SIO 207A: Fundamentals of Digital Signal Processing
Class 6

Florian Meyer

Scripps Institution of Oceanography
Electrical and Computer Engineering Department
University of California San Diego

SCRIPPS INSTITUTION OF OCEANOGRAPHY

UC San Diego
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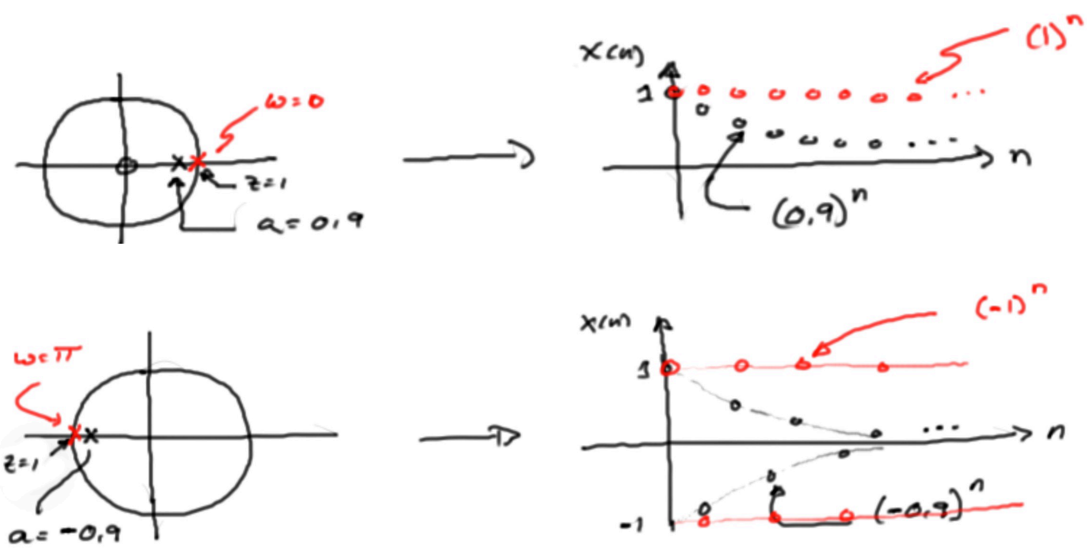
Z-Plane Pole Locations and Their Corresponding Time Series



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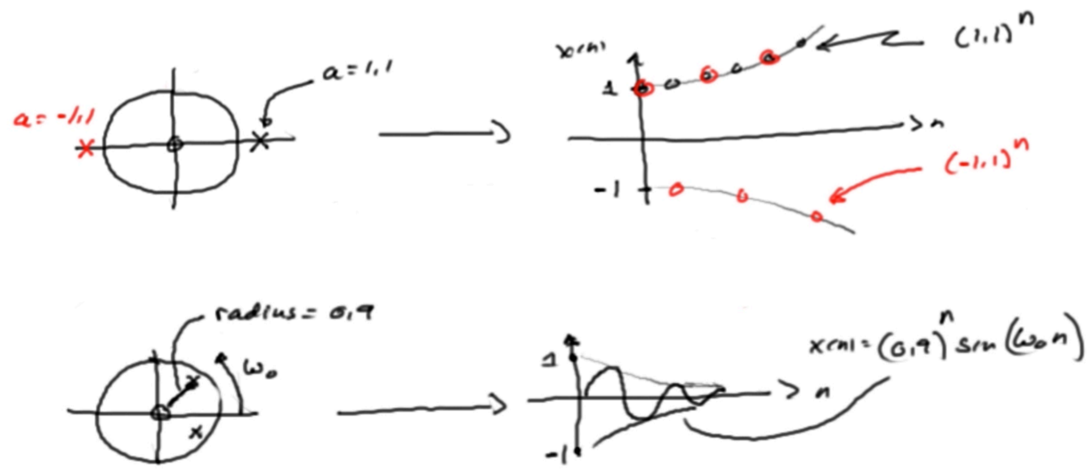
Z-Plane Pole Locations and Their Corresponding Time Series



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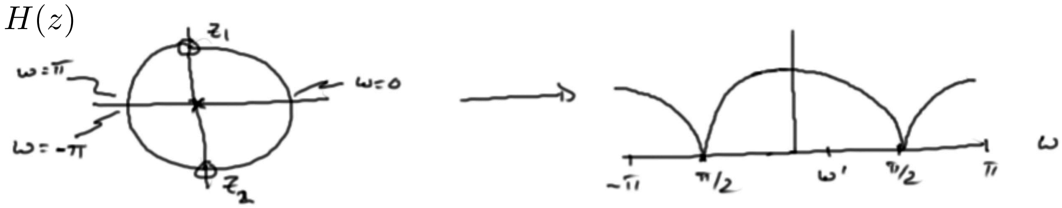
Z-Domain Analysis

$H(z)$



$$H(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2} = \frac{(z - z_1)(z - z_2)}{z^2}$$

$$H(z)\Big|_{z=e^{j\omega'}} = \frac{(e^{j\omega'} - z_1)(e^{j\omega'} - z_2)}{\boxed{(e^{j\omega'})^2}}$$ magnitude equal to 1

$$\left|H(e^{j\omega'})\right| = \left|e^{j\omega'} - z_1\right|\left|e^{j\omega'} - z_2\right|$$

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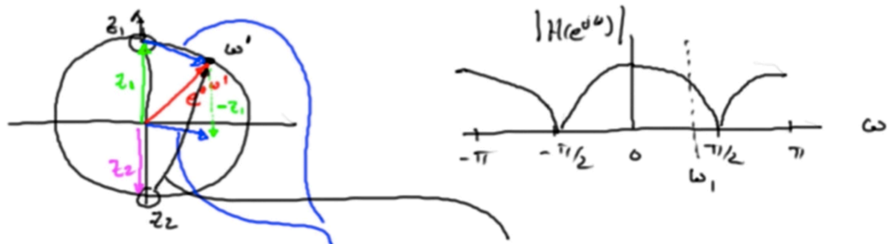
Z-Domain Analysis



$$|H(e^{j\omega'})| = |e^{j\omega'} - z_1|\,|e^{j\omega'} - z_2|$$

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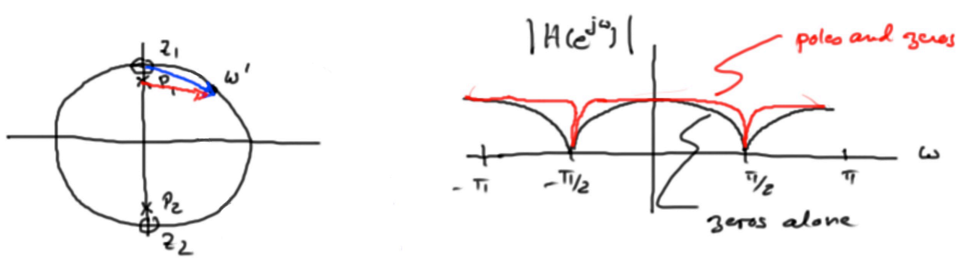
Z-Domain Analysis



$$H(z) = \frac{(z - z_1)(z - z_2)}{(z - p_1)(z - p_2)}$$

$$\left|H(e^{j\omega'})\right| = \frac{|e^{j\omega'} - z_1||e^{j\omega'} - z_2|}{|e^{j\omega'} - p_1||e^{j\omega'} - p_2|}$$

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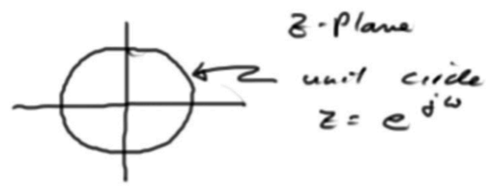
Recall Fourier Transform

- Fourier transform is z-transform evaluated on the unit circle

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$



$$X(z)\big|_{z=e^{j\omega}} = X(e^{j\omega})$$

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Recall Fourier Transform

- Analysis:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} \qquad (1)$$

- Synthesis:

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, \mathrm{d}\omega \qquad (2)$$

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Discrete Fourier Series

- **A.** Discrete Fourier Series

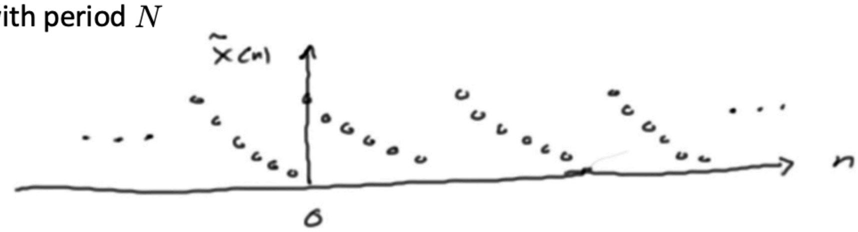
Assume $x[n]$ has finite length N, i.e.,

$$x[n] = 0, \quad n < 0 \text{ and } n \geq N$$

Form periodic replication $\tilde{x}[n]$ such that $\tilde{x}[n + lN] = x[n]$

$$n = 0, \ldots, N-1 \quad \text{and } l \in \mathbb{N}$$

$\tilde{x}[n]$ is periodic with period N



Discrete Fourier Series

$\tilde{x}[n]$ can be represented by a (complex exponential) Fourier series

- harmonically related sequences $e^{j\frac{2\pi}{N}kn}$ $\begin{cases} n \text{ is a sample index} \\ k \text{ is a frequency index} \end{cases}$

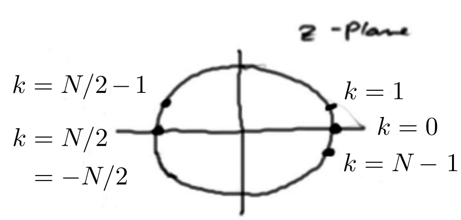
frequency:

$$\omega = \frac{2\pi}{N}k$$

- is periodic in k with period N

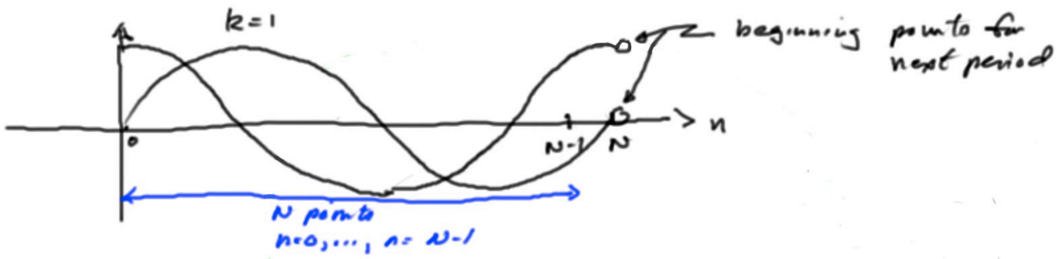
  only sequences for $k = 0, \ldots, N-1$ are required

- interpret k as the number of cycles per period



$$k = \{0, 1, \ldots, N-1\}$$
$$= \{-N/2, \ldots, 0, \ldots, N/2 - 1\}$$

Discrete Fourier Series



$$e^{j\frac{2\pi}{N}kn} = \cos\left(\underbrace{\boxed{\frac{2\pi}{N}}}_{}^{=\omega} kn\right) + j\sin\left(\boxed{\frac{2\pi}{N}}^{=\omega} kn\right)$$

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