First Order Systems

Let $z_m = re^{j\theta}$. Then $1 - z_m e^{-j\omega} = 1 - re^{-j(\omega-\theta)}$. Noting that $|a|^2 = aa^*$, we have the magnitude response

$$|1 - z_k e^{-j\omega}|^2 = (1 - re^{-j(\omega-\theta)})(1 - re^{j(\omega-\theta)}) = 1 + r^2 - 2r\cos(\omega - \theta)$$

$$20\log_{10}|1 - z_k e^{-j\omega}| = 10\log_{10}(1 + r^2 - 2r\cos(\omega - \theta))$$

Phase: $1 - re^{-j(\omega-\theta)} = 1 - r\cos(\omega - \theta) + jr\sin(\omega - \theta)$.

$$\mathrm{ARG}(1 - re^{-j(\omega-\theta)}) = \arctan\frac{r\sin(\omega - \theta)}{1 - r\cos(\omega - \theta)}$$

Figure 5.9 Frequency response for a single zero, with $r = 0.9$ and the three values of θ shown. (a) Log magnitude. (b) Phase. (c) Group delay.

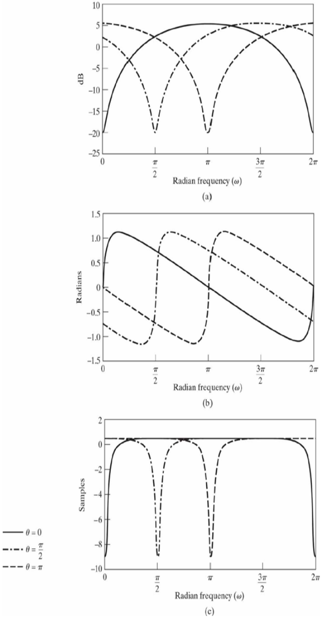
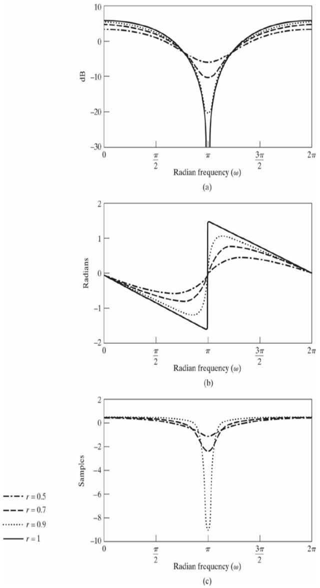


Figure 5.11 Frequency response for a single zero, with $\theta = \pi$, $r = 1$, 0.9, 0.7, and 0.5. (a) Log magnitude. (b) Phase. (c) Group delay for $r = 0.9$, 0.7, and 0.5.

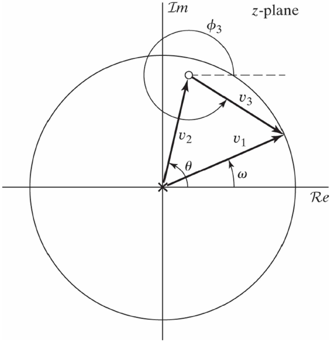


First Order System

$$H(e^{j\omega}) = 1 - re^{j\theta}e^{-j\omega} = \frac{e^{j\omega} - re^{j\theta}}{e^{j\omega}} = \frac{v_1 - v_2}{v_1} = \frac{v_3}{v_1}$$

$$|H(e^{j\omega})| = \frac{|v_3|}{|v_1|} \text{ and } \angle H(e^{j\omega}) = \angle v_3 - \angle v_1 = \phi_3 - \omega$$

Figure 5.10 z -plane vectors for a 1st-order system function evaluated on the unit circle, with $r < 1$.



Second Order System

$$H(e^{j\omega}) = \frac{1}{(1 - re^{j\theta}e^{-j\omega})(1 - re^{-j\theta}e^{-j\omega})} = \frac{e^{j2\omega}}{(e^{j\omega} - re^{j\theta})(e^{j\omega} - re^{-j\theta})} = \frac{v_3^2}{v_1 v_2}$$

$$|H(e^{j\omega})| = \frac{|v_3|^2}{|v_1||v_2|} \quad \text{and} \quad \angle H(e^{j\omega}) = 2\angle v_3 - \angle v_1 - \angle v_2$$

Figure 5.12 Pole–zero plot for Example 5.6.

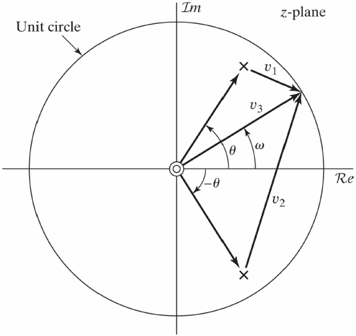


Figure 5.13 Frequency response for a complex-conjugate pair of poles as in Example 5.6, with $r = 0.9$, $\theta = \pi/4$. (a) Log magnitude.