Evergreening

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Abstract

We develop a simple model of relationship lending where lenders have an incentive to evergreen loans by offering better terms to less productive and more indebted firms. We detect such lending distortions using loan-level supervisory data for the United States. Low-capitalized banks systematically distort their risk assessments of firms to window-dress their balance sheets and extend relatively more credit to underreported borrowers. Consistent with our theoretical predictions, these effects are driven by larger outstanding loans and low-productivity firms. We incorporate the theoretical mechanism into a dynamic heterogeneous-firm model to show that evergreening can affect aggregate outcomes, resulting in lower interest rates, higher levels of debt, and lower aggregate productivity.

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JEL Codes: E32, E43, E44, E52, E60, G21, G32

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"Owe your banker £1,000 and you are at his mercy; owe him £1 million and the position is reversed." — J. M. Keynes (1945)

1 Introduction

Following the outbreak of COVID-19 in early 2020, firm profits declined sharply and governments supported businesses through a number of programs that provided firms with subsidized credit. In the short-run, such interventions can stabilize the economy since they prevent firms from laying off workers and declaring bankruptcy, mitigating adverse aggregate demand externalities during a recession. However, in the medium-run, they may contribute to less productive firms being kept alive, potentially hindering efficient restructuring and depressing aggregate productivity. Related to these government programs, concerns emerged that banks would "evergreen" loans, with similar short-run benefits, but potentially leading to the creation of "zombie" firms and lowering economic growth after an immediate crisis passes (Peek and Rosengren, 2005; Caballero, Hoshi and Kashyap, 2008). However, at least in the United States, such worries were frequently dismissed on the basis that such evergreening is typically associated with economies experiencing depressions with severely undercapitalized banking systems—with Japan in the 1990s used as a prime example—and the U.S. economy was not thought to be in such a position (e.g., Gagnon, 2021).

To assess whether banks evergreen loans requires a general theory that formalizes such lending behavior. In this paper, we illustrate the economic mechanism that results in evergreening using a stylized model of bank lending. Equipped with this basic framework, we address the following questions. Instead of being specific to economies that resemble Japan in the 1990s, is evergreening in fact a general feature of financial intermediation? If so, can we find empirical evidence for such lending distortions even for the U.S. economy over recent years, when banks were operating with relatively high capital ratios? And lastly, what are the macroeconomic implications of evergreening for aggregate productivity and output?

To begin our analysis, we modify a benchmark model of bank-firm lending along two realistic dimensions. First, we assume that a bank owns a firm's legacy debt, resulting in bank losses in the case of firm default. Second, we posit that the bank has market power and internalizes how the offered lending terms influence a firm's decision to default and therefore the likelihood of repayment of existing liabilities. In the presence of such relationship banking and market power, typical lending incentives can be reversed. In contrast to standard intuition, lenders may offer *better* terms to less productive and more indebted firms. That is because such firms are closer to the default boundary. By offering more attractive conditions on a new loan contract, a bank can raise the continuation value of a firm, thereby reduce the likelihood of default, and increase the chance of repayment of existing debt. All else equal, larger outstanding debt raises the threat of default and improves a borrower's position vis-à-vis its lender, as captured by the quote above. Within our static framework, firms with "worse" fundamentals—more debt and lower productivity—pay lower interest rates and invest relatively more. As a result, these firms have lower marginal products of capital, leading to capital misallocation across firms. Importantly, our proposed mechanism is distinct from well-known corporate finance theories, such as risk-shifting or debt overhang, and does not hinge on lending frictions such as asymmetric information.

With these theoretical predictions, we turn to the data to test whether such lending behavior can be found in practice. To this end, we use the Federal Reserve's Y-14 data set that provides us with detailed loan-level information for the United States. For our analysis, we make use of the fact that the data include banks' risk assessments for each individual loan and that banks have an incentive to assess similar loans differently due to the regulator design. Specifically, we show that banks with low capital buffers systematically understate their credit risk exposure, confirming previous findings by Plosser and Santos (2018). Such "window-dressing" can arise because the loan risk assessments either directly or indirectly affect bank capital positions. In the cross-section of banks, low-capitalized banks therefore have a stronger incentive to lend more to their underreported borrowers to avoid further declines in their capital ratios and to reconcile their reporting. Using the approach by Khwaja and Mian (2008), we confirm such differential evergreening behavior across banks. However, we show that these results are only present for larger preexisting debt and for low productivity firms, confirming our theoretical mechanism which predicts that evergreening should occur in these instances. Illustrating the generality of the theoretical incentives, these effects are found even outside of a recession when U.S. banks were thought to be well capitalized, operating with relatively high capital ratios but smaller capital buffers above regulatory requirements.

Building on this empirical evidence, we embed the theoretical mechanism into a dynamic model to study the macroeconomic implications of evergreening. We augment the frameworks developed by Hopenhayn (1992), Hennessy and Whited (2005), and Gomes and Schmid (2010) with the relationship lending behavior that we describe in the static, two-period model. Unlike the static model, the dynamic one endogenizes the joint distribution of firm productivity, debt, and capital. Based on a calibration that targets moments related to U.S. firms, we show that evergreening is an equilibrium outcome that affects firm borrowing and investment decisions. In the aggregate, two forces largely work against each other. On the one hand, evergreening allows lenders to recover their investments more frequently and these benefits are passed on to borrowers through lower interest rates. In turn, incumbent firms increase their debt and investment. On the other hand, the firms that are saved and invest more are the ones that are less productive and they prevent new firms from entering. In turn, this results in a shift in the distribution of firm productivity with aggregate TFP losses of around 0.3% relative to an economy with competitive lenders. On net, the two forces—higher capital but lower TFP—largely offset each other, such that aggregate output remains similar in economies with or without relationship lenders. One important insight from our framework is that most evergreening is associated with riskier firms that are paying relatively high interest rates. This suggests that attempts to empirically identify zombies as the ones with funding costs below benchmark risk-free rates may underestimate the extent of this phenomenon.

Related Literature. Our paper relates to the literature on evergreening and zombie lending that emerged from Japan's "lost decade," which started with the collapse of stock and real estate markets in the early 1990s. For this period, Peek and Rosengren (2005) provide evidence of evergreening by showing that poor-performing firms typically experienced an increase in their credit. Lending surges were also associated with banks that were weakly capitalized or if banks and firms had strong corporate affiliations.[1] Similarly, Caballero, Hoshi and Kashyap (2008) document a rise in the share of zombie firms, which they define as businesses that pay interest rates below comparable prime rates. Consistent with a model of creative destruction, they show that in industries that experienced an increase in the share of zombie firms, job creation and destruction declined and productivity growth stalled. The presence of zombie firms also spilled over to other firms. In industries with a higher share of zombies, healthy firms experienced a fall in their investment and employment, while their productivity relative to zombies increased.

Building on these seminal contributions, a number of papers have documented similar evidence of evergreening and real economy effects of zombie firms subsequently.[2] These studies span several countries with varying economic conditions, but they generally share the findings that evergreening is more prevalent among weakly capitalized banks during severe recessions, and that zombie firms adversely impact healthy firms and impede firm exit and entry, hindering productivity growth within industries. Throughout, this literature faces two key identification challenges. First, identifying the credit supply effects of evergreening. Second, quantifying the spillover effects of zombie firms onto other firms and broader economic indicators.

We contribute to the literature by addressing these two challenges with the following two approaches. We isolate the credit supply effects with the described empirical strategy that exploits the regulatory environment in the United States.[3] Low capitalized banks have incentives to underreport their credit risk exposure and we use this setting to test for the existence of lending distortions.[4] Related to our empirical analysis, Blattner, Farinha and Rebelo (2020) use data from Portugal to show that low-capitalized banks extended relatively more credit to borrowers with underreported loan losses following an unexpected increase in capital requirements.

To assess the real effects of zombie lending, the common approach follows Caballero, Hoshi and Kashyap (2008) in first defining what a zombie firm is and, based on this definition, testing for spillover effects within industries or beyond. This approach isolates only extreme forms of evergreening by design—those that lead to the creation of zombie firms—and has led to a number of distinct zombie-firm definitions that may affect the identification of the spillover effects as pointed out by Schivardi, Sette and Tabellini (2020).[5] Given these empirical challenges, we depart from the

[1] Within the bank, loan officers may engage in evergreening if they face a lower likelihood of being exposed (e.g., Hertzberg, Liberti and Paravisini, 2010). Related to this explanation, banks are found to reduce zombie-lending after on-site inspections (e.g., Bonfim et al., 2020; Angelini et al., 2021).

[2] Among others, examples are Giannetti and Simonov (2013), Storz et al. (2017), McGowan, Andrews and Millot (2018), Banerjee and Hofmann (2018), Acharya et al. (2019), Andrews and Petroulakis (2019), Acharya et al. (2020), Acharya et al. (2021), Bittner, Fecht and Georg (2021), Blattner, Farinha and Rebelo (2020), Chari, Jain and Kulkarni (2021) and Schivardi, Sette and Tabellini (2021).

[3] In this regard, we connect to an extensive body of work that measures how bank health affects the allocation of firm credit (e.g., Khwaja and Mian, 2008) and firm outcomes (e.g., Chodorow-Reich, 2014). Related to our application, Berrospide and Edge (2019), Favara, Ivanov and Rezende (2021), and Ma, Paligorova and Peydro (2021) have used the Y-14 data in this context to investigate the effects of bank capitalization and lender expectations.

[4] Underreporting of risk has been found for various bank assets and to be linked to bank capital positions in a number of circumstances (see, e.g., Behn, Haselmann and Vig, 2016; Begley, Purnanandam and Zheng, 2017; Plosser and Santos, 2018; and Behn et al., 2019).

[5] For example, zombie firms have been defined according to their interest expenses, profitability, age, investment rates, leverage, ratings, and often based on a combination of several measures (see, e.g., Caballero, Hoshi and Kashyap,

common practice and take a theoretical approach instead, embedding our mechanism into a dynamic model which allows us to investigate the spillover effects and also study the aggregate implications of evergreening.

The theoretical mechanism is distinct from existing models of zombie-lending, though it shares similarities with some mechanisms in distantly related literatures. Thus far, relatively few papers formalize the idea of zombie-lending theoretically. Previous theories have relied on information asymmetries (Rajan, 1994; Puri, 1999; Hu and Varas, 2021) or on the premise that banks gamble for resurrection or evergreen loans to meet regulatory limits (Bruche and Llobet, 2013; Acharya, Lenzu and Wang, 2021). In contrast, our model assumes full information and excludes the possibility of bank default and regulation. Related to our dynamic model, Tracey (2021) considers a heterogeneous-firm setting, in which firms have the option to enter a loan forbearance state, which results in a larger number of less productive firms in equilibrium compared with an economy without this option. In contrast, in our model, lenders choose to offer better loan conditions to less productive firms to keep them alive, recover their outstanding debt, firms do not enter explicit states of bankruptcy or restructuring to be subsidized by the lender.

The mechanism is also different from the classic problem of debt overhang (Myers, 1977). This theory posits that equity holders are reluctant to invest in profitable investment projects since the benefits could be reaped by existing debt holders, hindering further borrowing. In our static framework, more indebted firms receive better loan conditions, enabling them to borrow and invest relatively more, yielding strikingly different predictions than the debt overhang theory. Similar to the prediction of our model, less competition among banks is found to be related with fewer firms that are on average larger (e.g., Cetorelli and Strahan, 2006) and a higher indebtness of banks to certain industries is associated with stronger incentives to provide credit in times of distress (e.g., Giannetti and Saidi, 2019). In the sovereign debt literature, similar mechanisms illustrate comparable results as our model, showing that more indebted governments are able to obtain more favorable conditions once they restructure their debt (e.g., Dvorkin et al., 2021).

Last, our paper relates to an extensive literature that studies factor misallocation (e.g., Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009). For Spain around the early 2000s, Gopinath et al. (2017) show that the dispersion of the return to capital increased, at the same time as real interest rates declined and aggregate productivity growth stalled.[6] Using a heterogeneous firm model, they show that these facts can be explained by a misallocation of capital inflows towards less productive firms. Our model shares the feature that lower interest rates lead to an increase of the capital stock of less productive firms. However, such a decline in interest rates is the result of evergreening in our framework and is constrained to the set of indebted and less productive firms.

Overview. The next section illustrates the economic mechanism of evergreening using a static two-period model. Section 3 contains the empirical analysis and provides evidence for the mecha-

2008; Storz et al., 2017; McGowan, Andrews and Millot, 2018; Banerjee and Hofmann, 2018; Acharya et al., 2019; Acharya et al., 2020; and Schivardi, Sette and Tabellini, 2021).

[6]The connection between the secular decline in interest rates and aggregate productivity and output has also recently been studied by Liu, Mian and Sufi (2021), Asriyan et al. (2021), and López-Salido, Goldberg and Chikis (2021).

nism. Section 4 embeds the static two-period framework to a dynamic infinite-horizon model and studies the macroeconomic consequences of evergreening. Section 5 concludes.

2 Static Model

In this section, we develop a simple model of bank-firm lending. We begin by presenting the problem of a firm that decides how much to borrow and invest, taking the interest rate on new credit as given. The firm has some pre-existing liabilities and may decide to default on its outstanding debt instead of investing and producing. Given the firm's problem, we compare the equilibrium outcomes of two economies: (i) one with competitive lenders and (ii) an economy with relationship banking. The latter features a single lender that owns the firm's outstanding debt and internalizes how the offered lending terms affect the firm's decision to default on its legacy debt. In equilibrium, the relationship lender may offer better terms to firms that are more indebted and less productive to save these firms from defaulting and thereby recover its previous investment. However, the dispersion in lending conditions also leads to differences in marginal products of capital across firms, and thus capital misallocation.

Environment. Time is discrete and finite with two periods $t = 0, 1$. The economy features two types of agents: firms, which are indexed by their pre-determined states (b, z), where b are pre-existing liabilities and z is productivity, and lenders, who are risk-neutral and have deep pockets. In the competitive lending economy, there is a continuum of lenders for each firm. In the relationship banking economy, each firm borrows from a single lender.

2.1 Firm Problem

At the beginning of the first period $t = 0$, the firm may choose to default. If the firm defaults, it obtains a zero value. If it remains in business, the firm has a continuation value equal to $V(z, b; Q)$, which is a function of the legacy debt b, productivity z, and the price of new debt Q that is offered by the lender at $t = 0$, and which the firm takes as a given. The firm therefore defaults if and only if $V(z, b; Q) < 0$. For simplicity, we assume that there is no default at $t = 1$.[7]

If the firm does not default, it has to repay its existing liabilities b, borrows Qb', and invests k' at $t = 0$. At $t = 1$, the firm produces according to a decreasing returns to scale technology $z(k')^\alpha$, where $\alpha \in (0, 1)$, and repays debt b' borrowed at $t = 0$. Additionally, the firm faces a borrowing constraint at $t = 0$ that takes the form $b' \leq \theta k'$, where $\theta > 0$.[8] The firm's value, conditional on not

[7]Given this assumption, lenders price the new debt as if they would always be fully repaid. No default is an equilibrium outcome under the additional restriction that $Q \leq 1/\theta + \beta^f(1-\alpha)$.

[8]Our results hold for other specifications of the borrowing constraint, such as $b' \leq z(k')^\alpha$, which guarantees no default in the second period, or a constraint that includes the price of debt, $Qb' \leq \theta k'$. Appendix A.1 discusses general constraints of the type $b' \leq g(k')$, with g positive and increasing, for which we can still prove our main results.

defaulting, is then given by

$$V(z,b;Q) = \max_{b',k' \geq 0} -b - k' + Qb' + \beta^f [z(k')^\alpha - b'] \tag{2.1}$$
$$\text{s.t.}$$
$$b' \leq \theta k' \quad ,$$

where β^f is the firm's discount factor.[9] The firm's first-order condition (FOC) with respect to b' is simply

$$Q - \beta^f - \lambda \leq 0 \quad ,$$

where $\lambda \geq 0$ is the Lagrange multiplier on the borrowing constraint. Clearly, the constraint binds as long as $Q \geq \beta^f$, implying that $\lambda = Q - \beta^f$. We assume that $\lambda > 0$ for now, and later impose restrictions on the model's parameters to ensure that this is the case. The FOC for capital investment is

$$-1 + \beta^f z\alpha(k')^{\alpha-1} + \lambda\theta \leq 0 \quad .$$

Substituting in $\lambda = Q - \beta^f$, we obtain a closed-form expression for the optimal capital stock,

$$k'(z;Q) = \left(\frac{\beta^f \alpha z}{1 - \theta(Q - \beta^f)}\right)^{\frac{1}{1-\alpha}} \quad . \tag{2.2}$$

With a binding borrowing constraint, the optimal level of new debt is

$$b'(z;Q) = \theta k'(z;Q) = \theta \left(\frac{\beta^f \alpha z}{1 - \theta(Q - \beta^f)}\right)^{\frac{1}{1-\alpha}} \quad , \tag{2.3}$$

and, finally, the value function can be written in closed-form

$$V(z,b;Q) = -b + \left(\frac{1}{\alpha} - 1\right) \frac{(\beta^f \alpha z)^{\frac{1}{1-\alpha}}}{[1 - \theta(Q - \beta^f)]^{\frac{\alpha}{1-\alpha}}} \quad . \tag{2.4}$$

This characterizes the firm's problem for an arbitrary price of debt Q, which is taken as given. We restrict $Q \leq \beta^f + \frac{1}{\theta}$ to ensure that policy and value functions are well-defined and later confirm that this restriction is satisfied in equilibrium. From equations 2.2-2.4, it is easy to see that the firm's policies and value are all strictly increasing in productivity z and the price of debt Q. Additionally, firm value is strictly decreasing in the amount of legacy debt b. The fact that firm value is strictly increasing in Q also allows us to characterize the firm's default decision. In particular, we can show that there exists $Q^{\min}(z,b)$ such that the firm chooses to default if it is offered a Q that is lower than this threshold, as illustrated with the following proposition.

[9] We assume that the firm owns no pre-existing stock of capital that would allow it to produce at $t = 0$ and faces no costs of issuing equity. These assumptions are made without loss of generality, and to keep the framework as simple as possible. Pre-existing capital and production in the first period are equivalent to re-scaling the net liabilities b, and therefore do not change our results. Adding a linear equity issuance cost also rescales/increases net liabilities in the first period and introduces an additional investment distortion (as the marginal cost of investment rises), but does not affect our results.

Proposition 1 *There exists a $Q^{\min}(z,b)$ such that the firm defaults if and only if $Q < Q^{\min}(z,b)$. The threshold is given by*

$$Q^{\min}(z,b) = \beta^f + \frac{1}{\theta} - \frac{(\beta^f \alpha z)^{\frac{1}{\alpha}}}{\theta} \left(\frac{1-\alpha}{\alpha b}\right)^{\frac{1-\alpha}{\alpha}} \quad . \tag{2.5}$$

The threshold is:

1. *Strictly increasing in b*
2. *Strictly decreasing in z*
3. *Satisfies* $\lim_{b\to\infty} Q^{\min}(z,b) = \beta^f + \frac{1}{\theta}$

The last condition ensures that $Q^{\min} < \beta^f + \frac{1}{\theta}$ for finite values of b. Equipped with the solution to the firm's problem for a given price of debt Q, we now proceed to study two different forms of determining Q and characterize the equilibria that result from each of them.

2.2 Competitive Lending

In the first economy that we consider, there is a continuum of lenders that are willing to lend to the firm. These lenders are risk-neutral, have unlimited resources, and discount payoffs with factor $\beta^k > \beta^f$, as in Kiyotaki and Moore (1997). Additionally, we also assume that $\beta^k < \beta^f + \frac{1}{\theta}$, as otherwise there is never default at $t = 0$ for $b < \infty$. Since we assume that there is no default at $t = 1$, perfect competition in the lending market imposes that the offered contract, conditional on no default at $t = 0$, satisfies

$$Q = \begin{cases} \beta^k & \text{if } \beta^k \geq Q^{\min}(z,b) \\ 0 & \text{otherwise} \end{cases} \quad . \tag{2.6}$$

The equilibrium allocation is then obtained by evaluating 2.2, 2.3, and 2.4 at $Q = \beta^k$.[10] In particular, the firm's FOC for capital can be rewritten as

$$z\alpha(k')^{\alpha-1} \equiv MPK = \frac{1-\theta(Q-\beta^f)}{\beta^f} = \frac{1-\theta(\beta^k-\beta^f)}{\beta^f} \quad \forall (z,b) \quad ,$$

which implies that all non-defaulting firms borrow at the same interest rate, regardless of their initial states (z,b). Marginal products of capital (MPKs) are therefore equalized across all surviving firms. Hence, there is no misallocation in this economy, as one could not redistribute capital from one firm to another and thereby increase overall output.

To illustrate the equilibrium outcomes, we use a standard parametrization and plot policies and prices in Figure 2.1, as a function of the pre-existing liability b and for different levels of productivity z.[11] The bottom left panel shows that all firms borrow at the same price of new debt Q, up to the point where b becomes sufficiently large and the firm decides to default instead.

[10] Since $\beta^k = Q > \beta^f$, our conjecture that the constraint is always binding is confirmed.
[11] All plots are based on the model parametrization described in Appendix A.2.

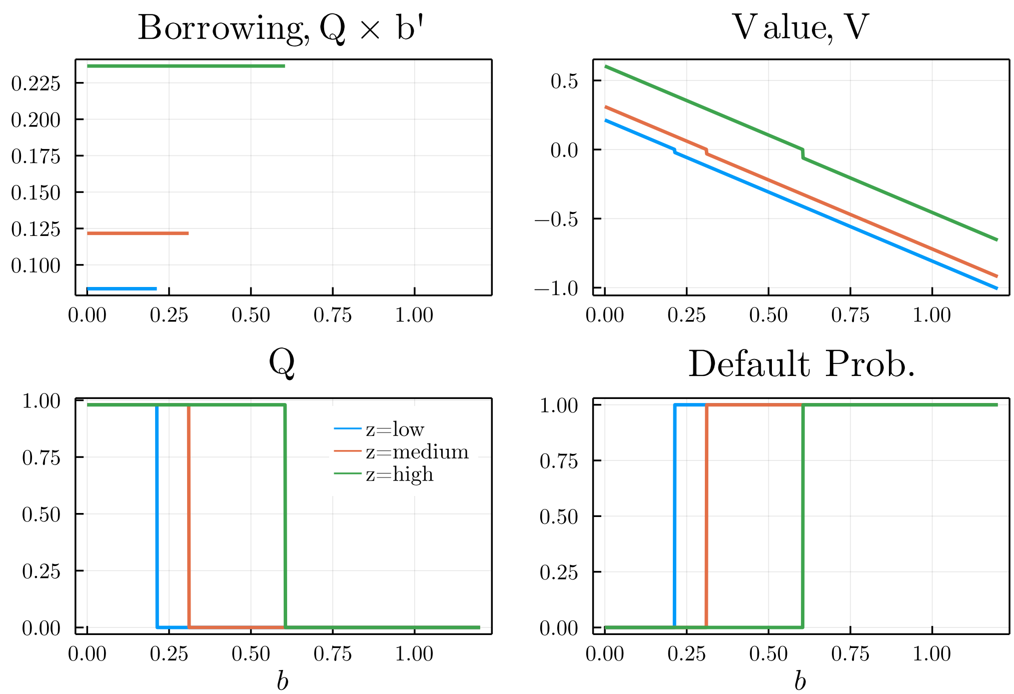

Figure 2.1: **Competitive Lending Economy.** Equilibrium policies and prices for the competitive lending economy as a function of b, for different levels of z.

Default occurs at lower values of b for less productive firms, as visible form the shape of the $Q^{\min}(z,b)$ function. The top left panel shows that more productive firms also borrow and invest more, as MPKs are equalized. Furthermore, investment and borrowing policies, as well as prices, are independent of b up to default.

2.3 Relationship Lending

We now proceed to analyze the equilibrium under a different institutional setting that resembles relationship banking. Compared with the competitive lending economy, there are two key differences. First, the lender has market power, and behaves like a Stackelberg leader, internalizing how its choice of Q affects the firm's policies and values $(b', k', V)(z, b; Q)$. Second, lending is non-anonymous in the sense that the lender owns the pre-existing debt b and understands that this debt is lost in the case of default. In the context of relationship lending, we use the terms "lender" and "bank" interchangeably. The lender's problem is now given by

$$W = \max_{Q \geq \beta^k} \mathbb{I}[V(z, b; Q) \geq 0] \times \left[b - Qb'(z; Q) + \beta^k b'(z; Q)\right] \quad , \tag{2.7}$$

where $\mathbb{I}$ is the indicator function. If the firm defaults at $t = 0$, the lender makes zero profits. Otherwise, the lender recovers b, lends Qb', and obtains b' at $t = 1$, which is discounted at the factor β^k. Finally, the lender's choice of Q is constrained to be above β^k, as we assume that the firm may access a competitive debt market as the one previously described if the lender tries to offer terms that are worse than those. Note that we can equivalently write the bank's problem as

$$W = \max_{Q \geq \max\{\beta^k, Q^{\min}(z,b)\}} \left[b + b'(z;Q)(\beta^k - Q) \right] \quad . \tag{2.8}$$

From this formulation, and the fact that $\partial b'(z;Q)/\partial Q > 0$, it is evident that the bank's objective function is strictly decreasing in Q (subject to the constraint on Q). For this reason, it is optimal for the bank to offer the lowest possible Q as long as $W \geq 0$. The next propositions characterize the bank's optimal lending policy.

Proposition 2 *Let $Q^{\max}(z,b)$ denote the maximum Q at which the bank is willing to lend,*

$$Q^{\max}(z,b) : W(z,b;Q^{\max}) = 0 \tag{2.9}$$

$Q^{\max}(z,b)$ solves the implicit equation

$$b + [\beta^k - Q^{\max}(z,b)]\theta \left(\frac{\beta^f \alpha z}{1 - \theta(Q^{\max}(z,b) - \beta^f)} \right)^{\frac{1}{1-\alpha}} = 0 \tag{2.10}$$

and satisfies the properties:

1. *$Q^{\max}(z,b) > \beta^k$ iff $b > 0$*
2. *It is increasing in b*
3. *It is decreasing in z*

Proposition 3 *The bank's optimal policy can be written as*

$$Q^*(b,z) = \begin{cases} \beta^k & \text{if } Q^{\min}(z,b) \leq \beta^k \leq Q^{\max}(z,b) \\ Q^{\min}(z,b) & \text{if } \beta^k \leq Q^{\min}(z,b) \leq Q^{\max}(z,b) \\ 0 & \text{otherwise} \end{cases} \tag{2.11}$$

Let $\bar{b}(z)$ be such that $Q^{\min}(\bar{b}(z),z) = \beta^k$ and $\hat{b}(z)$ such that $Q^{\min}(\hat{b}(z),z) = Q^{\max}(\hat{b}(z),z)$, with closed-form expressions given by

$$\bar{b}(z) = \frac{1-\alpha}{\alpha} \left[\frac{\alpha \beta^f z}{(1 - \theta(\beta^k - \beta^f))^{\alpha}} \right]^{\frac{1}{1-\alpha}}$$
$$\hat{b}(z) = (1-\alpha) \left[\frac{\beta^f z}{(1 - \theta(\beta^k - \beta^f))^{\alpha}} \right]^{\frac{1}{1-\alpha}}$$

then:

1. *$\bar{b}(z) < \hat{b}(z), \forall z$*
2. *$Q^*(b,z)$ is increasing in b, strictly if $b \in [\bar{b}(z), \hat{b}(z)]$*

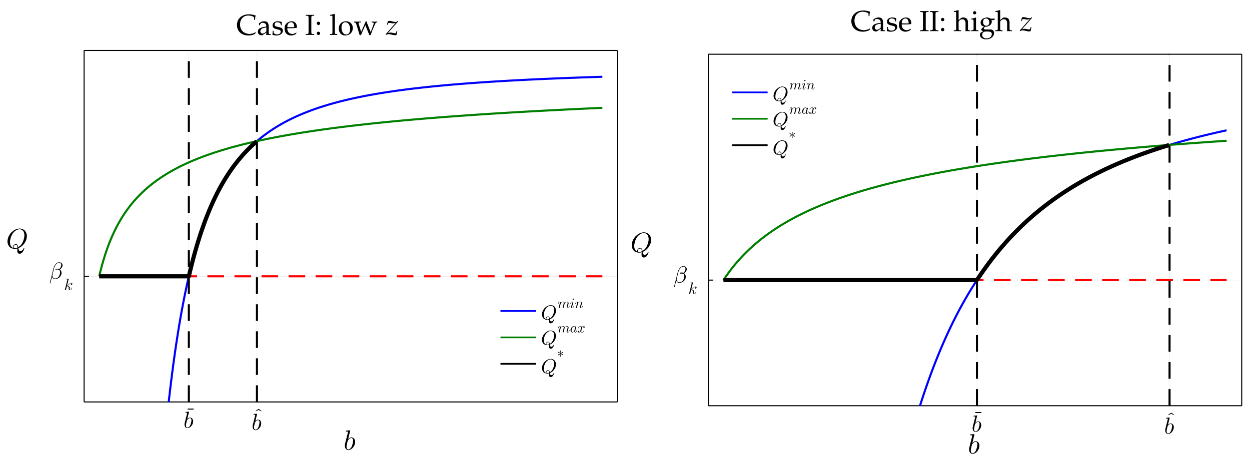

Figure 2.2: **Relationship Lending Economy.** Equilibrium allocation as a function of b, for low z (left) and high z (right). The solid blue line is $Q^{\min}(z, b)$, the solid green line is $Q^{\max}(z, b)$, the dashed red line is β^k, and the thick black line is the bank's chosen policy Q^*.

3. *$Q^*(b, z)$ is decreasing in z, strictly if $b \in [\bar{b}(z), \hat{b}(z)]$*

Proposition 3 states that as long as the legacy debt is nonzero, $b > 0$, the bank is willing to offer terms that are better than those in the competitive market to the firm. Offering more favorable lending conditions allows the bank to recover b by preventing the firm from defaulting. The optimal price of debt Q^* consists of three regions. First, as long as $Q^{\min}(z, b) < \beta^k$, the bank can offer $Q^* = \beta^k$ and guarantee that the firm does not default. In this case, the allocation in the relationship economy coincides with the competitive lending economy. Second, Proposition 1 states that $Q^{\min}(z, b)$ is increasing in b and decreasing in z. Therefore, for sufficiently high b or low z, $Q^{\min}(z, b)$ exceeds β^k. If that is the case, the firm would simply exit in the competitive economy. In the relationship economy, however, and as long as $Q^{\min}(z, b) < Q^{\max}(z, b)$, the bank is willing to keep the firm alive by offering $Q^* = Q^{\min}(z, b) > \beta^k$. These terms are strictly better than those that the firm could obtain in the competitive market, and become more favorable as b increases or z falls. In the third region, $Q^{\min}(z, b)$ exceeds the maximum price that the bank is willing to offer to break-even, and the bank decides to simply liquidate the firm.[12]

These three regions are shown in Figure 2.2, for a low productivity level on the left, and a high productivity level on the right. Comparing the two panels illustrates that the intermediate "evergreening region" with $Q^{\min}(z, b) > \beta^k$ starts at a higher level of b if productivity is higher. Furthermore, conditional on the same level of legacy debt b, the amount of surplus that the bank needs to transfer to the firm to prevent it from defaulting is lower the higher z, since firm value is increasing in productivity.

Misallocation. Recall that the firm's FOC implies that

$$z\alpha(k')^{\alpha-1} \equiv MPK = \frac{1 - \theta[Q^*(z, b) - \beta^f]}{\beta^f} \quad .$$

[12]Note that this confirms our conjecture that $Q^*(b, z) < \beta^f + 1/\theta$.

The results in Proposition 3 establish that $Q^*(b, z)$ is weakly increasing in b and decreasing in z. More indebted firms and less productive firms are therefore offered better lending terms $Q^*(z, b)$ and choose larger levels of capital, implying lower MPKs. Unlike the competitive lending case, where MPKs are equalized across all surviving firms, the relationship lending economy features MPK dispersion, with more capital flowing to firms that are more indebted and less productive. Thus, one could increase overall output by simply redistributing capital across firms.

2.4 Discussion

The two-period model isolates potential advantages and disadvantages of evergreening. On the one hand, evergreening saves firms with too much debt, but otherwise viable investment projects that have a positive net present value (NPV) and generate additional production. On the other hand, low-productive firms are kept alive, MPKs differ across firms, and capital could therefore be reallocated to increase overall output. However, the static model also leaves several questions unanswered. First and foremost, does the mechanism reflect well how banks make lending decisions in practice? We address this question in the next section using detailed loan-level data.

But beyond the empirical relevance, the static model falls short in assessing the broader macroeconomic consequences. Thus far, we assumed that firms start with certain levels of debt and productivity, while setting the amount of pre-existing capital to zero. But how often do firms actually end up with levels of debt, capital, and productivity that give rise to evergreening? Do firms potentially acquire more debt today if they know that they could be saved tomorrow, a form of moral hazard? And last, by saving some firms, are more-productive ones kept out of the economy? To answer these questions, a macroeconomic framework is needed that allows for endogenous firm entry and exit, aggregation across firms, and a counterfactual analysis between relationship and competitive economies. We propose such a model in Section 4.

Before proceeding with the empirical analysis, we compare our mechanism to some well-known corporate finance theories and discuss the contracting protocol in our benchmark model.

2.5 Relation to Existing Corporate Finance Theories

Our proposed mechanism is distinct from phenomena such as risk-shifting or debt overhang. According to the risk-shifting or gamble for resurrection theory, distressed borrowers have an incentive to invest in risk-increasing negative NPV projects under limited liability (e.g., Jensen and Meckling, 1976). That is because they can reap the benefits if the investments go well, but creditors bear the costs otherwise. In contrast, in our framework, banks do not borrow, and firms do not default *following* their investments, preventing such risk-shifting to occur.

According to the debt overhang theory, highly indebted borrowers underinvest since the potential profits would largely accrue to the current creditors, hindering further borrowing (e.g., Myers, 1977). The debt overhang theory relies on the timing that the outstanding (long-term) debt matures *after* the investment decision takes place. In contrast, in our framework, the timing of these decisions is reversed, legacy debt is short-term, and highly indebted firms "overinvest," in the sense that their MPKs are lower than the ones of less indebted firms.

2.6 Contracting Protocol

Our benchmark model assumes a specific contracting protocol that is based on a Stackelberg game. The relationship lender is the leader (offering Q) and the firm is the follower (choosing b', k' based on the offered Q). One could think of alternative arrangements, where the lender sets both the price Q and the quantity of debt b' in a take-it-or-leave-it offer. Appendix A.3 derives the solution to such a contracting protocol. In this case, the lender implicitly chooses the firm's investment which is linked to credit quantities via the firm's borrowing constraint, while extracting the maximum surplus from the firm by setting its continuation value to zero. The solution to such a protocol is therefore equivalent to a scenario where the lender owns the firm, and we show that such an arrangement eliminates misallocation across firms. Intuitively, the lender restricts the quantity of credit to less productive firms, while offering a more favorable price of debt to raise a firm's continuation value, just enough to prevent it from defaulting. In contrast to this prediction, we show in the next section that evergreening is characterized by more favorable lending conditions both with respect to credit quantities and interest rates in the data. We therefore view our benchmark model as the empirically relevant case.

3 Empirical Analysis

3.1 Identification Approach

In this section, we investigate whether the described mechanism plays a role in banks' lending behavior. Identifying such lending practices empirically poses some well-known, as well as some more subtle identification challenges. To begin with, we are interested in isolating the credit supply effects of evergreening. To this end, we apply the common approach of using a "credit registry" and a sample of firms that borrow from multiple banks, which allows us to control for a firm's common credit demand across lenders using firm-time fixed effects (Khwaja and Mian, 2008). To test our theory, we could distinguish banks according to the share of a firm's debt that they hold, with more indebted banks having a stronger incentive to evergreen loans. However, differentiating bank-firm pairs by their ex-ante lending intensity would likely violate the common credit demand assumption of the Khwaja and Mian (2008) approach as firms' credit demand plausibly depends on how much they have borrowed from lenders in the past.[13] Further, the effects of any firm-specific characteristics, like measures of firm productivity, could not be estimated separately in the presence of firm-time fixed effects, rendering any empirical design that builds on such differences infeasible to test our theory.

We therefore pursue a different empirical strategy. Due to the regulatory environment in the United States, the incentives to report their credit risk exposure correctly differs in the cross-section of banks. We document that low capitalized banks systematically underreport their risk exposure relative to better capitalized banks for similar loans to the same firm. In turn, this creates

[13] Similarly, regressions that additionally allow for the estimated relations to change over time (e.g., crisis vs. normal times) are only valid if the firms' credit demand does not shift over the same time periods. For example, if firms adjust their credit demand during a crisis towards their relationship lender from which they have borrowed more in the past, then such regressions would not be able to credibly distinguish between credit demand and supply.

heterogeneity in the incentives to evergreen loans in the cross-section of banks. We test whether the differential risk assessments influence lending decisions, and if so, whether the patterns match the predictions our theory. We find that low capitalized banks lend relatively more to underreported borrowers to avoid potential losses and to reconcile their reporting. This lending behavior is only observed for the subset of firms with lower productivity and larger legacy debt, which is consistent with the predictions from our theoretical model that evergreening should occur for low productivity and indebted firms. However, it is worth stressing that neither our static nor our dynamic model explicitly include bank capital or risk reporting, but distinguishing banks along these dimensions in the data helps us to isolate a set of bank-firm pairs, for which the incentives to evergreen loans are stronger than for others, and to use this cross-sectional heterogeneity to test our theory empirically.

3.2 Data

The main data set of our analysis is the corporate loan schedule H.1 of the Federal Reserve's Y-14Q collection (Y14 for short). These data were introduced as part of the Dodd-Frank Act following the 2007-09 financial crisis. They are typically used for stress-testing and cover large bank holding companies (BHCs).[14] For the BHCs within our sample, the data contain quarterly updates on the universe of loan facilities with commitments in excess of $1 million and include detailed information about the credit arrangements. Important for our analysis, the data cover risk assessments for each individual loan, allowing us to compare evaluations for the same borrower across banks, as explained in the next section.

We identify a firm using the Taxpayer Identification Number (TIN). The vast majority of firms within our data are private ones. For these firms, we rely on the banks' own collections of firm balance sheet and income statements that are also part of the Y14 data. To reduce measurement error and to increase the number of observations, we take the median of firm financial variables across all banks and loans for a particular firm-date observation since these data are firm-specific. For the public firms, we instead use information from Compustat on firm financials. We further apply several sample restrictions. First, we exclude lending to financial and real estate firms. Second, we restrict the start of the sample to 2012:Q3 to allow for a short phase-in period for the structure of the collection and variables to stabilize, though most of our analysis is constrained to begin in 2014:Q4, when loan risk assessments were required for all banks. We include information up until 2020:Q4. Over this sample, we cover 4,904,321 loan facility observations, 216,661 distinct firms, and we identify 3,217 of those as public ones, since they can be matched to Compustat. Last, we apply a number of filtering steps that are described in Appendix B, which also includes an overview of the variables that are used from the various sources.

[14] Until 2019, BHCs with more than $50 billion in assets were required to participate in the collection, and the size threshold was changed to $100 billion subsequently.

3.3 Risk-Reporting and Bank Capital

For each loan, banks have to report several risk measures: the probability of default (PD), a loan rating, the loss given default, and the exposure at default. Among those, we use the PD for our analysis, which measures whether a loan is non-performing over the course of the next year, that is, it is not repaid in full or the borrower is sufficiently late on its payments.[15] In contrast to the other risk measures, the PD has the advantage that it is a continuous measure and approximately borrower- rather than loan-specific.[16] That is, a borrower is typically late on several outstanding payments or defaults on a number of loans at the same time. In support of this approximation, Appendix Figure C.1 shows that individual banks assign virtually the same PD across multiple loans to the same firm, even if those loans have distinct characteristics. In contrast, there is substantial dispersion of PDs across banks, even when considering loans with similar characteristics to the same firm.

To understand the origin of this dispersion across banks, we conduct a similar analysis as Plosser and Santos (2018). Weighted by all outstanding loans, we denote the probability of default that bank j reports for firm i at time t by $PD_{i,j,t}$. To compare risk-reporting across banks, we further define the difference between this variable and the average reported PD by all other banks as PD-Gap$_{i,j,t} = PD_{i,j,t} - \overline{PD}_{i,t}$ where $\overline{PD}_{i,t} = (1/M)\sum_m^M PD_{i,m,t}$ for all $m \neq j$. In practice, there are many reasons why banks differ in their risk assessments. For example, some banks may possess private information about a borrower, resulting in a more accurate and potentially different forecast relative to other banks. To assess whether bank capital positions can explain the dispersion across banks, we estimate different versions of the regression

$$PD_{i,j,t} = \beta Capital_{j,t-1} + \gamma X_{j,t-1} + \alpha_{i,t} + \kappa_j + u_{i,j,t} \qquad , \tag{3.1}$$

where either PD$_{i,j,t}$ or PD-Gap$_{i,j,t}$ is used as a dependent variable, $X_{j,t-1}$ is a vector of bank characteristics, $\alpha_{i,t}$ is a firm-time fixed effect, and κ_j is a bank fixed effect. The variable of interest is $Capital_{j,t-1}$ and we use the buffer over the common equity Tier 1 (CET1) requirement to measure bank capital positions.[17]

Before estimating the regression, it is useful to consider various explanations for different values of β. First, assume that some banks possess private information and therefore have more

[15] According to the Basel Committee, a loan is in default, if either one or both of the following events have taken place: (1) the bank considers that the obligor is unlikely to pay its credit obligations to the banking group in full, without recourse by the bank to actions such as realizing security (if held); and (2) the obligor is past due more than 90 days on any material credit obligation to the banking group. Source: https://www.bis.org/basel_framework/chapter/CRE/36.htm

[16] Our data does not cover information on the debt seniority and we might therefore compare loans with different seniority levels to the same borrower. However, that is unlikely to affect our results for two reasons. First, the PD only captures the likelihood that a borrower is late on payments or does not repay the loan in full without seizing collateral, and both events are likely similar across loans with different seniority. Second, the debt seniority would have to correlate with the bank capital positions over time to affect our regressions (3.1) and (3.2).

[17] Throughout our analysis, we use the CET1 buffer since CET1 is the most "costly" type of capital for banks. It covers common stock, stock surplus, retained earnings, minority interest, and accumulated other comprehensive income. We define the capital buffer as the difference between the capital ratio and the required capital, consisting of a minimum and a capital conservation buffer requirement (GSIB surcharge + stress capital buffer + countercyclical capital buffer). In addition to the CET1 requirement, banks face requirements on their Tier 1 and their total capital.

accurate forecasts than others. All else equal, such an explanation should not result in a systematic relation between bank capital and reported PDs but rather yield $\beta \approx 0$. Second, assume that a bank has downward-biased PDs. If that bank's risk-weighted assets (RWAs) are computed according to the internal ratings-based approach (IRB), then such a bank would assign relatively lower risk-weights and therefore lower RWAs. The ratio of capital-to-RWAs should therefore be higher, resulting in $\beta < 0$. Similarly, imagine that a bank learns that its loan portfolio is riskier than previously anticipated. This should raise PDs, risk-weights, and RWAs, and therefore lower the ratio of capital to RWAs, again giving $\beta < 0$. And third, there are two relevant explanations that can instead result in $\beta > 0$. Assume that a bank's overall risk-perception is low or its risk-taking is high. Such a bank may assign low PDs but also operate with a high leverage (or low capital buffers). Similarly, if banks specialize in risky lending, they may assign high PDs but also operate with high capital buffers to support potential losses. To account for this "business-model" explanation, we include bank fixed effects and total portfolio risk variables into our regressions, controlling for time-invariant and time-varying factors, respectively.

The final explanation for why we should find $\beta > 0$ is that low-capitalized banks systematically underreport their credit risk exposure due to regulatory incentives. In the United States, banks may have such incentives for the following three reasons. First, around half of the banks in our sample were subject to the IRB approach, which allows banks to use their own risk measures to compute loan-specific risk weights.[18] The PDs that we use directly enter those calculations, and banks with low capital buffers may underreport PDs to avoid further declines in their capital ratios and potential penalties for violating capital requirements.[19] Second, the Federal Reserve's stress tests also make use of the banks' own risk measures.[20] Institutions with low capital buffers may therefore have an incentive to underreport their credit risk exposure to increase the chance of passing the tests. And third, low-capitalized banks attract supervisory attention and may therefore window-dress their balance sheets to avoid further regulatory scrutiny (e.g., through on-site inspections). With these explanations in mind, we expect to find that $\beta \leq 0$ when accounting for the business-model explanation and absent any regulatory incentives.

Table 3.1 reports the estimation results for various setups of regression (3.1). Columns (i) and (ii) use $\text{PD}_{i,j,t}$ as a dependent variable, whereas columns (iii) and (iv) show the results for

[18]According to the advanced IRB approach, banks' own risk measures determine risk weights (PD, exposure at default, loss given default, expected credit loss, and loan maturities). Pre-2020, banks with >\$250 billion assets or >\$10 billion in foreign exposure were required to use the advanced IRB approach. Post-2020, the requirement changed to cover all GSIBs or firms with >\$700 billion assets or >\$75 billion cross-jurisdictional activity. In the United States, banks that are subject to the advanced IRB approach also have to compute their capital ratios based on the standardized approach and must comply with the capital requirements under both approaches. Source: https://www.federalreserve.gov/aboutthefed/boardmeetings/files/board-memo-20181031.pdf

[19]When bridging the capital conservation buffer requirement, banks may face limitations such as restrictions on dividend payouts, retained earnings, and share buybacks. When violating the minimum requirement, regulators may, for example, force a bank to issue capital or restrict asset growth ("prompt corrective action"). Sources: https://www.bis.org/basel_framework/chapter/RBC/30.htm and https://www.occ.gov/news-issuances/bulletins/2018/bulletin-2018-33.html

[20]Specifically, banks' corporate loan ratings are one of the inputs that are used to compute potential losses under the various scenarios. These ratings are directly related to the PDs (see the Y14 data description, Appendix Table B.2). Starting in 2020:Q4, the bank-specific stress capital buffer requirement is also based on the outcome of the stress tests, providing an additional incentive for low-capitalized banks to underreport their credit risk exposures. Source: https://www.federalreserve.gov/publications/files/2019-march-supervisory-stress-test-methodology.pdf

Table 3.1: Reported PDs and Bank Capital.

	(i) PD	(ii) PD	(iii) PD-Gap	(iv) PD-Gap
Capital	0.10***	0.06**	0.10**	0.08***
	(0.04)	(0.03)	(0.04)	(0.02)
Fixed Effects				
Firm × Time	✓	✓		
Time			✓	✓
Bank		✓		✓
Bank Controls	✓	✓	✓	✓
Portfolio Risk Controls		✓		✓
R-squared	0.80	0.80	0.00	0.01
Observations	412,537	401,790	419,060	407,362
Number of Firms	12,189	12,065	12,489	12,347
Number of Banks	32	32	32	32

Notes: Estimation results for regression (3.1), where the dependent variable is either given by $\text{PD}_{i,j,t}$ in columns (i) and (ii) or by $\text{PD-Gap}_{i,j,t}$ in columns (iii) and (iv). Bank controls: bank size (natural log of total assets), return on assets (net income/total assets), deposit share (total deposits/total assets), and banks' income gap (see Table B.3 in Appendix B for details on the data). Portfolio risk controls: RWA/total assets, weighted portfolio PD. All specifications are estimated using OLS. Standard errors in parentheses are clustered by bank. Sample: 2014:Q4-2020:Q4. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

$\text{PD-Gap}_{i,j,t}$ instead. To account for the business-model explanation, we include bank fixed effects and total portfolio risk controls into the regressions reported in columns (ii) and (iv).[21] Across the various specifications, we find that β is positive and statistically significant at either the 1 percent or the 5 percent confidence level. These results are also economically sizable. A 1 percentage point higher capital buffer is related to a 6-10 basis points higher PD of a bank's entire loan portfolio, a substantial effect given that the average PD across all loans is around 2.5 percent. The magnitude of the effects are also comparable to the ones by Plosser and Santos (2018) who estimate similar regressions for syndicated loans.

We interpret these findings as providing evidence that low-capitalized banks systematically underreport their credit risk exposure. The quantitative magnitude of our results are likely a lower bound for two reasons. First, the described alternative explanations may push β in the opposite direction, such that the effect originating from regulatory incentives may be even larger. Second, our findings are conservative if all banks are misreporting, even the ones with the largest capital buffers in our sample.[22]

[21] Following Plosser and Santos (2018), we use the ratio of risk-weighted assets to total assets and the PD of the total loan portfolio based on the average reported PDs of other banks, given by $PD_{j,t} = \sum_i \overline{PD}_{i,t} Loan_{i,j,t} / \sum_i Loan_{i,j,t}$ where $\overline{PD}_{i,t} = (1/K) \sum_k PD_{i,k,t}$ where $k \neq j$.

[22] Appendix C collects additional evidence. Table C.1 shows that our results extend to local projections that consider how PDs adjust following changes in bank capital buffers. Table C.2 illustrates that the positive relation between bank capital and PDs is driven by riskier credit types, such as loans with higher PDs, that are syndicated, and held by private firms.

3.4 PDs, Bank Capital, and Credit Supply

Next, we exploit these differential risk assessments and test whether they also result in lending distortions. Specifically, we are interested in whether low-capitalized banks not only understate their credit risk exposure, but also lend relatively more to underreported borrowers to reconcile their reporting and avoid losses. At a first pass, we analyze credit movements following the outbreak of COVID-19 in 2020:Q1, an adverse macroeconomic shock that was largely unexpected. For firm i, bank j, and loan type k, we estimate

$$\frac{L^k_{i,j,t+2} - L^k_{i,j,t}}{0.5 \cdot (L^k_{i,j,t+2} + L^k_{i,j,t})} = \alpha^k_{i,t} + \beta_1 \text{Capital}_{j,t} + \beta_2 \text{Low-PD}^k_{i,j,t} + \beta_3 \text{Low-PD}^k_{i,j,t} \times \text{Capital}_{j,t} + \gamma X_{j,t} + u^k_{i,j,t} \quad (3.2)$$

where t denotes 2019:Q4 and we consider movements in credit $L^k_{i,j,t}$ over two quarters. As a dependent variable, we use the symmetric growth rate as an approximation of a percentage change in credit.[23] Following Khwaja and Mian (2008), we include firm-time fixed effects $\alpha^k_{i,t}$ into our regressions, and the sample is therefore restricted to firms that borrow from multiple banks. This approach accounts for potential links between bank-firm selection and firm demand. The fixed effects control for credit demand under the assumption that firms have a common demand across their lenders.

The coefficients of interest β_1, β_2, and β_3 therefore capture credit supply effects, conditional on other bank-specific controls that are collected in the vector $X_{j,t}$. The variable $\text{Capital}_{j,t}$ again denotes bank j's CET1 capital buffer in period t. $\text{Low-PD}^k_{i,j,t}$ is a binary indicator variable that takes the value of one if $PD^k_{i,j,t}$ is lower than the average reported PDs by other banks for the same firm and zero otherwise.[24] The interpretation of β_1, β_2, and β_3 is as follows. If $\beta_1 > 0$, banks that are better capitalized lend relatively more to firms to which they assign high PDs. If $\beta_2 > 0$, banks with zero-capital buffers extend relatively more credit to firms if they also consider those firms to have relatively low PDs. Last, if $\beta_3 < 0$, lowering capital predicts a relative increase in lending from low-PD banks in comparison with high-PD banks.

We restrict the sample in three additional ways. First, we exclude loans that are guaranteed by a third party since the associated PD may not be representative of the firm itself. Second, we consider only term loans and omit credit lines which were largely demand-driven after the COVID outbreak (Greenwald, Krainer and Paul, 2020). To account for the variation of credit line drawdowns across banks at the time, we also include the bank-specific ratio of unused credit lines to total assets before the outbreak into $X_{j,t}$. Third, we consider adjustable- and fixed-rate loans as separate types k since the demand for these loans may differ when short-term rates adjust suddenly and may be correlated with the bank-specific variables of interest.[25]

The estimation results for regression (3.2) are shown in Table 3.2. The first three columns

[23] The symmetric growth rate is the second-order approximation of the log-difference for growth rates around zero. It is bounded in the range [-2,2], robust to outliers, and is able to include changes in credit from a starting level of zero.

[24] That is, $\text{Low-PD}^k_{i,j,t}$ is one if $PD^k_{i,j,t} < \overline{PD}_{i,t}$. After excluding credit lines as described in the text, $\overline{PD}_{i,t} = (1/M)(1/K)\sum_m^M \sum_k^K PD^k_{i,m,t}$ is the average PD for firm i at time t across all non-j lenders and loan types.

[25] To avoid that our results are explained by a possible switching effect between credit lines and term loans, as well as between loans that differ in the flexibility of interest rates, we exclude bank-firm pairs that cover multiple types. If a bank issues multiple loans of a single type to the same firm, then we aggregate these loans at each date.

Table 3.2: COVID-19 — Credit Supply.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Capital	0.78	0.96	1.77*	2.27**	3.80***	
	(0.59)	(0.70)	(0.86)	(0.92)	(1.04)	
Low-PD		2.63*	6.51**	9.86***	11.56***	8.29**
		(1.51)	(2.74)	(2.93)	(2.70)	(3.44)
Capital × Low-PD			-1.23*	-2.16***	-2.19**	-1.43**
			(0.63)	(0.68)	(0.78)	(0.68)
Fixed Effects						
Firm × Rate	✓	✓	✓			✓
Firm × Rate × Syn.				✓		
Firm × Rate × Pur.					✓	
Bank						✓
Bank Controls	✓	✓	✓	✓	✓	
R-squared	0.53	0.53	0.53	0.53	0.55	0.55
Observations	892	667	667	612	510	663
Number of Firms	412	309	309	286	240	307
Number of Banks	24	23	23	21	23	21

Notes: Estimation results for regression (3.2). All specifications include firm fixed effects that additionally vary by rate type (adjustable- or fixed-rate) and whether the loan is syndicated (column iv) or the loan purpose (column v). Columns (i)-(v) include various bank controls for 2019:Q4: bank size (natural log of total assets), return on assets (net income/total assets), deposit share (total deposits/total assets), banks' income gap, and the ratio of unused credit lines to assets (see Table B.3 in Appendix B for details on the data). Column (vi) includes bank fixed effects. All specifications are estimated using OLS. Standard errors in parentheses are clustered by bank. Sample: 2019:Q4 - 2020:Q2. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

introduce the regressors of interest sequentially. In column (iii), β_1 and β_2 are estimated to be positive, while β_3 is negative, corresponding to the interpretation of the coefficients above.[26] The three coefficients are statistically different from zero at either the 5 percent or the 10 percent level. In comparison with columns (i) and (ii), β_1 increases in magnitude and statistical significance, highlighting the importance of the interaction term that is included in column (iii).

Columns (iv)-(vi) consider alternative specifications that address several identification concerns. First, the demand for syndicated and non-syndicated loans may have changed during the COVID crisis as some firms may have chosen to borrow from their main relationship lender. In turn, the supply of these different types of credit may depend on bank capitalization and potentially relative risk assessments, leading us to interpret shifts in credit demand as supply effects. To account for this possibility, we extend $\alpha^k_{i,t}$ by a loan's syndication type in column (iv). Similarly, if banks specialize in certain types of lending and firm demand across the lending types differs, then β_1, β_2, and β_3 may again capture demand rather than supply effects if such bank specialization is correlated with $\text{Capital}_{j,t}$ or $\text{Low-PD}^k_{i,j,t}$ (Paravisini, Rappoport and Schnabl, 2020). To

[26] Appendix Figure D.1 provides a graphical illustration of the estimates in column (iv) of Table 3.2 over the range of the observed capital buffers in 2019:Q4 among the Y14-banks.

address this possibility, we extend $\alpha_{i,t}^k$ by categories of loan purposes that firms report in column (v).[27] The estimation results show that the findings actually strengthen in magnitude and statistical significance with the more granular fixed effects. Last, in column (vi), we include a bank fixed effect. While the impact of other bank characteristics cannot be estimated separately in the presence of such a fixed effect, our findings with respect to β_2 and β_3 remain intact. Taken together, our results show that bank capitalization and relative risk assessments jointly determine credit availability. Low-capitalized banks not only underreport their credit risk exposure, but they also lend relatively more to underreported borrowers.

Bank Capital Buffers. While the outbreak of COVID-19 represents a unique setting with a sharp adverse macroeconomic shock, the mechanism that we identify may not be specific to this episode but can also be present during other periods. To explore this possibility, we exploit the historical evolution of bank capital buffers that is specific to the sample for which our data are available. As shown in Figure 3.1, the typical bank in our sample operates with a capital buffer of 3 percent or more in "normal times" such as during the early 2000s until the financial crisis of 2007-09, during which bank capital buffers sharply increased. In the following years, capital buffers remained elevated, possibly in anticipation of the higher capital requirements, which increased step-by-step from 2013:Q1 until the end of our sample, while bank capital ratios stayed high (see Appendix Figures D.2 and D.3). This allows us to split our sample into two parts: one running from 2014:Q4 to 2017:Q4 when typical capital buffers were relatively high (marked by the two vertical lines in Figure 3.1), and one starting in 2018:Q1 with typical capital buffers close to the ones in the early 2000s.[28]

For these two subsamples, we reestimate regression (3.2) and the results are shown in Tables 3.3 and 3.4. For the earlier sample with high capital buffers, the estimated coefficients are relatively small compared with the ones in Table 3.2, sometimes with opposite signs, and largely statistically insignificant. In contrast, for the later sample with low capital buffers, the estimated coefficients are slightly smaller in absolute magnitude but close to the ones in Table 3.2 and highly statistically significant. In comparison, Table 3.4 covers a substantially larger sample with close to 7,000 observations. Furthermore, the results in Table 3.4 do not depend on the inclusion of the COVID episode but also hold for a shorter sample that excludes this period and ends in 2019:Q4 (see Appendix Table D.1). Overall, these findings suggest that economies may be more prone to the documented lending distortions when the banking sector has relatively low *capital buffers* but may be present even when banks have high *capital ratios*, such as the banks in our sample that were generally perceived to be "well-capitalized" around the onset of the COVID crisis.

Robustness. Appendix D collects additional evidence and robustness checks of our findings for the extended "low-capital-buffer" sample. First, Table D.2 shows that the effects are not only

[27]Specifically, we consider the categories "Mergers and Acquisition," "Working Capital (permanent or short-term)," "Real estate investment or acquisition," and "All other purposes" as separate types (see also Appendix Table B.2).

[28]We end the low capital buffer sample in 2020:Q2, such that the latest capital ratios that enter the estimations are the ones in 2019:Q4. This avoids that the capital ratios during the COVID crisis enter our analysis, which were subject to a number of regulatory changes to make it easier for banks to meet the requirements.

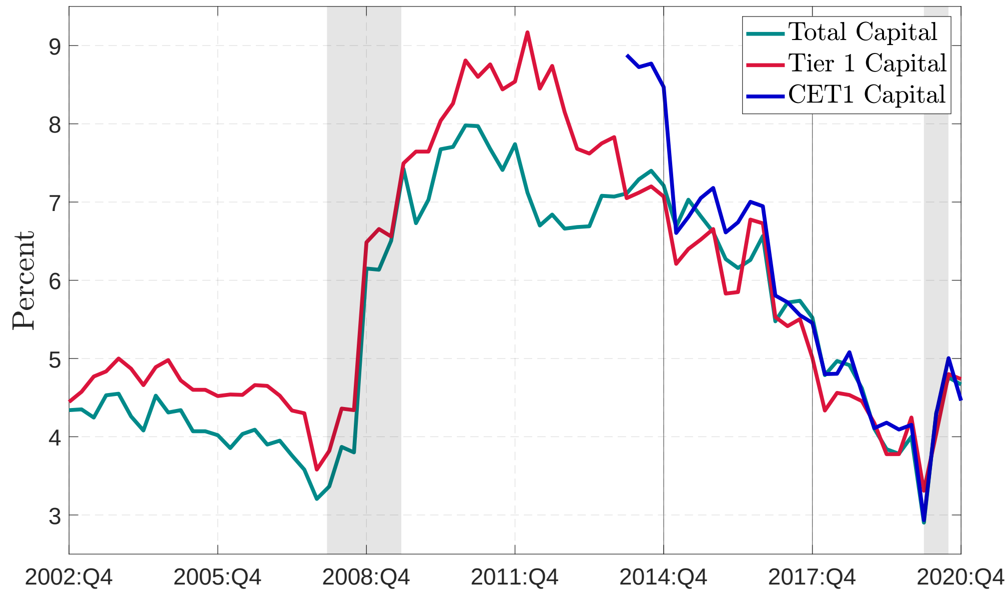

Figure 3.1: Bank Capital Buffers.

Notes: For each date, the figure shows the median of the CET1, Tier 1, and total capital buffer across the Y14-banks. Capital buffers are defined as the difference between capital ratios and requirements. Gray bars denote NBER recessions.

present for loan quantities but also for interest rates. This additional finding is in accordance with our static model which predicts that evergreening leads to both lower interest rates and larger quantities of credit, providing support for the contracting protocol that we assume. Second, we test whether our findings depend on the inclusion of the firm fixed effects. Table D.3 omits the firm-specific component of the fixed effect and Table D.4 uses time, location, industry, and firm-size fixed effects instead. Across the various alternative specifications, our results remain largely unchanged.[29] Third, we investigate whether our findings can be explained by an alternative channel, as opposed to the mechanism working through underreporting and lending distortions. For example, it may the case that low-capitalized banks favor safer borrowers. To test for this hypothesis, we replace Low-PD$^k_{i,j,t}$ with PD$^k_{i,j,t}$ itself in regression (3.2). As shown in Table D.5, we do not find evidence that low-capitalized banks favor safer borrowers since the coefficient β_3 is statistically not distinguishable from zero across the various regressions. Alternatively, it may be the case that lending supply is jointly determined by Low-PD$^k_{i,j,t}$ and another bank characteristic. To account for this possibility, we include various interaction terms between Low-PD$^k_{i,j,t}$ and the bank controls into regression (3.2). The estimation results in Table D.6 show that the original size and significance of the coefficient β_3 remains much the same. Last, we include credit lines into our regressions. However, we consider loan commitments rather than used credit amounts to minimize the possibility that we pick up demand rather than supply effects. The estimated coefficients reported in Table D.7 are similar to our baseline results.

[29] Even though these regressions increase the sample size in comparison with Table 3.4, they do not include firms that borrow from a single lender in our data. That is because we require a multi-bank sample to compute relative risk assessments and the variable Low-PD$^k_{i,j,t}$.

Table 3.3: High Capital Buffers — Credit Supply.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Capital	-0.17	0.09	0.10	-0.19	0.40	
	(0.29)	(0.25)	(0.32)	(0.36)	(0.52)	
Low-PD		0.88	0.92	-1.22	-1.16	5.22**
		(0.80)	(1.87)	(2.37)	(4.12)	(2.18)
Capital × Low-PD			-0.01	0.26	0.27	-0.62
			(0.38)	(0.44)	(0.71)	(0.39)
Fixed Effects						
Firm × Rate × Time	✓	✓	✓			✓
Firm × Rate × Syn. × Time				✓		
Firm × Rate × Pur. × Time					✓	
Bank × Time						✓
Bank Controls	✓	✓	✓	✓	✓	
R-squared	0.54	0.55	0.55	0.56	0.55	0.58
Observations	10,309	6,606	6,606	6,135	3,160	6,535
Number of Firms	835	581	581	551	307	574
Number of Banks	32	26	26	26	25	23

Notes: Estimation results for regression (3.2). All specifications include firm-time fixed effects that additionally vary by rate type (adjustable- or fixed-rate) and whether the loan is syndicated (column iv) or the loan purpose (column v). Columns (i)-(v) include various bank controls at time t: bank size (natural log of total assets), return on assets (net income/total assets), deposit share (total deposits/total assets), banks' income gap, and the ratio of unused credit lines to assets (see Table B.3 in Appendix B for details on the data). Column (vi) includes bank-time fixed effects. All specifications are estimated using OLS. Standard errors in parentheses are clustered by bank. Sample: 2014:Q4 - 2017:Q4. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Sample Splits. Next, we return to the theoretical predictions based on the "Static Model" in Section 2. Accordingly, the lending distortions should be stronger for low-productivity firms with larger preexisting debt since relationship lenders have stronger incentives to keep such firms alive. Hence, in the cross-section of banks, we should observe that low-capitalized banks lend relatively more to underreported borrowers particularly for these cases. To test whether the data aligns with our theoretical model in this way, we split the sample in Table 3.4 into high- and low productivity firms and small and large loans (relative to total firm debt).[30] We use several measures of firm productivity: (i) return on assets (net income to total assets), (ii) an approximation of firm marginal revenue product of capital (MRPK, see Appendix E for details), and (iii) the average interest rate that firms pay on their total debt as a proxy for MRPK based on a typical optimality condition.

The results are shown in Table 3.5. We find that our previous findings are driven by the subsamples of low-productivity firms and larger legacy debt. That is consistent with our theory as

[30] We distinguish small and large loans by the ex-ante share of a firm's total debt, as opposed to the absolute loan amount. This choice is guided by our theory. The relative payoffs of recovering existing debt and new lending determine whether a bank evergreens a loan. All else equal, the smaller the ex-ante share of a firm's total debt, the lower the gains of recovery relative to the necessary credit amount to keep a firm alive, and therefore the lower the evergreening incentives. Nevertheless, the estimation results are much the same if we consider the absolute loan size instead.

Table 3.4: Low Capital Buffers — Credit Supply.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Capital	0.18	0.17	0.95**	1.13***	1.68**	
	(0.30)	(0.34)	(0.40)	(0.40)	(0.64)	
Low-PD		0.63	5.46***	5.92***	6.82**	5.24**
		(1.30)	(1.89)	(1.86)	(2.58)	(2.25)
Capital × Low-PD			-1.29***	-1.64***	-1.63**	-1.14**
			(0.36)	(0.35)	(0.63)	(0.41)
Fixed Effects						
Firm × Rate × Time	✓	✓	✓			✓
Firm × Rate × Syn. × Time				✓		
Firm × Rate × Pur. × Time					✓	
Bank × Time						✓
Bank Controls	✓	✓	✓	✓	✓	
R-squared	0.51	0.54	0.54	0.54	0.54	0.57
Observations	6,977	4,674	4,674	4,188	3,617	4,649
Number of Firms	683	495	495	455	396	491
Number of Banks	29	27	27	26	27	24

Notes: Estimation results for regression (3.2). All specifications include firm-time fixed effects that additionally vary by rate type (adjustable- or fixed-rate) and whether the loan is syndicated (column iv) or the loan purpose (column v). Columns (i)-(v) include various bank controls at time t: bank size (natural log of total assets), return on assets (net income/total assets), deposit share (total deposits/total assets), banks' income gap, and the ratio of unused credit lines to assets (see Table B.3 in Appendix B for details on the data). Column (vi) includes bank-time fixed effects. All specifications are estimated using OLS. Standard errors in parentheses are clustered by bank. Sample: 2018:Q1 - 2020:Q2. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

low-capitalized banks have stronger incentives to evergreen lending to underreported borrowers for these cases. In contrast, the regression estimates for high-productivity firms and small loans are largely statistically insignificant.

Effects at the Firm Level. In a last exercise, we test whether the lending distortions also persist at the firm level, affecting total firm debt and investment. To this end, we estimate

$$\frac{y_{i,t+2} - y_{i,t}}{0.5 \cdot (y_{i,t+2} + y_{i,t})} = \alpha_i + \tau_{m,t} + \beta_1 \widetilde{\text{Capital}}_{i,t} + \beta_2 \widetilde{\text{Low-PD}}_{i,t} + \beta_3 \widetilde{\text{Low-PD} \times \text{Capital}}_{i,t} + \gamma X_{i,t} + u_{i,t} \tag{3.3}$$

where $y_{i,t}$ denotes an outcome for firm i, α_i is a firm fixed effect, $\tau_{m,t}$ is an industry-time fixed effect, and $X_{i,t}$ is a vector of firm controls. As dependent variables, we consider changes in total firm debt and fixed assets as an approximation for investment. The regressors associated with β_1, β_2, and β_3 represent exposures to bank capitalization and risk assessments that firms have through their term borrowing. That is, each regressor is defined as $\widetilde{R}_{i,t} = \sum_j R_{i,j,t} \times \text{Term Loan}_{i,j,t}/\text{Debt}_{i,t}$ where $R_{i,j,t}$ is given by $\text{Capital}_{j,t}$, $\text{Low-PD}^k_{i,j,t}$, or the interaction of the two, and firms' term-loan-

Table 3.5: Low Capital Buffers — Sample Splits.

	(i) Low ROA	(ii) High ROA	(iii) Low MRPK	(iv) High MRPK	(v) Low Rate	(vi) High Rate	(vii) Large Loans	(viii) Small Loans
Capital	3.39***	0.54	3.51***	0.77	2.33**	0.76	1.77	1.22
	(1.06)	(0.73)	(1.02)	(0.83)	(0.98)	(0.90)	(1.08)	(0.96)
Low-PD	15.23**	8.83*	19.04***	9.29	17.18***	4.49	13.61***	8.49
	(6.57)	(4.46)	(3.86)	(6.60)	(5.06)	(4.03)	(4.30)	(8.31)
Capital × Low-PD	-3.20***	-0.81	-3.87***	-1.25	-3.01***	-0.33	-2.77***	-1.02
	(1.02)	(1.06)	(0.97)	(0.96)	(0.78)	(0.75)	(0.85)	(1.22)
Fixed Effects								
Firm × Rate × Time	✓	✓	✓	✓	✓	✓	✓	✓
Bank Controls	✓	✓	✓	✓	✓	✓	✓	✓
R-squared	0.56	0.64	0.56	0.67	0.61	0.56	0.51	0.69
Observations	632	618	674	653	640	633	549	547
Number of Firms	116	103	111	107	132	110	104	88
Number of Banks	24	20	24	23	24	23	22	20

Notes: Estimation results for regression (3.2). The samples are split at the median at time t according to the return on assets (ROA) in columns (i) & (ii), the MRPK in columns (iii) & (iv), the average interest rate in columns (v) & (vi), and the size of the loan relative to total firm debt in columns (vii) & (viii). All specifications include firm-time fixed effects that additionally vary by rate type (adjustable- or fixed-rate) and various bank controls at time t: bank size (natural log of total assets), return on assets (net income/total assets), deposit share (total deposits/total assets), banks' income gap, and the ratio of unused credit lines to assets (see Table B.3 in Appendix B for details on the data). All specifications are estimated using OLS. Standard errors in parentheses are clustered by bank. Sample: 2018:Q1 - 2020:Q2. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

to-debt ratios are used as weights to aggregate the exposures across lenders.[31]

The estimation results for regression (3.3) are reported in Appendix Table D.8. Columns (ii) and (iv) show that the credit supply effects persist at the firm level and firms do not alter their credit across pre-existing or new lenders, such that their total debt adjusts by a similar amount as for the regressions reported in Table (3.4). These debt changes also translate into investment adjustments, indicating that firms do not alter other resources like their cash-holdings in response.

Taken together, our empirical results show that large U.S. banks with low capital buffers systematically underreport their credit risk exposure. To avoid potential losses and to reconcile their reporting, such banks favor underreported borrowers in their credit decisions, affecting real firm outcomes like investment. Consistent with the theoretical mechanism in Section 2, the lending distortions are only present among low-productivity firms and firms with larger outstanding debts. Building on this empirical validation, we next embed the mechanism into a dynamic model to study whether such lending incentives also affect aggregate capital allocation, productivity, and output.

[31] We note three details about regression (3.3). First, we include firm fixed effects to capture time-invariant firm-specific changes of debt and investment, and to estimate these effects consistently, we extend the estimation back to 2016:Q3 to allow for a sufficiently long sample covering four years of data. Second, the variable Low-PD$^{k}_{i,j,t}$ takes either values of zero or one and the associated coefficients are only identified because of the relative size shares of term borrowing across lenders. Third, apart from the exclusion of credit lines, we lift all other sample restrictions in comparison with regression (3.2), such as the exclusion of bank-firm observations with multiple credit types.

4 Dynamic Model

The structure of the dynamic model is based on the one developed by Hopenhayn (1992), augmented with debt and default. Firms are heterogeneous with respect to their productivity, holdings of physical capital, and debt. Firm entry and exit are endogenous, as well as the joint distribution of physical capital and debt, which is essential to study misallocation in this context. We first present the model setup and the problem of the firm. We then define a stationary industry equilibrium (SIE) for an arbitrary debt price function. We proceed to describe two potential institutional arrangements, as in the static model, that give rise to different debt price functions and therefore to different SIE. Finally, we calibrate the model and compare equilibria under the two arrangements.

4.1 Setup

Environment. Time is discrete and infinite, $t = 0, 1, 2, \ldots$. The economy is populated by a continuum of firms whose mass is endogenous. The distribution of firms is denoted by $\lambda(z, b, k)$, where z denotes productivity, b is debt, and k is capital. Firms endogenously enter and exit the economy, with the mass of entrants denoted by m. For now, we simply assume that the price of debt is described by some arbitrary function $Q(z, b, k)$ that firms take as given. In the following sections, we present alternative institutional arrangements that provide microfoundations for this function. There is a fixed and constant supply of labor equal to $\bar{N}$, and the supply of physical capital is perfectly elastic. The wage rate w is endogenous, and the price of capital is constant and equal to 1.

Timing. The timing within each period is as follows:

1. Firm productivity z is realized.
2. The lending contract Q is determined.
3. Firm draws i.i.d. extreme value preference shocks $\varepsilon^P, \varepsilon^D$, choosing to default or not.
4. Non-defaulting firms and new entrants invest, produce, repay, and borrow.

Besides endogenous entry, a new feature with respect to the static model is the introduction of the i.i.d. preference shocks for the firm. This feature is primarily introduced for computational tractability as it smoothens the expectation and probability functions for the firm and the lender.

4.2 Firm Problem

As in the static model, we assume that firms take the terms of the contract Q as given, and decide to repay, how much to borrow, and how much to invest. The firm has access to a decreasing returns to scale production technology with the production function given by $z^{1-\eta}(k^{\alpha}n^{1-\alpha})^{\eta}$, where z is current productivity, k is current capital, and n is labor. The capital share is denoted by α and η is the degree of returns to scale. The firm hires labor at wage w and invests in new capital k'

at a constant unit cost. Capital depreciates at rate δ. Additionally, the firm pays a fixed cost of operation equal to $\mathbf{c}$. The firm's value, after receiving an offer Q and upon realizing the extreme value shocks $\varepsilon^P, \varepsilon^D$ can be written as

$$V_0(z,b,k,\varepsilon^P,\varepsilon^D;Q) = \max\left\{V^P(z,b,k;Q) + \varepsilon^P, 0+\varepsilon^D\right\} \quad , \tag{4.1}$$

where $V^P(z,b,k)$ is the value of repaying (net of the preference shock), and we normalize the value of default to zero. One way to motivate these preference shocks is that they represent a stochastic outside option for the entrepreneur who runs the firm. We assume that these shocks follow a type I extreme value distribution (Gumbel), which implies that the difference between the two $\varepsilon = \varepsilon^P - \varepsilon^D$ follows a logistic distribution with scale parameter κ. This has the following implications:

1. Conditional on today's states and the offered contract $(z,b,k;Q)$, we can write the probability of repayment as

$$\mathcal{P}(z,b,k;Q) = \frac{\exp\left[\frac{V^P(z,b,k;Q)}{\kappa}\right]}{1+\exp\left[\frac{V^P(z,b,k;Q)}{\kappa}\right]} \tag{4.2}$$

2. Conditional on today's states and the offered contract $(z,b,k;Q)$, we can write the expected value as

$$\mathcal{V}(z,b,k;Q) = \mathbb{E}_{\varepsilon^P,\varepsilon^D} V_0(z,b,k,\varepsilon^P,\varepsilon^D;Q) = \kappa \log\left\{1+\exp\left[\frac{V^P(z,b,k;Q)}{\kappa}\right]\right\} \tag{4.3}$$

The value of repaying conditional on today's state $s = (z,b,k)$ and the offered contract Q is given by

$$V^P(z,b,k;Q) = \max_{b',k',n} div - \mathbb{I}[div<0][e_{con} + e_{slo} \times div^2] + \beta^f \mathbb{E}_{z'}[\mathcal{V}(z',b',k')|z] \tag{4.4}$$

s.t.

$$div = z^{1-\eta}(k^\alpha n^{1-\alpha})^\eta - wn - k' + (1-\delta)k + Qb' - b - \mathbf{c} \tag{4.5}$$

$$b' \le \theta k' \tag{4.6}$$

$$k', b', n \ge 0 \tag{4.7}$$

The value of repayment is equal to current dividends div plus the continuation value, which is given by the expectation of 4.3 over productivity in the next period z', conditional on productivity today z. Additionally, the firm is subject to equity issuance costs, which consist of a fixed cost component e_{con} and a quadratic cost scaled by e_{slo}. Equation 4.5 defines the firm dividend: it is equal to the value of production, minus the wage bill, minus new investment net of undepreciated capital, plus new borrowings, minus debt repayments, and minus the fixed cost. Equation 4.6 is the borrowing constraint, which states that repayments on newly borrowed debt may not exceed a fraction of newly chosen capital. Finally, 4.7 is a non-negativity constraint on the choices of debt, capital, and labor.

Characterizing the Firm's Problem. For simplicity and tractability, let us ignore for now the fixed cost of equity issuance, which introduces a non-differentiability in the firm's problem, $e_{con} = 0$. Let $\mu(div) \equiv 1 + 2e_{slo} \max\{0, -div\}$ denote the marginal value of equity for the firm, and let λ denote the Lagrange multiplier on the borrowing constraint. Let $\pi(z,k)$ denote the profit function at the optimal labor choice:

$$\pi(z,k) = \max_{n} z^{1-\eta}(k^{\alpha}n^{1-\alpha})^{\eta} - wn \quad . \tag{4.8}$$

Given these definitions, the firm's FOCs are

$$\begin{aligned} k' &: \beta^f \mathbb{E}_{z'}\left\{\mathcal{P}(z',b',k')\pi_k(z',k')[1+\mu(div')]\right\} - [1+\mu(div)] + \lambda\theta \leq 0 \quad , \\ b' &: -\beta^f \mathbb{E}_{z'}\left\{\mathcal{P}(z',b',k')[1+\mu(div')]\right\} + Q[1+\mu(div)] - \lambda \leq 0 \quad . \end{aligned}$$

When the borrowing constraint binds, we can write the FOC for capital as

$$\mathbb{E}_{z'}\left\{\mathcal{P}(z',b',k')\left(\beta^f \frac{1+\mu(div')}{1+\mu(div)}\right)\left[\pi_k(z',k') - \theta\right]\right\} = 1 - \theta Q \quad . \tag{4.9}$$

This condition establishes a relationship between the choice of capital k' and the price of debt Q. In particular, $\pi_k(z',k')$ is related to the marginal product of capital next period and is decreasing in k'. Hence, as long as the constraint binds, firms that receive better lending terms (higher Q) will tend to choose more capital k', everything else constant, and borrow more since $b' = \theta k'$. This is the dynamic version of the expression that established a tight link between the MPK and Q in the static model. When does the constraint bind? Rewrite the FOC for b' as

$$\lambda \geq Q[1+\mu(div)] - \beta^f \mathbb{E}_{z'}\left\{\mathcal{P}(z',b',k')[1+\mu(div')]\right\} \quad . \tag{4.10}$$

This condition states that the constraint will tend to bind when the offered price of debt Q, adjusted by the marginal value of equity today, is relatively high compared with the cost of repayment, which is adjusted by the probability of repayment and by the marginal value of equity tomorrow. Thus, the constraint is also more likely to bind when the marginal value of equity is high today relative to tomorrow.

4.3 Entry and Industry Equilibrium

Firm Entry. Let $\Gamma(z)$ denote the exogenous distribution from which potential entrants draw their starting productivity level. New entrants have to pay a fixed cost ω to take a productivity draw and start operating. The free-entry condition for firms is

$$\mathbb{E}_{\Gamma}[\mathcal{V}(z',0,0)] = \omega \quad . \tag{4.11}$$

Firm Distribution and Law of Motion. Let $\lambda(z,b,k)$ be the distribution of firms after entry and exit have taken place. Then, the law of motion for the distribution is given by

$$\begin{aligned}\lambda(z',b',k') = & \int_{z,b,k} \Pr(z'|z)\mathbb{I}[b'(z,b,k)=b']\mathbb{I}[k'(z,b,k)=k']\mathcal{P}(z,b,k)\mathrm{d}\lambda(z,b,k) \\ & + \mathrm{m}\int_z \Gamma(z)\Pr(z'|z)\mathbb{I}[b'(z,0,0)=b']\mathbb{I}[k'(z,0,0)=k']\mathcal{P}(z,0,0)\mathrm{d}z \quad ,\end{aligned} \tag{4.12}$$

where $\mathbb{I}$ is the indicator function, equal to 1 if the condition in brackets is satisfied and 0 otherwise, and m is the mass of new entrants.

Labor Market Equilibrium. The mass of entrants in each period must be such that the total amount of labor that is demanded by active firms equals the exogenous labor supply:

$$\bar{N} = \int_{z,b,k} \mathcal{P}(z,b,k)n(z,b,k)\mathrm{d}\lambda(z,b,k) \quad . \tag{4.13}$$

Stationary Industry Equilibrium. Given a contract function $Q(z,b,k)$, a stationary industry equilibrium (SIE) is a collection of policy and value functions (k^p, b^p, V^p), an equilibrium wage w, a stationary distribution $\lambda(z,b,k)$, and a mass of entrants m such that:

1. The policy and value functions solve the firm's problem in 4.4 given the lending function Q and the wage rate w.

2. The wage rate w ensures that the free-entry condition 4.11 is satisfied.

3. λ is a fixed point of the law of motion 4.12.

4. The mass of entrants is such that the labor market clears as in 4.13.

Note that we define a SIE for an arbitrary function of the price of debt $Q(z,b,k)$. The exact nature of how this function is specified is not crucial for the definition of the equilibrium, as long as firms take Q as given when solving their problem. We now explore two different institutional arrangements for the credit market that give rise to two different Q functions, and study the properties of the SIE under each of those.

4.4 Competitive Lending

The first institutional arrangement consists of a purely competitive credit market. It can be thought of as a bond market with atomistic lenders. The assumption that there is large mass of potential lenders, who are willing to lend to the firm with states $s=(z,b,k)$, implies that the price of debt Q is determined by a free-entry condition for lenders. We use the notation $Q^{zero}(z,b',k')$ to refer to the price at which lenders would make zero profits when a firm with productivity z chooses (b',k'); i.e.,

$$Q^{zero}(z,b',k') = \beta^k \mathbb{E}_{z'}[\mathcal{P}(z',b',k')] \quad . \tag{4.14}$$

This expression resembles the price used in models of sovereign default in the tradition of Eaton and Gersovitz (1981). Here, since firms choose (b', k') after they are offered the price Q as explained above, we must consider the policy functions from the firm's problem; i.e, $b'(s; Q)$ and $k'(s; Q)$. Using these functions, the equilibrium price with competitive lenders, $Q^{comp}(s)$, solves this equation

$$Q^{zero}(z, b'(s; Q^{comp}), k'(s; Q^{comp})) = Q^{comp}(s) \quad . \tag{4.15}$$

This condition simply states that, in equilibrium, lenders make zero expected profits at the price Q^{comp} when firms choose (b', k') taking as given the price Q^{comp}.[32]

4.5 Relationship Lending

The second type of credit market that we study is one where lenders internalize the possibility of default on current claims b when choosing lending terms, and, as a consequence, may offer a different Q. There is still a large mass of potential lenders that are willing to start a new relationship with the firm, which limits the degree of market power that the existing lender can exercise.[33]

Similar to the competitive case, we first describe the price of zero profits as

$$\tilde{Q}^{zero}(z, b', k') = \frac{\beta^k \mathbb{E}_{z'}[W(z', b', k')|z]}{b'} \quad , \tag{4.16}$$

where $W(z', b', k')$ is what it is worth for a lender to have a relationship with a firm with states (z', b', k'), which we explain below. Competition among potential lenders to start relationships with firms implies that profits at the beginning of a relationship must be zero. This means that for a firm with an arbitrary state s the equilibrium price with a new relationship lending can be obtained in a similar manner as in the competitive case; i.e.,

$$\tilde{Q}^{zero}(z, b'(s; Q^{new}), k'(s; Q^{new})) = Q^{new}(s) \quad . \tag{4.17}$$

In what follows, we write $V(s; Q^{new}(s))$ to represent the value that a firm obtains if it starts a new relationship. With this notation, we can write the problem of a lender that is already in a relationship as

$$W(s) = \max_{Q} \mathcal{P}(s; Q) \left[b - Qb'(s; Q) + \beta^k \mathbb{E}_{z'}[W(z', b'(s; Q), k'(s; Q))|z] \right] \tag{4.18}$$

$$\text{s.t.} \qquad V(s; Q) \geq V(s; Q^{new}(s)) \quad . \tag{4.19}$$

Accordingly, the lender can choose a value of Q subject to a participation constraint. This constraint implies that the firm is better off taking the deal than starting a relationship with a new

[32] In Appendix F.1, we explain how potential issues about existence or multiplicity are addressed.

[33] In contrast to the static model in Section 2, we assume that a firm can start a new contract with other relationship lenders, as opposed to resorting to a competitive bond market as an outside option.

Table 4.1: Model Parameters and Values

Parameter	Value	Target / Source
β^f	0.900	Firm Leverage
θ	0.700	Firm Leverage
κ	0.125	Firm Exit Rates
$\mathbf{c}$	0.125	Firm Exit Rates
$\tilde{z}$	1.483	Firm Exit Rates
$e_{constant}$	0.100	Equity Issuance
e_{slope}	40.00	Equity Issuance
ω	0.344	Normalize $w = 1$
ρ_z	0.767	Gourio and Miao (2010)
σ_u	0.211	Gourio and Miao (2010)
η	0.800	Clementi and Palazzo (2016)
β^k	0.970	Standard
α	0.330	Standard
δ	0.09	Standard

lender. It is possible to rewrite this problem as

$$W(s) = \max_{Q} \mathcal{P}(s;Q) \left\{ b - b'(s;Q) \left[Q - \tilde{Q}^{zero}(z, b'(s;Q), k'(s;Q)) \right] \right\}$$
$$\text{s.t.} \qquad V(s;Q) \geq V(s;Q^{new}(s)) \quad .$$

This simplified formulation of the relationship lender's problem highlights the trade-offs clearly. On the one hand, the lender would like to exploit its market power to extract as much surplus from the relationship as it can. This induces the lender to reduce Q by as much as possible, but the lender is constrained in its ability to do this by the outside option because other lenders could start a new relationship. On the other hand, the lender also understands that Q affects the probability of survival today $\mathcal{P}(s;Q)$ and hence the likelihood of b being repaid. This induces the lender to potentially offer a Q that is strictly higher than the one that the firm could obtain by borrowing the same amount from a new lender.

4.6 Calibration

We calibrate the model to an annual frequency and the parameters that we pick are summarized in Table 4.1. As our benchmark economy, we choose the model under competitive lending. Table 4.2 compares moments from the SIE of the model to the data.

We assume that firm productivity follows an AR(1) process in logs,

$$\log z' = \mu_z + \rho_z \log z + \sigma_z \epsilon_z \quad , \tag{4.20}$$

and the associated parameters are taken from Gourio and Miao (2010), with $\mu_z = 0$. We set the firm discount factor $\beta = 0.90$ and the borrowing constraint parameter $\theta = 0.7$ to match average book and market leverage of 0.67 and 0.29, respectively, taken from Gomes and Schmid (2010). The

Table 4.2: Model Moments vs. Data

Moment	Data	Model	Source
Book leverage	0.67	0.54	Gomes and Schmid (2010)
Market leverage	0.29	0.30	Gomes and Schmid (2010)
Investment/Assets (median)	0.16	0.09	Compustat
Exit rate	0.09	0.09	Hopenhayn, Neira and Singhania (2018)
Exit rate, new firms	0.25	0.25	Hopenhayn, Neira and Singhania (2018)
Freq. of equity issuance	0.09	0.10	Gomes and Schmid (2010)
Size of equity issuance	0.09	0.17	Hennessy and Whited (2007)

depreciation rate is set to a standard annual value $\delta = 0.09$, which implies that our investment rate slightly understates the one in Compustat. The scale parameter κ, the fixed cost $\mathbf{c}$, and the productivity distribution of new entrants F^e are chosen to match average exit rates over the last 40 years for all firms (0.09) and for new entrants (0.25), from Hopenhayn, Neira and Singhania (2018). The productivity distribution is assumed to be uniform between 0 and $\tilde{z} = 1.483$. The equity issuance cost parameters are chosen to target the frequency and size of equity issuances, from Gomes and Schmid (2010) and Hennessy and Whited (2007), respectively. The production function parameters (α, η) are standard and taken from the literature. Finally, the discount factor of lenders is set to target a risk-free rate of 3%, a standard value, and the entry cost ω is chosen to normalize the wage to $w = 1$ in the benchmark case of competitive lending.

4.7 Firm Choices and Debt Pricing

Figure 4.1 plots policy functions, continuation values, and debt prices for a firm with the same (z, k) in the two economies, as a function of pre-existing debt b. We begin by describing the competitive case illustrated by the blue dashed lines, where results are perhaps more standard and intuitive. The firm's value is strictly decreasing in b, which implies the same relation for the probability of repayment (panel a). Similarly, k' is strictly decreasing in b as visible in panel (d). That is because firms with more debt are more likely to realize negative profits, forcing them to issue costly equity. When the marginal value of equity is high, investment is lower which implies less borrowing due to the borrowing constraint, as shown in panel (c). Finally, panel (b) plots the equilibrium price $Q^{comp}(z, b, k)$. As legacy debt increases, the probability of default in the following period rises, leading to a fall in the competitive price. For high levels of legacy debt, the equilibrium price rises slightly as the firm strongly cuts down on its borrowing but still invests.

The red lines correspond to the same policy functions under relationship lending. For low enough debt, the policies are much the same. However, after a certain point, they begin to diverge. Specifically, panel (b) shows that the price of debt rises earlier with more legacy debt. The higher price of debt reflects the relationship lender's subsidy who attempts to prevent firm default. The price function is discontinuous. When the probability of repayment approaches zero, the required subsidy to keep the firm alive is so high that the lender prefers to liquidate the borrower instead. As panels (a), (c) and (d) show, the subsidy affects the probability of repayment, as well as firm choices of capital and debt, which are all larger compared with the competitive case.

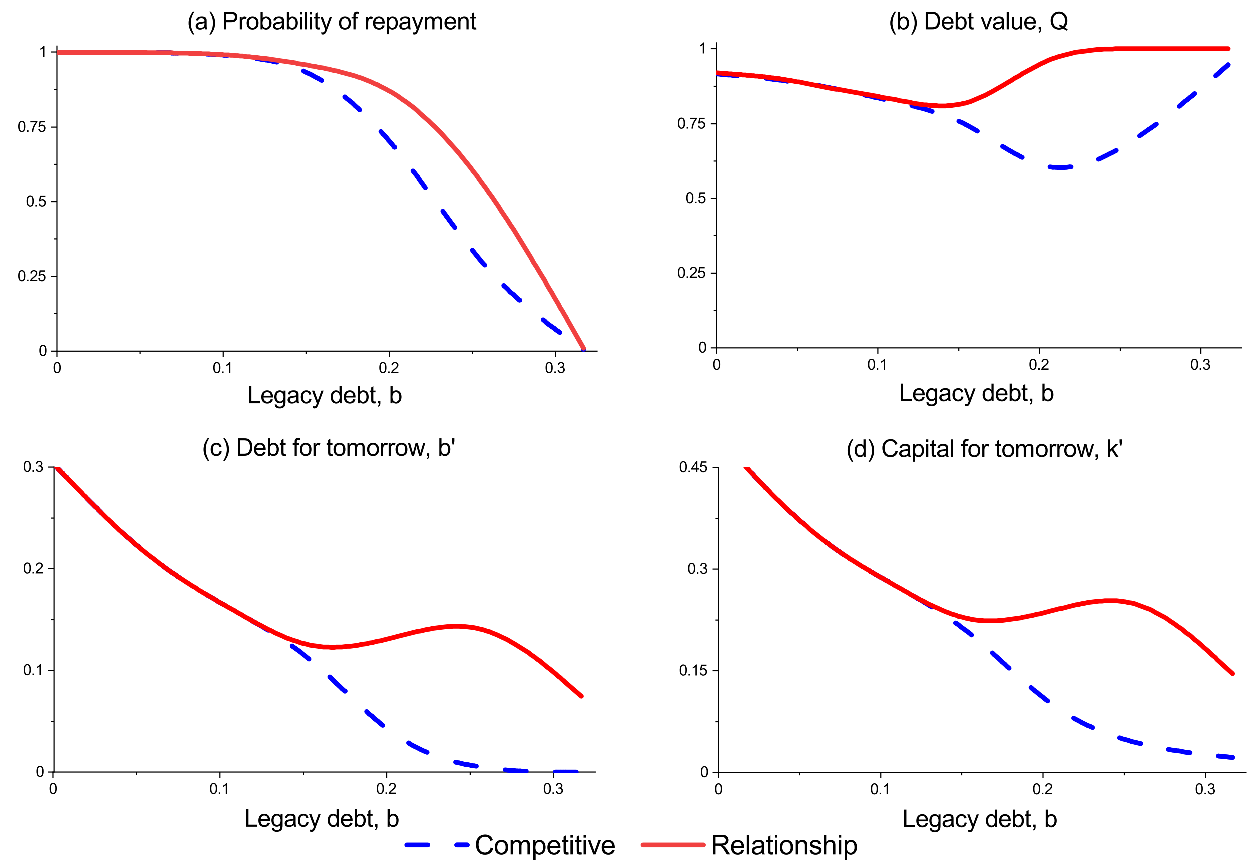

Figure 4.1: **Comparison of Policy Functions.** Policy functions and values for a firm with the same set of (z, k), as a function of b, competitive lending (blue, dashed) vs. relationship lending (red, solid) economies

4.8 Aggregate Effects

We now compare the SIE for the two economies, as described in section 4.3. The wage rate w in each economy is adjusted so that the free-entry condition 4.11 is satisfied, and the distribution for each economy is computed by solving for the stationary distribution as the fixed point of 4.12. The mass of entrants m is computed to ensure that the stationary distribution λ is such that the labor market clears.

Firm Distributions. Figure 4.2 plots cumulative distribution functions for the survival probability and interest rates in the SIE for the competitive lending economy (CLE) and the relationship lending economy (RLE). The survival probability CDF for the RLE first-order stochastically dominates the CDF of the CLE (panel a). Put differently, firms are uniformly less likely to exit in the RLE stemming from the equilibrium effects of relationship lending. The subsidy is visible in the panel (b), which shows that the distribution of interest rates in the CLE first-order stochastically dominates that of the RLE. That is, interest rates are uniformly lower in the RLE than in the CLE, throughout the firm distribution. One interesting take away from panel (b) is the following. Firms that are paying relatively high interest rates are primarily the ones that are benefiting from the

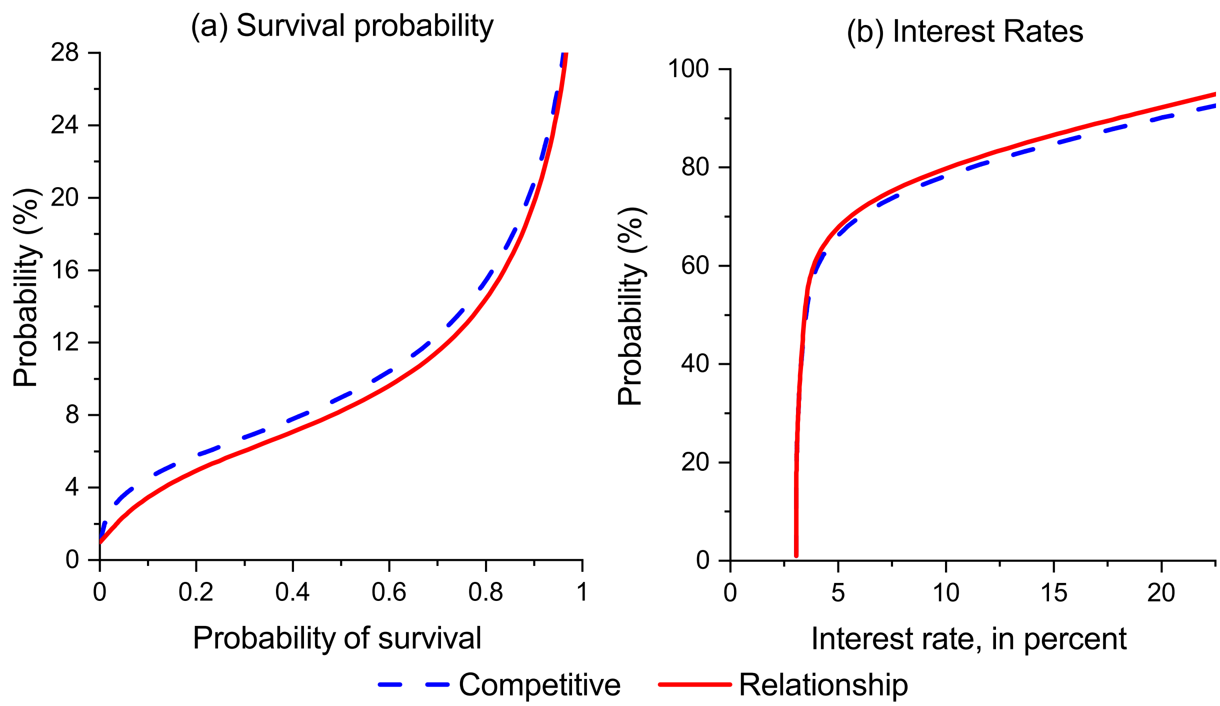

Figure 4.2: Equilibrium Cumulative Distribution Functions

interest rate subsidy—we do not observe a large discrepancy between the two CDFs for low levels of interest rates. However, even the subsidized interest rates are still well above the safe rate in our model. In contrast, previous papers have identified "zombie firms" as the ones that are paying extremely low interest—typically below comparable prime rates (e.g., Caballero, Hoshi and Kashyap, 2008). Our results therefore suggest that such definitions may severely underestimate the number of firms that are actually being subsidized.

Aggregate Moments. Table 4.3 presents several moments from the SIE of the competitive and the relationship lending economies, as well as percentages differences between the two. The first part of the table corresponds to averages across firms and the second part presents aggregates. By steering a firm's default decision through the offered lending terms, a relationship lender is able to recover its previous investment more often, benefiting the lender all else equal. However, under the assumption that lenders make zero profits in expectation, incumbent firms reap these benefits by borrowing at lower rates which decrease by around 0.6% in the RLE compared with the CLE. The average firm in the RLE is therefore more indebted with book and market leverage rising by 3.5% and 5%, respectively, and owns a larger stock of capital, which increases by around 0.2%. However, relationship lending also keeps less-productive firms alive, such that output for the average firm declines by around 0.8%. The aggregate numbers resemble the ones of the average firm, with aggregate capital and debt rising by 1% and 1.4%. The more frequent survival of low-productive firms that invest relatively more impedes the entry of other firms and leads to a shift in the distribution of firm productivity. As a result, aggregate TFP falls by around 0.3%. On net, the benefits of evergreening, stemming from the lender's enhanced ability to recover its previous

Table 4.3: Comparison of Stationary Equilibria.

Parameter	Competitive	Relationship	Δ%
Averages			
Market leverage	0.363	0.382	5.050
Book leverage	0.513	0.531	3.522
Interest rate	5.180	5.147	-0.643
Average capital	2.563	2.568	0.167
Average productivity	1.170	1.164	-0.486
Average output	1.467	1.456	-0.749
Aggregates			
Aggregate labor	1.000	1.000	0.000
Aggregate TFP	1.257	1.254	-0.273
Aggregate capital	3.257	3.288	0.975
Aggregate output	1.863	1.864	0.050
Aggregate debt	2.092	2.120	1.357
Wage	0.9995	1.000	0.048
Exit rate	0.087	0.084	-3.343
Measure entrants	0.173	0.161	-7.051

investment, are offset by the reduction in productivity, such that aggregate output remains much the same. Furthermore, aggregate labor is constant across the two economies by construction and the wage rate is slightly higher in the RLE, which is a consequence of the fact that relationship lending raises firm values of entrants in the absence of any wage changes.

5 Conclusion

Up to this point, the literature has largely associated zombie lending or evergreening with economies that are in a depression and have severely under-capitalized banks. The main empirical contributions focus on cases that fit these descriptions—Japan in the 1990s and periphery countries during the Eurozone crisis more recently. In this paper, we take a different perspective. We argue, both theoretically and empirically, that evergreening is in fact a general feature of financial intermediation—taking place even outside of depressions and within economies that have well-capitalized banks.

Our proposed theoretical mechanism builds on an intuitive idea. To recover its previous investment, a relationship lender has an incentive to offer more favorable lending conditions to a firm that is close to default in order to keep the firm alive. Turning standard intuition on its head, firms with worse fundamentals—that have more debt and are less productive—can borrow at better terms. Equipped with this generic theory of evergreening, we explore both its empirical relevance and its macroeconomic consequences.

For our empirical analysis, we use loan-level supervisory data for the United States and exploit the fact that the data include detailed information on banks' reported risk assessments for each individual loan. In the cross-section of banks, the incentives to evergreen loans differ due to the regulatory environment. Low-capitalized banks tend to understate their credit risk exposure and lend relatively more to underreported borrowers, but only if the loan is sufficiently large or a firm's productivity is depressed. These findings match our theory, which predicts that evergreening should occur in these instances.

To investigate the broader macroeconomic consequences, we turn to a dynamic heterogeneous-firm model in the tradition of Hopenhayn (1992), augmented with debt, default, and our evergreening mechanism. The framework provides additional insights beyond the intuition that our static model offers and what the data can tell us. First and foremost, evergreening is an equilibrium outcome that occurs frequently within a well-calibrated macroeconomic model and affects firm borrowing and investment decisions. Second, it takes place throughout the firm distribution, with firms that are closest to default and pay the highest interest rates enjoying the largest subsidies from their relationship lenders. However, the subsidized interest rates are still well above safe rates, implying that previous definitions of zombie firms as the ones with rates below safe rates may have understated the extent of this phenomenon. And third, evergreening affects macroeconomic aggregates. On the one hand, it depresses aggregate TFP since low-productivity firms are kept alive that invest relatively more, shifting the distribution of firm productivity relative to an economy without relationship lending. On the other hand, relationship lenders are able to recover their investments more frequently and pass on these benefits to their borrowers in the form of lower interest rates, leading to a rise in aggregate debt and capital. On net, these two forces—higher capital but lower TFP—largely offset each, such that aggregate output is similar with or without evergreening.

A number of fascinating avenues for future research result from our analysis. First, while our theory of evergreening differs from well-known corporate finance mechanisms such as debt overhang or risk-shifting, interesting interactions between them could arise. For example, firms with long-term outstanding debt could experience a fall in future productivity due to debt overhang and underinvestment. In turn, the debt burden and the additional loss in productivity may push them into the region where banks evergreen their loans and keep them alive. Second, our dynamic model focuses on the long-run implications of evergreening by analyzing stationary equilibria. However, in the short run, the effects of evergreening may differ. For example, after a large adverse macroeconomic shock like the COVID-19 crisis, evergreening may have similar effects as credit subsidies to firms, potentially preventing them from laying off workers and mitigating aggregate demand externalities. How do those potential short-run benefits trade off against the long-run effects that we document? And third, it would be interesting to consider policy interventions with respect to both firms and banks that can improve macroeconomic outcomes. We regard all of these explorations as valuable starting points for further analysis.

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APPENDIX

A Static Model

A.1 General Form of Borrowing Constraint

In this appendix, we show that some of the main results for the static model hold for the case where the firm faces a general constraint of the type

$$b' \leq g(k') \quad ,$$

with $g, g' \geq 0$ and $g'' \leq 0$. Note that many types of borrowing constraints, such as no default constraints, are special cases of this general form. With the constraint $b' \leq g(k')$, the firm's choice of capital cannot be solved in closed form, and is implicitly given by the FOC

$$\beta^f z\alpha(k')^{\alpha-1} - 1 + (Q - \beta^f)g'(k') = 0 \quad .$$

Note that as long as the constraint binds, all the comparative statics for k' extend to b' due to monotonicity of g. Furthermore, the optimal choices of capital and debt do not depend on Q for the case where $g' = 0$. We can use the above expression to obtain the implicit derivatives

$$\begin{aligned}
\frac{\partial k'(z;Q)}{\partial Q} &= \frac{g'(k')}{\beta^f z\alpha(1-\alpha)(k')^{\alpha-2} - Qg''(k')} \geq 0 \\
\frac{\partial b'(z;Q)}{\partial Q} &= \frac{[g'(k')]^2}{\beta^f z\alpha(1-\alpha)(k')^{\alpha-2} - Qg''(k')} \geq 0 \\
\frac{\partial k'(z;Q)}{\partial z} &= \frac{\beta^f \alpha(k')^{\alpha-1}}{\beta^f z\alpha(1-\alpha)(k')^{\alpha-2} - Qg''(k')} > 0 \\
\frac{\partial b'(z;Q)}{\partial z} &= \frac{g'(k')\beta^f \alpha(k')^{\alpha-1}}{\beta z\alpha(1-\alpha)(k')^{\alpha-2} - Qg''(k')} \geq 0 \quad .
\end{aligned}$$

It is also straightforward to show that

$$\begin{aligned}
\frac{\partial V(z,b;Q)}{\partial Q} &= b' \geq 0 \\
\frac{\partial V(z,b;Q)}{\partial z} &= \beta^f (k')^\alpha \geq 0 \\
\frac{\partial V(z,b;Q)}{\partial b} &= -1 < 0 \quad .
\end{aligned}$$

The following derivations show that it is still possible to prove Proposition 1-3 and the misallocation across firms depending on their initial level of debt for the general borrowing constraint $b' \leq g(k')$.

Proof of Proposition 1. Given that $V(z,b,;Q)$ is increasing in Q, the threshold $Q^{\min}(z,b)$ exists for $b>0$. $Q^{\min}$ is now implicitly defined by

$$0=-b+Q^{\min}b'(z,Q^{\min})-k'(z,Q^{\min})+\beta^f[z(k'(z,Q^{\min}))^{\alpha}-b'(z,Q^{\min})]$$

Combining the results of Proposition 1 with the above equation and the implicit function theorem allows us to derive the comparative statics

$$\begin{aligned}\frac{\partial Q^{\min}(z,b)}{\partial z}&=-\frac{\beta^f(k'(z,Q^{\min}))^{\alpha}}{b'(z,Q^{\min})}<0\\ \frac{\partial Q^{\min}(z,b)}{\partial b}&=\frac{1}{b'(z,Q^{\min})}>0\quad.\end{aligned}$$

This concludes the proof of Proposition 1.

Misallocation with Competitive Lending. With the general borrowing constraint, there may be misallocation in the competitive lending economy if $g''<0$. The FOC for capital implies that

$$z\alpha(k')^{\alpha-1}\equiv MPK=\frac{1-(Q-\beta^f)g'(k')}{\beta^f}\quad.$$

Even if Q is the same for all (b,z), the MPK will depend on the size of the firm k', which in turn is a function of the initial productivity state z. In particular, our assumptions imply that more productive firms are larger, and hence have a lower $g'(k')$ and a higher MPK. If $g''=0$, the $g'(k')$ term is independent of size and there is no misallocation in this economy. Importantly, misallocation in the competitive lending economy is independent of b.

Proof of Proposition 2. $Q^{\max}$ now solves the implicit equation

$$b+[\beta^k-Q^{\max}]b'(z;Q^{\max})=0\quad.$$

Clearly, $Q^{\max}\geq\beta^k$ for $b\geq 0$, as $b'(z;Q)\geq 0$. Additionally, applying the implicit function theorem allows us to derive the relationships

$$\begin{aligned}\frac{\partial Q^{\max}(z,b)}{\partial b}&=\frac{1}{b'(z;Q^{\max})+(Q^{\max}-\beta^k)\frac{\partial b'(z;Q^{\max})}{\partial Q}}>0\\ \frac{\partial Q^{\max}(z,b)}{\partial z}&=-\frac{(Q^{\max}-\beta^k)\frac{\partial b'(z;Q^{\max})}{\partial z}}{b'(z;Q^{\max})+(Q^{\max}-\beta^k)\frac{\partial b'(z;Q^{\max})}{\partial Q}}<0\quad.\end{aligned}$$

Proof of Proposition 3. Proposition 3 follows the same arguments as in the main text. The comparative statics with respect to $Q^*(b,z)$ follow from those of $Q^{\min}(z,b)$.

Misallocation with Relationship Lending The firm's choice of k' now follows

$$z\alpha(k')^{\alpha-1} \equiv MPK = \frac{1-(Q^*(b,z)-\beta^f)g'(k')}{\beta^f} \quad .$$

Notice that dispersion in b causes misallocation in an economy where all firms have the same productivity z, due to the lending subsidy $Q^*(z,b)$, vis-a-vis the competitive lending case. In particular, more indebted firms receive better lending terms and are thus larger and invest more.

A.2 Parametrization for Numerical Examples

The static model has four parameters: $\alpha, \beta^f, \beta^k, \theta$. All plots are based on the parametrization in Table A.1.

Table A.1: Static Model Parametrization

Parameter	Description	Value
α	Returns to scale	0.35
β^f	Discount factor Firm	0.9
β^k	Discount factor Lender	0.98
θ	Borrowing constraint	0.70

A.3 Alternative Contracting Protocol

In this section, we relax the assumptions underlying the contract offered by the relationship lender in the baseline version of the model. Our benchmark is a Stackelberg game where the lender offers Q and the firm chooses how much to borrow for a given Q. Next, we consider an alternative case where the relationship lender offers a contract that specifies both an interest rate Q and a repayment amount b'. We focus on the interesting case where $\beta^k < Q^{\min}(z,b)$, so that the firm would exit if it borrowed from the competitive lenders. Thus the firm can either accept the (Q,b')-offer or default. Taking the firm's decision into account, the relationship lender is able to extract the maximum surplus from the contract, offering (Q,b') such that $V(z,b;Q)=0$. This is equivalent to

$$0 = -b + Qb' - k'(z,b;Q,b') + \beta^f \left[zk'(z,b;Q,b')^\alpha - b'\right] \quad ,$$

where $k'(z;b',Q)$ is the firm's optimal choice of capital, given the states (z,b) and the offered contract (Q,b'). We consider first the case where the firm is unconstrained, and show that it cannot be an equilibrium. We then characterize the equilibrium contract for the case where the firm's borrowing constraint is binding.

Firm is Unconstrained. First, assume that the firm is unconstrained, i.e. $b' < \theta k'(z,b;Q,b')$. Its capital policy is independent of lending terms and solves

$$k' = \left(\beta^f z\alpha\right)^{\frac{1}{1-\alpha}} \quad .$$

The relationship lender's problem is then

$$\max_{Q,b'} W = b - Qb' + \beta^k b'$$

s.t.

$$0 = -b + (Q - \beta^f)b' + (\beta^f z\alpha)^{\frac{1}{1-\alpha}}(1/\alpha - 1) \quad .$$

One can use the constraint to replace for Q

$$Q = \beta^f + \frac{b - (\beta^f z\alpha)^{\frac{1}{1-\alpha}}(1/\alpha - 1)}{b'} \quad ,$$

and turn the lender's problem into an univariate problem over b'

$$\max_{b'} \left(\beta^k - \beta^f\right) b' + (\beta^f z\alpha)^{\frac{1}{1-\alpha}}(1/\alpha - 1) \quad .$$

Clearly, the lender's problem is strictly increasing in b' as long as $\beta^k > \beta^f$, which we assume. Thus the lender would like to choose $b' = \infty$, which cannot be an equilibrium.

Firm is Constrained. If the firm's borrowing constraint binds, the optimal capital policy must satisfy

$$k'(z; b', Q) = \frac{b'}{\theta} \quad .$$

As before, the relationship lender's problem can be written as

$$\max_{Q,b'} W = b - Qb' + \beta^k b'$$

s.t.

$$0 = -b + Qb' - b'/\theta + \beta^f \left[z(b'/\theta)^\alpha - b'\right] \quad .$$

Using the constraint to replace for Q

$$Q = \frac{b + b'/\theta + \beta^f b' - \beta^f z(b'/\theta)^\alpha}{b'} = \beta^f + \frac{1}{\theta} - \beta^f z\theta^{-\alpha}(b')^{\alpha-1} + \frac{b}{b'} \quad ,$$

one can turn the lender's problem into a univariate problem over b'

$$\max_{b'} \left(\beta^k - \beta^f - \frac{1}{\theta}\right) b' + \beta^f z\theta^{-\alpha}(b')^\alpha \quad .$$

The solution to this problem is

$$
\begin{aligned}
(b')^* &= \theta \left(\frac{\beta^f z\alpha}{1-\theta(\beta^k-\beta^f)} \right)^{\frac{1}{1-\alpha}} \\
(k')^* &= \left(\frac{\beta^f z\alpha}{1-\theta(\beta^k-\beta^f)} \right)^{\frac{1}{1-\alpha}} \\
Q^* &= \beta^f + \frac{1}{\theta} \left[1 - \frac{1-\theta(\beta^k-\beta^f)}{\alpha} + b \left(\frac{1-\theta(\beta^k-\beta^f)}{\alpha z \beta^f} \right)^{\frac{1}{1-\alpha}} \right] \quad .
\end{aligned}
$$

In this case, the allocations are the same as in a competitive lending equilibrium. Hence, as long as $Q \leq Q^{\max}(z,b)$, the MPKs are equalized across firms. Thus, this case eliminates misallocation in the relationship lending economy. Effectively, it corresponds to the bank taking over ownership of the firm and indirectly choosing investment via the binding borrowing constraint. Since the firm has no outside option (other than exit), the bank is able to extract the maximum surplus while setting the firm's value to zero. We can therefore think of this case as a type of restructuring whereby the lender has full control of the firm and its project. Further, it holds that

$$
Q^{\min}(z,b) \geq \beta^k \Leftrightarrow Q^* \geq \beta^k \quad .
$$

Thus, as long as the firm's states (z,b) are such that the firm would default in the competitive case, which is the situation that we consider, the price of debt offered by the lender Q^* will always be larger than the competitive price β^k. Taken together, if the bank offers both Q and b', the allocations of b' and k' coincide with the ones of the competitive case (without default), but the bank offers a price Q^* that is strictly larger and therefore a lower quantity of debt Q^*b'. In contrast, our empirical analysis shows that evergreening is associated with both lower interest rates and larger credit amounts. We therefore view the contracting protocol of our benchmark as the empirically relevant setting since it is consistent with the data in this regard.

B Data

In Tables B.1-B.3, we provide names, definitions, and sources for all variables that are used in the empirical analysis. Table B.1 collects all variables that are used from Compustat, B.2 the ones from the FR Y-14Q H.1 data, and Table B.3 the variables from the FR Y-9C Filings. Section B.1 lists the sample restrictions and filtering steps that we apply.

Table B.1: Compustat Variable Definitions.

Variable Name	**Description**	**Compustat Name**
Total Assets	Total firm assets	atq
Cash and Short-Term Investments	Cash and short-term investments	cheq
Tangible Assets	Constructed from cash, fixed assets, receivables, and inventories	cheq + invtq + ppentq + rectq
Employer Identification Number	Used to match to TIN in Y14, successful merges are basis for publicly traded designation	ein
Total Liabilities	Total firm liabilities	ltq
Net Income	Firm net income (converted to 12-month trailing series)	niq
Total Debt	Debt in current liabilities + long-term debt	dlcq + dlttq
Sales	Total firm sales	saleq
Fixed Assets	Net property, plant, and equipment	ppentq

Notes: All data are obtained from the Wharton Research Data Services. Nominal series are converted into real series using the consumer price index for all items taken from St. Louis Fed's FRED database.

Table B.2: FR Y-14 Variable Definitions.

Variable Name	Description / Use	Field No.
Zip code	Zip code of headquarters	7
Industry	Derived 2-Digit NAICS Code	8
TIN	Taxpayer Identification Number	11
Internal Credit Facility ID	Used together with BHC and previous facility ID to construct loan histories	15
Previous Internal Credit Facility ID	Used together with BHC and facility ID to construct loan histories	16
Term Loan	Loan facility type reported as Term Loan, includes Term Loan A-C, Bridge Loans, Asset-Based, and Debtor in Possession.	20
Credit Line	Loan facility type reported as revolving or non-revolving line of credit, standby letter of credit, fronting exposure, or commitment to commit.	20
Purpose	Credit facility purpose	22
Committed Credit	Committed credit exposure	24
Used Credit	Utilized credit exposure	25
Line Reported on Y-9C	Line number reported in HC-C schedule of FR Y-9C	26
Participation Flag	Used to determine whether a loan is syndicated	34
Variable Rate	Interest rate variability reported as "Floating" or "Mixed"	37
Interest Rate	Current interest rate	38
Guarantor Flag	Used to determine whether a loan is guaranteed	44
Date Financials	Financial statement date used to match firm financials to Y-14 date	52
Net Sales Current	Firm sales over trailing 12-month period	54
Operating Income	Used for operation surplus = operating income + depreciation & amortization	56
Depreciation & Amortization	Used for operation surplus = operating income + depreciation & amortization	57
Interest Expense	Used in calculating average interest rate on all debt	58
Net Income	Current net income for trailing 12-months used to construct return on assets	59, 60
Cash and Securities	Cash and marketable securities	61
Tangible Assets	Tangible assets	68
Fixed Assets	Fixed assets	69
Total Assets	Total assets, current year and prior year	70
Short Term Debt	Used in calculating total debt	74
Long Term Debt	Used in calculating total debt	78
Total Liabilities	Total liabilities	80
Probability of Default	Probability of default for firms (corresponds to internal risk rating for non-advanced BHCs)	88
Syndicated Loan	Syndicated loan flag	100

Notes: Nominal series are converted into real series using the consumer price index for all items taken from St. Louis Fed's FRED database. The corresponding "Field No." can be found in the data dictionary (Schedule H.1, pp. 162-217): https://www.federalreserve.gov/reportforms/forms/FR_Y-14Q20200331_i.pdf

Table B.3: Variables from Y-9C filings.

Variable Code	**Variable Label**
BHCK 2170	Total Assets
BHCK 2948	Total Liabilities
BHCK 4340	Net Income
BHCK 3197	Earning assets that reprice or mature within one year
BHCK 3296	Interest-bearing deposit liabilities that reprice or mature within one year
BHCK 3298	Long-term debt that reprices within one year
BHCK 3408	Variable-rate preferred stock
BHCK 3409	Long-term debt that matures within one year
BHDM 6631	Domestic offices: noninterest-bearing deposits
BHDM 6636	Domestic offices: interest-bearing deposits
BHFN 6631	Foreign offices: noninterest-bearing deposits
BHFN 6636	Foreign offices: interest-bearing deposits
BHCK JJ33	Provision for loan and lease losses
BHCA P793	Common Tier 1 Capital Ratio

Notes: The table lists variables that are collected from the Consolidated Financial Statements or FR Y-9C filings for Bank-Holding Companies from the Board of Governors' National Information Center database. The one-year income gap is defined as $(\text{BHCK}\,3197 - (\text{BHCK}\,3296 + \text{BHCK}\,3298 + \text{BHCK}\,3408 + \text{BHCK}\,3409))\,/\text{BHCK}\,2170$. Total deposits are given by $(\text{BHDM}\,6631 + \text{BHDM}\,6636 + \text{BHFN}\,6631 + \text{BHFN}\,6636)$. Nominal series are converted into real series using the consumer price index for all items taken from St. Louis Fed's FRED database. The FR Y-9C form for March 2020 can be found at: https://www.federalreserve.gov/reportforms/forms/FR_Y-9C20200401_f.pdf.

B.1 Sample Restrictions and Filtering Steps

1. We restrict the sample to begin in 2012:Q3. The Y14 collection began in 2011:Q3, but there was a significant expansion in the number of BHCs required to submit Y14 commercial loan data until 2012:Q3. Moreover, the starting date in 2012:Q3 also affords a short phase-in period for the structure of the collection and variables to stabilize.

2. We constrain the sample to loan facilities with line reported on the HC-C schedule in the FR Y9-C filings as commercial and industrial loans, "other" loans, "other" leases, and owner-occupied commercial real estate (corresponding to Field No. 26 in the H.1 schedule of the Y14 to be equal to 4, 8, 9, or 10; see Table B.2). In addition, we drop all observations with NAICS codes 52 and 53 (loans to financial firms and real estate firms).

3. Observations with negative or zero values for committed exposure, negative values for utilized exposure, and with committed exposure less than utilized exposure are excluded.

4. When aggregating loans at the firm-level, we exclude observations for which the firm identifier "TIN" is missing. To preserve some of these missing values, we fill in missing TINs from a history where the non-missing TIN observations are all the same over a unique facility ID.

5. When using information on firms' financials in the analysis, we apply a set of filters to ensure that the reported information is sensible. We exclude observations (i) if total assets, total liabilities, short-term debt, long-term debt, cash assets, tangible assets, or interest expenses are negative, (ii) if tangible assets, cash assets, or total liabilities are greater than total assets, and (iii) if total debt (short term + long term) is greater than total liabilities.

6. A loan facility may include both credit lines and term loans. We assume that all unused credit (i.e., committed exposure - utilized exposure) takes the form of unused capacity on the firm's credit lines. That is, we include unused borrowing capacity on a firm's term loans in the total unused credit line measure.

7. When using the interest rate on loans in our calculations, we exclude observations with interest rates below 0.5 or above 50 percent to minimize the influence of data entry errors.

C Risk-Reporting and Bank Capital

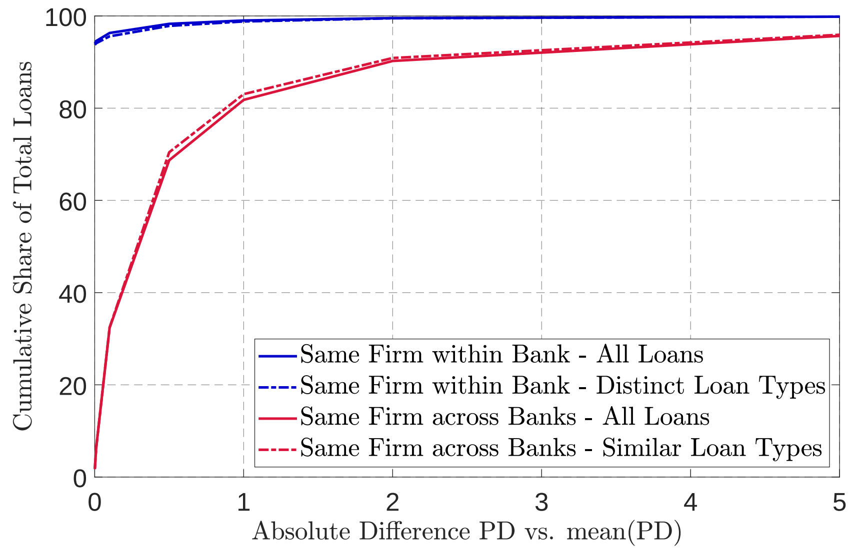

Figure C.1: Probability of Default Dispersion.

Notes: For different subsets of loans, the figure shows the cumulative share of total loans up to a specific absolute difference between the PD and the average PD for each respective subset of loans. For these calculations, firms with a single loan from a bank are excluded. The solid blue line considers all loans for a particular bank-firm pair. The dashed blue line additionally distinguishes loans by whether they are syndicated, adjustable-rate, and a credit line or a term loan. Similarly, the dashed red line compares loans to the same firm across banks that are similar across those three characteristics, whereas the solid red line considers all loans. Sample: 2014:Q4-2020:Q4.

Table C.1: Reported PDs and Bank Capital − Local Projections.

	(i) PD	(ii) PD	(iii) PD-Gap	(iv) PD-Gap
Capital	0.09*	0.07*	0.10**	0.09**
	(0.04)	(0.04)	(0.04)	(0.04)
Fixed Effects				
Firm × Time	✓	✓		
Time			✓	✓
Bank		✓		✓
Bank Controls	✓	✓	✓	✓
Portfolio Risk Controls		✓		✓
R-squared	0.66	0.66	0.00	0.00
Observations	278,319	278,319	284,686	284,684
Number of Firms	9,206	9,206	9,427	9,427
Number of Banks	32	32	32	32

Notes: Estimation results for $y_{i,j,t+2} - y_{i,j,t-1} = \beta \Delta Capital_{j,t-1} + \gamma X_{j,t-1} + \alpha_{i,t-1} + \kappa_j + u_{i,j,t+2}$, where $y_{i,j,t}$ is either given by $\text{PD}_{i,j,t}$ in columns (i) and (ii) or by $\text{PD-Gap}_{i,j,t}$ in columns (iii) and (iv). Bank controls: bank size (natural log of total assets), return on assets (net income/total assets), deposit share (total deposits/total assets), and banks' income gap (see Table B.3 in Appendix B for details on the data). Portfolio risk controls: RWA/total assets, weighted portfolio PD. All specifications are estimated using OLS. Standard errors in parentheses are clustered by bank. Sample: 2014:Q4-2020:Q4. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table C.2: Reported PDs and Bank Capital − Interactions.

	PD	PD	PD	PD	PD	PD
Capital × log(Loan)	-0.00					-0.00
	(0.01)					(0.01)
Capital × log(Assets)		-0.03***				-0.01
		(0.01)				(0.01)
Capital × mean(PD)			0.08***			0.06**
			(0.02)			(0.03)
Capital × Syndicated				0.12***		0.06**
				(0.02)		(0.03)
Capital × Public					-0.06***	-0.05*
					(0.02)	(0.03)
Fixed Effects						
Bank × Time	✓	✓	✓	✓	✓	✓
Firm × Time	✓	✓	✓	✓	✓	✓
R-squared	0.8	0.74	0.8	0.8	0.8	0.74
Observations	412,537	253,417	412,537	373,996	412,537	224,954
Number of Firms	12,189	8,599	12,189	11,889	12,189	8,318
Number of Banks	32	32	32	32	32	32

Notes: Estimation results for $PD_{i,j,t} = \beta Capital_{j,t-1} \times X_{i,j,t} + \alpha_{i,t} + \kappa_{j,t} + u_{i,j,t}$, where $X_{i,j,t}$ is either given by loan size (natural log of used credit), firm size (natural log of total assets), the average PD for firm i (weighted average across all loans), or binary variables indicating whether the loan is syndicated or the firm is publicly traded. All specifications include firm-time $\alpha_{i,t}$ and bank-time $\kappa_{j,t}$ fixed effects and are estimated using OLS. Standard errors in parentheses are clustered at the bank-firm level. Sample: 2014:Q4-2020:Q4. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

D PDs, Bank Capital, and Credit Supply

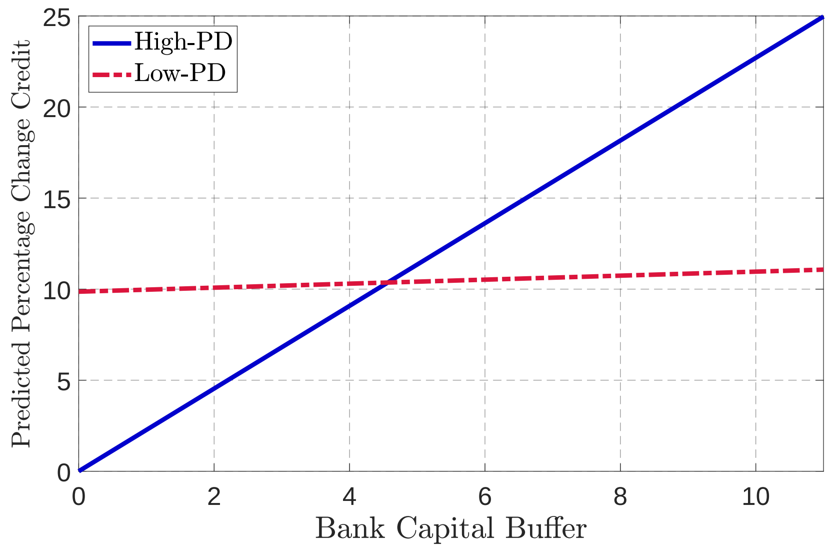

Figure D.1: Graphical Illustration of Regression Coefficients.

Notes: The figure plots the regression estimates from column (iv) of Table 3.2, $\beta_1 = 2.27$, $\beta_2 = 9.86$, $\beta_3 = -2.16$, constant=0. Bank capital buffers in 2019:Q4 range from 1.66 to 10.19 among the Y14-banks in our sample.

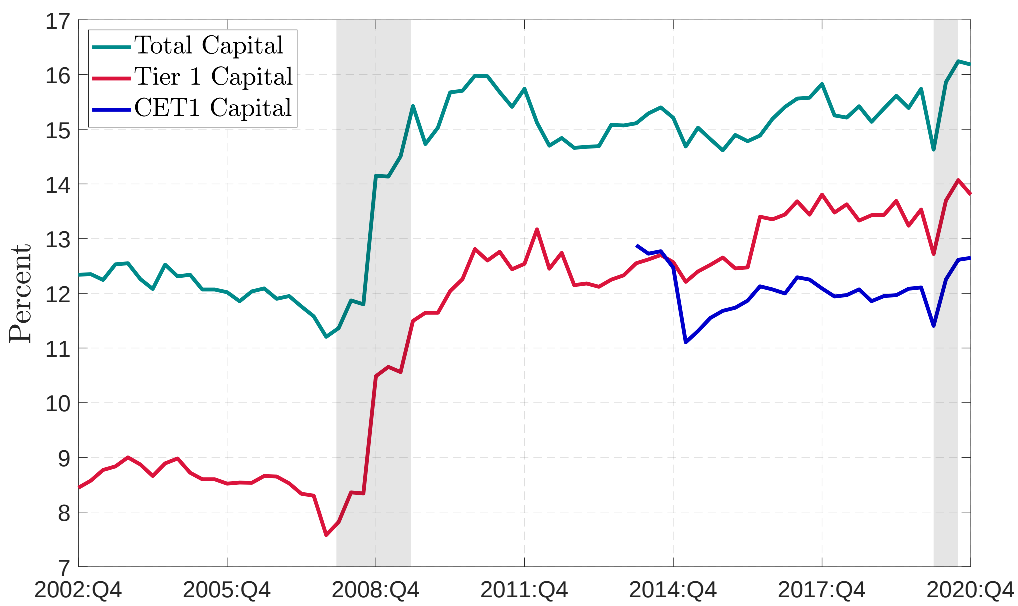

Figure D.2: Bank Capital Ratios.

Notes: For each date, the figure shows the median of the CET1, Tier 1, and total capital ratios across the Y14-banks. Gray bars denote NBER recessions.

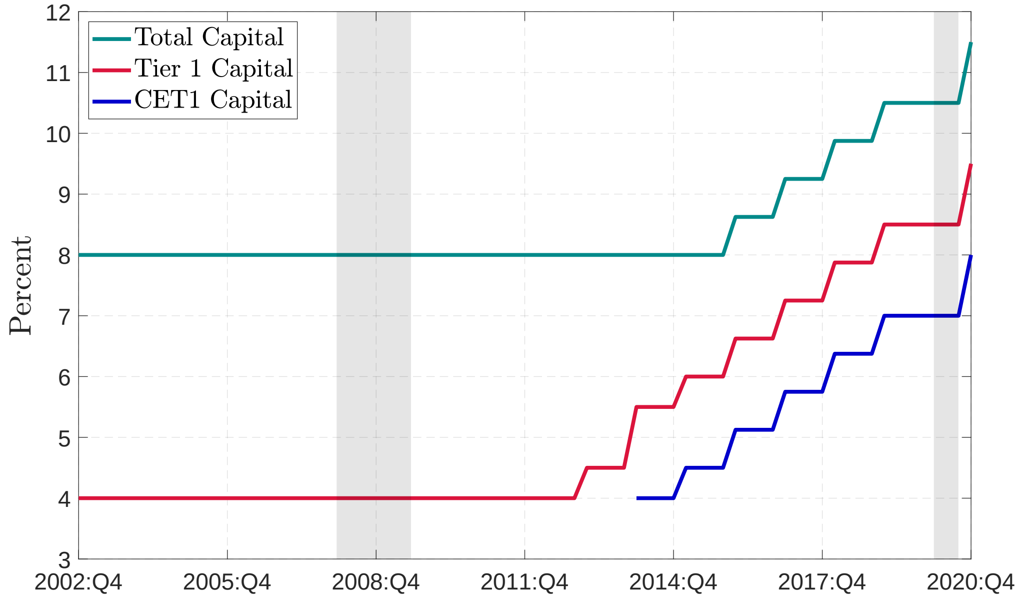

Figure D.3: Bank Capital Requirements.

Notes: For each date, the figure shows the median of the CET1, Tier 1, and total capital requirements across the Y14-banks. Gray bars denote NBER recessions.

Table D.1: Low Capital Buffers Excluding COVID − Credit Supply.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Capital	-0.20	-0.18	0.58	0.85*	1.09	
	(0.34)	(0.42)	(0.48)	(0.47)	(0.76)	
Low-PD		0.04	4.98**	4.95*	5.96*	3.71
		(1.38)	(2.39)	(2.53)	(3.23)	(2.89)
Capital × Low-PD			-1.27***	-1.54***	-1.55**	-0.93
			(0.43)	(0.46)	(0.69)	(0.54)
Fixed Effects						
Firm × Rate × Time	✓	✓	✓			✓
Firm × Rate × Syn. × Time				✓		
Firm × Rate × Pur. × Time					✓	
Bank × Time						✓
Bank Controls	✓	✓	✓	✓	✓	
R-squared	0.5	0.53	0.53	0.53	0.52	0.56
Observations	5,292	3,477	3,477	3,097	2,663	3,456
Number of Firms	606	422	422	386	335	420
Number of Banks	28	25	25	25	24	23

Notes: Estimation results for regression (3.2). All specifications include firm-time fixed effects that additionally vary by rate type (adjustable- or fixed-rate) and whether the loan is syndicated (column iv) or the loan purpose (column v). Columns (i)-(v) include various bank controls at time t: bank size (natural log of total assets), return on assets (net income/total assets), deposit share (total deposits/total assets), banks' income gap, and the ratio of unused credit lines to assets (see Table B.3 in Appendix B for details on the data). Column (vi) includes bank-time fixed effects. All specifications are estimated using OLS. Standard errors in parentheses are clustered by bank. Sample: 2018:Q1 - 2019:Q4. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table D.2: Low Capital Buffers − Interest Rates.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Capital	-0.00	-0.00	-0.01*	-0.01**	-0.01**	
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	
Low-PD		0.01**	-0.02**	-0.02**	-0.03**	-0.03***
		(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
Capital × Low-PD			0.01***	0.01***	0.01***	0.01***
			(0.00)	(0.00)	(0.00)	(0.00)
Fixed Effects						
Firm × Rate × Time	✓	✓	✓			✓
Firm × Rate × Syn. × Time				✓		
Firm × Rate × Pur. × Time					✓	
Bank × Time						✓
Bank Controls	✓	✓	✓	✓	✓	
R-squared	0.88	0.89	0.89	0.88	0.87	0.91
Observations	6,538	4,399	4,399	3,944	3,416	4,368
Number of Firms	652	474	474	433	379	470
Number of Banks	29	27	27	26	27	24

Notes: Estimation results for regression (3.2), where the dependent variable is given by changes in interest rates $i^k_{i,j,t+2} - i^k_{i,j,t}$. Interest rates are weighted by used credit and observations within the 1% tails of the dependent variable are excluded. All specifications include firm-time fixed effects that additionally vary by rate type (adjustable- or fixed-rate) and whether the loan is syndicated (column iv) or the loan purpose (column v). Columns (i)-(v) include various bank controls at time t: bank size (natural log of total assets), return on assets (net income/total assets), deposit share (total deposits/total assets), banks' income gap, and the ratio of unused credit lines to assets (see Table B.3 in Appendix B for details on the data). Column (vi) includes bank-time fixed effects. All specifications are estimated using OLS. Standard errors in parentheses are clustered by bank. Sample: 2018:Q1 - 2020:Q2. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table D.3: Low Capital Buffers − Omitting Firm Fixed Effects.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Capital	0.13	0.54**	0.92***	1.05***	1.14***	
	(0.17)	(0.24)	(0.29)	(0.31)	(0.29)	
Low-PD		-0.07	2.37*	2.97**	2.85**	2.93**
		(0.97)	(1.22)	(1.22)	(1.29)	(1.07)
Capital × Low-PD			-0.66**	-0.81***	-0.73***	-0.65**
			(0.24)	(0.18)	(0.26)	(0.25)
Fixed Effects						
Rate × Time	✓	✓	✓			✓
Rate × Syn. × Time				✓		
Rate × Pur. × Time					✓	
Bank × Time						✓
Bank Controls	✓	✓	✓	✓	✓	
R-squared	0.01	0.02	0.02	0.02	0.03	0.05
Observations	84,274	8,033	8,033	7,529	7,996	8,022
Number of Firms	15,258	1,135	1,135	1,093	1,133	1,135
Number of Banks	31	27	27	27	27	27

Notes: Estimation results for regression (3.2). All specifications include time fixed effects that additionally vary by rate type (adjustable- or fixed-rate) and whether the loan is syndicated (column iv) or the loan purpose (column v). Columns (i)-(v) include various bank controls at time *t*: bank size (natural log of total assets), return on assets (net income/total assets), deposit share (total deposits/total assets), banks' income gap, and the ratio of unused credit lines to assets (see Table B.3 in Appendix B for details on the data). Column (vi) includes bank-time fixed effects. All specifications are estimated using OLS. Standard errors in parentheses are two-way clustered by bank and firm. Sample: 2018:Q1 - 2020:Q2. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table D.4: Low Capital Buffers — Alternative Fixed Effects.

	(i)	(ii)	(iii)	(iv)
Capital	1.02***	0.86***	0.73**	0.77**
	(0.25)	(0.29)	(0.34)	(0.36)
Low-PD	2.78*	2.60*	2.38	1.27
	(1.35)	(1.44)	(1.45)	(1.33)
Capital × Low-PD	-0.77***	-0.78**	-0.75**	-0.75**
	(0.25)	(0.29)	(0.31)	(0.30)
Fixed Effects				
Time	✓			
Location × Time		✓		
Location × Industry × Time			✓	
Location × Industry × Size × Time				✓
Bank Controls	✓	✓	✓	✓
R-squared	0.01	0.09	0.29	0.42
Observations	8,033	5,822	5,388	3,536
Number of Firms	1,135	833	736	570
Number of Banks	27	27	27	26

Notes: Estimation results for regression (3.2). All specifications include time fixed effects that additionally vary by location (state-level) in columns (ii)-(iv), industry (two-digit NAICS code) in columns (iii) and (iv), and firm size (deciles of the unconditional firm size distribution) in column (iv). All regressions include various bank controls at time t: bank size (natural log of total assets), return on assets (net income/total assets), deposit share (total deposits/total assets), banks' income gap, and the ratio of unused credit lines to assets (see Table B.3 in Appendix B for details on the data). All specifications are estimated using OLS. Standard errors in parentheses are two-way clustered by bank and firm. Sample: 2018:Q1 - 2020:Q2. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table D.5: Low Capital Buffers − Probability of Default.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Capital	0.07	0.11	0.07	0.13	0.36	
	(0.37)	(0.35)	(0.35)	(0.30)	(0.40)	
PD		-0.11	-0.27*	-0.27**	-0.21	-0.28
		(0.10)	(0.14)	(0.12)	(0.13)	(0.17)
Capital × PD			0.05	0.04	-0.01	0.05
			(0.04)	(0.04)	(0.03)	(0.05)
Fixed Effects						
Firm × Rate × Time	✓	✓	✓			✓
Firm × Rate × Syn. × Time				✓		
Firm × Rate × Pur. × Time					✓	
Bank × Time						✓
Bank Controls	✓	✓	✓	✓	✓	
R-squared	0.5	0.51	0.51	0.52	0.51	0.54
Observations	9,930	7,263	7,263	6,348	5,701	7,251
Number of Firms	969	754	754	674	606	752
Number of Banks	29	27	27	27	27	26

Notes: Estimation results for regression (3.2), where Low-PD$^k_{i,j,t}$ is replaced by PD$^k_{i,j,t}$. All specifications include firm-time fixed effects that additionally vary by rate type (adjustable- or fixed-rate) and whether the loan is syndicated (column iv) or the loan purpose (column v). Columns (i)-(v) include various bank controls at time t: bank size (natural log of total assets), return on assets (net income/total assets), deposit share (total deposits/total assets), banks' income gap, and the ratio of unused credit lines to assets (see Table B.3 in Appendix B for details on the data). Column (vi) includes bank-time fixed effects. All specifications are estimated using OLS. Standard errors in parentheses are clustered by bank. Sample: 2018:Q1 - 2020:Q2. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table D.6: Low Capital Buffers − Low-PD Interactions.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Capital	0.28	0.30	1.18*	1.29**	2.04**	
	(0.33)	(0.30)	(0.65)	(0.60)	(0.80)	
Low-PD		-23.52	29.03	20.58	68.99	44.40
		(58.28)	(71.36)	(87.25)	(72.53)	(63.60)
Capital × Low-PD			-1.62*	-1.93**	-2.23**	-1.69*
			(0.83)	(0.86)	(0.98)	(0.89)
Fixed Effects						
Firm × Rate × Time	✓	✓	✓			✓
Firm × Rate × Syn. × Time				✓		
Firm × Rate × Pur. × Time					✓	
Bank × Time						✓
Bank Controls	✓	✓	✓	✓	✓	
Bank Controls × Low-PD	✓	✓	✓	✓	✓	✓
R-squared	0.54	0.54	0.54	0.54	0.54	0.57
Observations	4,674	4,674	4,674	4,188	3,617	4,649
Number of Firms	495	495	495	455	396	491
Number of Banks	27	27	27	26	27	24

Notes: Estimation results for regression (3.2). All specifications include firm-time fixed effects that additionally vary by rate type (adjustable- or fixed-rate) and whether the loan is syndicated (column iv) or the loan purpose (column v). Columns (i)-(v) include various bank controls at time t: bank size (natural log of total assets), return on assets (net income/total assets), deposit share (total deposits/total assets), banks' income gap, and the ratio of unused credit lines to assets (see Table B.3 in Appendix B for details on the data). All specifications include interaction terms of each of the bank controls with Low-PD. Column (vi) includes bank-time fixed effects. All specifications are estimated using OLS. Standard errors in parentheses are clustered by bank. Sample: 2018:Q1 - 2020:Q2. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table D.7: Low Capital Buffers — Credit Lines and Loan Commitments.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Capital	0.15	0.13	0.36**	0.45**	0.61**	
	(0.13)	(0.14)	(0.17)	(0.19)	(0.26)	
Low-PD		0.34	2.20**	2.61***	3.07***	1.81*
		(0.50)	(0.82)	(0.81)	(1.08)	(0.96)
Capital × Low-PD			-0.50***	-0.68***	-0.66**	-0.44**
			(0.18)	(0.21)	(0.27)	(0.19)
Fixed Effects						
Firm × Rate × Time	✓	✓	✓			✓
Firm × Rate × Syn. × Time				✓		
Firm × Rate × Pur. × Time					✓	
Bank × Time						✓
Bank Controls	✓	✓	✓	✓	✓	
R-squared	0.6	0.63	0.64	0.63	0.63	0.64
Observations	21,712	15,152	15,152	11,193	10,233	15,146
Number of Firms	1,881	1,315	1,315	1,075	918	1,314
Number of Banks	30	28	28	27	28	27

Notes: Estimation results for regression (3.2), where the dependent variable covers term loan and credit line commitments. All specifications include firm-time fixed effects that additionally vary by rate type (adjustable- or fixed-rate) and whether the loan is syndicated (column iv) or the loan purpose (column v). Columns (i)-(v) include various bank controls at time t: bank size (natural log of total assets), return on assets (net income/total assets), deposit share (total deposits/total assets), banks' income gap, and the ratio of unused credit lines to assets (see Table B.3 in Appendix B for details on the data). Column (vi) includes bank-time fixed effects. All specifications are estimated using OLS. Standard errors in parentheses are clustered by bank. Sample: 2018:Q1 - 2020:Q2. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table D.8: Effects at the Firm Level − Credit Supply.

	Δ Total Debt		Investment	
	(i)	(ii)	(iii)	(iv)
Capital	0.14***	2.62**	-0.17***	2.08***
	(0.04)	(1.03)	(0.01)	(0.75)
Low-PD		6.11		9.25***
		(4.37)		(3.33)
Capital × Low-PD		-3.55***		-1.50**
		(0.86)		(0.62)
Fixed Effects				
Firm	✓	✓	✓	✓
Time × Industry	✓	✓	✓	✓
Firm Controls	✓	✓	✓	✓
R-squared	0.4	0.4	0.39	0.39
Observations	82,204	82,204	74,926	74,926
Number of Firms	13,861	13,861	12,081	12,081
Number of Banks	37	37	37	37

Notes: Estimation results for regression (3.3), where $y_{i,t}$ is either given by total firm debt in columns (i) and (ii) or fixed assets in columns (iii) and (iv). All specifications include firm fixed effects, industry-time fixed effects, and various firm controls dated at time t: cash, net income, tangible assets, liabilities (all relative to assets), firm size (natural log of total assets), public-firm-indicator, total term loans/debt, total observed unused credit/debt. Standard errors in parentheses are two-way clustered by main-bank and firm. All specifications are estimated using OLS. Sample: 2016:Q3-2020:Q4. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

E Marginal Revenue Product of Capital

This section outlines how we approximate firms' marginal revenue product of capital. Using standard notation, assume that firm i faces production function and isoelastic demand

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha_i} L_{i,t}^{\beta_i} \quad , \quad Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\sigma} Y_t \ .$$

Maximizing profits

$$\Pi_{i,t} = P_{i,t} Y_{i,t} - (1 + \tau_{i,t}^K)(r_t + \delta_{i,t}) K_{i,t} - (1 + \tau_{i,t}^L) w_{i,t} L_{i,t}$$

with respect to capital yields

$$MRPK_{i,t} = \alpha_i \left(\frac{\sigma - 1}{\sigma}\right)\left(\frac{P_{i,t} Y_{i,t}}{K_{i,t}}\right) = (1 + \tau_{i,t}^K)(r_t + \delta_{i,t}) \ ,$$

where $\tau^K_{i,t}$ and $\tau^L_{i,t}$ denote capital and labor distortions for firm i at time t (see, e.g., Hsieh and Klenow, 2009). To compare $MRPK_{i,t}$ across firms, we approximate

$$\frac{P_{i,t}Y_{i,t}}{K_{i,t}} \approx \frac{\text{Sales}_{i,t}}{\text{Fixed Assets}_{i,t}} \quad , \quad \alpha_i \approx Median_i(\frac{\text{Operating Surplus}_{i,t}}{\text{Sales}_{i,t}}) \, ,$$

which are based on firm balance sheet and income statements. The results are robust to using Compustat data to approximate α by industry s

$$\alpha_{i,s} \approx \alpha_s \approx Mean_s(\frac{\text{Operating Surplus}_{i,s,t}}{\text{Sales}_{i,s,t}}) \, .$$

F Dynamic model

F.1 Competitive lenders

For most points in the state space, the equilibrium can be characterized by the condition (4.15). However, for our computations, we employ the more general condition,

$$Q^{comp}(s) = \max_{Q} : Q^{zero}(z, b'(s;Q), k'(s;Q)) \geq Q \quad , \tag{F.1}$$

because it takes care of two potential issues. First, if there is more than one solution for which lenders would make zero profits, we pick the one with the larger Q because firms would be better off. Second, we allow for instances that lenders make positive profits, $Q^{zero}(z, b'(s;Q), k'(s;Q)) > Q$, only if a larger Q for which $Q^{zero}(z, b'(s;Q), k'(s;Q)) \geq Q$ does not exist, something that may occur if the policy functions are discontinuous in Q.