SIO 207A: Fundamentals of Digital Signal Processing
Class 18

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Recap: Multiplication in Time Domain

- Multiplication

$$y[n] = x_1[n]x_2[n] \qquad Y(z) = \sum_{n=\infty}^{\infty} x_1[n]x_2[n]z^{-n}$$

- Multiplication in time domain is convolution is frequency domain, i.e.,

$$Y(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_2(e^{j\omega'})X_1(e^{j(\omega-\omega')})\mathrm{d}\omega'$$

- *Note:*

1. This is a periodic or circular convolution (not linear)

$$Y(e^{j\omega}) = X_1(e^{j\omega}) \oplus X_2(e^{j\omega})$$

2. Think of circular convolution as a divided cylinder with Fourier transforms pointed on them

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Examples

- Multiplication by a^n

$$x_1[n] = a^n x[n]$$

$$X_1(z) = X(a^{-1}z)$$

$$\begin{aligned} X_1(z) &= \sum_{n=-\infty}^{\infty} \left(a^n x[n]\right) z^{-n} \\ &= \sum_{n=-\infty}^{\infty} x[n] \left(a^{-1}z\right)^{-n} \quad \text{let } z' = a^{-1}z \\ &= \sum_{n=-\infty}^{\infty} x[n] z'^{-n} \\ &= X(z') \end{aligned}$$

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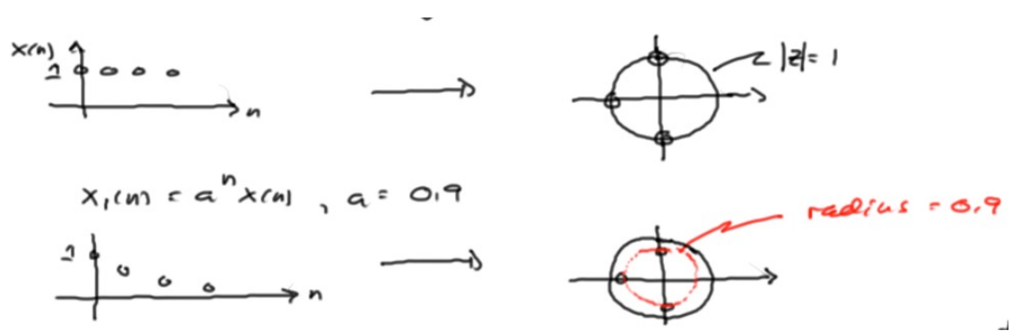
Examples

- Let a be real and positive with $a \leq 1$
- This has the effect of drawing the roots inward on radial paths



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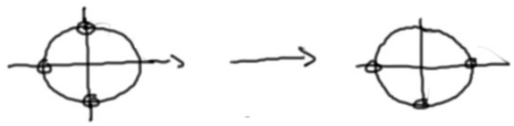
Examples

- Let a be complex and on the unit circle, i.e., $a = e^{j\omega_c}$
- This has the effect of rotating the original z-transform

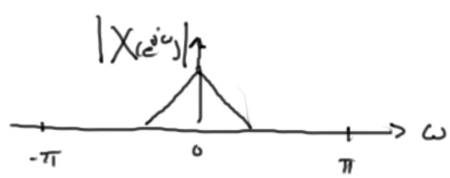
$x_1[n] = a^n x[n] = e^{j\omega n} x[n]$



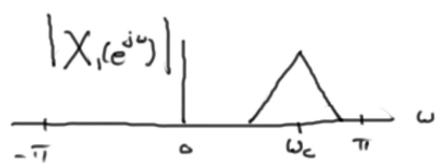
$a = e^{j\frac{\pi}{2}}$

Recap: Modulation

$x[n]$ is a low pass audio process

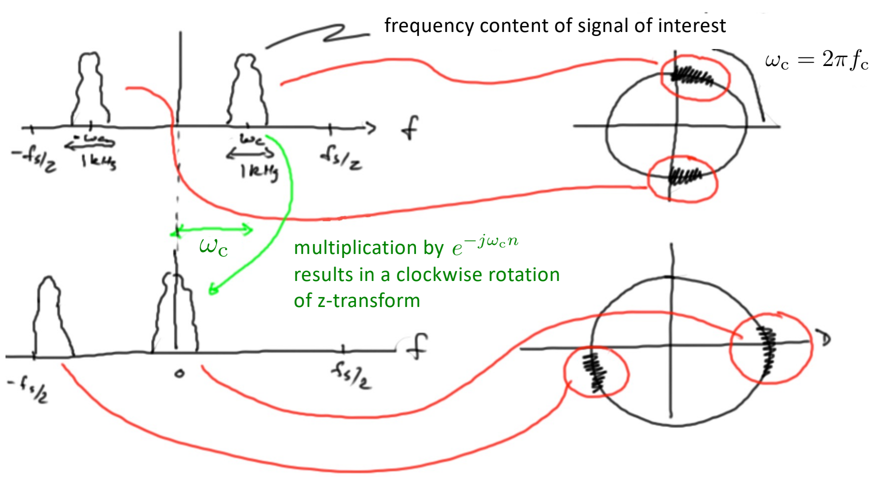


$x_1[n] = a^n x[n]$ where $a = e^{j\omega_c}$



amplitude modulation $x_1[n] = \cos(\omega_c n) x[n]$

Recap: Complex Basebanding



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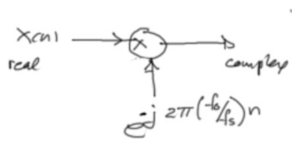
End Term Project: Complex Basebanding

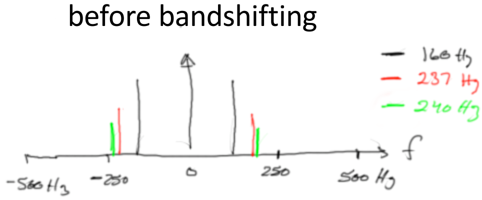


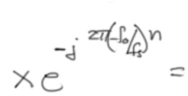
Bandshifting: Recall that multiplication by $e^{-j2\pi(f_0/f_s)n}$ results in a clockwise rotation of the z-transform (see class 4)

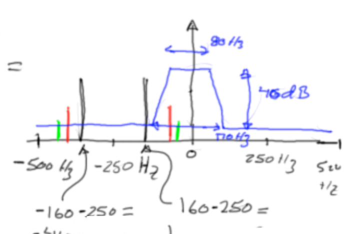
$$x[n]e^{-j2\pi f_0/f_s n}$$
$$= x[n]\cos 2\pi f_0/f_s n$$
$$- jx[n]\sin 2\pi f_0/f_s n$$

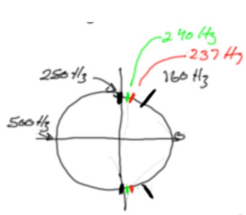
before bandshifting

after bandshifting

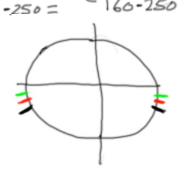








$$f_s = 1\text{kHz}$$
$$f_0 = 250\text{Hz}$$



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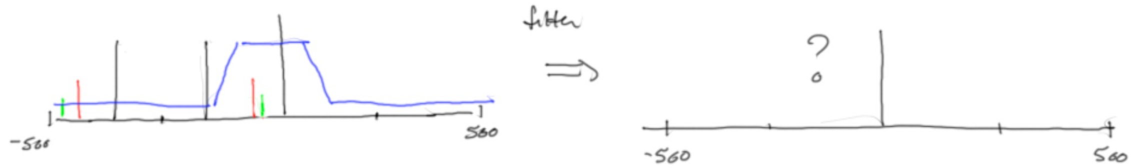
End Term Project: Low-Pass Filter



- Subsequently, desample the low-pass filtered complex bandshifted sequence so that $f_s' = f_s/8 = 125\text{Hz}$

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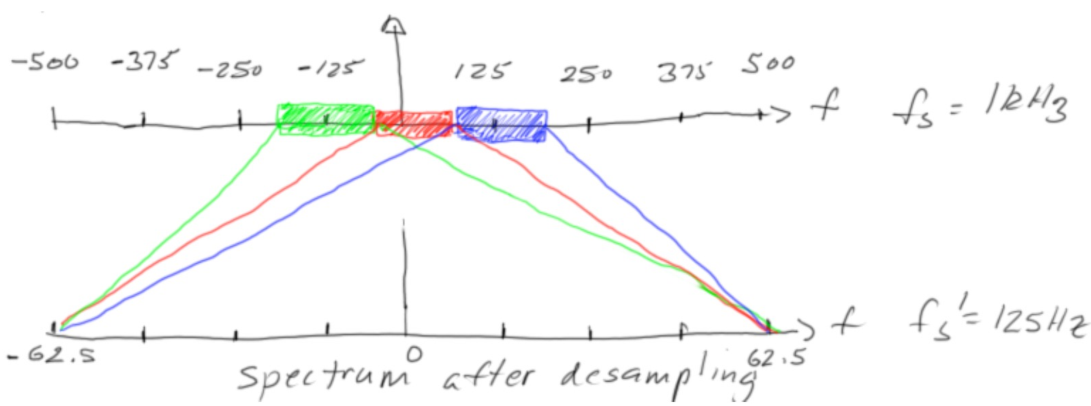
End Term Project: Mapping After Desampling

- Spectrum of complex bandshifted signal output of LPF prior to desampling



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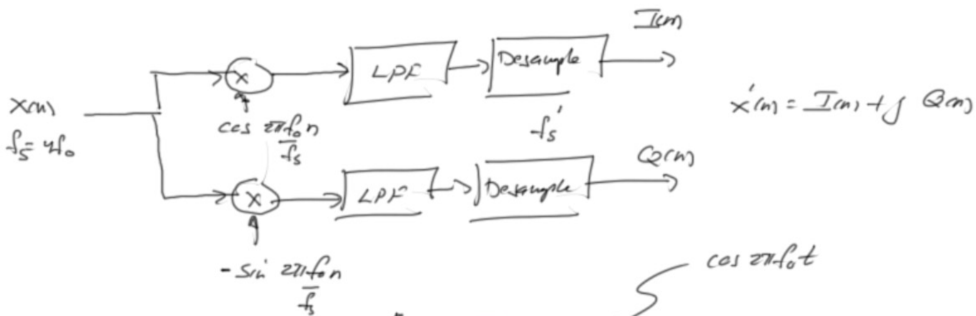
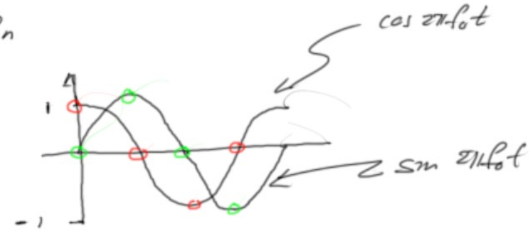
Related Topics: Hardware Simplifications

- Sampling at $4f_0 = f_s$



→ multiplication by $\cos 2\pi f_0/f_s n$ or $\sin 2\pi f_0/f_s n$ is either a change of sign or a multiplication of the input value by zero



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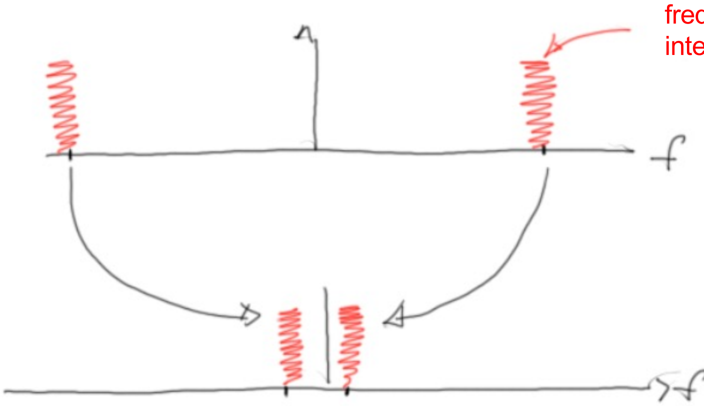
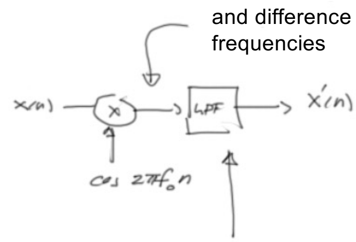
Related Topics: Real Bandshifting

- Heterodyne procedure



Downshift to an intermediate frequency (IF)



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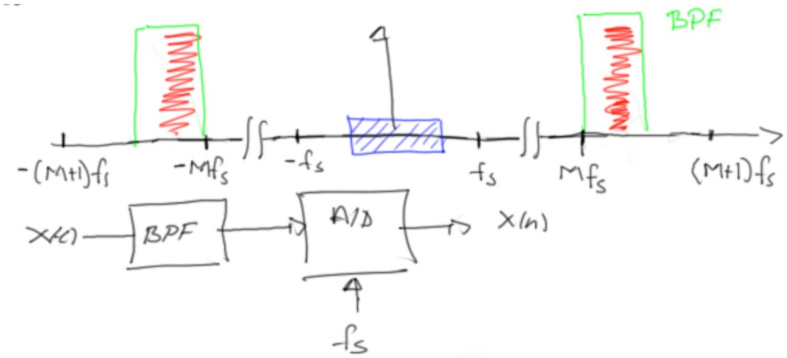
Related Topics: Bandpass Sampling

- Bandpass sampling



$x[n]$ will consist of all the aliased components take from $x(t)$ over subbands of width f_s. Thus the high frequency band of interest will be aliased into the lower band defined by $\boxed{-f_s/2 \text{ to } f_s/2}$

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