Algorithm of Conservative Convergent-Divergent Duct

1. Input nodes, parameters, and constant variables
2. Process Duct Geometry with equation:

$$A = 1 + 2.2\,(x - 1.5)^2 \qquad 0 \le x \le 3 \tag{1}$$

3. Calculate Δx
4. Calculate initial conditions for $i = 1 \sim i = n - 2$ (c-like array)

    (a) Initial condition for ρ' and T':

$$\left.\begin{aligned} \rho' &= 1.0 \\ T' &= 1.0 \end{aligned}\right\} \quad for \quad 0 \le x' \le 0.5 \tag{2}$$

$$\left.\begin{aligned} \rho' &= 1.0 - 0.366\,(x' - 0.5) \\ T' &= 1.0 - 0.167\,(x' - 0.5) \end{aligned}\right\} \quad for \quad 0.5 < x' \le 1.5 \tag{3}$$

$$\left.\begin{aligned} \rho' &= 0.634 - 0.702\,(x' - 1.5) \\ T' &= 0.833 - 0.4908\,(x' - 1.5) \end{aligned}\right\} \quad for \quad 1.5 < x' \le 2.1 \tag{4}$$

$$\left.\begin{aligned} \rho' &= 0.5892 + 0.10228\,(x' - 2.1) \\ T' &= 0.93968 + 0.0622\,(x' - 2.1) \end{aligned}\right\} \quad for \quad 2.1 < x' \le 3.0 \tag{5}$$

    (b) Initial condition for v' (use constant mass flow):

$$v' = \frac{U_2}{\rho' A'} = \frac{0.59}{\rho' A'} \tag{6}$$

    (c) Initial conditions for U_1, U_2, and U_3:

$$U_1 = \rho' A' \tag{7}$$

$$U_2 = \rho' A' v' \tag{8}$$

$$U_3 = \rho' \left(\frac{e'}{\gamma - 1} + \frac{\gamma}{2} v'^2 \right) A' \quad ; \quad e' = T' \tag{9}$$

5. Looping Process

    (a) Calculate a':

$$a' = \sqrt{\gamma R T'} \tag{10}$$

    (b) Calculate Δt from each grid, and choose the maximum value:

$$\Delta t' = C \frac{\Delta x'}{a_i' + v_i'} \quad ; \quad 0 < C \le 1 \tag{11}$$

(c) Predictor Step

i. Calculate boundary conditions (BC) for $i = n-1$ (subsonic outflow):
BC for U_1 and U_2:

$$U_{1_{i=n-1}} = 2U_{1_{i=n-2}} - U_{1_{i=n-3}} \tag{12}$$
$$U_{2_{i=n-1}} = 2U_{2_{i=n-2}} - U_{2_{i=n-3}} \tag{13}$$

BC for v':

$$v_{n-1}{}' = \frac{U_{2_{n-1}}}{U_{1_{n-1}}} \tag{14}$$

BC for U_3:

$$U_3 = \frac{P'_{n-1}A'}{\gamma - 1} + \frac{\gamma}{2} U_{2_{n-1}} v'_{n-1} \tag{15}$$

choose $P'_{n-1} = 0.6784$.

ii. Calculate F_1, F_2, F_3 for $i = 1 \sim i = n-1$ and J_2 for $i = 1 \sim i = n-2$:
F_1, F_2, and F_3:

$$F_1 = U_2 \tag{16}$$
$$F_2 = \frac{U_2^2}{U_1} + \frac{\gamma - 1}{\gamma}\left(U_3 - \frac{\gamma}{2}\frac{U_2^2}{U_1}\right) \tag{17}$$
$$F_3 = \gamma \frac{U_2 U_3}{U_1} - \frac{\gamma(\gamma - 1)}{2}\frac{U_2^3}{U_1^2} \tag{18}$$

J_2:

$$J_2 = \frac{\gamma - 1}{\gamma}\left(U_3 - \frac{\gamma}{2}\frac{U_2^2}{U_1}\right)\frac{\partial(lnA')}{\partial x'} \tag{19}$$
$$= \frac{1}{\gamma}\rho' T' \frac{\partial A'}{\partial x'} \tag{20}$$

iii. Calculate time derivative of U_1, U_2, and U_3 using forward difference from $i = 1 \sim i = n-2$:

$$\frac{\partial U_1}{\partial t'} = -\frac{\partial F_1}{\partial x'} \tag{21}$$
$$\frac{\partial U_2}{\partial t'} = -\frac{\partial F_2}{\partial x'} + J_2 \tag{22}$$
$$\frac{\partial U_3}{\partial t'} = -\frac{\partial F_3}{\partial x'} \tag{23}$$

In finite difference form:

$$\left(\frac{U_1}{\partial t'}\right)_i^{n+1} = -\left(\frac{(F_1)_{i+1} - (F_1)_i}{\Delta x'}\right) \tag{24}$$
$$\left(\frac{U_2}{\partial t'}\right)_i^{n+1} = -\left(\frac{(F_2)_{i+1} - (F_2)_i}{\Delta x'}\right) \tag{25}$$
$$\left(\frac{U_3}{\partial t'}\right)_i^{n+1} = -\left(\frac{(F_3)_{i+1} - (F_3)_i}{\Delta x'}\right) \tag{26}$$

iv. Compute the predicted value from $i = 1 \sim i = n-2$:

$$\left(\overline{U}_1\right)_i^{n+1} = (U_1)_i^n + \left(\frac{\partial U_1}{\partial t'}\right)_i^n \Delta t' \tag{27}$$

$$\left(\overline{U}_2\right)_i^{n+1} = (U_2)_i^n + \left(\frac{\partial U_2}{\partial t'}\right)_i^n \Delta t' \tag{28}$$

$$\left(\overline{U}_3\right)_i^{n+1} = (U_3)_i^n + \left(\frac{\partial U_3}{\partial t'}\right)_i^n \Delta t' \tag{29}$$

Primitive variables:

$$\left(\overline{\rho'}\right)_i^{n+1} = \frac{\left(\overline{U}_1\right)_i^{n+1}}{(A')_i} \tag{30}$$

$$\left(\overline{T'}\right)_i^{n+1} = (\gamma - 1)\left[\frac{\left(\overline{U}_3\right)_i^{n+1}}{\left(\overline{U}_1\right)_i^{n+1}} - \frac{\gamma}{2}\left(\frac{\left(\overline{U}_2\right)_i^{n+1}}{\left(\overline{U}_1\right)_i^{n+1}}\right)^2\right] \tag{31}$$

(d) Corrector Step

i. Calculate boundary conditions (BC) for $i = 0$ (subsonic inflow):
BC for ρ' and T':

$$\rho' = 1.0 \tag{32}$$

$$T' = 1.0 \tag{33}$$

BC for U_1:

$$U_1 = \rho' A' = A' = \text{fixed} \tag{34}$$

BC for U_2:

$$U_{2_{i=0}} = 2U_{2_{i=1}} - U_{2_{i=2}} \tag{35}$$

BC for v':

$$v' = \frac{U_2}{U_1} \tag{36}$$

BC for U_3:

$$U_3 = \rho'\left(\frac{e'}{\gamma - 1} + \frac{\gamma}{2} v^{2\prime}\right) A' \tag{37}$$

$$= U_1\left(\frac{T'}{\gamma - 1} + \frac{\gamma}{2} v^{2\prime}\right) \tag{38}$$

ii. Compute $\overline{F}_1$, $\overline{F}_2$, and $\overline{F}_3$ from $i = 0 \sim i = n-2$ and $\overline{J}_2$ from $i = 1 \sim i = n-2$:
$\overline{F}_1$, $\overline{F}_2$, and $\overline{F}_3$:

$$\overline{F}_1 = \overline{U}_2 \tag{39}$$

$$\overline{F}_2 = \frac{\overline{\overline{U}}_2^2}{\overline{U}_1} + \frac{\gamma - 1}{\gamma}\left(\overline{U}_3 - \frac{\gamma}{2}\frac{\overline{U}_2^2}{\overline{U}_1}\right) \tag{40}$$

$$\overline{F}_3 = \gamma \frac{\overline{U}_2 \overline{U}_3}{\overline{U}_1} - \frac{\gamma(\gamma - 1)}{2}\frac{\overline{U}_2^3}{\overline{U}_1^2} \tag{41}$$

$\overline{J}_2$:

$$\overline{J}_2 = \frac{\gamma-1}{\gamma}\left(\overline{U}_3 - \frac{\gamma}{2}\frac{\overline{U}_2^2}{\overline{U}_1}\right)\frac{\partial\left(lnA'\right)}{\partial x'} \tag{42}$$

$$= \frac{1}{\gamma}\rho' T' \frac{\partial A'}{\partial x'} \tag{43}$$

iii. Calculate time derivative of $\overline{U}_1$, $\overline{U}_2$, and $\overline{U}_3$ using backward difference from $i = 1 \sim i = n-2$:

$$\frac{\partial \overline{U}_1}{\partial t'} = -\frac{\partial \overline{F}_1}{\partial x'} \tag{44}$$

$$\frac{\partial \overline{U}_2}{\partial t'} = -\frac{\partial \overline{F}_2}{\partial x'} + \overline{J}_2 \tag{45}$$

$$\frac{\partial \overline{U}_3}{\partial t'} = -\frac{\partial \overline{F}_3}{\partial x'} \tag{46}$$

In finite difference form:

$$\left(\frac{\partial \overline{U}_1}{\partial t'}\right)_i^{n+1} = -\left(\frac{\left(\overline{F}_1\right)_i - \left(\overline{F}_1\right)_{i-1}}{\Delta x'}\right) \tag{47}$$

$$\left(\frac{\partial \overline{U}_2}{\partial t'}\right)_i^{n+1} = -\left(\frac{\left(\overline{F}_2\right)_i - \left(\overline{F}_2\right)_{i-1}}{\Delta x'}\right) \tag{48}$$

$$\left(\frac{\partial \overline{U}_3}{\partial t'}\right)_i^{n+1} = -\left(\frac{\left(\overline{F}_3\right)_i - \left(\overline{F}_3\right)_{i-1}}{\Delta x'}\right) \tag{49}$$

iv. Compute the average time derivative:

$$\left(\frac{\partial U_1}{\partial t}\right)_{avg} = \frac{1}{2}\left[\left(\frac{\partial U_1}{\partial t'}\right)_i^n + \left(\frac{\partial \overline{U}_1}{\partial t'}\right)_i^{n+1}\right] \tag{50}$$

$$\left(\frac{\partial U_2}{\partial t}\right)_{avg} = \frac{1}{2}\left[\left(\frac{\partial U_2}{\partial t'}\right)_i^n + \left(\frac{\partial \overline{U}_2}{\partial t'}\right)_i^{n+1}\right] \tag{51}$$

$$\left(\frac{\partial U_3}{\partial t}\right)_{avg} = \frac{1}{2}\left[\left(\frac{\partial U_3}{\partial t'}\right)_i^n + \left(\frac{\partial \overline{U}_3}{\partial t'}\right)_i^{n+1}\right] \tag{52}$$

(e) Calculate new time-step variables:

i. Calculate new U_1, U_2, and U_3:

$$(U_1)_i^{n+1} = (U_1)_i^n + \left(\frac{\partial U_1}{\partial t'}\right)_{avg}\Delta t' \tag{53}$$

$$(U_2)_i^{n+1} = (U_2)_i^n + \left(\frac{\partial U_2}{\partial t'}\right)_{avg}\Delta t' \tag{54}$$

$$(U_3)_i^{n+1} = (U_3)_i^n + \left(\frac{\partial U_3}{\partial t'}\right)_{avg}\Delta t' \tag{55}$$

ii. Calculate primitive variables:

$$(\rho')_i^{n+1} = (U_1)_i^{n+1} \tag{56}$$

$$(v')_i^{n+1} = \left(\frac{U_2}{U_1}\right)_i^{n+1} \tag{57}$$

$$(T')_i^{n+1} = (\gamma - 1)\left(\frac{U_3}{U_1} - \frac{\gamma}{2}v'^2\right)_i^{n+1} \tag{58}$$

(f) Go back to the first step of the loop, with the new variables.

(g) Finished Looping.

6. Print output.

7. Program exit.