ECE 161A: Discrete Time Sequences

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Discrete Time Sequences

Sequence of numbers denoted by

$$x = \{x[n]\}$$

where $x[n]$ is the nth number in the sequence, and n is an integer.

Sometimes we will just use $x[n]$ and use context to distinguish between the sequence and the value at time n.

Often these sequences arise by sampling a continuous time signal

$$x[n] = x_a(nT)$$

and so we refer to n as the time variable.

Typical Sequences

1. Unit Sample or Delta function (Kronecker) :

$$x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

2. Delayed delta function :

$$x[n] = \delta[n - n_0] = \begin{cases} 1 & n = n_0 \\ 0 & n \neq n_0 \end{cases}$$

3. Step function :

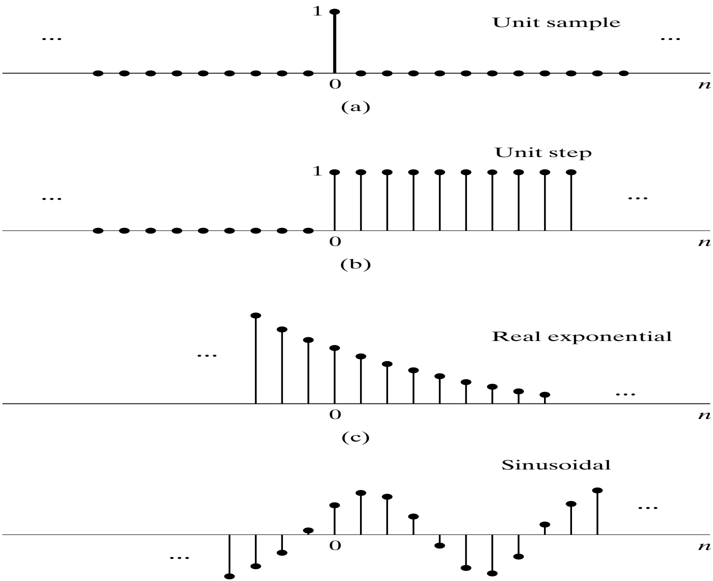
$$x[n] = u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

4. Real Exponential:

$$x[n] = A\alpha^n u[n].$$

5. Sines, Cosines, Complex Exponentials: $x[n] = A\cos(\omega_0 n + \phi)$

Plots of Typical Sequences



General Representation

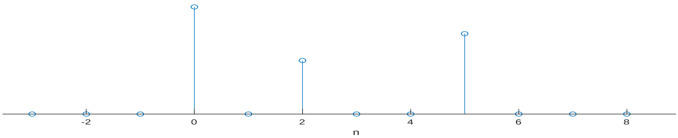
In general, any signal can be written as a weighted sum of delayed delta functions:

$$
\begin{aligned}
x[n] &= \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \\
&= \ldots + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \ldots
\end{aligned}
$$

Example of a Sequence

Consider a sequence

$$x[n] = \begin{cases} 4, & n = 0 \\ 2, & n = 2 \\ 3, & n = 5 \\ 0, & \text{otherwise} \end{cases}$$

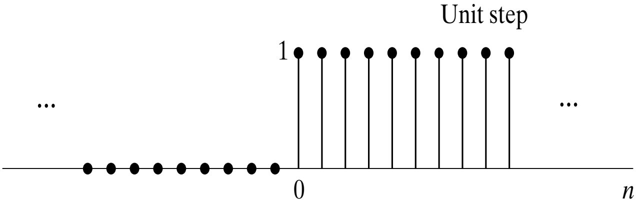


This sequence can be considered a sum of 3 sequences, one for each nonzero value

$$x[n] = 4\delta[n] + 2\delta[n-2] + 3\delta[n-5]$$

Another Example

$$x[n] = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$



This sequence can be considered a sum of infinite sequences, one for each nonzero value

$$x[n] = \sum_{k=0}^{\infty} \delta[n-k].$$

In general $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$.