ECE 175B: Probabilistic Reasoning and Graphical Models

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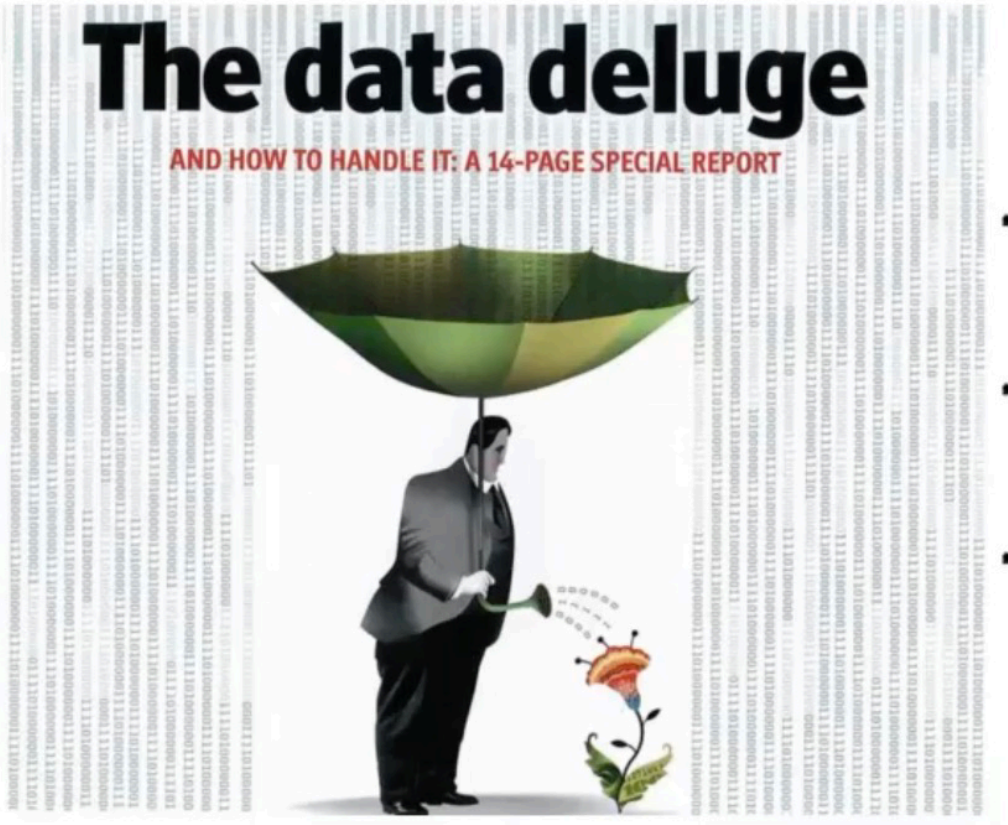
Big Data



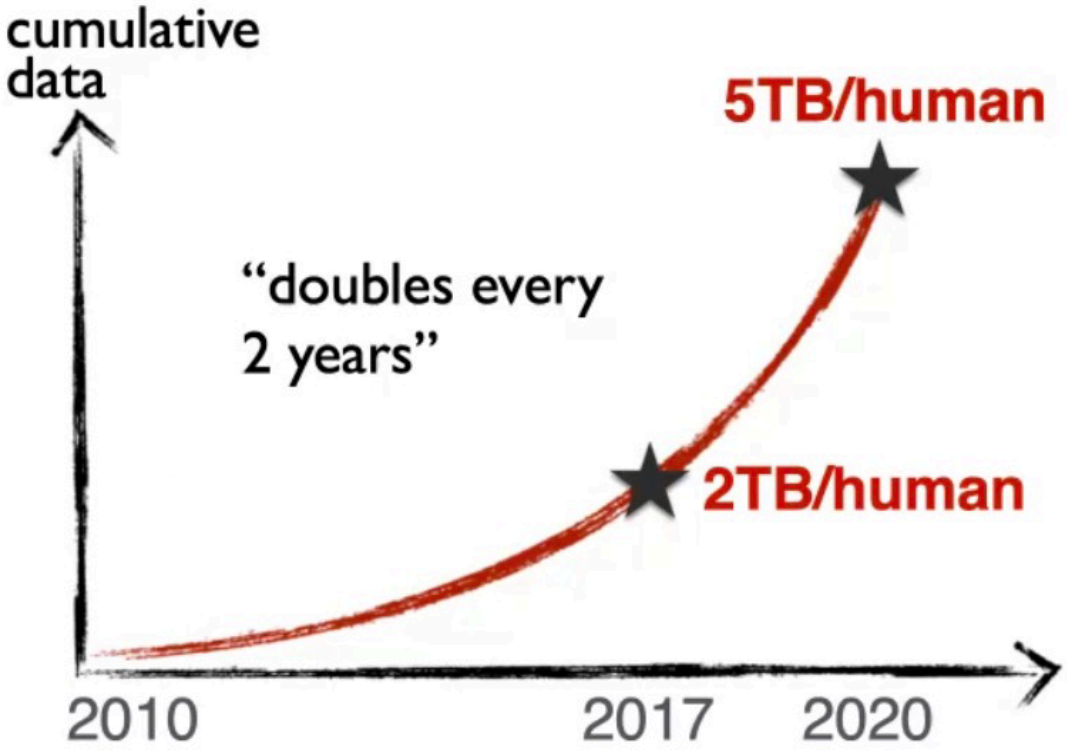
→ Volume

→ Velocity

→ Variety

The Economist
2010

Volume and Velocity



Source: IDC Digital Universe

Mathematical Formulation of a Graph

Graph: $\mathcal{G}=(\mathcal{V},\mathcal{E})$ such that $\mathcal{E}\subseteq[\mathcal{V}]^2$

Vertex (node) set $\mathcal{V}$: a collection of objects

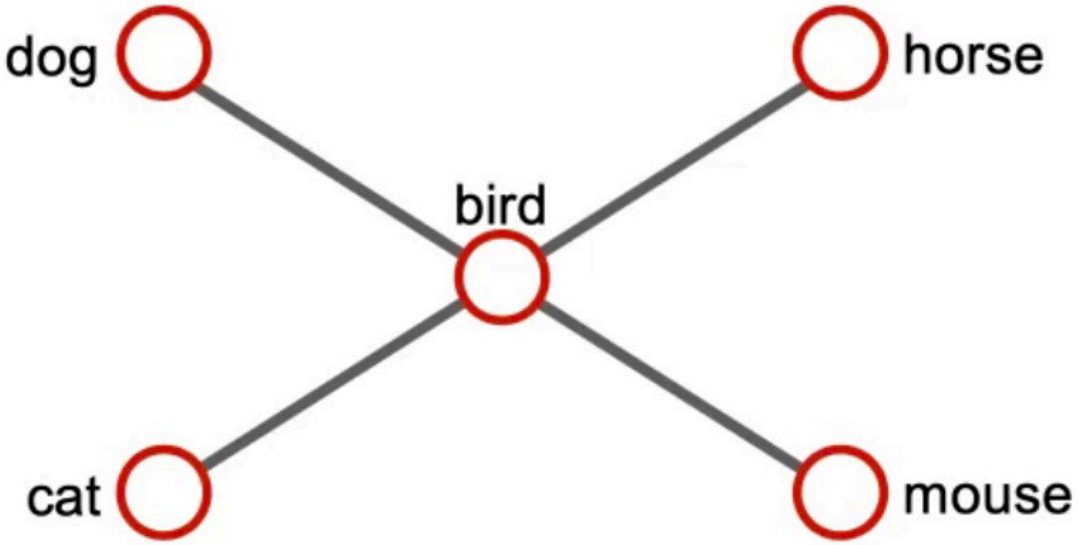
Edge set $\mathcal{E}$: relation between objects

Example:

$\mathcal{V}=\{\text{dog, cat, bird, horse, mouse}\}$

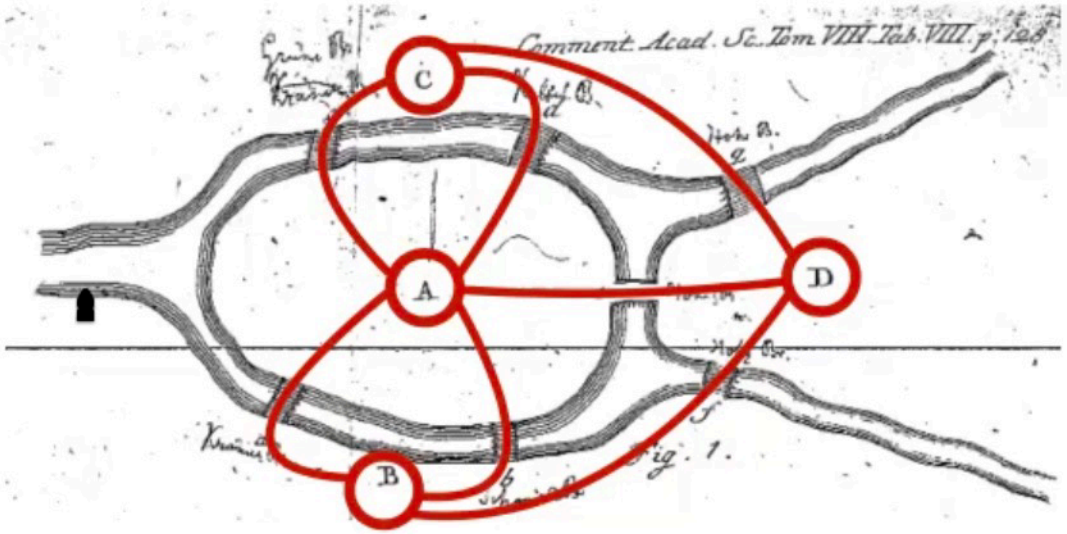
$\mathcal{E}=\{\{\text{dog, bird}\},\{\text{cat, bird}\},\{\text{bird, horse}\},\{\text{bird, mouse}\}\}$

Graph Representation

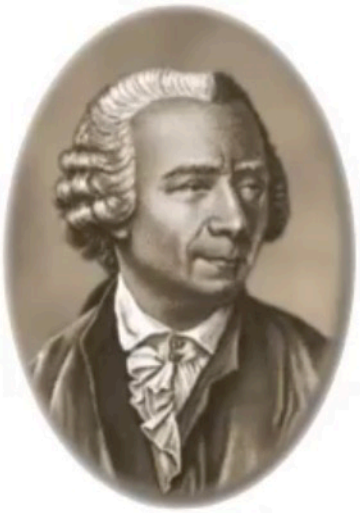


it's all about structure

History of Graphs



Seven Bridges of Königsberg



Leonhard
EULER

How to Use Graphs

- **data model** - captures abstract structure
- computational vehicle for **efficient algorithm**
- natural tool for **distributed** implementations

Tools and Applications

- **Graphical models** combine
  - graph theory: intuitive appeal
  - probability theory: interface between model and data
  - linear algebra & optimization: efficient algorithms
- Allow to deal with **uncertainty** and **complexity**
- **Application areas:** machine learning, information theory, communications, statistics, image processing, bioinformatics, statistical mechanics, social networks, sensor networks,
- **Unifying framework:** factor analysis, mixture models, hidden Markov models, Kalman filters, error correcting codes, Ising models, etc.

Example 1

- Consider binary (yes, no) **variables** relating to patient's health:

  asia, smoking, tuberculosis, cancer, bronchitis, xray, dyspnea

  Can you specify their joint distribution?

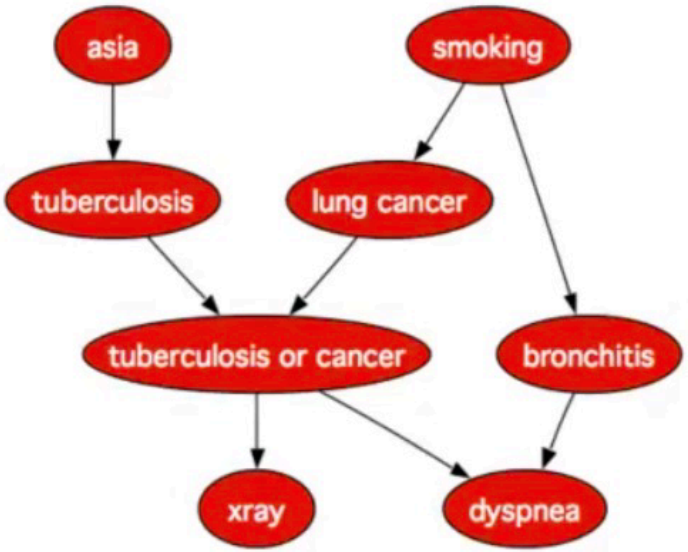
- Assume we know the **joint distribution**

  $p(\text{asia}, \text{smoking}, \text{tuberculosis}, \text{cancer}, \text{bronchitis}, \text{xray}, \text{dyspnea})$

  Can you efficiently calculate $p(\text{asia}|\text{dyspnea}=\text{no})$?

Example 1 cont.

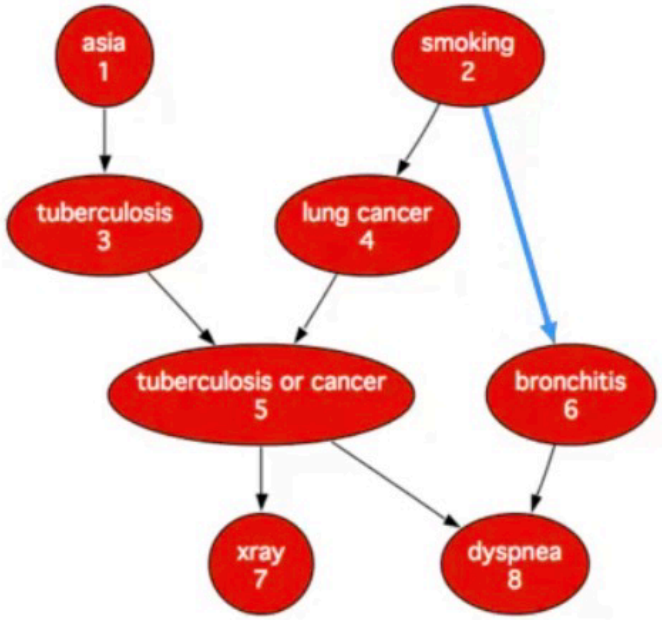
Bayesian network: directed acyclic graph (DAG)



- stay in asia increases risk of tuberculosis
- smoking can cause lung cancer and bronchitis
- tuberculosis, cancer, and bronchitis may result in dyspnea
- tuberculosis and cancer may lead to positive xray result

- 8 nodes (vertices): represent relevant variables
- 8 directed links (edges): represent interactions (causal influence)
- strength of interactions to be quantified via probabilities

Example 1 cont.



graph adjacency matrix

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \\ 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

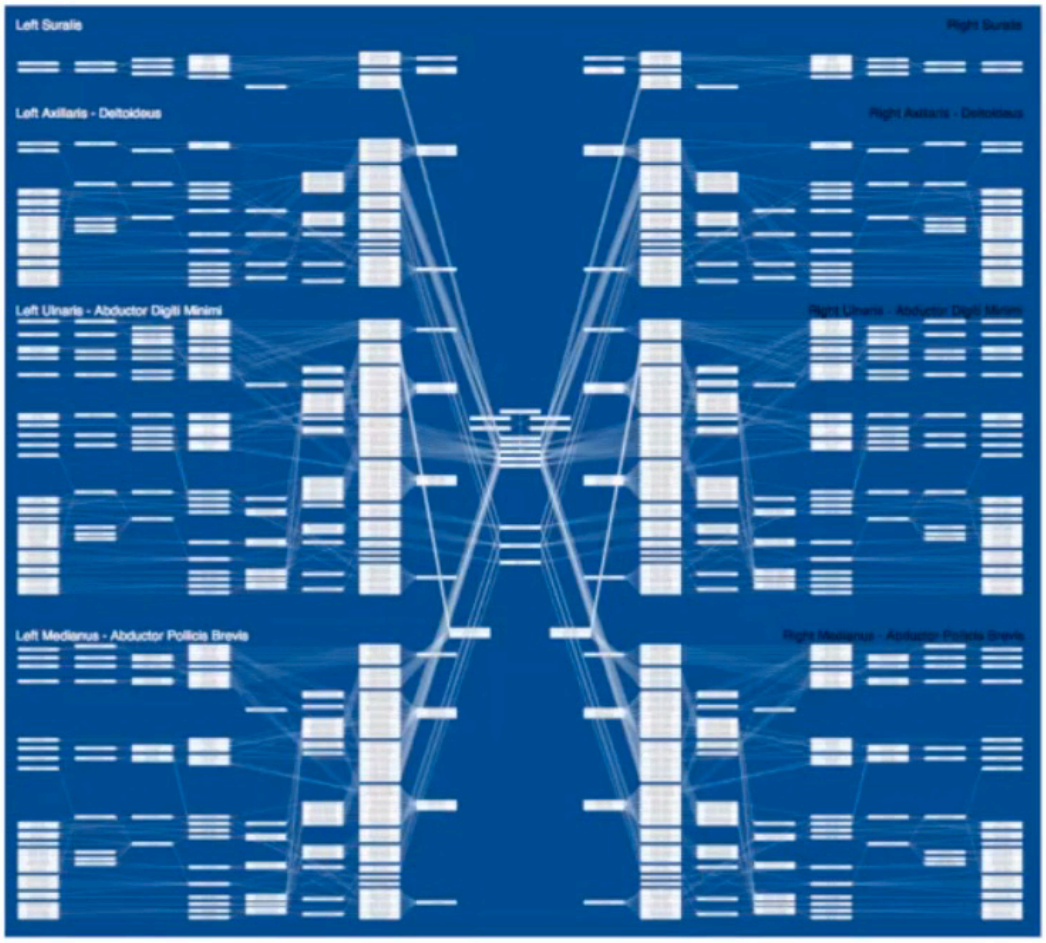
row sum: out-degrees (#children)

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 2 & 1 & 1 & 2 \end{pmatrix}$$

column sum: in-degrees (#parents)

- {asia, smoking}: in-degree zero; {xray, dyspnea}: out-degree zero
- **no directed cycle:** moving along arrows cannot get us back to any starting node

Example 2: MUNIN

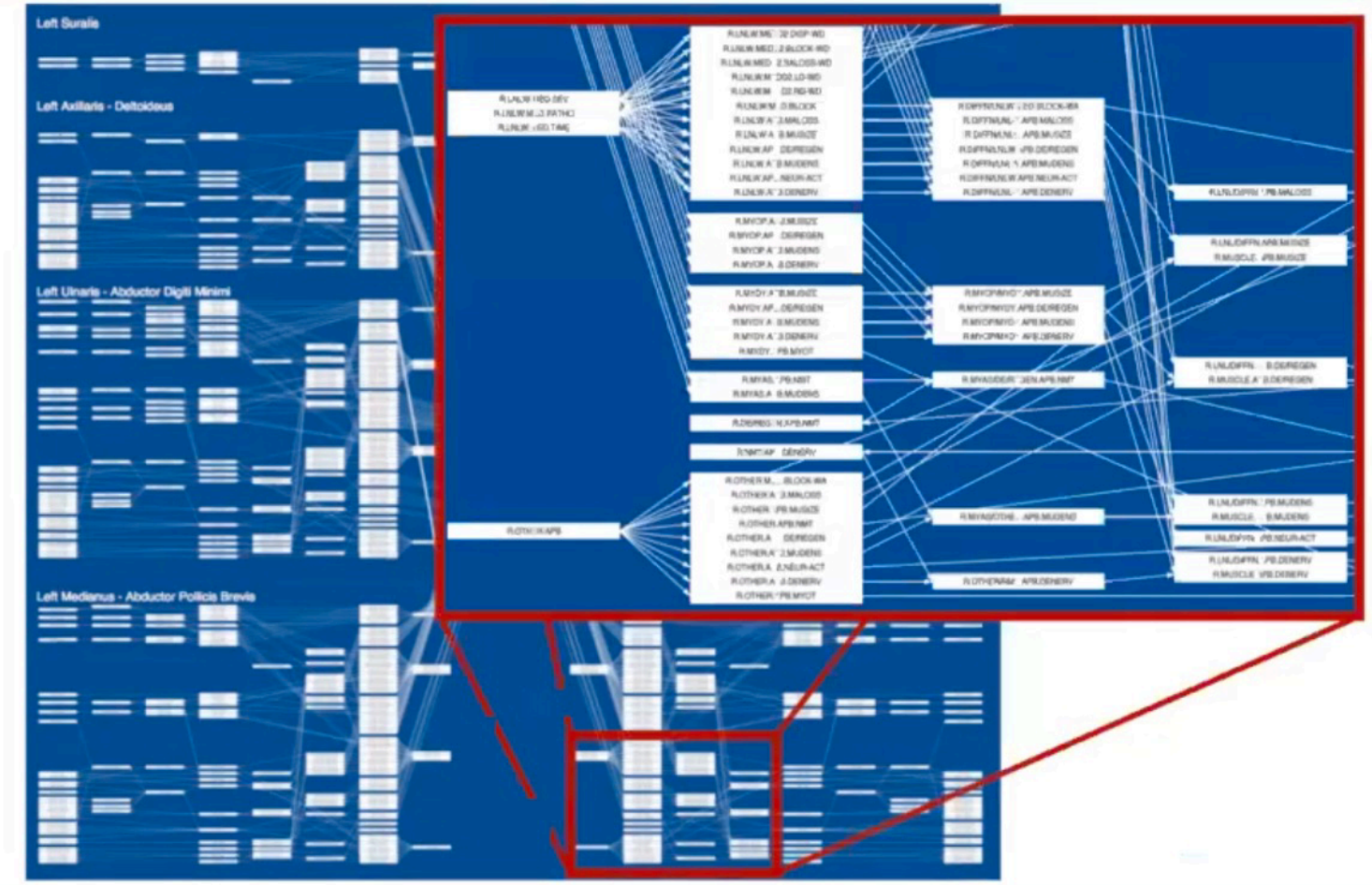


MUscle and **N**erve
Inference **N**etwork

Expert system for EMG-based diagnosis of neuromuscular disorder

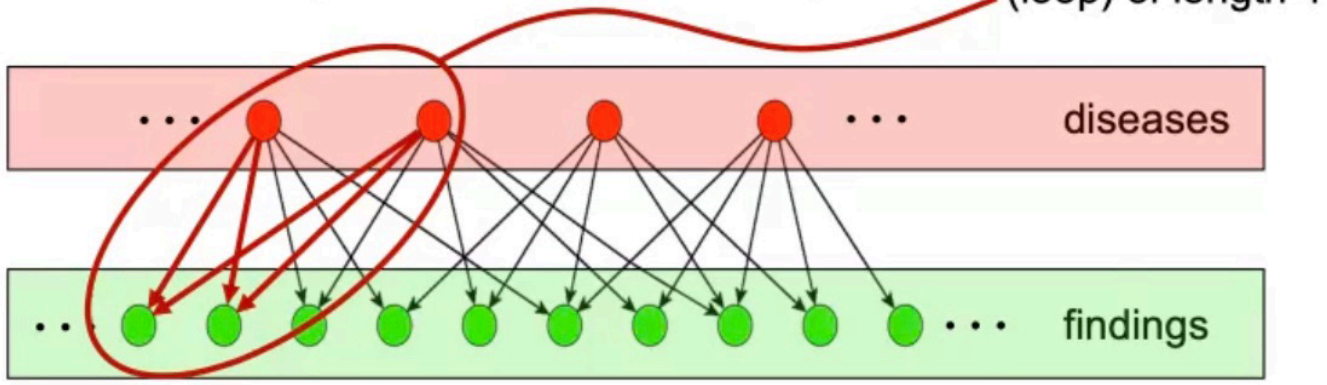
> 1000 nodes

Example 2: MUNIN



Example 3: Quick Medical Reference

- **Quick Medical Reference (QMR)**
  - bipartite (two-colorable) graph
  - variables represent diseases and findings

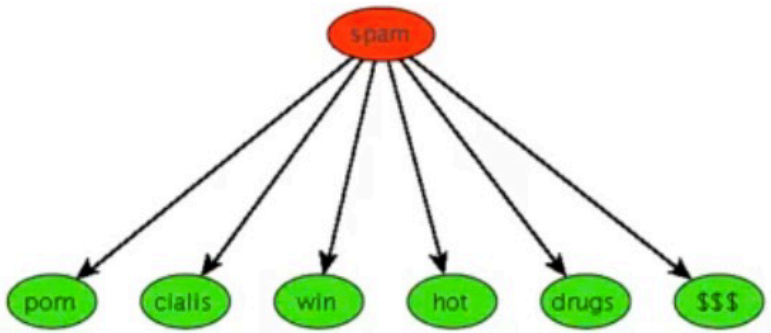


  - Univ. Pittsburgh QMR Database
    - ~40000 edges
    - ~400 diseases (average outdegree ~100)
    - ~4000 findings (average indegree ~10)

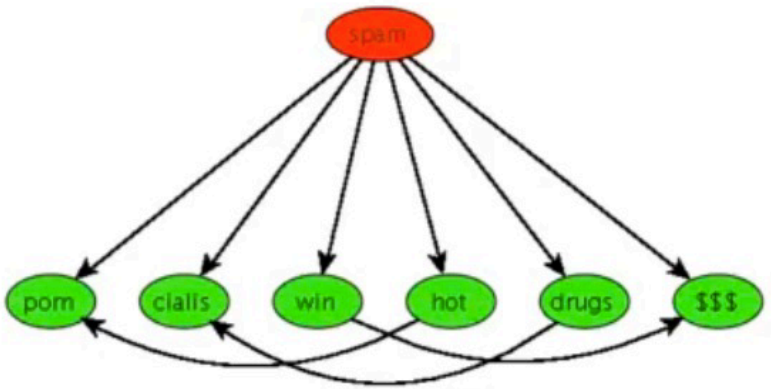
Example 4: Spam Filtering

Bayesian spam filters

- essentially text classifiers
- one root, many leaves representing words (or word fragments)

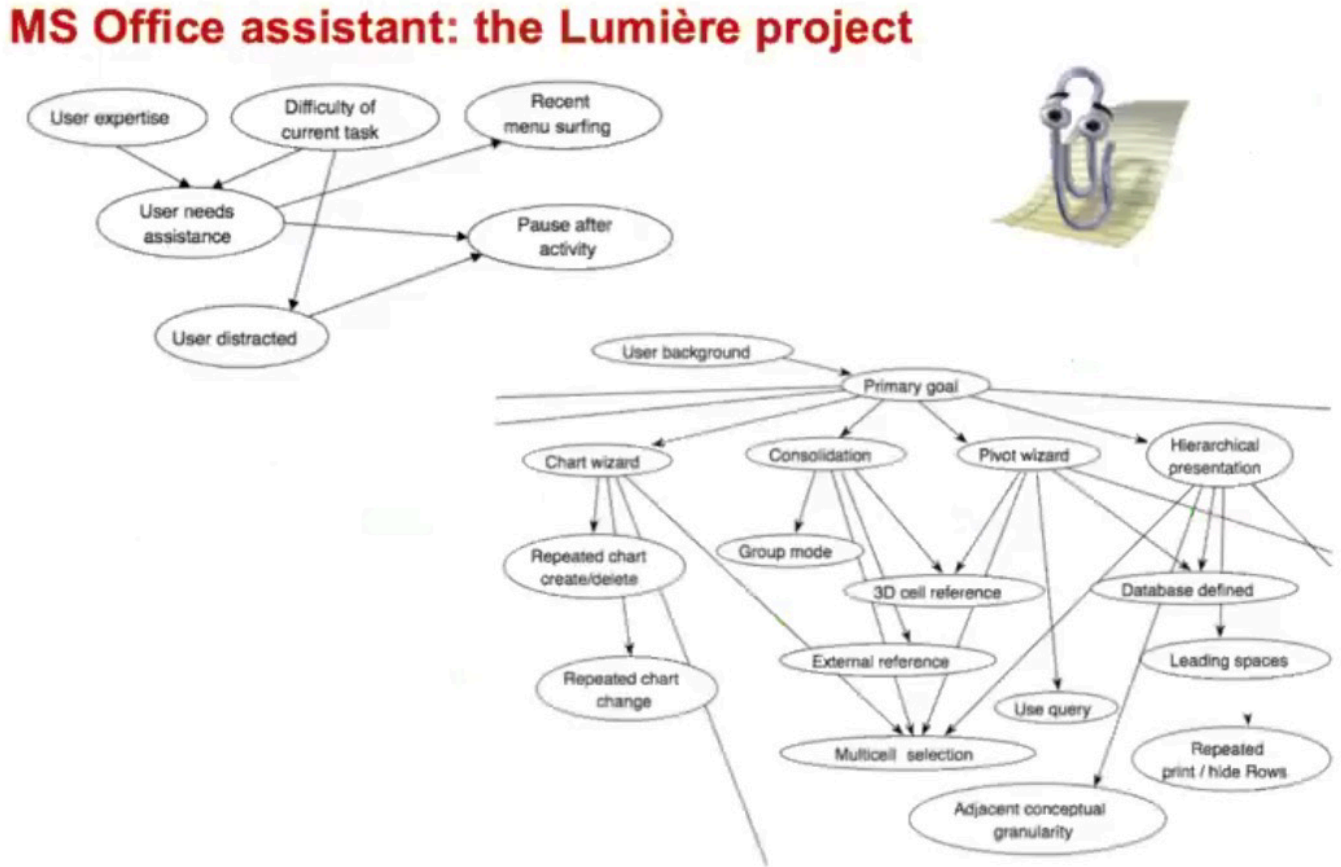


„naive Bayes“



Bayesian filter with dependencies

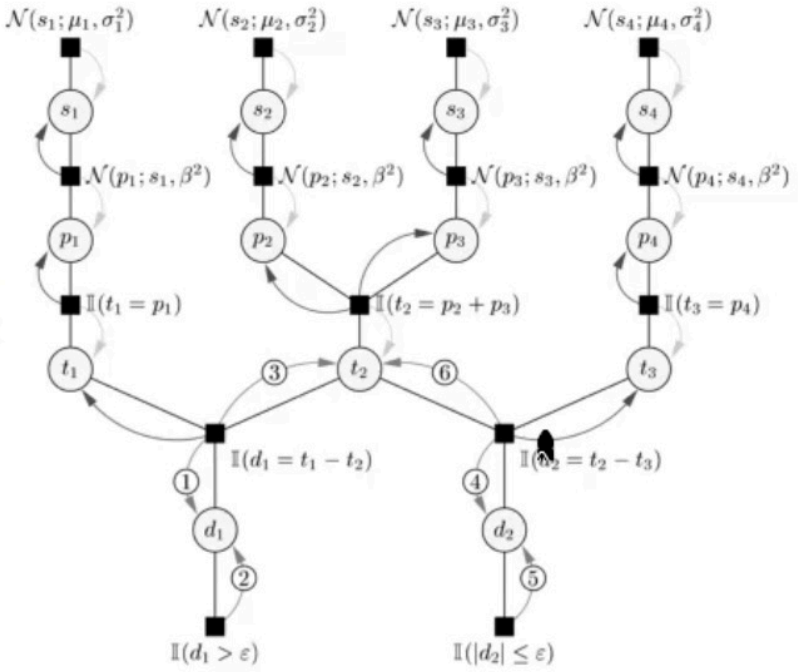
Example 5: MS Office Assistant



Example 6: Gaming

Microsoft's TrueSkill™: player skill rating

"planet-scale" application of Bayesian inference



Example 7: Autonomous Navigation

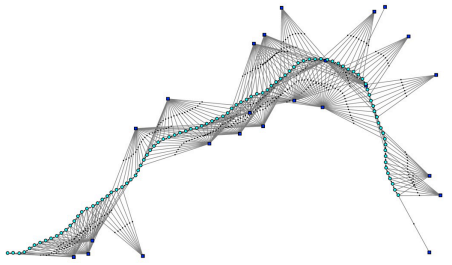
- **Data:** Stream of measurements from heterogeneous sensors




- **Goal:** Infer navigation information, objects in the environment, the floorplan, …

- Probabilistic reasoning and graphical models make it possible to consistently combine (``fuse'') data across time and sensors and provides explainable solutions

- Graphical representations are useful for
  - modeling large-scale problems
  - perform scalable inference



Course Overview

- **Online Platform:** Lecture notes, homework assignments, further handouts, and links to Zoom meetings will be posted on the *Canvas calendar.*
- **Teaching Assistant:** Parthasarathi Khirwadkar (pkhirwad@ucsd.edu)
- **Class Schedule:**
  - Lectures are Tuesdays and Thursdays 12:30PM – 1:50PM in WLH 2205
  - Discussion sessions are Mondays 12:00 – 12:50 in PETER 102
- **Office Hours:** Office hours are every *Friday at 3 PM via Zoom or in-person by appointment*
- **Grades:** Homework 30%, mid-term exam 30%, and final exam 40%

Homework

- **Homework assignments** and **their solution** will be posted approximately every 2-3 weeks on Canvas and will be due one week later. A scan or a high resolution photos of the solution must be uploaded to Canvas
- Collaborations are encouraged but the developed solutions you hand in should reflect your own understanding of the course material
- Homework is **graded using an "A for effort" scheme**. Individual homework problems are not corrected. Points are assigned proportionally to the percentage of work done
- Because homework is not graded for correctness, students must read the solutions, to determine if they performed the homework correctly or not

Exams

- The **mid-term and final exams** are graded in the traditional manner
- Both exams are closed book and notes
- Only nonprogrammable calculators can be used; all other electronic storage and communication devices are banned
- Cheating will result in penalties

Bibliography

- *Bayesian Reasoning & Machine Learning,* David Barber, Cambridge Univ. Press, 2012.

Lecture notes slides, and additional references will be posted on the Canvas website of the course.

Questions?