SIO 209: Signal Processing for Ocean Sciences
Class 11

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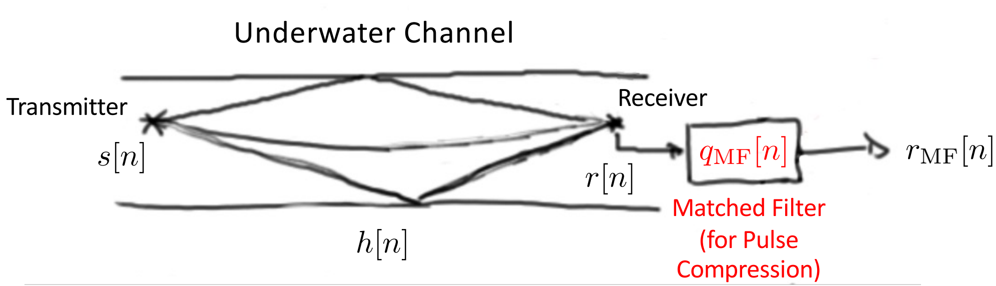
0

Matched Filtering

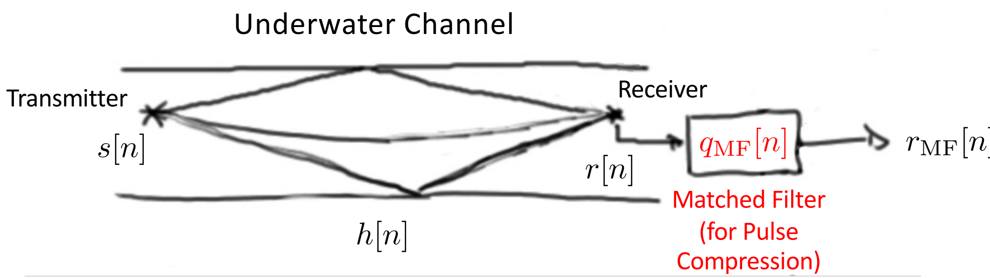


- A filter is "matched" to the know transmit signal $s[n]$ if $q_{\mathrm{mf}}[n] = s^*[-n]$
- Applying the matched filter to $s[n]$, results in the autocorrelation function of $s[n]$, i.e.,

$$s[n] * q_{\mathrm{mf}}[n] = \sum_{k=-\infty}^{\infty} q_{\mathrm{mf}}[k] s[n-k] = \sum_{k=-\infty}^{\infty} s^*[-k] s[m-k] = \sum_{k=-\infty}^{\infty} s^*[k] s[n+k] = \phi_{ss}[n]$$

1

Matched Filtering



- The output of the matched filter $r_{\mathrm{mf}}[n]$ is given by

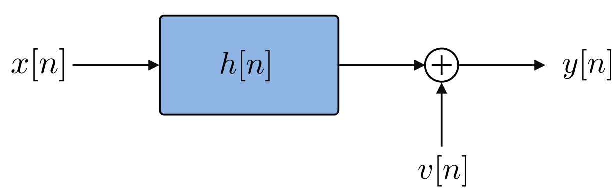
$$r_{\mathrm{mf}}[n] = (s[n] * h[n]) * q_{\mathrm{mf}}[n] = (s[n] * q_{\mathrm{mf}}[n]) * h[n] = \phi_{ss}[n] * h[n]$$

- If we use a transmit pulse $s[n]$ with an autocorrelation function $\phi_{ss}[n] \approx \delta[n]$, i.e., similar to the unit sample, we can use the matched filter to estimate $h[n]$

2

Coherence and Transfer Function Estimation

- **System Model**



- Functions of interest
  - *Transfer Function:* The relationship between $x[n]$ and $y[n]$
  - *Coherence Function:* The degree of causality between $x[n]$ and $y[n]$

P. R. Roth, "Effective Measurements Using Digital Signal Analysis." IEEE Spectrum, 1971

G. C. Carter, "Coherence and Time Delay Estimation." Proc. IEEE, 1987

3

Coherence and Transfer Function Estimation

- **Components and Their Definitions**

A. Auto-Power Spectra

conventional power spectral estimation

$$G_{xx}(\omega) \longrightarrow \overline{\hat{G}_{xx}(k)} = \overline{X(k)X^*(k)}$$ where $X(k) = \mathrm{DFT}\{x[n]\}$

$$G_{yy}(\omega) \longrightarrow \overline{\hat{G}_{yy}(k)} = \overline{Y(k)Y^*(k)}$$ where $Y(k) = \mathrm{DFT}\{y[n]\}$

B. Cross-Power Spectra

$$G_{yx}(\omega) \longrightarrow \overline{\hat{G}_{yx}(k)} = \overline{Y(k)X^*(k)}$$

$$\begin{aligned} G_{yx}(\omega) &= H(\omega)G_{xx}(\omega) + G_{vx}(\omega) \\ &= H(\omega)G_{xx}(\omega) \end{aligned}$$

assume $G_{vx}(\omega) = 0$

4

Coherence and Transfer Function Estimation

- A PSD estimate of $G_{yx}(\omega)$ can be developed as follows

$$\begin{aligned} \overline{\hat{G}_{yx}(k)} &= \overline{Y(k)X^*(k)} \\ &= \overline{[H(k)X(k) + V(k)]X^*(k)} \\ &= H(k)\overline{X(k)X^*(k)} + \overline{V(k)X^*(k)} \\ &= H(k)\overline{\hat{G}_{xx}(k)} + \boxed{\overline{V(k)X^*(k)}} \end{aligned}$$

not zero if a single or few records are used for estimation

5

Coherence and Transfer Function Estimation

- **Transfer Function Estimation**

$$H(\omega) = \frac{G_{yx}(\omega)}{G_{xx}(\omega)}$$

$$\hat{H}(k) = \frac{\overline{\hat{G}_{yx}(k)}}{\overline{\hat{G}_{xx}(k)}} = \frac{H(k)\overline{\hat{G}_{xx}(k)} + \overline{V(k)X^*(k)}}{\overline{\hat{G}_{xx}(k)}}$$

- The component $V(k)X^*(k)$ can be made arbitrarily small by averaging
- The effect of non-white $G_{xx}(\omega)$ is removed
- Statistical fluctuations in $\hat{G}_{xx}(k)$ are removed by averaging

6