Fiscal Policy during a Pandemic

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Abstract

I study the effects of the 2019-20 coronavirus outbreak in the United States and subsequent fiscal policy response in a nonlinear DSGE model. The pandemic is a shock to the utility of contact-intensive services that propagates to other sectors via general equilibrium, triggering a deep recession. I use a calibrated version of the model to analyze different types of fiscal policies. I find that UI benefits are the most effective tool to stabilize income for borrowers, who are the hardest hit, while savers may favor unconditional transfers. Liquidity assistance programs are effective if the policy objective is to stabilize employment in the affected sector. I also study the effects of the $2 trillion CARES Act of 2020.

JEL Codes: E6, G01 H00

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1 Introduction

The on-going COVID-19 outbreak is causing widespread disruption in the world's advanced economies. Monetary authorities were quick to react, with the Federal Reserve and other major central banks returning to their 2008-09 Financial Crisis toolkits. Following these steps, fiscal authorities around the globe are in the process of designing and approving stabilization packages to help sustain household and firm balance sheets.

In this paper, I adapt a macroeconomic model to simulate the macroeconomic effects of a pandemic and to study the effects of different types of fiscal policy instruments. The pandemic is modeled as a sudden stop of a contact-intensive services sector. Through aggregate demand externalities, the shutdown of this sector propagates to the non-services sector. Through balance sheet linkages, it also propagates to the financial sector. The rise in unemployment leads to a wave of defaults, disrupting financial intermediation and amplifying the recession. The pandemic scenario is pessimistic: the shock lasts for three quarters (through the end of 2020) and results in about a 20% unemployment rate. Borrower households, who derive most of their income from employment and rely on bank credit to fund consumption, are the most affected group.

I use a calibrated version of the model to study the effects of different types of discretionary fiscal policy: (i) an increase in non-service government purchases, (ii) a decrease in the income tax, (iii) an expansion of unemployment insurance (UI), (iv) an unconditional transfer, (v) payment of wages by the government to service firms.

In terms of measuring the effectiveness of different measures, it is not clear that the traditional concept of GDP multiplier is appropriate in this context. The shut down of economic activity is largely intentional, and part of pandemic suppression measures, and focus on GDP stabilization could be detrimental to fight the pandemic. For that reason, I evaluate different policies based on consumption and household income multipliers, which measure the dollar impact of fiscal spending on consumption of either type of household, and on labor income net of government transfers. I find that there is considerable variation in the distribution effects of different types of policies. Borrowers, who are most affected by the crisis, receive a larger consumption boost from policies that resemble cash transfers, such as an increase in UI benefits. I find that unconditional transfers of the type that are currently being proposed generate similar distributional effects, with the added benefit of a potentially less costly implementation. Finally, I find that liquidity assistance to firms has the longest-lasting effects and can be very effective in terms of stabilizing employment in the medium-run. An analysis of the preliminary plans for the CARES Act of 2020 (the $ 2 trillion coronavirus aid package) reveals that UI and liquidity assistance are the most effective

components of the package in terms of stabilizing income and employment, respectively.

Finally, it is worth noting that the pandemic scenario is assumed to be a completely exogenous shock to the economy. In practice, it is likely that the most effective type of fiscal policy would be one that targets the underlying source of the shock, i.e. investment in public health measures related to prevention, suppression, mitigation, and/or cure. The exercise also assumes away changes in the labor force due to deaths caused by the disease, and which could potentially be significant (**?**).

Literature   The exercise in this paper is very similar to the analysis conducted by **?** and **?** for the American Recovery and Reinvestment Act of 2009, where the authors use a DSGE model to simulate a recession scenario and then consider the effects of a policy package. **?** conducts a similar analysis, while also taking into account financial sector interventions that involved asset purchases such as TARP, among others. I mostly abstract from issues related to financial sector interventions in this paper.

This paper also contributes to the modeling of a pandemic in a macroeconomic model. **?** study how monetary and fiscal policy can be used to respond to the current pandemic by preventing the economy from falling into stagnation traps following persistent negative shocks to productivity growth. **?** embed a canonical epidemiology model (the SIR model) in a real business cycle model. Since they endogenize the dynamics of the epidemic, their model allows them to study optimal health policy responses. They find that a severe recession, generated by agents' optimal decision to cut back on consumption and hours worked, helps reduce the severity of their epidemic. My analysis is complementary to theirs: I take the epidemic as exogenous and given, and study how a fiscal authority can help stabilize income and consumption during the epidemic.

Section 2 presents the model, Section 3 explains the calibration and describes the modeling of a pandemic, Section 4 discusses the effects of different fiscal policies in the model, Section 5 estimates multipliers for the different components of the CARES Act of 2020, and Section 6 concludes with an extensive discussion of the caveats of the present analysis.

2   Model

Time is discrete and infinite. There are two types of households: borrowers and savers. Financial intermediaries use deposits raised from savers as well as their own retained earnings to finance loans to borrowers. There are two sectors in this economy: a non-services sector (sector n), and a services sector (sector a). Labor markets are frictional in reduced form, and employment is demand-determined in both sectors. A Central Bank sets the interest

rate, and a fiscal authority collects taxes and can undertake different types of discretionary interventions. The model is adapted from **?** and many of its elements are standard in TANK models. For this reason, I mostly focus on what is different.

2.1 Households

There are two types of households in fixed types: borrowers in mass χ and savers in mass $1 - \chi$.

2.1.1 Borrowers, Debt, and Default

There is a representative borrower family that consists of a continuum of agents $i \in [0, 1]$. Each of these agents can be employed in the n-sector, employed in the a-sector, or unemployed. Let $N_t^{n,b}, N_t^{a,b}$ denote the mass of agents working in the n- and a-sectors, respectively, and let $1 - N_t^{a,b} - N_t^{n,b}$ denote the mass of unemployed agents.

To generate realistic default rates in the context of a representative agent model, I assume that the members of the borrower household are subject to a cash-in-advance constraint and liquidity shocks. The borrower family enters the period with a stock of debt to be repaid equal to B_{t-1}^b. Each member of the household is responsible for repaying an equal amount B_{t-1}^b at the beginning of the period. At this point, the only available resources are labor income, net government transfers, and a liquidity shock $\varepsilon_t(i) \sim F^e, F^u$, where F^e, F^u are distributions with support in the real line.[1] Total cash in hand is therefore given by

$$\mathbb{1}\left[i \in N_t^{n,b}\right] w_t^n (1 - \tau_t^l) + \mathbb{1}\left[i \in N_t^{a,b}\right] w_t^a (1 - \tau_t^l) + \mathbb{1}\left[i \notin N_t^{n,b}, N_t^{a,b}\right] \mathrm{ui}_t + T_t^b + \varepsilon_t(i)$$

where T_t^b is an unconditional transfer from the government, and ui_t is unemployment insurance. Default is liquidity-based: agent i compares cash-in-hand to the required repayment B_{t-1}^b and defaults if she does not have enough resources to repay. This allows me to define three thresholds that determine default rates for each of the possible employment states,

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[1]I allow the distribution of liquidity shocks to differ for the employed and unemployed agents as this allows me to jointly match replacement rates and different default rates for employed and unemployed.

$$\varepsilon_t^a = \frac{B_{t-1}^b}{\Pi_t} - w_t^a(1 - \tau_t^l) - T_t^b$$

$$\varepsilon_t^n = \frac{B_{t-1}^b}{\Pi_t} - w_t^n(1 - \tau_t^l) - T_t^b$$

$$\varepsilon_t^u = \frac{B_{t-1}^b}{\Pi_t} - \text{ui}_t - T_t$$

The total default rate is then given by

$$F_t^b = N_t^{a,b} F^e(\varepsilon_t^a) + N_t^{n,b} F^e(\varepsilon_t^n) + (1 - N_t^{a,b} - N_t^{n,b}) F^u(\varepsilon_t^u)$$

After default decisions are made, the borrower household jointly takes all other relevant decisions at the household level. The borrower solves the following program,

$$V_t^b(B_{t-1}^b) = \max_{C_t^b, B_t^b} u(C_t^b) + \beta^b \mathbb{E}_t V_{t+1}^b(B_t^b)$$

s.t.

$$C_t^b + \frac{B_{t-1}^b}{\Pi_t}(1 - F_t^b) = N_t^{a,b} w_t^a(1 - \tau_t^l) + N_t^{n,b} w_t^n(1 - \tau_t^l) + (1 - N_t^{a,b} - N_t^{n,b})\text{ui}_t + T_t^b + Q_t^b B_t^b$$

$$B_t^b \le \Gamma$$

where C_t^b is non-service consumption, the first constraint is the budget constraint, and the second constraint is a borrowing constraint expressed in terms of a limit to total repayment.

2.1.2 Savers

Savers also supply labor to both sectors. They save in government bonds and bank deposits, and own all firms and banks in this economy. Additionally, they derive utility from consumption in the services sector, C_t^a. They solve the following problem,

$$V_t^s(D_{t-1}, B_{t-1}^g) = \max_{C_t^s, C_t^a, B_t^g, D_t} u(C_t^s) + \alpha_t \frac{(C_t^a)^{1-\sigma_a}}{1 - \sigma_a} + \beta^s \mathbb{E}_t V_{t+1}^s(D_t, B_t^g)$$

s.t.

$$C_t^s + p_t^a C_t^a + Q_t(D_t + B_t^g) = N_t^{a,s} w_t^a(1 - \tau_t^l) + N_t^{n,s} w_t^n(1 - \tau_t^l)$$

$$+ (1 - N_t^{a,s} - N_t^{n,s})\text{ui}_t + \frac{B_{t-1}^g + D_{t-1}}{\Pi_t} + (1 - \tau^k)\mathcal{P}_t - T_t + T_t^b$$

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where p_t^a is the price of a-sector goods in terms of the numeraire (final n-goods), D_t is bank deposits, B_t^g is government debt, and Π_t is the inflation rate in terms of non-service goods. $\mathcal{P}_t$ is total profits from firms and banks, which are taxed at some flat rate τ^k. I assume that deposits are safe, and so they pay the same return as government bonds. T_t is a lump-sum tax paid to the government. It is useful to define the stochastic discount factor (SDF) of savers as

$$\Lambda_{t+1}^s = \beta^s \frac{u'(C_{t+1}^s)}{u'(C)t^s}$$

Finally, α_t is a shock to the utility derived from the consumption of services. This shock plays an important role in what follows. Demand for services is given by

$$C_t^a = \left[\alpha_t \frac{1}{p_t^a u'(C_t^s)} \right]^{1/\sigma_a}$$

2.2   Financial Intermediaries

Financial intermediaries are based on a version of **?**. There is a continuum of intermediaries indexed by j that take deposits from savers and originate loans to borrowers. Intermediation is subject to two important frictions: first, there is a market leverage constraint that imposes that the value of the intermediary's assets not exceed a multiple of its market value. Second, the intermediary must pay a fraction $1 - \theta$ of its earnings as dividends every period. The intermediary problem is

$$V_t^k(D_{t-1}(j), B_{t-1}^b(j)) = \max_{B_t(j), D_t(j)} (1 - \theta)\pi_t(j) + \mathbb{E}_t \Lambda_{t+1}^s V_{t+1}^k(D_t(j), B_t^b(j))$$

$$\text{s.t.}$$

$$Q_t^b B_t^b(j) = \theta \pi_t(j) + Q_t D_t(j)$$

$$\kappa Q_t^b B_t^b(j) \leq \mathbb{E}_t \Lambda_{t+1}^s V_{t+1}^k(D_t(j), B_t^b(j))$$

$$\pi_t(j) = (1 - F_t^b)\frac{B_{t-1}^b(j)}{\Pi_t} - \frac{D_{t-1}(j)}{\Pi_t}$$

The value of the intermediary is equal to dividends paid today, a fraction $1 - \theta$ of its earnings, plus the continuation value. The first constraint is a balance sheet constraint: assets must be financed with either retained earnings or deposits. The second constraint is a market leverage constraint: bank assets cannot exceed a multiple $1/\kappa$ of ex-dividend bank value. Finally, the third constraint is the law of motion for earnings: the bank earns revenues for non-defaulted loans and must pay out previously borrowed deposits.

It is possible to show that the value function is homogeneous of degree one in earnings,

thus allowing for aggregation. That is, letting π_t be the relevant state variable, we can show that $V_t^k(\pi_t(j)) = \Phi_t \theta \pi_t(j)$, and that Φ_t is the same for all banks. Define aggregate retained earnings as

$$E_t = \theta \left[(1 - F_t^b) \frac{B_{t-1}^b(j)}{\Pi_t} - \frac{D_{t-1}(j)}{\Pi_t} \right] + \varpi$$

where ϖ is a small (gross) equity injection from savers. Then, we can work with a representative bank that has retained earnings equal to E_t.

The first-order condition for lending takes the form

$$\mathbb{E}_t \frac{\Lambda_{t+1}^s}{\Pi_{t+1}} (1 - \theta + \theta \Phi_{t+1}) \left[\frac{1 - F_{t+1}^b}{Q_t^b} - \frac{1}{Q_t} \right] = \mu_t \kappa$$

where μ_t is the Lagrange multiplier on the leverage constraint, and $\frac{\Lambda_{t+1}^s}{\Pi_{t+1}}(1 - \theta + \theta \Phi_{t+1}) \equiv \Omega_{t+1}$ is the bank's SDF. When the constraint binds $\mu_t > 0$, this generates excess returns on lending over and above what would be warranted by pure credit risk. The constraint will typically bind when the bank is undercapitalized, i.e. when its value is low. Binding constraints allow the bank to recapitalize itself by generating a positive wedge between the cost of borrowing $1/Q_t$ and the return on lending $(1 - F_{t+1}^b)/Q_t^b$. This means that when banks are in bad shape, they tend to lend less and at higher interest rates.

2.3 Production

There are two sectors in this economy: non-services and services.

2.3.1 Non-Services Sector

The n-sector is the largest sector in this economy, and n-sector final goods work as the numeraire. This sector operates like the single sector in a standard New Keynesian model. Goods in the n-sector are produced by a continuum of producers that operate under monopolistic competition and are subject to costs of adjusting their prices. The final-goods aggregator for n-sector intermediates is

$$Y_t = \left[\int_0^1 Y_t(l)^{\frac{\epsilon-1}{\epsilon}} \, \mathrm{d}l \right]^{\frac{\epsilon}{\epsilon-1}}$$

Firms in the n-sector operate a linear technology that produces variety l using labor,

$$Y_t(l) = A_t N_t^n(l)$$

where A_t is an aggregate TFP shock. They sell their good at price $P_t(l)$ and face adjustment costs a la **?**,

$$d[P_t(l), P_{t-1}(l)] = Y_t \frac{\eta}{2} \left[\frac{P_t(l)}{P_{t-1}(l)\Pi} - 1 \right]^2$$

where η measures the degree of nominal rigidity and Π is steady state inflation (indexing). From the aggregator, each producer faces a demand curve given by $Y_t(l) = [P_t(l)/P_t]^{-\varepsilon} Y_t$, where P_t is the price level for n-sector goods. Standard derivations and imposing a symmetric equilibrium in price-setting yield a New-Keynesian Phillips Curve

$$\eta \mathbb{E}_t \left\{ \Lambda_{t+1}^s \frac{Y_{t+1}}{Y_t} \frac{\Pi_{t+1}}{\Pi} \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \right\} - \epsilon \left(\frac{\epsilon - 1}{\epsilon} - \frac{w_t^n}{A_t} \right) = \eta \frac{\Pi_t}{\Pi} \left(\frac{\Pi_t}{\Pi} - 1 \right)$$

where $\frac{w_t^n}{A_t}$ is the real marginal cost. Aggregate production in this sector is

$$Y_t^n = A_t N_t^n [1 - d(\Pi_t)]$$

where $d(\Pi_t)$ is resource costs from price adjustment.

2.3.2 Services Sector

The services sector operates differently. There is a continuum of firms indexed by k; the total mass of active firms is denoted by F_t. At the beginning of the period, each firm observes the aggregate state and draws an idiosyncratic cost shock $c \sim H \in [0, \infty)$. It may choose to exit or operate and produce. If it exits, it receives a payoff of zero. If it operates, it hires one unit of labor and produces one unit of services output. Its value is

$$V_t^a(A_t) = p_t^a A_t - w_t^a + T_t^a w_t^a + \mathbb{E}_t \Lambda_{t+1}^s \int_c \max\{0, V_{t+1}^a(A_{t+1}) - c\} \mathrm{d}H(c)$$

It is possible to show that there exists a threshold $\bar{c}_t(A_t)$ such that a firm decides to operate if its cost is below this threshold, and exit otherwise. This threshold can be shown to be equal to the value of the firm, $\bar{c}_t(A_t) = V_t^a(A_t)$.

Every period, there is an endogenous mass of entrant firms ν_t that pay a fixed cost to enter this sector. The cost is increasing in the mass of entrants so as to capture some type of congestion and is given by $\kappa \nu_t^{\psi}$. The free-entry condition determines the mass of entrants,

$$V_t^a(A_t) \le \kappa \nu_t^{\psi} \perp \nu_t \ge 0$$

Implicitly, I am assuming that entrants do not draw an operating cost and that they can

start hiring/producing in the period they enter.

The total mass of service firms in the economy at any given point in time is then given by surviving firms that did not exit plus firms that entered this period. The law of motion for the mass of firms is

$$F_t = H[\bar{c}_t(A_t)]F_{t-1} + \nu_t$$

Since each firm hires one worker, this will also be total demand for labor in this sector. Total output from this sector is therefore given by

$$Y_t^a = A_t F_t$$

2.3.3 Labor Markets

Since there is no disutility of work, I assume that both savers and borrowers supply as much labor as firms demand. For simplicity, I assume that labor is perfectly mobile across sectors and there is a single wage. I assume a reduced-form rule for wages,

$$w_t = \xi A_t (N_t^n + N_t^a)^\zeta$$

where ξ is a constant. Wages comove with labor productivity A_t, and also respond to total employment as a proxy for labor market tightness.[2] Similar wage rules could be derived from more complicated models that make labor market frictions explicit (**??**). I assume that labor is rationed in equal proportion among savers and borrowers so that

$$N_t^{b,a} = N_t^{s,a} = N_t^a$$
$$N_t^{b,n} = N_t^{s,n} = N_t^n$$

2.4 Fiscal and Monetary Policy

2.4.1 Central Bank

The Central Bank (CB) follows a standard Taylor Rule subject to an explicit zero lower bound,

$$\frac{1}{Q_t} = \max\left\{1, \left(\frac{\Pi_t}{\bar{\bar{\Pi}}}\right)^{\phi_\Pi} \left(\frac{p_t^a}{p_{t-1}^a}\right)^{\phi_a} \left(\frac{GDP_t}{G\bar{D}P}\right)^{\phi_{GDP}}\right\}$$

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[2]A previous version of this paper featured different wages across sectors. This did not affect the results in any meaningful way.

I allow the CB to respond to fluctuations in inflation in the n (numeraire) sector and in the services sector. GDP is defined as

$$GDP_t = Y_t^n + p_t^a Y_t^a$$

2.4.2 Fiscal Authority

The fiscal authority has outflows related to non-service consumption G_t, unemployment insurance ui_t, and debt repayments B_{t-1}^g/Π_t. Its inflows are labor income/payroll taxes $\tau_t^l(w_t^a N_t^a + w_t^n N_t^n)$, capital income/profit taxes $\tau^k \mathcal{P}_t$, debt issuance B_t^g, and lump-sum taxes T_t. Additionally, the fiscal authority can engage in a variety of other types of spending. Net spending of other types is denoted $\mathcal{N}_t$. The government budget constraint is

$$G_t + \frac{B_{t-1}^g}{\Pi_t} + ui_t(1 - N_t^a - N_t^n) + \mathcal{N}_t = \tau_t^l w_t(N_t^a + N_t^n) + \tau^k \mathcal{P}_t + B_t^g + T_t$$

Lump-sum taxes adjust to ensure government solvency in the long-run. The adjustment rule is standard (?),

$$T_t = \left[\frac{B_{t-1}^g}{\bar{B}^g}\right]^{\phi_\tau} - 1$$

and ϕ_τ controls the speed of adjustment. A low value means that current spending is mostly deficit-financed.

Discretionary Fiscal Policy I assume that the fiscal authority has access to an additional set of instruments. Given their extraordinary nature, these interventions will be treated as one-time shocks that are completely unexpected, but once deployed their paths are perfectly anticipated. These components of $\mathcal{N}_t$ are: (i) unconditional transfers to all agents in the economy, T_t^b, and (ii) transfers to service-sector firms that are proportional to their wages, $T_t^a w_t^a$. Thus,

$$\mathcal{N}_t = T_t^b + T_t^a w_t F_t$$

Additionally, I assume that the government can also conduct one-time changes to existing fiscal instruments: (i) an increase in non-service consumption G_t, (ii) an increase in unemployment insurance transfers ui_t, and (iii) a reduction in the income tax τ_t^l.

2.5 Resource Constraints

The resource constraint for non-service goods is

$$\chi C_t^b + (1 - \chi)C_t^s + G_t + \Psi[\bar{c}_t(A_t)]F_{t-1} = A_t N_t^n[1 - d(\Pi_t)]$$

where $\Psi[\bar{c}_t(A_t)] \equiv \int_0^{\bar{c}(A_t)} c \, dH(c)$ is total operating costs paid by non-exiting service sector firms, expressed in terms of non-service goods. I assume that firm-entry costs are rebated to savers.[3] The resource constraint for service goods is

$$(1 - \chi)C_t^a = A_t F_t$$

A full list of equilibrium conditions is in Appendix **??**.

3 Numerical Experiment

3.1 Model Calibration

The model steady state is calibrated to the US economy in the eve of the coronavirus pandemic. The calibration is summarized in Table **??**.

In terms of functional forms, the utility of non-service consumption is isoelastic, $u(C) = \frac{C^{1-\sigma}}{1-\sigma}$. The distributions of liquidity shocks F^e, F^u are gaussian with mean zero and variances σ^e, σ^u, which are calibrated to match total average charge-off rates and default rates for unemployed households. The distribution of cost shocks for service sector firms is assumed to be log-normal with mean 1 and variance σ_k.

Most saver parameters are standard, with the exception of σ_a, which I assume to be equal to 1 — equal to the value for non-services— as a benchmark and since there is no consensus on estimates for the EIS of nondurable services. Naturally, some of the results are sensitive to this parameter, as it affects the price-elasticity of demand for service goods, the level of complementarity between service and non-service goods and, consequently, the employment effects of interventions in that sector.[4] Borrower parameters are also set to match standard targets.

With regards to production and labor markets, I set the share of labor in contact-intensive services to be 40% based on the data for 2018 on Table 2.1 of Employment Projections from the Bureau of Labor Statistics. The classification is more or less manual, but I consider this

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[3]This avoids artificial demand-driven expansions/recessions due to waves of high or low entry.

[4]In particular, a lower value for this parameter means that the price of the services good will be less responsive in equilibrium to changes in quantities.

type of services to be comprised of: 50% of wholesale trade, 100% of retail trade, 50% of transportation and warehousing, 50% of professional services, 50% of educational services, 33% of healthcare and social assistance, 100% of leisure and hospitality, and 100% of other services. This generates an employment share close to 40% (39.3%). In practice, much informal labor is likely to be in contact-intensive industries so it is possible that this may be an underestimate. The elasticity of wages to total employment is chosen to be 0.05, a relatively low level so that wages do not move by much. Raising this parameter helps stabilize employment in the services sector (as wages fall upon a shock), but it makes spillovers to the non-services sector worse (due to aggregate demand externalities). Since it is not clear at this point what will happen to wages across sectors, I believe that this is an agnostic assumption. The entry cost constant κ is set so as to generate an entry rate of 8% yearly, consistent with recent studies on US business dynamism. The elasticity with respect to the number of entrants is also not easily calibrated, I set it to 1 so as to generate what seem plausible entry dynamics (results are also robust with respect to this parameter). The markup in the services sector is set at 1%. This parameter, along with σ_k and κ jointly determine the entry and unemployment rates in this sector.

Regarding the banking system and government, parameters are reasonably standard. ϕ_τ is set to 0.25 to ensure that government debt peaks right after the crisis, and starts decreasing in the following quarters, but most results are robust to alternative values of this parameter (except for the path of public debt, naturally). The value of the unemployment subsidy is chosen to be 35% of the steady state wage. **?** estimate that only 25% of unemployed workers in the US receive UI. I choose a slightly higher value to account for informal and home production.

Parameter	Description	Value	Target
	Saver Parameters		
β^s	Discount factor saver	0.9951	Annualized real interest rate of 2%
σ	Elasticity of intertemporal substitution	1	Standard/log utility
α	Utility of services	2.5557	Implied by other parameters
σ_a	EIS for services	1	Same as for non-services
	Borrower Parameters		
β^b	Borrower discount factor	0.9453	Constrained at steady state
Γ	Borrowing constraint	0.1769	Payment to income ratio of 30%
χ	Fraction of borrowers	0.475	?
σ^e	SD of liquidity shock, employed	0.2315	Default rate of 8%, yearly
σ^u	SD of liquidity shock, unemployed	0.0742	Default rate of 40%, yearly
	Production/Labor Market Parameters		
ϵ	Elasticity of subst. sector n	6	20% markup in SS
η	Rotemberg menu cost	59.12	$\simeq$ Calvo parameter of 0.75
ϕ	Labor in a-sector	0.40	BLS: % of employment in contact-intensive industries
N	Employment at SS	0.925	SS unemployment rate of 7.5%
ζ	Sector elasticity of wage to employment	0.05	See text
κ	Entry cost constant	0.20	Entry rate of 8% yearly
ψ	Elasticity of entry costs to entrants	1.00	See text
p^a/w^a	a-sector markup at SS	1.01	See text
σ_k	Variance of a-sector shock	4.7617	Employment in the a-sector
	Banking Parameters		
θ	Retained earnings	0.90	Net payouts of 3.5% (?)
κ	Leverage constraint	0.10	Leverage of money center banks
ϖ	Transfer to new banks	0.0004	Annual lending spread of 1%
	Policy Parameters		
$\bar{\Pi}$	Trend inflation	$1.02^{0.25}$	2% for the U.S.
ϕ_Π	Taylor rule: Inflation sector n	1.5	Standard
ϕ_a	Taylor rule: Inflation sector a	0.0	
ϕ_Y	Taylor rule: Output	0.5/4	Standard
$\bar{G}$	Govt Consumption of n-goods	$0.2 \times Y^n$	Standard
$\bar{B}^g$	Govt debt at SS	$0.9 \times Y^n$	US, 2019
ϕ_τ	Fiscal rule parameter	0.25	
$\bar{\text{ui}}$	Unemployment insurance	$0.35 \times w$	25% covered by UI + home production
τ^l	Labor income tax rate	15%	Avg for the US
τ^k	Tax rate on profits	28%	Implied by other parameters

Table 1: Summary of the calibration.

3.2 What is a Pandemic in a DSGE model?

The main purpose of this paper is to study the dynamic response of the economy to different types of fiscal policy instruments during a pandemic event. It is not obvious, in principle, how to model a pandemic in an otherwise standard DSGE model. It seems to be widely accepted that a highly contagious pandemic results in a reduction in economic activity as households start isolating themselves from others. This leads to a sharp reduction in activity in sectors of the economy that are contact-intensive, such as hospitality and leisure, as well as certain types of retail (brick and mortar) and transportation (air travel).

Arguments can be made for a negative shock to the marginal utility of consumption

/ discount factor, or a positive shock to the disutility of labor (**?**). Neither of these is ideal in isolation, for different reasons. A shock to the marginal utility of consumption leads to a fall in aggregate demand that results in unemployment in this model. But this could be easily counteracted with an increase in non-service government consumption, for example. In practice, it is very unlikely that any type of stimulus based on government consumption can restore activity in, say, leisure. A shock to the marginal disutility of labor, on the other hand, generates counterfactual implications in terms of wages and potentially welfare. A more sophisticated approach is taken by **?**, who embed an epidemiology model in a real business cycle framework. In their model, agents can become infected by "meeting" other infected agents while purchasing consumption goods or working. For this reason, the outbreak of an epidemic results in a contraction of consumption and hours worked.

Since I want to be able to preserve some tractability so as to be able to talk about different types of stabilization policies, I decide to model a pandemic as a shock to the marginal utility of one particular sector in the economy. For technical reasons, I assume that only savers are subject to this type of shock. A sufficiently large shock to α_t leads to a large drop in employment in this sector. This affects mostly borrowers, who are constrained and have a very high marginal propensity to consume. As their income falls due to loss of employment, default rates rise. This constrains banks, which in turn demand higher interest rates on their lending. These two effects contribute to a decline in non-service consumption, which in turn triggers a fall in inflation and a fall in the demand for non-service labor. The central bank responds to these shocks by lowering interest rates. This helps banks by lowering their cost of funding, but eventually interest rates are constrained by the zero lower bound. If the shock is sufficiently severe, the economy hits the zero lower bound (ZLB) and a large recession can ensue.

3.3   Size and duration of the pandemic

To calibrate the intensity and duration of the shock, I adopt a pessimistic approach. The size of the shock is chosen so that the unemployment rate rises to 20%, following the worst-case scenario put forward by Treasury Secretary Mnuchin to Members of Congress on March 17, 2020.[5] This can be achieved with a drop in α_t of 60%. I assume that the shock lasts for three quarters: from 2020Q1 through 2020Q3. Finally, I assume that there is an equal shock in each quarter, as it is highly unlikely that people will start using services again as long as the pandemic is active, but that the shock has no persistence. Once the pandemic is gone, saver utility from consuming services returns to normal.

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[5]Source: https://blogs.wsj.com/economics/2020/03/18/newsletter-the-layoffs-are-starting/

Throughout, I assume that the pandemic is an exogenous shock. That is, I take the intensity and duration of the pandemic as given; I do not explicitly model government investment in healthcare and mitigation or how it could potentially reduce both of these characteristics, that is outside the scope of this exercise. I also abstract from mortality and how it could affect the size of the labor force.[6]

3.4 Pandemic Experiment

Figure ?? plots the response of selected variables to the α_t shock. The path of the shock is plotted in the first panel. The shock causes a 40% drop in employment in the services sector (4th panel). The loss of these jobs affects borrowers, whose consumption falls by almost 10%. This drop in non-service consumption also leads to a drop in employment in the other sector, of about 6%. Combined, these drops in employment lead to a 20% contraction in GDP that lasts for the full three quarters. The 6th panel shows that this recession pushes the economy to the zero lower bound for the entire period. The bottom two panels show that the loss in employment leads to a doubling of (quarterly) default rates. This in turn affects the financial sector, and lending spreads rise. This further amplifies the drop in borrower consumption and rise in defaults. Persistence arises from the only slow-moving state variable, the number of firms in the affected sector. Due to entry costs, the economy takes a while to recover from the shock.

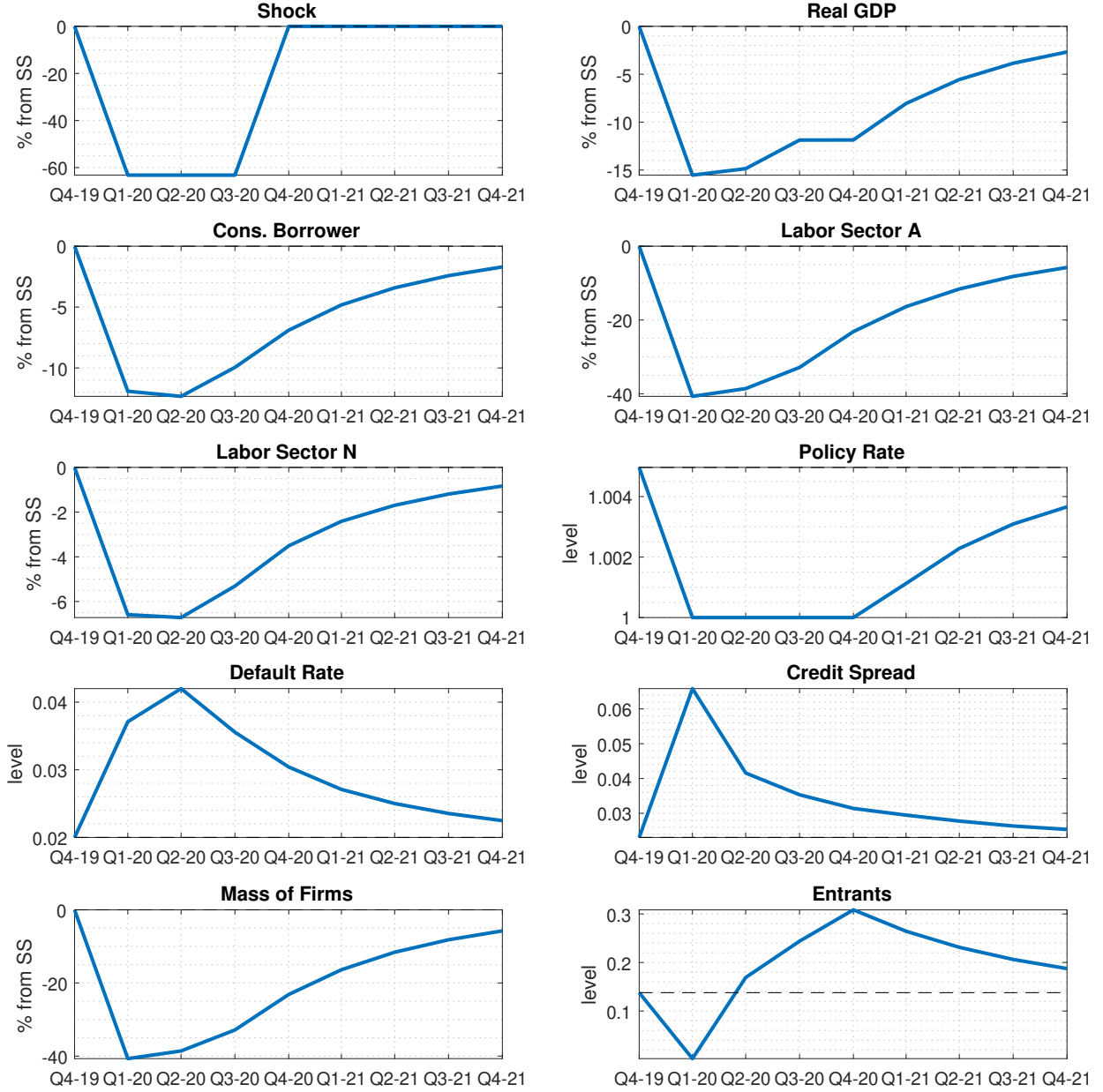[6] ? use data from the 1918-1920 Great Influenza Epidemic to estimate mortality rates of 2%.

Figure 1: Response to a 60% negative shock to α_t that lasts for 3 quarters and has zero persistence.

4 Fiscal Policy Response to the Pandemic

I consider, separately, the effects of deploying the following instruments:

1. Increase in government consumption in sector n, G_t

2. Labor income tax cut, τ_t^l

3. Increase in unemployment insurance, ui_t

4. Unconditional transfers to all agents, T_t^b

5. Transfers to service sector firms, T_t^a

In all cases, I consider a one-time impulse with zero persistence for each instrument. The impulse arrives at the beginning of 2020Q2, as the pandemic starts. This is a very rough and simplistic exercise, but the point is to try to isolate the different effects of these policies. A richer analysis would consider policy packages consisting of multiple instruments, and more persistent policies. That is work in progress.

The model responses are nonlinear and computed with perfect foresight. This means that shocks are completely unanticipated, but once they hit, their path is perfectly anticipated.

I choose the impulses so that the resulting deficits are somewhat comparable, of similar magnitudes. I focus on packages that involve a quarterly increase in the deficit on impact of $200 billion, or roughly 3.7% of quarterly GDP. The size and intensity of the interventions certainly matter since the model features nonlinearities such as the zero lower bound. A deeper exploration into the ideal size of each impulse is left for further research. At the end of this section, I present tables with present-value fiscal multipliers, which partly account for differing sizes of the interventions.

Next, I describe in more detail the effects of these policies. Many of them generate similar effects from a qualitative perspective. The quantitative effects are different, however. For a summary of the quantitative effects, feel free to skip to the next section where I compare measures of fiscal multipliers.

4.1   The Effects of Different Policies

Government Consumption of Non-Services   This is comparable to the traditional increase in G_t in one-sector New Keynesian models. I assume that it is not feasible for the government to purchase services directly: this would be roughly equivalent to a transfer to those firms, which is considered separately.

Figure **??** plots the effects of this policy on selected variables. The blue line corresponds to the crisis absent intervention (as in Figure **??**), while the orange line includes the intervention. The key effect of the policy is seen in the 5th panel: a large increase in government consumption helps sustain employment in the non-service sector. This, in turn, somewhat moderates the drop in borrower consumption and in GDP. Finally, the fact that employment does not fall by as much also helps contain default rates and, via the banking system, credit

spreads. This policy has no direct effect on the services sector; in fact, if anything, it makes things slightly worse by driving up wages for affected firms.
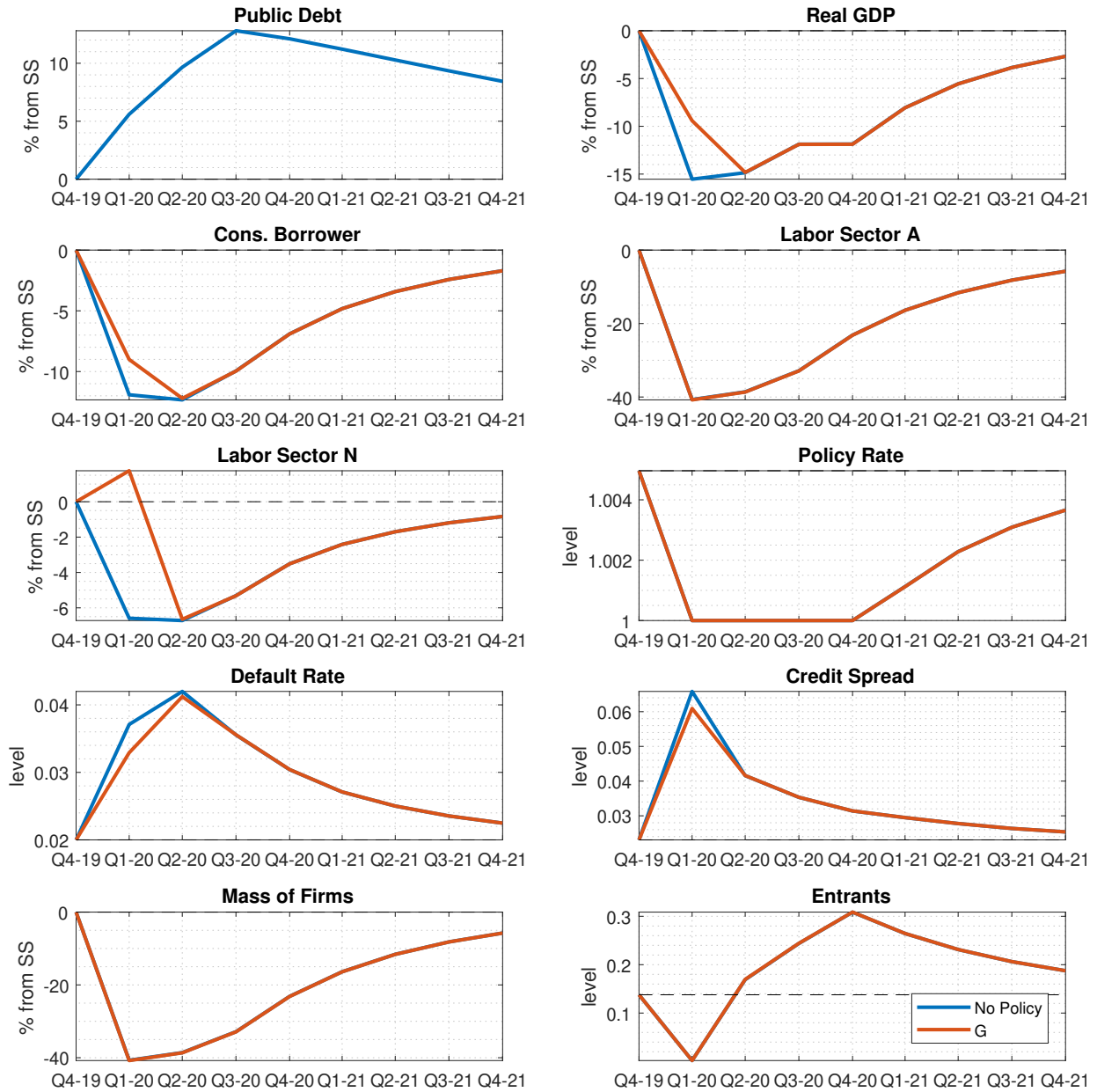

Figure 2: Response to a $\sim$\$200 bn increase in G_t, government consumption of non-service goods.

Labor Income Tax Cuts   To achieve a total deficit of the same size, the intervention consists of a one-time tax cut of 50%, i.e. the tax rate is cut by half. The effects of the income tax cut look relatively similar, in Figure **??**, with the main exception being that they do not stimulate labor in the non-service sector as much as the more targeted policy

of government consumption. Tax cuts still help sustain borrower income, which in turn results in a slightly lower drop in GDP and a decrease in default rates. One important thing to notice is that this model may underestimate the effectiveness of tax cuts: due to the assumption of labor market rationing, there are no direct benefits from removing labor market distortions.
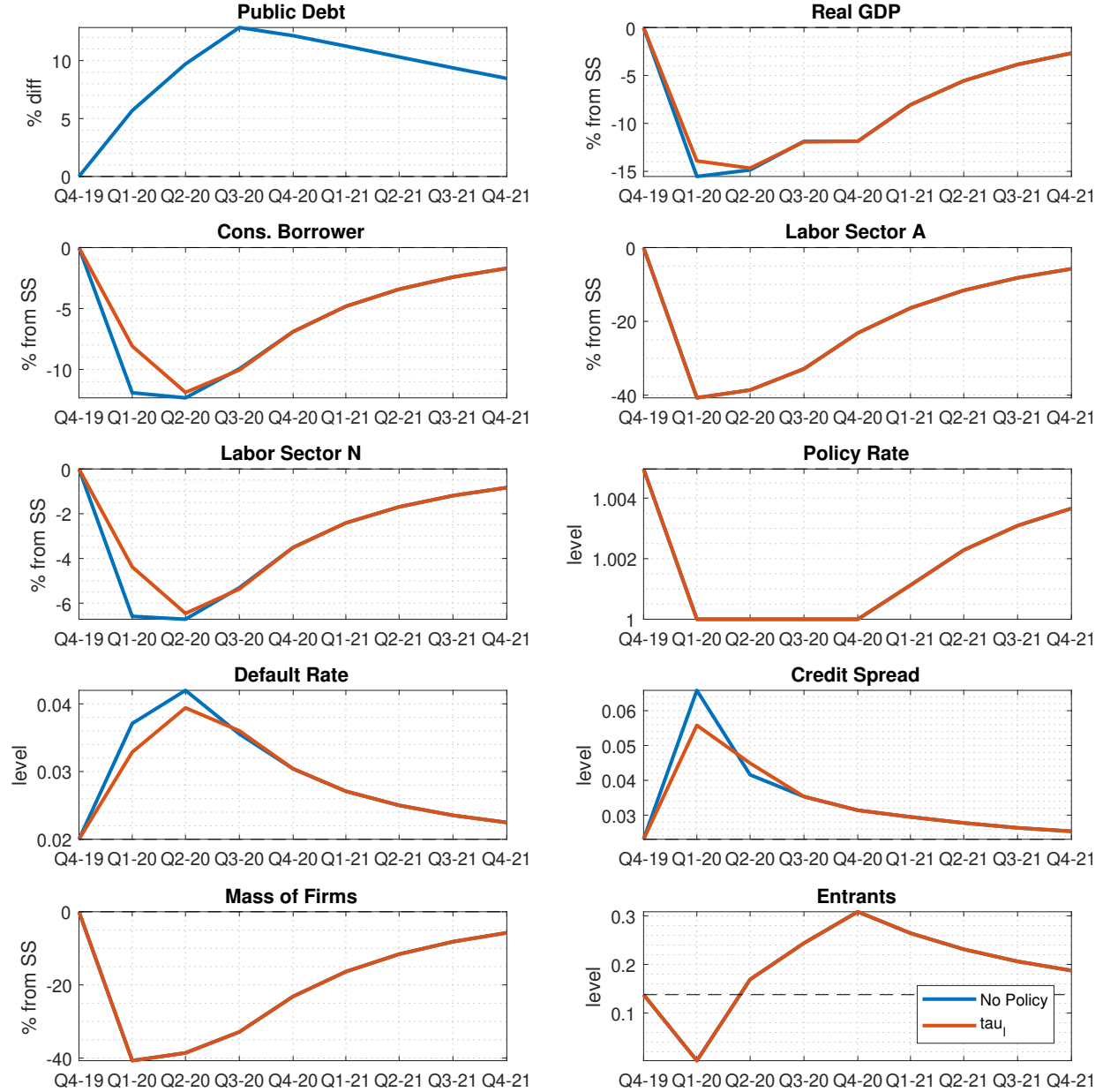

Figure 3: Response to a ∼$200 bn income tax cut.

Unemployment Insurance   Next, we consider a one-time increase in unemployment insurance payments. To achieve a $200 bn intervention, the unemployment insurance transfer

per agent is raised by 75%. The effects are noticeably larger on borrower consumption, as seen in Figure ??, which is now sustained on impact. This is somewhat predictable: income tax cuts benefit agents who remain employed, at a time when a large fraction of agents becomes unemployed. With unemployment insurance, it's the opposite: it helps unemployed agents at a time when a large fraction of agents becomes unemployed. The rise in borrower consumption helps sustain demand in the non-service sector, as seen in the fifth panel. This, in turn, results in a roughly 2.5% gain in GDP. Also note that while the intervention happens only in one quarter, the effects are relatively persistent. This has to do with the fact that borrowing costs remain low, as this increase in unemployment insurance considerably lowers default rates (as unemployed agents tend to have higher default rates than employed ones), and this results in an implicit recapitalization of the banking system.
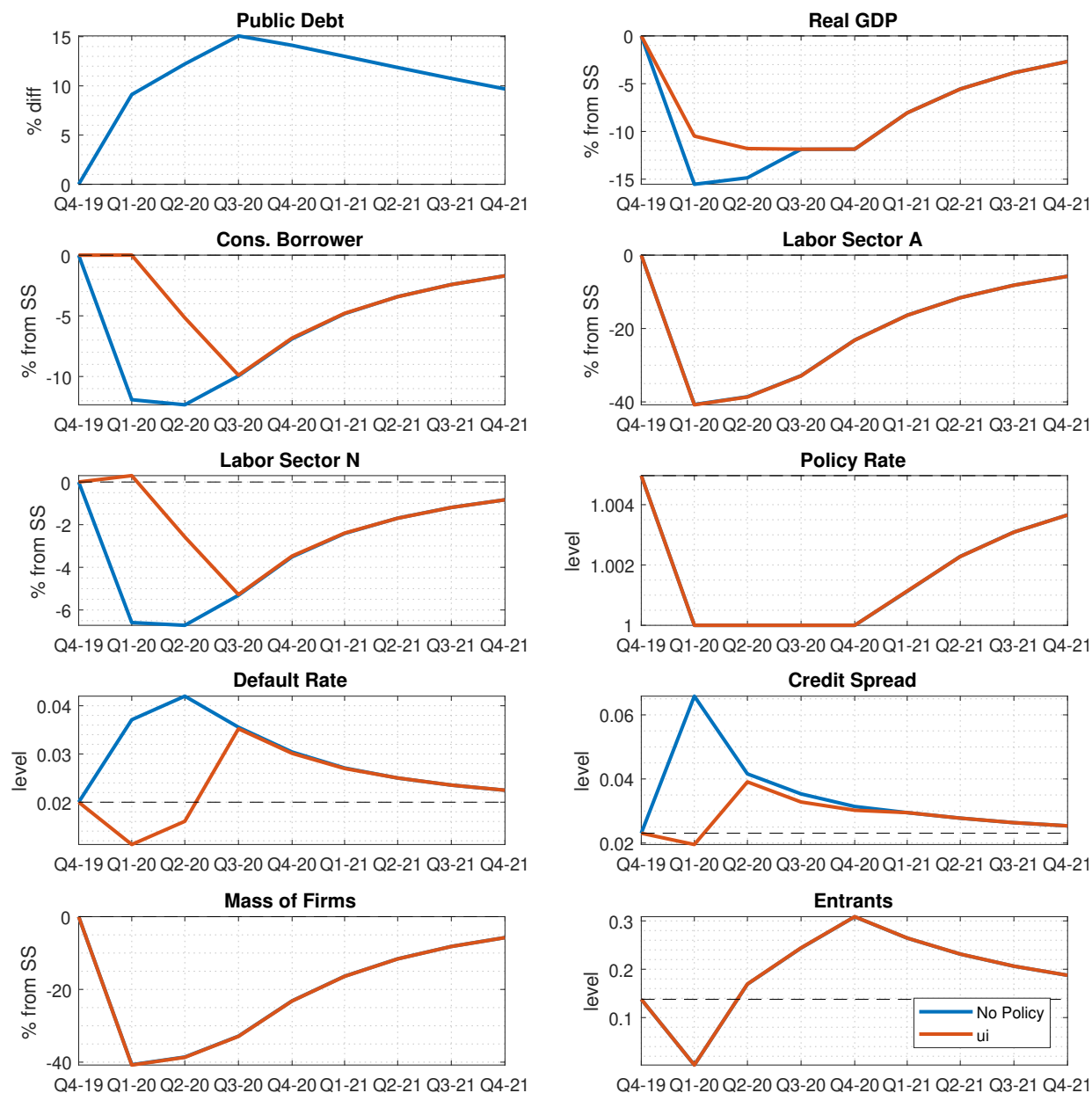
Figure 4: Response to a ∼$200 bn increase in UI.

Unconditional Transfers   Figure **??** plots the effect of a transfer that is given to everyone in this economy, including savers. The effects are similar to those of the payroll tax cut, which is not surprising as the incidence is effectively the same.
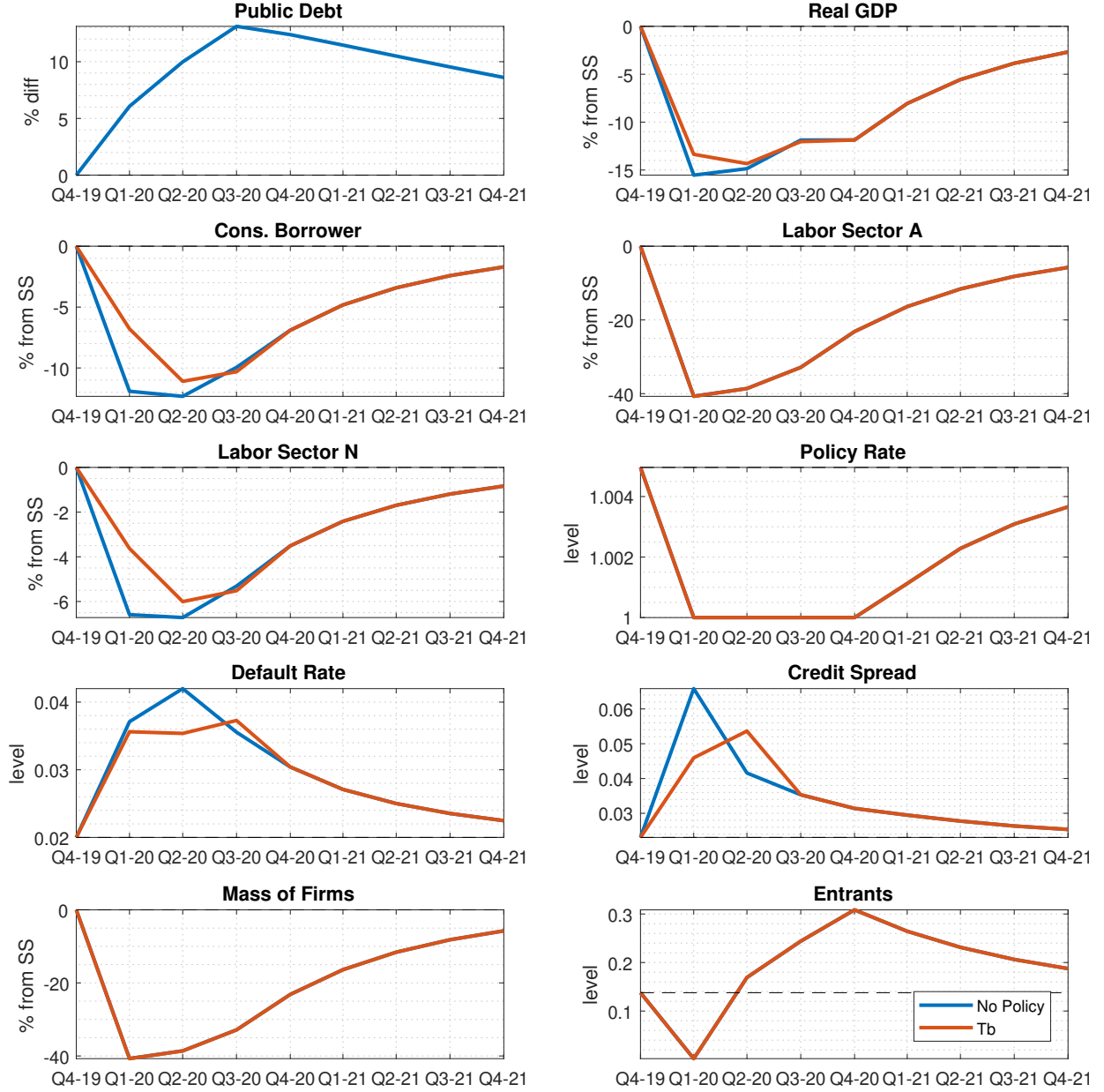
Figure 5: Response to a ∼$200 bn unconditional transfer.

Liquidity Assistance to Service Firms Figure **??** shows the effects of a per-wage subsidy to firms in the service sector. Unlike other interventions, this type of intervention (i) helps mitigate the fall in employment in the services sector, and (ii) has longer-lasting effects that result from less firm exit. The general equilibrium effects are reflected in borrower consumption and labor in the no-service sector. This experiment is not totally fair to this policy, to the extent that this is the only policy that explicitly targets the *a*-sector but does so for only one period, while agents expect the negative demand shock to last for an extra two periods. The remaining two periods without assistance affect the value of service firms,

$V_t(A_t)$, which does not rise by as much as it would should the assistance last for the duration of the pandemic.
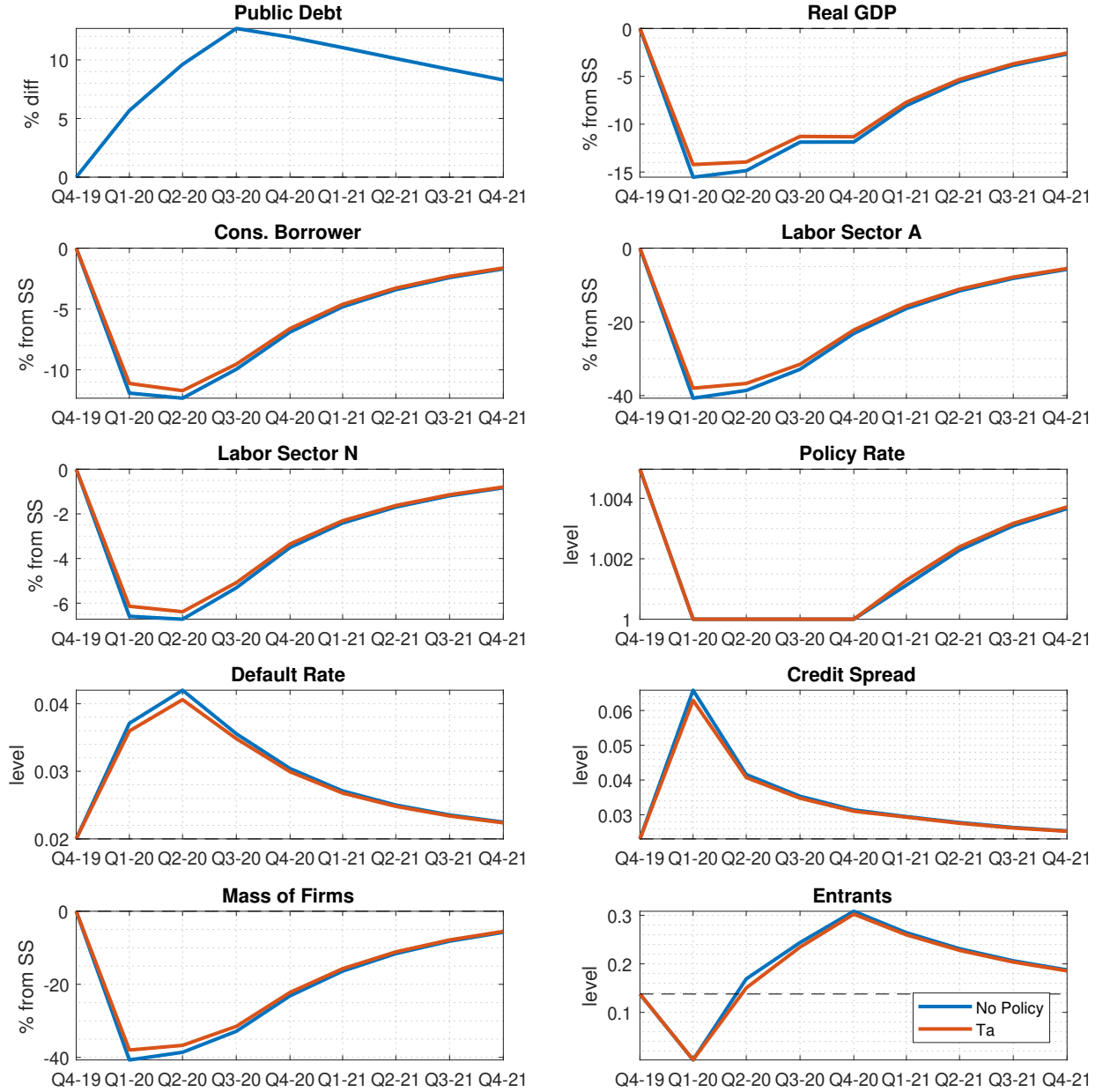

Figure 6: Response to a ∼$200 bn transfer to service firms.

4.2 Fiscal Multipliers

While the sizes of the interventions are calibrated to be of around $200 bn, or 3.7% of quarterly GDP, there are dynamic and general equilibrium effects that influence the path of government expenditure and revenue and that differ across instruments. One common way to control for these effects along with the size of the intervention, is to compute present

value discounted multipliers as in ? or ?. For a given outcome variable of interest x, the multiplier is computed as

$$\mathcal{M}_T(\omega) = \frac{\sum_{t=1}^{T} \prod_{j=1}^{t} R_j^{-1} \left(x_t^{\text{Stimulus}} - x_t^{\text{No Stimulus}}\right)}{\sum_{t=1}^{T} \prod_{j=1}^{t} R_j^{-1} \left(\text{Spending}_t^{\text{Stimulus}} - \text{Spending}_t^{\text{No Stimulus}}\right)}$$

The multiplier is computed for a given instrument $\omega \in \{G_t, \tau_t^l, \text{ui}_t, T_t^b, T_t^a\}$ and at a given horizon T. I set T equal to 20 quarters: this is a typical value for the horizon, but may underestimate the effects of some of the policies that have more persistent effects, such as liquidity assistance. Since the discount rate R_j differs across the economies with policy and with no policy, it is not obvious which one to use. I use the interest rate in the no-policy economy so as to keep the comparison between different tools as fair as possible.

Instrument	Description	$\mathcal{M}_{20}(\omega)$, Employment	$\mathcal{M}_{20}(\omega)$, Income	$\mathcal{M}_{20}(\omega)$, C_t^b	$\mathcal{M}_{20}(\omega)$, C_t^s	$\mathcal{M}_{20}(\omega)$, GDP
G	Govt. Consumption	1.2488	0.5553	0.5532	0.0004	1.2763
τ_t^l	Income Tax	0.6792	1.3896	1.3890	0.0003	0.6944
ς	UI	0.7609	1.5602	1.5537	0.0007	0.7770
T_t^b	Uncond. Transfer	0.6286	1.2790	1.2853	0.0003	0.6426
T_t^a	Liquidity Assist.	2.1796	0.9724	0.9709	-0.0258	0.4212

Table 2: Fiscal multipliers.

Table ?? compares multipliers for a variety of variables: total employment, income net of government transfers, borrower consumption, saver consumption of non-service goods, and GDP. Income net of transfers is defined as

$$\text{Income}_t = (1 - \tau_t^l)(w_t^a N_t^a + w_t^n N_t^n) + (1 - N_t^n - N_t^a)\text{ui}_t + T_t^b$$

In terms of income, the largest multipliers are generated by UI. Income tax cuts and unconditional transfers are also effective, but generate lower multipliers as they are less well-targeted to agents with lower incomes. UI is, furthermore, very well targeted in terms of its timing, as this transfer arrives precisely at a time when unemployment surges. Multipliers on borrower consumption are very similar to those of income, which is to be expected since borrowers are constrained and therefore have a high marginal propensity to consume out of their current income. Any differences reflect changes in the cost of credit from banks.

Multipliers on saver consumption are very low. Savers react relatively little to fiscal policy as they are unconstrained. Savers are "Ricardian" in the sense that they purchase public debt and pay lump-sum taxes and, therefore react to changes in the present value of government liabilities. Note however that the GE effects are strong enough to offset the usual fall in consumption for savers. Naturally, they react more positively to the unconditional

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transfer and more negatively to the increase in unemployment insurance, which is the closest instrument to a targeted transfer to borrowers in this environment.

Employment multipliers are particularly high for liquidity assistance. This is mostly due to the long-lasting effects of this policy. While its effects on impact are smaller than those of other policies, liquidity assistance prevents firm exit and ensures a faster recovery.

GDP multipliers are reported in the last column. As argued before, it is not clear that adopting measures that stabilize GDP is appropriate in this situation. Still, I report the multipliers for completeness. he measure that yields the largest GDP multiplier is government consumption. It is well known that it is "hard to beat" government consumption in this class of models (**?**), especially in the absence of very strong links between the balance sheets of households and the financial system. Income tax cuts, increases in unemployment insurance and unconditional transfers all deliver somewhat similar results. UI performs the best as it is the most well-targeted, while unconditional transfers perform the worst of those three as it is the least well-targeted. Liquidity assistance to firms seems to be the worst-performing policy, subject to the caveats pointed out in the next subsection.

4.3   Dissecting the Effect on Borrower Income

The change on borrower income, on impact, is shown in Figure **??**. This Figure confirms that UI increases have the largest effect. Note that in this picture (and in subsequent pictures), we are comparing % changes for a given impulse, and not adjusting for dollars spent as in the previous paragraphs. Transfers generate better results than income tax cuts. It all boils down to how well targeted a policy is.
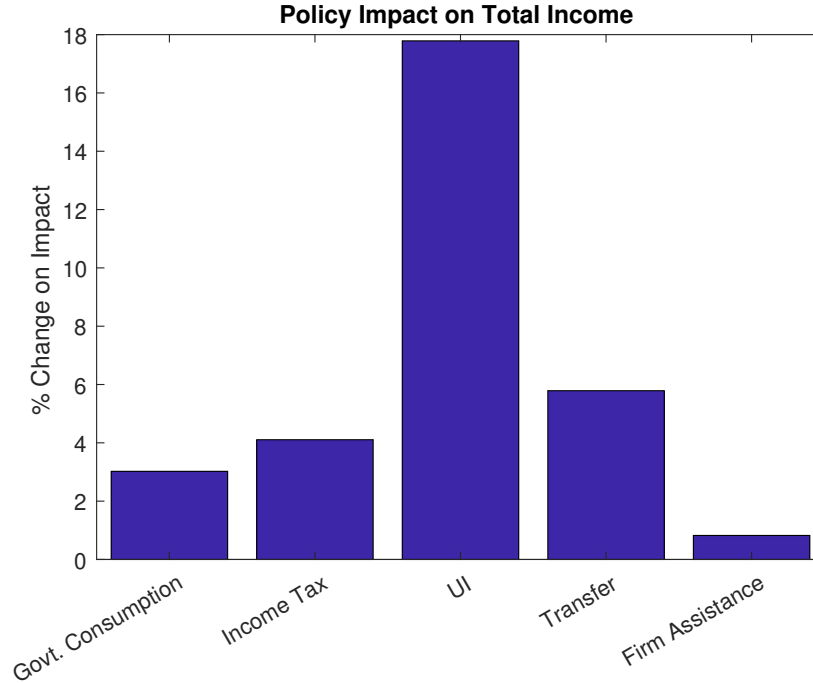
Figure 7: % change on income due to policy, on impact.

Figures **??** and **??** help us understand how well/poorly targeted each type of policy is, by decomposing the effect of each policy on prices and quantities (on impact). Figure **??** plots net income per worker (employed or unemployed), across policies. It shows, for example, that income tax cuts raise incomes for employed workers exclusively, while UI raises incomes for unemployed workers almost exclusively (there is a small increase in employed income that is not visible due to the scale). Transfers and government consumption of non-services operate via traditional aggregate demand effects, thus raising demand for n-sector goods and therefore earnings in this sector, but having no effect in other types of workers. Finally, liquidity assistance to a-sector firms helps sustain wages in this sector somewhat. Figure **??** plots absolute changes in number of workers in each sector, in the baseline economy with no policy (blue bar) and in the economy with the policy impulse (yellow bar).[7] While there are minor variations across policies, the overall pattern is the same across policies: the shock leads to a large reduction in sector a employment, a moderate reduction in sector n employment and large increase in unemployment. These two figures combined show very clearly why UI is the superior policy to stabilize household income, as they target the category of households that increases the most due to the shock.

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[7]Absolute changes are easier to compare since the steady state/initial distribution across sectors is very uneven, with relatively few unemployed agents.
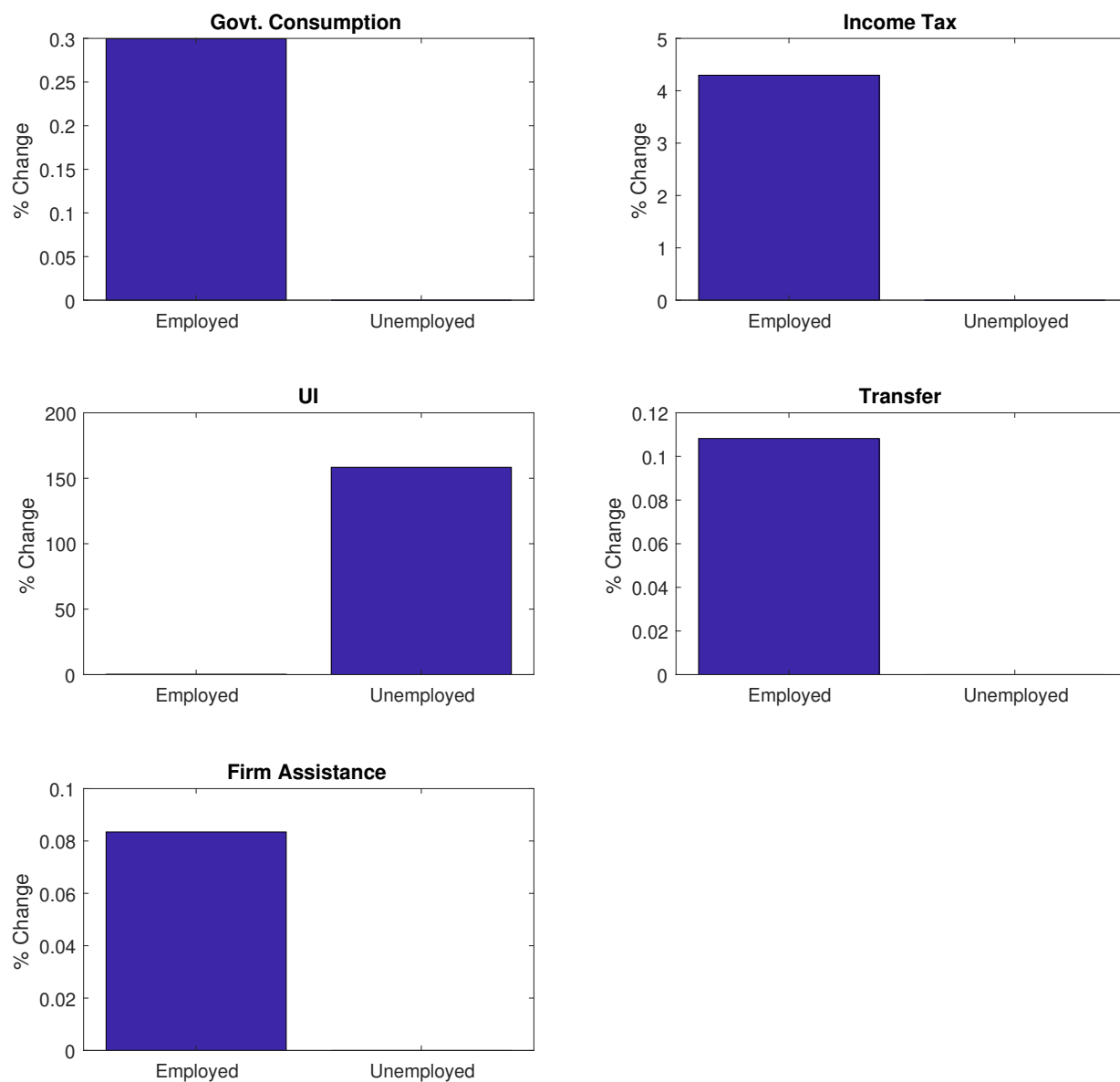
Net Income per Worker

Figure 8: % change on net income per worker due to policy, across sectors. Note that each panel has a different scale.
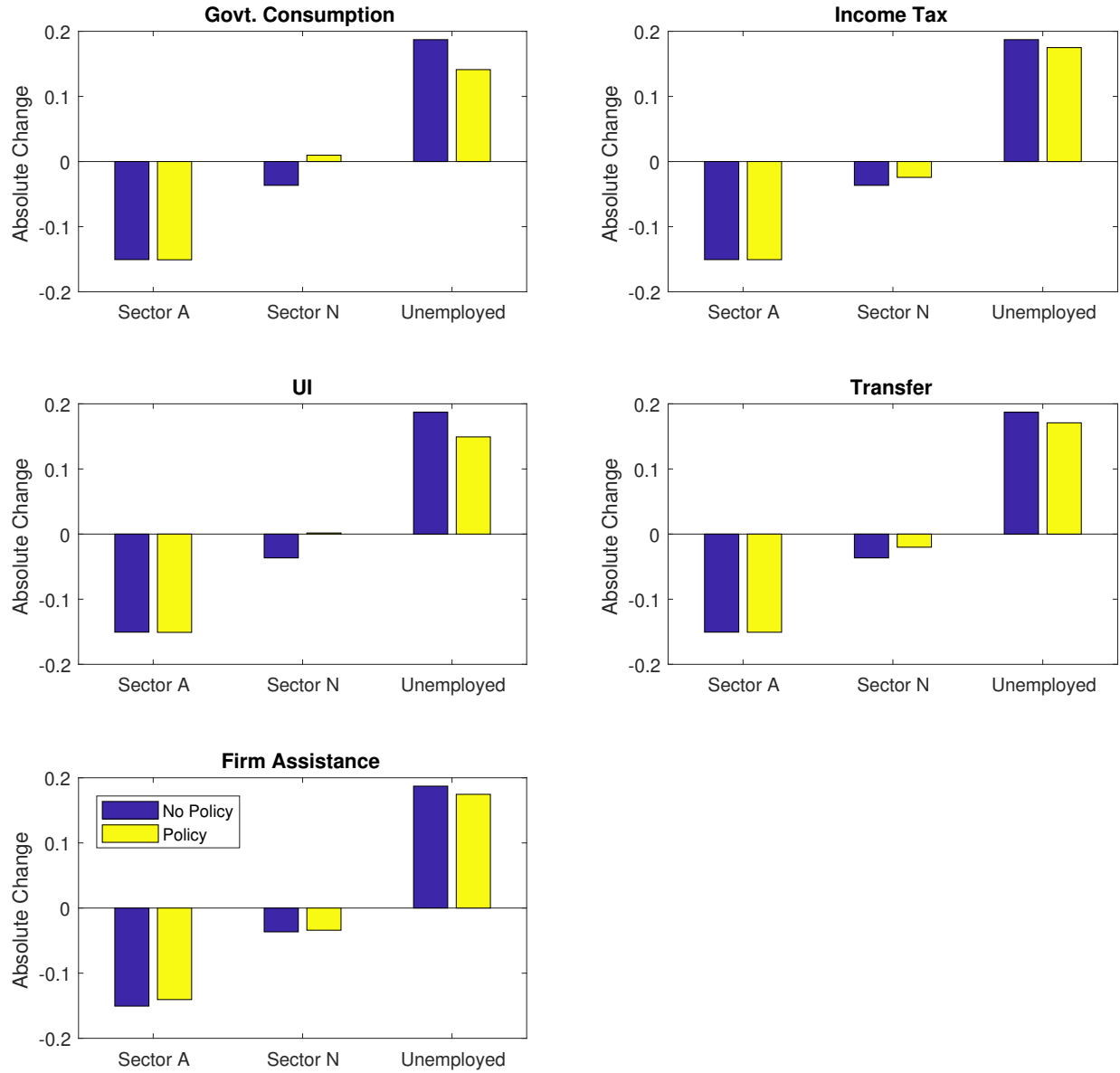
Mass of Workers

Figure 9: Total change in workers, across sectors.

5   The Effects of the CARES Act of 2020

I use the model to quantify the effects of the Coronavirus Aid, Relief, and Economic Security (CARES) Act of 2020 — the $2 trillion package that was by the House on March 27, 2020. As of the time of this writing, the main components of the bill are:

1. $423 billion (2% of GDP) in small business loans, payroll subsidies, and relief for

affected industries (T_t^a)

2. $250 billion (1.2% of GDP) in payments to individuals in the form of rebates to tax-payers (T_t^b)

3. $250 billion (1.2% of GDP) in expanded unemployment insurance (ui_t)

4. $490 billion (2.3% of GDP) in state fiscal aid and federal spending across departments and programs (G_t)

The bill does not explicitly include direct tax cuts, even though it does include tax relief measures such as the delaying of filing dates. For that reason, I do not explicitly model any τ_t^l intervention as part of this package. Excluded from the analysis are $454 billion that are allocated as a backstop to Federal Reserve credit facilities. I jointly simulate interventions of these sizes in the model. For liquidity assistance, unemployment insurance, and government purchases, I assume that the spending is spread across 4 quarters (i.e., a fiscal year) starting on the quarter of the shock (which has a duration of 3 quarters). For transfer payments, I assume that they are a one-time shock happening on the first quarter of the shock (2020Q2).

The result for the aggregate multipliers are show in Table **??**. The fiscal package has an income multiplier of 1.33 and an employment multiplier of 1.30. The following rows de-compose the multiplier across different policies. These numbers are obtained by considering one policy at a time, similar to the exercise in previous sections. Even though the inter-ventions have different sizes and lengths, the results from the baseline exercise are virtually unchanged, with UI and transfer payments providing most of the income and consumption stabilization, and liquidity assistance to firms providing most of the employment stabilization due to its long-run effects.

Instrument	Description	$\mathcal{M}_{20}(\omega)$, Employment	$\mathcal{M}_{20}(\omega)$, Income	$\mathcal{M}_{20}(\omega)$, C_t^b	$\mathcal{M}_{20}(\omega)$, C_t^s	$\mathcal{M}_{20}(\omega)$, GDP
All Policies		1.3045	1.3322	1.3250	-0.0293	0.9965
G	Govt. Consumption	1.1609	0.5174	0.5118	-0.0305	1.2039
ς	UI	0.6683	1.4926	1.4851	-0.0120	0.6896
T_t^b	Uncond. Transfer	0.5897	1.2619	1.2690	0.0004	0.6027
T_t^a	Liquidity Assist.	1.8589	0.8308	0.8294	-0.0320	0.3343

Table 3: Aggregate multipliers for the CARES Act of 2020 and decomposition.

6 Caveats and Discussion

In the context of a simple DSGE model, for an intervention of a fixed size equal to 3.7% of GDP, the most effective tool to stabilize household income and borrower consumption in the

context of an exogenous shock that leads to the shut down of the services sector seems to be an increase in unemployment insurance benefits. Overall, programs that involve transfers of some kind to households seem to be effective, with UI being the best targeted. Unconditional transfers are likely to be less costly in terms of implementation, may be favored by savers, and deliver somewhat similar (weaker) results. Firm liquidity assistance programs are effective at maintaining employment overall and have the longer-lasting effects.

The analysis in this paper is very simple, takes many shortcuts, and abstracts from many important things. Many of these caveats were already mentioned in the main analysis but are worth repeating. First, for the sake of comparison, I consider only one-time "fiscal impulses." In practice, fiscal policy packages are likely to be persistent and implemented over a certain horizon. As discussed, this is especially important for the case of liquidity assistance to service sector firms, and potentially for unemployment insurance. Second, these impulses are of a fixed size. In practice, size does matter and multipliers can be nonlinear (**?**). Third, I consider each policy separately, in single-instrument packages. There can be strong complementarities and substitutabilities between policies. In a previous paper (**?**), I argue that there were strong complementarities between financial sector bailouts and transfers to households during the 2008-09 Financial Crisis and subsequent recession. None of that is considered here. Fourth, the absence of an endogenous labor supply decision tends to underestimate the effects of an income tax cut and overestimate the effects of UI, as it does not consider the efficiency gains/losses from these policies. Fifth, the macroeconomic scenario caused by the pandemic is possibly too extreme, with a complete shutdown of the services sector for three full quarters and a GDP contraction of 15% per quarter. I completely abstract from the possibility that fiscal policy can be deployed to reduce the duration and intensity of the shock caused by the pandemic. Finally, I also abstract from the fact that stimulating economic activity may actually be *detrimental* in fighting the pandemic.

There are also other important caveats that were not previously discussed. There are implementation lags that can be made worse by attempts to better target policies. Better targeted policies may additionally entail extra costs associated with bureaucracy. It may sometimes be better to undertake a slightly worse policy whose implementation requires less information and time, i.e. unconditional transfers vs. expansion of unemployment insurance eligibility. Also, I completely abstract from other potential policies that have been part of the debate: the role of state fiscal policy, health insurance, debt forgiveness and restructuring, moratoria on debt (and bill) repayments, etc. For a detailed discussion of some of these policies, see **?**.

Household-banking interactions are extremely simplified and abstract from many important feedback effects. In particular, I abstract from endogenous collateral, which can have

a large effect on the consumption response to shocks and stimuli. As I show in previous research, many interventions that look like transfers to borrowers serve as implicit recapitalizations of the banking system and can have very strong spillovers to other sectors. For this reason, I am likely understating the effects of this type of interventions. Finally, I abstract from any direct intervention in the financial system. This is the main reason I abstract from unconventional monetary policy as well as the extraordinary measures taken by the Federal Reserve to restore confidence in financial markets. I also abstract from linkages between the financial system and the corporate sector. These are likely to be very important, especially at a time when corporate debt is at unprecedented levels in the US. This would be a natural first step in terms of extending the model.

A Full List of Equilibrium Conditions

Borrowers (λ_t is the Lagrange multiplier on the borrowing constraint),

$$\varepsilon_t^a = \frac{B_{t-1}^b}{\chi \Pi_t} - w_t(1 - \tau_t^l)$$

$$\varepsilon_t^n = \frac{B_{t-1}^b}{\chi \Pi_t} - w_t(1 - \tau_t^l)$$

$$\varepsilon_t^u = \frac{B_{t-1}^b}{\chi \Pi_t} - \mathrm{ui}_t$$

$$F_t^b = N_t^a F^e(\varepsilon_t^a) + N_t^n F^e(\varepsilon_t^n) + (1 - N_t^a - N_t^n)F^u(\varepsilon_t^u)$$

$$m_{t+1}^b \equiv \beta^b \frac{u'(C_{t+1}^b)}{u'(C_t^b)}$$

$$Q_t^b - \lambda_t = \mathbb{E}_t \frac{m_{t+1}^b}{\Pi_{t+1}}(1 - F_{t+1}^b)$$

$$C_t^b + \frac{B_{t-1}}{\chi \Pi_t}(1 - F_t^b) \le (N_t^a + N_t^n)w_t(1 - \tau_l) + (1 - N_t^a - N_t^n)\mathrm{ui}_t + Q_t^b B_t^b/\chi + T_t^b$$

$$Q_t^b B_t^b \chi \le \Gamma \perp \lambda_t \ge 0$$

Banks (μ_t is the Lagrange multiplier on the leverage constraint),

$$\mathbb{E}_t \frac{m_{t+1}^s}{\Pi_{t+1}}(1 - \theta + \theta\Phi_{t+1})\left[\frac{1 - F_{t+1}^b}{Q_t^b} - \frac{1}{Q_t}\right] = \mu_t \kappa$$

$$\mathbb{E}_t \frac{m_{t+1}^s}{\Pi_{t+1}}(1 - \theta + \theta\Phi_{t+1}) = \Phi_t(1 - \mu_t)Q_t$$

$$Q_t^b B_t^b = E_t + Q_t^d D_t$$

$$\kappa Q_t^b B_t^b \le \Phi_t E_t \perp \mu_t \ge 0$$

$$E_t = \Pi_t^{-1}\theta((1 - F_t^b)B_{t-1}^b - D_{t-1}) + \varpi$$

Savers,

$$m_{t+1}^s \equiv \beta^s \frac{u'(C_{t+1}^s)}{u'(C_t^s)}$$

$$Q_t = \mathbb{E}_t \frac{m_{t+1}^s}{\Pi_{t+1}}$$

$$C_t^a = \left[\alpha_t \frac{1}{p_t^a u'(C_t^s)} \right]^{1/\sigma_a}$$

Non-services sector,

$$\eta \frac{\Pi_t}{\Pi} \left(\frac{\Pi_t}{\Pi} - 1 \right) + \epsilon \left[\frac{\epsilon - 1}{\epsilon} - \frac{w_t^n}{A_t} \right] = \eta \mathbb{E}_t m_{t+1}^s \frac{Y_{t+1}^n}{Y_t^n} \frac{\Pi_{t+1}}{\Pi} \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right)$$

$$Y_t^n = A_t N_t^n \left[1 - 0.5\eta \left(\frac{\Pi_t}{\Pi} - 1 \right)^2 \right]$$

$$C_t = C_t^s + C_t^b$$

$$C_t + G_t + \Psi(\bar{c}_t) F_{t-1} = Y_t^n$$

$$w_t = \xi A_t (N_t^n + N_t^a)^\zeta$$

Services sector,

$$\bar{c}_t = A_t p_t^a - w_t^a + w_t^a T_t^a + \mathbb{E}_t m_{t+}^s \left[H_{t+1}^a \bar{c}_{t+1} - \Psi[\bar{c}_{t+1}] \right]$$

$$N_t^a = F_t$$

$$F_t = H_t^a F_{t-1} + \nu_t$$

$$\bar{c}_t \leq \kappa \nu_t^\psi \perp \nu_t \geq 0$$

$$H_t^a = \int_0^{\bar{c}_t} dH(c)$$

$$\Psi[\bar{c}_t] = \int_0^{\bar{c}_t} c\,dH(c)$$

$$(1-\chi)C_t^a = A_t N_t^a$$

Government and Central Bank

$$G_t + \frac{B^g_{t-1}}{\Pi_t} + (1 - N^a_t - N^n_t)\text{ui}_t + T^b_t + T^a_t w^a_t H^a_t$$

$$= (N^n_t + N^a_t)w_t\tau^l + \tau^k\left[Y^n_t(1 - d_t) - w_t N^n_t + (p^a_t - w_t)N^a_t - \Psi(\bar{c}_t)F_{t-1}\right] + Q_t B^g_t + T_t$$

$$T_t = \left(\frac{B^g_{t-1}}{\bar{B}^g}\right)^{\phi_\tau} - 1$$

$$\frac{1}{Q_t} = \max\left\{1, \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\Pi}\left(\frac{p^a_t}{p^a_{t-1}}\right)^{\phi_a}\left(\frac{GDP_t}{G\bar{D}P}\right)^{\phi_{GDP}}\right\}$$

$$GDP_t = Y^n_t + p^a_t Y^a_t$$