# Algorithm of Conservative Convergent-Divergent Duct

1. Input nodes, parameters, and constant variables

2. Process Duct Geometry with equation:

$$A = 1 + 2.2\,(x - 1.5)^2 \qquad 0 \leq x \leq 3 \tag{1}$$

3. Calculate $\Delta x$

4. Calculate initial conditions

   (a) Initial condition for $\rho'$ and $T'$:

$$\left.\begin{aligned} \rho' &= 1.0 \\ T' &= 1.0 \end{aligned}\right\} \quad for \quad 0 \leq x' \leq 0.5 \tag{2}$$

$$\left.\begin{aligned} \rho' &= 1.0 - 0.366\,(x' - 0.5) \\ T' &= 1.0 - 0.167\,(x' - 0.5) \end{aligned}\right\} \quad for \quad 0.5 < x' \leq 1.5 \tag{3}$$

$$\left.\begin{aligned} \rho' &= 0.634 - 0.702\,(x' - 1.5) \\ T' &= 0.833 - 0.4908\,(x' - 1.5) \end{aligned}\right\} \quad for \quad 1.5 < x' \leq 2.1 \tag{4}$$

$$\left.\begin{aligned} \rho' &= 0.5892 + 0.10228\,(x' - 2.1) \\ T' &= 0.93968 + 0.0622\,(x' - 2.1) \end{aligned}\right\} \quad for \quad 2.1 < x' \leq 3.0 \tag{5}$$

   (b) Initial condition for $v'$ (use constant mass flow):

$$v' = \frac{U_2}{\rho' A'} = \frac{0.59}{\rho' A'} \tag{6}$$

   (c) Initial conditions for $U_1$, $U_2$, and $U_3$:

$$U_1 = \rho' A' \tag{7}$$

$$U_2 = \rho' A' v' \tag{8}$$

$$U_3 = \rho' \left( \frac{e'}{\gamma - 1} + \frac{\gamma}{2} v'^2 \right) A' \quad ; \quad e' = T' \tag{9}$$

5. Looping Process

   (a) Calculate $a'$:

$$a' = \sqrt{\gamma R T'} \tag{10}$$

   (b) Calculate $\Delta t$ from each grid, and choose the minimum value:

$$\Delta t' = C \frac{\Delta x'}{a_i' + v_i'} \quad ; \quad 0 < C \leq 1 \tag{11}$$

(c) Predictor Step

i. Calculate boundary conditions (BC) for subsonic outflow:
BC for $U_1$ and $U_2$:

$$U_{1_{i=n-1}} = 2U_{1_{i=n-2}} - U_{1_{i=n-3}} \tag{12}$$
$$U_{2_{i=n-1}} = 2U_{2_{i=n-2}} - U_{2_{i=n-3}} \tag{13}$$

BC for $v'$:

$$v'_{i=n-1} = \frac{U_{2_{n-1}}}{U_{1_{n-1}}} \tag{14}$$

BC for $U_3$:

$$U_{3_{i=n-1}} = \frac{P'_{n-1}A'}{\gamma - 1} + \frac{\gamma}{2}U_{2_{n-1}}v'_{n-1} \tag{15}$$

choose $P'_{n-1} = 0.6784$ for default case.

ii. Calculate $F_1$, $F_2$, $F_3$, and $J_2$:
$F_1$, $F_2$, and $F_3$:

$$F_1 = U_2 \tag{16}$$
$$F_2 = \frac{U_2^2}{U_1} + \frac{\gamma - 1}{\gamma}\left(U_3 - \frac{\gamma}{2}\frac{U_2^2}{U_1}\right) \tag{17}$$
$$F_3 = \gamma\frac{U_2U_3}{U_1} - \frac{\gamma\left(\gamma - 1\right)}{2}\frac{U_2^3}{U_1^2} \tag{18}$$

$J_2$:

$$J_2 = \frac{\gamma - 1}{\gamma}\left(U_3 - \frac{\gamma}{2}\frac{U_2^2}{U_1}\right)\frac{\partial\left(lnA'\right)}{\partial x'} \tag{19}$$
$$= \frac{1}{\gamma}\rho'T'\frac{\partial A'}{\partial x'} \tag{20}$$

iii. Calculate time derivative of $U_1$, $U_2$, and $U_3$ using forward difference:

$$\frac{\partial U_1}{\partial t'} = -\frac{\partial F_1}{\partial x'} \tag{21}$$
$$\frac{\partial U_2}{\partial t'} = -\frac{\partial F_2}{\partial x'} + J_2 \tag{22}$$
$$\frac{\partial U_3}{\partial t'} = -\frac{\partial F_3}{\partial x'} \tag{23}$$

In finite difference form:

$$\left(\frac{U_1}{\partial t'}\right)_i^n = -\left(\frac{(F_1)_{i+1} - (F_1)_i}{\Delta x'}\right) \tag{24}$$
$$\left(\frac{U_2}{\partial t'}\right)_i^n = -\left(\frac{(F_2)_{i+1} - (F_2)_i}{\Delta x'}\right) + J_2 \tag{25}$$
$$\left(\frac{U_3}{\partial t'}\right)_i^n = -\left(\frac{(F_3)_{i+1} - (F_3)_i}{\Delta x'}\right) \tag{26}$$

iv. Compute artificial velocity:

$$S_i^{t'} = \frac{C_x \left| (p')_{i+1}^{t'} - 2\,(p')_i^{t'} + (p')_{i-1}^{t'} \right|}{(p')_{i+1}^{t'} + 2\,(p')_i^{t'} + (p')_{i-1}^{t'}} \left( U_{i+1}^{t'} - 2U_i^{t'} + U_{i-1}^{t'} \right) \tag{27}$$

v. Compute the predicted value with usage of the artificial velocity:

$$\left(\overline{U}_1\right)_i^{n+1} = (U_1)_i^n + \left(\frac{\partial U_1}{\partial t'}\right)_i^n \Delta t' + (S_1)_i^{t'} \tag{28}$$

$$\left(\overline{U}_2\right)_i^{n+1} = (U_2)_i^n + \left(\frac{\partial U_2}{\partial t'}\right)_i^n \Delta t' + (S_2)_i^{t'} \tag{29}$$

$$\left(\overline{U}_3\right)_i^{n+1} = (U_3)_i^n + \left(\frac{\partial U_3}{\partial t'}\right)_i^n \Delta t' + (S_3)_i^{t'} \tag{30}$$

Primitive variables:

$$\left(\overline{\rho'}\right)_i^{n+1} = \frac{\left(\overline{U}_1\right)_i^{n+1}}{(A')_i} \tag{31}$$

$$\left(\overline{T'}\right)_i^{n+1} = (\gamma - 1) \left[ \frac{\left(\overline{U}_3\right)_i^{n+1}}{\left(\overline{U}_1\right)_i^{n+1}} - \frac{\gamma}{2} \left( \frac{\left(\overline{U}_2\right)_i^{n+1}}{\left(\overline{U}_1\right)_i^{n+1}} \right)^2 \right] \tag{32}$$

(d) Corrector Step

i. Calculate boundary conditions (BC) for subsonic inflow:
BC for $\rho'$ and $T'$:

$$\rho'_{i=0} = 1.0 \tag{33}$$

$$T'_{i=0} = 1.0 \tag{34}$$

BC for $U_1$:

$$U_{1_{i=0}} = \rho' A' = A' = \text{fixed} \tag{35}$$

BC for $U_2$:

$$U_{2_{i=0}} = 2U_{2_{i=1}} - U_{2_{i=2}} \tag{36}$$

BC for $v'$:

$$v'_{i=0} = \frac{U_{2_{i=0}}}{U_{1_{i=0}}} \tag{37}$$

BC for $U_3$:

$$U_{3_{i=0}} = \rho'_{i=0} \left( \frac{e'_{i=0}}{\gamma - 1} + \frac{\gamma}{2} v^{2\prime}{}_{i=0} \right) A' \tag{38}$$

$$= U_{1_{i=0}} \left( \frac{T'_{i=0}}{\gamma - 1} + \frac{\gamma}{2} v'^2_{i=0} \right) \tag{39}$$

ii. Compute $\overline{F}_1$, $\overline{F}_2$, $\overline{F}_3$, and $\overline{J}_2$:
$\overline{F}_1$, $\overline{F}_2$, and $\overline{F}_3$:

$$\overline{F}_1 = \overline{U}_2 \tag{40}$$

$$\overline{F}_2 = \frac{\overline{U}_2^2}{\overline{U}_1} + \frac{\gamma - 1}{\gamma}\left(\overline{U}_3 - \frac{\gamma}{2}\frac{\overline{U}_2^2}{\overline{U}_1}\right) \tag{41}$$

$$\overline{F}_3 = \gamma\frac{\overline{U}_2\overline{U}_3}{\overline{U}_1} - \frac{\gamma(\gamma-1)}{2}\frac{\overline{U}_2^3}{\overline{U}_1^2} \tag{42}$$

$\overline{J}_2$:

$$\overline{J}_2 = \frac{\gamma - 1}{\gamma}\left(\overline{U}_3 - \frac{\gamma}{2}\frac{\overline{U}_2^2}{\overline{U}_1}\right)\frac{\partial\,(lnA')}{\partial x'} \tag{43}$$

$$= \frac{1}{\gamma}\rho' T' \frac{\partial A'}{\partial x'} \tag{44}$$

iii. Calculate time derivative of $\overline{U}_1$, $\overline{U}_2$, and $\overline{U}_3$ using backward difference:

$$\frac{\partial \overline{U}_1}{\partial t'} = -\frac{\partial \overline{F}_1}{\partial x'} \tag{45}$$

$$\frac{\partial \overline{U}_2}{\partial t'} = -\frac{\partial \overline{F}_2}{\partial x'} + \overline{J}_2 \tag{46}$$

$$\frac{\partial \overline{U}_3}{\partial t'} = -\frac{\partial \overline{F}_3}{\partial x'} \tag{47}$$

In finite difference form:

$$\left(\frac{\partial \overline{U}_1}{\partial t'}\right)_i^{n+1} = -\left(\frac{\left(\overline{F}_1\right)_i - \left(\overline{F}_1\right)_{i-1}}{\Delta x'}\right) \tag{48}$$

$$\left(\frac{\partial \overline{U}_2}{\partial t'}\right)_i^{n+1} = -\left(\frac{\left(\overline{F}_2\right)_i - \left(\overline{F}_2\right)_{i-1}}{\Delta x'}\right) + \overline{J}_2 \tag{49}$$

$$\left(\frac{\partial \overline{U}_3}{\partial t'}\right)_i^{n+1} = -\left(\frac{\left(\overline{F}_3\right)_i - \left(\overline{F}_3\right)_{i-1}}{\Delta x'}\right) \tag{50}$$

iv. Compute the average time derivative:

$$\left(\frac{\partial U_1}{\partial t}\right)_{avg} = \frac{1}{2}\left[\left(\frac{\partial U_1}{\partial t'}\right)_i^n + \left(\frac{\partial \overline{U}_1}{\partial t'}\right)_i^{n+1}\right] \tag{51}$$

$$\left(\frac{\partial U_2}{\partial t}\right)_{avg} = \frac{1}{2}\left[\left(\frac{\partial U_2}{\partial t'}\right)_i^n + \left(\frac{\partial \overline{U}_2}{\partial t'}\right)_i^{n+1}\right] \tag{52}$$

$$\left(\frac{\partial U_3}{\partial t}\right)_{avg} = \frac{1}{2}\left[\left(\frac{\partial U_3}{\partial t'}\right)_i^n + \left(\frac{\partial \overline{U}_3}{\partial t'}\right)_i^{n+1}\right] \tag{53}$$

v. Compute artificial velocity:

$$\overline{S}_i^{t'+\Delta t'} = \frac{C_x \left| \left(\overline{p}'\right)_{i+1}^{t'+\Delta t'} - 2\left(\overline{p}'\right)_i^{t'+\Delta t'} + \left(\overline{p}'\right)_{i-1}^{t'+\Delta t'} \right|}{\left(\overline{p}'\right)_{i+1}^{t'+\Delta t'} + 2\left(\overline{p}'\right)_i^{t'+\Delta t'} + \left(\overline{p}'\right)_{i-1}^{t'+\Delta t'}} \left(\overline{U}_{i+1}^{t'+\Delta t'} - 2\overline{U}_i^{t'+\Delta t'} + \overline{U}_{i-1}^{t'+\Delta t'}\right) \tag{54}$$

(e) Calculate new time-step variables:

i. Calculate new $U_1$, $U_2$, and $U_3$:

$$(U_1)_i^{n+1} = (U_1)_i^n + \left(\frac{\partial U_1}{\partial t'}\right)_{avg} \Delta t' + \left(\overline{S}_1\right)_i^{t'} \tag{55}$$

$$(U_2)_i^{n+1} = (U_2)_i^n + \left(\frac{\partial U_2}{\partial t'}\right)_{avg} \Delta t' + \left(\overline{S}_1\right)_i^{t'} \tag{56}$$

$$(U_3)_i^{n+1} = (U_3)_i^n + \left(\frac{\partial U_3}{\partial t'}\right)_{avg} \Delta t' + \left(\overline{S}_1\right)_i^{t'} \tag{57}$$

ii. Calculate primitive variables:

$$(\rho')_i^{n+1} = (U_1)_i^{n+1} \tag{58}$$

$$(v')_i^{n+1} = \left(\frac{U_2}{U_1}\right)_i^{n+1} \tag{59}$$

$$(T')_i^{n+1} = (\gamma - 1)\left(\frac{U_3}{U_1} - \frac{\gamma}{2}v'^2\right)_i^{n+1} \tag{60}$$

(f) Calculate errors in RMS for both $\rho$, $v$, and $T$.

(g) If the error is bigger than desired error, start again with the new variables.

(h) Finish computation.

6. Print output.

7. Program exit.