SIO 209: Signal Processing for Ocean Sciences
Class 4

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Homomorphic Signal Processing

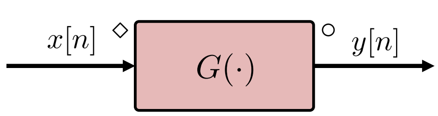
- Let us recall the additive superposition of two signals and the multiplication by a scalar of a signal prior to linear transformation $T(\cdot)$, i.e.,

$$T\big(x_1[n] + x_2[n]\big) = T\big(x_1[n]\big) + T\big(x_2[n]\big) \qquad T\big(cx_1[n]\big) = cT\big(x_1[n]\big)$$

- In homomorphic signal processing these operations from linear system theory are generalized by a general superposition and multiplication operators applied before and after the ***homomorphic transformation*** $G(\cdot)$, i.e.,

$$G\big(x_1[n] \diamond x_2[n]\big) = G\big(x_1[n]\big) \circ G\big(x_2[n]\big) \qquad G\big(c \odot x_1[n]\big) = c \star G\big(x_1[n]\big)$$

general superposition *general scaling*



see also Chapter 13 in
Oppenheim & Schafer, 2009

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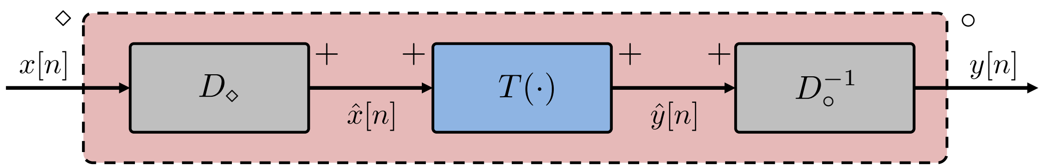
Homomorphic Signal Processing

- The system transformation $G(\cdot)$ is an algebraically linear transformation from the input vector space to the output vector space
- All homomorphic systems can be represented as a cascade of three systems; this is also known as the canonic representation



$D_\diamond$ is the characteristic system for the operation $\diamond$
$D_\circ$ is the characteristic system for the operation $\circ$
$T(\cdot)$ is a linear filter

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Homomorphic System – Multiplication

- The multiplicative superposition is defined as

conventional multiplication

$$x[n] = x_1[n]^\alpha \cdot x_2[n]^\beta$$

(Note that in a conventional additive superposition of a linear system reads $x[n] = \alpha x_1[n] + \beta x_2[n]$.)

- For multiplicative homomorphic systems we have $\circ = \diamond = \cdot$, and a characteristic system that satisfies

$$D_\cdot\big(x_1[n]^\alpha x_2[n]^\beta\big) = \alpha D_\cdot\big(x_1[n]\big) + \beta D_\cdot\big(x_2[n]\big)$$

  – the complex logarithm has the desired property
  – the inverse function is the exponential

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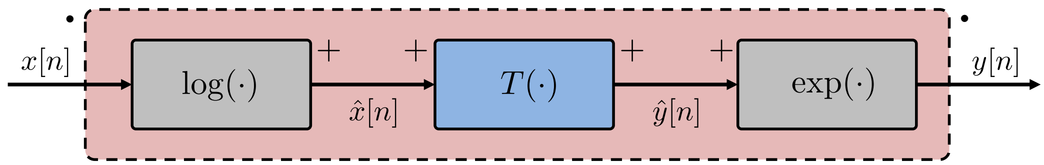
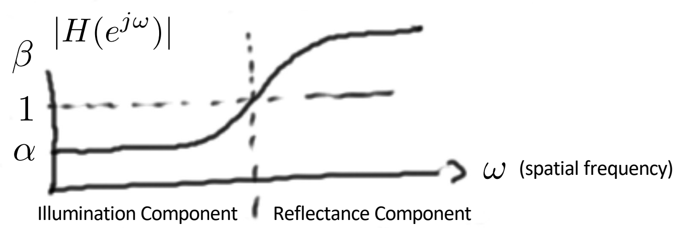
Example: Homomorphic Image Processing

- Image Model: $f(u,v) = f_{\mathrm{i}}(u,v) \cdot f_{\mathrm{r}}(u,v)$ ⟵ continuous function
  - u, v are continuous spatial coordinates
  - f_{i} is the illumination function
  - f_{r} is the reflectance function
- In digital image processing, we have $x(m,n) = f(m\Delta u, n\Delta v)$ ⟵ discrete function (2-D digital signal)
- The image model is thus given by $x(m,n) = x_{\mathrm{i}}(m,n) \cdot x_{\mathrm{r}}(m,n)$
- This model can be represented by a homomorphic system



Example: Homomorphic Image Processing

- If the illumination and reflectance functions are separable in $\hat{x}[n]$, meaningful processing can be done by the linear spatial filter
- In particular, assume that $x_{\mathrm{i}}(m,n)$ (and its logarithm) varies slowly across the scene, but $x_{\mathrm{r}}(m,n)$ (and its logarithm) varies rapidly
- We are interested in both contrast enhancement (i.e., increasing the dynamic range of reflectance) and range compression (i.e., decreasing the dynamic range of illumination)
- This could be done by a linear spatial filter with the following frequency response



Example: Homomorphic Image Processing

- Thus, conceptually there is the possibility of simultaneously enhancing contrast and reducing dynamic range, i.e.,

$$y(m,n) = x_{\mathrm{i}}(m,n)^{\alpha} \cdot x_{\mathrm{r}}(m,n)^{\beta}$$

where $\alpha < 1$ and $\beta > 1$

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Example: Homomorphic Image Processing

- Examples of an image enhanced by the homomorphic process. Note that, compared to the original image, the enhanced image has been sharpened while details in the dark interior of the building have been made more visible.

Original Image

Enhanced Image

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Homomorphic System – Convolution

- Recall convolution

$$x[n] = \sum_{k=-\infty}^{\infty} x_1[k]x_2[n-k] = x_1[n] * x_2[n]$$

- We consider homomorphic systems where $\circ = \diamond = *$, and a characteristic system that satisfies

$$D_*\big(x_1[n] * x_2[n]\big) = D_*\big(x_1[n]\big) + D_*\big(x_2[n]\big)$$

- A characteristic system with this property can be obtained as

$$D_* = \Gamma^{-1}\big\{\ln\big(\Gamma\{\cdot\}\big)\big\}$$

where $D_* = \Gamma\{\cdot\}$ denotes the z-transform (or Fourier transform)

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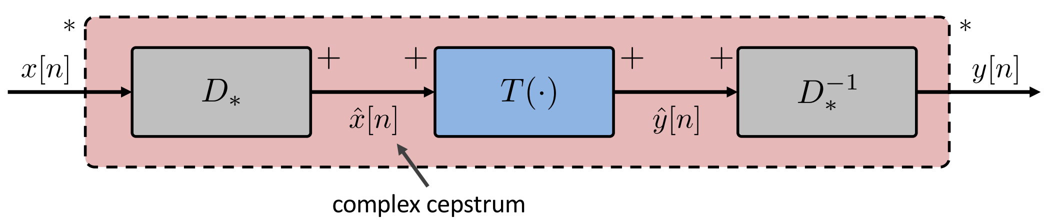
Homomorphic System – Convolution

- The homomorphic system is thus obtained as



where $D_* = \Gamma^{-1}\big\{\ln\big(\Gamma\{\cdot\}\big)\big\}$

convolution $\rightarrow$ multiplication

multiplication $\rightarrow$ addition

addition $\rightarrow$ addition

Note that before the $\ln(\cdot)$ is computed, the phase has to be unwrapped

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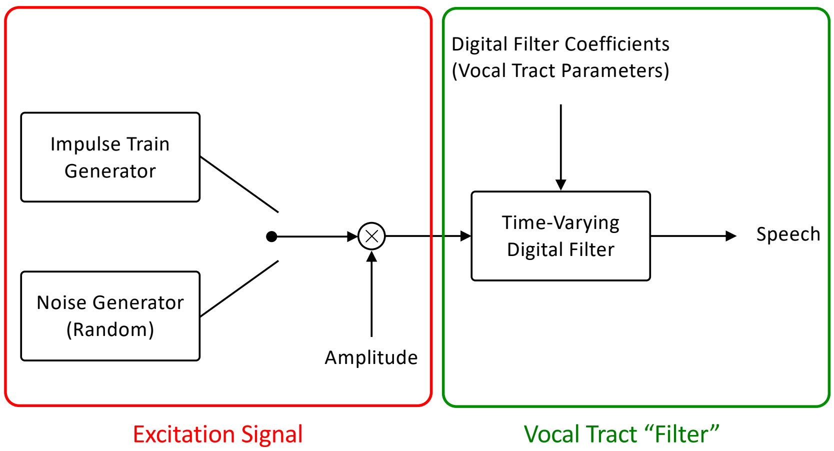
Example: Homomorphic Speech Processing

- Model of speech production



A vocoder extracts parameters of excitation signal and the vocal tract filter; transmitting these parameters (instead of the sampled speech signal) can reduce data rate by a factor of 20

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Example: Homomorphic Speech Processing

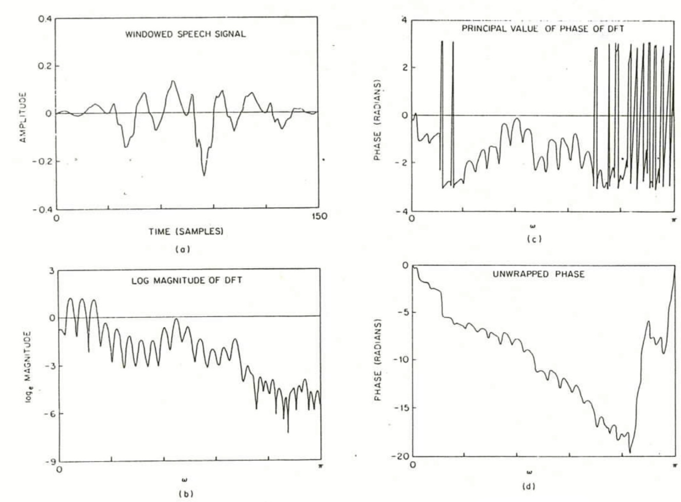
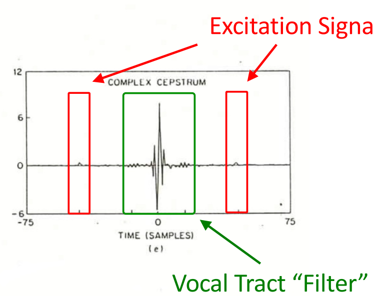


Fig. 7.8 Homomorphic analysis of voiced speech; (a) windowed time waveform; (b) log of magnitude of short-time Fourier transform; (c) principal value of phase; (d) "unwrapped" phase; (e) complex cepstrum.



Vocal tract and excitation signal are separable in complex cepstrum domain!

L. R. Rabiner and R. W. Schafer, *Digital Processing of Speech Signals,* Prentice-Hall, 1978.

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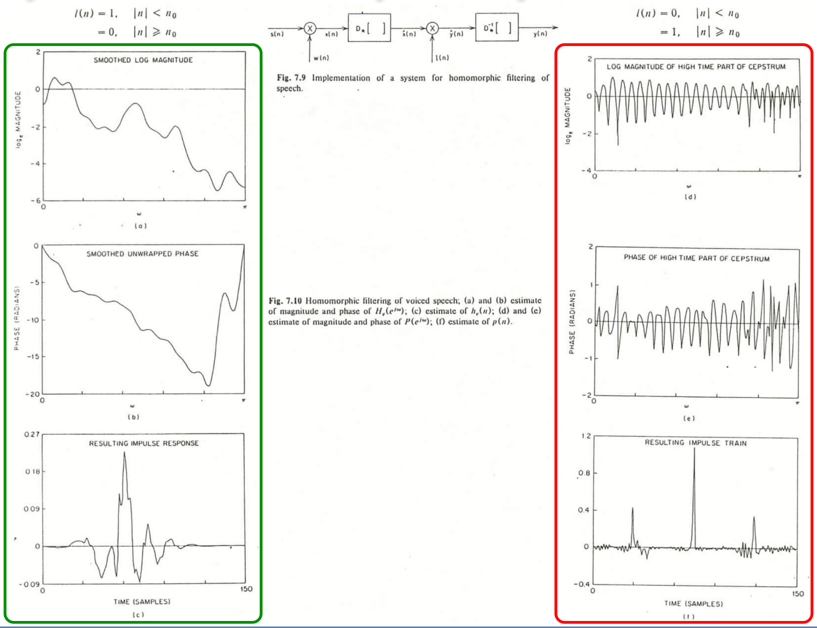
Example: Homomorphic Speech Processing

Vocal Tract “Filter”



Excitation Signal

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