



Leap

A Fast, Lattice-based OPRF with Application to Private Set Intersection

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Sebastian Ramacher[◊] Christian Rechberger^{†,♣}

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[◊] Austrian Institute of Technology

Main Results

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- ✓ 1st modification of 2HashDH UC proof for Naor-Reingold OPRF

Leap is an Oblivious Pseudorandom Function (OPRF)



input $c \in \mathcal{C}$
gets y

compute

$$y := F_k(c)$$



key $k \in \mathcal{K}$

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$$\mathcal{F}_K(c_1, \dots, c_m) = S \left(\text{iNTT} \left(\mathbf{k}_0 \cdot \prod_{i=1}^m \mathbf{k}_i^{c_i} \right) \right)$$

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$$\mathcal{F}_{\mathbf{k}}(c_1, \dots, c_m) = S \left(\text{iNTT} \left(\text{lookup} \left(\mathbf{k}_0 + \sum_{i=1}^m c_i \mathbf{k}_i \right) \right) \right)$$

A little help from MPC friends: Oblivious Transfer

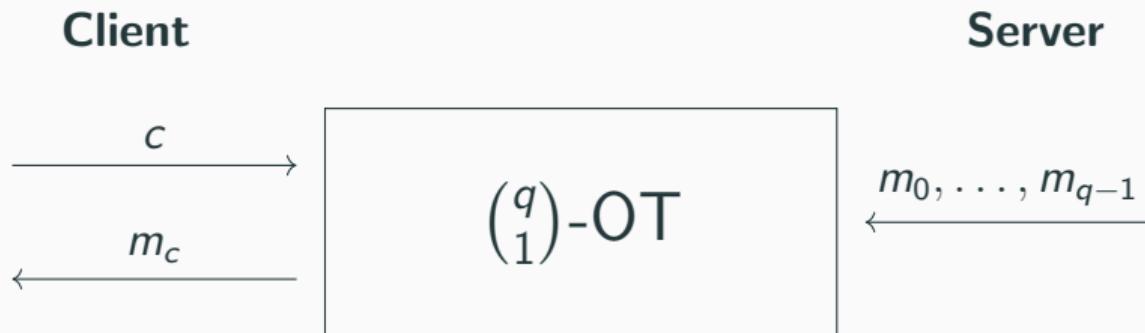


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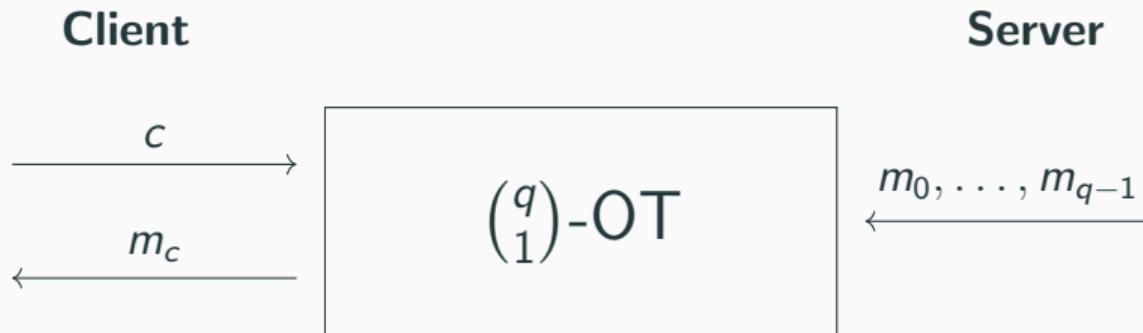


Extend with symmetric operations!

A little help from MPC friends: Oblivious Transfer (cont.)

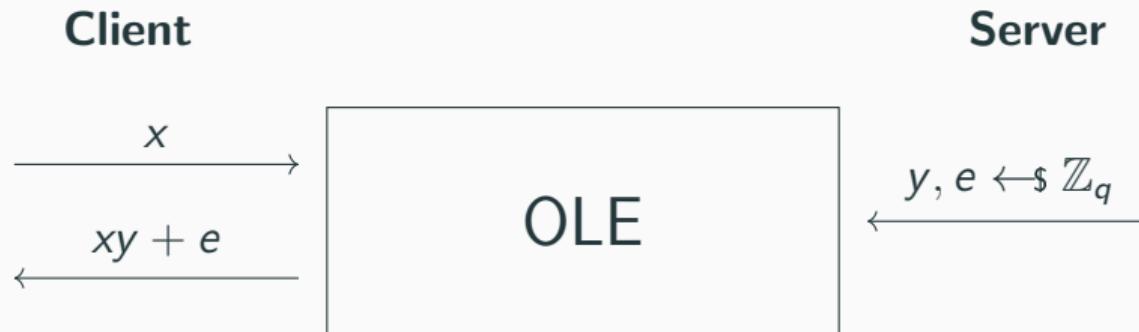


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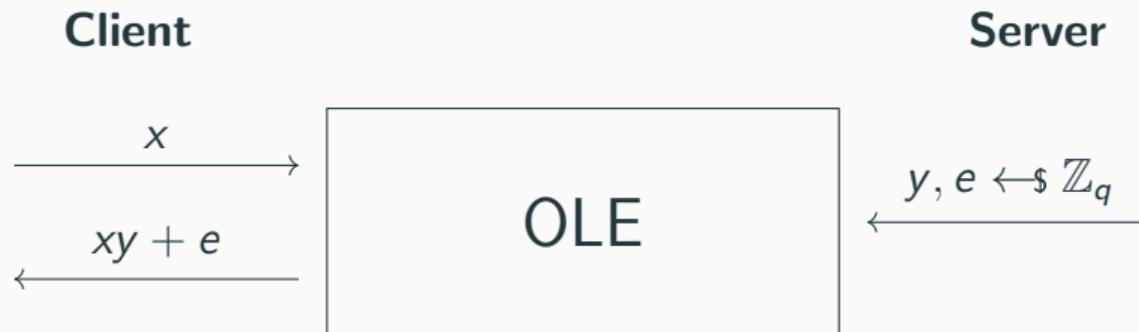


Generic construction from $\lceil \log q \rceil$ $\binom{2}{1}\text{-OTs}$!

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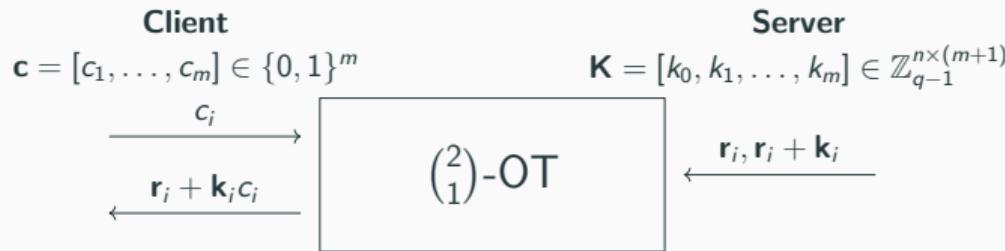


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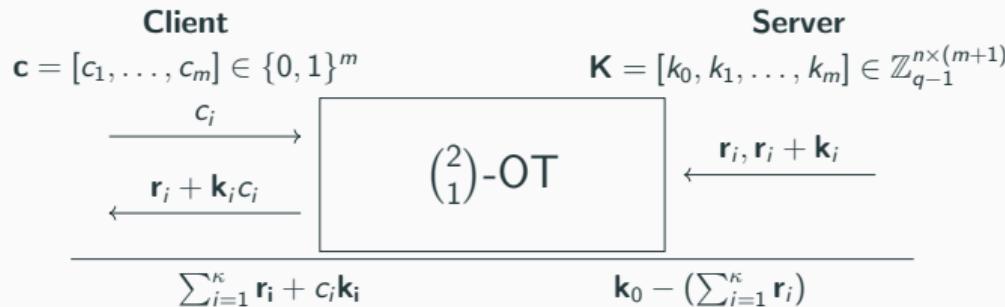
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An OPRF from Spring



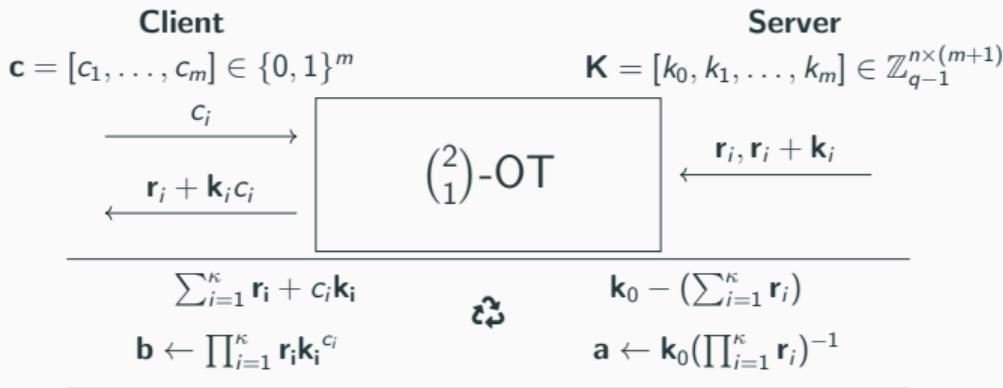
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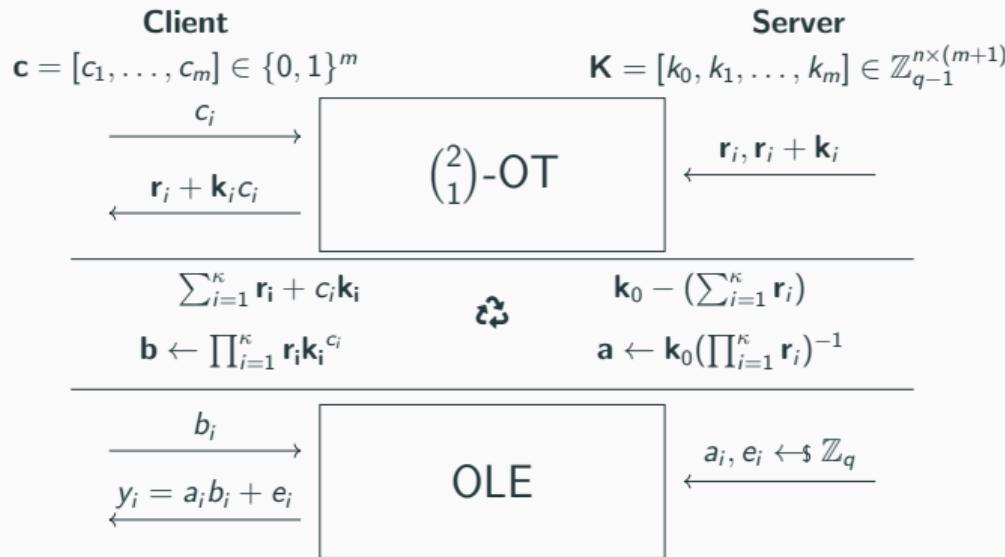
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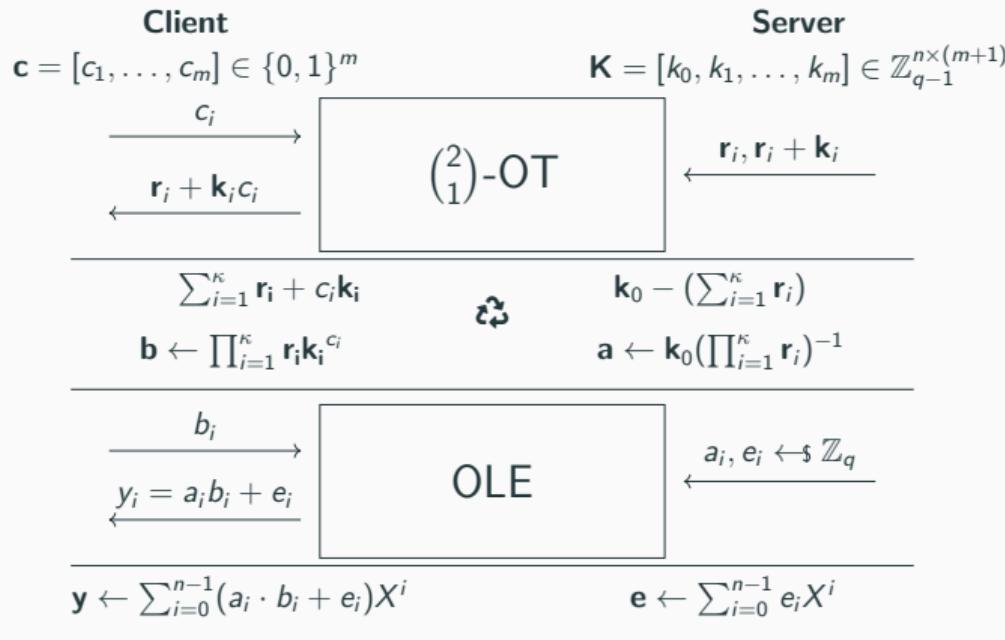
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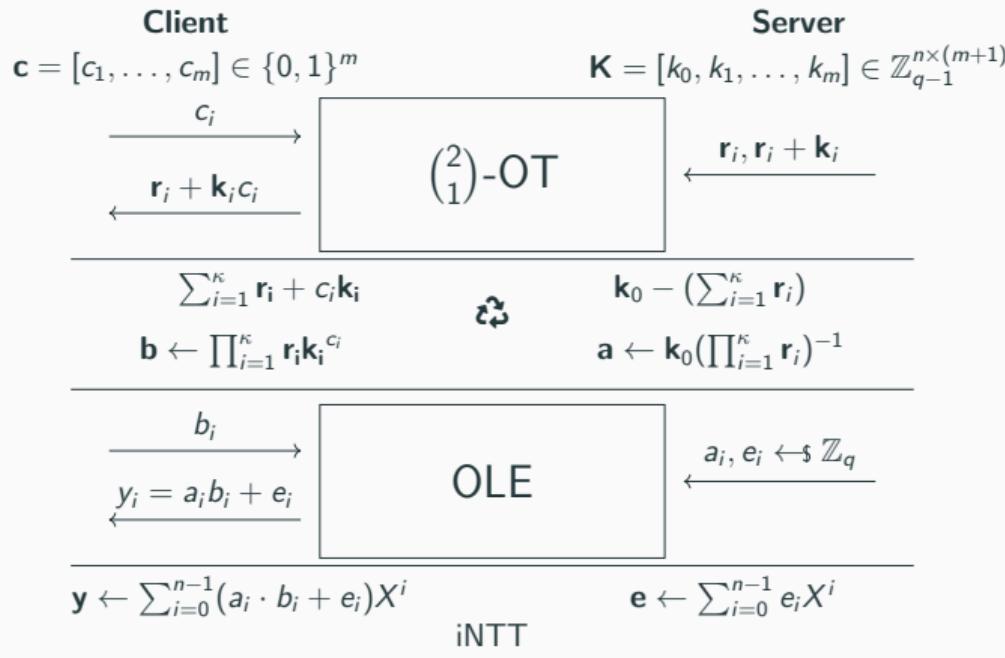
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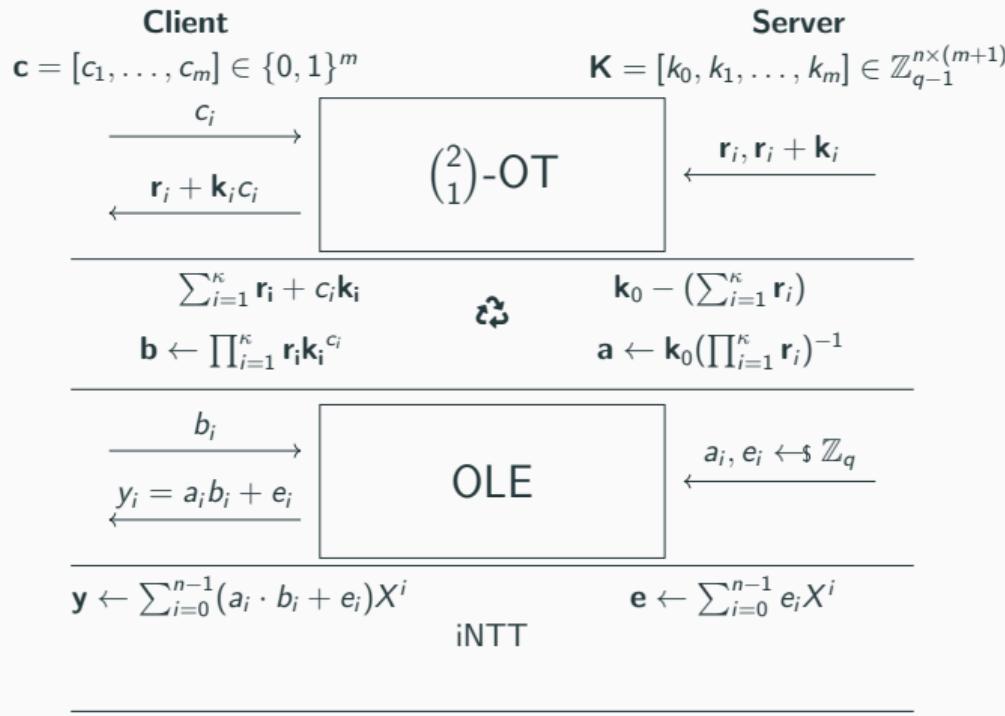
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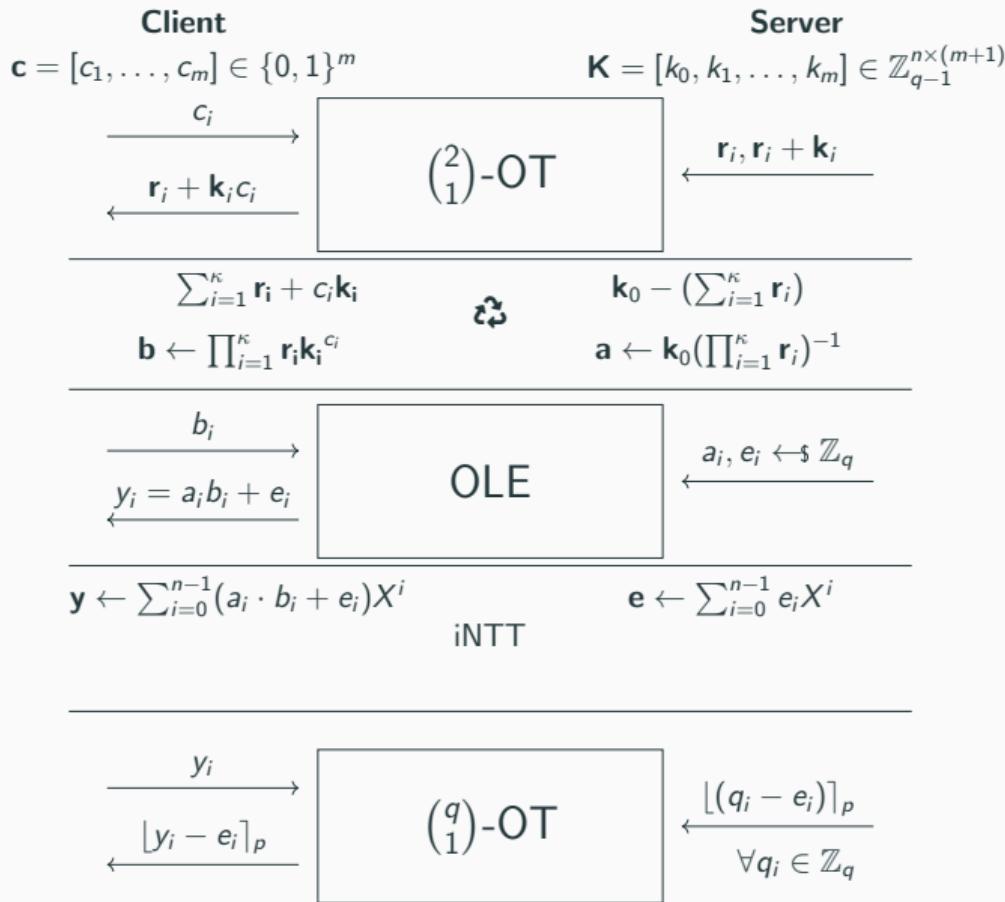
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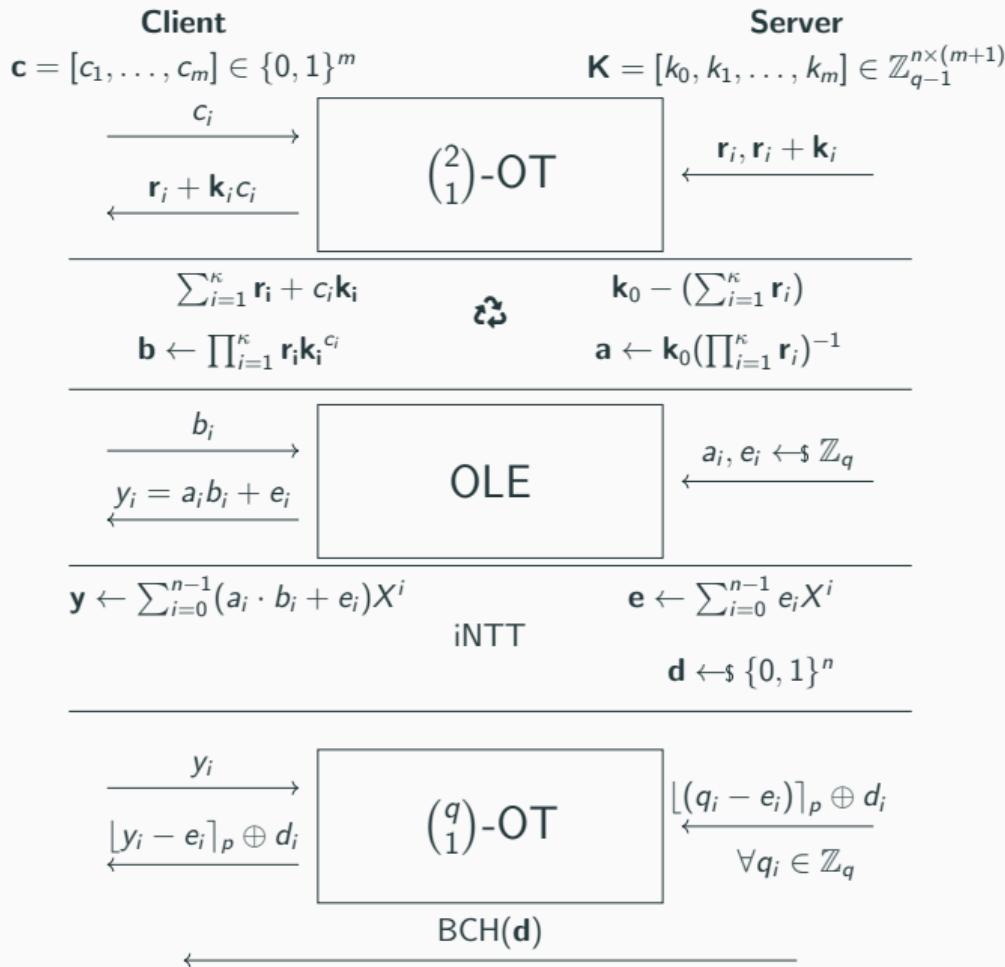
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Precomputation

Client

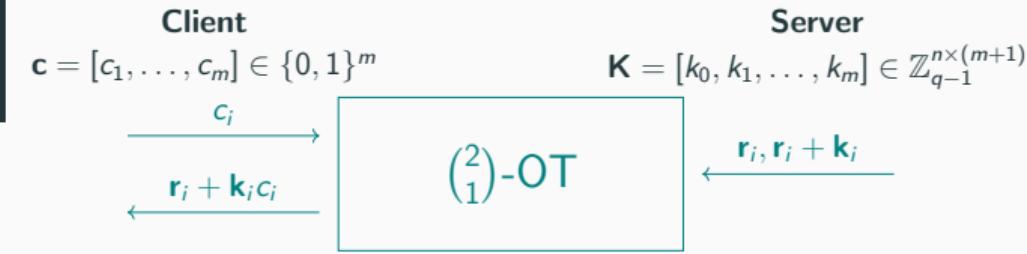
$$\mathbf{c} = [c_1, \dots, c_m] \in \{0, 1\}^m$$

Server

$$\mathbf{K} = [k_0, k_1, \dots, k_m] \in \mathbb{Z}_{q-1}^{n \times (m+1)}$$

- input length $m = 128$

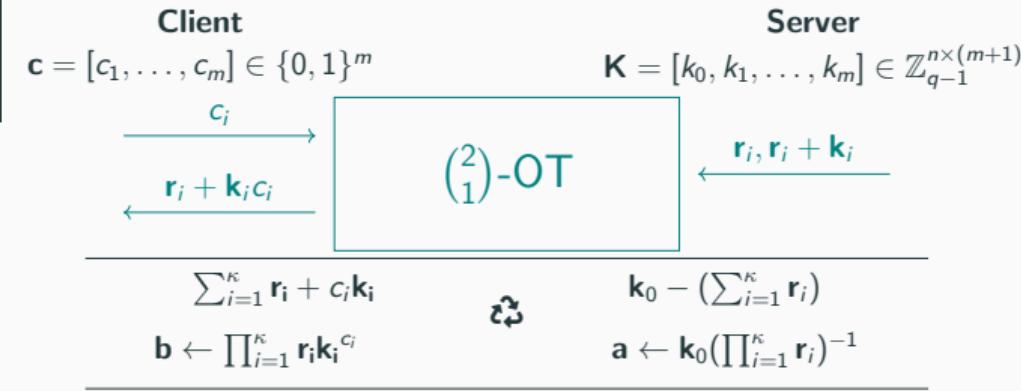
Precomputation



- input length $m = 128$
- m $\binom{2}{1}$ -OTs

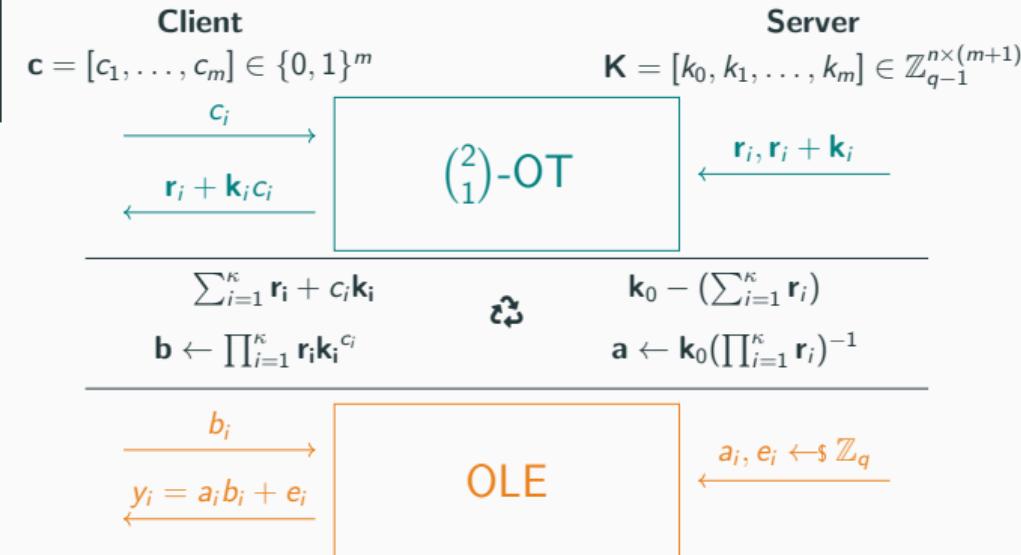
Precomputation

- input length $m = 128$
- $m \binom{2}{1}$ -OTs
- polynomial degree $n = 128$,
modulus $q = 257$, $\lceil \log q \rceil = 9$



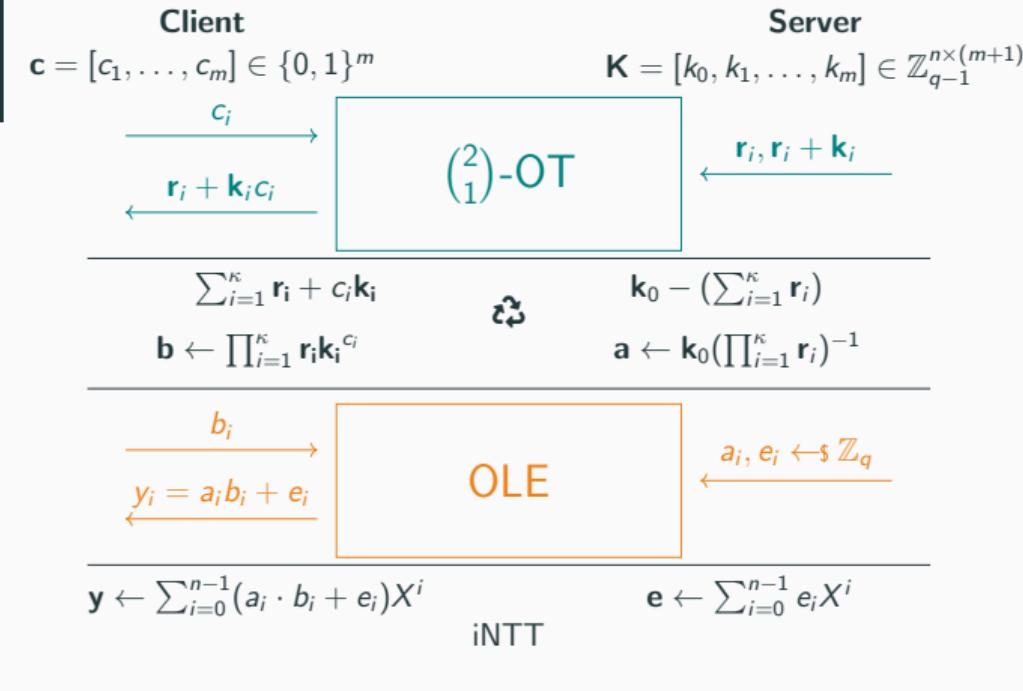
Precomputation

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- $N \log q \binom{2}{1}$ -OTs = 1152



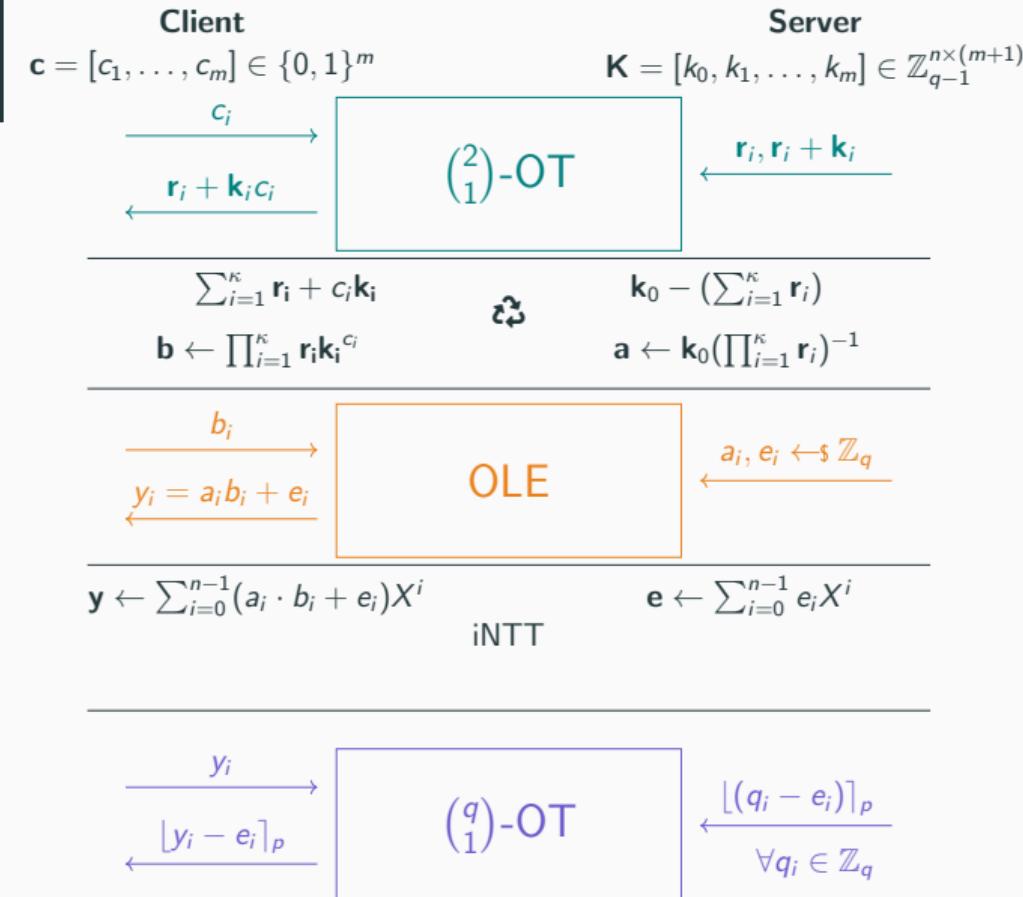
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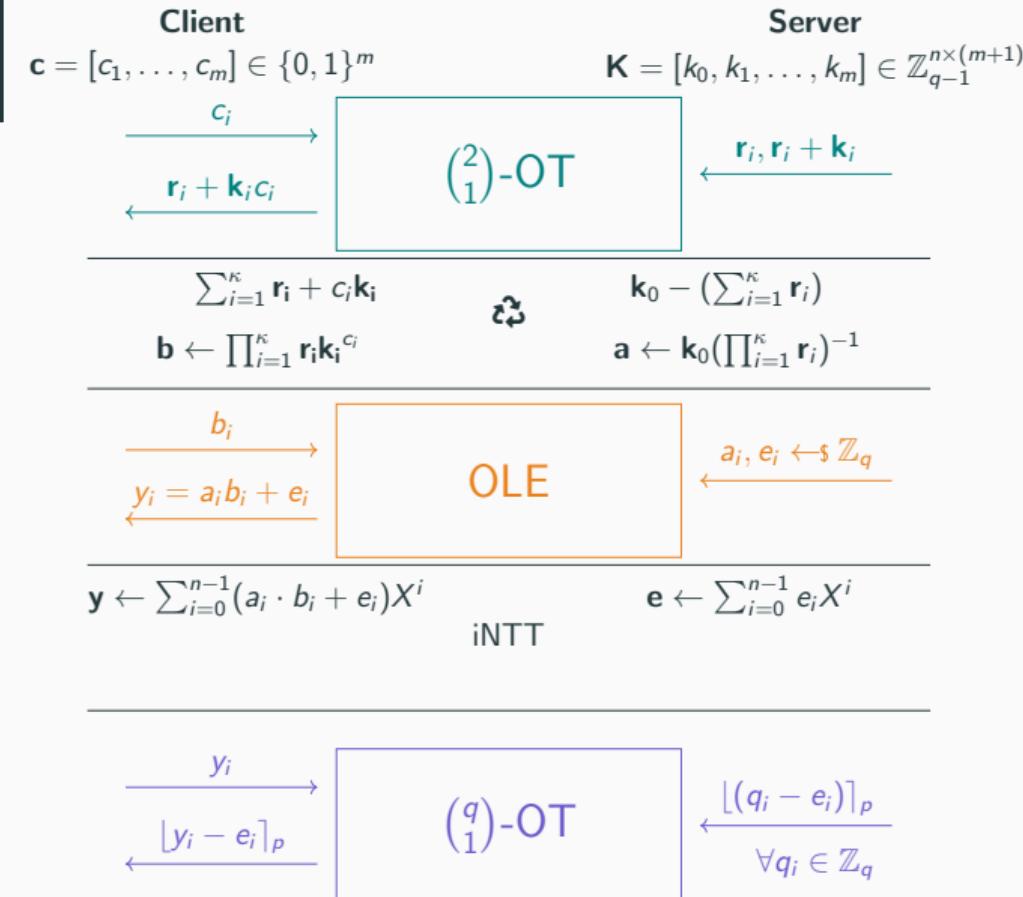
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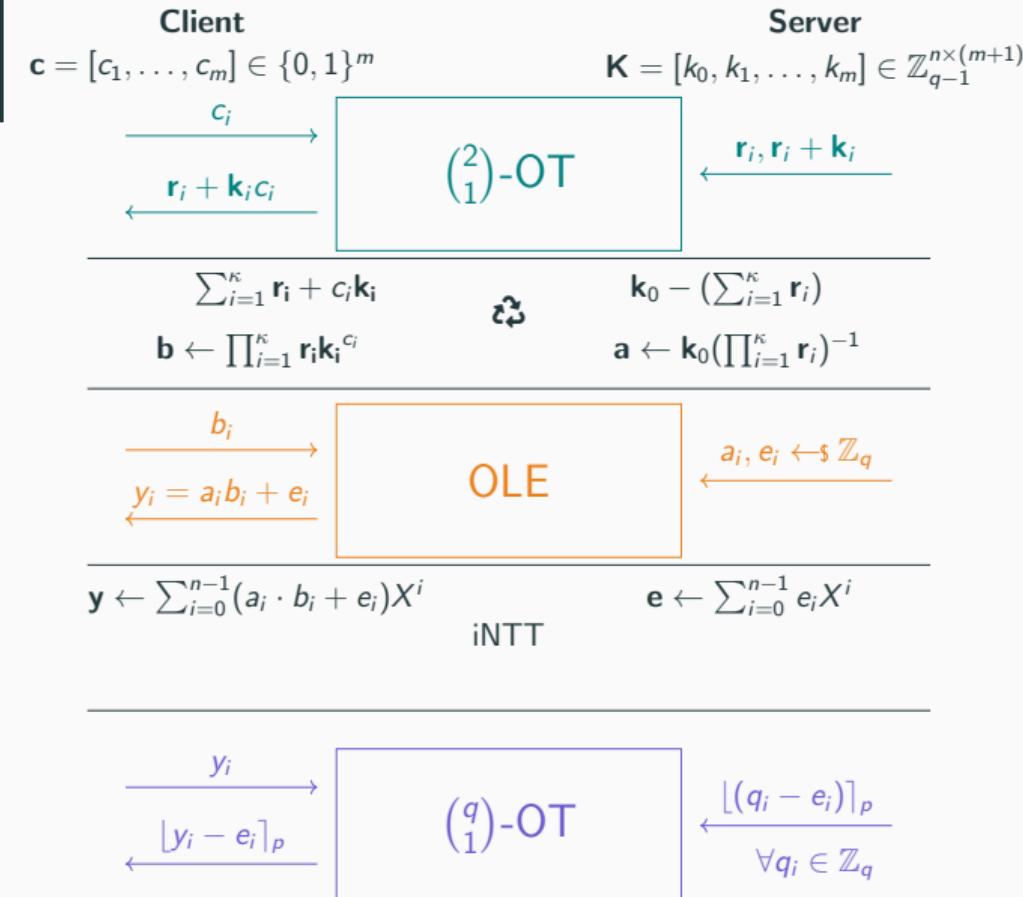
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- 2432 OTs



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- $N \log q \binom{2}{1}$ -OTs = 1152
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- 2432 128 base OTs + OT
extension



Online Communication

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$$\mathbf{c} = [c_1, \dots, c_m] \in \{0, 1\}^m$$

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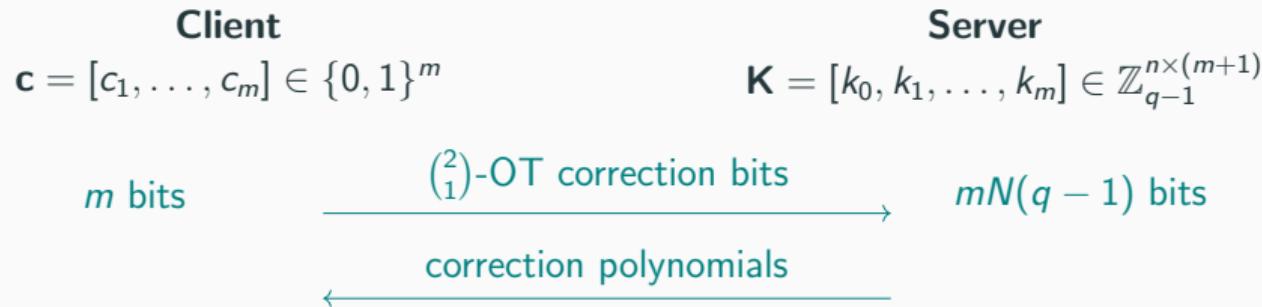
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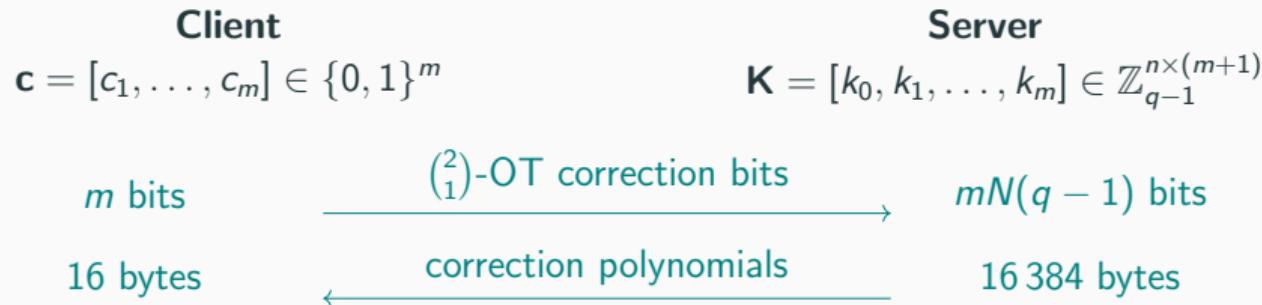
$\binom{2}{1}$ -OT correction bits



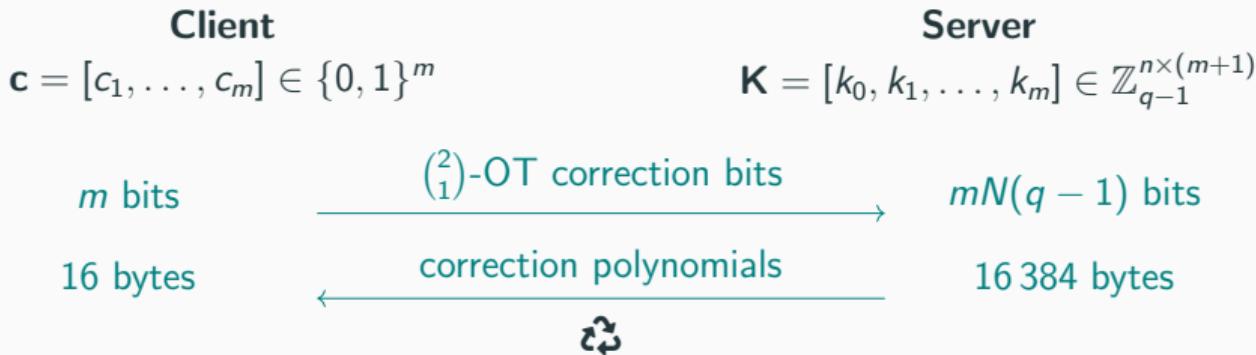
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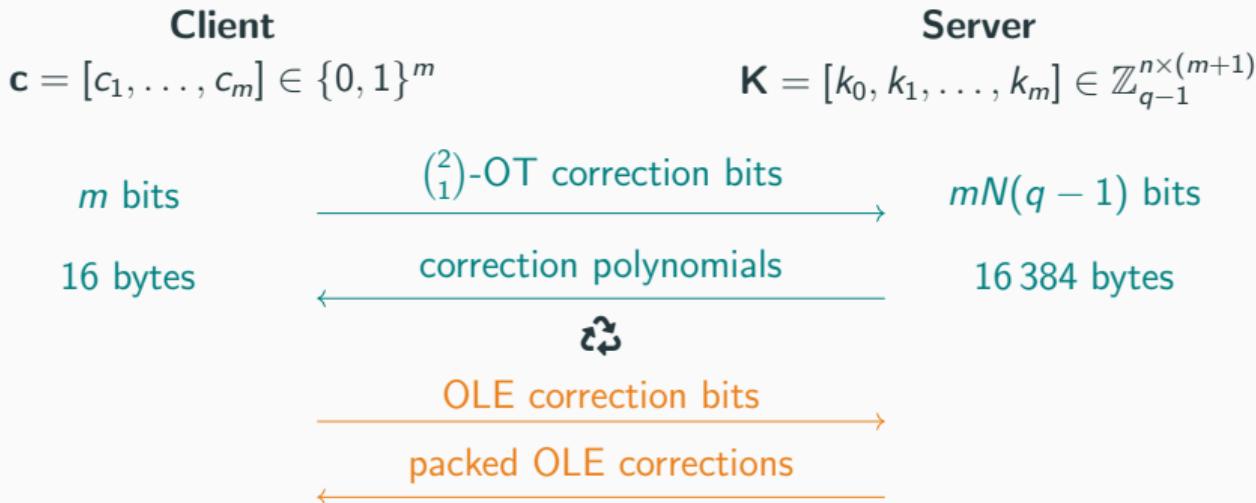
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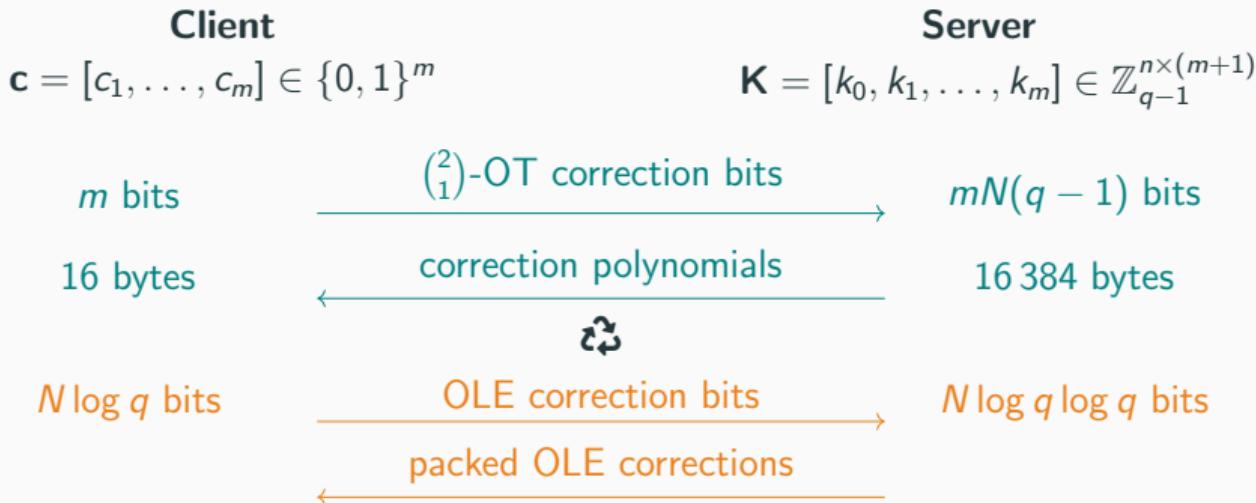
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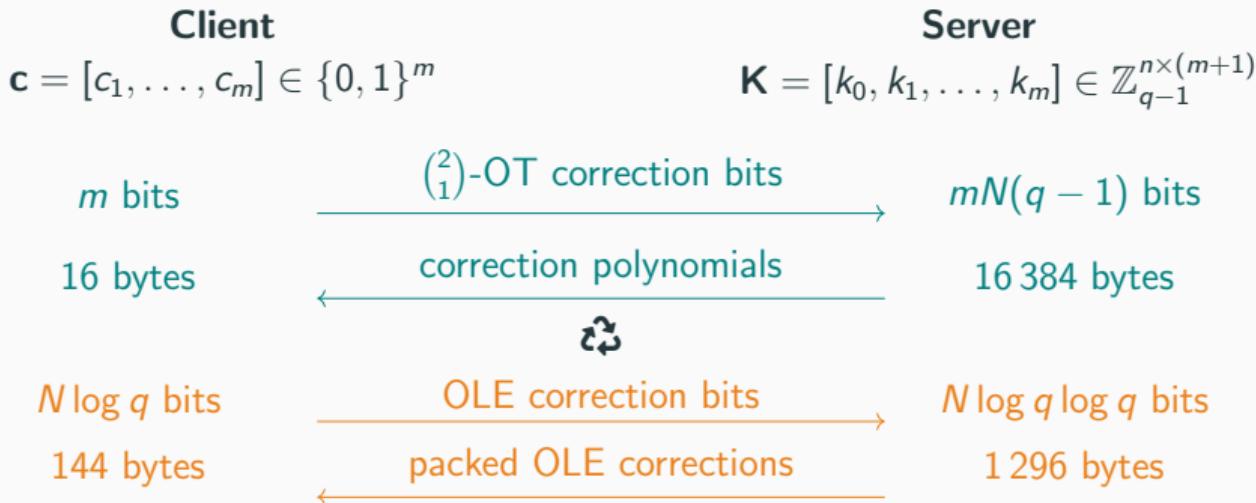
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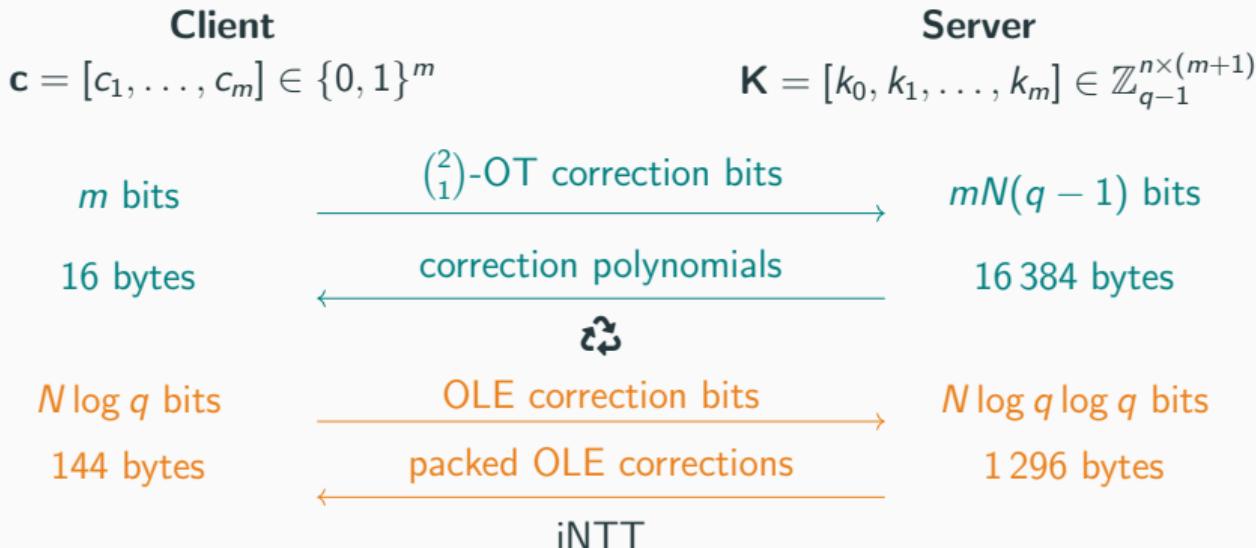
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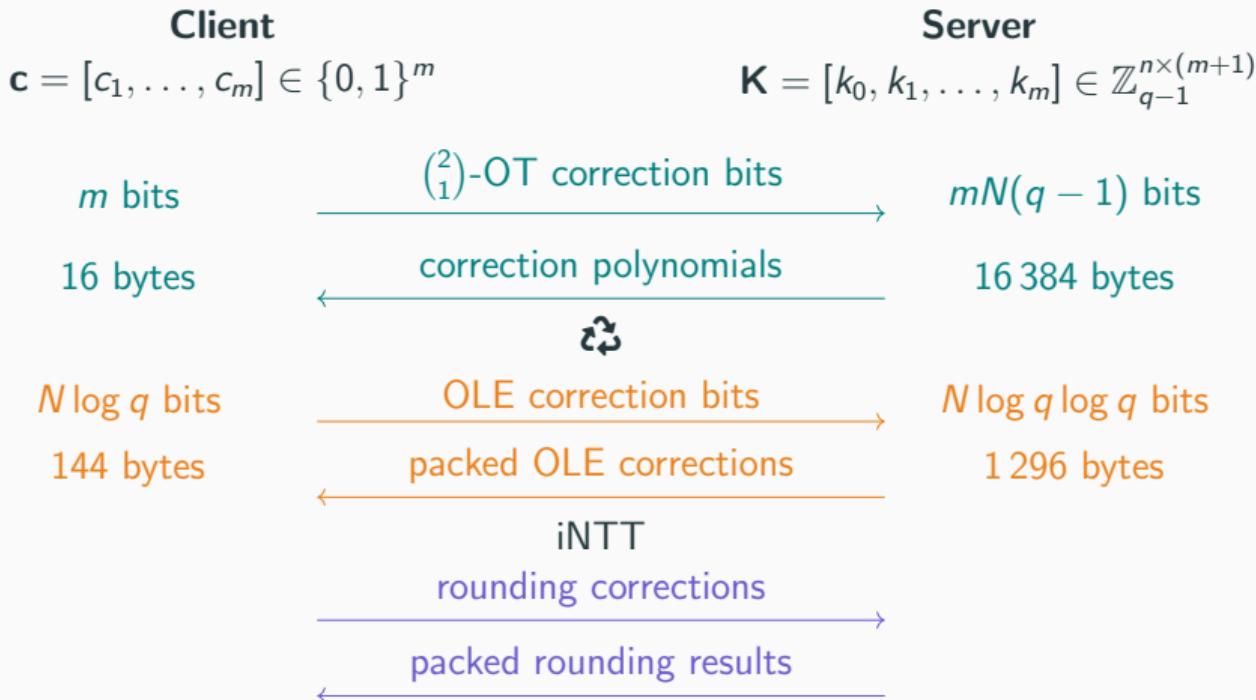
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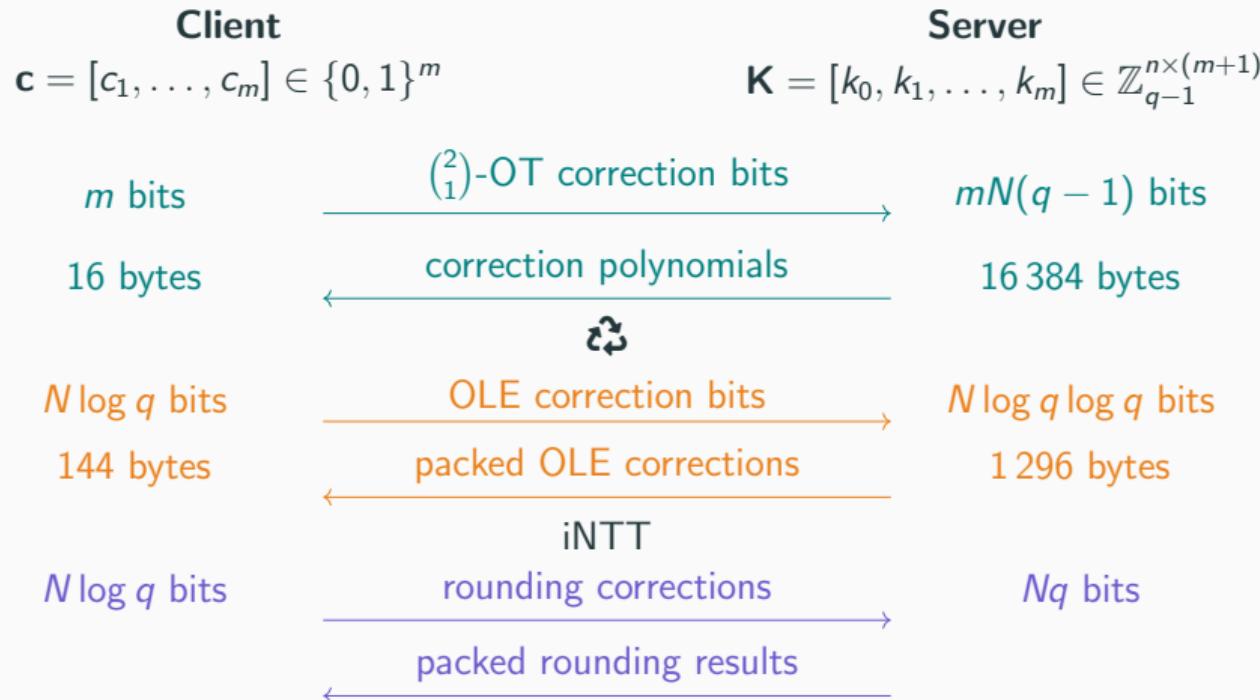
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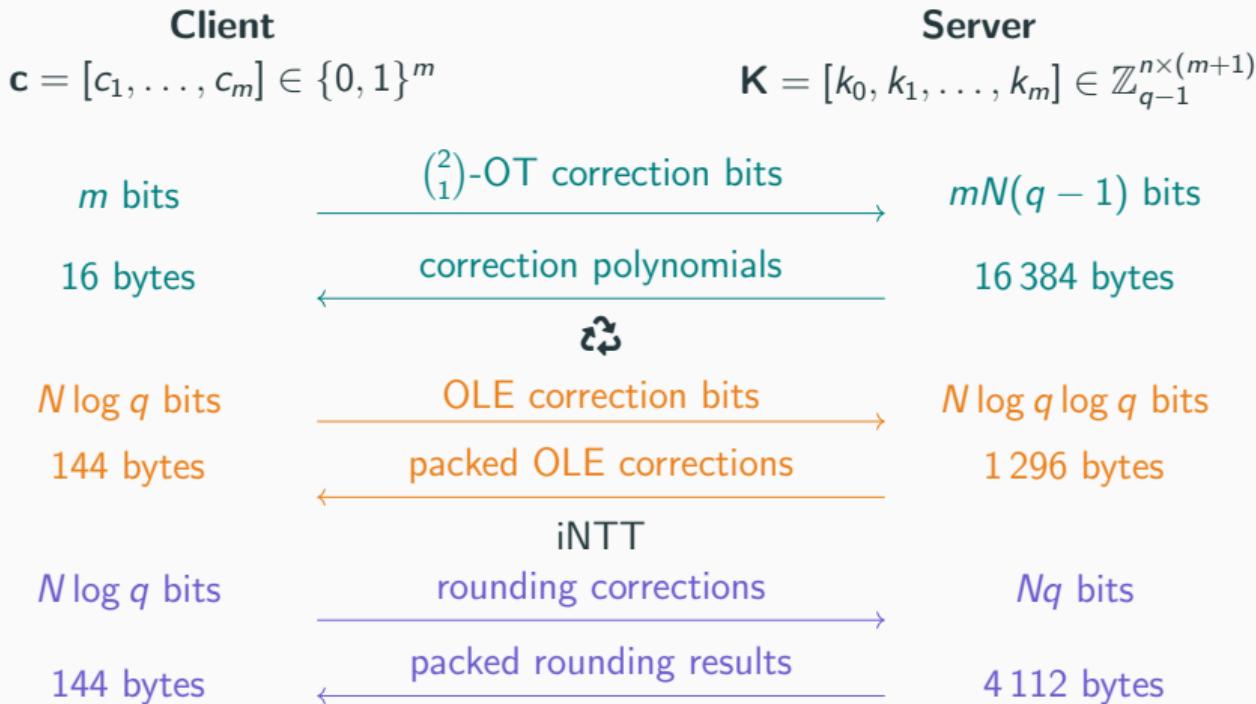
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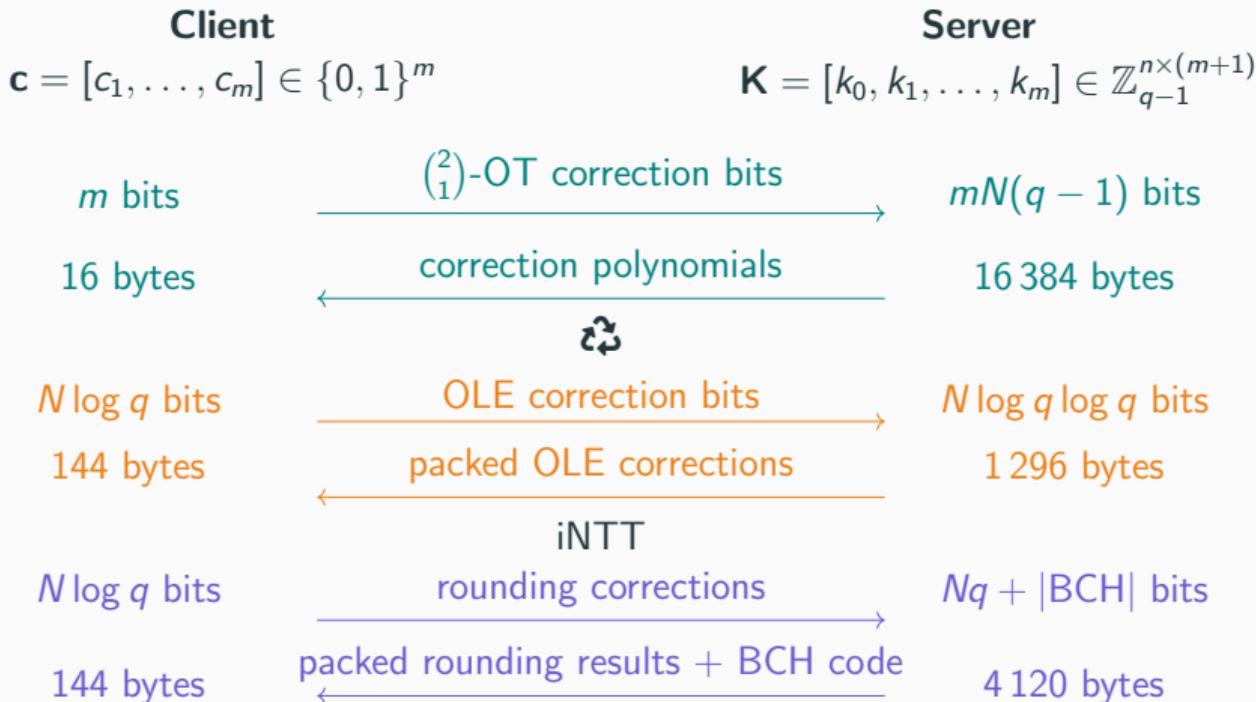
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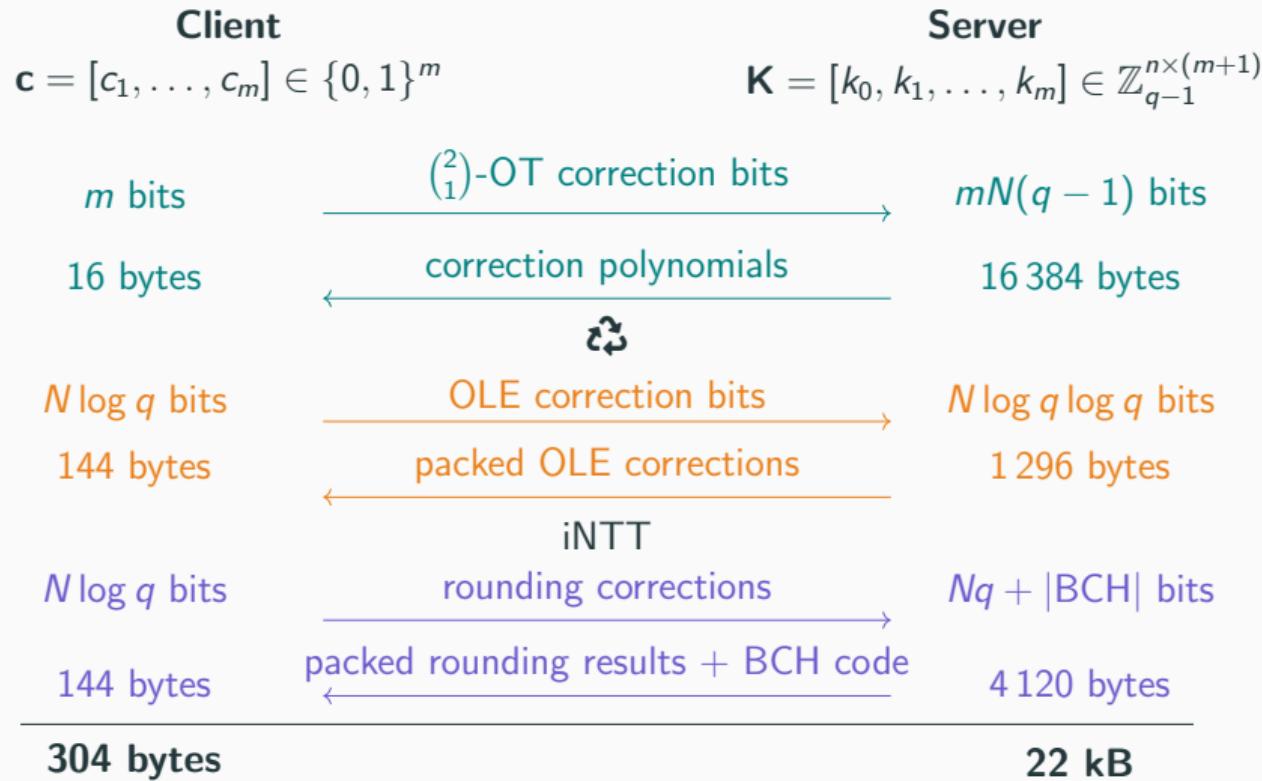
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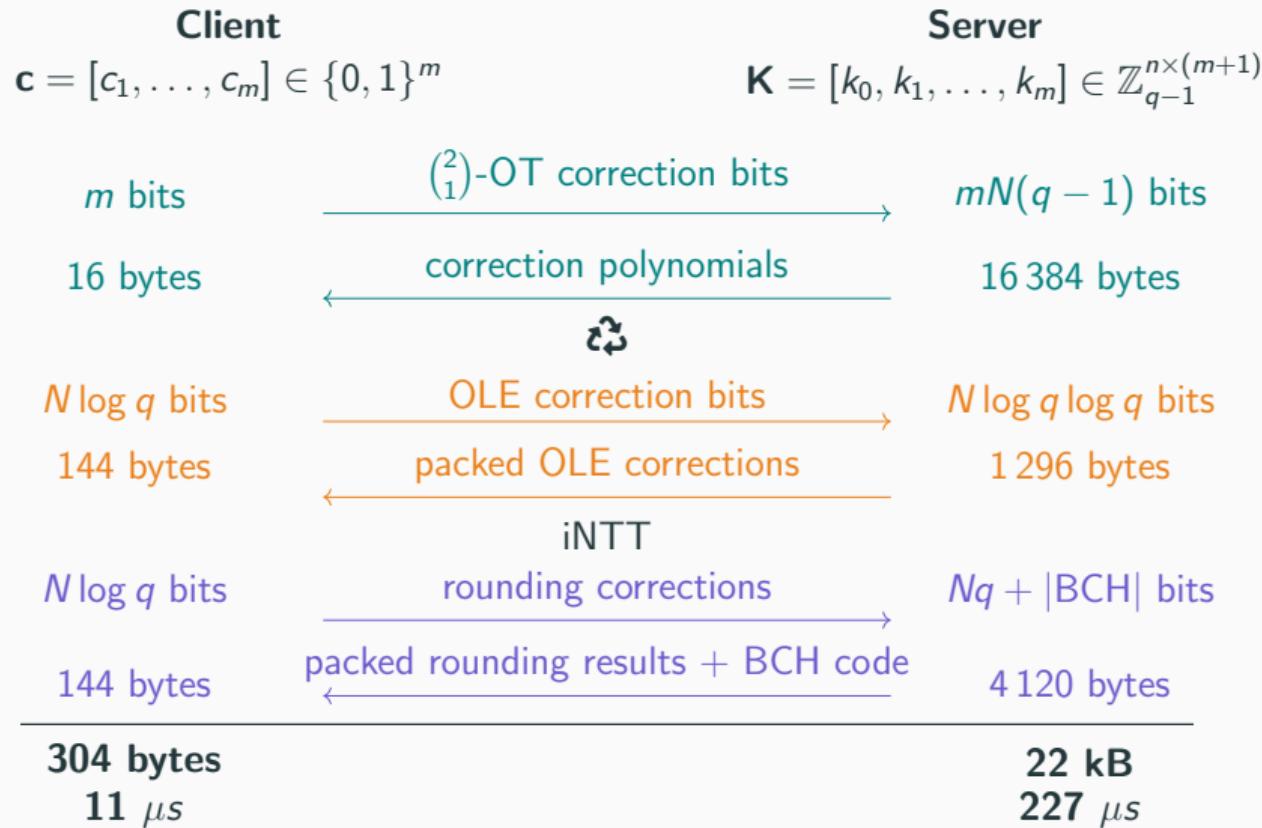
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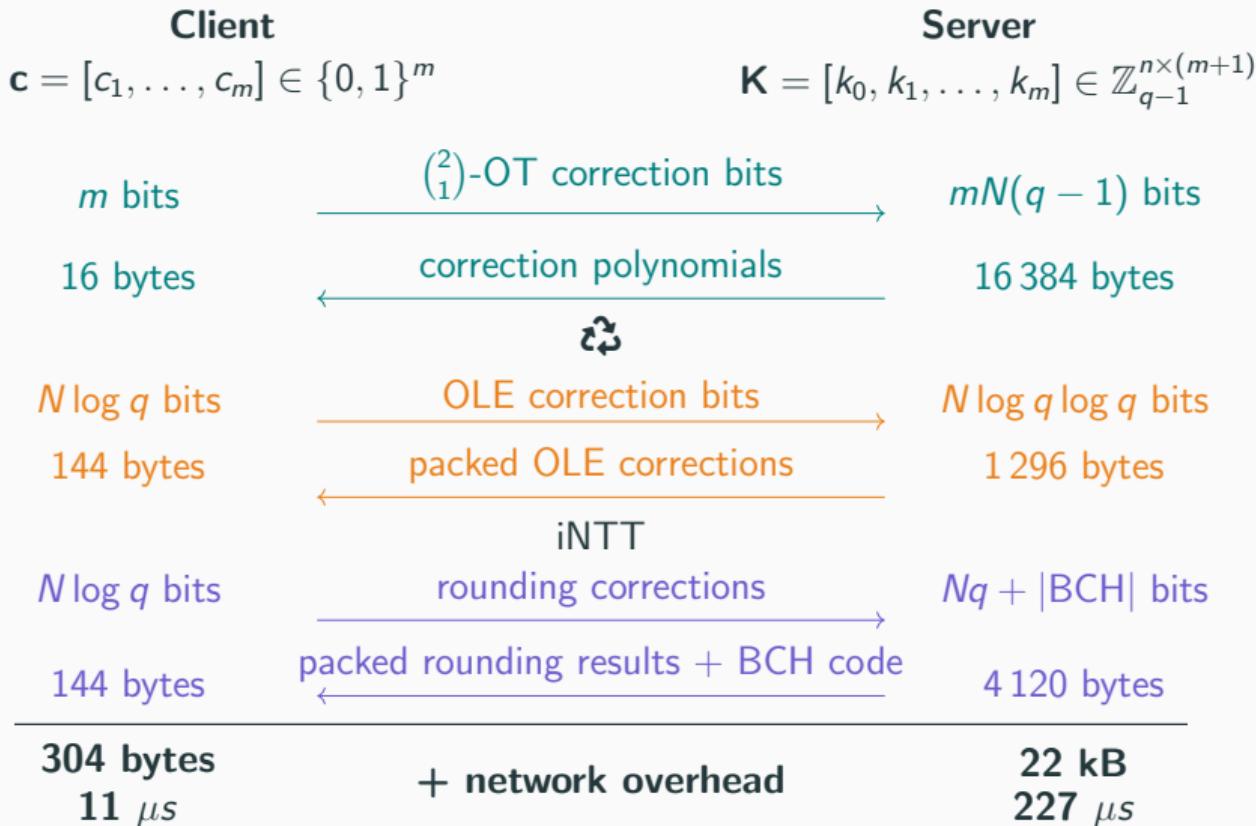
Online Communication



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Interchangeable Building Blocks: Precomputation Numbers

#	Protocol	Communication		Computation	
		Client	Server	Client	Server
2^0	Simplest OT+IKNP	39 kB	4 kB	63 ms	63 ms
	Kyber OT+IKNP	465 kB	328 kB	10 ms	10 ms
	Simplest OT+Silent	22 kB	22 kB	68 ms	68 ms
	Kyber OT+Silent	448 kB	392 kB	10 ms	10 ms
2^{13}	Simplest OT+IKNP	319 MB	4.26 kB	420 ms	463 ms
	Kyber OT+IKNP	319 MB	328 kB	311 ms	423 ms
	Simplest OT+Silent	46 kB	155 kB	2065 ms	3371 ms
	Kyber OT+Silent	487 kB	559 kB	2130 ms	3496 ms

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Client (x_1, \dots, x_m)

$\{F_k(x_i)\}_{i \in [m]}$

Server $(y_1, \dots, y_n), k$

OPRF

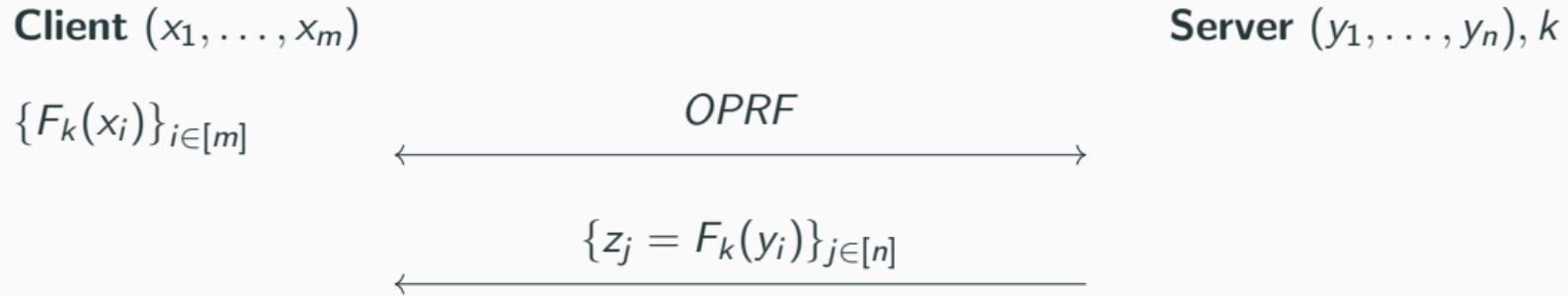


$\{z_j = F_k(y_i)\}_{j \in [n]}$



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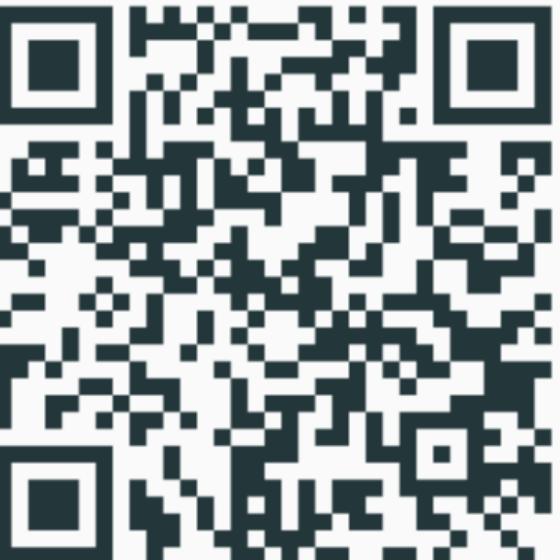


- If $x_i = y_j$ then $F_k(x_i) = z_j$
- Otherwise $F_k(y_j)$ is pseudorandom.

	Parameters		Setup		Online	
	$ S $	$ C $	$ S $	$ C $	$ S $	$ C $
Leap (IKNP+Kyber)	2^0	2^0	1 ms 117 bytes	1 ms 0 bytes	0.2s 73.4 kB	6 ms 20 kB
	2^5	2^5	1 ms 246 bytes	1 ms 0 bytes	0.08 s 802 kB	0.07 s 47.5 kB
	2^{10}	2^{10}	4 ms 4.30 kB	5 ms 0 bytes	2.4 s 22.39 MiB	2.46 s 646.5 kB
NR-OT (FHE OT) [Hei+24]	2^0	2^0	0.26 s 134 bytes	0.51 s 1 byte	0.06 s 128 kB	0.10 s 0.75MiB
	2^5	2^5	1.63 s 263 bytes	1.88 s 1 byte	3.11 s 4MiB	3.15 s 8.5 MiB
	2^{10}	2^{10}	45.04 s 4.31 MiB	45.28 s 1 byte	99.66 s 128 MiB	99.71 s 256.6 MiB
OPUS [Hei+24]	2^0	2^0	0.26 s 133 bytes	0.26 s 0 bytes	15.47 s 17.07 kB	15.91 s 9.04 kB
	2^5	2^5	8.71 s 262 bytes	8.71 s 0 bytes	328.46 s 546.25 kB	329.14 s 290.26 kB
	2^{10}	2^{10}	303.38 s 4.31 kB	303.38 s 0 bytes	16367.12 s 34.14 MiB	16367.60 s 18.08 MiB
ECNR (Simplest OT + IKNP) [Kal+19]	2^0	2^0	10 ms 133 bytes	0 s 0 bytes	0.23 s 12.04 kB	0.05 s 16 bytes
	2^5	2^5	0.02 s 262 bytes	0 s 0 bytes	0.21 s 137.05 kB	0.06 s 512 bytes
	2^{10}	2^{10}	0.3 s 4.36 kB	0 s 0 bytes	0.64 s 4.04 MiB	0.57 s 16 kB

work	assumption	rounds	comm.	cost	security(c-s)	preprocessing	no	trusted	setup	verifiable	available
ADDs21	(R)LWE+SIS	2	2 MB	semihonest-semihonest	-	YES	NO	YES			
ADDs21	(R)LWE+SIS	2	128 GB	malicious-malicious	-	YES	YES	YES	YES	NO	
AG24	(R)LWE+SIS	2	316 kB	malicious-malicious	221.5 kB	YES	YES	YES	YES	NO	
ADDG23	mod(2,3)+lattices	2	10 kB	malicious-semihonest	2.5 MB	YES	NO	NO	NO		
ADDG23	mod(2,3)+lattices	2	160 kB	malicious-semihonest	2.5 MB	YES	YES	YES	YES	NO	
HKL+25	heuristic LWR	6	23 kB	semihonest-semihonest	793 kB	YES	NO	YES		YES	
ESTX24	iMLWER-RU+MLE+SIS	2	126 kB*	malicious-malicious	10 kB	YES	YES	YES	NO		
APRR24	mod(2,3)	2	916 bits	malicious-semihonest	38 bits	YES	YES	YES	NO		
DGH+21	mod(2,3)	2	641 bits	semihonest-semihonest	1836 bits	NO	NO	NO	NO		
SHB23	Legendre PRF	3	13 kB	semihonest-semihonest	?	YES	YES	YES	NO		
KCM24	Legendre PRF	2	?	semihonest-semihonest	?	YES	YES	YES	YES	YES	
BDFH24	Legendre PRF as 2HashDH	9	356 kB	malicious-malicious	392 kB	YES	YES	YES	YES	YES	
YBHKR24	generalized power residue (Legendre) PRF	3	774 kB	malicious-semihonest	-	YES	YES	YES	NO		
YBHKR24	generalized power residue (Legendre) PRF	3	970 kB	malicious-malicious	-	YES	YES	YES	NO		
F0023	AES+Garbled Circuits	2	6.79MB	semihonest-semihonest	-	YES	NO	NO	YES		
HKLS24	Minicrypt	?	22 bytes [†]	malicious-malicious	-	YES	NO	NO	NO		
Basso23	Isogenies F_p^2	2	3.0 MB	malicious-malicious	-	NO	NO	NO	NO		
Basso23	Isogenies F_p^2	2	8.7 MB	malicious-malicious	-	NO	YES	YES	NO		
Basso24	Higher-Dimensional isogenies	2	28.9 kB	malicious-malicious	-	YES	YES	YES	YES	YES	
BKW20	Isogenies F_p + lattices	2	20.54 kB	semihonest-semihonest	-	NO	NO	NO	NO		
BKW20	Isogenies F_p + lattices	4	34.88 kB	malicious-semihonest	-	NO	NO	NO	NO		
HHM+23	Isogenies F_p + lattices + HE OT	2	640 kB	semihonest-semihonest	-	YES	NO	NO	YES		
HHM+23	Isogenies F_p	258	24.7 kB	semihonest-semihonest	-	YES	NO	YES		YES	
dSP23	Isogenies F_p	2	384 bytes	malicious-semihonest	68.4 kB	YES	YES	YES	NO		
dSP23	Isogenies F_p	2	16.38 kB	malicious-semihonest	-	YES	YES	YES	NO	13	

work	assumption	rounds	comm.	cost	security(c-s)	preprocessing	no	trusted	setup	verifiable	available
ADDS21	(R)LWE+SIS					YES	NO	YES			
ADDS21	(R)LWE+SIS					YES	YES	YES	NO		
AG24	(R)LWE+SIS					KB	YES	YES	NO		
ADDG23	mod(2,3)+lattices					MB	YES	NO	NO		
ADDG23	mod(2,3)+lattices					MB	YES	YES	NO		
HKL+25	heuristic LWR					KB	YES	NO	YES		
ESTX24	iMLWER-RU+MLE+SIS					B	YES	YES	NO		
APRR24	mod(2,3)					ts	YES	YES	NO		
DGH+21	mod(2,3)					its	NO	NO	NO		
SHB23	Legendre PRF						YES	YES	NO		
KCM24	Legendre PRF						YES	YES	YES	YES	
BDFH24	Legendre PRF as 2HashDH					KB	YES	YES	YES	YES	
YBHKR24	generalized power residue (Legendre						YES	YES	NO		
YBHKR24	generalized power residue (Legendre						YES	YES	NO		
F0023	AES+Garbled Circuits						YES	NO	YES		
HKLS24	Minicrypt						YES	NO	NO		
Basso23	Isogenies \mathbb{F}_{p^2}						NO	NO	NO		
Basso23	Isogenies \mathbb{F}_{p^2}						NO	YES	NO		
Basso24											YES
BKW20											NO
BKW20											NO
HHM+23											YES
HHM+23											YES



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dSP23	Isogenies \mathbb{F}_p	2	384 bytes	malicious-semihonest	68.4 kB	YES	YES	NO
dSP23	Isogenies \mathbb{F}_p	2	16.38 kB	malicious-semihonest	-	YES	YES	NO 13



Leap

A Fast, Lattice-based OPRF with Application to Private Set Intersection

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Performance

Phase	Client			Server		
	Comm.	Comp.	Idle	Comm.	Comp.	Idle
Subset-Sum	16 bytes	5 μ s	21 μ s	16 384 bytes	863 μ s	46 μ s
OLE	144 bytes	4 μ s	3 μ s	1 296 bytes	72 μ s	88 μ s
Rounding	144 bytes	2 μ s	726 μ s	4 118 bytes	814 μ s	67 μ s
BCH	0 bytes	1 μ s	0 μ s	8 bytes	/	26 μ s
Overall (Network)	304 bytes (328 bytes)	11 μ s	750 μ s	21 806 bytes (22 840 bytes)	1.7 ms	227 μ s

References

- [Ban+15] Abhishek Banerjee et al. “SPRING: Fast Pseudorandom Functions from Rounded Ring Products”. In: *FSE 2014*. Ed. by Carlos Cid and Christian Rechberger. Vol. 8540. LNCS. Springer, Berlin, Heidelberg, Mar. 2015, pp. 38–57. DOI: [10.1007/978-3-662-46706-0_3](https://doi.org/10.1007/978-3-662-46706-0_3).
- [Hei+24] Lena Heimberger et al. “OPRFs from Isogenies: Designs and Analysis”. In: *ASIACCS 24*. Ed. by Jianying Zhou et al. ACM Press, July 2024. DOI: [10.1145/3634737.3645010](https://doi.org/10.1145/3634737.3645010).
- [Kal+19] Daniel Kales et al. “Mobile Private Contact Discovery at Scale”. In: *USENIX Security 2019*. Ed. by Nadia Heninger and Patrick Traynor. USENIX Association, Aug. 2019, pp. 1447–1464.