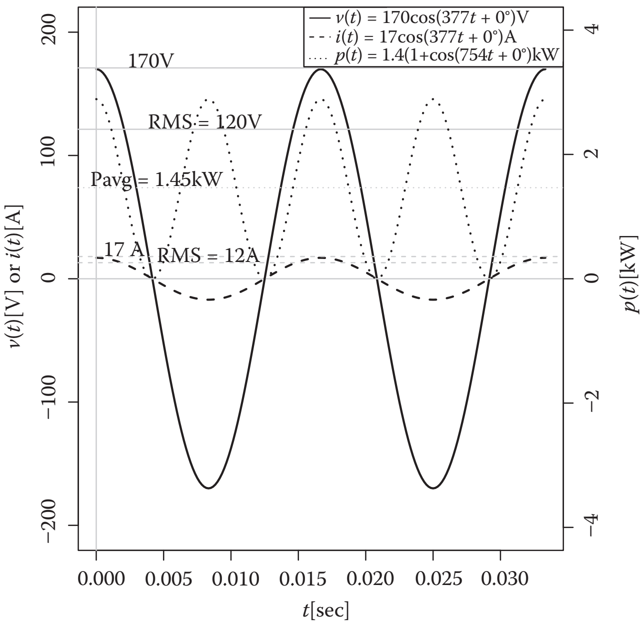

FIGURE 8.17 Voltage, current, and power in the time domain, showing rms values for voltage and current as well as average power.

$\mathbf{Z} = 1 + j1.118 = 1.5\angle 48.176°$, which has inductive reactance. To calculate current use $\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{170\angle 10°}{1.5\angle 48.176°} = 113.33\angle -38.176°$. Current lags the voltage. The rms values of voltage and current are $V = \frac{170}{\sqrt{2}}$ V $\simeq 120$ V and $I = \frac{113.33}{\sqrt{2}}$ A $\simeq 80$ A. From Equation 8.17 $p(t) \simeq (120 \times 80)$[cos
+ ($\cancel{-}$48.176°)] + cos($2\omega t - 28.176°$)] = 9.63 (0.667 + cos($754t - 28.176°$)) kW. The average power is the constant term $P = 9.63 \times 0.667 = 6.42$kW. If we wanted to calculate directly from Equation 8.18 we have $P = 120 \times 80\cos(-48.176°) = 9.63 \times 0.667 = 6.42$kW. All of these calculations can be expedited by using the following script, which produces the plots shown in Figure 8.18:

```
# RLC
w= 377; V.s=c(170,10)
v.lab <- c("v(t)","i(t)"); v.units <- c("V","A")
R=1;C=1000*10^-6;L=10*10^-3
Z.r <- c(R,w*L-1/(w*C)); Z.p <- polar(Z.r)
I.p <- div.polar(V.s,Z.p)
x <- list(V.s,I.p)
print(Z.r); print(Z.p); print(I.p)
inst.pow.plot(x,rms=TRUE)
```

Note that the average power is reduced by the factor $\cos\theta$ with respect to the one for a purely resistive impedance $P = VI$.

8.5 COMPLEX POWER

Besides instantaneous power in the time domain and its average, it is of great importance to consider AC power in the frequency domain using phasors. *Complex power* is defined as a complex number

Another way of seeing this is to use the tangent of both angles $\Delta Q = P(\tan(\cos^{-1}\theta) - \tan(\cos^{-1}\theta_c))$ where the quantity $\tan(\cos^{-1}\theta) - \tan(\cos^{-1}\theta c)$ is sometimes called the kW-factor, kWf, then $\Delta Q = P \times kWf$.

This required reduction ΔQ in kVAR is provided by a capacitor, which has a leading reactance. Thus we set $\Delta Q = Q_{cap}$. Recall that for a capacitor $X = 1/(\omega C)$ and that from Equation 8.24 $Q = V^2/X$. Therefore, $Q_{cap} = V^2/X = V^2(\omega C)$. In other words, solving for C we can calculate the capacitance C needed to correct the power factor:

$$C = \frac{Q_{cap}}{\omega V^2} \tag{8.26}$$

These calculations and graphics are implemented as functions pf.corr() and pf.corr.tri() in the package renpow taking arguments of load in kW, rms voltage V in V, existing pf, and desired pfc. We will see how to apply these using an example.

Example 8.18

A plant draws 40 A from a 240 V line (RMS) at 60 Hz to supply a 5 kW lagging load. Calculate apparent power, power factor, phase angle, reactive power, and capacitance to improve power factor to $pf = 0.9$. Answer: Apparent power $S = 40 \times 240\text{VA} = 9.6\text{kVA}$. Power factor $pf = \frac{P}{S} = \frac{5\text{ kW}}{9.6\text{ kVA}} = 0.52$. The phase angle is $\theta = \cos^{-1} 0.52 = 58.61°$. Reactive power is $Q = S \sin\theta = 9.6\text{kVA}$ x0.853 $= 8.2$ kVAR. To improve pf to 0.9 we need a new phase angle of $\theta_c = \cos^{-1} 0.9 = 25.842°$. The new apparent power is $S_c = \frac{P}{pf_c} = \frac{5\text{ kW}}{0.9} = 5.56$ kVA and the new reactive power of $Q_c = S_c \sin\theta_C = 5.55\text{kVA} \times \sin 25.842° = 2.42$ kVAR.

Therefore, we need a reactive power reduction of $Q_{cap} = 8.2 - 2.42 = 5.78$kVAR. Use Equation 8.26 to get $C = \frac{5.78 \times 1000}{377 \times 240^2} \simeq 266.1\ \mu\text{F}$.

We can solve the example using functions of renpow in the following manner:

```
P=5; V=240; I=40; pfc=0.9
pf <- P*1000/(V*I)
# call pf correction function
pfcorr <- pf.corr(P,V,pf,pfc)
```

In this particular example, we know the current and use it to calculate existing pf so that we can use it as argument to pf.corr function. The output pfcorr has the relevant results of the correction:

```
> pfcorr <- pf.corr(P,V,pf,pfc)
P=5kW, V=240V, pf=0.52, pfc=0.9
S=9.6kVA, theta=58.61°, Q=8.2kVAR, I=40
Sc=5.56kVA, theta=25.84°, Qc=2.42kVAR, Ic=23.15
kWf=1.15, Qcap=5.78kVAR, Cap=266.1uF
>
```

In addition, we can visualize the changes in the power triangle yielding Figure 8.24, which we already described while discussing the aforementioned process.

```
pf.corr.tri(pfcorr)
```

8.6.2 Power Factor Correction and Peak Demand

Suppose monthly utility rates are given in \$ per kWh consumed in a month plus \$ per peak kVA. The latter is the highest demand of apparent power during the month. We will discuss this matter following an example based on problem 3.12 of Masters [5].