$$F_k(s) = F_{k-1}(s) - \sum_{y=2}^{N/2} p(y) \cdot p_{\text{still in pool}}(s) \cdot \underbrace{\begin{cases} 2(s-1) & , s<y \\ 2(y-1) & , s \geq y \end{cases}}_{\substack{\text{number of effectively} \\ \text{removed links of length } s \\ \text{if } s_k = y}}$$

where
$$p(y) = \frac{F_{k-1}(y)}{\sum_z F_{k-1}(z)}$$
and
$$p_{\text{still in pool}}(s) = \frac{F_{k-1}(s)}{N}$$

$$= F_{k-1}(s) - p_{\text{still in pool}}(s) \cdot 2 \cdot \sum_y p(y) \begin{cases} s-1 & s<y \\ y-1 & s \geq y \end{cases}$$

this is a problem, it makes the whole thing non linear

$$\boxed{\Rightarrow \underline{\text{Approximation}} \quad p(y) = \frac{1}{N/2-1} \approx \frac{2}{N}}$$

$$\approx F_{k-1}(s) - \frac{F_{k-1}(s)}{N} \cdot 2 \cdot \frac{2}{N} \cdot \sum_{y=2}^{N/2} \begin{cases} s-1 & , s<y \\ y-1 & , s \geq y \end{cases}$$

$$= F_{k-1}(s) - \frac{4}{N^2} F_{k-1}(s) \cdot \left[\sum_{y=2}^{s} y-1 + \sum_{y=s+1}^{N/2} s-1 \right]$$

$$= F_{k-1}(s) - \frac{4}{N^2} F_{k-1}(s) \cdot \left[\sum_{y=1}^{s-1} y + (N/2 - s) \cdot (s-1) \right]$$

$$= F_{k-1}(s) - \frac{4}{N^2} F_{k-1}(s) \cdot \left[\frac{s \cdot (s-1)}{2} + \frac{(N-2s)(s-1)}{2} \right]$$

$$= F_{k-1}(s) - \frac{2}{N^2} F_{k-1}(s) \cdot \left[s^2 - s + Ns - N - 2s^2 + 2s \right]$$

$$= F_{k-1}(s) - \frac{2}{N^2} F_{k-1}(s) \cdot \left[-s^2 + s + Ns - N \right]$$

$$\boxed{s' := s/N} \qquad \boxed{s' \in [0, \tfrac{1}{2}]}$$

$$= F_{k-1}(s) - \frac{2}{N^2} F_{k-1}(s) \cdot N^2 \cdot \left[-s'^2 + \frac{s'}{N} + s' - \frac{1}{N} \right]$$

$$\approx F_{k-1}(s) + 2 \, F_{k-1}(s) \; s'(s'-1)$$

Hence the recursion rule is

$$F_k(s) = F_{k-1}(s) \cdot \underbrace{\left(1 - 2\, s'(1-s')\right)}$$

remark: this factor is always $\in [0,1]$ as it should
$\Rightarrow$ $F_k(s)$ decreases with k.

with $F_0(s') = N$

we get

$$\boxed{F_k(s') = N \cdot \left[1 - 2\, s'(1-s')\right]^k}$$

Finally: $P(s) \propto \sum_{k=0}^{N-4} \frac{F_k(s)}{C_k}$ where $C_k = \sum_{s=2}^{N/2} F_k(s)$

$$\boxed{P(s) \propto \sum_{k=0}^{N-4} \frac{\left[1 - 2\, s/N\,(1 - s/N)\right]^k}{\sum_{y=2}^{N/2} \left[1 - 2\, y/N\,(1 - y/N)\right]^k}}$$

ugly but plottable

Good news: - look power-law ish
- same "exponent" for different N

Bad news: - exponent ~ -2.3 or so

$\Rightarrow$ "uniform" approximation bad ?