Square-law detector

$$I_1 = W(1 + m\cos(tx)), I_2 = W(1 + m\cos(tx + \frac{2\pi}{3})), I_2 = W(1 + m\cos(tx - \frac{2\pi}{3}))$$

$$I_r^2 = (I_1 - I_2)^2 + (I_1 - I_3)^2 + (I_2 - I_3)^2$$

$$= W^2m^2\Bigg(\Big(\cos(tx) - \cos(tx + \frac{2\pi}{3})\Big)^2 + \Big(\cos(tx) - \cos(tx - \frac{2\pi}{3})\Big)^2 + \Big(\cos(tx + \frac{2\pi}{3}) - \cos(tx - \frac{2\pi}{3})\Big)^2\Bigg)$$

$$=W^2m^2\Big(2\cos^2(tx) + 2\cos^2(tx + \frac{2\pi}{3}) + 2\cos^2(tx - \frac{2\pi}{3}) - 2\cos(tx)\cos(tx + \frac{2\pi}{3}) - 2\cos(tx)\cos(tx - \frac{2\pi}{3}) - 2\cos(tx + \frac{2\pi}{3})\cos(tx - \frac{2\pi}{3})\Big)$$

Using double angle formulae on $\cos^2$ *and* $\cos(A)\cos(B)$ *terms:*

$$= W^2m^2\Big(3 + \cos(2tx) + \cos(2tx + \frac{4\pi}{3}) + \cos(2tx - \frac{4\pi}{3}) - \cos(2tx + \frac{2\pi}{3}) - \cos(-\frac{2\pi}{3}) - \cos(2tx - \frac{2\pi}{3}) - \cos(\frac{2\pi}{3}) - \cos(2tx) - cos(0)\Big)$$

Noting $\cos(x + \phi) = \cos(x + 2n\pi\phi), n \in \mathbb{Z}$*:*

$$= W^2m^2(3 + 0.5 + 0.5 - 1)$$

$$= 3W^2m^2$$

or

$$I_r = \sqrt{3}Wm$$

Signal of out in-focus plane is directly proportional to modulation depth.

Homodyne detector

$$I_1 = W(1 + m\cos(tx)), I_2 = W(1 + m\cos(tx + \frac{2\pi}{3})), I_2 = W(1 + m\cos(tx - \frac{2\pi}{3}))$$

$$I_r = \left| I_1 + I_2 \exp\left(\frac{2\pi j}{3}\right) + I_3 \exp\left(-\frac{2\pi j}{3}\right) \right|$$

$$= W\left|(1 + \exp\left(\frac{2\pi j}{3}\right) + \exp\left(-\frac{2\pi j}{3}\right)\right|$$
$$+ Wm\left|\left(\cos(tx) + \cos(tx + \frac{2\pi}{3})(-0.5 + \frac{\sqrt{3}j}{2}) + \cos(tx - \frac{2\pi}{3})(-0.5 - \frac{\sqrt{3}j}{2})\right)\right|$$

Noting $\left|(1 + \exp\left(\frac{2\pi j}{3}\right) + \exp\left(-\frac{2\pi j}{3}\right)\right| = 0$*:*

$$= Wm\left|\left(\cos(tx) + \cos(tx + \frac{2\pi}{3})(-0.5 + \frac{\sqrt{3}j}{2}) + \cos(tx - \frac{2\pi}{3})(-0.5 - \frac{\sqrt{3}j}{2})\right)\right|$$

$$= Wm\Bigg| \cos(tx) - 0.5\left(\cos(tx + \frac{2\pi}{3}) + (\cos(tx - \frac{2\pi}{3})\right)$$
$$+ \frac{\sqrt{3}j}{2}\left(\cos(tx + \frac{2\pi}{3}) - (\cos(tx - \frac{2\pi}{3})\right)\Bigg|$$

Using double angle formulae for $2\cos(A)\cos(B)$ *and* $2\sin(A)\sin(B)$*:*

$$= Wm\left|\cos(tx) - \left(\cos(tx)\cos(\frac{2\pi}{3})\right) + \sqrt{3}j\left(\cos(tx)\cos(\frac{2\pi}{3})\right)\right|$$

$$= Wm\left|\cos(tx) + 0.5\cos(tx) - \frac{3j}{2}\sin(tx)\right|$$

$$= \frac{3}{2}Wm|\cos(tx) - j\sin(tx)|$$

$$I_r = \frac{3}{2}Wm$$

Which is the same result as for square law detection, albeit a different scaling factor

Example