SIO 207A: Fundamentals of Digital Signal Processing
Class 8

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DFT / FFT Bins

$$X(k) = \sum_{n=0}^{N-1} x[n]\, e^{-j\underbrace{\boxed{\frac{2\pi}{N}nk}}_{\omega_k n}} \qquad \omega_k = \frac{2\pi}{N}k$$

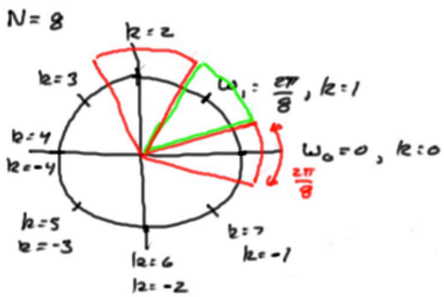
spacing in frequency domain

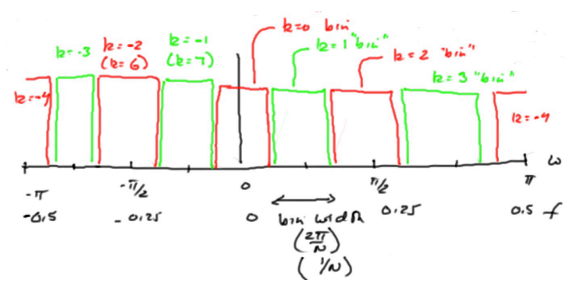
integer k $\quad \{0, \ldots, N-1\}$

"bin index" $\quad \{-N/2, \ldots, 0, \ldots, N/2-1\}$

each "bin" covers a bandwidth equal to $2\pi/N \;\; \mathrm{rad/samples}$ (or $1/N \;\; \mathrm{cycles/samples}$)

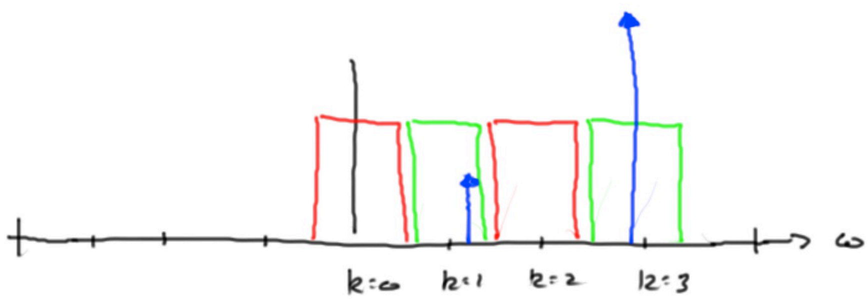
edges of bins are $\pm\pi/N \;\; \mathrm{rad/samples}$ from each center frequency



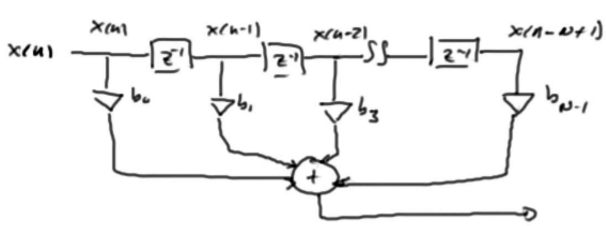


DFT / FFT Bins

- With "cookie cutter" viewpoint, there is no leakage of signal energy from one bin region with another, i.e., high level signal in bin region $k = 3$ will not show up in bin $k = 1$ after DFT – <u>not</u> true in practice



Interpretation of FFT/DFT Bin Calculation as a Filter



DFT: $X(k) = \sum_{n=0}^{N-1} \boxed{w[n]} x[n] e^{-j\frac{2\pi}{N}nk}$

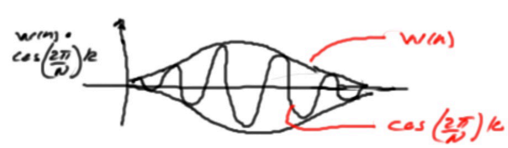
window function

"b" coefficients are $w[n]e^{-j\frac{2\pi}{N}nk}$

$$y[n] = \sum_{i=0}^{N-1} b_i x[n-i]$$

$e^{-j\frac{2\pi}{N}nk}$ (complex exponential)

$$= \cos\frac{2\pi}{N}nk - j\sin\frac{2\pi}{N}nk$$



Real Impulse Response of the “FFT Filter”

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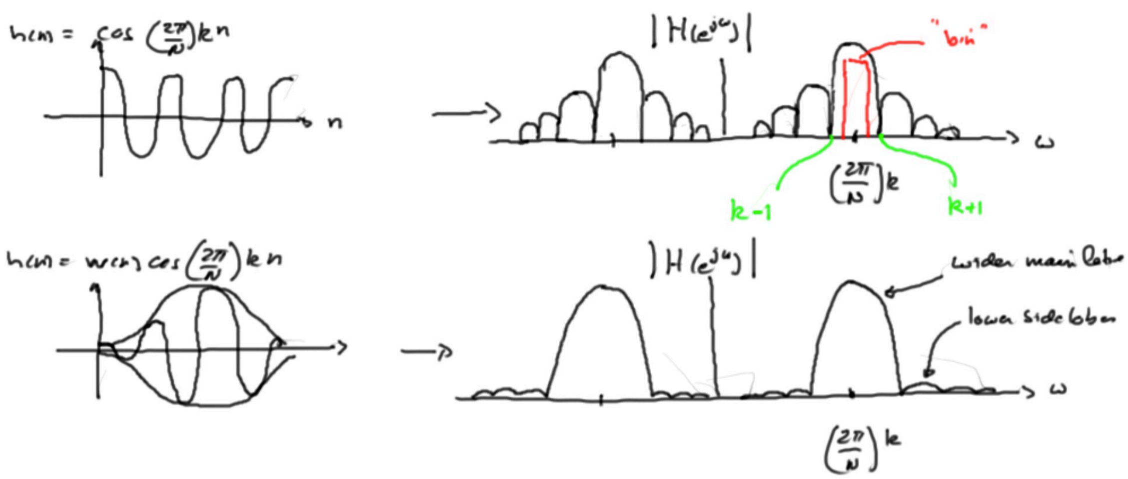
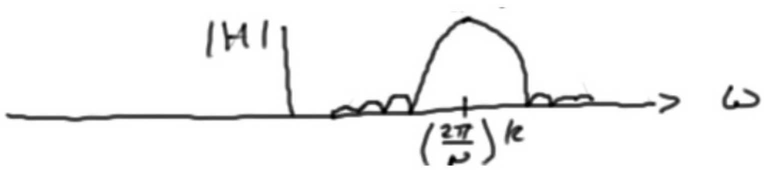


Complex Impulse Response of a Filter

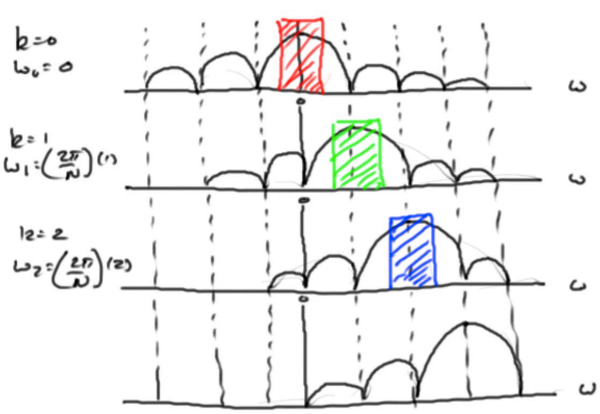
- From previous slide, each k defines a different center frequency
- In terms of complex exponentials defined in the DFT, the filter has a complex impulse response and corresponding transfer function that is one-sided



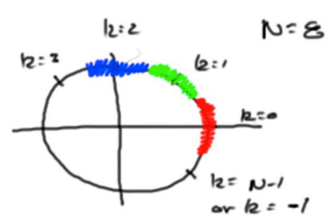
- FFT equivalent to a bank of complex bandpass filters

Interpretation of FFT/DFT as Bank of Filters

- Consider the frequency response of the entire set of complex filters defined by the DFT – in this case for a rectangular window function



Rectangular window
$w[n] = [1, 1, \ldots, 1]$

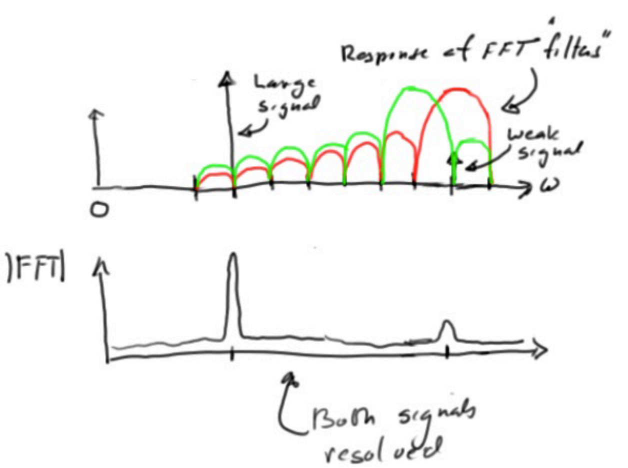


Spectral Leakage

- We consider a low frequency sinusoid much larger in amplitude than the higher frequency sinusoid

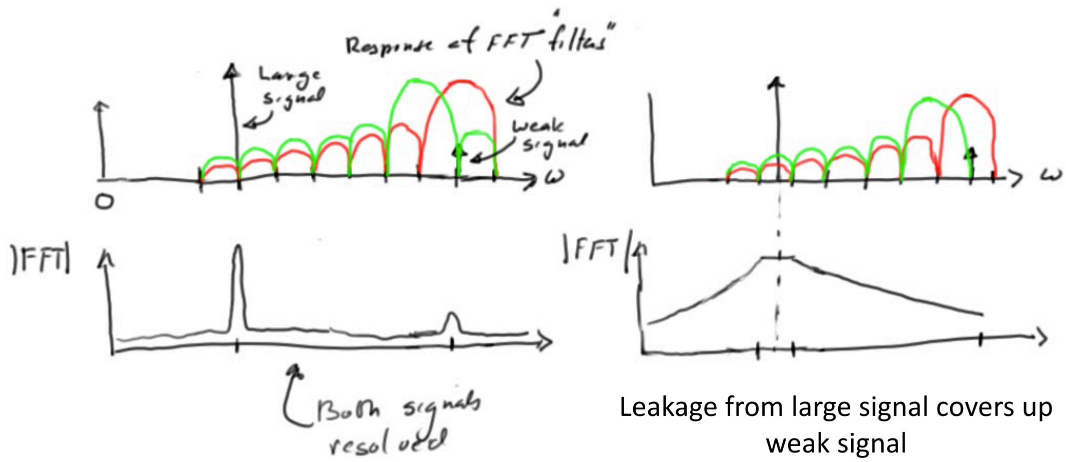
Spectral Leakage

A. Sinusoid separated 6 "bins" in frequency



Spectral Leakage

A. Sinusoid separated 6 “bins” in frequency **B. Sinusoid separated 5.5 “bins” in frequency**



Leakage from large signal covers up weak signal