SIO 209: Signal Processing for Ocean Sciences
Class 13

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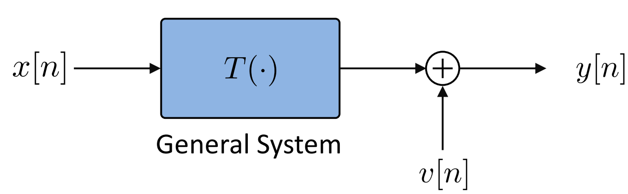
Coherence and Transfer Function Estimation

- Note that in homework 5, $r[n]$ is used to represent the measurement time series and $y[n]$ to represent the output of the linear system



- Note that we have $Y(\omega)^2 < 1$ when
  - noise is contaminating the output of the system
  - the system is nonlinear (power is transferred from one frequency to another frequency)
  - there are additional inputs to the system

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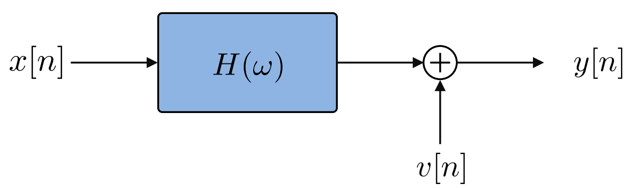

Power Spectrum of $y[n]$: $G_{yy}(\omega)$

- Recall that if the system is linear and time-invariant (LTI), the signal to noise ratio is given by

SNR: $$\frac{|H(\omega)|^2 G_{xx}(\omega)}{G_{vv}(\omega)} = \frac{Y^2(\omega)}{1 - Y^2(\omega)}$$

component due to system input $Y^2(\omega)G_{yy}(\omega)$

component due to additive noise $\big(1 - Y^2(\omega)\big)G_{yy}(\omega)$

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Coherence and Transfer Function Estimation

- Coherence function estimates are biased
- Averaging can reduce bias and variance
- Note that for $Y^2(\omega) = 0$ and $L \geq 32$ we have

$$\text{bias}\big[\hat{Y}^2(\omega)\big] \approx \frac{1}{L} \qquad \text{var}\big[\hat{Y}^2(\omega)\big] \approx \frac{1}{L^2}$$

- For $0 < Y^2(\omega) \leq 1$ and $L \geq 32$

$$\text{bias}\big[\hat{Y}^2(\omega)\big] \approx \frac{1}{L}\big(1 - Y^2(\omega)\big)^2$$

Number of averages: L

Averaging is critical; for $L = 1$

$$\hat{Y}^2(k) = \frac{\left|\overline{\hat{G}_{yx}(k)}\right|^2}{\overline{\hat{G}_{xx}(k)}\;\overline{\hat{G}_{yy}(k)}} = 1$$

$$\text{var}\big[\hat{Y}(\omega)\big] \approx \frac{2}{L} Y^2(\omega)\big(1 - Y^2(\omega)\big)^2$$

J. Bendat and A. Piersol, "Random Data: Analysis and Measurement Procedures." Wiley, 2000

G. Carter, C. Knapp and A. Nuttall, "Estimation of the magnitude-squared coherence function via overlapped fast Fourier transform processing." IEEE Trans. Audio Electroacoust., 1973

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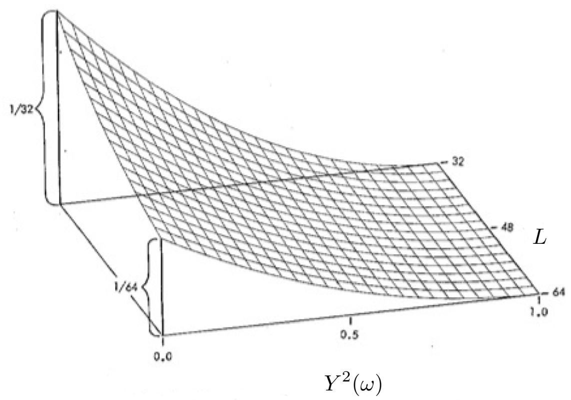
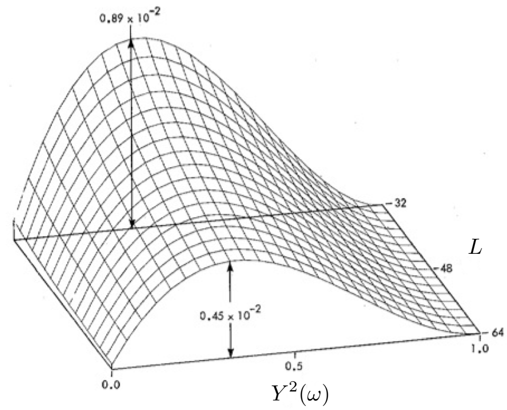
Coherence and Transfer Function Estimation

Bias of $\hat{Y}^2(\omega)$ versus $Y^2(\omega)$ and L



Variance of $\hat{Y}^2(\omega)$ versus $Y^2(\omega)$ and L



G. Carter, C. Knapp and A. Nuttall, "Estimation of the magnitude-squared coherence function via overlapped fast Fourier transform processing." IEEE Trans. Audio Electroacoust., 1973

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Coherence and Transfer Function Estimation

TABLE 3: 90% CONFIDENCE LIMITS ON COHERENCE FUNCTION MEASUREMENTS (MIN, MAX LIMITS)

Measured value of coherence function	Number of averages				
	16	32	64	128	256
0.4	0.15, 0.59	0.23, 0.54	0.28, 0.50	0.32, 0.47	0.34, 0.45
0.5	0.25, 0.67	0.33, 0.63	0.39, 0.59	0.42, 0.57	0.45, 0.55
0.6	0.36, 0.74	0.45, 0.71	0.50, 0.68	0.53, 0.66	0.55, 0.64
0.7	0.50, 0.81	0.57, 0.78	0.61, 0.76	0.64, 0.75	0.66, 0.73
0.8	0.65, 0.88	0.70, 0.86	0.74, 0.84	0.76, 0.83	0.77, 0.82
0.9	0.81, 0.94	0.85, 0.93	0.87, 0.92	0.88, 0.92	0.88, 0.91

TABLE 1: 90% CONFIDENCE LIMITS FOR RMS AVERAGING

	Number of averages						
	4	8	16	32	64	128	256
Upper limit(dB)	+ 4.7	+ 3.0	+ 2.0	+ 1.4	+ 1.0	+ 0.7	+ 0.5
Lower limit(dB)	− 2.9	− 2.2	− 1.6	− 1.2	− 0.8	− 0.6	− 0.4

N. Pendergrass, "Coherence function and averaging boost confidence in spectrum measurements." Electronics, 1978

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Coherence and Transfer Function Estimation

TABLE 2: 90% CONFIDENCE LIMITS ON THE MEASUREMENT OF THE AMPLITUDE |H| AND PHASE ϕ OF TRANSFER FUNCTIONS

Measured value of coherence function		Number of averages				
		16	32	64	128	256
0.2	|H| (dB)	+ 5.2, − 14.6	+ 3.8, − 7.1	+ 2.8, − 4.2	+ 2.1, − 2.7	+ 1.5, − 1.8
	ϕ (°)	± 54	± 34	± 23	± 16	± 11
0.3	|H| (dB)	+ 4.2, − 8.4	+ 3.1, − 4.8	+ 2.2, − 3.0	+ 1.6, − 2.0	+ 1.2, − 1.4
	ϕ (°)	± 38	± 25	± 17	± 12	± 8
0.4	|H| (dB)	+ 3.5, − 6.0	+ 2.6, − 3.6	+ 1.8, − 2.3	+ 1.3, − 1.6	+ 1.0, − 1.1
	ϕ (°)	± 30	± 20	± 14	± 10	± 7
0.5	|H| (dB)	+ 3.0, − 4.5	+ 2.1, − 2.8	+ 1.5, − 1.9	+ 1.1, − 1.3	+ 0.8, − 0.9
	ϕ (°)	± 24	±16	± 11	± 8	± 5
0.6	|H| (dB)	+ 2.5, − 3.5	+ 1.8, − 2.2	+ 1.3, − 1.5	+ 0.9, − 1.0	+ 0.7, − 0.7
	ϕ (°)	± 19	± 13	± 9	± 6	± 4
0.7	|H| (dB)	+ 2.1, − 2.7	+ 1.5, − 1.7	+ 1.0, − 1.2	+ 0.7, − 0.8	+ 0.5, − 0.6
	ϕ (°)	± 15	± 10	± 7	± 5	± 4
0.8	|H| (dB)	+ 1.6, − 2.0	+ 1.1, − 1.3	+ 0.8, − 0.9	+ 0.6, − 0.6	+ 0.4, − 0.4
	ϕ (°)	± 12	± 8	± 6	± 4	± 3
0.9	|H| (dB)	+ 1.1, − 1.3	+ 0.8, − 0.8	+ 0.5, − 0.6	+ 0.4, − 0.4	+ 0.3, − 0.3
	ϕ (°)	± 8	± 5	± 4	± 3	± 2

N. Pendergrass, "Coherence function and averaging boost confidence in spectrum measurements." Electronics, 1978

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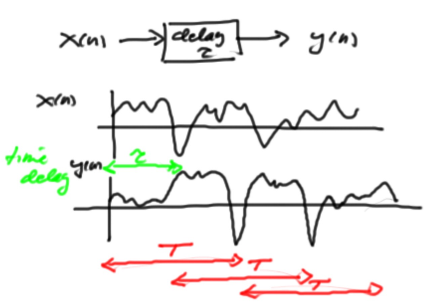
Coherence and Transfer Function Estimation

- Bias due to time delay between sample records of $x[n]$ and $y[n]$

$$\hat{Y}^2(\omega) \approx \hat{Y}^2(\omega)\Big(1 - \frac{\tau}{T}\Big)$$

τ time delay

T segment length of each FFT



– In order to minimize this effect, first compute cross-correlation of $x[n]$ and $y[n]$ the delay/advance $y[n]$ with respect to $x[n]$ to minimize the lag in the peak correlation

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Transfer Function Estimation: Example 1

Underdamped second-order system with white random noise excitation

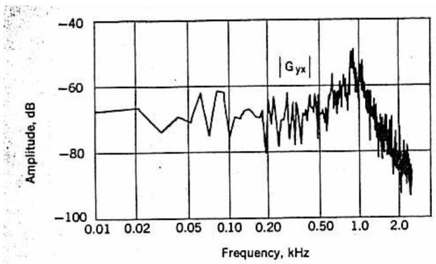


FIGURE 2. The cross power spectrum for an underdamped second-order system with white random noise excitation.

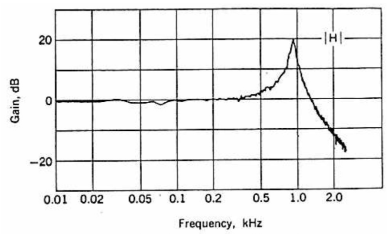
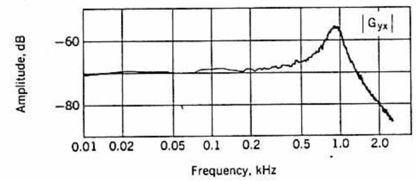


FIGURE 3. Transfer function computed from the same data as in Fig. 2. Variability error is greatly reduced, and the ratio of low-frequency response to peak response is 20 dB, indicating a Q of 10 for the system.

FIGURE 4. The cross power spectrum for the system of Fig. 2 from 100 times the data (40.96 seconds). Variability is about the same as the transfer function computed from 0.4096 second of data, and the Q is only 5.6 (computed from a low-frequency-to-peak ratio of 15 dB). This error is attributed to the nonwhite character of the input caused by the reactive nature of the noise-generator load.



P. R. Roth, "Effective Measurements Using Digital Signal Analysis." IEEE Spectrum, 1971

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Transfer Function Estimation: Example 2

Path-delay measurements problem

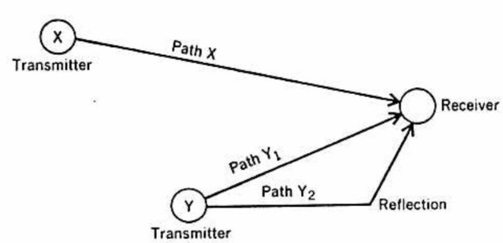

FIGURE 9. One form of the path-delay measurement problem. Transmitters X and Y radiate a pulse with a known delay, and by measuring the difference in transmission time the position of the receiver may be determined.

P. R. Roth, "Effective Measurements Using Digital Signal Analysis." IEEE Spectrum, 1971

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Transfer Function Estimation: Example 2

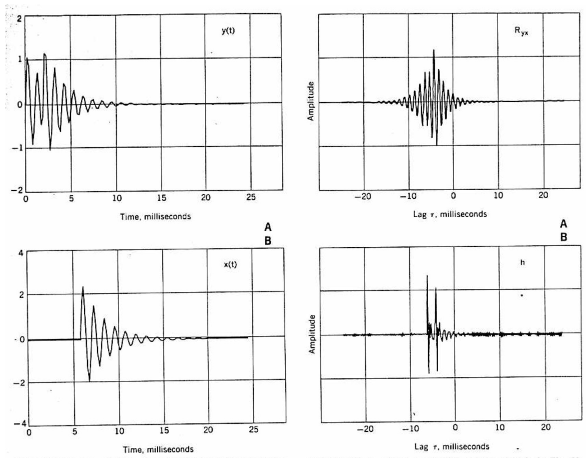

FIGURE 10. A—The double pulse received from transmitter Y for the situation shown in Fig. 9. B—The single pulse received from transmitter X.

FIGURE 11. A—Cross correlation of the signals in Fig. 10. Note that, although two delays are physically present, only one (at −4.3 ms) is clearly resolved. B—The impulse response of the same signals as in A. Note that both delays are shown and that the correct longer delay (at −6.2 ms), not apparent in A, is now clearly shown.

P. R. Roth, "Effective Measurements Using Digital Signal Analysis." IEEE Spectrum, 1971

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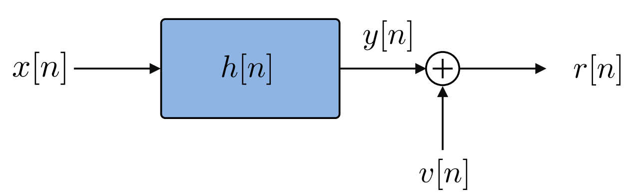
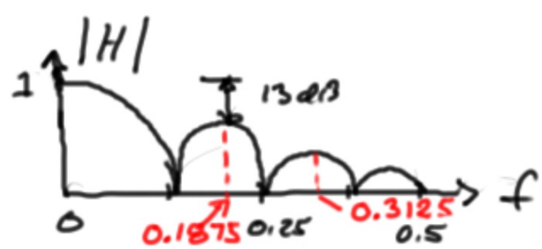
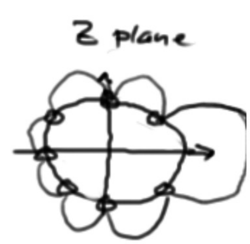
Homework 5 – Transfer and Coherence Function Est.

- Note that in homework 5, $r[n]$ is used to represent the measurement time series and $y[n]$ to represent the output of the linear system



- **Part A:**





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Coherence and Transfer Function Estimation

- **Part B: Power Spectra (2-sided)** $\hat{S}_{xx}(f) = \frac{1}{f_s MU}\overline{|X(k)|^2}$

$\mathrm{var}(w_1[n]) = 1 = 0\mathrm{dB}$

$\mathrm{var}(w_2[n]) = \frac{1}{32} = 2^{-5} = -15\,\mathrm{dB}$