ECE 286 Class 7:
Sequential Bayesian Estimation on the Factor Graph

Florian Meyer

Electrical and Computer Engineering Department
University of California San Diego

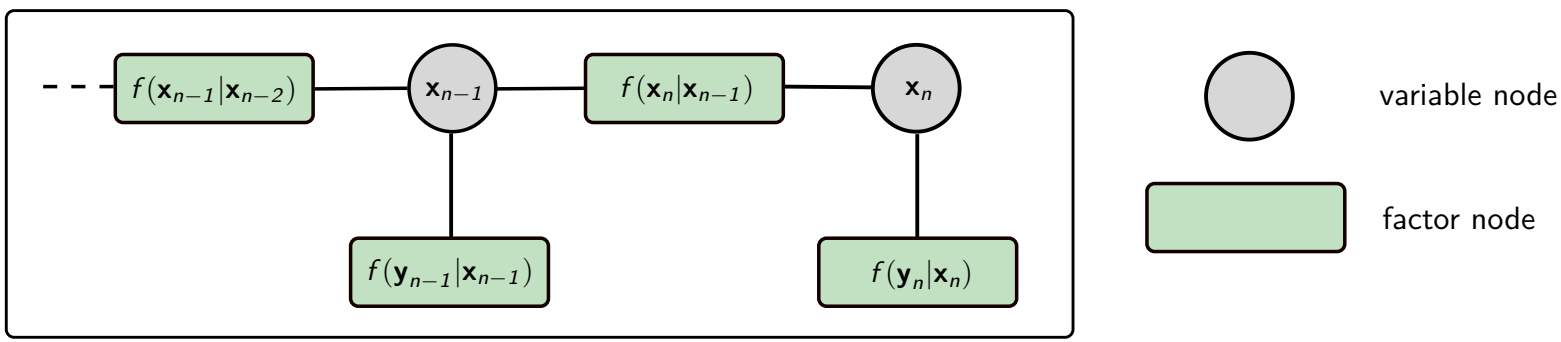
UC San Diego
JACOBS SCHOOL OF ENGINEERING

Factor Graph

- Recall factorization:

$$f(\mathbf{x}_{0:n}|\mathbf{y}_{1:n}) \propto f(\mathbf{x}_0) \prod_{n'=1}^{n} f(\mathbf{y}_{n'}|\mathbf{x}_{n'})\, f(\mathbf{x}_{n'}|\mathbf{x}_{n'-1})$$

- Representation by **factor graph**:



[**Kschischang et al., 01**] F. R. Kschischang, B. J. Frey, and H.-A. Loeliger, "Factor graphs and the sum-product algorithm," *IEEE Trans. Inf. Theory*, 2001.

Message Passing

Prediction step $\rightarrow$ message filtering

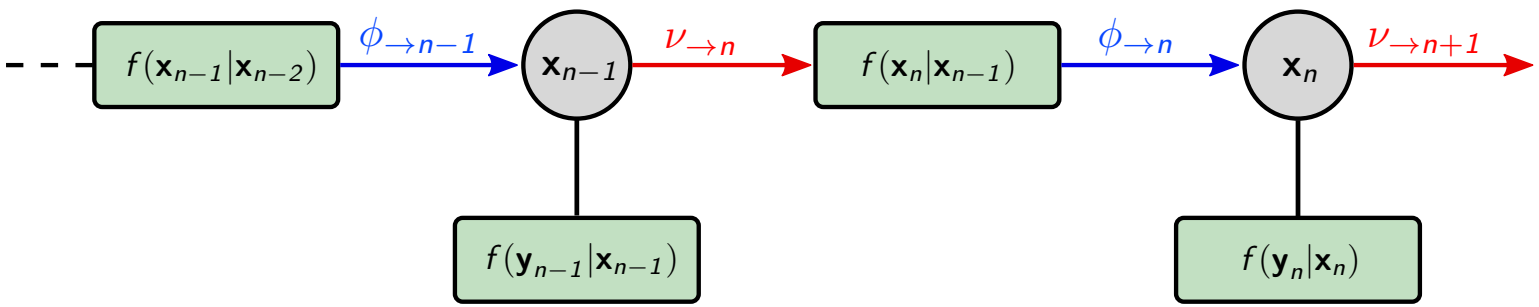
$$f(\mathbf{x}_n|\mathbf{y}_{1:n-1}) = \int f(\mathbf{x}_n|\mathbf{x}_{n-1})\, f(\mathbf{x}_{n-1}|\mathbf{y}_{1:n-1})\, \mathrm{d}\mathbf{x}_{n-1}$$

$$\phi_{\rightarrow n}(\mathbf{x}_n) = \int f(\mathbf{x}_n|\mathbf{x}_{n-1})\, \nu_{\rightarrow n}(\mathbf{x}_{n-1})\, \mathrm{d}\mathbf{x}_{n-1}$$

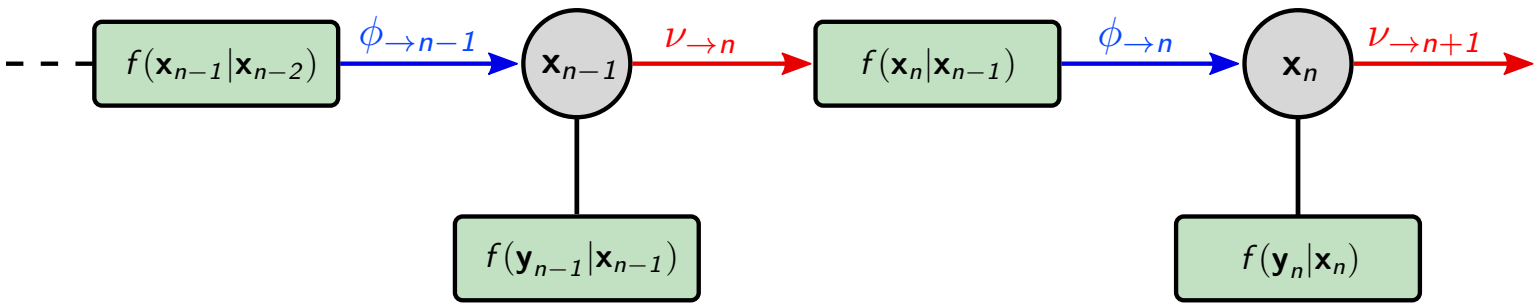
Measurement update step $\rightarrow$ message multiplication

$$f(\mathbf{x}_n|\mathbf{y}_{1:n}) \propto f(\mathbf{y}_n|\mathbf{x}_n)\, f(\mathbf{x}_n|\mathbf{y}_{1:n-1})$$

$$\nu_{\rightarrow n+1}(\mathbf{x}_n) = f(\mathbf{y}_n|\mathbf{x}_n)\, \phi_{\rightarrow n}(\mathbf{x}_n)$$



Message Passing



- Sequential calculation of the marginal posterior pdf $f(\mathbf{x}_n|\mathbf{y}_{1:n})$ can be formulated as **message passing on a factor graph**