SIO 207A: Fundamentals of Digital Signal Processing
Class 17

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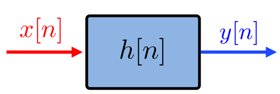
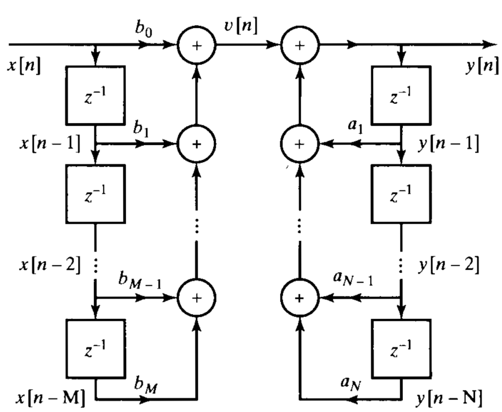
Infinite Impulse Response (IIR) Filter Structures

System Input/Output Description:





Linear Constant Difference Equation:

$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{r=0}^{M} b_r x[n-r]$$

Z-Transform:

$$Y(z) = \sum_{k=1}^{N} a_k z^{-k} Y(z) + \sum_{r=0}^{M} b_r z^{-r} X(z)$$
$$= H(z)X(z)$$

Z-Domain Representation:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{r=0}^{M} b_r z^{-r}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$
$$= \frac{b_0 z^M + b_1 z^{M-1} + \cdots + b_M}{z^N + a_1 z^{N-1} + \cdots + a_N}$$

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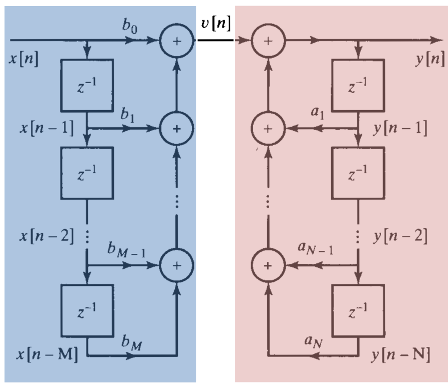
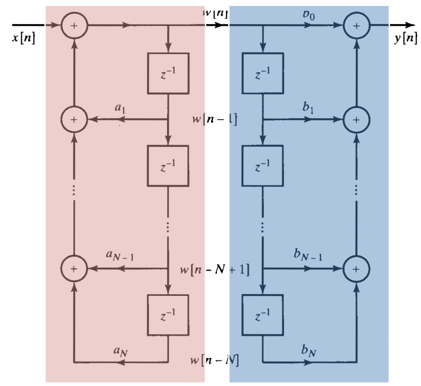
Infinite Impulse Response (IIR) Filter Structures

Direct Form I:



Direct Form II:



$$v[n] = \sum_{k=0}^{M} b_k x[n-k] \qquad y[n] = \sum_{k=1}^{N} a_k y[n-k] + v[n]$$

$$w[n] = \sum_{k=1}^{N} a_k w[n-k] + x[n] \qquad y[n] = \sum_{k=0}^{M} b_k w[n-k]$$

Since we have a linear system we can interchange subsystem one with subsystem two

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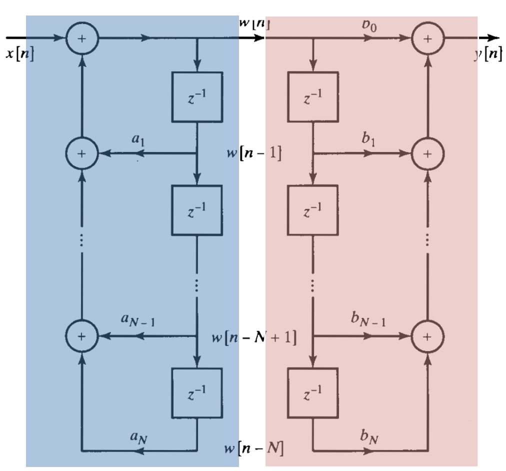
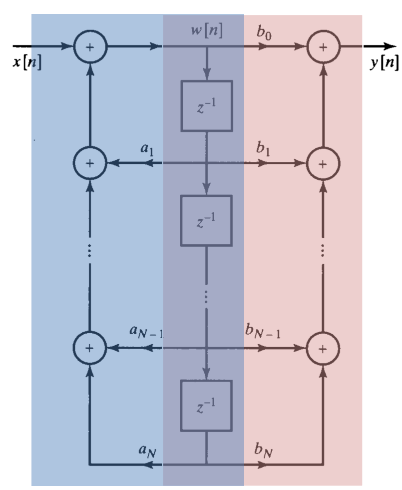
Infinite Impulse Response (IIR) Filter Structures

In **direct form II**, we can save half of the delays by combining the vertical "delay line" of the two subsystems





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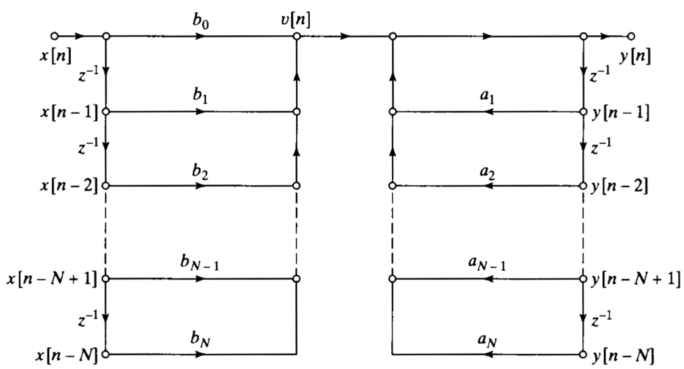
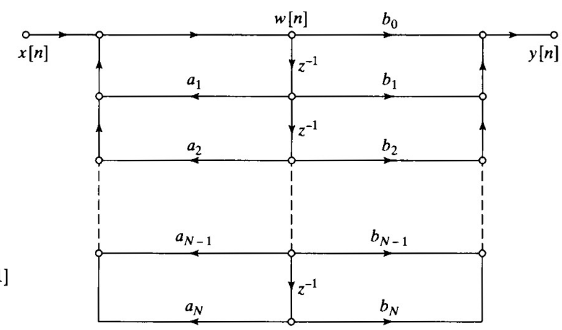
Representation as Flow Diagrams

- If multiple signals enter a node, this represents a summation
- Every branch is a linear operation indicated by a coefficient (no coefficient is equal to a multiplication by 1)

Direct Form I:



Direct Form II:



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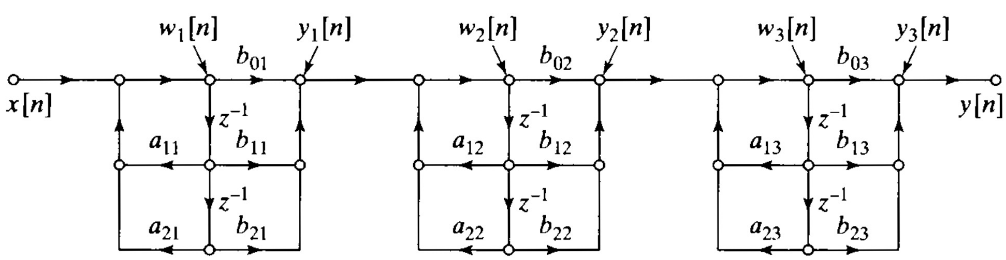
Cascade of 2nd Order Filters

$$H(z) = \prod_{k=1}^{N_s} \frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$



Note that possible pole and zero locations after quantization depend on filter structure

This implementation is typically used for numerical stability

see also Section 6.3, 6.4, and 6.8 in *Oppenheim & Schafer, 1999*

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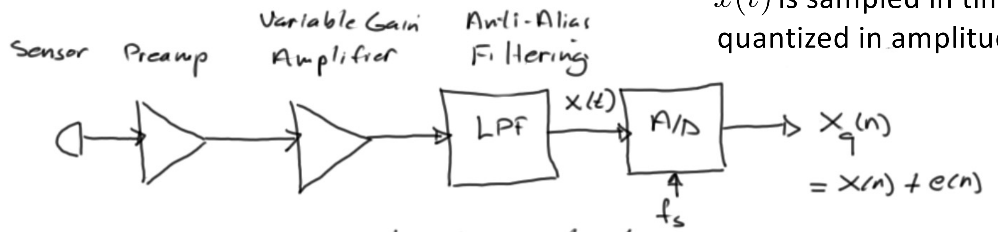
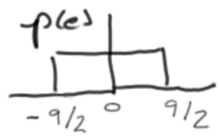
Quantization Noise at the A/D Converter Output

- Recall the block diagram of a data acquisition system



$x(t)$ is sampled in time and quantized in amplitude

- $e[n]$ is error due to A/D amplitude quantization also known as quantization noise
- $e[n]$ can be modeled of a sequence of independent random variables with uniform probability density function



q quantization level in volt (V)

$\sigma_e^2 = q^2/12$ variance/power

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Noise Sources

- There are three sources of noise in the output signal of an analog to digital (A/D) converter
  1. Ambient noise in the environment
  2. Electronics-related self-noise (typically dominated by the preamp immediately after the sensors)
  3. A/D converter quantization noise (governed by the aperture of the A/D and number of bits)

- Quantization noise example: an A/D with aperture [-5V,+5V] (10V in total) and 10 bits (1024 levels) has a resolution of $q = 10\mathrm{V}/1024 \approx 10\mathrm{mV}$

  Note: An additional source of error is clipping in case the analog signal $x(t)$ exceeds the A/D aperture

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Noise Sources

- It is desired that the ambient noise dominates over electronics self-noise and A/D converter quantization noise

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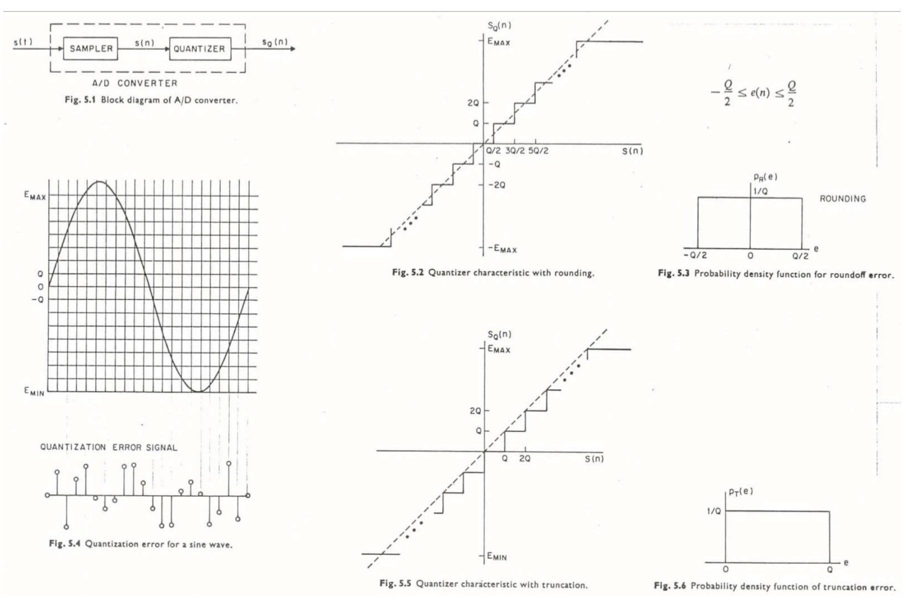
Quantization Noise



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Quantization Noise

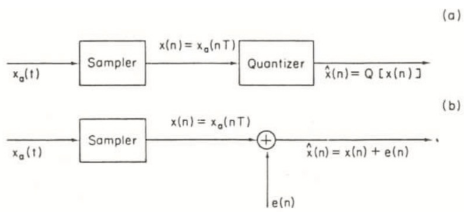


Fig. 9.2 Representation of sampling of an analog signal: (a) nonlinear model; (b) statistical model.

$$-\frac{\Delta}{2} < e(n) \leq \frac{\Delta}{2}$$

1. The sequence of error samples $\{e(n)\}$ is a sample sequence of a stationary random process.
2. The error sequence is uncorrelated with the sequence of exact samples $\{x(n)\}$.
3. The random variables of the error process are uncorrelated; i.e., the error is a white-noise process.
4. The probability distribution of the error process is uniform over the range of quantization error.

$$\hat{x}(n) = Q[x(n)] = x(n) + e(n)$$

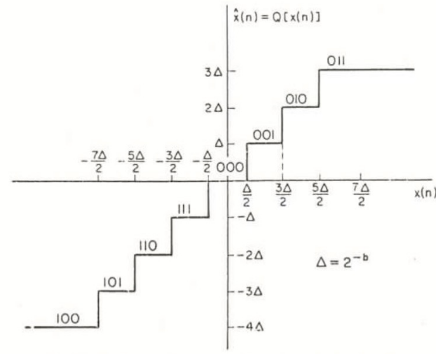
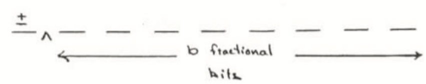


Fig. 9.3 Two's-complement rounding of samples for $b = 2$.



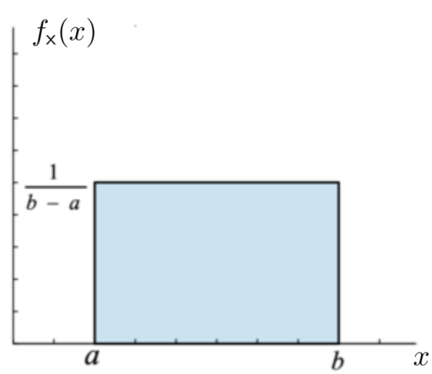
Uniformly Distributed Random Variable

Probability Density Function:

$$f_x(x) = \begin{cases} \frac{1}{b-a} & \text{for } a < x < b \\ 0 & \text{for } a < x \text{ or } x > b \end{cases}$$

Mean: $\mu_x = \dfrac{b+a}{2}$

Standard Deviation: $\sigma_x = \dfrac{b-a}{\sqrt{12}}$



Quantization Noise – SNR Calculations

$$\frac{\sigma_x^2}{\sigma_e^2} = \frac{\sigma_x^2}{2^{-2b}/12} = \left(12 \cdot 2^{2b}\right) \sigma_x^2$$

$$\text{SNR} = 6.02b + 10.79 + 10\log \sigma_x^2 \ \text{dB}$$

for: $-1 \le x(n) < 1$

$$= 6.02b + 10.79 + 10\log \sigma_x^2 + 20\log A \ \text{dB}$$

for: $-1 \le A\,x(n) \le 1$; A = attenuation factor

A. $x(n)$ sinusoidal $\longrightarrow$ max. amplitude $= 1 - 2^{-b} < 1$

$\therefore\ \sigma_x^2 \simeq 1^2/2$

$$\text{SNR}_{\text{max, sinusoid}} = 6.02b + 7.79 \ \text{dB}$$

B. $x(n)$ random process with $P(x(n) > 4\sigma_x) \ll 1$

$\therefore$ let $A = 1/4\sigma_x$

$$\text{SNR}_{\text{random process}} = 6.02b - 1.24 \ \text{dB}$$

Note: a 9.03 dB (= 1½ bits) drop from sinusoidal signal case

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