ECE 175B: Probabilistic Reasoning and Graphical Models
Lecture 9: Final Comments on Causality, Confounding, and Simpson's Paradox

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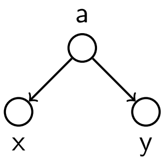
"Guaranteed" ($\square$) versus "Possibly" ($\diamond$)

g_0 :

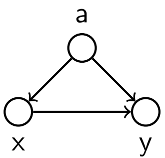


$P_0(x, y, a) = P_0(x|a)P_0(y|a)P_0(a)$

or

which model?

g_1 :



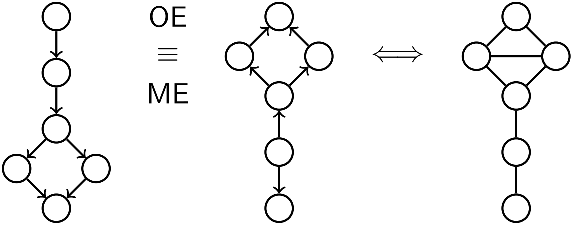
$P_1(x, y, a) = P_1(y|x, a)P_1(x|a)P_1(a)$

(The shown compatibilities are denoted $P_0 \sim g_0$ and $P_1 \sim g_1$)

- $P_0(x, y|a) = P_0(x|a)P_0(y|a) \implies \square(x \perp\!\!\!\perp y|a)$ guaranteed for all $P_0 \sim g_0$
- $P_1(x, y|a) = P_1(y|x, a)P_1(x|a) \implies \diamond(x \top\!\!\!\top y|a)$ possibly that $P_1(y|x, a) = P_1(y|a)$
- Suppose from observational data we learn that $P_1(y|x, a) = P_1(y|a)$ then $P_1(x, y|a) = P_1(x|a)P_1(y|a) \Leftrightarrow x \perp\!\!\!\perp_{P_1} y \mid a$
- Thus, until we verify otherwise, it is safest to take
  - $\diamond$ = "it is possible that, but not guaranteed" for a compatible distribution

Markov Equivalence

- Two BNs are markov equivalent (ME) iff they encode the same conditional independencies
- All ME graphs have the same moral graph and the same immoralities
  - An immorality is a node that has two or more parents without edge between them
- Graphs that are ME are also called observationally equivalent (OE) because from observational data alone they can't be distinguished



(A) (B) (M)

Learning Causal Structure

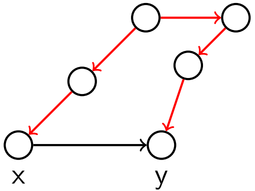
- Algorithms exist that can learn one of many possible BNs from observational data
- From the learned BN all OE graphs can be found
- But which one is causally relevant?
- Causal structure is always an "extra ingredient"

Learning Causal Structure

- A human can often provide this "extra ingredient"
- Note that (A) $\overset{\text{OE}}{\equiv}$ (B), but are not causally equivalent
- A human can suggest a likely causal model that can be tested by doing $\mathrm{x} = x$ and testing against the prediction $P(y|\mathrm{do}(\mathrm{x} = x))$ given by the model
- To learn causal directions "from scratch" requires a controlled intervention: $\mathrm{do}(\delta y) \Rightarrow \delta x \neq 0$? or $\mathrm{do}(\delta x) \Rightarrow \delta y \neq 0$?
- This determines the direction: $\mathrm{x} \to \mathrm{y}$ or $\mathrm{y} \to \mathrm{x}$

Backdoor Paths (BDPs) and Deconfounding

- Assume a Causal BN: A backdoor path from x to y is a path beginning with an arrow into x and ending at y which is unblocked (active)

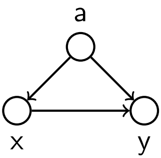


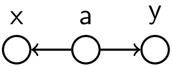
- A BDP (shown in red) allows information flow from x to y, and vice versa, because it is active (unblocked)

- We say that confounding exists when $P(y|\mathrm{do}(\mathrm{x} = x)) \neq P(y|x)$
- There is a BDP $\implies$ confounding exists
- To control for confounding, we can condition on a node that blocks the BDP; such a node is a deconfounder

Backdoor Paths (BDPs) and Deconfounding

Example:





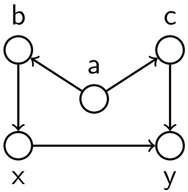
confounding

is a BDP $\implies P(y|\mathrm{do}(\mathrm{x}=x)) \neq P(y|x)$, i.e., we have

- By conditioning on random variable a we block the BDP $\implies$ a is a deconfounder
- We find that $P\big(y\,|\,\mathrm{do}(\mathrm{x}=x), a\big) = P(y|\mathrm{x}, a)$
- Fixing a "level" $\mathrm{a} = a$ controls for confounding
- Finally, we average over the levels, i.e.,

$$P(y|\mathrm{do}(\mathrm{x}=x)) = \sum_a P(y|x,a)P(a)$$

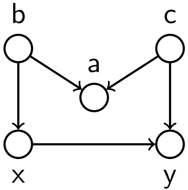
Examples

1. 

BDP: $x \leftarrow b \leftarrow a \rightarrow c \rightarrow y$

Confounding exists: $P(y|\mathrm{do}(x)) \neq P(y|x)$

Any variable a, b, or c can be used as a deconfounder

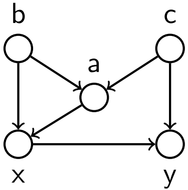
2. 

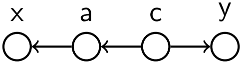
There is NO BDP; therefore, do nothing

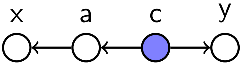
$P(y|\mathrm{do}(x)) = P(y|x)$; no confounding exists

Controlling for variable a ruins the situation!

Examples

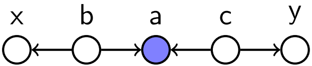
3. 

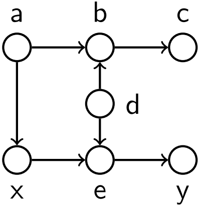
BDP: 

Block using c = c: 

Confounding exists: $P(y|\text{do}(x)) \neq P(y|x)$

Do NOT condition of a, because this then opens a new confounding path:



4. 

There is no BDP; thus do nothing

$P(y|\text{do}(x)) = P(y|x)$; there is no confounding

$\Rightarrow$ "Controlling for b" ruins the situation

$\Rightarrow$ Controlling for a, c, d is wasted effort

Simpson's Paradox – Barber Section 3.4

- A drug has become widely used on an "off-label" basis because doctors have heard anecdotal claims that it increases the recovery rate of a disease.

- An *observational study* is set up asking for N_f = 40 female and N_m = 40 male participants. Observational data already exists, and subjects are found by randomly making phone calls until the required number of 40 male and 40 female subjects are found who agree to release their data to the study.

- Below is the tabulated aggregated data, including recovery rate data, from the N = 80 subjects.

Males	Recovered	Not Recovered	Rec. Rate
Given Drug	18	12	60%
Not Given Drug	7	3	70%

For the **male subjects** it ***seems*** better **to not take** the drug.

Females	Recovered	Not Recovered	Rec. Rate
Given Drug	2	8	20%
Not Given Drug	9	21	30%

For the **female subjects** it ***seems*** better **to not take** the drug.

Combined	Recovered	Not Recovered	Rec. Rate
Given Drug	20	20	50%
Not Given Drug	16	24	40%

For **all subjects** it ***seems*** better **to take** the drug.

What is going on? Note that "seems" refers to *"seeing"*, not *"doing"*

Simpson's Paradox

$$\text{Males} \left\{ \begin{array}{l} P(\mathrm{r}=y|\mathrm{d}=y,\mathrm{g}=m)=60\% \\ P(\mathrm{r}=y|\mathrm{d}=n,\mathrm{g}=m)=70\% \end{array} \right\} \Delta_m = -10\% \quad \text{bad for males}$$

$$\text{Females} \left\{ \begin{array}{l} P(\mathrm{r}=y|\mathrm{d}=y,\mathrm{g}=f)=20\% \\ P(\mathrm{r}=y|\mathrm{d}=n,\mathrm{g}=f)=30\% \end{array} \right\} \Delta_f = -10\% \quad \text{bad for females}$$

$$\begin{aligned}
\text{Total: } P(\mathrm{r}=y|\mathrm{d}=y) &= \sum_g P(\mathrm{r}=y,g|\mathrm{d}=y) \\
&= \sum_g P(\mathrm{r}=y|g,\mathrm{d}=y)P(g|\mathrm{d}=y) \\
&= \underbrace{P(\mathrm{r}=y|\mathrm{g}=m,\mathrm{d}=y)}_{60\%}\underbrace{P(\mathrm{g}=m|\mathrm{d}=y)}_{3/4} \\
&\quad + \underbrace{P(\mathrm{r}=y|\mathrm{g}=f,\mathrm{d}=y)}_{20\%}\underbrace{P(\mathrm{g}=f|\mathrm{d}=y)}_{1/4} \\
&= 50\%
\end{aligned}$$

Simpson's Paradox

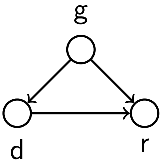
$$\begin{aligned} P(\mathrm{r}=y|\mathrm{d}=n) &= \underbrace{P(\mathrm{r}=y|\mathrm{g}=m,\mathrm{d}=n)}_{70\%}\underbrace{P(\mathrm{g}=m|\mathrm{d}=n)}_{1/4} \\ &\quad + \underbrace{P(\mathrm{r}=y|\mathrm{g}=f,\mathrm{d}=n)}_{30\%}\underbrace{P(\mathrm{g}=f|\mathrm{d}=n)}_{3/4} \\ &= 40\% \end{aligned}$$

$\boxed{\Delta_{\text{total}} = +10\%}$ better for $\mathrm{d} = y$ cohort which is true, i.e., "see" probabilities are good for prediction

Simpson's Paradox

- If someone is drawn at random from the 40 members of cohort "D=y" (3/4 male), they have a 50% chance of recovery
- If someone is drawn at random from the 40 members of cohort "D=n" (3/4 female), they have a 40% chance of recovery
- But that is not our interest; we want to know $P\big(\mathrm{r} = y \big| \mathrm{do}(\mathrm{d} = y)\big)$

Simpson's Paradox

For the model 

we have shown that

- $P(\mathrm{r}=y|\mathrm{do}(\mathrm{d}=y)) = \sum_g P(\mathrm{r}=y|\mathrm{d}=y,g)\underbrace{P(g)}_{1/2} = 40\%$

  Note: $P(\mathrm{g}=m) = P(\mathrm{g}=f) = \frac{1}{2}$ since there is no gender imbalance in population

- $P(\mathrm{r}=y|\mathrm{d}=y) = \sum_g P(\mathrm{r}=y|\mathrm{d}=y,g)\underbrace{P(g|\mathrm{d}=y)}_{\neq 1/2} = 50\%$

  Note: $\begin{cases} P(\mathrm{g}=m|\mathrm{d}=y) = 3/4 \\ P(\mathrm{g}=f|\mathrm{d}=y) = 1/4 \end{cases}$ gender imbalance in cohort $\mathrm{d}=y$

- $P(\mathrm{r}=y|\mathrm{do}(\mathrm{d}=n)) = 50\%$

- $\boxed{\Delta_{\mathrm{do}} = 40\% - 50\% = -10\%}$