Graphs with views

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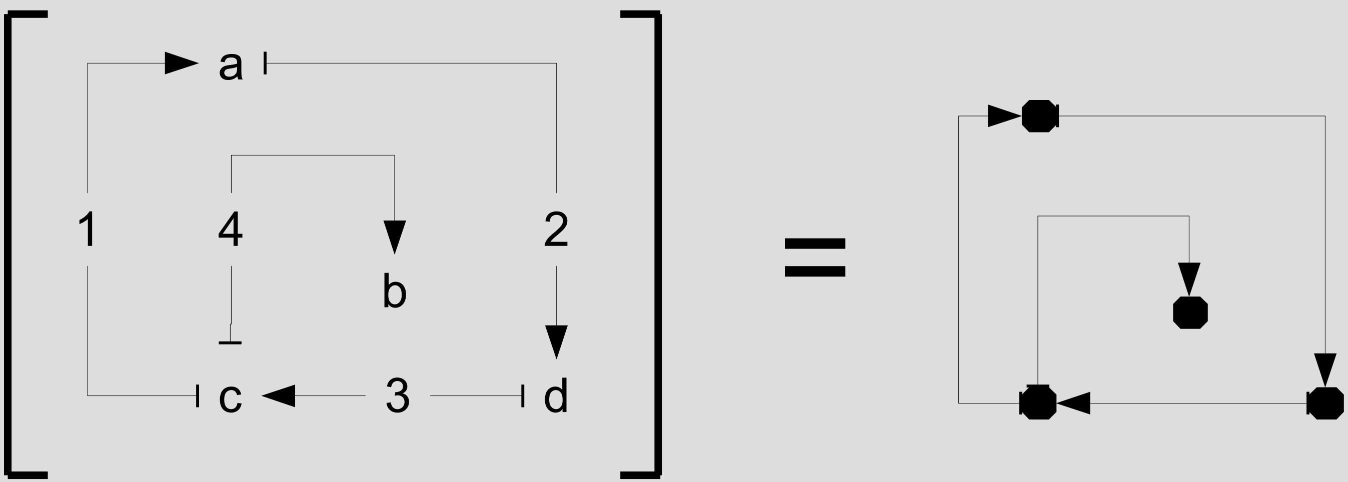
Plan

- Standard structures
- Mapping between trees
- The introduced model
- Factorisation capabilities
- Conclusion

Standard directed graphs

- *A directed graph is an ECUTI of $(V, E, src: E\to V, tgt: E\to V)$*

$$[(\{a,b,c,d\}, \{1,2,3,4\},$$
$$\{1\to c,\ 2\to a,\ 3\to d,\ 4\to c\},$$
$$\{1\to a,\ 2\to d,\ 3\to c,\ 4\to b\})]$$

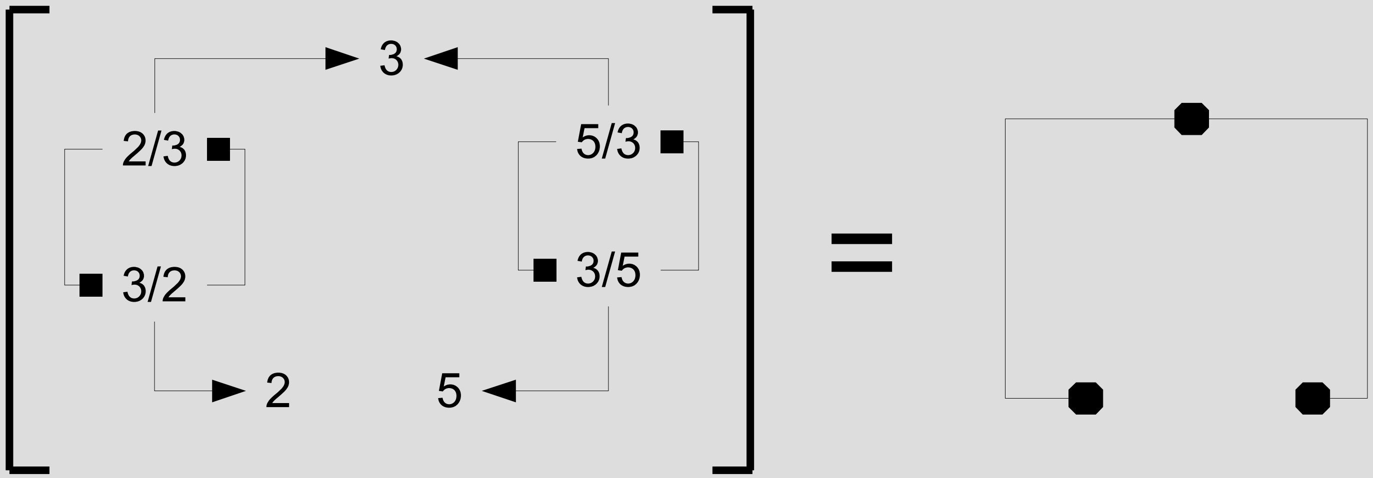


ECUTI = equivalence class up to isomorphism

Standard undirected graphs

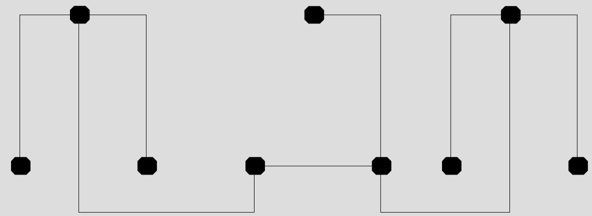
- *An undirected graph is an ECUTI of $(V, E, tgt\colon E \to V, dir\colon V \to V)$ where $dir^2 = Id_V$*

  [({2,3,5}, {2/3, 3/5, 5/3, 3/2}, inverse, dénominateur]



Standard Trees

- Minimal connected graph



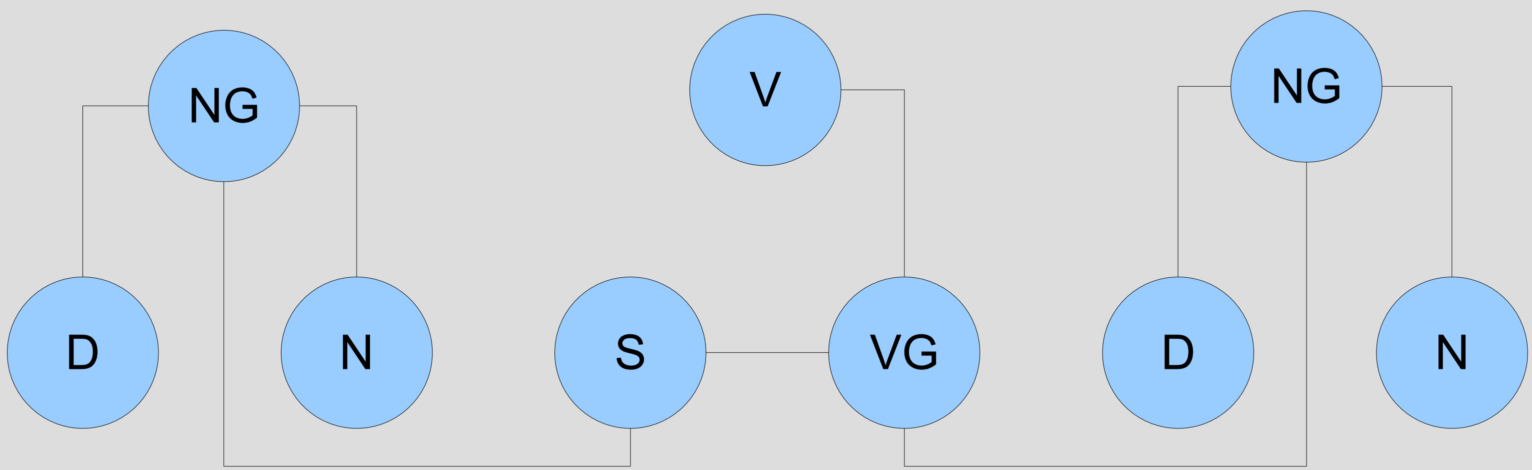
- Syntactic expression

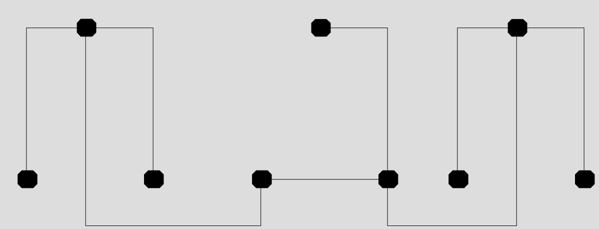
$[_S[_{NG}[_D \textit{the}_{/D}][_N \textit{cat}_{/N}]_{/NG}][_{VG}[_V \textit{eats}_{/V}][_{NG}[_D \textit{the}_{/D}][_N \textit{mouse}_{/N}]_{/NG}]_{/VG}]_{/S}]$

- Is there a mapping?

Mapping (1) : The labelling

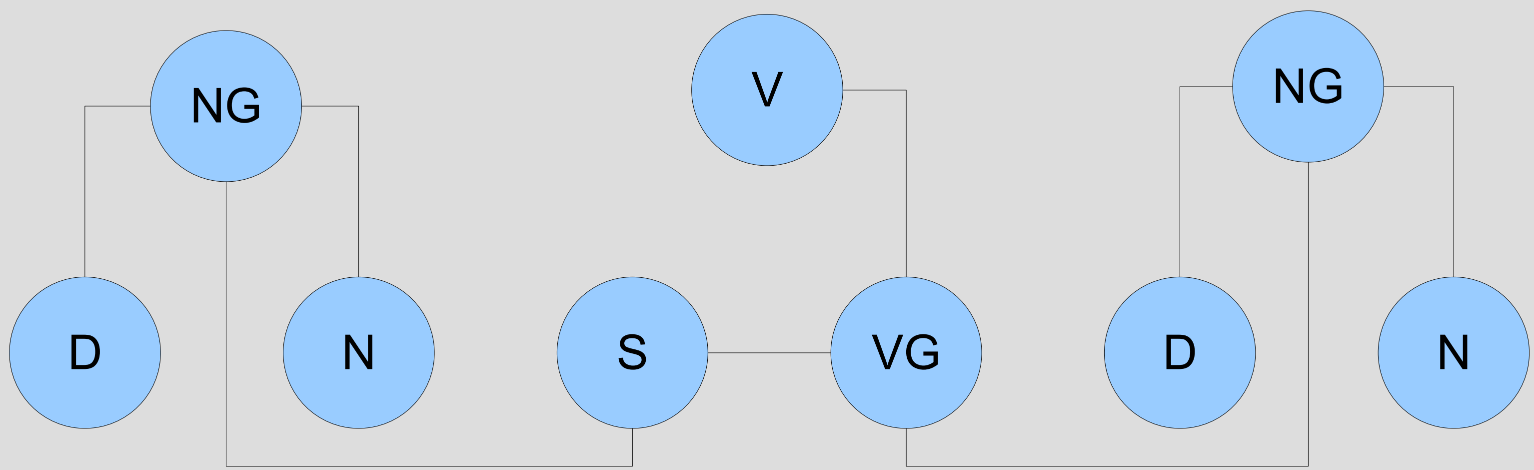
$[_S[_{NG}[_D \textit{the}_{/D}][_N \textit{cat}_{/N}]_{/NG}][_{VG}[_V \textit{eats}_{/V}][_{NG}[_D \textit{the}_{/D}][_N \textit{mouse}_{/N}]_{/NG}]_{/VG}]_{/S}]$

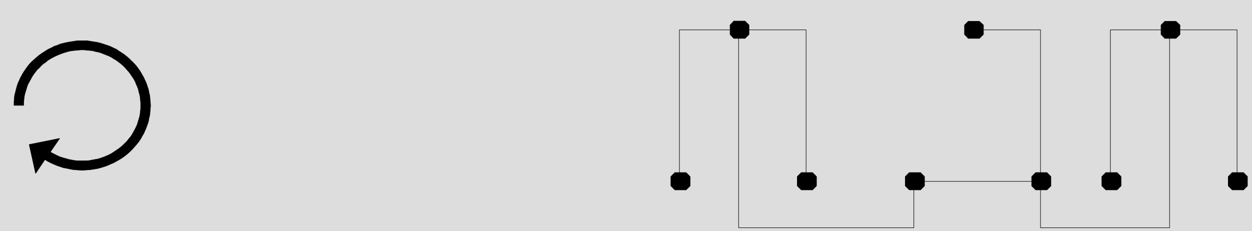




Mapping (2) : The ordering

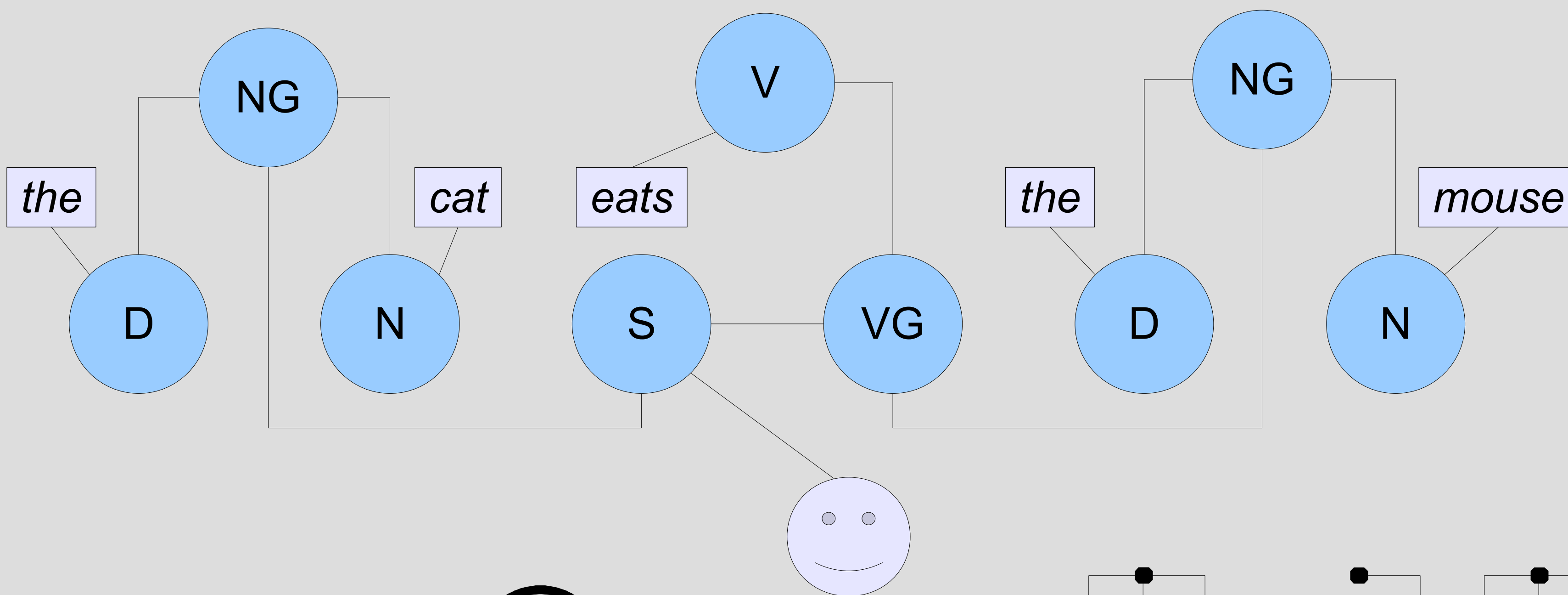
$[_{S}[_{NG}[_{D}\textit{the}_{/D}][_{N}\textit{cat}_{/N}]_{/NG}][_{VG}[_{V}\textit{eats}_{/V}][_{NG}[_{D}\textit{the}_{/D}][_{N}\textit{mouse}_{/N}]_{/NG}]_{/VG}]_{/S}]$

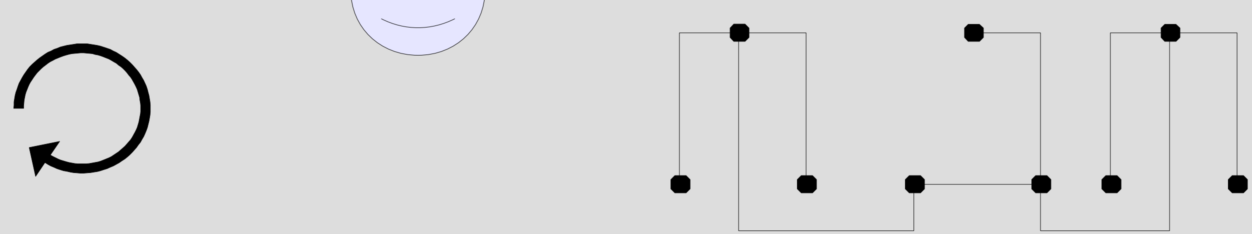




Mapping (3) : The « *views* »

$[_{S}[_{NG}[_{D}\textit{the}_{/D}][_{N}\textit{cat}_{/N}]_{/NG}][_{VG}[_{V}\textit{eats}_{/V}][_{NG}[_{D}\textit{the}_{/D}][_{N}\textit{mouse}_{/N}]_{/NG}]_{/VG}]_{/S}]$



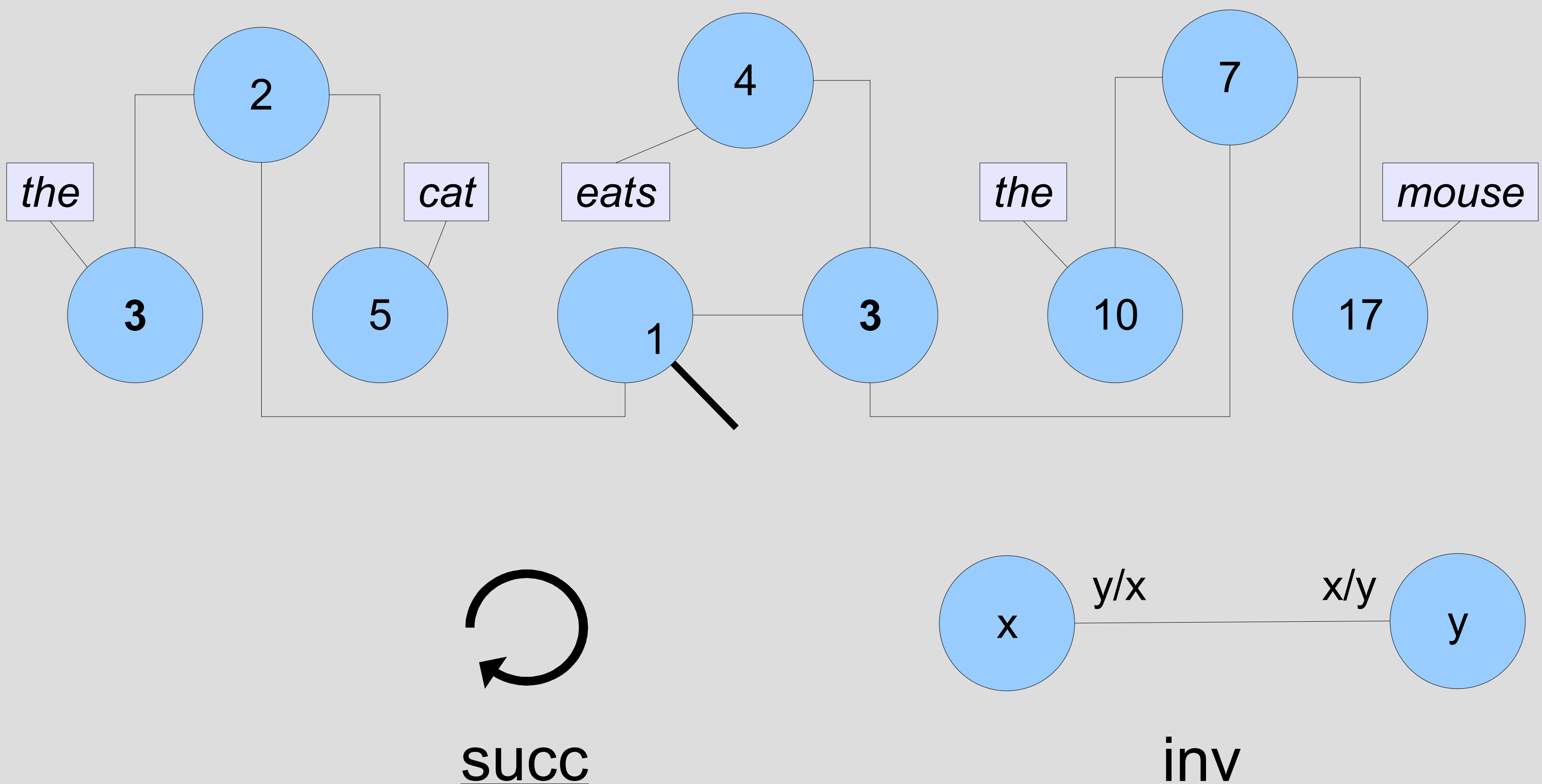


The model (1) : BDOG

- A bidirectional oriented graph (BDOG) is an ECUTI on (**Q**, succ, inv) where:
  - succ is a bijection (succ^{-1} is a function)
  - inv is a convolution ($\text{inv} \circ \text{inv} = \text{Id}_{\mathbf{Q}}$)
- The set of fractions with standard functions (succ = λx: x+1, inv = λx: 1/x) is a BDOG.
- When **Q** is finite, it gives a candidate for our problem.

Mapping (4) : The core BDOG

- Q = {1, 2, 1/2, 3/2, 2/3, *5/3*, 5/2, 2/5, *7/5*, 3, 1/3, 4/3, 3/4, *7/4*, *7/3*, 3/7, 10/7, 7/10, *17/10*, *17/7*, 7/17, *24/17*}

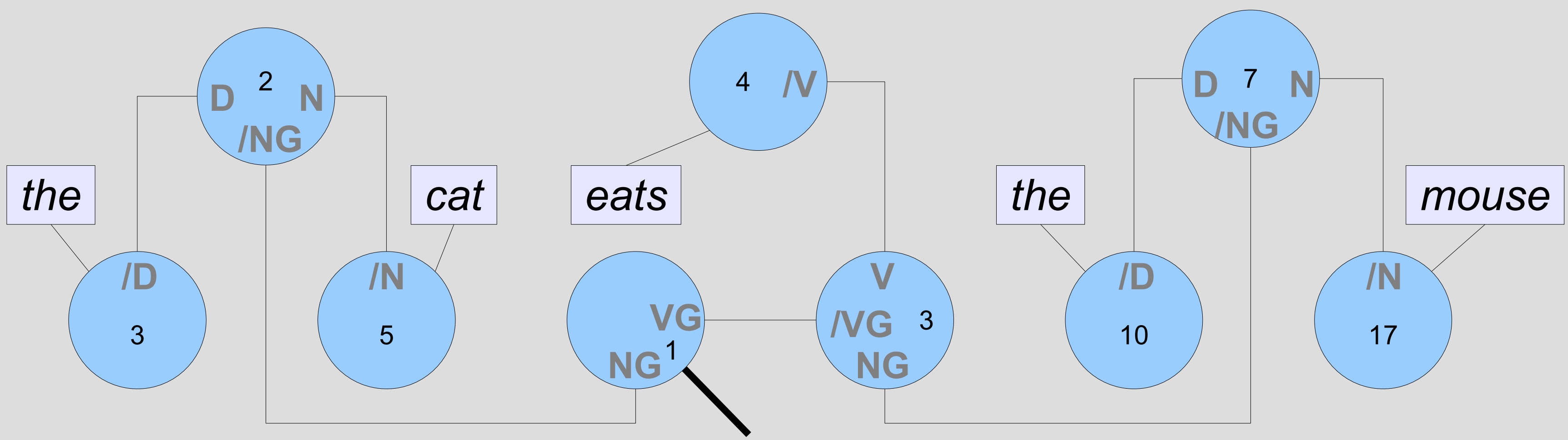


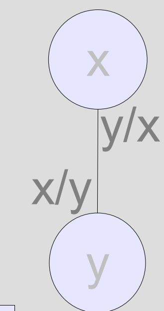
The model (2) : the ordering

- From [(**Q**, succ, inv)] where:
  - succ is a bijection ($succ^{-1}$ is a function)
  - inv is a convolution ($inv \circ inv = Id_{\mathbf{Q}}$)
- To a bijection f : *X X* we can associate a function $\mathbf{O}_f$: X X_f, which associate any element to its orbit (i.e. equivalence class of transitive cloture of f as a relation).
- [($\mathbf{Q}_{succ}$, **Q**, $\mathbf{O}_{succ}$, inv)] is an undirected graph

Mapping (5) : « edge » labels

$[_{NG}[_{D}\textit{the}_{/D}][_{N}\textit{cat}_{/N}]_{/NG}][_{VG}[_{V}\textit{eats}_{/V}][_{NG}[_{D}\textit{the}_{/D}][_{N}\textit{mouse}_{/N}]_{/NG}]_{/VG}]$





$\{1^{@}, 2_{NG}, 1/2_{/NG}, 3/2_{D}, 2/3_{/D}, 5/3^{[the]}, 5/2_{N}, 2/5_{/N}, 7/5^{[cat]}, 3_{VG},$
$1/3_{/VG}, 4/3_{V}, 3/4_{/V}, 7/4^{[eats]}, 7/3_{NG}, 3/7_{/NG}, 10/7_{D}, 7/10_{/D},$
$17/10^{[the]}, 17/7_{N}, 7/17_{/N}, 24/17^{[mouse]}\}$

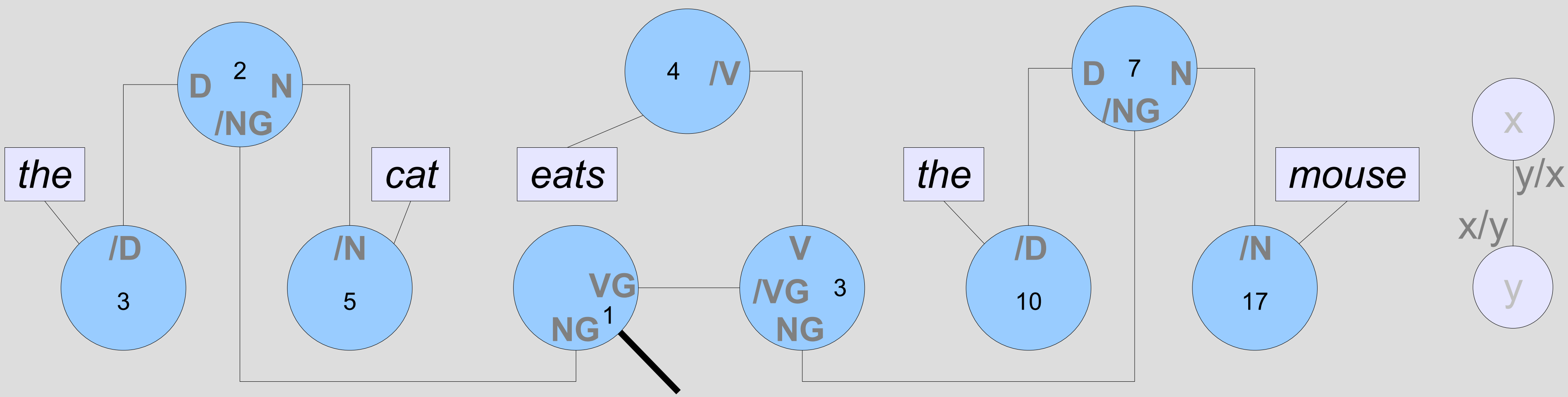
The model (3) : the « dual »

- The dual is an operation on BDOG
  - $[(\mathbf{Q}, \underline{\mathrm{succ}}, \mathrm{inv})] \rightarrow [(\mathbf{Q}, \underline{\mathrm{succ}} \circ \mathrm{inv}, \mathrm{inv})]$
- On our example it gives:
  - All nodes in a single $\underline{\mathrm{succ}} \circ \mathrm{inv}$ orbit:

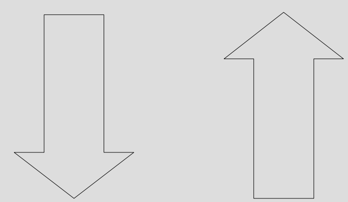
$[1^{@}, 2_{NG}, 3/2_{D}, 5/3^{[the]}, 2/3_{/D}, 5/2_{N}, 7/5^{[cat]}, 2/5_{/N}, 1/2_{/NG}, 3_{VG}, 4/3_{V}, 7/4^{[eats]}, 3/4_{/V}, 7/3_{NG}, 10/7_{D}, 17/10^{[the]}, 7/10_{/D}, 17/7_{N}, 24/17^{[mouse]}, 7/17_{/N}, 3/7_{/NG}, 1/3_{/VG}]$

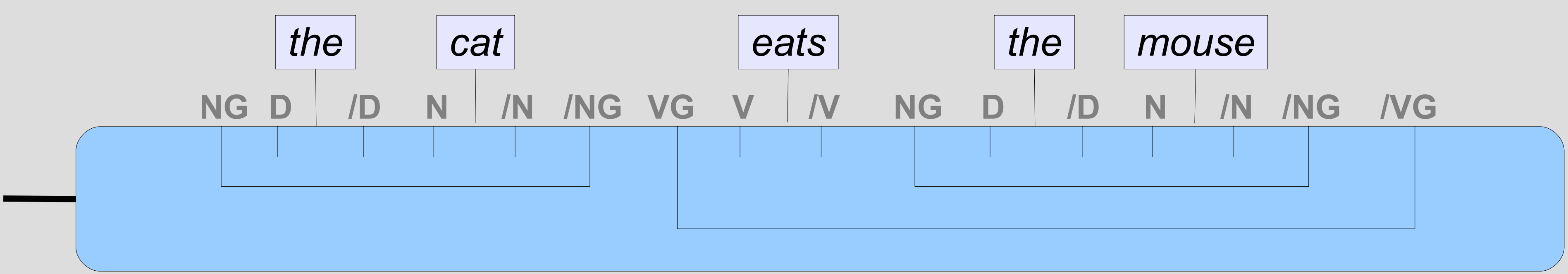
  - inv gives parenthesis association two by two!

Mapping (final) : « dual » forms



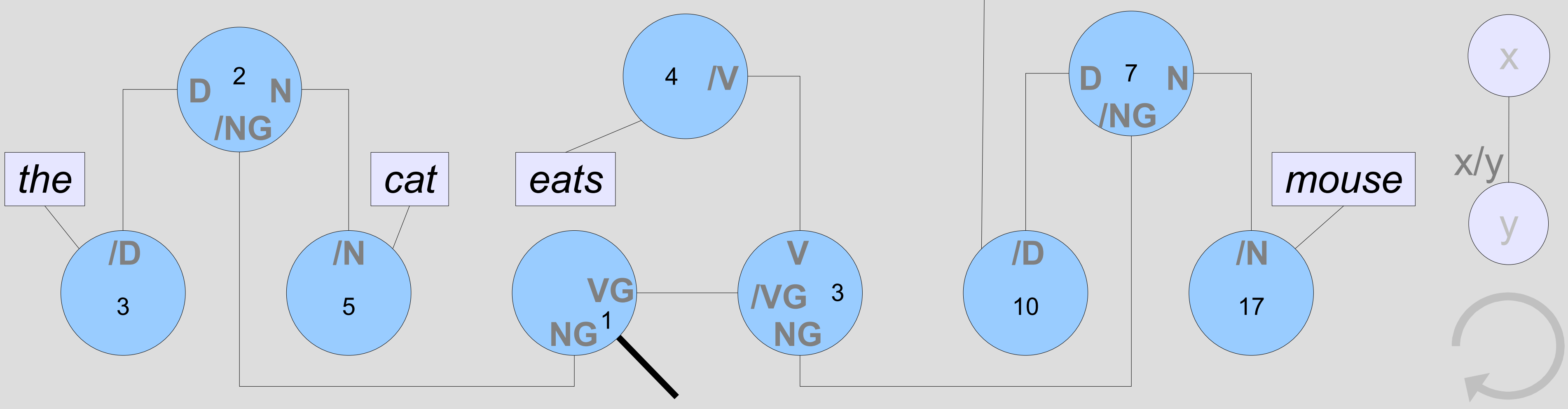
(**Q**, <u>succ</u>, inv) → (**Q**, <u>succ</u> o inv, inv)





Factorization (1) : Excel

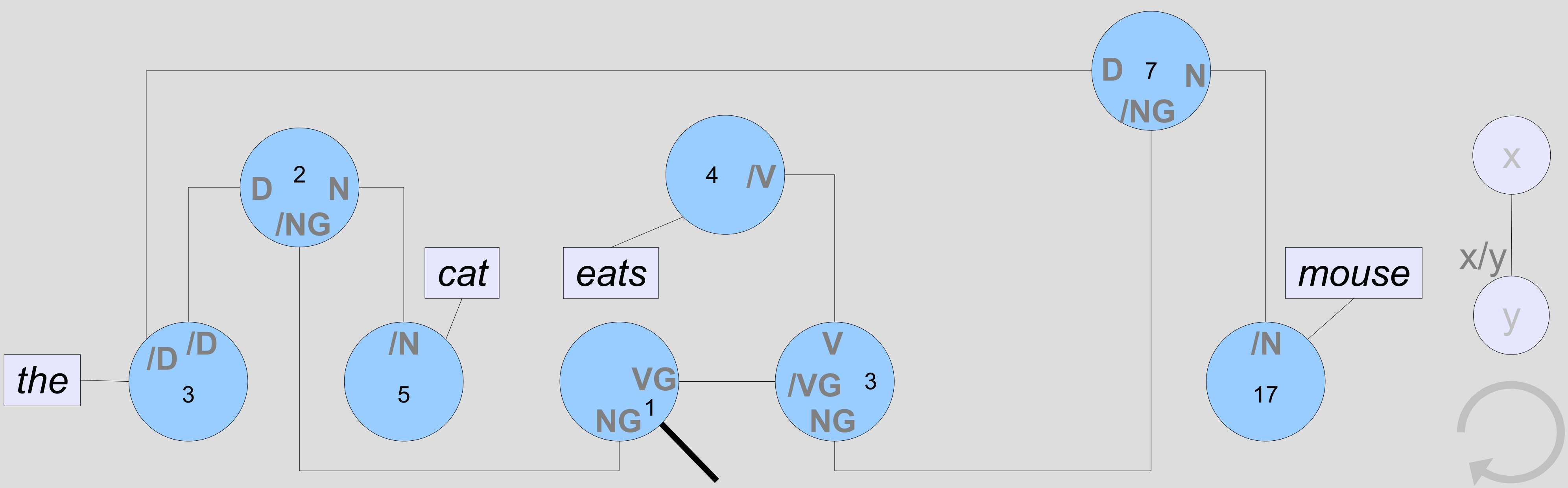
- We can interpret the « *white rectangles* » as Excel cells and use labelling to navigate throught the structure.
- Locally the structure behaves as a tree and therefore « *standard* » request language can be used for example : *=/D./NG./VG.NG.D.«get white rectangle value»*



Factorization (2) : Compiler

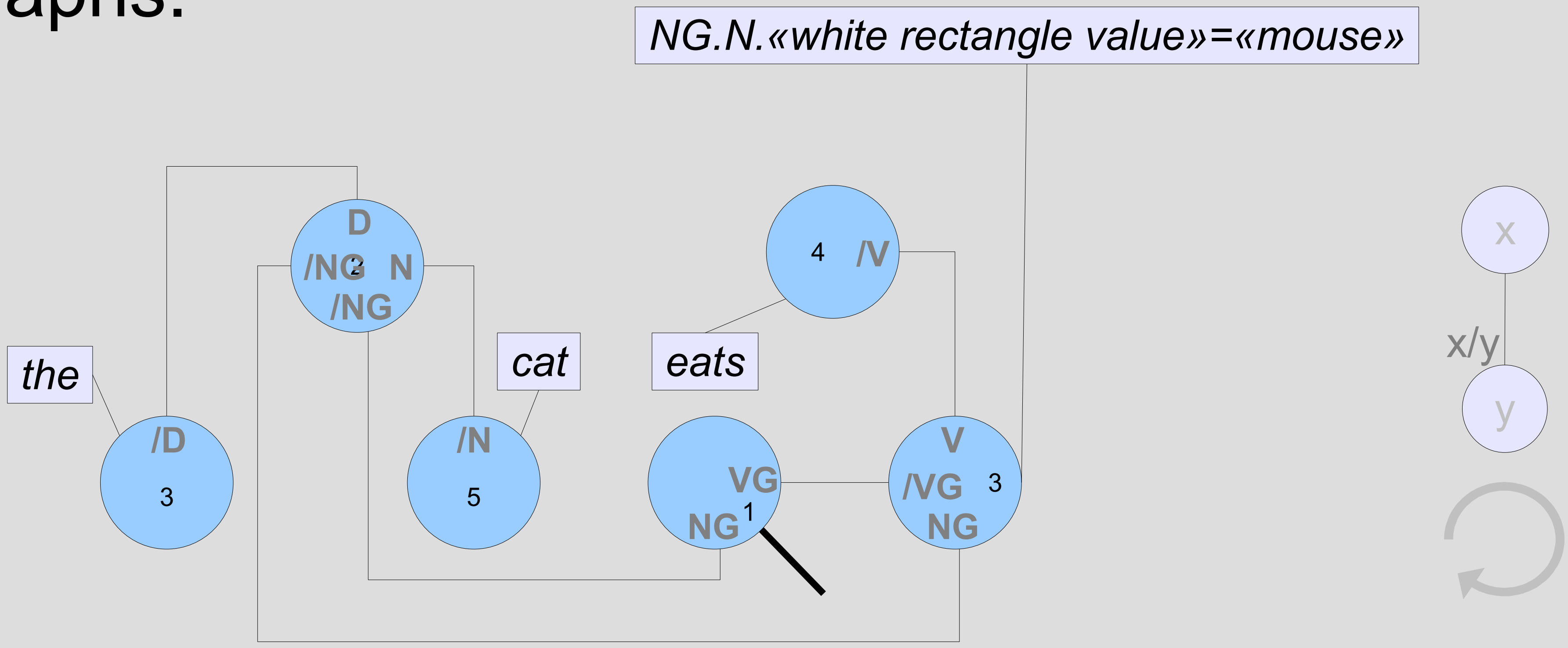
- The classical factorization consist in regrouping equivalent terms, e.g.

$[_{NG}[_{D}\mathit{the}_{/D}][_{N}\mathit{cat}_{/N}]_{/NG}][_{VG}[_{V}\mathit{eats}_{/V}][_{NG}[_{D}\mathit{the}_{/D}][_{N}\mathit{mouse}_{/N}]_{/NG}]_{/VG}]$



Factorization (3): Overload

- Another possibility for factorizing resides in « *overloading* » which allows to define different context of interpreation of sub-graphs.



Conclusion

- In order to make a mapping between syntactic graphs and graphical graphs we propose a new construction for graphs
- In this construction, at any place the graphs locally behaves as a tree but can embed any graph reality normally unreachable to trees without external reference system.
- Among capability of factorization, « *overloading* » is the most powerful and offers a new paradigm to functional call.