1)

Ⓐ $\int_1^e \frac{\sqrt{x}+\ln(x)}{x^3}\,dx =$

$$\int \frac{x^{1/2}+\ln(x)}{x^3}\,dx = \int \frac{x^{1/2}}{x^3}\,dx + \int \frac{\ln(x)}{x^3}\,dx = \int x^{-5/2}\,dx + \int \frac{\ln(x)}{x^3}\,dx =$$

$$\boxed{\int x^{-5/2}\,dx = \frac{-2}{3}x^{-3/2} = -\frac{2}{3x^{3/2}} + k} \quad \boxed{= -\frac{2}{3x^{3/2}} - \frac{\ln(x)}{2x^2} - \frac{1}{4x^2}}$$

$$\int \frac{\ln(x)}{x^3}\,dx = \int \ln(x).x^{-3}\,dx = -\frac{\ln(x)}{2x^2} - \int -\frac{1}{2x^3}\,dx = \boxed{-\frac{\ln(x)}{2x^2} - \frac{1}{4x^2} + k}$$

$F(x) = \ln(x) \qquad F'(x) = \frac{1}{x}$

$g'(x) = x^{-3} \qquad g(x) = \frac{x^{-2}}{-2} = -\frac{1}{2x^2}$

$$\int -\frac{1}{2x^3}\,dx = -\frac{1}{2}\int x^{-3}\,dx = -\frac{1}{2}\cdot\frac{x^{-2}}{2} \boxed{= \frac{1}{4x^2} + k}$$

Barrow

$$\int_1^e \frac{\sqrt{x}+\ln(x)}{x^3} = \left[-\frac{2}{3.e^{3/2}} - \frac{\ln(e)}{2.e^2} - \frac{1}{4.e^2}\right] - \left[\frac{-2}{3.1^{3/2}} - \cancel{\frac{\ln(1)}{2.1^2}} - \frac{1}{4.1^2}\right]$$

$$= \frac{-2}{3.\sqrt{e^3}} - \frac{1}{2.e^2} - \frac{1}{4.e^2} + \left[\frac{2}{3} - \frac{1}{4}\right]$$

$$\boxed{= \frac{-2}{3.\sqrt{e^3}} - \frac{1}{2.e^2} - \frac{1}{4.e^2} + \frac{11}{12}}$$ → Resultado Final

1) B

$$\int \frac{2x+1}{x^2+4}\,dx = \int \frac{2x}{x^2+4}\,dx + \int \frac{1}{x^2+4}\,dx \boxed{= \ln|x^2+4| + \frac{1}{2}\cdot \arctan\left(\frac{x}{2}\right) + k}$$

$$\int \frac{2x}{x^2+4}\,dx = \int 2x\cdot(x^2+4)^{-1}\,dx = \int 2x\cdot u^{-1}\,\frac{du}{2x} = \ln|u| = \boxed{\ln|x^2+4| + k}$$

$u = x^2+4$

$du = 2x\,dx$

$\boxed{\frac{du}{2x} = dx}$

$$\int \frac{1}{x^2+4}\,dx = \int \frac{1}{2u^2+4}\,du.2 = \int \frac{1}{2(u^2+1)}\,du = \frac{1}{2}\int \frac{1}{u^2+1}\,du =$$

$u = \frac{x}{2} = x = 2u$

$du = \frac{1}{2}dx$

$2du = dx$

$$= \frac{1}{2}\arctan(u) = \boxed{\frac{1}{2}\cdot\arctan\left(\frac{x}{2}\right) + k}$$

C $$\int \frac{dx}{x^2(x+1)} = \int \frac{1}{x^2}\,dx - \int \frac{1}{x} + \int \frac{1}{x+1} \boxed{= -\frac{1}{x} - \ln|x| + \ln|x+1| + k}$$

$$\frac{1}{x^2(x+1)} = \frac{A_1}{x^2} + \frac{A_2}{x} + \frac{B}{(x+1)}$$

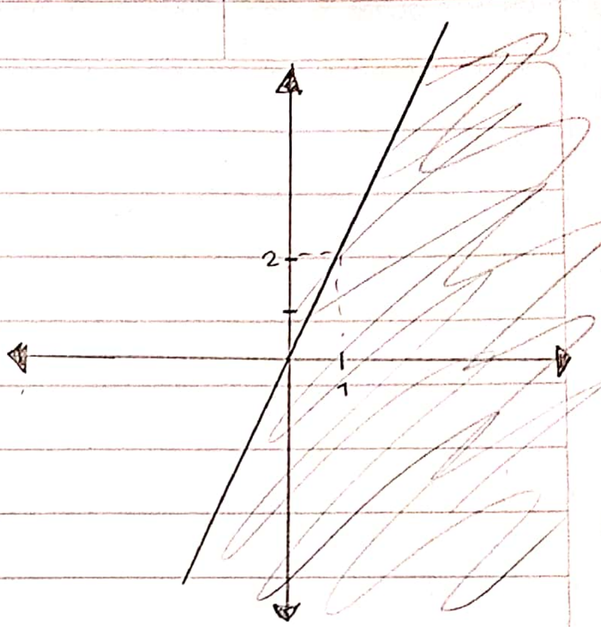
$$1 = A_1\cdot(x+1) + A_2(x(x+1)) + B\cdot x^2$$

si $x=-1$ $\quad \boxed{1 = B}$ $\quad x=2 \quad 1 = 3 + A_2\cdot 6 + 4$

$-6 = A_2\cdot 6$

si $x=0$ $\quad \boxed{1 = A_1}$ $\quad \boxed{-1 = A_2}$

2) $F(x,y) = \sqrt{x-2y} \qquad \mathbb{R}^2 \to \mathbb{R}$



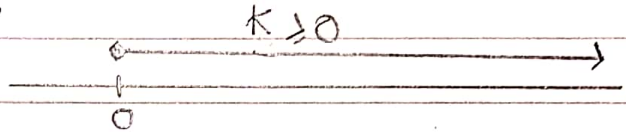
a) $\text{Dom}F = \{(x,y) \in \mathbb{R}^2 / x-2y \geq 0$

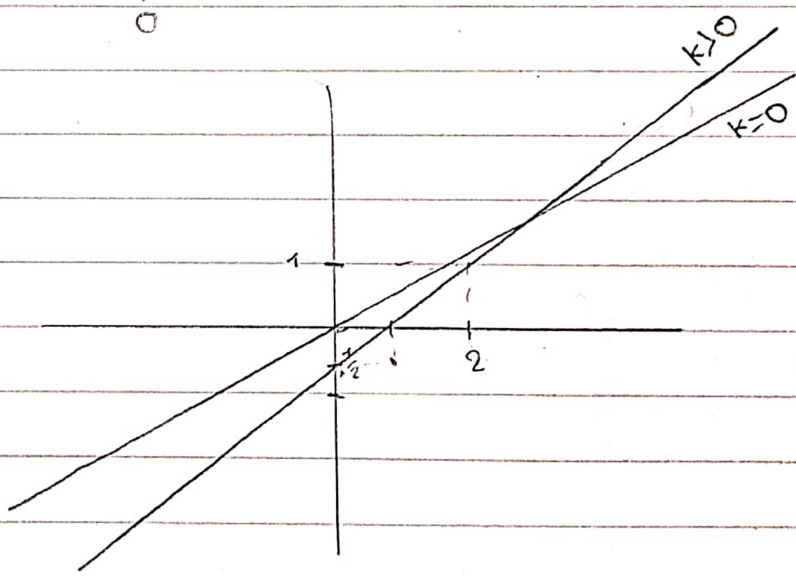
$x - 2y \geq 0$

$y \geq \frac{1}{2}x$

B) $\text{Img } F = \{(x,y) \in \mathbb{R} / k \geq 0$

$k = \sqrt{x-2y}$





c) $k = \sqrt{x-2y}$

$\cancel{k^2 + x = 2y}$

$$\boxed{\frac{-k^2+x}{2} = y}$$

$$\frac{1}{2} = \frac{-k^2+10}{2}$$

$1 = -k^2 + 10$

$9 = k^2$

$\sqrt{9} = k \qquad S = \{\vec{x} \in \mathbb{R}^2 / F(\vec{x}) = (3;-3) \text{ con } k \in \text{Img } F\}$