measurement-oriented" DA vector $\mathbf{b}_{k,s} = \big[\mathsf{b}_{k,s}^{(1)} \cdots \mathsf{b}_{k,s}^{(\mathsf{M}_{k,s})}\big]^{\mathrm{T}}$ with measurement-oriented association variables $\mathsf{b}_{k,s}^{(m)} = j \in \{0, 1, \ldots, \mathsf{J}_{k,s-1}\}$. In particular, $\mathsf{b}_{k,s}^{(m)} = j \in \{1, \ldots, \mathsf{J}_{k,s-1}\}$ if measurement m originated from legacy PO j, and $\mathsf{b}_{k,s}^{(m)} = 0$ if it originated from a new PO or is a FP. Since only existing new POs can generate a measurement, the fact that $\overline{\mathsf{r}}_{k,s}^{(m)} = 0$ and $\mathsf{b}_{k,s}^{(m)} = 0$ implies that measurement m is a FP.

1.3 Joint Posterior pdf and Factor Graph

Let us denote by $\mathbf{y}_{0:k}$, $\mathbf{a}_{1:k}$, $\mathbf{b}_{1:k}$, and $\boldsymbol{z}_{1:k}$ all the PO states, object-oriented association variables, measurement-oriented association variables, and measurements of all sensors up to time k. For example, $\mathbf{z}_{1:k} = [\mathbf{z}_1^{\mathrm{T}} \cdots \mathbf{z}_k^{\mathrm{T}}]^{\mathrm{T}}$ with $\mathbf{z}_k = \big[\mathbf{z}_{k,s}^{(1)\mathrm{T}} \cdots \mathbf{z}_{k,s}^{(\mathsf{M}_{k,s})\mathrm{T}}\big]^{\mathrm{T}}$. Using common assumptions [3], the joint posterior pdf of $\mathbf{y}_{0:k}$, $\mathbf{a}_{1:k}$, and $\mathbf{b}_{1:k}$ conditioned on observed $\boldsymbol{z}_{1:k}$ can be obtained as

$$
\begin{aligned}
f\big(\boldsymbol{y}_{0:k}, \boldsymbol{a}_{1:k}, \boldsymbol{b}_{1:k} \big| \boldsymbol{z}_{1:k}\big) \propto & \left(\prod_{j''=1}^{J_0} f\big(\boldsymbol{y}_0^{(j'')}\big) \right) \prod_{k'=1}^{k} \left(\prod_{j'=1}^{J_{k'-1}} f\big(\underline{\boldsymbol{y}}_{k'}^{(j')} \big| \boldsymbol{y}_{k'-1}^{(j')}\big) \right) \prod_{s=1}^{S} \\
& \times \left(\prod_{j=1}^{J_{k',s-1}} q\big(\underline{\boldsymbol{x}}_{k',s}^{(j)}, \underline{r}_{k',s}^{(j)}, a_{k',s}^{(j)}; \boldsymbol{z}_{k',s}\big) \prod_{m'=1}^{M_{k',s}} \Psi_{j,m'}\big(a_{k',s}^{(j)}, b_{k',s}^{(m')}\big) \right) \\
& \times \prod_{m=1}^{M_{k',s}} v\big(\overline{\boldsymbol{x}}_{k',s}^{(m)}, \overline{r}_{k',s}^{(m)}, b_{k',s}^{(m)}; \boldsymbol{z}_{k',s}^{(m)}\big).
\end{aligned}
\tag{1}
$$

Here, the legacy PO factor $q\big(\underline{\boldsymbol{x}}_{k,s}^{(j)}, \underline{r}_{k,s}^{(j)}, a_{k,s}^{(j)}; \boldsymbol{z}_{k,s}\big)$ is given by

$$
q\big(\underline{\boldsymbol{x}}_{k,s}^{(j)}, 1, a_{k,s}^{(j)}; \boldsymbol{z}_{k,s}\big) \triangleq \begin{cases} \frac{p_{\mathrm{d}}}{\mu_{\mathrm{fp}} f_{\mathrm{fp}}\left(\boldsymbol{z}_{k,s}^{(m)}\right)} f\big(\boldsymbol{z}_{k,s}^{(m)} \big| \underline{\boldsymbol{x}}_{k,s}^{(j)}\big), & a_{k,s}^{(j)} = m \in \{1, \ldots, M_{k,s}\} \\ 1 - p_{\mathrm{d}}, & a_{k,s}^{(j)} = 0 \end{cases}
\tag{2}
$$

and $q\big(\underline{\boldsymbol{x}}_{k,s}^{(j)}, 0, a_{k,s}^{(j)}; \boldsymbol{z}_{k,s}\big) \triangleq 1(a_{k,s}^{(j)})$. Furthermore, the new PO factor $v\big(\overline{\boldsymbol{x}}_{k,s}^{(m)}, \overline{r}_{k,s}^{(m)}, b_{k,s}^{(m)}; \boldsymbol{z}_{k,s}^{(m)}\big)$ reads

$$
v\big(\overline{\boldsymbol{x}}_{k,s}^{(m)}, 1, b_{k,s}^{(m)}; \boldsymbol{z}_{k,s}^{(m)}\big) \triangleq \begin{cases} 0, & b_{k,s}^{(m)} \in \{1, \ldots, J_{k,s-1}\} \\ \frac{p_{\mathrm{d}} \, \mu_{\mathrm{b}} \, f_{\mathrm{b}}\left(\overline{\boldsymbol{x}}_{k,s}^{(m)}\right)}{\mu_{\mathrm{fp}} f_{\mathrm{fp}}\left(\boldsymbol{z}_{k,s}^{(m)}\right)} f\big(\boldsymbol{z}_{k,s}^{(m)} \big| \overline{\boldsymbol{x}}_{k,s}^{(m)}\big), & b_{k,s}^{(m)} = 0 \end{cases}
\tag{3}
$$

and $v\big(\overline{\boldsymbol{x}}_{k,s}^{(m)}, 0, b_{k,s}^{(m)}; \boldsymbol{z}_{k,s}^{(m)}\big) \triangleq f_{\mathrm{D}}\big(\overline{\boldsymbol{x}}_{k,s}^{(m)}\big)$.

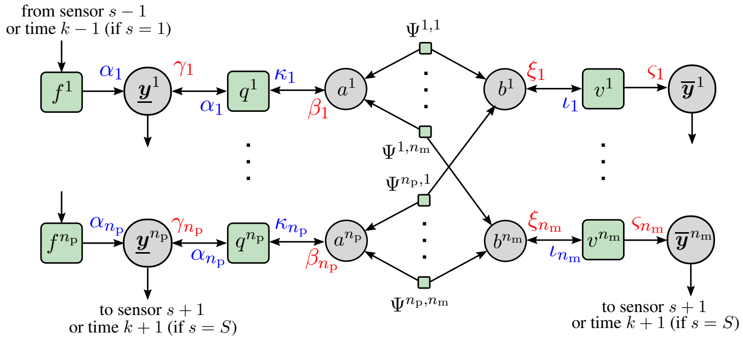

Figure 1: Factor graph for MOT of an unknown, time-varying number of objects, corresponding to the propagation of the joint pdf $f\big(\boldsymbol{y}_{1:k}, \boldsymbol{a}_{1:k}, \boldsymbol{b}_{1:k} \big| \boldsymbol{z}_{1:k}\big)$ in (1). The part of the factor graph corresponding to one sensor update is shown. Messages calculated using Particle flow (PFL) are depicted in red. These messages are calculated based on messages depicted in blue. The time index k and sensor index s are omitted, and the following short notation is used: $n_{\mathrm{m}} \triangleq m_k$, $n_{\mathrm{p}} \triangleq j_{k-1}$, $\underline{\boldsymbol{y}}^j \triangleq \underline{\boldsymbol{y}}_k^{(j)}$, $\overline{\boldsymbol{y}}^m \triangleq \overline{\boldsymbol{y}}_k^{(m)}$, $\boldsymbol{a}^j \triangleq \boldsymbol{a}_{k,s}^{(j)}$, $\boldsymbol{b}^m \triangleq \boldsymbol{b}_{k,s}^{(m)}$, $f^j \triangleq f\big(\underline{\boldsymbol{y}}_{k,1}^{(j)} \big| \boldsymbol{y}_{k-1}^{(j)}\big)$ (for $s=1$), $f^j \triangleq \delta\big(\underline{\boldsymbol{y}}_{k,s}^{(j)} - \boldsymbol{y}_{k,s-1}^{(j)}\big)$ (for $s>1$), $q^j \triangleq q\big(\underline{\boldsymbol{x}}_{k,s}^{(j)}, \underline{r}_{k,s}^{(j)}, a_{k,s}^{(j)}; \boldsymbol{z}_{k,s}\big)$, $v^m \triangleq v\big(\overline{\boldsymbol{x}}_{k,s}^{(m)}, \overline{r}_{k,s}^{(m)}, b_{k,s}^{(m)}; \boldsymbol{z}_{k,s}^{(m)}\big)$, $\Psi^{j,m} \triangleq \Psi_{j,m}\big(a_{k,s}^{(j)}, b_{k,s}^{(m)}\big)$, $\gamma_j \triangleq \gamma_{k,s}^{(j)}\big(\underline{\boldsymbol{y}}_{k,s}^{(j)}\big)$, $\beta_j \triangleq \beta_{k,s}^{(j)}\big(a_{k,s}^{(j)}\big)$, $\xi_m \triangleq \xi_{k,s}^{(m)}\big(b_{k,s}^{(m)}\big)$, $\varsigma_m \triangleq \varsigma_{k,s}^{(m)}\big(\overline{\boldsymbol{y}}_{k,s}^{(m)}\big)$, $\alpha_j \triangleq \alpha_{k,s}^{(j)}\big(\underline{\boldsymbol{y}}_{k,s}^{(j)}\big)$, $\kappa_j \triangleq \kappa_{k,s}^{(j)}\big(a_{k,s}^{(j)}\big)$, and $\iota_m \triangleq \iota_{k,s}^{(m)}\big(b_{k,s}^{(m)}\big)$.

Finally, the binary indicator function $\Psi_{j,m}\big(a_{k,s}^{(j)}, b_{k,s}^{(m)}\big)$ checks association consistency of a pair of object-oriented and measurement-oriented variables $\big(a_{k,s}^{(j)}, b_{k,s}^{(m)}\big)$ in that $\Psi_{j,m}\big(a_{k,s}^{(j)}, b_{k,s}^{(m)}\big)$ is zero if $a_{k,s}^{(j)} = m, b_{k,s}^{(m)} \neq j$ or $b_{k,s}^{(m)} = j, a_{k,s}^{(j)} \neq m$ and one otherwise (see [2, 3, 7] for details). If and only if all pairs of object-oriented and measurement-oriented association variables are consistent, the data association assumption is satisfied. Formulating the MOU measurement model for multiple objects using both object-oriented and measurement-oriented association variables makes it possible to significantly reduce computational complexity and increase the scalability of probabilistic data association [2,3,7] by making use of the loopy sum-product algorithm (SPA). A detailed derivation of this joint posterior is provided in [3, Section VIII-G]. The joint posterior in (1) can be represented by the factor graph in Fig. 1.

References

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