Implementation of Filters

Start with the difference equation

$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$

Corresponding transfer function

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

Hardware Requirements

- Adders
- Multipliers
- Storage

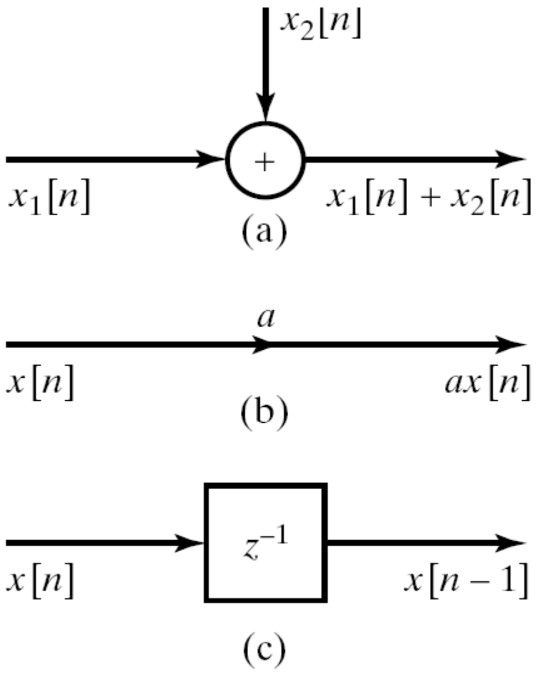
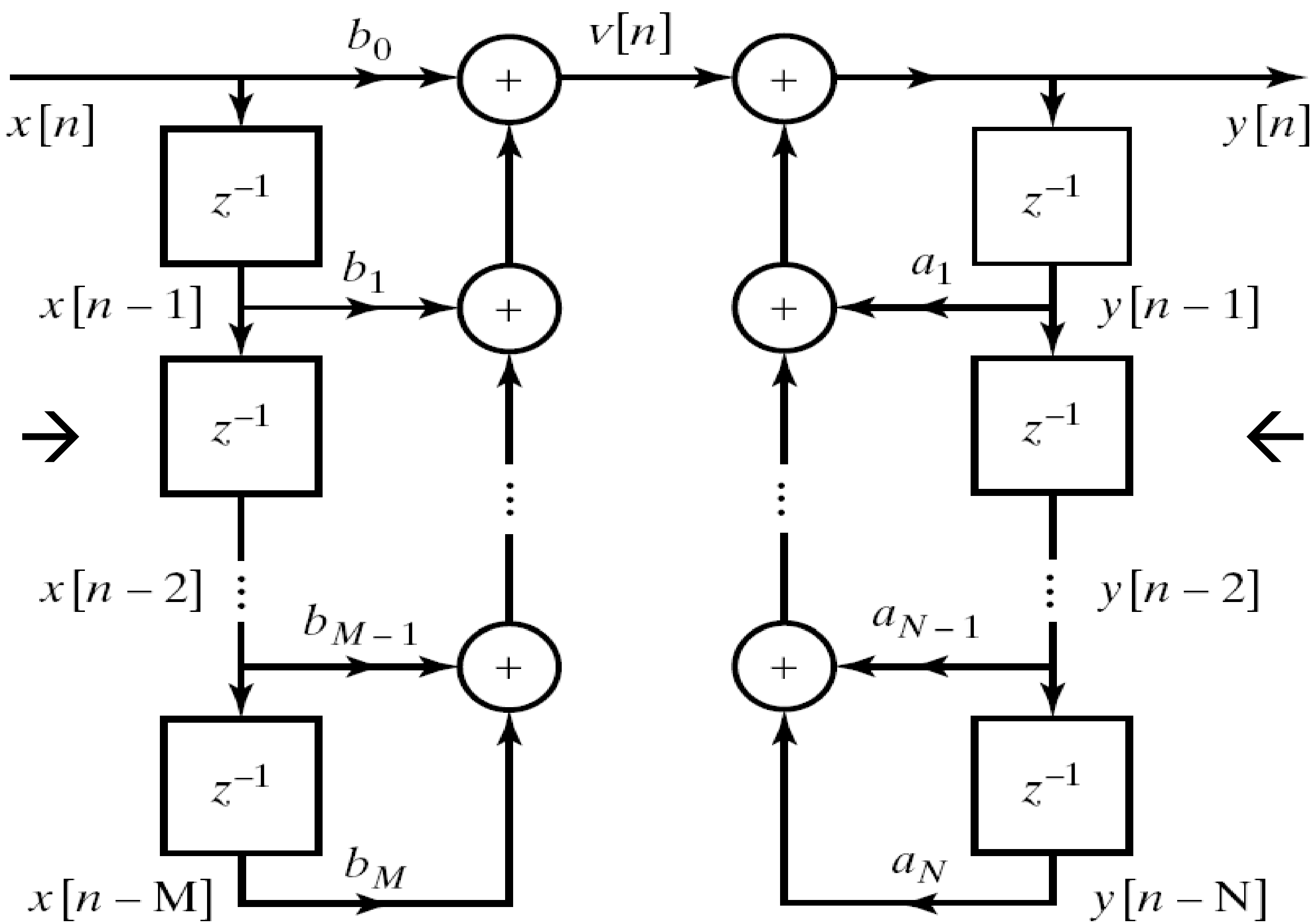


Figure 6.1 Block diagram symbols. (a) Addition of two sequences. (b) Multiplication of a sequence by a constant. (c) Unit delay.

Direct Form I

$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$



B(z) →   ← 1/A(z)

Direct Form II

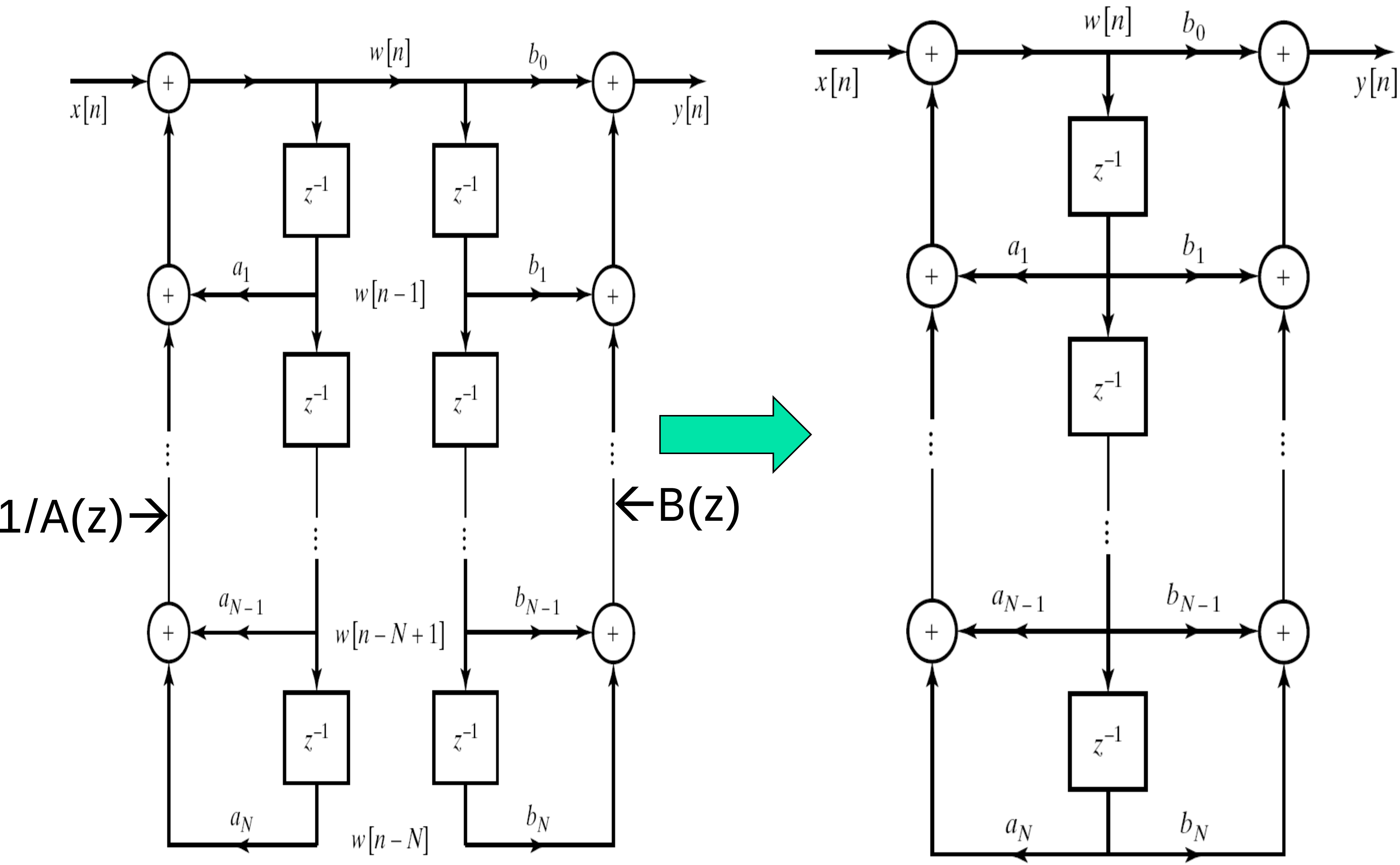
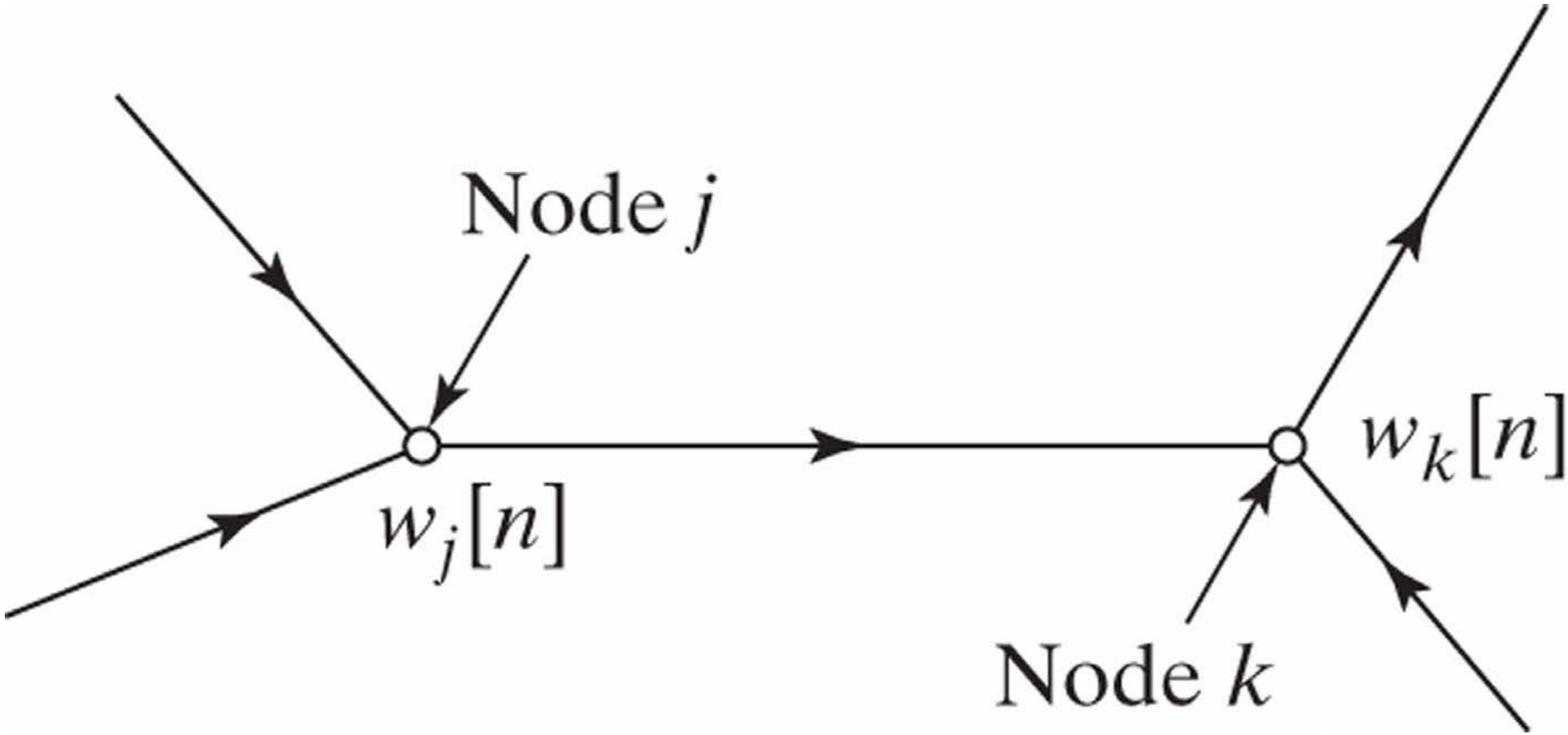


Figure 6.8 Example of nodes and branches in a signal flow graph.



Signal Flow Graph: 1) a network of directed branches that connect at nodes. 2) Each node is associated with a variable. 3) Each branch has an input value and an output values.

Figure 6.9 Example of a signal flow graph showing source and sink nodes.

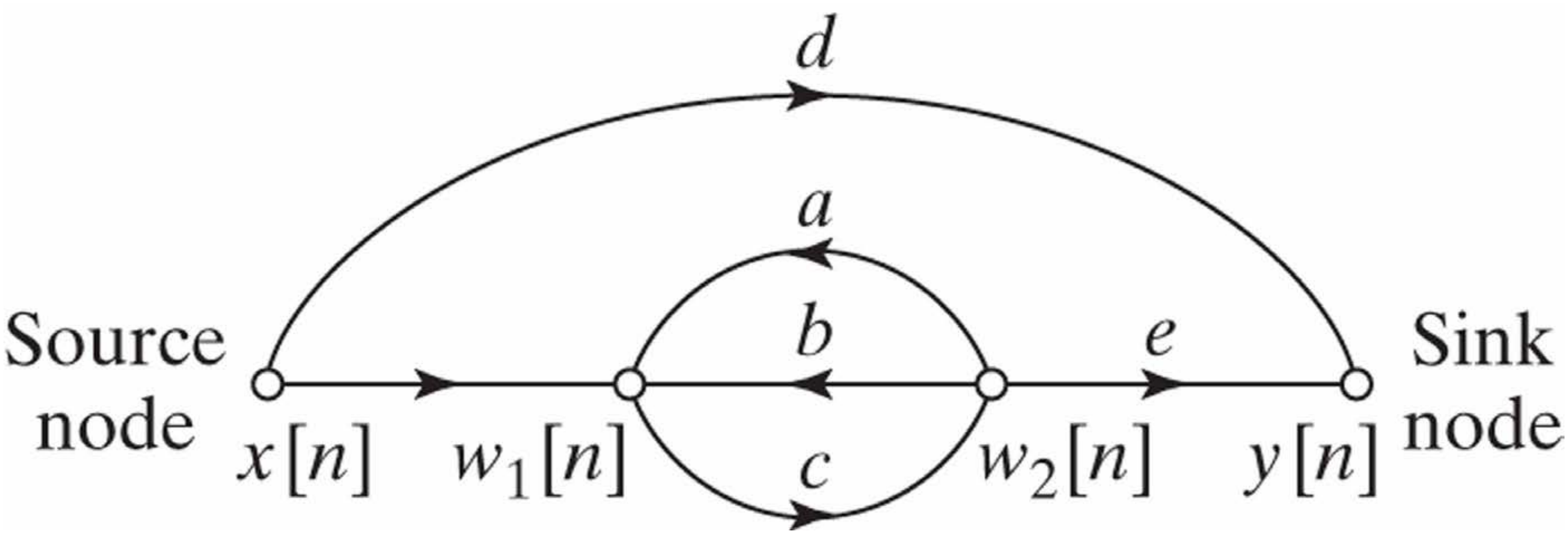


Figure 6.10 (a) Block diagram representation of a 1st-order digital filter. (b) Structure of the signal flow graph corresponding to the block diagram in (a).

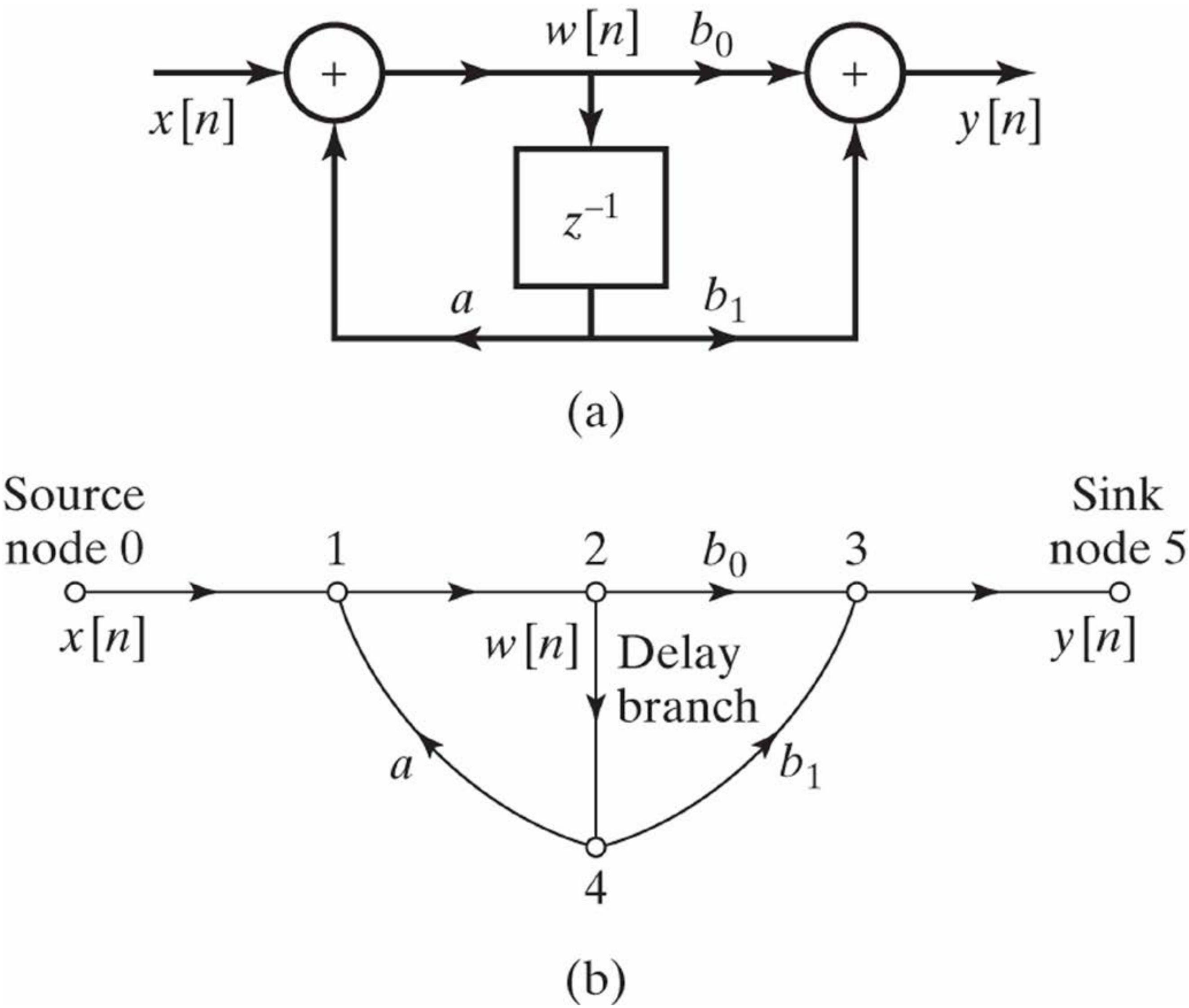
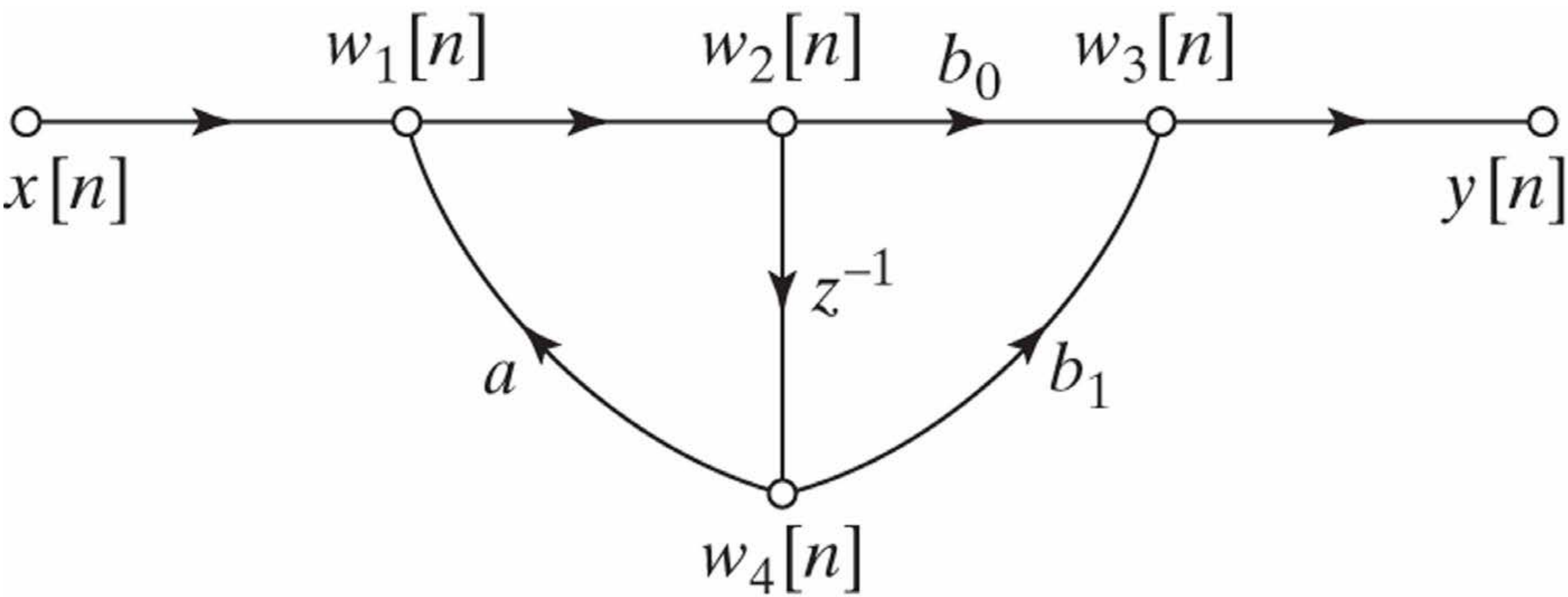
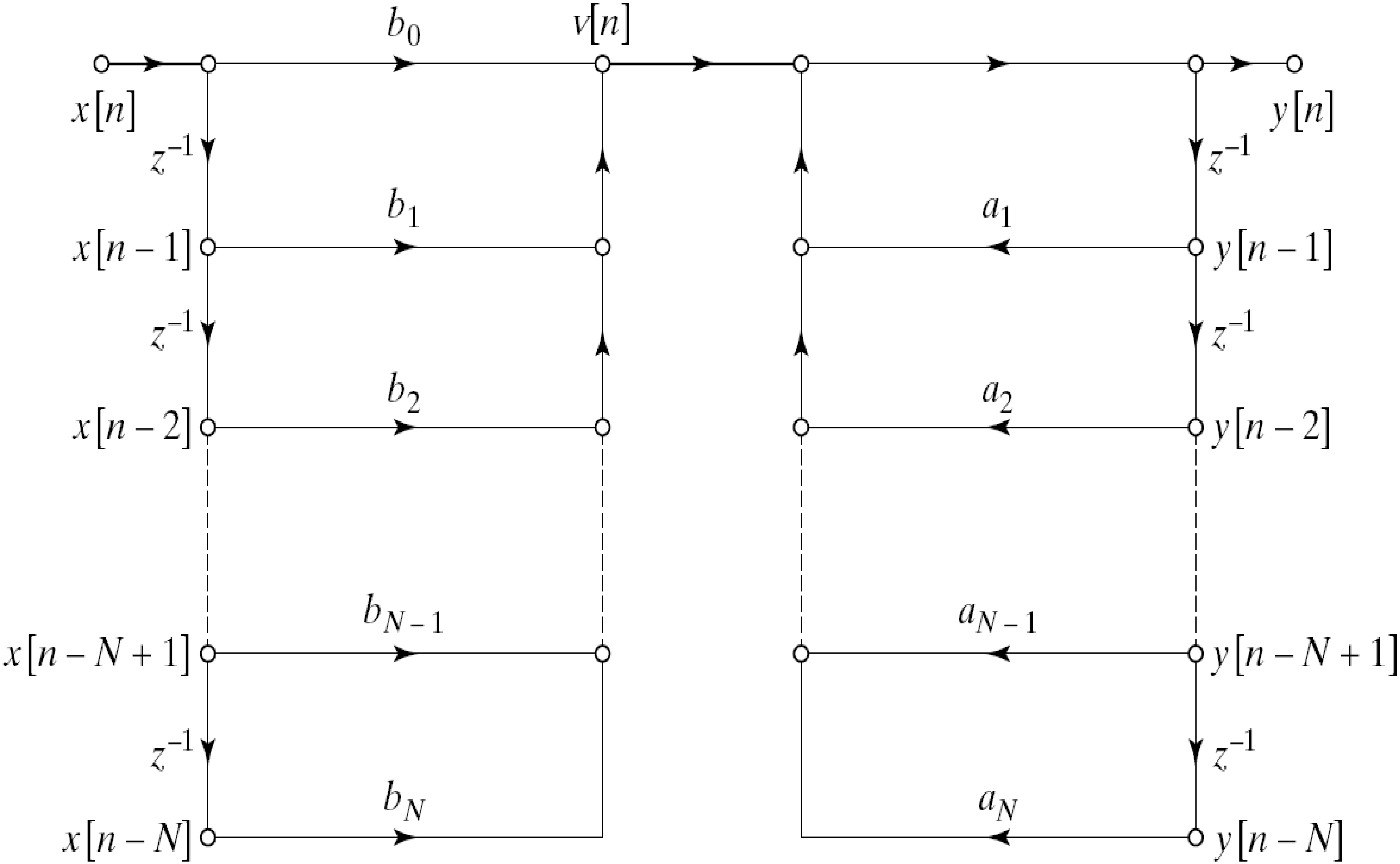


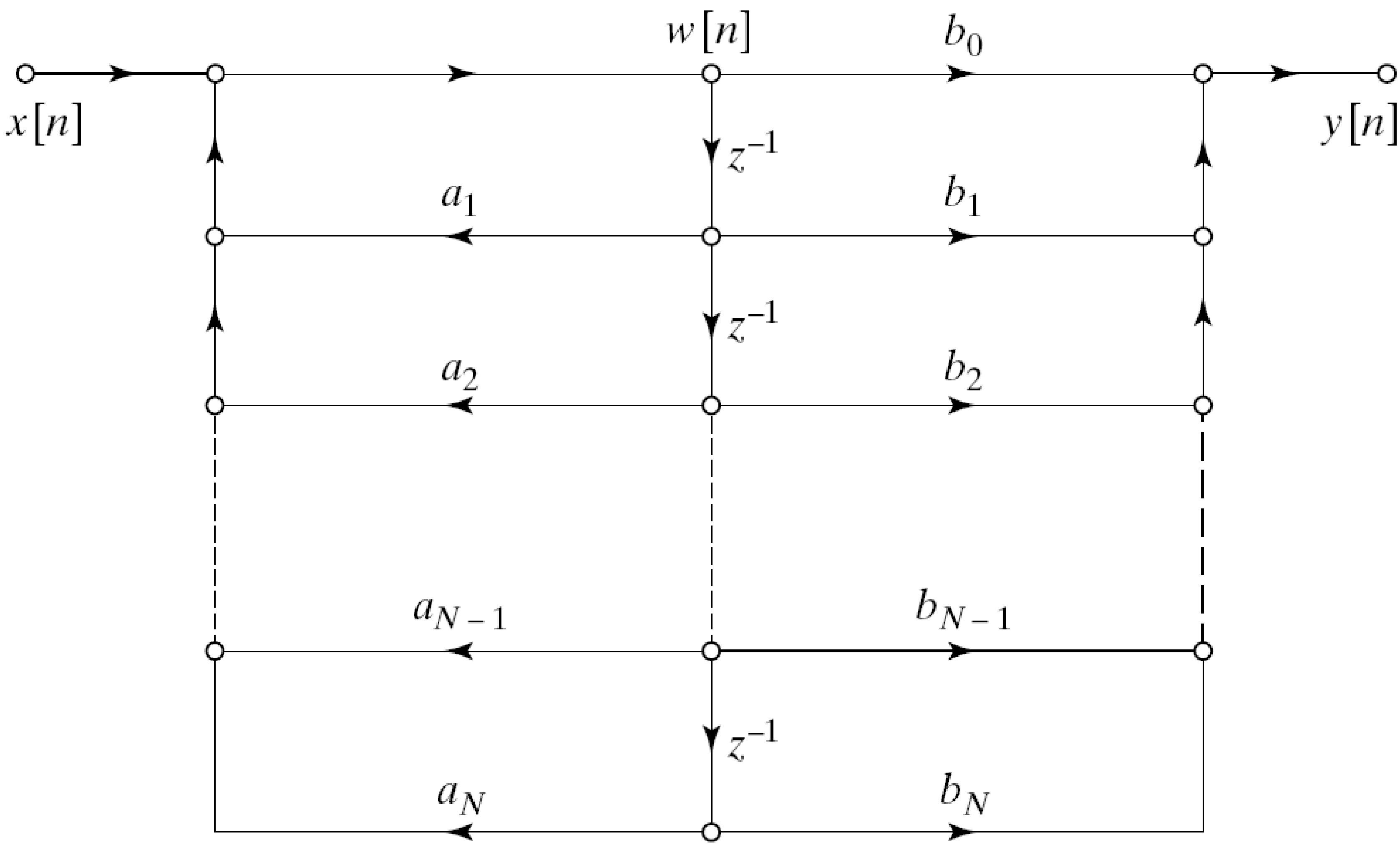
Figure 6.11 Signal flow graph of Figure 6.10(b) with the delay branch indicated by z^{-1}.



Direct Form I (Signal Flow Graph)



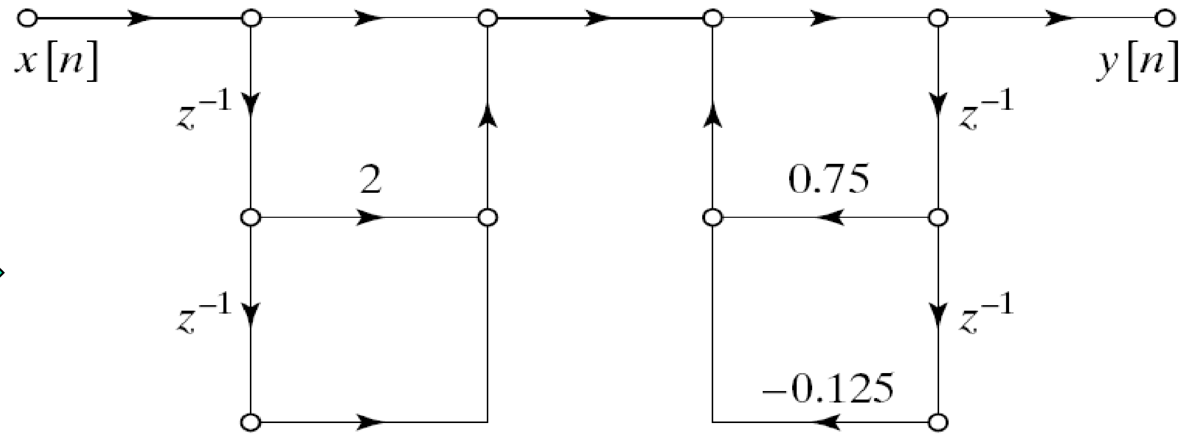
Direct Form II (Signal Flow Graph)



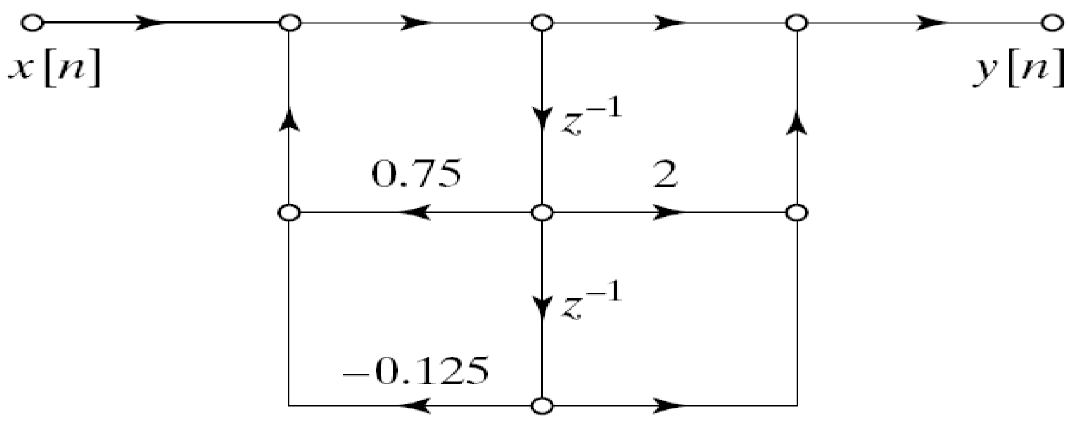
Example:

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - .75z^{-1} + .125z^{-2}}$$

Direct Form I



Direct Form II



Transposed Form

1. Reverse the direction of all branches
2. Interchange input and Output

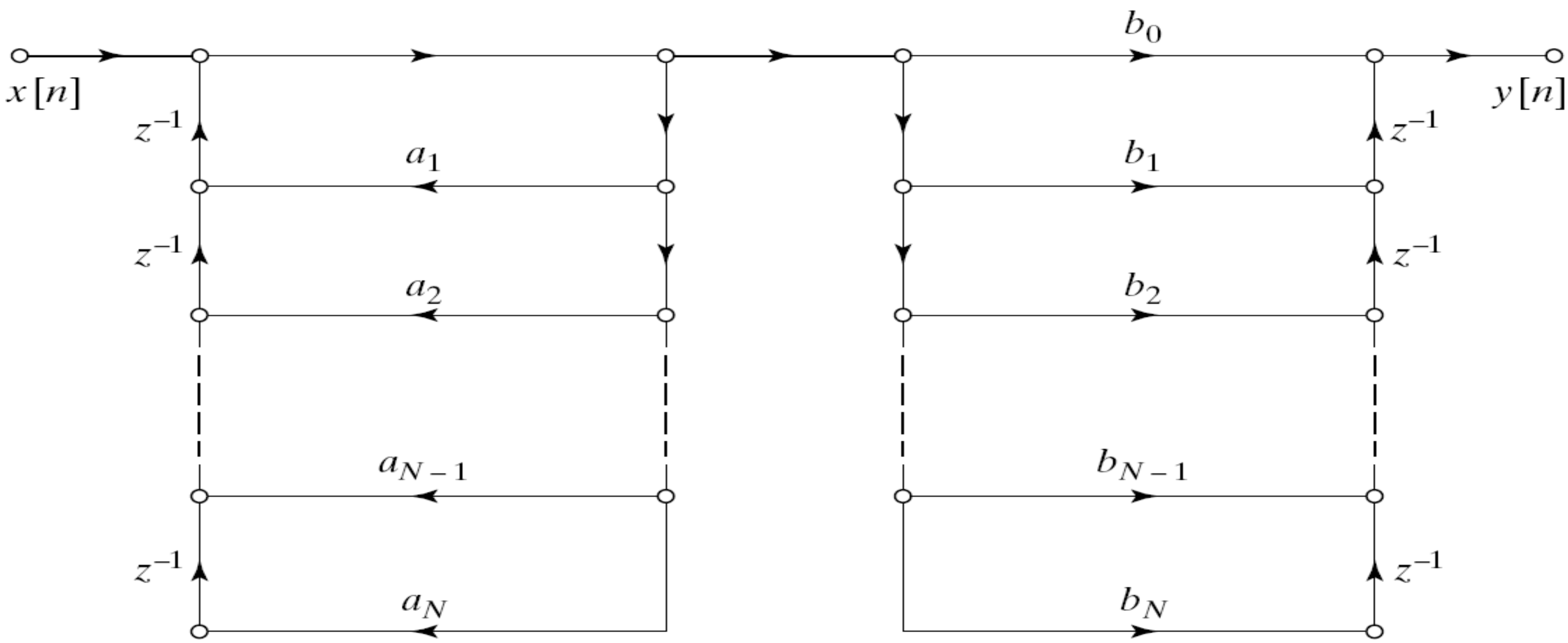


Figure 6.29 General flow graph resulting from applying the transposition theorem to the direct form I structure of Figure 6.14.

Finite Precision Arithmetic

- Coefficient Quantization
  - Representing the coefficients of the filters using a finite number of bits and the related degradation in filter transfer function

- Roundoff Noise
  - Relates to the errors resulting from the arithmetic operations carried out in finite precision
  - Nature of the errors depend on fixed point arithmetic versus floating point arithmetic

Coefficient Quantization

$$\hat{H}(z) = \frac{\sum_{k=0}^{M} \hat{b}_k z^{-k}}{1 - \sum_{k=1}^{N} \hat{a}_k z^{-k}}$$

- Where $\hat{a}_k = a_k + \Delta a_k$ and $\hat{b}_k = b_k + \Delta b_k$
- For coefficient sensitivity, we need to understand how errors in coefficients affect the zeros and poles

$$A(z) = 1 - \sum_{k=1}^{N} a_k z^{-k} = \prod_{l=1}^{N} (1 - z_l z^{-1})$$

$$\Delta z_i = \sum_{k=1}^{N} \frac{\partial z_i}{\partial a_k} \Delta a_k, i = 1,2,..N$$

Coefficient Quantization Cont'd

$$\frac{\partial z_i}{\partial a_k} = \frac{z_i}{\prod_{j=1,\, j\neq i}^{N} (z_i - z_j)}$$

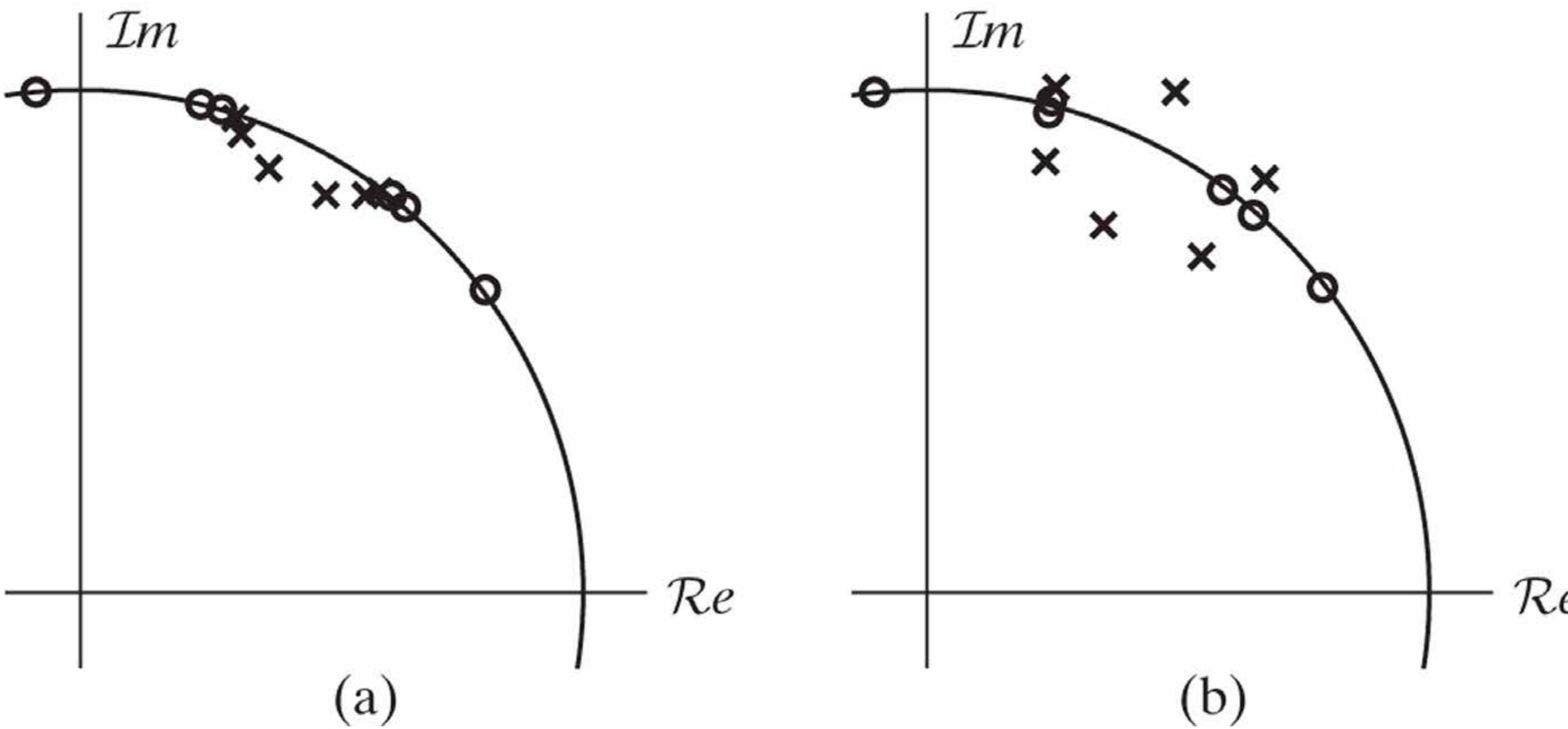
Conclusion: Closely spaced roots is a problem. Small perturbation in coefficients can cause large perturbation of roots.

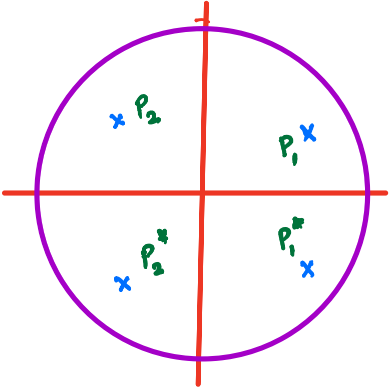
Table 6.1 UNQUANTIZED DIRECT-FORM COEFFICIENTS FOR A 12TH-ORDER ELLIPTIC FILTER

TABLE 6.1 UNQUANTIZED DIRECT-FORM COEFFICIENTS FOR A 12TH-ORDER ELLIPTIC FILTER

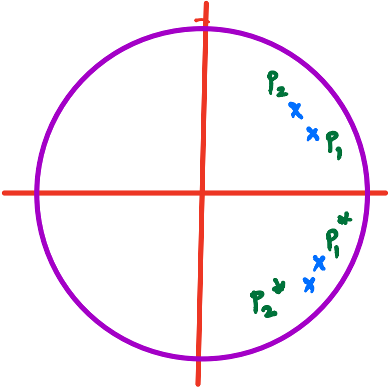
k	b_k	a_k
0	0.01075998066934	1.00000000000000
1	-0.05308642937079	-5.22581881365349
2	0.16220359377307	16.78472670299535
3	-0.34568964826145	-36.88325765883139
4	0.57751602647909	62.39704677556246
5	-0.77113336470234	-82.65403268814103
6	0.85093484466974	88.67462886449437
7	-0.77113336470234	-76.47294840588104
8	0.57751602647909	53.41004513122380
9	-0.34568964826145	-29.20227549870331
10	0.16220359377307	12.29074563512827
11	-0.05308642937079	-3.53766014466313
12	0.01075998066934	0.62628586102551

Figure 6.48 IIR coefficient quantization example. (a) Poles and zeros of $H(z)$ for unquantized coefficients. (b) Poles and zeros for 16-bit quantization of the direct form coefficients. (Bandpass Filter)

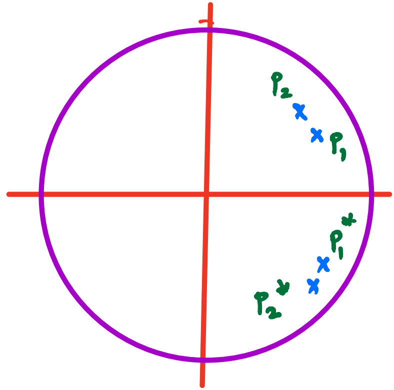




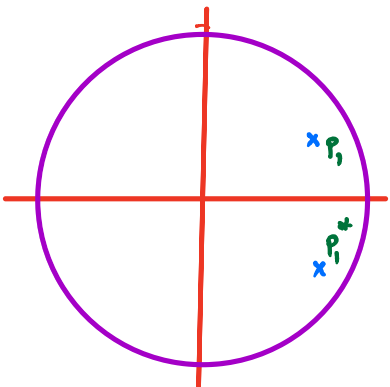
Filter 1



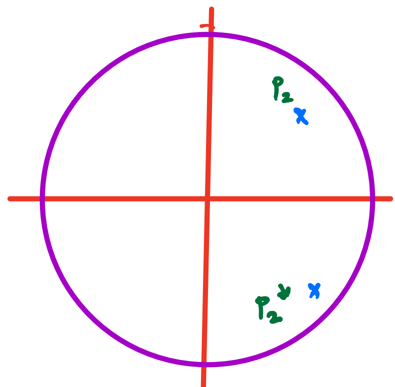
Filter 2



Filter 2



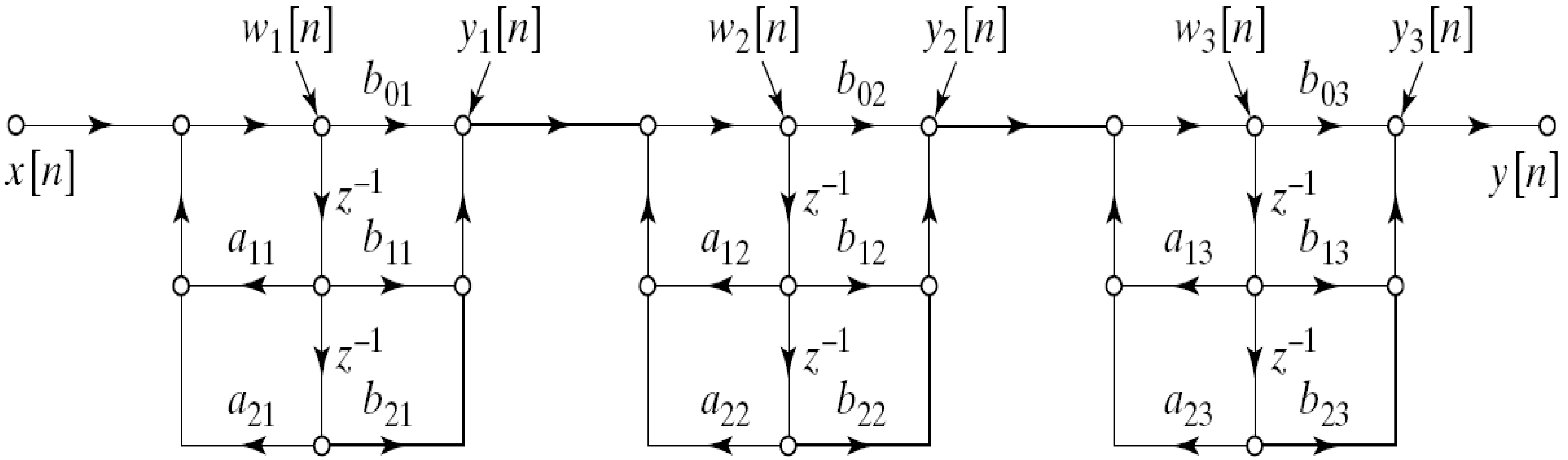
Implement Pole 1



Implement Pole 2

Cascade Form

$$H(z)=\prod_{k=1}^{N}\frac{b_{0k}+b_{1k}z^{-1}+b_{2k}z^{-2}}{1-a_{1k}z^{-1}-a_{2k}z^{-2}}$$



Parallel Form

$$H(z) = \sum_{k=0}^{N_p} C_k z^{-k} + \sum_{k=0}^{N_s} \frac{e_{0k} + e_{1k} z^{-1}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$

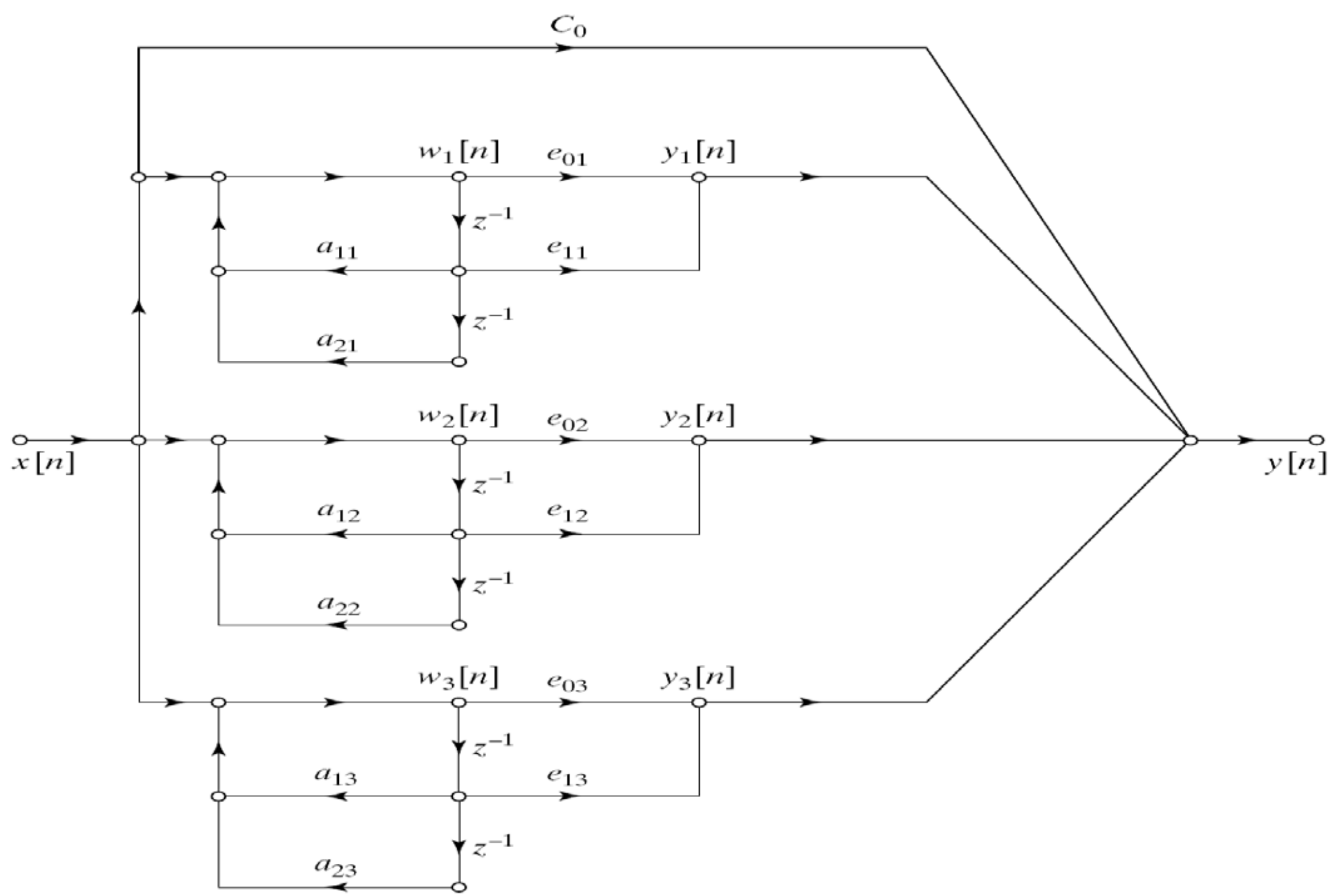
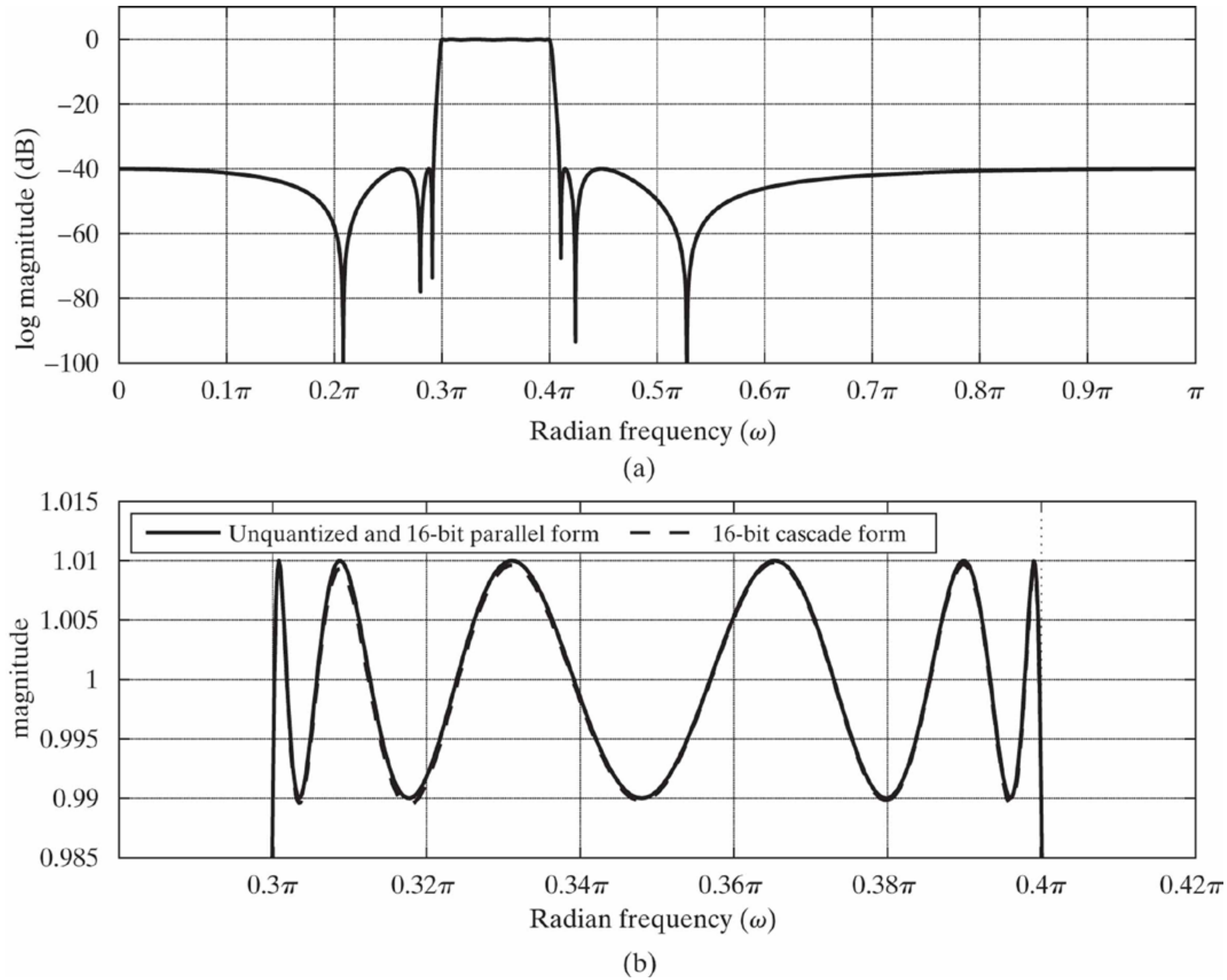
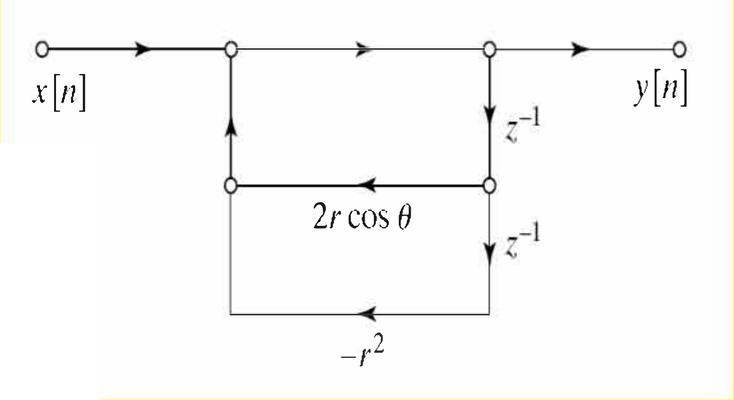
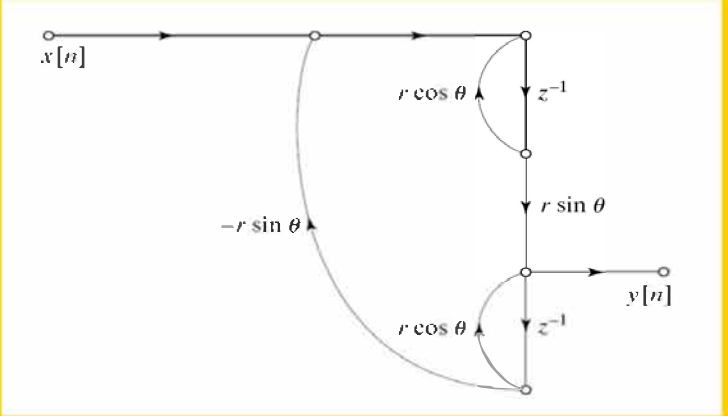


Figure 6.47 IIR coefficient quantization example. (a) Log magnitude for unquantized elliptic bandpass filter. (b) Magnitude in passband for unquantized (solid line) and 16-bit quantized cascade form (dashed line).







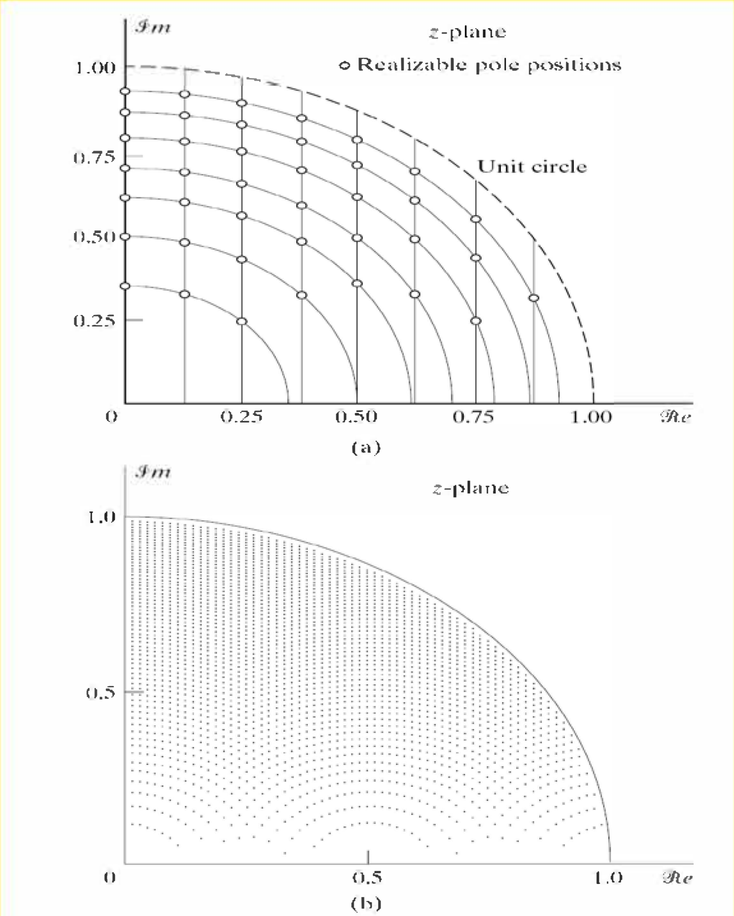


Figure 6.42 Pole-locations for the second-order IIR direct-form system of Figure 6.41. (a) Four-bit quantization of coefficients. (b) Seven-bit quantization.