SIO 207A: Fundamentals of Digital Signal Processing
Class 13

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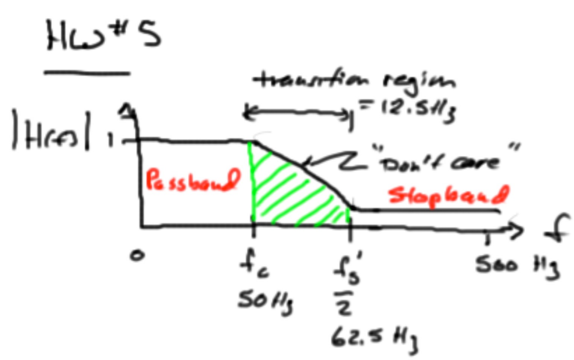
Filter Design – Homework 5 & 6

- In Homework 5, transition width specified as $50\text{Hz} \to 62.5\text{Hz}$ to protect against aliasing (all frequencies above $f_s' = 62.5\text{Hz}$ will alias into region below 62.5Hz)
- However, since the transition region is a "don't care" region, might as well allow aliasing of signals above 62.5Hz into this region



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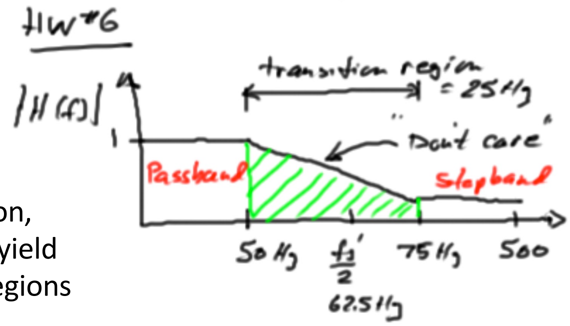
Filter Design – Homework 5 & 6

- In Homework 5, transition width specified as $50\mathrm{Hz} \to 62.5\mathrm{Hz}$ to protect against aliasing (all frequencies above $f_s' = 62.5\mathrm{Hz}$ will alias into region below $62.5\mathrm{Hz}$)
- However, since the transition region is a "don't care" region, might as well allow aliasing of signals above $62.5\mathrm{Hz}$ into this region
- Thus expanding the transition region to $50\mathrm{Hz} \to 75\mathrm{Hz}$ will not result in aliasing into the region we care about $(0\mathrm{Hz} \to 50\mathrm{Hz})$
- The benefit is that the wider the transition region, the easier it is for the filter design algorithm to yield smaller ripples in the passband and stopband regions



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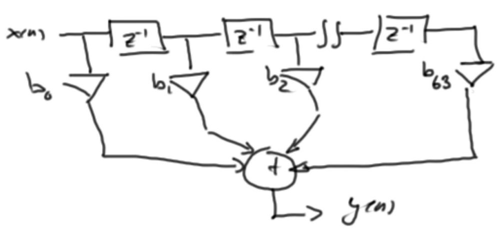
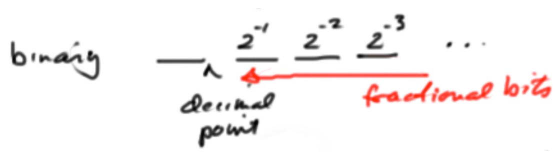
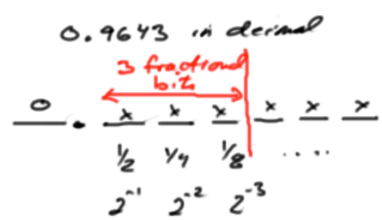
Filter Design – Homework 6





- Normalize impulse response $h[n]$ so that largest coefficients is 1
- Multiply coefficients by 2^b, i.e., $b = 3, 2^3 = 8$
- Round to nearest integer part
- Multiply by 2^{-b}
- Look at the characteristics of quantized impulse response



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Frequency Sampling Approach to FIR Filter Design

- Let us assume we design an ideal filter in discrete frequency domain by setting all samples in stopband to 0

$$H(e^{j\omega}) = e^{-j\omega\frac{N-1}{2}} \sum_{k=0}^{N-1} \tilde{H}(k)\, e^{j\frac{2\pi}{N}k\frac{N-1}{2}} \frac{\sin\left[N\frac{\omega-\frac{2\pi}{N}k}{2}\right]}{\sin\left[\frac{\omega-\frac{2\pi}{N}k}{2}\right]}$$

samples of desired frequency response

$$\tilde{h}[n] = \mathrm{IFFT}\left(\tilde{H}(k)\right)$$

impulse response with N samples period

see also Section 7.4 in *Oppenheim & Schafer, 1999*: “Optimal Approximations of FIR Filters”

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Frequency Sampling Approach to FIR Filter Design

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interpolation function

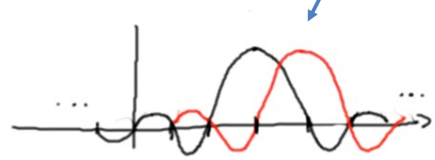
samples of desired frequency response

$$\tilde{h}[n] = \mathrm{IFFT}\left(\tilde{H}(k)\right)$$

impulse response with N samples period

FIR filter response $h[n]$ is equal to the first N samples of $\tilde{h}[n]$ and zero otherwise

$\longrightarrow$ multiplication of $\tilde{h}[n]$ with rectangular window



$$k'' \rightarrow \omega'' = \frac{2\pi}{N}k''$$

$$k' \rightarrow \omega' = \frac{2\pi}{N}k'$$

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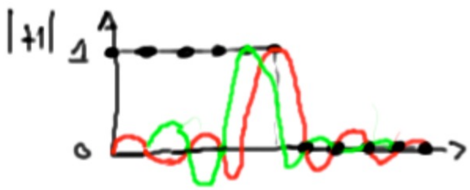
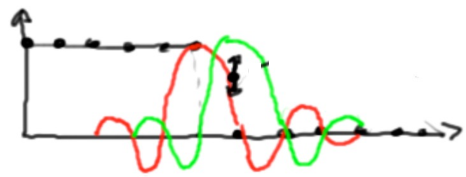
Frequency Sampling Approach to FIR Filter Design

- Instead of just using samples of the ideal response, include 2-3 transition regions samples to mitigate the high sidelobes

Continuous Frequency Response Synthesis of “Ideal” Discrete Filter



Continuous Frequency Response Synthesis: Equiripple Design



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Frequency Sampling Approach to FIR Filter Design

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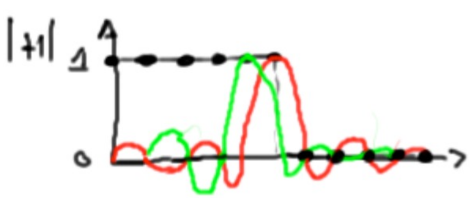
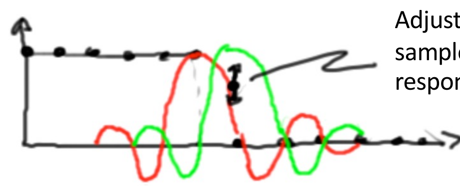


Continuous Frequency Response Synthesis: Equiripple Design



Adjust value of transition sample to minimize stopband response levels

see also Section 7.4.3 in *Oppenheim & Schafer, 1999*: “The Parks-McClellan algorithm” (Equiripple design)

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