SHAHID BEHESHTI UNIVERSITY

Problem Set 1

Course: *Mathematical Physics II (Winter 2022)*

Due Date: *5:00 PM - Thursday, 5th of Esfand 1400*

Analytical Problems

Problem 1. Find the eigenvalues and corresponding normalized eigenvectors of the following matrices:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 2 & -2 & -1 \\ -1 & 3 & 1 \\ 2 & -4 & -1 \end{bmatrix}$$

Problem 2. For a three dimentional ket space of a set of orthonormals kets - say $|1\rangle, |2\rangle, |3\rangle$ - are used as the base kets. Consider operators A and B with a and b both real.

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{bmatrix} \quad , \quad B = \begin{bmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{bmatrix} \tag{1}$$

1. Show A exhibits a degenerate spectrum. Does B exhibits a degenerate spectrum?
2. show that A and B share common eigenstates without calculating their eigenstates.
3. Find a new set of orthonormal kets that are simultaneous eigenkets of both A and B. Specify the eigenvalues of A and B for each of three eigenkets. Does your specification of eigenvalues completely characterize each eigenket?

Problem 3. For matrix A with $x \neq 0$

$$\begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} \tag{2}$$

Show that its not possible to find an invertive matrix like R so that RAR^{-1} be diagonalize.

Problem 4. Using bra-ket algebra, prove

1. $tr(XY) = tr(YX)$,where X and Y are operators.
2. $tr(XY)^\dagger = tr(YX)^\dagger$, where X and Y are operators.
3. $\sum_{a'} \psi^*_{a'}(\mathbf{x}')\psi_{a'}(\mathbf{x}'')$, where $\psi^*_{a'}(\mathbf{x}') = \langle \mathbf{x}' | a' \rangle$.

Problem 5. Find eigenvalues and eigenvectors of operator $-id/dx$.

Problem 6. Operators A and B satisfy the commutation relation [A,B]=1. let $|b\rangle$ be an eigenvector of B with eigenvalue λ. Show that $e^{-\tau A}|b\rangle$ is also an eigenvector of B, but with eigenvalue $\lambda + \tau$.
Hint: First find $[B, e^{-\tau A}]$.

Computational Problems

Problem 7 (bonus). For two 3 by 3 matrices M and N of your own choice find

1. Trace of M.
2. Transpose of M.
3. Show if N and M commute.
4. Show if N is orthogonal or not.
5. Show if N is symmetric or not.