each side? Answer: Turns ratio $n = \frac{N_2}{N_1} = \frac{v_2}{v_1} = \frac{240}{120} = 2$ and at the primary $R_{in} = \left(\frac{N_1}{N_2}\right)^2 R = \left(\frac{1}{2}\right)^2 200 = 50\ \Omega$. Current entering the primary is $i_1 = \frac{v_1}{R_{in}} = \frac{120}{50} = 2.4$ A, then reflect this current to the secondary using the reciprocal of turns ratio $i_2 = \frac{N_1}{N_2} i_1 = \frac{1}{2} 2.4 = 1.2$ A or simply use Ohm's law at the secondary $i_2 = \frac{v_2}{R} = \frac{240}{200} = 1.2$ A. Finally, power is the same for both sides $v_1 i_1 = 120\text{ V} \times 2.4\text{ A} = 288\text{ W} = 240\text{ V} \times 1.2\text{ A} = v_2 i_2$.

When considering more complicated circuits (i.e., more components connected to the transformer), we proceed similarly but have to take into account that the primary and secondary voltages (or currents) must obey Kirchhoff's voltage law (KVL) (or Kirchhoff's current law, KCL) for each of the circuits connected to these coils.

Example 10.4

Refer to Figure 10.5 with $\mathbf{V}_s = 12 \angle 30°$ V and $N_1 = 2$, $N_2 = 1$. Assume impedances connected to the transformer are $\mathbf{Z}_s = 1\ \Omega$, $\mathbf{Z}_o = 0\ \Omega$, $\mathbf{Z}_{load} = 1\ \Omega$. Is this transformer step-up or step-down? What are primary and secondary voltage, current, and power? What is the load voltage, current, and power? Answer: Turns ratio is $n = \frac{N_2}{N_1} = \frac{1}{2}$, therefore the transformer is step-down. Impedance seen from the secondary is $\mathbf{Z}_2 = \mathbf{Z}_o + \mathbf{Z}_{load} = 0 + 1 = 1\ \Omega$. Reflect secondary impedance (purely resistive) back to the primary side using $\mathbf{Z}_1 = \frac{\mathbf{Z}_2}{n^2} = \frac{1}{(1/2)^2} = 4\ \Omega$, which is also purely resistive. The impedance seen from the source is $\mathbf{Z}_s + \mathbf{Z}_1 = 1 + 4 = 5\ \Omega$, again purely resistive. Calculate primary current $\mathbf{I}_1 = \frac{\mathbf{V}_s}{5} = \frac{12}{5} \angle 30°\text{ A} = 2.4 \angle 30°\text{ A}$. This current times $\mathbf{Z}_1$ determines voltage drop across primary $\mathbf{V}_1 = 4 \times \mathbf{I}_1 = 4 \times 2.4 \angle 30°\text{ V} = 9.6 \angle 30°$ V. Now reflect primary current to the secondary using
$\mathbf{I}_2 = \frac{1}{n}\mathbf{I}_1 = 2 \times 2.4 \angle 30°\text{ A} = 4.8 \angle 30°$ A and primary voltage using $\mathbf{V}_2 = n\mathbf{V}_1 = (1/2) \times 9.6 \angle 30°$ 12
V $= 4.8 \angle 30°$ V. Complex power at primary $\mathbf{V}_1 \mathbf{I}_1^* = 9.6 \angle 30°\text{ V} \times 2.4 \angle(-30°)\text{ A} = 23.04$ W, which is all real power. Note that power supplied by the source is $\mathbf{V}_s \mathbf{I}_1^* = \cancel{10} \angle 30°\text{ V} \times 2.4 \angle(-30°)\text{ A} = 24$ W, also all real power. So there is a loss of $24 - 23.04 = 0.96$ W at $\mathbf{Z}_s$. The current through the load is the same as $\mathbf{I}_2$. Then voltage across the load is $\mathbf{V}_{load} = \mathbf{Z}_{load}\mathbf{I}_2 = 1\ \Omega \times 4.8 \angle 30°$ A $= 4.8 \angle 30°$ V, and real power consumed by the load is $P = I_{load}^2 R_{load} = 4.8^2 \times 1 = 23.04$ W.

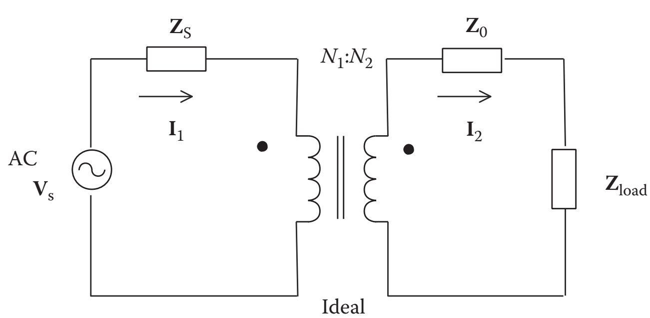

FIGURE 10.5 Transformer circuit showing primary and secondary circuits.

With the notation shown in Figure 10.12, the induced voltages are

$$\mathbf{V}_{an} = \sqrt{2}V_p\angle 0^\circ,\ \mathbf{V}_{bn} = \sqrt{2}V_p\angle -120^\circ,\ \mathbf{V}_{cn} = \sqrt{2}V_p\angle 120^\circ \tag{10.10}$$

where V_p is the rms of phase voltage. In the time domain $v_{an}(t) = \sqrt{2}V_p\cos(\omega t)$, $v_{bn}(t) = \sqrt{2}V_p\cos(\omega t - 120^\circ)$, and $v_{cn}(t) = \sqrt{2}V_p\cos(\omega t + 120^\circ)$. These voltages are in *a, b, c sequence*; or *positive phase sequence*. This means that a leads, b follows a by 120°, and c follows b by 120°.

Example 10.7

Use $V_p = 120$ V and functions ac.plot and phasor.plot of renpow to illustrate three-phase voltages. Answer: $V_p = 120$ V corresponds to a magnitude of $Vm = \sqrt{2} \times 120 \simeq 170$ V

```
Vp <- 120; Vm <- 170; Van.p <-c(Vm,0); Vbn.p <- c(Vm,-120); Vcn.p <- c(Vm,120)
v3.p  x <- list(Van.p,Vbn.p,Vcn.p); v3.t <- waves(x)  v3.p
v3t.lab <- c("van(t)","vbn(t)","vcn(t)"); v.units <- rep("V",3)
ac.plot(v3.t,v3t.lab,v.units)
v3p.lab <- c("Van","Vbn","Vcn")
v3.p  phasor.plot(x,v3p.lab,v.units,lty.p=rep(1,3))
```

The time-domain results are shown in Figure 10.14, whereas the phasors are illustrated in Figure 10.15.

The three-phase voltages add up to zero at all values of t. That is to say, $\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$. We can see this in the time domain $v_{an}(t) + v_{bn}(t) + v_{cn}(t) = 0$ by using the trigonometric identity $\frac{1}{2} \times [\cos(x+y) + \cos(x-y)] = \cos x \cos y$ in $v_{bn}(t) + v_{cn}(t)$

$$\begin{aligned} v_{bn}(t) + v_{cn}(t) &= \sqrt{2}V_p(\cos(\omega t - 120^\circ) + \cos(\omega t + 120^\circ)) \\ &= 2\sqrt{2}V_p\cos\omega t\cos(120^\circ) = -\sqrt{2}V_p\cos\omega t \end{aligned}$$

Note that when one wave is zero, the other two have the same magnitude but opposite signs. Thus, canceling and totaling zero.

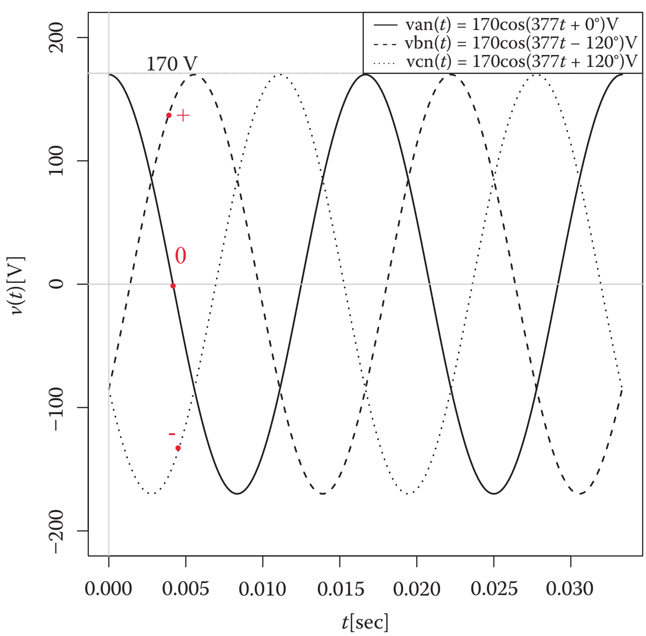

FIGURE 10.14 Time-domain plots for phase voltages. Three-phase sequence *abc*.

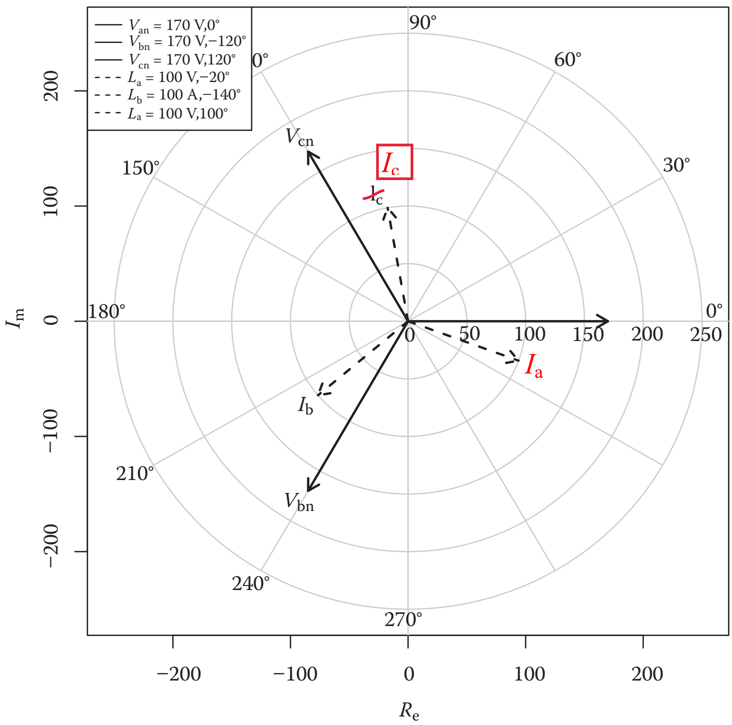

FIGURE 10.18 Phasor diagram of phase voltages and currents.

The angles are 0°, –120°,120° for voltages and –20°,–140°,100° for currents. To draw a phasor diagram use

```
Vp <- 120; Vm<-170; Van.p <-c(Vm,0); Vbn.p <- c(Vm,-120); Vcn.p <- c(Vm,120)
Z.p <- c(1.7,20)
Ia.p <- div.polar(Van.p,Z.p)
Ib.p <- div.polar(Vbn.p,Z.p)
Ic.p <- div.polar(Vcn.p,Z.p)
v3pl <- list(Van.p,Vbn.p,Vcn.p,Ia.p,Ib.p,Ic.p)
v.units <- c(rep("V",3),rep("A",3))
v3pl.lab <- c("Van","Vbn","Vcn","Ia","Ib","Ic")
phasor.plot(v3pl,v3pl.lab,v.units,lty.p=c(rep(1,3),rep(2,3)))
```

The result is shown in Figure 10.18. We can see how each current lags the corresponding voltage by 20° and it is reduced in magnitude according to impedance.

10.2.3 Power

To calculate power, we can treat each phase independently of the others and then sum the results. Each phase is calculated as for single-phase circuits (Chapter 8). Taking into account that all impedances are the same, and the angles are 0°,–120°,120°, the instantaneous power in each phase is

$$
\begin{aligned}
p_a(t) &= v_{an}(t)i_a(t) = V_p I_p[\cos\theta + \cos(2\omega t + 0^\circ - \theta)] \\
p_b(t) &= v_{bn}(t)i_b(t) = V_p I_p[\cos\theta + \cos(2\omega t - 120^\circ - \theta)] \\
p_c(t) &= v_{cn}(t)i_c(t) = V_p I_p[\cos\theta + \cos(2\omega t + 120^\circ - \theta)]
\end{aligned}
\tag{10.14}
$$

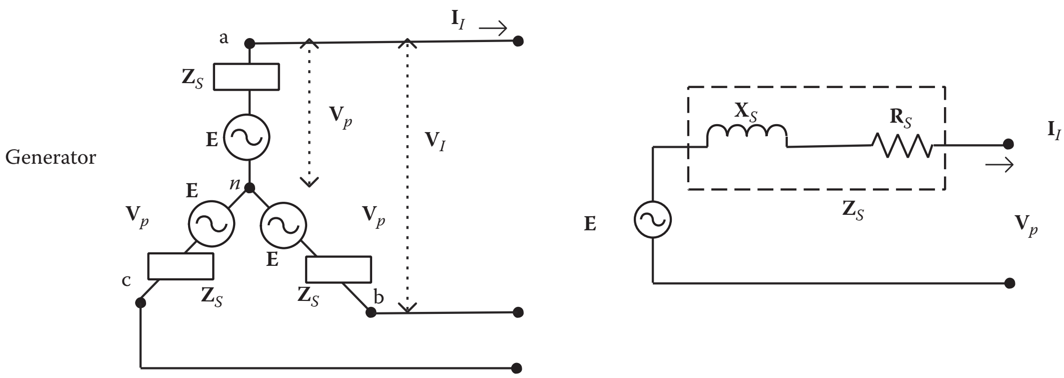

FIGURE 10.21 Synchronous generator model.

output voltage or phase voltage, and $\mathbf{I}_l$ is the line current delivering power at a given *pf*. Therefore, $\mathbf{E} = \mathbf{I}_p\mathbf{Z}_s + \mathbf{V}_p$.

The magnitude ~~E~~ $\boxed{E}$ or **E** is controlled by the amount of field current provided to the generator, whereas the angle δ of **E** is controlled by the torque provided by the prime mover to the generator. Recall from Chapters 1 and 5 that the power provided by the prime mover is torque multiplied by mechanical angular speed $P_{mech} = \tau\omega_{mech} \times \frac{\pi}{30}$ W, where torque τ is in Nm, and angular speed ω_{mech} is in revolutions per minute (rpm). For a fixed speed in rpm, to obtain 60 Hz increasing the torque will require more power from the prime mover. For a Y-Y connected three-phase generator with rated $S_{3\phi}$ and V_l, the real power is $P_{3\phi} = S_{3\phi} \cos\theta$. The mechanical power required for a given electrical generator efficiency η_e is $P_{mech} = \frac{P_{3\phi}}{\eta_e}$. We can calculate torque for a desired rpm.

As we mentioned in the previous section, the *rated* rms voltage and current are given as the line voltage V_l and current I_l. For Y-connected, $V_l = \sqrt{3}V_p$, $I_l = I_p$, and also $P_{3\phi} = S_{3\phi}\cos\theta = \sqrt{3}V_l I_l \cos\theta$. As a reference we can assume $\mathbf{V}_p = V_p \angle 0°$ and then $\mathbf{I}_p = I_p \angle \cos^{-1}(pf)$.

Example 10.12

Consider a 15 MVA, 13.8 kV, 1800 rpm, 60 Hz, 98% efficient, Y-connected three-phase generator with $\mathbf{Z}_s = 0.1 + j2\ \Omega$ per phase, delivering rated current at $pf = 0.85$ lagging.

What is the torque in Nm? What is the three-phase real power? What is the armature current? What is the phase voltage? What is the **E** required to provide this phase voltage? Note: The values given are the rated three-phase apparent power and line voltage. Answer: We are given rated $S_{3\phi}$ = 15 MVA and V_l = 13.8 kV. The real power is $P_{3\phi} = S_{3\phi}\cos\theta = 15 \times 0.85 = 12.75$ MW. The mechanical power required is $P_{mech} = \frac{P_{3\phi}}{\eta_e} = \frac{12.75\text{ MW}}{0.98} = 13.01$ MW. Torque $\tau = \frac{P_{mech}}{\omega_{mech} \times \frac{\pi}{30}}$ and substituting values $\tau = \frac{13.01 \times 10^6}{1800 \times \frac{\pi}{30}}$ Nm = 69.02 kNm. We can use $S_{3\phi} = \sqrt{3}V_l I_l$ to solve for the rms of line current (which will be the same as the phase and the armature current) $I_l = \frac{S_{3\phi}}{\sqrt{3}V_l} = \frac{15000\text{ kVA}}{\sqrt{3} \times 13.8\text{ kV}} = 627.55$ A. Thus, $I_p = 627.55$ A. The phase voltage is solved from $V_l = \sqrt{3}V_p$ to get $V_p = \frac{13.8}{\sqrt{3}} = 7.967$ kV. Then, $\mathbf{E} = \mathbf{I}_p\mathbf{Z}_s + \mathbf{V}_p = 627.55\text{ A} \angle \cos^{-1}(0.85)(0.1 + j2\ \Omega) + 7967 \angle 0°$V. Now, convert impedance to polar, and multiply $\mathbf{E} = (627.55\text{ A} \angle$ ~~31.79~~ -31.79$°)(2.002 \angle 87.13°) + 7967 \angle$

0° = 8.743 ∠ 6.789° kV. We have expressed the phasors with magnitude given by the rms. We could multiply by $\sqrt{2}$ to express as amplitude of the wave.

The **E** result from this example is interpreted to be the induced voltage **E** required to provide the required real power at $\mathbf{V_p}$ for this line and load. The field current of the generator controls the magnitude of **E**, whereas the angle δ of **E** is controlled by the torque provided by the prime mover.

These calculations are expedited with function generator of renpow

```
x <- list(S3p = 15*10^6, Vl.rms = 13.8*10^3, pf=0.85, lead.lag=-1, Zs.r = c(0.1,2))
generator(x)
```

which gives us the **E**, $\mathbf{V_p}$, and $\mathbf{I_l}$ in polar form and produces a phasor diagram (Figure 10.22)

```
> generator(x)
$E.p
[1] 8743.058   6.789
$Vp.p
[1] 7967.434   0.000
$Il.p
[1] 627.55464 -31.78833
```

This phasor diagram represents conditions when E is larger than V_p and the current lags $\mathbf{V_p}$.

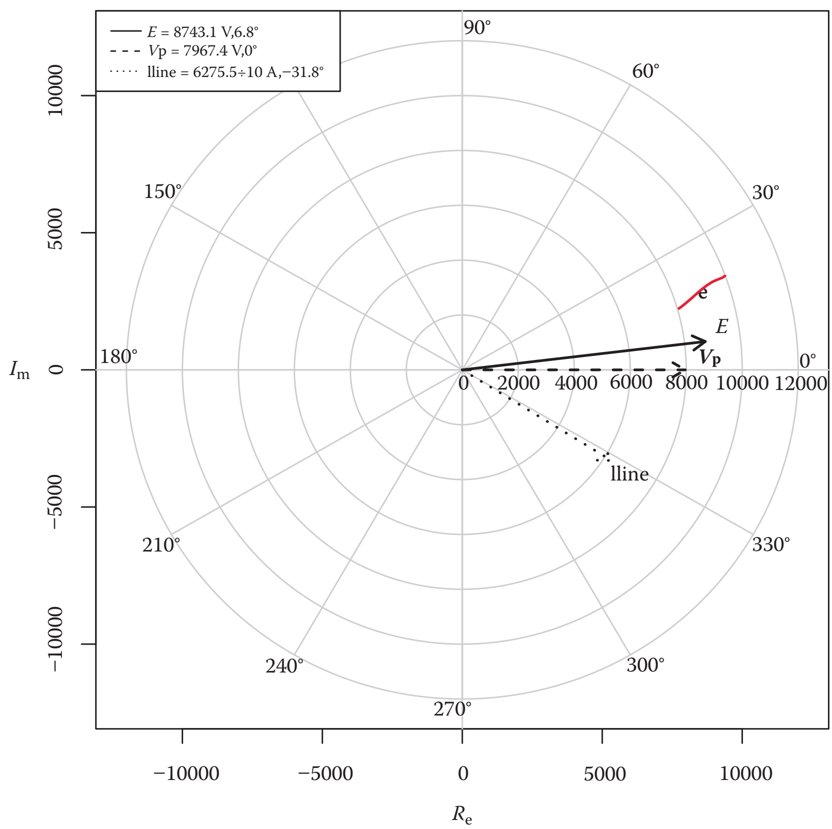

FIGURE 10.22 Phasor diagram of synchronous generator.