Algorithm of Conservative Convergent-Divergent Duct

1. Input nodes, parameters, and constant variables
2. Process Duct Geometry with equation:

$$A = 1 + 2.2\,(x - 1.5)^2 \qquad 0 \leq x \leq 3 \tag{1}$$

3. Calculate Δx
4. Calculate initial conditions

   (a) Initial condition for ρ' and T':

$$\left.\begin{aligned} \rho' &= 1.0 \\ T' &= 1.0 \end{aligned}\right\} \quad for \quad 0 \leq x' \leq 0.5 \tag{2}$$

$$\left.\begin{aligned} \rho' &= 1.0 - 0.366\,(x' - 0.5) \\ T' &= 1.0 - 0.167\,(x' - 0.5) \end{aligned}\right\} \quad for \quad 0.5 < x' \leq 1.5 \tag{3}$$

$$\left.\begin{aligned} \rho' &= 0.634 - 0.702\,(x' - 1.5) \\ T' &= 0.833 - 0.4908\,(x' - 1.5) \end{aligned}\right\} \quad for \quad 1.5 < x' \leq 2.1 \tag{4}$$

$$\left.\begin{aligned} \rho' &= 0.5892 + 0.10228\,(x' - 2.1) \\ T' &= 0.93968 + 0.0622\,(x' - 2.1) \end{aligned}\right\} \quad for \quad 2.1 < x' \leq 3.0 \tag{5}$$

   (b) Initial condition for v' (use constant mass flow):

$$v' = \frac{U_2}{\rho' A'} = \frac{0.59}{\rho' A'} \tag{6}$$

   (c) Initial conditions for U_1, U_2, and U_3:

$$U_1 = \rho' A' \tag{7}$$

$$U_2 = \rho' A' v' \tag{8}$$

$$U_3 = \rho' \left(\frac{e'}{\gamma - 1} + \frac{\gamma}{2} v'^2 \right) A' \quad ; \quad e' = T' \tag{9}$$

5. Looping Process

   (a) Calculate a':

$$a' = \sqrt{\gamma R T'} \tag{10}$$

   (b) Calculate Δt from each grid, and choose the minimum value:

$$\Delta t' = C \frac{\Delta x'}{a_i' + v_i'} \quad ; \quad 0 < C \leq 1 \tag{11}$$

(c) Predictor Step

i. Calculate boundary conditions (BC) for subsonic outflow:
BC for U_1 and U_2:

$$U_{1_{i=n-1}} = 2U_{1_{i=n-2}} - U_{1_{i=n-3}} \tag{12}$$
$$U_{2_{i=n-1}} = 2U_{2_{i=n-2}} - U_{2_{i=n-3}} \tag{13}$$

BC for v':

$$v'_{i=n-1} = \frac{U_{2_{n-1}}}{U_{1_{n-1}}} \tag{14}$$

BC for U_3:

$$U_{3_{i=n-1}} = \frac{P'_{n-1}A'}{\gamma - 1} + \frac{\gamma}{2} U_{2_{n-1}} v'_{n-1} \tag{15}$$

choose $P'_{n-1} = 0.6784$ for default case.

ii. Calculate F_1, F_2, F_3, and J_2:
F_1, F_2, and F_3:

$$F_1 = U_2 \tag{16}$$
$$F_2 = \frac{U_2^2}{U_1} + \frac{\gamma - 1}{\gamma}\left(U_3 - \frac{\gamma}{2}\frac{U_2^2}{U_1}\right) \tag{17}$$
$$F_3 = \gamma \frac{U_2 U_3}{U_1} - \frac{\gamma\left(\gamma - 1\right)}{2}\frac{U_2^3}{U_1^2} \tag{18}$$

J_2:

$$J_2 = \frac{\gamma - 1}{\gamma}\left(U_3 - \frac{\gamma}{2}\frac{U_2^2}{U_1}\right)\frac{\partial\left(lnA'\right)}{\partial x'} \tag{19}$$
$$= \frac{1}{\gamma}\rho' T' \frac{\partial A'}{\partial x'} \tag{20}$$

iii. Calculate time derivative of U_1, U_2, and U_3 using forward difference:

$$\frac{\partial U_1}{\partial t'} = -\frac{\partial F_1}{\partial x'} \tag{21}$$
$$\frac{\partial U_2}{\partial t'} = -\frac{\partial F_2}{\partial x'} + J_2 \tag{22}$$
$$\frac{\partial U_3}{\partial t'} = -\frac{\partial F_3}{\partial x'} \tag{23}$$

In finite difference form:

$$\left(\frac{U_1}{\partial t'}\right)_i^n = -\left(\frac{(F_1)_{i+1} - (F_1)_i}{\Delta x'}\right) \tag{24}$$
$$\left(\frac{U_2}{\partial t'}\right)_i^n = -\left(\frac{(F_2)_{i+1} - (F_2)_i}{\Delta x'}\right) + J_2 \tag{25}$$
$$\left(\frac{U_3}{\partial t'}\right)_i^n = -\left(\frac{(F_3)_{i+1} - (F_3)_i}{\Delta x'}\right) \tag{26}$$

iv. Compute artificial velocity:

$$S_i^{t'} = \frac{C_x \left| (p')_{i+1}^{t'} - 2\,(p')_i^{t'} + (p')_{i-1}^{t'} \right|}{(p')_{i+1}^{t'} + 2\,(p')_i^{t'} + (p')_{i-1}^{t'}} \left(U_{i+1}^{t'} - 2U_i^{t'} + U_{i-1}^{t'} \right) \tag{27}$$

v. Compute the predicted value with usage of the artificial velocity:

$$\left(\overline{U}_1\right)_i^{n+1} = (U_1)_i^n + \left(\frac{\partial U_1}{\partial t'}\right)_i^n \Delta t' + (S_1)_i^{t'} \tag{28}$$

$$\left(\overline{U}_2\right)_i^{n+1} = (U_2)_i^n + \left(\frac{\partial U_2}{\partial t'}\right)_i^n \Delta t' + (S_2)_i^{t'} \tag{29}$$

$$\left(\overline{U}_3\right)_i^{n+1} = (U_3)_i^n + \left(\frac{\partial U_3}{\partial t'}\right)_i^n \Delta t' + (S_3)_i^{t'} \tag{30}$$

Primitive variables:

$$\left(\overline{\rho'}\right)_i^{n+1} = \frac{\left(\overline{U}_1\right)_i^{n+1}}{(A')_i} \tag{31}$$

$$\left(\overline{T'}\right)_i^{n+1} = (\gamma - 1)\left[\frac{\left(\overline{U}_3\right)_i^{n+1}}{\left(\overline{U}_1\right)_i^{n+1}} - \frac{\gamma}{2}\left(\frac{\left(\overline{U}_2\right)_i^{n+1}}{\left(\overline{U}_1\right)_i^{n+1}} \right)^2 \right] \tag{32}$$

(d) Corrector Step

i. Calculate boundary conditions (BC) for subsonic inflow:
BC for ρ' and T':

$$\rho'_{i=0} = 1.0 \tag{33}$$

$$T'_{i=0} = 1.0 \tag{34}$$

BC for U_1:

$$U_{1_{i=0}} = \rho' A' = A' = \text{fixed} \tag{35}$$

BC for U_2:

$$U_{2_{i=0}} = 2U_{2_{i=1}} - U_{2_{i=2}} \tag{36}$$

BC for v':

$$v'_{i=0} = \frac{U_{2_{i=0}}}{U_{1_{i=0}}} \tag{37}$$

BC for U_3:

$$U_{3_{i=0}} = \rho'_{i=0} \left(\frac{e'_{i=0}}{\gamma - 1} + \frac{\gamma}{2} v^{2\prime}{}_{i=0} \right) A' \tag{38}$$

$$= U_{1_{i=0}} \left(\frac{T'_{i=0}}{\gamma - 1} + \frac{\gamma}{2} v'^2_{i=0} \right) \tag{39}$$

ii. Compute $\overline{F}_1$, $\overline{F}_2$, $\overline{F}_3$, and $\overline{J}_2$:
$\overline{F}_1$, $\overline{F}_2$, and $\overline{F}_3$:

$$\overline{F}_1 = \overline{U}_2 \tag{40}$$

$$\overline{F}_2 = \frac{\overline{U}_2^2}{\overline{U}_1} + \frac{\gamma - 1}{\gamma}\left(\overline{U}_3 - \frac{\gamma}{2}\frac{\overline{U}_2^2}{\overline{U}_1}\right) \tag{41}$$

$$\overline{F}_3 = \gamma\frac{\overline{U}_2\overline{U}_3}{\overline{U}_1} - \frac{\gamma(\gamma - 1)}{2}\frac{\overline{U}_2^3}{\overline{U}_1^2} \tag{42}$$

$\overline{J}_2$:

$$\overline{J}_2 = \frac{\gamma - 1}{\gamma}\left(\overline{U}_3 - \frac{\gamma}{2}\frac{\overline{U}_2^2}{\overline{U}_1}\right)\frac{\partial(lnA')}{\partial x'} \tag{43}$$

$$= \frac{1}{\gamma}\rho' T'\frac{\partial A'}{\partial x'} \tag{44}$$

iii. Calculate time derivative of $\overline{U}_1$, $\overline{U}_2$, and $\overline{U}_3$ using backward difference:

$$\frac{\partial\overline{U}_1}{\partial t'} = -\frac{\partial\overline{F}_1}{\partial x'} \tag{45}$$

$$\frac{\partial\overline{U}_2}{\partial t'} = -\frac{\partial\overline{F}_2}{\partial x'} + \overline{J}_2 \tag{46}$$

$$\frac{\partial\overline{U}_3}{\partial t'} = -\frac{\partial\overline{F}_3}{\partial x'} \tag{47}$$

In finite difference form:

$$\left(\frac{\partial\overline{U}_1}{\partial t'}\right)_i^{n+1} = -\left(\frac{\left(\overline{F}_1\right)_i - \left(\overline{F}_1\right)_{i-1}}{\Delta x'}\right) \tag{48}$$

$$\left(\frac{\partial\overline{U}_2}{\partial t'}\right)_i^{n+1} = -\left(\frac{\left(\overline{F}_2\right)_i - \left(\overline{F}_2\right)_{i-1}}{\Delta x'}\right) + \overline{J}_2 \tag{49}$$

$$\left(\frac{\partial\overline{U}_3}{\partial t'}\right)_i^{n+1} = -\left(\frac{\left(\overline{F}_3\right)_i - \left(\overline{F}_3\right)_{i-1}}{\Delta x'}\right) \tag{50}$$

iv. Compute the average time derivative:

$$\left(\frac{\partial U_1}{\partial t}\right)_{avg} = \frac{1}{2}\left[\left(\frac{\partial U_1}{\partial t'}\right)_i^n + \left(\frac{\partial\overline{U}_1}{\partial t'}\right)_i^{n+1}\right] \tag{51}$$

$$\left(\frac{\partial U_2}{\partial t}\right)_{avg} = \frac{1}{2}\left[\left(\frac{\partial U_2}{\partial t'}\right)_i^n + \left(\frac{\partial\overline{U}_2}{\partial t'}\right)_i^{n+1}\right] \tag{52}$$

$$\left(\frac{\partial U_3}{\partial t}\right)_{avg} = \frac{1}{2}\left[\left(\frac{\partial U_3}{\partial t'}\right)_i^n + \left(\frac{\partial\overline{U}_3}{\partial t'}\right)_i^{n+1}\right] \tag{53}$$

v. Compute artificial velocity:

$$\overline{S}_i^{t'+\Delta t'} = \frac{C_x \left| (\overline{p}')_{i+1}^{t'+\Delta t'} - 2\,(\overline{p}')_i^{t'+\Delta t'} + (\overline{p}')_{i-1}^{t'+\Delta t'} \right|}{(\overline{p}')_{i+1}^{t'} + 2\,(\overline{p}')_i^{t'} + (\overline{p}')_{i-1}^{t'}} \left(\overline{U}_{i+1}^{t'+\Delta t'} - 2\overline{U}_i^{t'+\Delta t'} + \overline{U}_{i-1}^{t'+\Delta t'} \right) \tag{54}$$

(e) Calculate new time-step variables:

i. Calculate new U_1, U_2, and U_3:

$$(U_1)_i^{n+1} = (U_1)_i^n + \left(\frac{\partial U_1}{\partial t'}\right)_{avg} \Delta t' + \left(\overline{S}_1\right)_i^{t'} \tag{55}$$

$$(U_2)_i^{n+1} = (U_2)_i^n + \left(\frac{\partial U_2}{\partial t'}\right)_{avg} \Delta t' + \left(\overline{S}_1\right)_i^{t'} \tag{56}$$

$$(U_3)_i^{n+1} = (U_3)_i^n + \left(\frac{\partial U_3}{\partial t'}\right)_{avg} \Delta t' + \left(\overline{S}_1\right)_i^{t'} \tag{57}$$

ii. Calculate primitive variables:

$$(\rho')_i^{n+1} = (U_1)_i^{n+1} \tag{58}$$

$$(v')_i^{n+1} = \left(\frac{U_2}{U_1}\right)_i^{n+1} \tag{59}$$

$$(T')_i^{n+1} = (\gamma - 1)\left(\frac{U_3}{U_1} - \frac{\gamma}{2}v'^2\right)_i^{n+1} \tag{60}$$

(f) Calculate errors in RMS for both ρ, v, and T.

(g) If the error is bigger than desired error, start again with the new variables.

(h) Finish computation.

6. Print output.

7. Program exit.