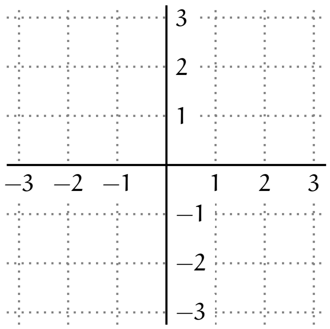
Calculus II Week 04 (Quiz 03) Problem Bank

Quiz 03 will be administered in recitation during Week 04. Two questions chosen at random will be selected for the quiz. The actual quiz questions will be variants of these practice quiz questions. In other words, the same concepts will be addressed, but numbers, expressions, or specific questions asked may differ.

1. Determine all t when the parameterized curve $\left(2t - \sin^2 t,\, t^2 + 2\cos^2 t + 1\right)$ intersects the line $y = 2x$.
2. On the given axes, sketch the parameterized curve: $\left(t^2 + t,\, 2t - 1\right)$ for $-1 \leqslant t \leqslant 1$. Show the proper general shape and label at least 3 points on the curve with their x, y, and t values.



3. Find the length of the curve.
$x = \dfrac{t^2}{2}, \quad y = \dfrac{(8t+16)^{3/2}}{12}, \quad 0 \leqslant t \leqslant 6$
4. Find a positive value of a so that the region $0 \leqslant y \leqslant 5x^a$ with $0 \leqslant x \leqslant 1$ and constant density has $\overline{x} = \dfrac{8}{13}$.
5. Find S, the area of the surface generated, when the following parametric curve is rotated about the x-axis:
$$x = \sin t, \quad y = 1 + \cos t, \quad \frac{\pi}{2} \leqslant t \leqslant \frac{3\pi}{2}$$
6. Find an equation for the tangent line to $(t^2 e^t,\, 3(t+1)e^t)$ at $t = 2$.
7. Let $x = t + \cos t$ and $y = 2 - \sin t$.
Find the value of $\dfrac{d^2y}{dx^2}$ at $t = \dfrac{\pi}{6}$.
8. Find M_x, the moment about the x-axis, of a thin plate bounded by the curve $y = \sec x$ and the x-axis for $0 \leqslant x \leqslant \dfrac{\pi}{6}$ if the density is given by $\delta = 4$.
9. Consider the curve given parametrically by
$x(t) = \displaystyle\int_0^t -6\sin^2\tau\cos\tau\, d\tau$ and
$y(t) = \displaystyle\int_0^t -6\sin\tau\cos^2\tau\, d\tau.$
Find the length of the curve for $0 \leqslant t \leqslant \dfrac{\pi}{3}$.
10. Find M_x, the moment about the x-axis, of a wire with constant density $\delta = 1$ that lies along the curve $y = \dfrac{1}{2}\sqrt{x}$ from $x = 0$ to $x = 1$.

Math 1660 Quiz 03 hints

1. The parametric form of the curve is a condensed way of specifying two equations – one involving x and the other involving y.
2. For the general shape, eliminate the parameter to get an equation involving just x and y.
3. What changes in the previous form of the arc length integral when the curve is given parametrically?
4. Integrating a polynomial term works the same way regardless of whether the exponent is a known number or an unknown constant.
5. The "arc length" part of the integral simplifies nicely.
6. $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$
7. $y' = \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} =$ "some expression in terms of t".
Then $\dfrac{d^2y}{dx^2}$ is found by repeating that process – just with y' in place of y.
8. Be careful specifying the distance of the mass of the strips from the x-axis.
9. The Fundamental Theorem of Calculus might be helpful in finding some derivatives.
10. M_x is the sum of "distance from the x-axis" times "amount of mass lying at that distance". Convert those phrases into mathematical expressions to set up the integral. (Don't memorize formulas.)

Math 1660 Quiz 03 final answers

1. $t = 1, 3$
2. $x = \dfrac{y^2 + 4y + 3}{4}$ parabola opening to the right beginning at $(0, -3)$ at $t = -1$, to $(0, -1)$ at $t = 0$, to $(2, 1)$ at $t = 1$.
3. 42
4. $a = \dfrac{3}{5}$
5. $S = 2\pi(\pi - 2)$
6. $y - 9e^2 = \dfrac{3}{2}(x - 4e^2)$
7. -4
8. $\dfrac{2\sqrt{3}}{3}$
9. $\dfrac{9}{4}$
10. $M_x = \dfrac{1}{192}\left[17\sqrt{17} - 1\right]$