

Algorithm of Conservative Convergent-Divergent Duct

1. Input nodes, parameters, and constant variables
2. Process Duct Geometry with equation:

$$A = 1 + 2.2(x - 1.5)^2 \quad 0 \leq x \leq 3 \quad (1)$$

3. Calculate Δx
4. Calculate initial conditions for $i = 1 \sim i = n - 2$ (c-like array)
 - (a) Initial condition for ρ' and T' :

$$\left. \begin{array}{l} \rho' = 1.0 \\ T' = 1.0 \end{array} \right\} \quad for \quad 0 \leq x' \leq 0.5 \quad (2)$$

$$\left. \begin{array}{l} \rho' = 1.0 - 0.366(x' - 0.5) \\ T' = 1.0 - 0.167(x' - 0.5) \end{array} \right\} \quad for \quad 0.5 < x' \leq 1.5 \quad (3)$$

$$\left. \begin{array}{l} \rho' = 0.634 - 0.702(x' - 1.5) \\ T' = 0.833 - 0.4908(x' - 1.5) \end{array} \right\} \quad for \quad 1.5 < x' \leq 2.1 \quad (4)$$

$$\left. \begin{array}{l} \rho' = 0.5892 + 0.10228(x' - 2.1) \\ T' = 0.93968 + 0.0622(x' - 2.1) \end{array} \right\} \quad for \quad 2.1 < x' \leq 3.0 \quad (5)$$

- (b) Initial condition for v' (use constant mass flow):

$$v' = \frac{U_2}{\rho' A'} = \frac{0.59}{\rho' A'} \quad (6)$$

- (c) Initial conditions for U_1 , U_2 , and U_3 :

$$U_1 = \rho' A' \quad (7)$$

$$U_2 = \rho' A' v' \quad (8)$$

$$U_3 = \rho' \left(\frac{e'}{\gamma - 1} + \frac{\gamma}{2} v'^2 \right) A' \quad ; \quad e' = T' \quad (9)$$

5. Looping Process

- (a) Calculate a' :

$$a' = \sqrt{\gamma R T'} \quad (10)$$

- (b) Calculate Δt from each grid, and choose the minimum value:

$$\Delta t' = C \frac{\Delta x'}{a_{i'}' + v_{i'}'} \quad ; \quad 0 < C \leq 1 \quad (11)$$

(c) Predictor Step

i. Calculate boundary conditions (BC) for $i = n - 1$ (subsonic outflow):

BC for U_1 and U_2 :

$$U_{1_{i=n-1}} = 2U_{1_{i=n-2}} - U_{1_{i=n-3}} \quad (12)$$

$$U_{2_{i=n-1}} = 2U_{2_{i=n-2}} - U_{2_{i=n-3}} \quad (13)$$

BC for v' :

$$v_{n-1}' = \frac{U_{2_{n-1}}}{U_{1_{n-1}}} \quad (14)$$

BC for U_3 :

$$U_3 = \frac{P'_{n-1}A'}{\gamma - 1} + \frac{\gamma}{2}U_{2_{n-1}}v'_{n-1} \quad (15)$$

choose $P'_{n-1} = 0.6784$.

ii. Calculate F_1 , F_2 , F_3 for $i = 1 \sim i = n - 1$ and J_2 for $i = 1 \sim i = n - 2$:

F_1 , F_2 , and F_3 :

$$F_1 = U_2 \quad (16)$$

$$F_2 = \frac{U_2^2}{U_1} + \frac{\gamma - 1}{\gamma} \left(U_3 - \frac{\gamma}{2} \frac{U_2^2}{U_1} \right) \quad (17)$$

$$F_3 = \gamma \frac{U_2 U_3}{U_1} - \frac{\gamma(\gamma - 1)}{2} \frac{U_2^3}{U_1^2} \quad (18)$$

J_2 :

$$J_2 = \frac{\gamma - 1}{\gamma} \left(U_3 - \frac{\gamma}{2} \frac{U_2^2}{U_1} \right) \frac{\partial (\ln A')}{\partial x'} \quad (19)$$

$$= \frac{1}{\gamma} \rho' T' \frac{\partial A'}{\partial x'} \quad (20)$$

iii. Calculate time derivative of U_1 , U_2 , and U_3 using forward difference from $i = 1 \sim i = n - 2$:

$$\frac{\partial U_1}{\partial t'} = -\frac{\partial F_1}{\partial x'} \quad (21)$$

$$\frac{\partial U_2}{\partial t'} = -\frac{\partial F_2}{\partial x'} + J_2 \quad (22)$$

$$\frac{\partial U_3}{\partial t'} = -\frac{\partial F_3}{\partial x'} \quad (23)$$

In finite difference form:

$$\left(\frac{U_1}{\partial t'} \right)_i^n = - \left(\frac{(F_1)_{i+1} - (F_1)_i}{\Delta x'} \right) \quad (24)$$

$$\left(\frac{U_2}{\partial t'} \right)_i^n = - \left(\frac{(F_2)_{i+1} - (F_2)_i}{\Delta x'} \right) + J_2 \quad (25)$$

$$\left(\frac{U_3}{\partial t'} \right)_i^n = - \left(\frac{(F_3)_{i+1} - (F_3)_i}{\Delta x'} \right) \quad (26)$$

iv. Compute the predicted value from $i = 1 \sim i = n - 2$:

$$(\overline{U}_1)_i^{n+1} = (U_1)_i^n + \left(\frac{\partial U_1}{\partial t'} \right)_i^n \Delta t' \quad (27)$$

$$(\overline{U}_2)_i^{n+1} = (U_2)_i^n + \left(\frac{\partial U_2}{\partial t'} \right)_i^n \Delta t' \quad (28)$$

$$(\overline{U}_3)_i^{n+1} = (U_3)_i^n + \left(\frac{\partial U_3}{\partial t'} \right)_i^n \Delta t' \quad (29)$$

Primitive variables:

$$(\overline{\rho'})_i^{n+1} = \frac{(\overline{U}_1)_i^{n+1}}{(A')_i} \quad (30)$$

$$(\overline{T'})_i^{n+1} = (\gamma - 1) \left[\frac{(\overline{U}_3)_i^{n+1}}{(\overline{U}_1)_i^{n+1}} - \frac{\gamma}{2} \left(\frac{(\overline{U}_2)_i^{n+1}}{(\overline{U}_1)_i^{n+1}} \right)^2 \right] \quad (31)$$

(d) Corrector Step

i. Calculate boundary conditions (BC) for $i = 0$ (subsonic inflow):
BC for ρ' and T' :

$$\rho' = 1.0 \quad (32)$$

$$T' = 1.0 \quad (33)$$

BC for U_1 :

$$U_1 = \rho' A' = A' = \text{fixed} \quad (34)$$

BC for U_2 :

$$U_{2i=0} = 2U_{2i=1} - U_{2i=2} \quad (35)$$

BC for v' :

$$v' = \frac{U_2}{U_1} \quad (36)$$

BC for U_3 :

$$U_3 = \rho' \left(\frac{e'}{\gamma - 1} + \frac{\gamma}{2} v'^2 \right) A' \quad (37)$$

$$= U_1 \left(\frac{T'}{\gamma - 1} + \frac{\gamma}{2} v'^2 \right) \quad (38)$$

ii. Compute \overline{F}_1 , \overline{F}_2 , and \overline{F}_3 from $i = 0 \sim i = n - 2$ and \overline{J}_2 from $i = 1 \sim i = n - 2$:
 \overline{F}_1 , \overline{F}_2 , and \overline{F}_3 :

$$\overline{F}_1 = \overline{U}_2 \quad (39)$$

$$\overline{F}_2 = \frac{\overline{U}_2^2}{\overline{U}_1} + \frac{\gamma - 1}{\gamma} \left(\overline{U}_3 - \frac{\gamma}{2} \frac{\overline{U}_2^2}{\overline{U}_1} \right) \quad (40)$$

$$\overline{F}_3 = \gamma \frac{\overline{U}_2 \overline{U}_3}{\overline{U}_1} - \frac{\gamma(\gamma - 1)}{2} \frac{\overline{U}_2^3}{\overline{U}_1^2} \quad (41)$$

\bar{J}_2 :

$$\bar{J}_2 = \frac{\gamma - 1}{\gamma} \left(\bar{U}_3 - \frac{\gamma}{2} \frac{\bar{U}_2^2}{\bar{U}_1} \right) \frac{\partial (\ln A')}{\partial x'} \quad (42)$$

$$= \frac{1}{\gamma} \rho' T' \frac{\partial A'}{\partial x'} \quad (43)$$

iii. Calculate time derivative of \bar{U}_1 , \bar{U}_2 , and \bar{U}_3 using backward difference from $i = 1 \sim i = n - 2$:

$$\frac{\partial \bar{U}_1}{\partial t'} = - \frac{\partial \bar{F}_1}{\partial x'} \quad (44)$$

$$\frac{\partial \bar{U}_2}{\partial t'} = - \frac{\partial \bar{F}_2}{\partial x'} + \bar{J}_2 \quad (45)$$

$$\frac{\partial \bar{U}_3}{\partial t'} = - \frac{\partial \bar{F}_3}{\partial x'} \quad (46)$$

In finite difference form:

$$\left(\frac{\partial \bar{U}_1}{\partial t'} \right)_i^{n+1} = - \left(\frac{(\bar{F}_1)_i - (\bar{F}_1)_{i-1}}{\Delta x'} \right) \quad (47)$$

$$\left(\frac{\partial \bar{U}_2}{\partial t'} \right)_i^{n+1} = - \left(\frac{(\bar{F}_2)_i - (\bar{F}_2)_{i-1}}{\Delta x'} \right) + \bar{J}_2 \quad (48)$$

$$\left(\frac{\partial \bar{U}_3}{\partial t'} \right)_i^{n+1} = - \left(\frac{(\bar{F}_3)_i - (\bar{F}_3)_{i-1}}{\Delta x'} \right) \quad (49)$$

iv. Compute the average time derivative:

$$\left(\frac{\partial U_1}{\partial t} \right)_{avg} = \frac{1}{2} \left[\left(\frac{\partial U_1}{\partial t'} \right)_i^n + \left(\frac{\partial \bar{U}_1}{\partial t'} \right)_i^{n+1} \right] \quad (50)$$

$$\left(\frac{\partial U_2}{\partial t} \right)_{avg} = \frac{1}{2} \left[\left(\frac{\partial U_2}{\partial t'} \right)_i^n + \left(\frac{\partial \bar{U}_2}{\partial t'} \right)_i^{n+1} \right] \quad (51)$$

$$\left(\frac{\partial U_3}{\partial t} \right)_{avg} = \frac{1}{2} \left[\left(\frac{\partial U_3}{\partial t'} \right)_i^n + \left(\frac{\partial \bar{U}_3}{\partial t'} \right)_i^{n+1} \right] \quad (52)$$

(e) Calculate new time-step variables:

i. Calculate new U_1 , U_2 , and U_3 :

$$(U_1)_i^{n+1} = (U_1)_i^n + \left(\frac{\partial U_1}{\partial t} \right)_{avg} \Delta t' \quad (53)$$

$$(U_2)_i^{n+1} = (U_2)_i^n + \left(\frac{\partial U_2}{\partial t} \right)_{avg} \Delta t' \quad (54)$$

$$(U_3)_i^{n+1} = (U_3)_i^n + \left(\frac{\partial U_3}{\partial t} \right)_{avg} \Delta t' \quad (55)$$

ii. Calculate primitive variables:

$$(\rho')_i^{n+1} = (U_1)_i^{n+1} \quad (56)$$

$$(v')_i^{n+1} = \left(\frac{U_2}{U_1} \right)_i^{n+1} \quad (57)$$

$$(T')_i^{n+1} = (\gamma - 1) \left(\frac{U_3}{U_1} - \frac{\gamma}{2} v'^2 \right)_i^{n+1} \quad (58)$$

(f) Go back to the first step of the loop, with the new variables.

(g) Finished Looping.

6. Print output.

7. Program exit.