Etude 10 – Epidemic

COSC326 Etude #10 – Epidemic Toby Meyrick

How many initially sick individuals are required for a final universe to become all sick (unless immune)? The problem is broken down into universes without any immune cells and universes with immune cells (and of differing levels).

Universes w/out immune cells
Universes without any immune cells will require a minimal sick population proportional to N and M where N is the number of horizontal cells and M is the number of vertical cells. If N=M, so the grid has N^2 cells, the minimum number of sick cells required is N. I show below an example of this with a 4x4 grid.

S	.	.	.
.	S	.	.
.	.	S	.
.	.	.	S

Initial (4x4 grid)

S	S	S	S
S	S	S	S
S	S	S	S
S	S	S	S

Final (4x4 grid)

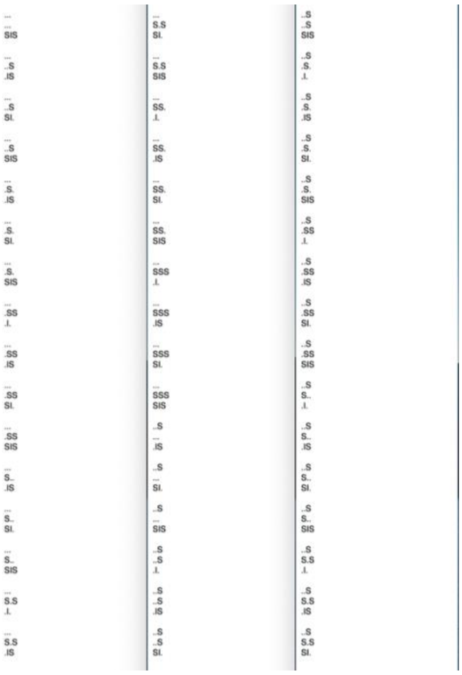
If it is a rectangular, instead of a square grid, the formula is not the same, however. There must be sick cells from the very start on the edge columns and rows, otherwise you will inevitably have an edge without any sick cells. This will mean you need N/2 cells and M/2 cells, rounded up respectively if N or M happens to be an odd number. For example, a 7x5 grid will require a minimal number of 7 initially sick cells. The formula is as follows, where MS = minimal sick and ceiling(x) is a ceiling function:

MS(N, M) = ceil((M+N)/2)

Universe w/ immune cells
Immune cells are a trickier case mathematically and will require a computational approach. Working out the number of possible grids (grids that could conceivably yield an entirely sick grid), there will be 2^(X-I) grids for any X sized population with I cells. So, if there is a 2x2 grid, there will be 4 cells and say 1 immune cell so 2^3 possible grids = 8 combinations. One of these grids will have zero sick cells which isn't very interesting, another grid will be all sick, which defeats the purpose, and another 3 grids will have one sick cell which doesn't allow any progression of the epidemic either. This only leaves 3 possible grids. If we had a 3x3 grid, we would have 9 cells, with say 1 immune cell. This results in 8 cells, 2^8 combinations/grids = 256. We can rule out a grid with zero sick cells, and grids with only sick cell, of which there are X-I or 8 possible grids. We can also rule out the grid where they are all sick for the same reason stated above. This rules out 10 possible grids. There are 246 remaining. Thus, there are:

> $2^{(X-I)}$ - (X-I)-2 interesting grids/universes.
> Where X = cells (NxM) and I = immune cells

For a grid of 20x20 with 5 immune cells, this becomes an expensive search – (2^(400-5))-(400-5)-2 = 8.06953087e118 possible grids we would have to search through to see the minimal number of sick cells needed. To generate every possible solution, we would need to write a program to generate every possible grid. To do this, I've used a recursive algorithm. For a 3x3 grid, there are 9 cells. I simply add on a "." or a "S" and recursively call this function until the string has a length of 9. This gives all possible permutations. At the index of the immune cell, I print "I". Grids are added to a TreeSet to ensure no duplicates and I only print the out the grids that have 2 or more sick cells and one or more healthy cells. This gives all possible grids (equation above). I show the first 48 grid results of the 3x3 grid, as printed out to a text file. There were 246 grids all up, as predicted above.



Now that we have all the possible grids, we can evolve each grid. If the evolved grid is entirely sick, print out the number of individuals that were initially sick and take the minimum.

Beyond a small grid size, the search becomes computationally expensive. Instead of searching every possible grid (O(2^N), where N is the size of the grid – the no. of immune individuals, assuming we know where the immune individuals are placed), in practice we would require some sort of heuristic for grids above a size of about 20 cells. Brute force search through the possibilities will not be efficient.