SIO 207A: Fundamentals of Digital Signal Processing
Class 11

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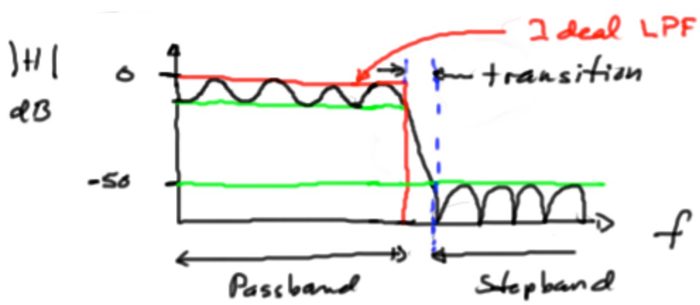
Filter Design

- Parameters available in filter design
  - ripple level in passband and stopband
  - transition region width
  - number of filter coefficients

see also Section 7 in Oppenheim & Schafer, 1999

- Number of filter coefficients
  - FIR filter: length of impulse response
  - IIR filter: number of poles and zeros

Generic low-pass filter (LPF)



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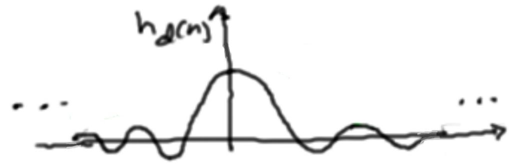
FIR Filter Design – Windowing Approach

$$H_{\mathrm{d}}(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_{\mathrm{d}}[n] e^{-j\omega n}$$

where $h_{\mathrm{d}}[n]$ is the corresponding impulse response of the desired or ideal LPF

$$h_{\mathrm{d}}[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_{\mathrm{d}}(e^{j\omega}) e^{j\omega n} \mathrm{d}\omega$$



Note: $h_d(n)$ is infinite in extent

see also Section 7.2 in
Oppenheim & Schafer, 1999

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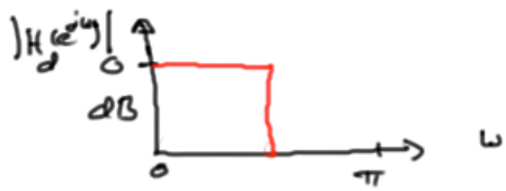

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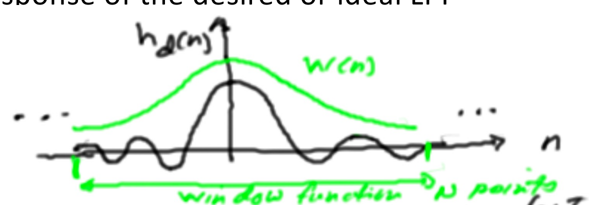

Note: $h_d(n)$ is infinite in extent

$w(n)$ is finite in extent
(N points)

see also Section 7.2 in
Oppenheim & Schafer, 1999

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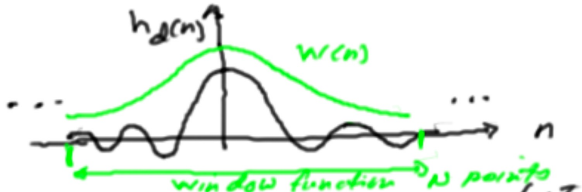
FIR Filter Design – Windowing Approach

$$h[n] = w[n]h_{\mathrm{d}}[n]$$

$$H(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{\mathrm{d}}(e^{j\theta})W(e^{j(\omega-\theta)})\mathrm{d}\theta$$

Fourier transform of $w[n]$



Note: $h_d(n)$ is infinite in extent

$w(n)$ is finite in extent (N points)

$H(e^{j\omega})$ will be a "smeared" version of desired LPF $H_{\mathrm{d}}(e^{j\omega})$

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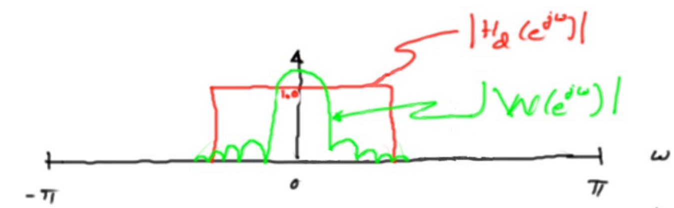
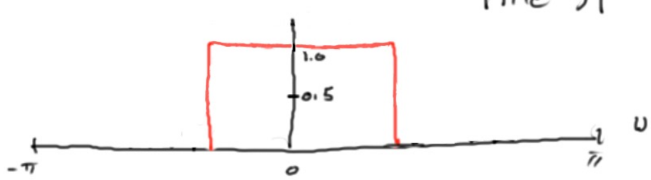
FIR Filter Design – Windowing Approach

- Impact of truncation of $h_{\mathrm{d}}[n]$ by $w[n]$ is to smear the ideal transfer function $H_{\mathrm{d}}(e^{j\omega})$ by Fourier transform of window function



$$|H(e^{j\omega})| = |H_d(e^{j\omega}) * W(e^{j\omega})|$$



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FIR Filter Design – Windowing Approach

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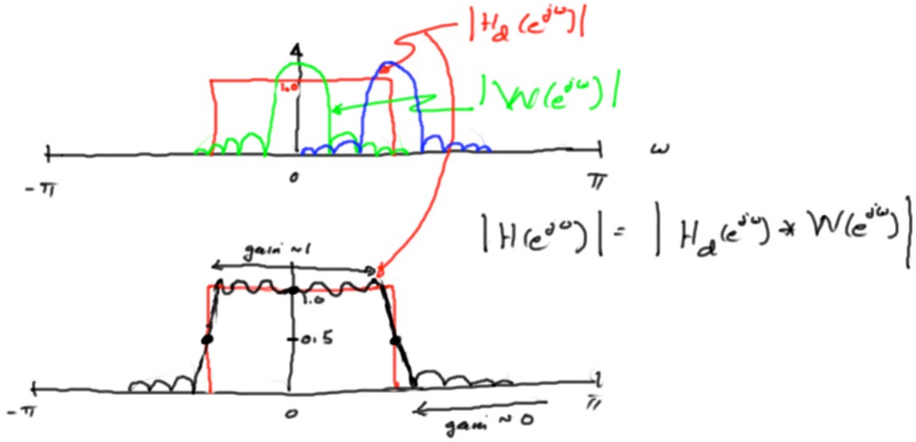

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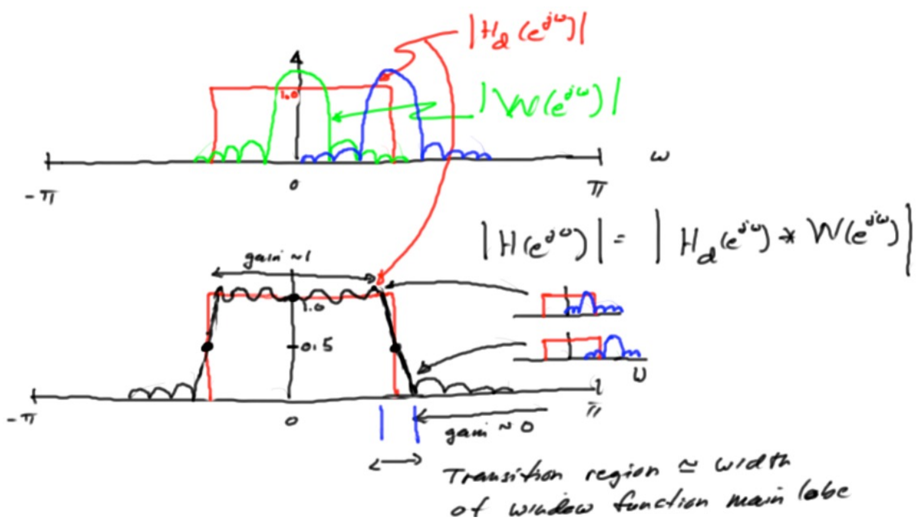
FIR Filter Design – Windowing Approach

- Impact of truncation of $h_{\mathrm{d}}[n]$ by $w[n]$ is to smear the ideal transfer function $H_{\mathrm{d}}(e^{j\omega})$ by Fourier transform of window function



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FIR Filter Design – Windowing Approach

- Result is that the transition width of the filter is width of main lobe of the window function
- In general most “good” window functions have main lobes that are ~3-4 FFT bins wide of the equivalent length FFT, i.e., N points where N is the length of the filter

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