SIO 207A: Fundamentals of Digital Signal Processing
Class 16

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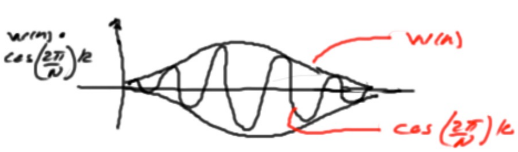
0

Windowing

- Windows are applied only before we perform a DFT/FFT for analysis in the frequency domain
- We always apply the window if the DFT/FFT length is shorter than the length of the signal
- We never apply the window if the DFT/FFT length is longer than the length of the signal (transient signal case)
- Windows are never applied if we perform a DFT/FFT for filtering purposes, i.e., to replace the convolution in the time domain by multiplication in the frequency domain



DFT/FFT: $X(k) = \sum_{n=0}^{N-1} \boxed{w[n]}\, x[n]\, e^{-j\frac{2\pi}{N}nk}$

window function

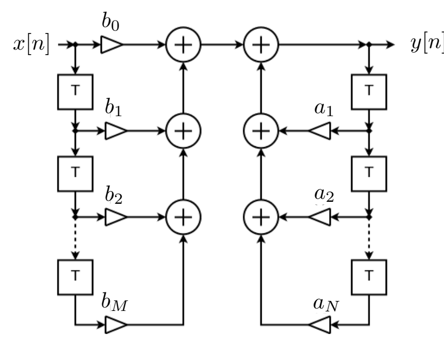
1

1

IIR Filters

Homework 7 – Matlab functions
“buttord”, “butter”,
“cheb1ord”, “cheby1”,
“cheb2ord”, “cheby2”,
“ellipord”, “ellip”

see also Sections 7.1.1, 7.1.2, 7.1.3, and Appendix B in *Oppenheim & Schafer, 1999*



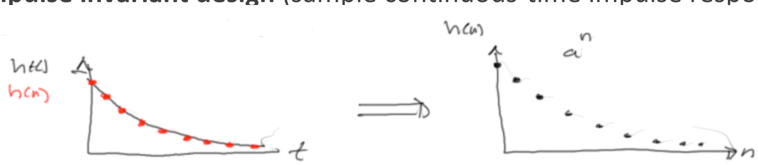
$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{r=0}^{M} b_r x[n-r]$$

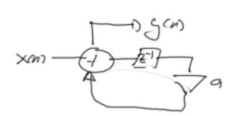
Matlab filter design functions will provide coefficients a_k and b_r

2

Transformation Techniques

- **Transformation Techniques:** Take advantage of an analog filter design and transform it into discrete time domain
  - **Impulse invariant design** (sample continuous-time impulse response)

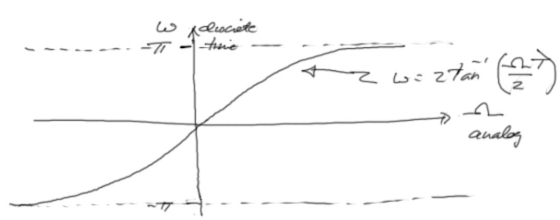




Sampling causes aliasing; thus, this approach is never used in practice

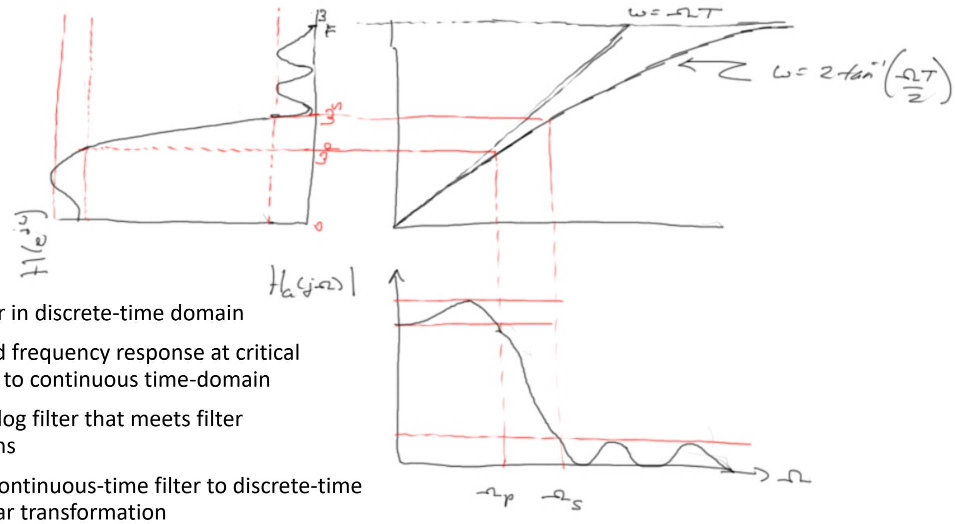
  - **Bilinear transformation**

$$H(z) = H_a(s)\Big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$$



3

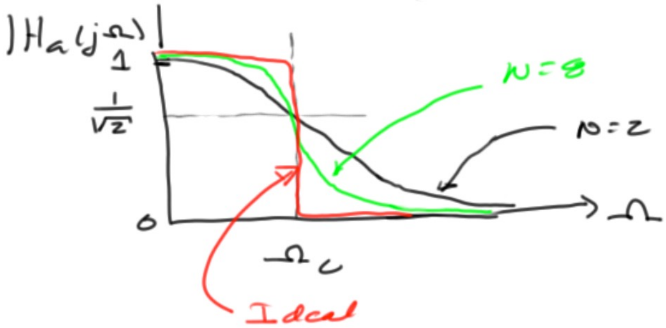
Bilinear Transformation



- Specify filter in discrete-time domain
- Map desired frequency response at critical frequencies to continuous time-domain
- Choose analog filter that meets filter specifications
- Transform continuous-time filter to discrete-time using bilinear transformation

4

Butterworth Filter



Nth order filter
Critical parameters N and Ω_c

$$|H_\mathrm{a}(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_\mathrm{c}}\right)^{2N}}$$

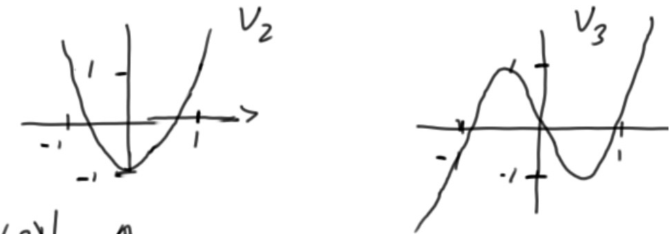
Note: No ripples passband or stopband

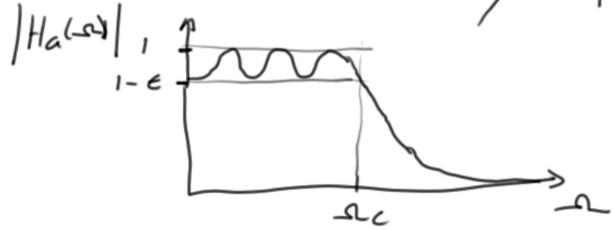
5

Chebyshev Filter

$$|H_{\mathrm{a}}(\Omega)|^2 = \frac{1}{1+\epsilon^2 V_n^2(\frac{\Omega}{\Omega_{\mathrm{c}}})} \quad (\epsilon > 0)$$

V_n are Chebyshev polynomials



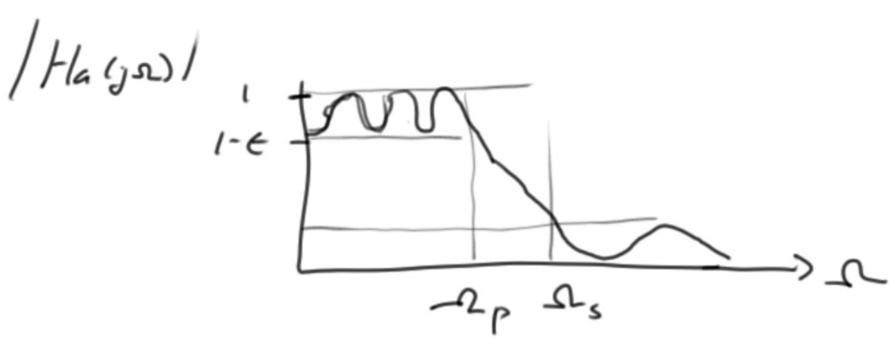


6

Elliptic Filter

$$|H_{\mathrm{a}}(\Omega)|^2 = \frac{1}{1+\epsilon^2 U_n^2(\Omega)}$$

where $U_n(\Omega)$ is a Jacobian elliptic function



7