ECE 286: Bayesian Machine Perception

Class 14: Track Management and Partitioning of Measurements

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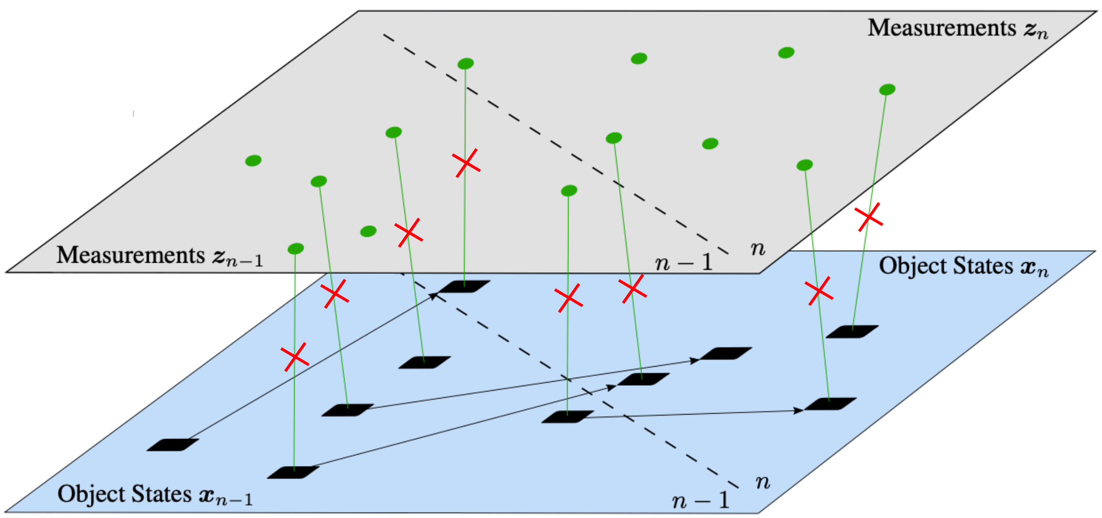
The Multiobject Tracking Problem

- At each time n: localize and track an unknown number of objects $\boldsymbol{x}_n = [\boldsymbol{x}_{n,1}^{\mathrm{T}} \ldots \boldsymbol{x}_{n,I_n}^{\mathrm{T}}]^{\mathrm{T}}$ from measurements $\boldsymbol{z}_n = [\boldsymbol{z}_{n,1}^{\mathrm{T}} \ldots \boldsymbol{z}_{n,M_n}^{\mathrm{T}}]^{\mathrm{T}}$ with uncertain origin



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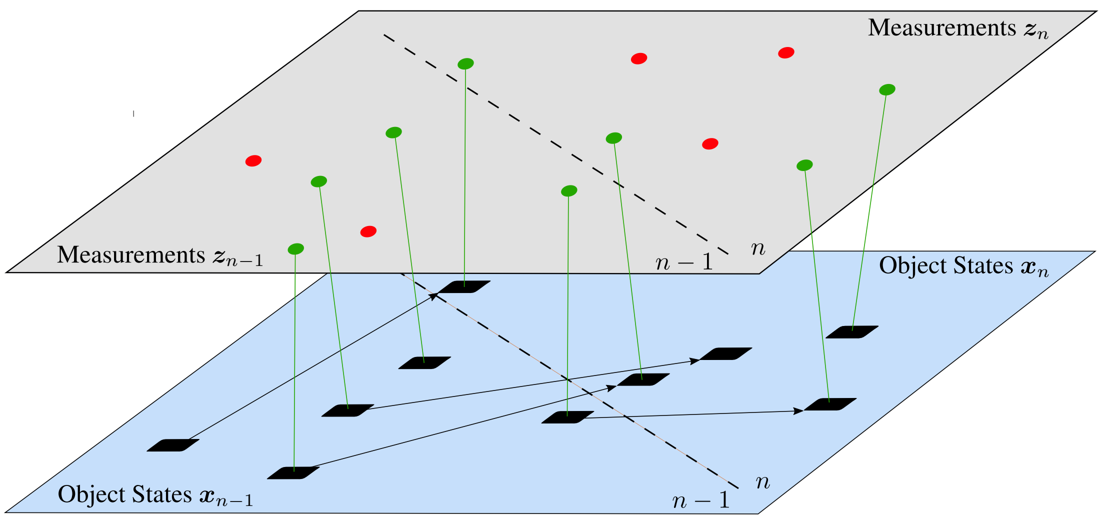
- Data association is challenging because of clutter measurements, missing measurements, object births, and object deaths



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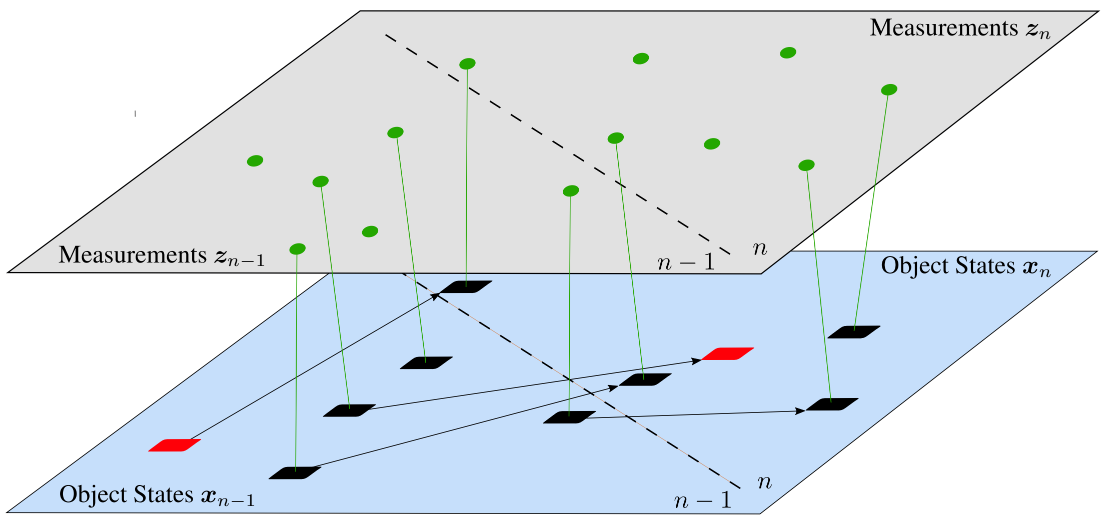

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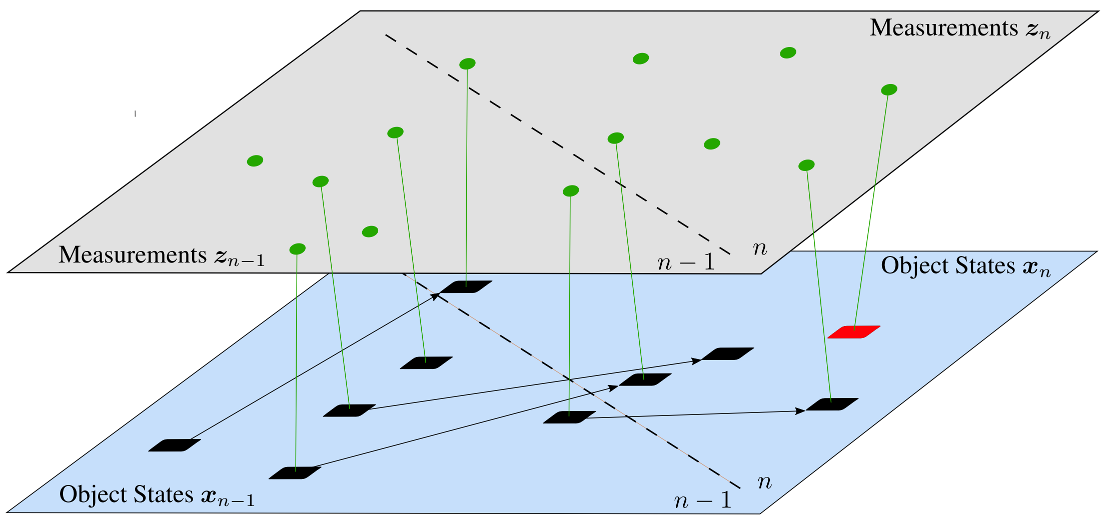

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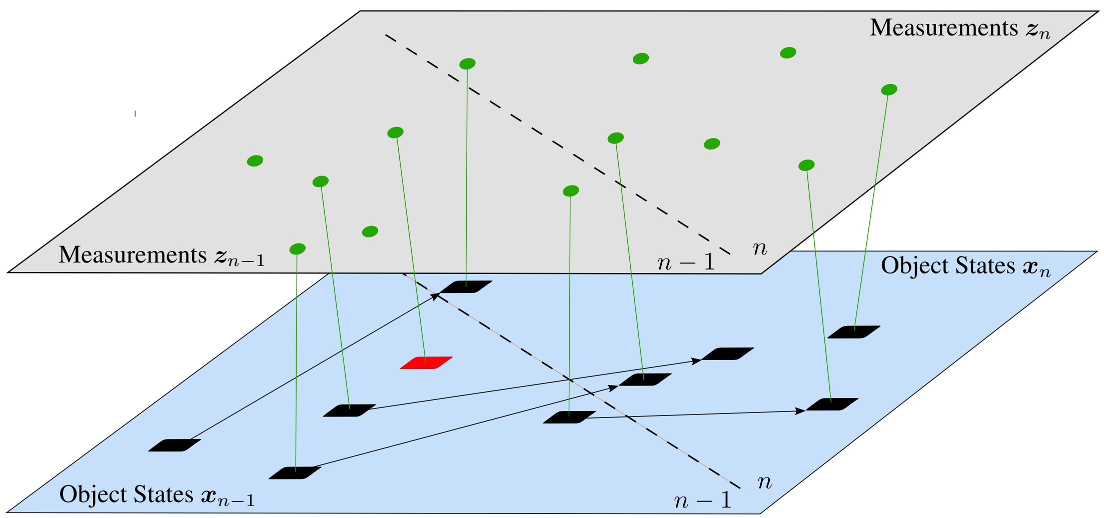
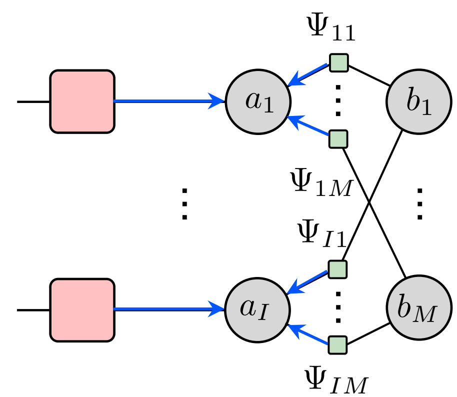


Association Probabilities

- Approximate object-oriented marginal association probabilities after $\ell = L$ iterations

$$\tilde{p}(a_i|\boldsymbol{z}) \propto \phi_{a_i}(a_i) \prod_{m=1}^{M} \phi^{[L]}_{\Psi_{im} \to a_i}(a_i)$$



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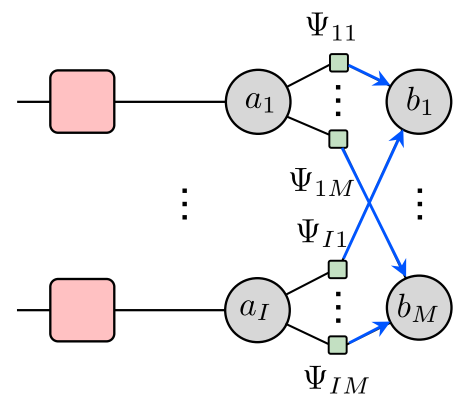


- Approximate measurement-oriented marginal association probabilities after $\ell = L$ iterations

$$\tilde{p}(b_m|\boldsymbol{z}) \propto \prod_{i=1}^{I} \phi^{[L]}_{\Psi_{im}\rightarrow b_m}(b_m)$$

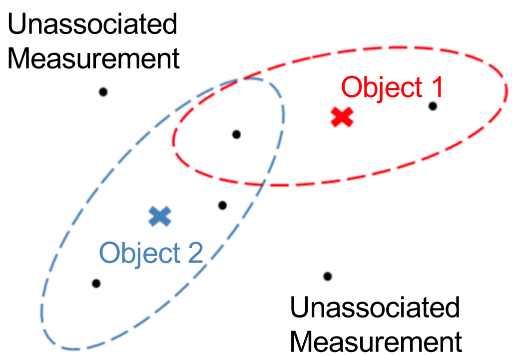
- Note that $\tilde{p}(a_i = 0|\boldsymbol{z})$ is the probability that object i did not generate a measurement and $\tilde{p}(b_m = 0|\boldsymbol{z})$ is the probability that measurement m was not generated by an object

$\longrightarrow$ potentially useful for generating or terminating tracks

Unassociated Measurements

- Unassociated measurements are measurements that with high probability have not been originated by an object

- Can be determined by
  - hard measurement evaluation: measurements that are outside the gates of all the objects are declared unassociated
  - joint probabilistic data association: all measurements with $\tilde{p}(b_m = 0|\boldsymbol{z})$ larger than a certain threshold are declared unassociated



Track Formation and Termination Heuristics

- A heuristic to initialize a new track for a newborn object is referred to as track formation in clutter
- The logic-based approach uses gates to search for sequence of measurements that are not associated to any existing object
- If a requirement is satisfied, then the measurement sequence is accepted as a valid track and initialized by increasing the state space and extracting a prior distribution from the sequence of measurements
- A track is terminated if for a number of time steps N no measurement is associated to it

K/N Formation Heuristic

1. Every unassociated measurement is an ``initiator'' -- it yields a tentative track

2. At the time step following the detection of an initiator, a gate is set up based on the
   - assumed maximum and minimum object motion parameters
   - the measurement noise variances

   such that, if there is a target that gave rise to the initiator, the measurement from it in this second time step (if detected) will fall in the gate with nearly unity probability

3. If there is a measurement, this tentative track becomes a preliminary track. If there is no measurements, this track is dropped

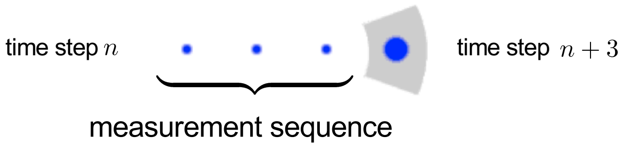
4. Since a preliminary track has two measurements, a sequential Bayesian estimation can be initialized and used to set up a gate for the next (third) time step

K/N Formation Heuristic

5. Starting from the third scan a logic of K detections out of N time steps is used for subsequent gates

6. If at the end (scan $N+2$ at the latest) the logic requirement is satisfied, the track becomes a confirmed track; otherwise it is dropped

- The requirement of two initial detections reduces the probability of false tracks
- Typical values for K/N: 3/5, 4/6, …



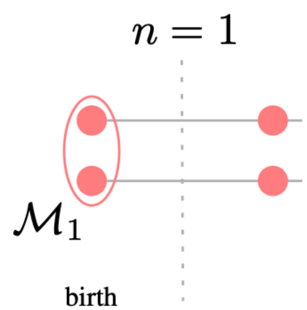
- **Advantages:** Easy to implement
- **Disadvantages:** Heuristic, performance analysis difficult

Y. Bar-Shalom, P. K. Willett, and X. Tian, *Tracking and Data Fusion: A Handbook of Algorithms*. YBS, 2011.

Bayesian Initialization of New Tracks

- Consider time n and *potential object states* $\boldsymbol{y}_{i,n} = [\boldsymbol{x}_{i,n}^{\mathrm{T}}\ r_{i,n}]^{\mathrm{T}},\ i \in \mathcal{I}_n$ where existence is modeled by a Bernoulli variable $r_{i,n} \in \{0, 1\}$

- **Potential object birth:** For each measurement $z_{m,n}, m \in \mathcal{M}_n$ introduce a new state



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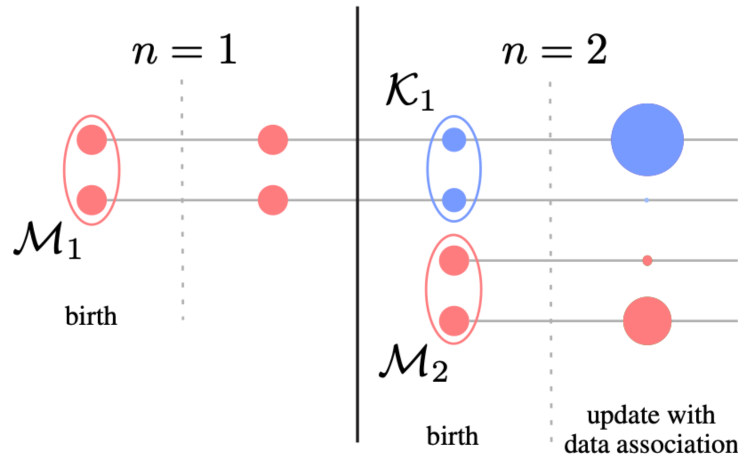
- **Potential object death:** Remove states i with low probability of existence $p(r_{i,n} = 1 | \boldsymbol{z}_{1:n})$



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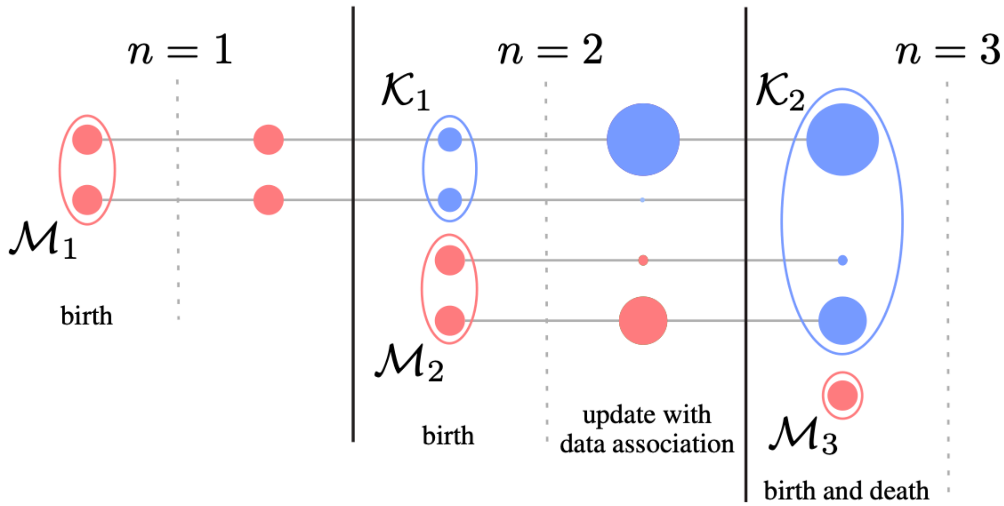
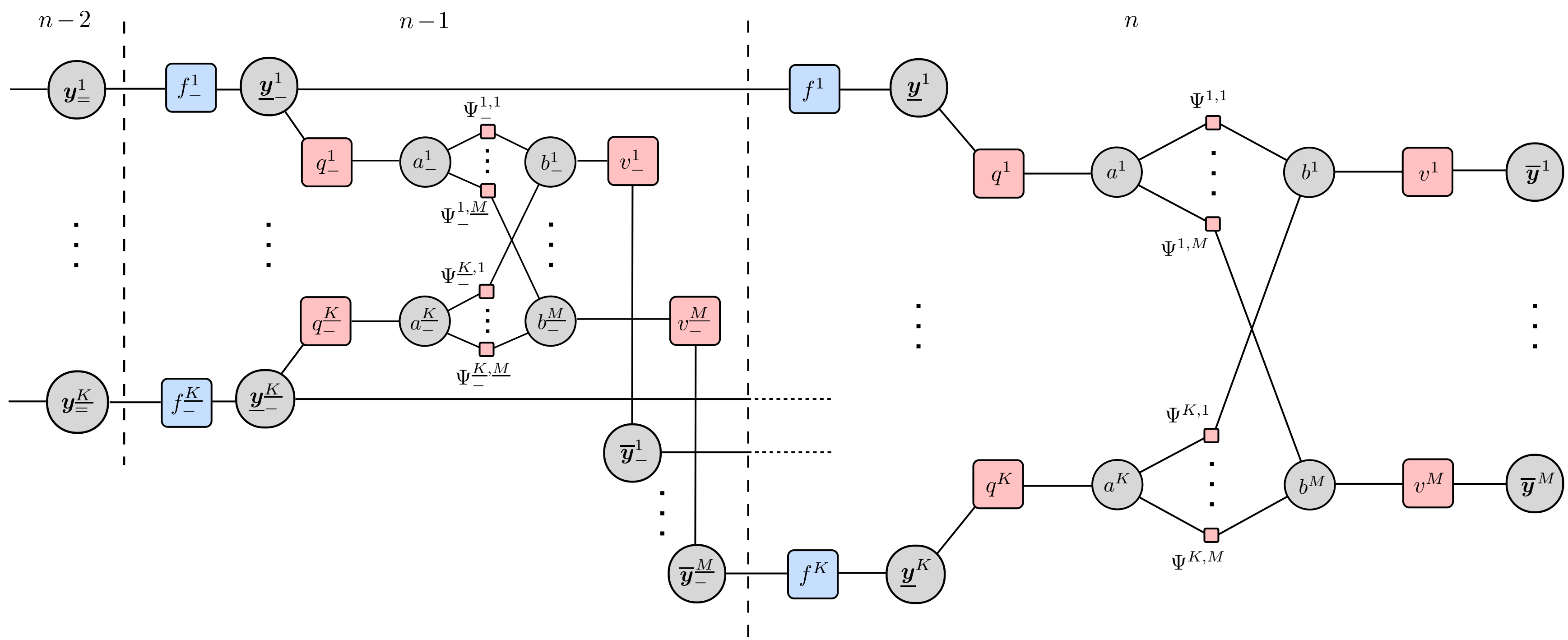


- **Localizing an unknown number of objects:**
  - determine existence of object i by comparing $p(r_{i,n} = 1 | \boldsymbol{z}_{1:n})$ to threshold, e.g., $P_{\mathrm{th}} = 0.5$
  - estimate the states $\boldsymbol{x}_{i,n}$ of existing objects by using, e.g., the MMSE estimator

Multiobject Tracking – Factor Graph

- Complete graph for a sequence of measurements:



F. Meyer, T. Kropfreiter, J. L. Williams, R. A. Lau, F. Hlawatsch, P. Braca, and M. Z. Win, "Message passing algorithms for scalable multitarget tracking," *Proc. IEEE*, Feb. 2018.

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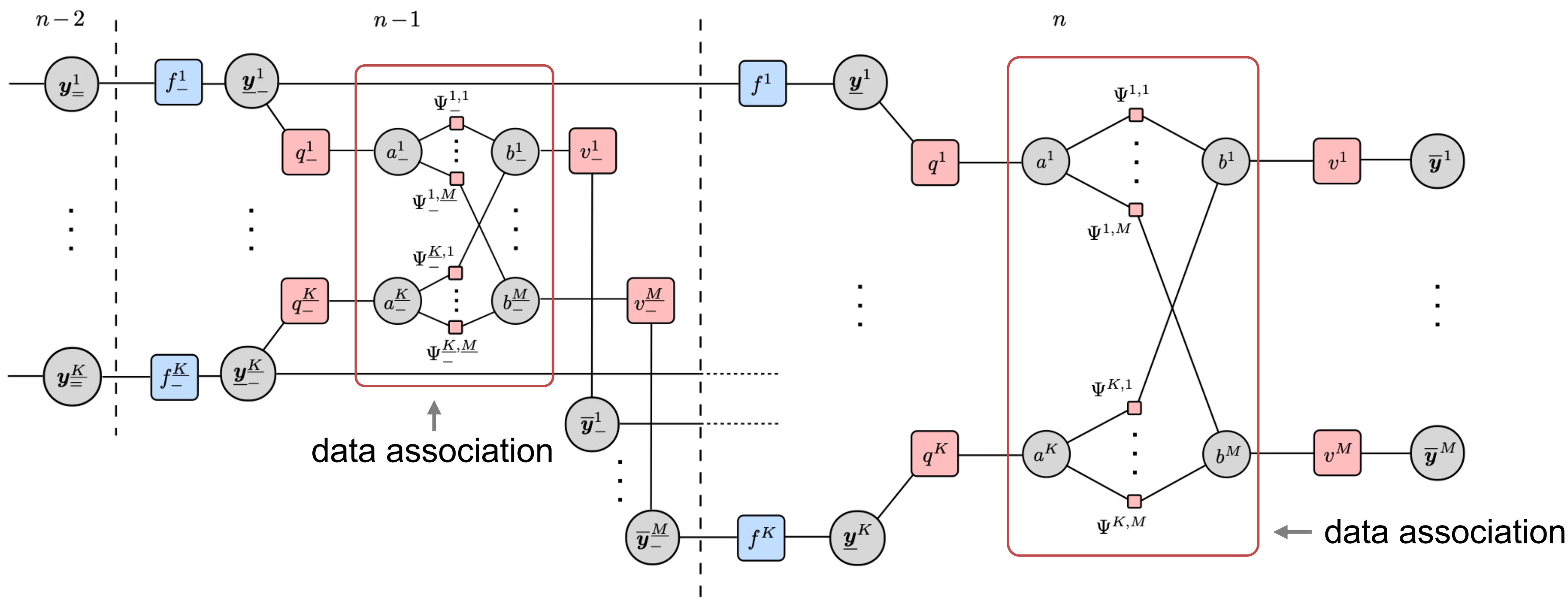

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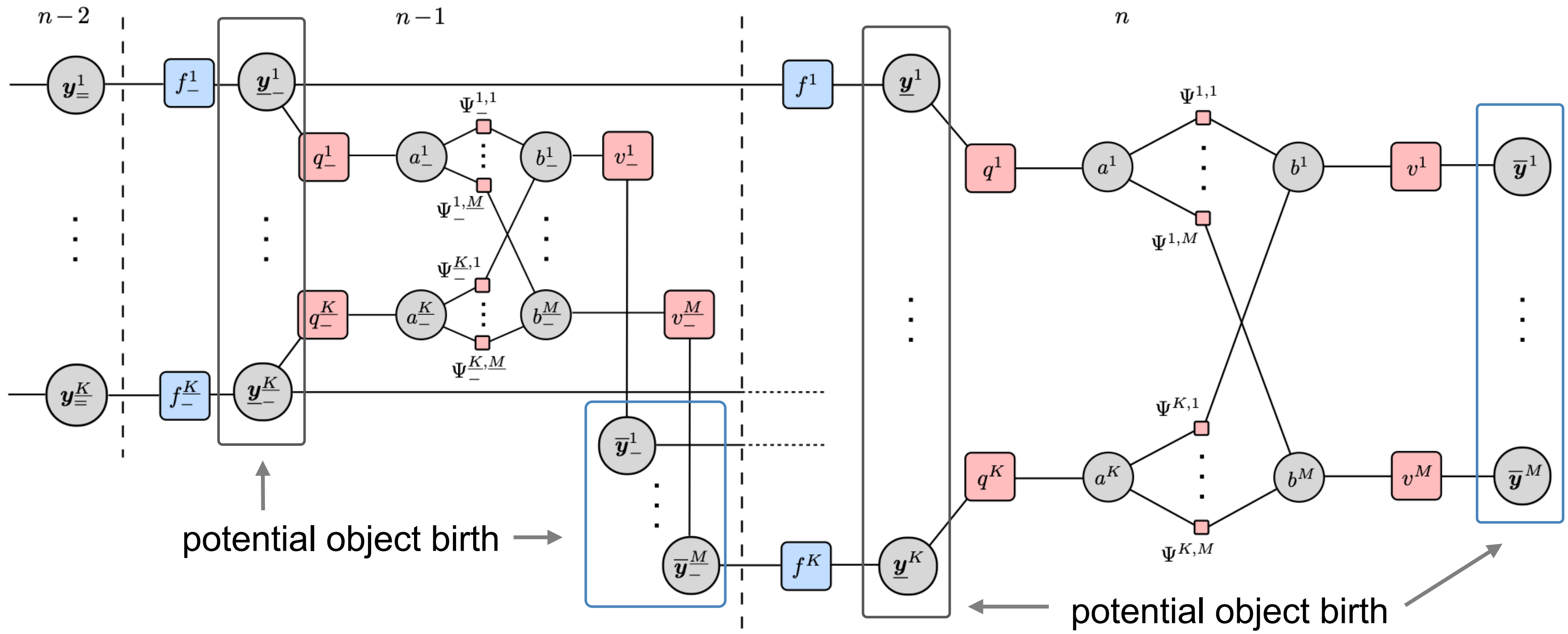

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Distance Partitioning

- Recall data association assumptions: An (i) object can generate at most one measurement and a (ii) measurement can be generated by at most one object
- For high-resolution sensors (e.g., LIDAR), (i) is typically not satisfied
- Statistical model for case where (i) is not satisfied results in very challenging data association problem $\longrightarrow$ extended object tracking
- Heuristic preprocessing stage that aims to enforce (i):
  - Partition the set of measurements $\mathcal{Z} = \{\boldsymbol{z}_m \,|\, m \in \{1, \ldots, M\}\}$ into disjoint subsets (cells) $\mathcal{Z}^{(c)}, \; c \in 1, \ldots, C$, where each subset contains spatially close measurements that are likely to be generated by the same object $(C \leqslant M)$
  - Use "hyper measurements" $\boldsymbol{z}^{(c)}$ related to cells $\mathcal{Z}^{(c)}$ as measurements for multiobject tracking

Distance Partitioning

- Let us assume $d(\cdot,\cdot)$ is a distance measure and Δ_{m_1,m_2} is the distance of measurement pair $\boldsymbol{z}_{m_1}$ and $\boldsymbol{z}_{m_2}$
- The set the measurements $\mathcal{Z} = \{\boldsymbol{z}_m \big| m \in \{1,\dots,M\}\}$ can be partitioned into disjoint subsets (cells) based on the following theorem
- **Theorem:** A distance threshold d_ℓ defines a unique partition of that leaves all pairs of measurements (m_1, m_2) satisfying $\Delta_{m_1,m_2} < d_\ell$ in the same cell (see references for detailed version of theorem)
- If the measurements noise is additive Gaussian, the Mahalanobis distance can be used

$$d(\boldsymbol{z}_{m_1}, \boldsymbol{z}_{m_2}) = \sqrt{(\boldsymbol{z}_{m_1} - \boldsymbol{z}_{m_2})^{\mathrm{T}} \boxed{\boldsymbol{\Sigma}_{\boldsymbol{v}}^{-1}} (\boldsymbol{z}_{m_1} - \boldsymbol{z}_{m_2})}$$

Measurement noise covariance matrix

K. Granström, C. Lundquist, and O. Orguner, “Extended target tracking using a Gaussian-mixture PHD filter,” *IEEE Trans. Aerosp. Electron. Syst.,* Oct. 2012.

K. Granström, O. Orguner, R. Mahler, and C. Lundquist, Corrections on: “Extended target tracking using a Gaussian-mixture PHD filter,” *IEEE Trans. Aerosp. Electron. Syst.,* Apr. 2017.

Distance Partitioning

- Distance threshold: d_ℓ
- Number of measurements: $N_{\boldsymbol{z}}$
- Distance between measurement $\boldsymbol{z}_i$ and measurement $\boldsymbol{z}_j$: $\Delta_{i,j}$

Distance Partitioning

Require: d_ℓ, $\Delta_{i,j}$, $1 \leq i \neq j \leq N_z$.

1: CellNumber(i) = 0, $1 \leq i \leq N_z$ {Set cells of all measurements to null}
2: CellId = 1 {Set the current cell id to 1}
%Find all cell numbers
3: **for** $i = 1 : N_z$ **do**
4: **if** CellNumbers(i) = 0 **then**
5: CellNumbers(i) = CellId
6: CellNumbers = FindNeighbors(i,CellNumbers,CellId)
7: CellId = CellId+1
8: **end if**
9: **end for**

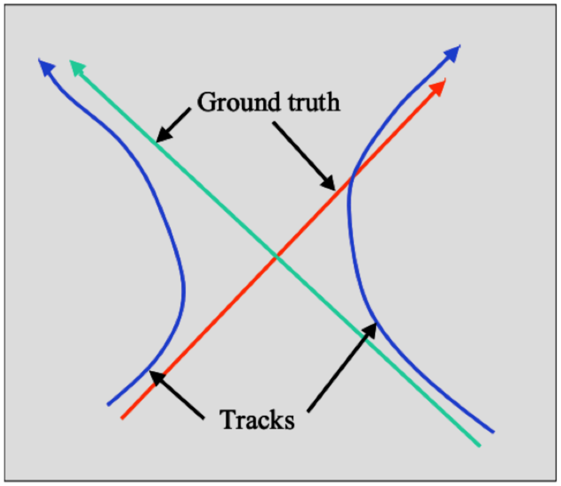
The recursive function FindNeighbors($\cdot,\cdot,\cdot$) is given as

1: **function** CellNumbers = FindNeighbors(i,CellNumbers,CellId)
2: **for** $j = 1 : N_z$ **do**
3: **if** $j \neq i$ & $\Delta_{ij} \leq d_\ell$ & CellNumbers(j) = 0 **then**
4: CellNumbers(j) = CellId
5: CellNumbers = FindNeigbors(j,CellNumbers,CellId)
6: **end if**
7: **end for**

Optimal Subpattern Assignment Metric

- Mean square error is not a suitable metric for many multiobject tracking applications
  - not defined if estimated number of objects is different than the true number of objects
  - track swapping leads to large errors

- Parameters
  - Metric order p
  - Cutoff parameter η
  - Inner metric $d(\boldsymbol{x}_i, \boldsymbol{x}_j)$



D. Schuhmacher, B.-T. Vo, B.-N. Vo, "A Consistent Metric for Performance Evaluation of Multi-Object Filters," *IEEE Trans. Signal Process., Jul*. 2008.

Optimal Subpattern Assignment Metric

- Let $\boldsymbol{x} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_I]^{\mathrm{T}}$ be the true joint object state vector and $\hat{\boldsymbol{x}} = [\hat{\boldsymbol{x}}_1, \hat{\boldsymbol{x}}_2, \ldots, \hat{\boldsymbol{x}}_{\hat{I}}]^{\mathrm{T}}$ be the estimated joint object state vector

- Simple version for the case $I = \hat{I}$, $p = 2$, $\eta = \infty$, and $d(\boldsymbol{x}_i, \boldsymbol{x}_j) = \|\boldsymbol{x}_i - \boldsymbol{x}_j\|$

$$d_2^{(\infty)} = \frac{1}{I}\Big(\min_{\pi \in \Pi_I} \sum_{i=1}^{I} \|\boldsymbol{x}_i - \hat{\boldsymbol{x}}_{\pi(i)}\|^2 \Big)^{1/2}$$

  - Π_I is the set of all permutations of $[1, 2, \ldots, I]^{\mathrm{T}}$

Optimal Subpattern Assignment Metric

- General version for $\hat{I} \leqslant I$

$$d_p^{(\eta)}(\boldsymbol{x}, \hat{\boldsymbol{x}}) = \frac{1}{I}\Big(\min_{\pi \in \Pi_I} \sum_{i=1}^{\hat{I}} d^{(\eta)}(\boldsymbol{x}_i, \hat{\boldsymbol{x}}_{\pi(i)})^p + \eta^p (I - \hat{I})\Big)^{1/p}$$

where $d^{(\eta)}(\boldsymbol{x}_i, \hat{\boldsymbol{x}}_j) = \boxed{\min(\eta, d(\boldsymbol{x}_i, \hat{\boldsymbol{x}}_j))}$

Individual object state errors are cutoff at η

Optimal Subpattern Assignment Metric

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Penalty for dimension mismatch

where $d^{(\eta)}(\boldsymbol{x}_i, \hat{\boldsymbol{x}}_j) = \min(\eta, d(\boldsymbol{x}_i, \hat{\boldsymbol{x}}_j))$

Optimal Subpattern Assignment Metric

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where $d^{(\eta)}(\boldsymbol{x}_i, \hat{\boldsymbol{x}}_j) = \min(\eta, d(\boldsymbol{x}_i, \hat{\boldsymbol{x}}_j))$

- General version for $\hat{I} > I$

$$d_p^{(\eta)}(\hat{\boldsymbol{x}}, \boldsymbol{x}) = \frac{1}{\hat{I}}\Big(\min_{\pi \in \Pi_{\hat{I}}} \sum_{i=1}^{I} d^{(\eta)}(\hat{\boldsymbol{x}}_i, \boldsymbol{x}_{\pi(i)})^p + \eta^p (I - \hat{I}) \Big)^{1/p}$$

Summary

- Marginal association probabilities and gating are useful to introduce and remove objects states from the state space (initiate and terminate tracks)
- Distance partitioning can be used to as a preprocessing stage to ``enforce'' the property that each object just produces one measurement
- The very general optimal subpattern assignment metric makes it possible to quantify estimation errors in arbitrary multiobject tracking problems