This result should remind you of the simple example of complete conversion of potential energy to kinetic energy of a mass m at height z, $mgz = \frac{1}{2}mv^2$, which solves $v = \sqrt{2gz}$.

Bernoulli's equation can be normalized using the concept of *energy head* or energy per unit weight. Dividing Equation 12.1 by ρg, which is *weight density*, we obtain

$$\underbrace{\frac{p}{\rho g} + z}_{static} + \underbrace{\frac{v^2}{2g}}_{dynamic} = \frac{\text{constant}}{\rho g} = H \tag{12.2}$$

where the constant, which was energy density in $\frac{\text{Nm}}{\text{m}^3}$, is now total energy head H in m. We can see this by looking at the units $\frac{\frac{\text{Nm}}{\text{m}^3}}{\frac{\text{kg}}{\text{m}^3}\frac{\text{m}}{\text{s}^2}} = \text{m}$. In Equation 12.2, we have separated what we considered static and dynamic terms. The term $\underbrace{\frac{p}{\rho g} + z}_{static} = h$ is the hydraulic head or static head, or the sum of elevation and pressure head. Using these terms, we can rewrite Bernoulli's equation succinctly as

$$\underbrace{\frac{p}{\rho g} + z}_{h} + \frac{v^2}{2g} = h + \frac{v^2}{2g} = H \tag{12.3}$$

Stating that the sum of static head and dynamic head is the total head H, which remains constant. In this parameterization of Bernoulli's equation, the units of head are m.

Example 12.2

Consider another simple analogy of hydroelectric generation consisting of a large tank full of water as in Figure 12.8 and water flowing out of the bottom. The tank is 3 m tall and both points 1 and 2 are open to atmospheric pressure. What is the water velocity flowing out of the bottom of the tank?

Answer: We can solve this problem using Equation 12.1, but let us apply Equation 12.3 to practice V_1^2 calculations using hydraulic head. At points 1 and 2, we have $\frac{p_1}{\rho g} + z_1 + \frac{\cancel{v_2^2}}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g}$. Both pressure values are the same and cancel. We take the tank bottom as datum, and therefore $z_2 = 0$, $z_1 = 3$ m, thus head at 1 is $h_1 = z_1$ and $h_2 = 0$. At point 1, we can assume that velocity $v_1 = 0$ is zero

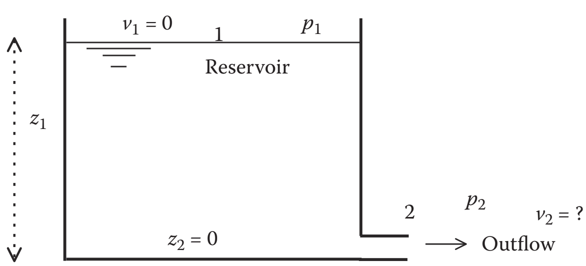

FIGURE 12.8 Illustration of Bernoulli's equation applied to water flowing out of a tank.

```
 Head loss(m) Roughness
1     11.84    150
>
```

Power is $P = \eta g\rho Q(h - h_L) = 0.9 \times 9800 \times 0.01 \times (30 - 11.84) = 1.6$ kW, or simply use P.Qh:

```
> hL <- pipe.loss(x)[1,1]
> x <- list(h=30-hL,Q=0.01~~,nu=0.9~~)
> P.Qh(x)
 Head(m) Flow(m3/s) Eff PowWater(kW) ~~PowGen(kW)~~
1  18.16     0.01 0.9     1.78     ~~1.6~~
> then multiply by efficiency
  > 0.9*P.Qh(x)[3]
    Power(kW)
  1      1.602
```

12.2.8 Turbine: Converting Pressure to Kinetic Energy and Force

Let us look a little closer at what happens at the turbine. The water at the inlet of the turbine is at pressure p_i and velocity v_i. We can apply Bernoulli's equation assuming water at the surface of the reservoir is at atmospheric pressure p_a and zero velocity. Then,

$$p_i \simeq p_a + \rho g h - \frac{1}{2}\rho v_i^2 \tag{12.11}$$

Any pressure drop along the turbine $dp = p_i - p$ will produce an increase in kinetic energy given by Bernoulli's equation. This increased kinetic energy is an increased velocity that can exert a force on the turbine blades and cause rotational movement. To simplify the discussion, we will derive the equations for a specific type of turbine (impulse turbines) where the relationship between water velocity, and force and power is easy to see. In these turbines, pressure drop is converted to a water jet with constant velocity v.

First, recall from Chapter 1 that momentum is $\mathbf{p} = mv$ and $F = \frac{d\mathbf{p}}{dt} = m\frac{dv}{dt}$, which is to say that force is rate of change of momentum, or product of mass and rate of change of velocity for constant mass. The letter p is the usual symbol for momentum; we are using **p** boldface to avoid confusion with p for pressure. A water jet of constant velocity v (product of the pressure drop) applies a force to a turbine blade given by $F = \frac{d\mathbf{p}}{dt} = \frac{dm}{dt}v = \dot{m}v$, or the product of mass flow rate and velocity. Recall that $\dot{m} = \rho A v$ and therefore $F = \rho A v^2$.

But the blade is moving at velocity u, therefore the force applied to the blade is $F = 2\rho A\,(v - u)^2$. The factor 2 occurs in these turbines because the water jet returns at the same velocity but in opposite direction. Power given to the blade is the product of force and its velocity $P = Fu = 2\rho A(v - u)^2 u$. At $u = v$ (freewheeling) and at $u = 0$ (blade at rest), the power is zero. For a blade velocity between 0 and v, we should have maximum power. Taking the derivative of power $\frac{dP}{du} = 2\rho A \times (v - u)^2 - 2(v - u)u$, and making it equal to zero, we obtain $u = \frac{1}{2}v$ for maximum power. The rotational speed ω in rad/s will be $\omega = u/r$, where r is the radius of the turbine, or $N = \omega/2\pi$ in Hz. Also, power is $P = \tau\omega$, where τ is torque.