SIO 207A: Fundamentals of Digital Signal Processing
Class 14

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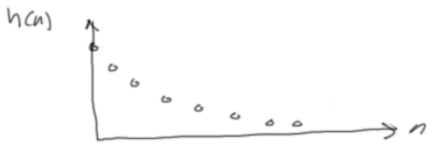
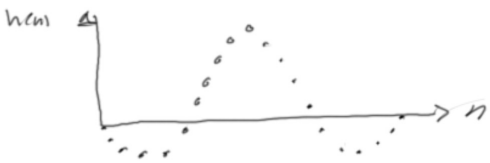
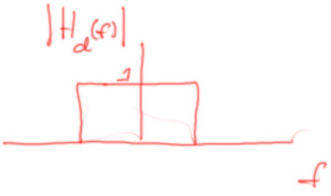
Linear Phase FIR Filters

- In general, the impulse response of a FIR is <u>not</u> symmetric



- Filter designed by "firpm" are symmetric





Why do we get a linear phase for symmetric FIR filters?

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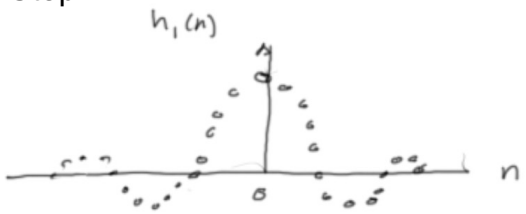
Linear Phase FIR Filters

- In the case of a symmetric filter, think of two steps

Step I



$\Rightarrow$

Note: Non-causal impulse response

Pure real / no imaginary part $\longrightarrow$ phase is zero

$$H_1(f) = \sum_{n=-\infty}^{\infty} h_1[n]e^{-j2\pi fn}$$

$$= \sum_{n=-\infty}^{\infty} h_1[n]\left[\cos(2\pi fn) - \boxed{j\sin(2\pi fn)}\right]$$

$\longleftarrow$ Cancelled out due to odd nature of $\sin$ function

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Linear Phase FIR Filters

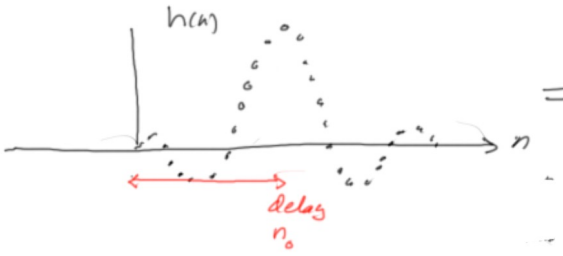
- Delay $h_1[n]$ so that $h[n] = 0$ for $n < 0$

$H_1(z)$ $\quad z^{-n_0}$ $\quad H(z) = z^{-n_0}H_1(z)$

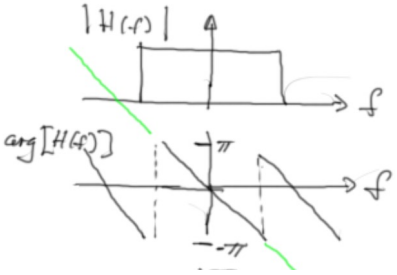
$h_1[n]$ — n_0 units delay — $h[n]$

$$z^{-n_0}\big|_{z=e^{2\pi f}} = e^{-j2\pi fn_0}$$

Step II



$\Rightarrow$



The impact of the delay is to yield a causal impulse response for $h[n]$ and also results in the Fourier transformation of $h[n]$ having a linear phase characteristic $H(f) = e^{-j2\pi fn_0}H_1(f)$

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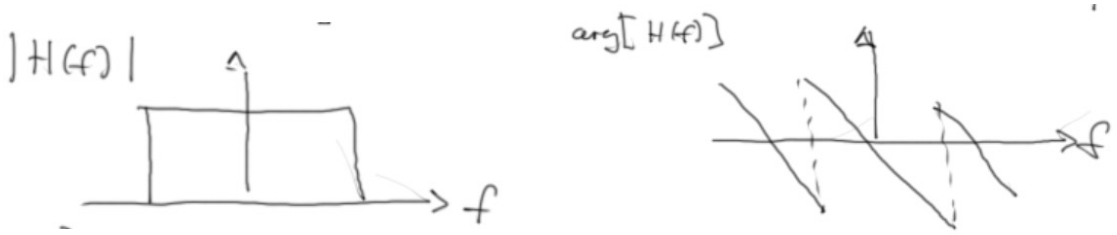
Linear Phase FIR Filters

- Symmetric causal FIR filter (e.g., output of "firpm")



- Why do we care about linear phase?
  1. Enables keeping track of absolute time reference
  2. IIR filters have nonlinear phase characteristics and that can result in difficulties with interpreting of filtering results (e.g. near the critical frequencies)

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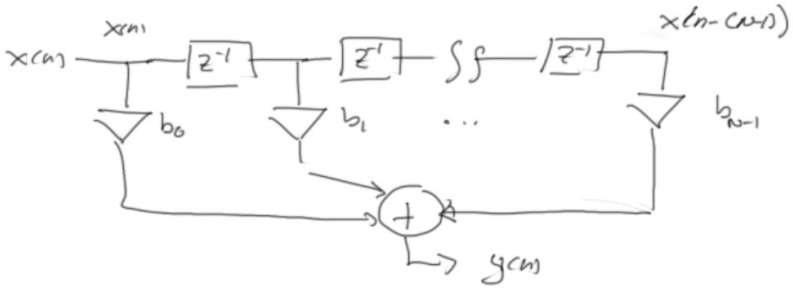
Linear Phase FIR Filters

- Another aspect of symmetric FIR filters is that a factor of 2 in computational savings is possible



- When filter coefficients are symmetric, then use common filter coefficient values and add together the time series being multiplied by those coefficients first, i.e., $x[n]$ and $x[n-(N-1)]$ then multiply by $b_0 = b_{N-1}$

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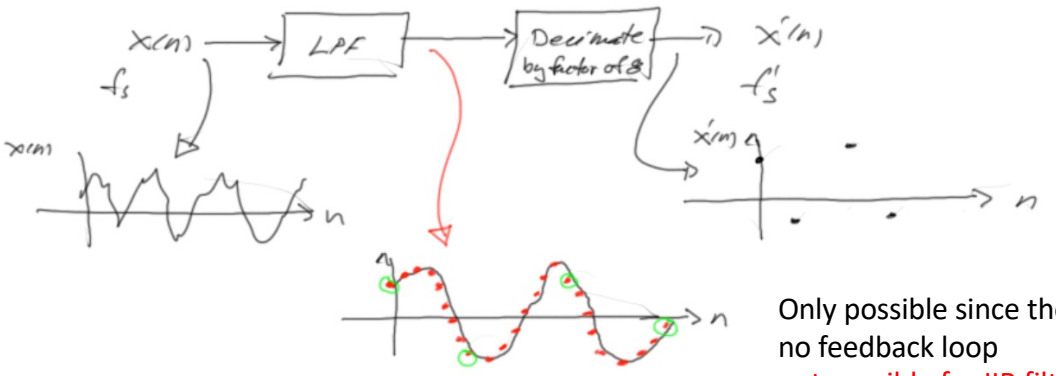
Linear Phase FIR Filters

- Lastly, one other advantage of using FIR decimation filters (as opposed to IIR filters) is that there is no need to compute points that are just going to be thrown away

Decimation:



Only possible since there is no feedback loop

→ not possible for IIR filters

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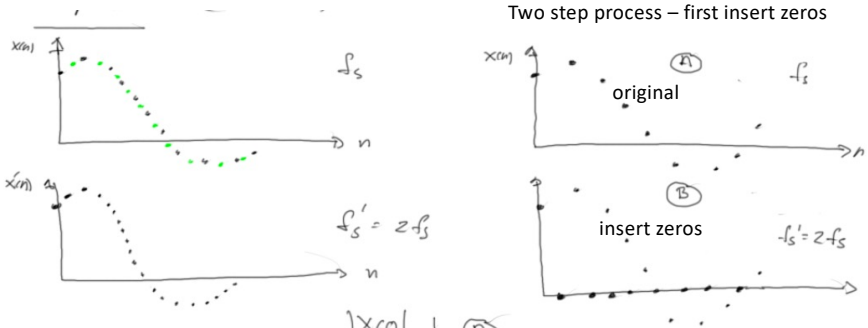
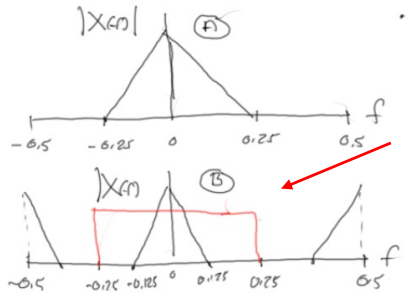
Interpolation

Two step process – first insert zeros





see also Section 4.6.2 in *Oppenheim & Schafer, 1999*

Ideal LPF which will attenuate replicas of desired spectrum

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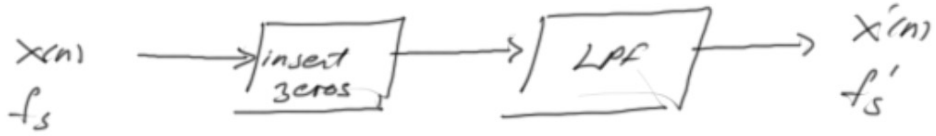
Interpolation

- Second step is to pass the time series (8) through a LPF to get rid of the high frequency spectral content (i.e., the replicas of the desired spectrum)



- Computational savings can be achieved by not carrying out multiplies of time series values that are 0

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