ECE 286: Bayesian Machine Perception

Class 15: Cooperative Localization

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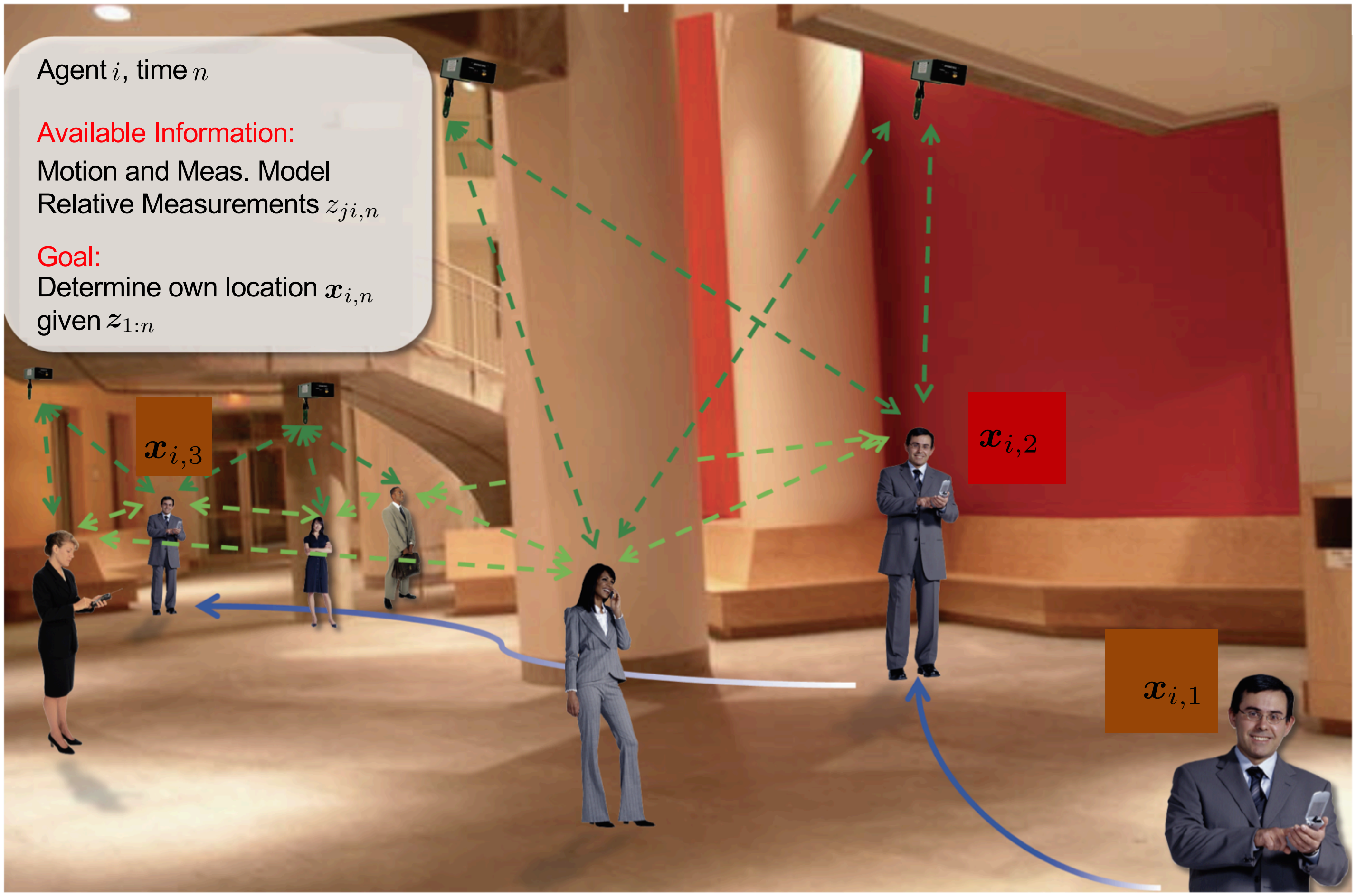
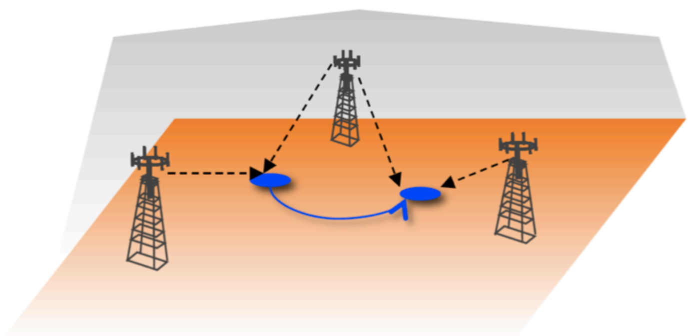
The Cooperative Localization Problem

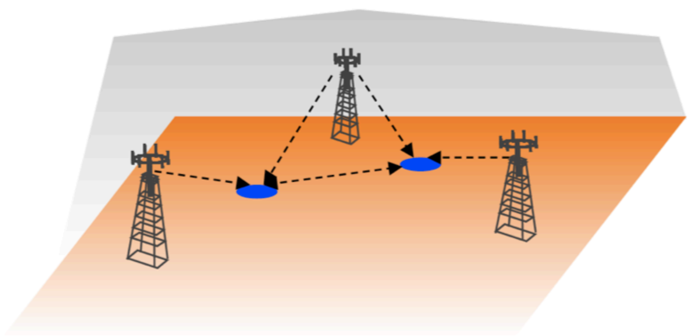


Simplest Nontrivial Cases



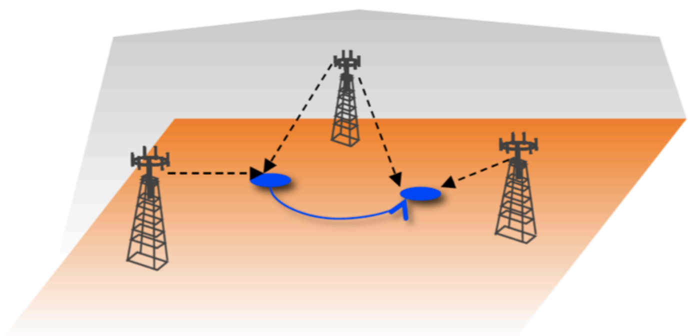
One Mobile Agent

→ Sequential Bayesian Estimation

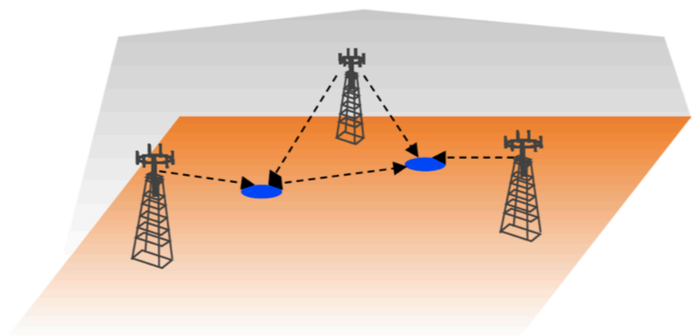


Multiple Static Agents

Simplest Nontrivial Cases



One Mobile Agent



Multiple Static Agents

Prior Distribution

- Let us consider a network that consists of I_a agents (with index set $\mathcal{I}_\mathrm{a}$) and of I_b anchors (with index set $\mathcal{I}_\mathrm{b}$)

- The edge set $\mathcal{E}$ of the network consists of links (i, j) over which pairwise measurements $\boldsymbol{z}_{ij}$ are performed

- Agents perform measurements with anchors and with other agents

- Joint prior pdf:

$$f(\boldsymbol{x}) = \prod_{i \in \mathcal{I}_\mathrm{a} \cup \mathcal{I}_\mathrm{b}} f(\boldsymbol{x}_i)$$

with $f(\boldsymbol{x}_i)$ informative (e.g., $f(\boldsymbol{x}_i) = \delta(\boldsymbol{x}_i - \boldsymbol{p}_i)$) for anchors $i \in \mathcal{I}_\mathrm{b}$ and uninformative (e.g., uniform) for agents $i \in \mathcal{I}_\mathrm{a}$

Statistical Model

- Joint likelihood function:

$$f(\boldsymbol{z}|\boldsymbol{x}) = \prod_{(i,j)\in\mathcal{E}} f(\boldsymbol{z}_{ij}|\boldsymbol{x}_i, \boldsymbol{x}_j)$$

- Joint posterior pdf:

$$f(\boldsymbol{x}|\boldsymbol{z}) \propto f(\boldsymbol{z}|\boldsymbol{x}) f(\boldsymbol{x}) = \prod_{i\in\mathcal{I}_\mathrm{a}\cup\mathcal{I}_\mathrm{b}} f(\boldsymbol{x}_i) \left(\prod_{(i,j)\in\mathcal{E}} f(\boldsymbol{z}_{ij}|\boldsymbol{x}_i, \boldsymbol{x}_j) \right)$$

- Recall that estimates $\hat{\boldsymbol{x}}_i$ for the individual agent states $\boldsymbol{x}_i$, rely on the marginal posteriors

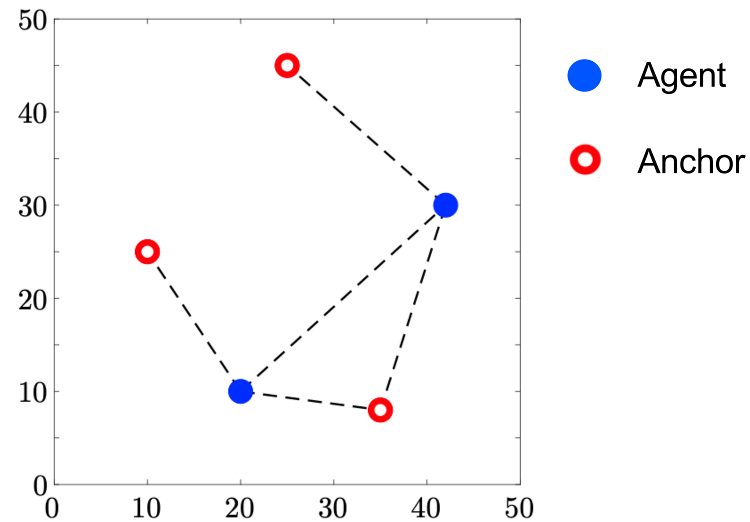
$$f(\boldsymbol{x}_i|\boldsymbol{z}) = \int f(\boldsymbol{x}|\boldsymbol{z}) \mathrm{d}\boldsymbol{x}_{\sim i}$$

Example

- The network of agents consists of $I_\mathrm{a} = 2$ agents (with index set $\mathcal{I}_\mathrm{a} = \{1, 2\}$) and $I_\mathrm{b} = 3$ anchors (with index set $\mathcal{I}_\mathrm{b} = \{3, 4, 5\}$)
- The edge set of the network is given by $\mathcal{E} \triangleq \big\{(1,2)\,(1,3)\,(1,4)\,(2,3)\,(2,5)\big\}$
- Anchors have perfect prior location information, i.e.,

$$f(\boldsymbol{x}_i) = \delta(\boldsymbol{x}_i - \boldsymbol{p}_i), \quad i \in \{3, 4, 5\}$$

where $\boldsymbol{p}_i$ is the true position of anchor i



Example

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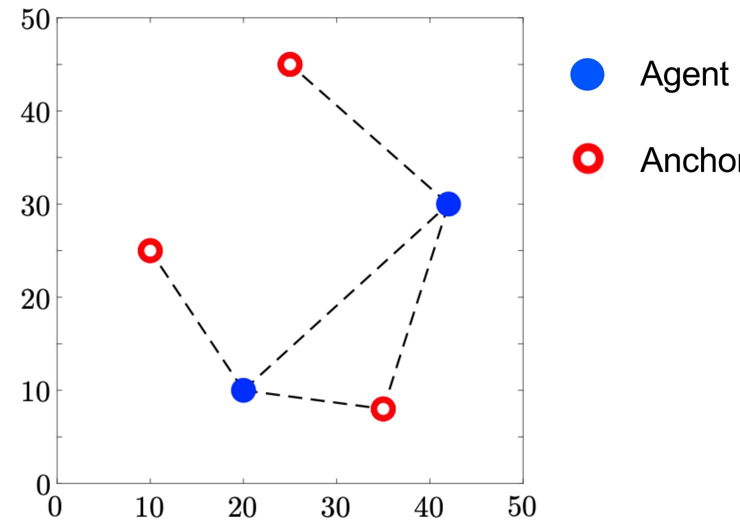
- The edge set of the network is given by $\mathcal{E} \triangleq \{(1,2)\,(1,3)\,(1,4)\,(2,3)\,(2,5)\}$
- Agents $i \in \{1, 2\}$ have a prior location information $f(\boldsymbol{x}_i)$ that is uniform on the area of interest $[0, 50)^2$
- The agent perform distance measurements with their neighbors in the network, i.e.,

$$z_{ij} = \|\boldsymbol{x}_i - \boldsymbol{x}_j\| + \boxed{v_{ij}}$$

with $v_{ij} \sim \mathcal{N}(0, \sigma_{\mathrm{v}}^2)$

Statistical independent across edges (i, j)



Marginal Posterior Pdfs

- For agent $i = 1$ the marginal posterior pdf can be calculated as

$$f(\boldsymbol{x}_1|\boldsymbol{z}) = \int f(\boldsymbol{x}|\boldsymbol{z})\,\mathrm{d}\boldsymbol{x}_{\sim 1} \propto \int\!\!\int\!\!\int\!\!\int f(\boldsymbol{z}|\boldsymbol{x})\,f(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x}_2\,\mathrm{d}\boldsymbol{x}_3\,\mathrm{d}\boldsymbol{x}_4\,\mathrm{d}\boldsymbol{x}_5$$

Shifting property of $\delta(\boldsymbol{x}_i - \boldsymbol{p}_i), i \in \{3,4,5\}$

$$\longrightarrow \; = f(\boldsymbol{x}_1)\,f(z_{13}|\boldsymbol{x}_1,\boldsymbol{p}_3)\,f(z_{14}|\boldsymbol{x}_1,\boldsymbol{p}_4)\,\tilde{f}(z_{12}|\boldsymbol{x}_1)$$

where the likelihood function related to the inter-agent measurement z_{12} is given by

$$\tilde{f}(z_{12}|\boldsymbol{x}_1) = \int f(\boldsymbol{x}_2)\,f(z_{23}|\boldsymbol{x}_2,\boldsymbol{p}_3)\,f(z_{24}|\boldsymbol{x}_2,\boldsymbol{p}_5)\,f(z_{12}|\boldsymbol{x}_2,\boldsymbol{x}_1)\,\mathrm{d}\boldsymbol{x}_2$$

Marginal Posterior Pdfs

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$$= f(\boldsymbol{x}_1)\,f(z_{13}|\boldsymbol{x}_1,\boldsymbol{p}_3)\,f(z_{14}|\boldsymbol{x}_1,\boldsymbol{p}_4)\,\tilde{f}(z_{12}|\boldsymbol{x}_1)$$

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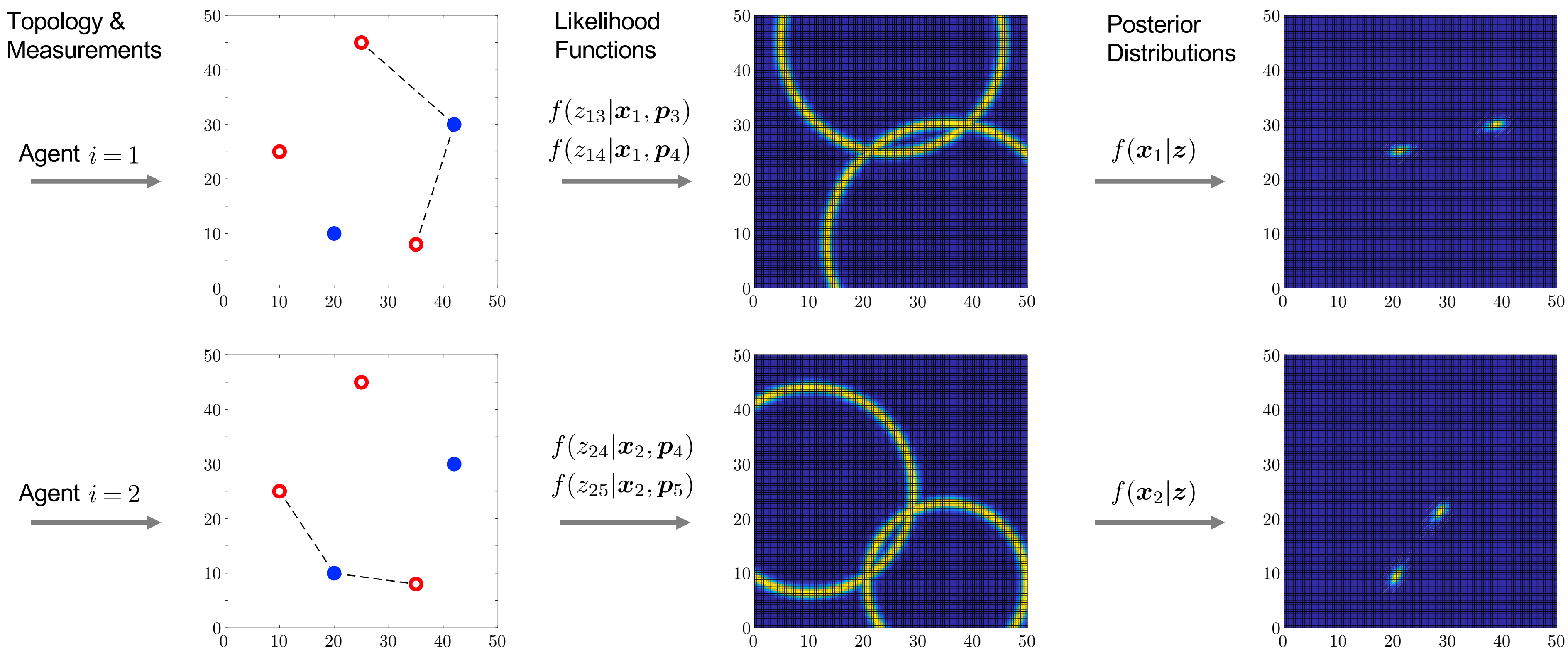
- The marginal posterior consists of the noncooperative solution and the likelihood function related to cooperation

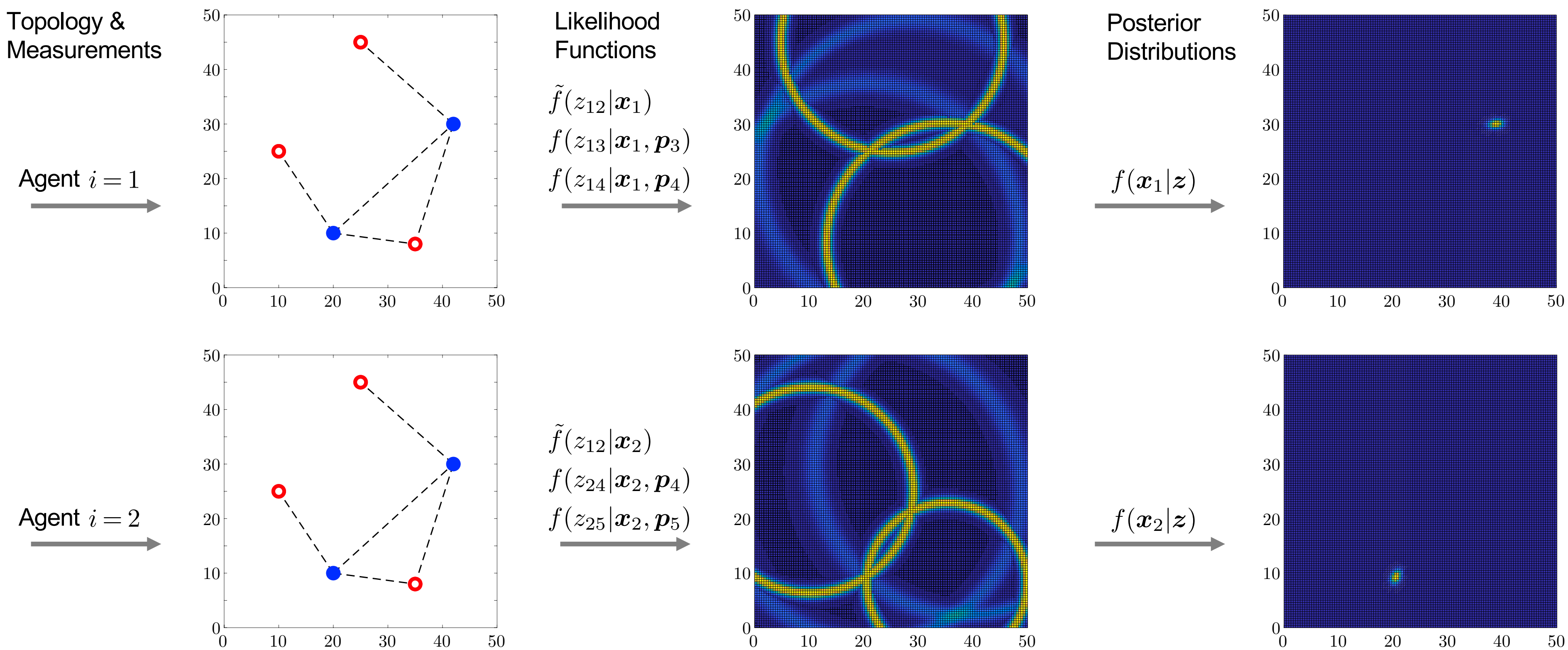
Noncooperative Case

- Location information of anchor-to-agent measurements



Cooperative Case

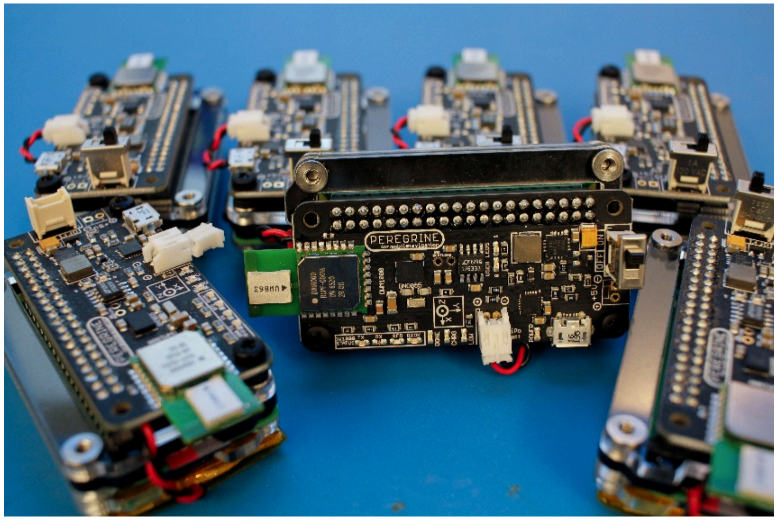
- **Location information of anchor-to-agent measurements**



Peregrine Localization System

- In-house hardware nodes for 3D network localization and navigation
- Key components:
  - inexpensive *Raspberry Pi Zero* PCU running Linux OS
  - commercial ultrawide-bandwith (UWB) radio
  - software written in C and Python
- Key features:
  - easily deployable and highly scalable
  - automatic neighbor discovery
  - holistic and opportunistic medium access
  - channel quality sensing
  - fully *distributed* and *asynchronous* operation

2018 R&D 100 Award



B. Teague, Z. Liu, F. Meyer, A. Conti, and M. Z. Win, “Peregrine: Network localization and navigation with scalable inference and efficient operation,” 2019. https://winslab.lids.mit.edu/

BRIAN BARRETT GEAR 09.12.2019 07:00 AM

The Biggest iPhone News Is a Tiny New Chip Inside It

By embracing ultra-wideband location tech, Apple has a chance to reshape experiences way beyond AirDrop.

Apple CEO Tim Cook introduces the new iPhone at the Steve Jobs Theater in Cupertino, California, on Tuesday. DAVID PAUL MORRIS/BLOOMBERG/GETTY IMAGES

Summary

- In cooperative localization, each agent of a networks aims to estimate its own state (that includes the agent's position) from all available measurements in the network
- The complexity of this nonlinear and high-dimensional estimation problem can be dramatically reduced by means of the framework of factor graphs and the sum-product algorithm
- A distributed implementation of a dynamic message passing algorithm is possible if the communication graph of the agent network coincides with the factor graph