Consumption, Saving, and Investment

Econ 4021, Washington University in St. Louis

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Introduction

- Previous lectures: factors that determine production in the short-run
- That is also known as the "supply-side" of the economy
- We now move to how the demand for goods and services is determined
- Recall the income-expenditure identity

$$Y = C + I + G + X - M$$

  Output Y can be used to satisfy:
  - Consumption of goods and services C
  - Investment in capital goods I
  - Government consumption and investment G
  - Net foreign demand for goods and services $X - M$
- We will ignore the last component for now and focus on the simpler case of a **closed economy**

Introduction

This series of lectures:

1. Consumption and saving
2. Investment
3. Goods market equilibrium

1. Consumption and saving

Aggregate Consumption and Saving

- At an individual level, this is really one decision only

  $$\text{income} = \text{consumption} + \text{saving}$$

  (similar to the labor/leisure decision)
- **Desired Consumption**: aggregate quantity of goods and services that households want to consume given income, interest rates, and other factors

  $$C^d(Y, r) = \sum_{i=1}^{I} c_i^d(y_i, r)$$

  If individual incomes fall $y_i \downarrow$, desired individual consumption falls $c_i^d \downarrow$ and so aggregate desired consumption falls $C^d \downarrow$
- **Desired national saving**: level of national saving that occurs when consumption is at its desired level

  $$S^d = Y - C^d - G$$

Individual Consumption and Saving

- Where does $c_i^d(y_i, r)$ come from?
- Households solve an **intertemporal utility maximization problem** to determine their optimal consumption and saving, given income, wealth, and interest rates
- Available resources:

$$\text{Current income} = y$$
$$\text{Future income} = y^f$$
$$\text{Initial wealth} = a$$

- Choices to be made:

$$\text{Current consumption} = c$$
$$\text{Future consumption} = c^f$$
$$\text{Future wealth} = a^f$$

Individual Consumption and Saving

- In the current period, total available resources are equal to current income plus initial wealth, $y + a$
- Individual can use these resources to consume today or to accumulate wealth for next period
- This trade-off gives rise to today's budget constraint

$$c + a^f = y + a$$

- Tomorrow, total available resources are equal to future income plus saved wealth, compounded at the interest rate

$$c^f = y^f + a^f(1 + r)$$

- More savings today $a^f \uparrow$ means less consumption today $c \downarrow$, but more consumption in the future $c^f \uparrow$

Intertemporal Budget Constraint

- Note that we can rewrite today's budget constraint as

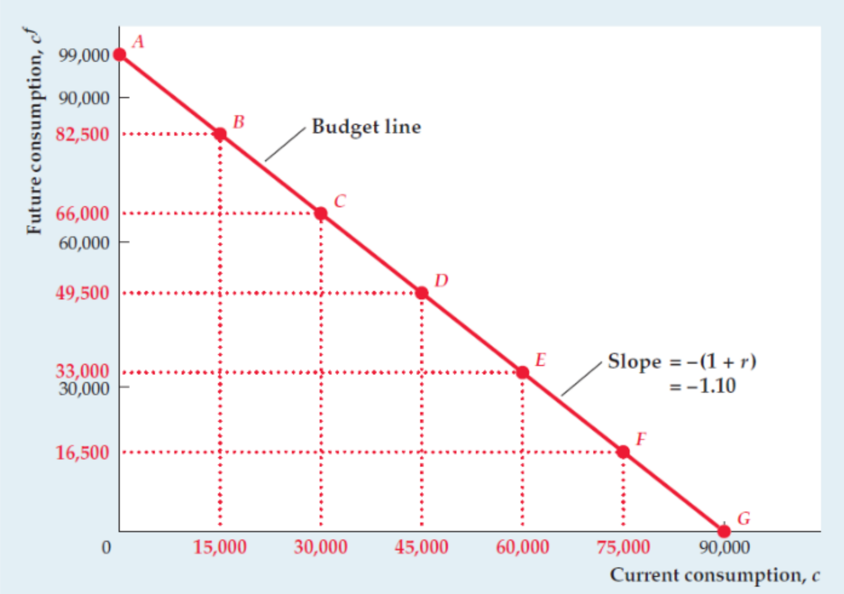
$$a^f = y + a - c$$

- Today's savings are equal to available resources minus today's consumption.
- If $a^f > 0$, we say that the individual is saving
- If $a^f < 0$, we say that the individual is dissaving, or borrowing
- Plug this in tomorrow's budget constraint to derive the **intertemporal budget constraint**

$$c^f = y^f + a^f(1 + r)$$
$$c^f = y^f + (y + a - c)(1 + r)$$

- Highlights trade-off between consuming today and tomorrow

Intertemporal Budget Constraint



Interest rates and Present Values

- The slope of the IBC is $-(1+r)$, the relative price of intertemporal consumption
- If I consume an extra \$1 today, I forego savings that would allow me to consume $\$(1+r)$ tomorrow
- This means that if you want to buy something that costs \$1 tomorrow, you have to save less than \$ 1 today: you only need to save $\$\frac{1}{1+r}$
- $\frac{1}{1+r}$ is the **present value** of \$ 1 tomorrow. PV's are often used by economists to compare values across time

$$\text{Present Value} = \frac{\text{Future Value}}{1+r}$$

- Example: if $r = 10\%$, then \$12,000 invested for one year is worth $\$12,000 \times (1+10\%) = \$13,200$. So the present value of \$13,200 in one year is \$12,000 today.

Interest rates and Present Values

- In our model, future income is y^f and the interest rate is r
- We can use this to write the **present value of lifetime resources** as

$$PVLR = a + y + \frac{y^f}{1+r}$$

- This is the total value of resources that are available for consumption, across time, in comparable units (present values)
- Recall the expression for the intertemporal budget constraint

$$c^f = y^f + (y + a - c)(1+r)$$

- We can rewrite it as

$$c + \frac{c^f}{1+r} = y + a + \frac{y^f}{1+r} = PVLR$$

- The **present value of lifetime consumption** must equal the PVLR
- The IBC intercepts the x-axis at $c = PVLR$ and the y-axis at $c^f = (1+r)PVLR$

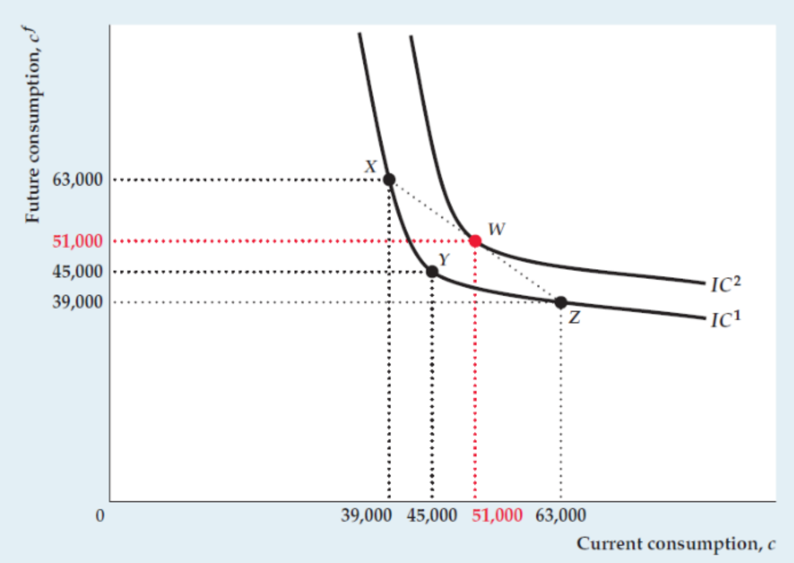
Intertemporal Preferences and Utility

- We have now described the **intertemporal budget constraint** (IBC): the set of possible combinations of current and future consumption that are feasible given income and interest rates
- So what pair (c, c^f) would the consumer like to choose?
- As with the labor supply decision, this will be the point that **maximizes utility** over current and future consumption

$$U = u(c, c^f)$$

- **Indifference Curves** for a given level of utility $\bar{U}$ are the different combinations of current and future consumption (c, c^f) that yield the same level of utility $\bar{U}$

Indifference Curves



Indifference Curves

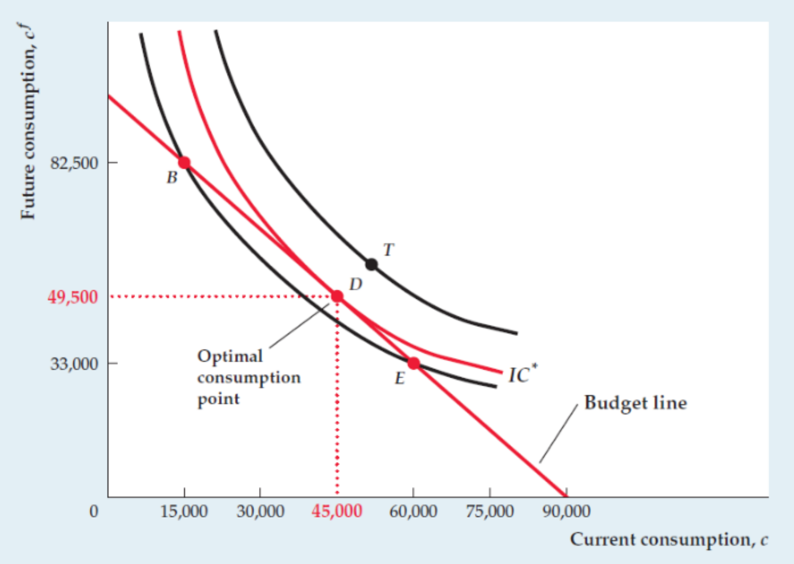
Properties:

1. Indifference curves slope downward from left to right, this reflects the fact that utility is increasing in both current and future consumption. So if $c^f \uparrow$, then $c \downarrow$ for utility to remain constant.

2. Indifference curves that are further up to the right represent higher levels of utility, again reflecting that utility is increasing in both c and c^f

3. Indifference curves are bowed towards the origin, reflecting the **consumption-smoothing motive**: people like to consume "intermediate bundles" and dislike very extreme combinations of current and future consumption

Optimal Level of Consumption



Optimal Level of Consumption

- The optimal bundle is the "best" bundle that is feasible given the intertemporal budget constraint
- The best bundle that is feasible is the point where the indifference curve is tangent to the IBC
- Mathematically, this is analogous to solving the following constrained optimization problem

$$\max_{c,c^f} u(c, c^f)$$

$$\text{s.t. } c + \frac{c^f}{1+r} = a + y + \frac{y^f}{1+r} (= PVLR)$$

- Use the IBC to replace $c^f = (1+r)PVLR - (1+r)c$ and solve

$$\max_{c} u(c, (1+r)PVLR - (1+r)c)$$

Optimal Level of Consumption

- Take first-order conditions:

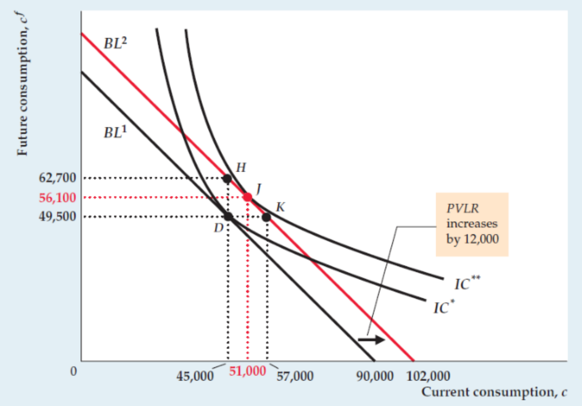
$$\frac{\partial u(c, c^f)}{\partial c} + \frac{\partial u(c, c^f)}{\partial c^f}(-1)(1+r) = 0$$

$$\Leftrightarrow -\frac{\frac{\partial u(c,c^f)}{\partial c}}{\frac{\partial u(c,c^f)}{\partial c^f}} = -(1+r)$$

- This is the same thing as saying that the slope of indifference curve = slope of the IBC at the optimal bundle
- Note that the optimal choice (c, c^f) depends on only two things:
  - The interest rate r, which determines the slope of the IBC
  - The PVLR, which determines the intercepts of the IBC
- This leads to a very important insight: **the effect on consumption of changes in current income, future income, or wealth depends only on how that change affects the consumer's PVLR**

Effects of Changes in Income and Wealth

- Any change in (y, y^f, a) simply expands or contracts the IBC
- If any of these increase, then $PVLR \uparrow$. Due to the consumption-smoothing motive, both current and future consumption increase



Permanent Income Theory

- Since only PVLR matters for the optimal choice of current and future consumption, it does not matter whether the change in PVLR is coming from current or future income (as long as it has the same present value)
- This irrelevance result is known as **Permanent Income Theory** (PIT), developed in the 1950s by Milton Friedman
- This theory posits that only permanent, not temporary, fluctuations in income have a significant effect on consumption over the lifecycle
- A temporary increase in income is such that $y\uparrow$ and y^f is unchanged $\Rightarrow$ PVLR does not change much, and so (c, c^f) do not change much
- A permanent increase in income is such that both $y\uparrow$ and $y^f\uparrow \Rightarrow$ PVLR $\uparrow\uparrow$ and so (c, c^f) both increase

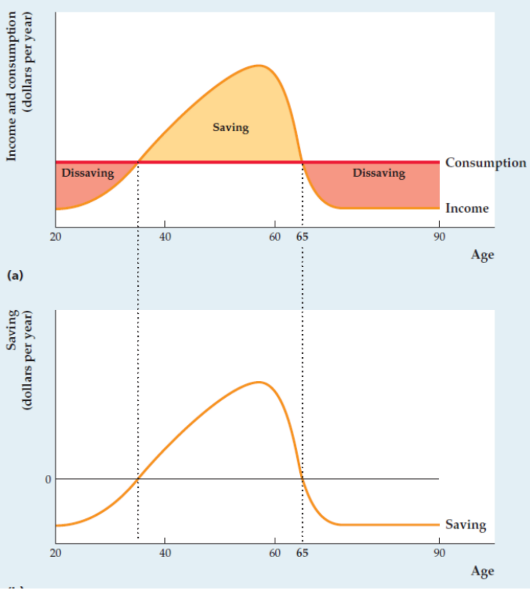
Permanent Income Theory

- Note that while temporary changes in income have no significant effect on current and future consumption, they do affect saving. Recall the static budget constraint:
$$a^f = a + y - c$$
- If $PVLR \uparrow$ because of an expected increase in future income and so $c \uparrow$, then $a^f \downarrow$ as current income has not changed

The Life-Cycle Model

- The life-cycle model was developed by Franco Modigliani in the 1950s and is a generalization of our simple model to many periods
- It tries to capture the life-cycle profile of income of the average individual:
  1. Low income when young and not working
  2. Higher income when prime age and working
  3. Lower income when older and retired
- PIT and the consumption smoothing motives tell us that consumption should be much smoother across the lifecycle, even if income is not
- This implies large fluctuations in saving: dissaving when young (borrowing), saving when working, and dissaving again when older

The Life-Cycle Model

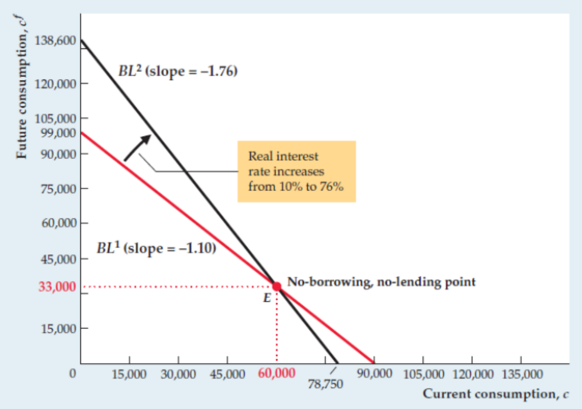


Borrowing Constraints and Excess Sensitivity

- The life-cycle model assumes **perfect capital markets**, i.e. people can freely borrow and save at the same interest rate all through their lives
- In practice, this is a big simplification as credit markets are not perfect:
  1. The interest rate for saving is usually much lower than the interest rate for borrowing, so borrowing is harder than saving
  2. Banks and other lenders impose limits on how much borrowing individuals can do, in particular they often ask for guarantees and collateral
  3. This is why it is harder for younger people and children to borrow and smooth their consumption!
- These **borrowing constraints** break the predictions of the life-cycle and permanent income models, as they imply **excess sensitivity** of consumption to current income
- While the theory predicts that c should depend on PVLR only, in practice it often depends on current income $(a + y)$ only, particularly for those that have more limited access to credit markets, such as lower income people

Changes in the Real Interest Rate

- How does the optimal consumption decision change hen r changes?
- Instead of expanding, the IBC now rotates around the endowment point



Changes in the Real Interest Rate

- When $r \uparrow$, the maximum amount of current consumption the individual can choose falls and the maximum amount of future consumption increases

- That happens because it becomes more expensive to borrow (bring future income to the present), and future consumption becomes cheaper ($ 1 saved allows for more consumption tomorrow)

- The change in the real interest rate triggers two opposing effects:

  1. A **substitution effect**

  2. An **income effect**

Increase in the Real Interest Rate

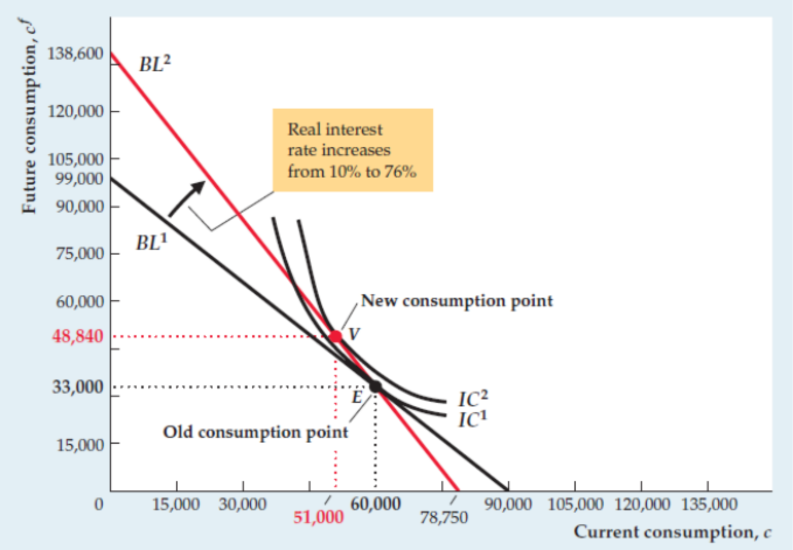
1. **Substitution effect**: as future consumption becomes cheaper, $\frac{1}{1+r} \downarrow$. This induces the individual to consume less today and more in the future, i.e. to save more.

2. **Income effect**: depends on whether the individual was a borrower or a lender before the interest rate change.

   - If the individual was a borrower, $c > a + y$, then they become poorer as their borrowing becomes more expensive. This leads to a fall in both current and future consumption.

   - If the individual was a saver, $c < a + y$, then they become richer as their savings yield more future income. This leads to a rise in both current and future consumption

Increase in the Real Interest Rate



Increase in the Real Interest Rate

Which effect dominates?

- Depends on the utility function, the change in the interest rate, and the values of current and future income
- For **savers**, both effects imply that $c^f \uparrow$ but the effect on c is ambiguous (substitution effect $\downarrow$, income effect $\uparrow$)
- For **borrowers**, both effects imply that $c \downarrow$, but the effect on c^f is ambiguous (SE implies $c^f \uparrow$ but IE implies $c^f \downarrow$)
- What happens to savings in the aggregate is ambiguous.
- Empirically, most evidence points towards a positive relationship between aggregate savings and the real interest rate
- We will assume this is the case for the rest of the course

Back to Aggregate Consumption and Saving

- We have studied how changes in income, wealth, and interest rates affect **individual consumption and saving decisions**
- Aggregate consumption and saving are simply the sum of individual consumption and saving decisions, for given levels of income and real interest rates

$$C = \sum_{i=1}^{I} c_i$$

$$S = \sum_{i=1}^{I} a_i^f$$

- Recall that total desired savings depend not just on private consumption but also on government consumption

$$S^d = Y - C^d - G$$

Government Purchases

- The government faces a budget constraint (GBC)

$$G = T$$

- For now let's ignore public debt and deficits: all government spending must be financed with taxes
- Assume that there is only one representative individual in this economy, who has to pay these taxes. The IBC is

$$C + \frac{C^f}{1+r} = A + Y - T + \frac{Y^f}{1+r}$$

- Replace the GBC in the IBC:

$$C + \frac{C^f}{1+r} = A + Y - G + \frac{Y^f}{1+r}$$

- An increase in G implies an increase in T, which reduces current income and the PVLR
- For that reason, current consumption falls, but by less than the increase in G (due to consumption smoothing)

Intertemporal Government Budget Constraint

- What if the government does not raise taxes today and borrows instead?
- We can write the government's budget constraint today and in the future as

$$G = T + B^g$$
$$(1+r)B^g + G^f = T^f$$

- The government may issue debt B^g, which needs to be paid for with future taxes T^f (plus interest)
- Like with the individual, we can collapse the two to a single **intertemporal government budget constraint** (IGBC)

$$G + \frac{G^f}{1+r} = T + \frac{T^f}{1+r}$$

- Present and future government spending must all be paid for with taxes - current and/or future

Ricardian Equivalence

- What happens then if $G \uparrow$ and the government issues debt instead of raising taxes today?
- Individuals understand that this debt (plus interest) will have to be repaid by raising future taxes
- Thus future income falls, which causes current consumption to fall and savings to rise even with no change in current taxes and income

Ricardian Equivalence

- Note that we can replace the IGBC in the individual's IBC

$$
\begin{aligned}
C + \frac{C^f}{1+r} &= A + Y - T + \frac{Y^f - T^f}{1+r} \\
&= A + Y + \frac{Y^f}{1+r} - \left(T + \frac{T^f}{1+r} \right) \\
&= A + Y + \frac{Y^f}{1+r} - \left(G + \frac{G^f}{1+r} \right)
\end{aligned}
$$

- Thus the timing of taxes does not matter for the individual's PVLR - only the timing of government expenditure matters

- This result is known as the **Ricardian Equivalence**

Ricardian Equivalence

- The RE is a useful theoretical benchmark, but depends on many strong assumptions that rarely hold in practice:

  1. Perfect capital markets: the ability of individuals to borrow and save at the same interest rate, and to access the same interest rate as the government
  2. Infinitely-lived agents: if the individual is no longer alive by the time taxes are raised, then there is no effect on their future expected income!
  3. Fixed government expenditures: the RE assumes that the path of government consumption is fixed, while in practice it changes all the time.

- In practice, economists have found positive effects of government spending on consumption (i.e. tax rebates), which cast doubt on the practical relevance of the RE

2. Investment

Investment

Investment spending by firms is important in macroeconomics for two main reasons:

1. It consists of the purchase of capital goods K, which enter the production function and thus expand the economy's future ability to produce goods and services. This makes it a crucial ingredient of economic growth.

2. It is an important component of total demand for goods and services, and so understanding investment is crucial for understanding the business cycle

- Investment is similar to saving in that it involves a trade-off between the present and the future
- When a firm decides to invest, it foregoes current profits to increase future profits
- For this reason, expectations about the future are very important to determine investment

Desired Capital Stock

- The **desired capital stock** is the optimal amount of capital that is chosen by firms, to maximize their profits
- This depends on the costs and benefits of investing in additional capital
  - The marginal benefit of investment is the future marginal product of capital, MPK
  - The marginal cost of investment is the **user cost of capital**
- Recall the problem of the firm, where f denotes future variables

$$\max_{K,N} A^f F(K,N) - uc \times K - w \times N$$

- The solution is

$$MPN = w$$
$$MPK = uc$$

- The level of K that solves this problem is the desired capital stock
- In practice we assume that firms choose capital one period in advance, and so the optimality condition is $MPK^f = uc$, where f stands for "future"

User Cost of Capital

- The user cost of capital, uc, is the expected real cost of using a unit of capital for a specific period of time
- It includes **depreciation costs** as well as the **opportunity cost of investment**
- Let p_k be the price of a machine (a capital good), then

$$uc = \underbrace{p_k \times r}_{\text{opportunity cost}} + \underbrace{p_k \times \delta}_{\text{depreciation cost}} = p_k \times (r + \delta)$$

- The true cost of a machine with a price tag p_k is:
  1. The fact that I am foregoing investing/lending p_k dollars, which could have earned me a net interest return equal to $r \times p_k$ for the length of time the machine is used
  2. The fact that the machine depreciates while I use it, which results in a loss equal to the depreciation rate δ times the value of the machine p_k

User Cost of Capital: Example

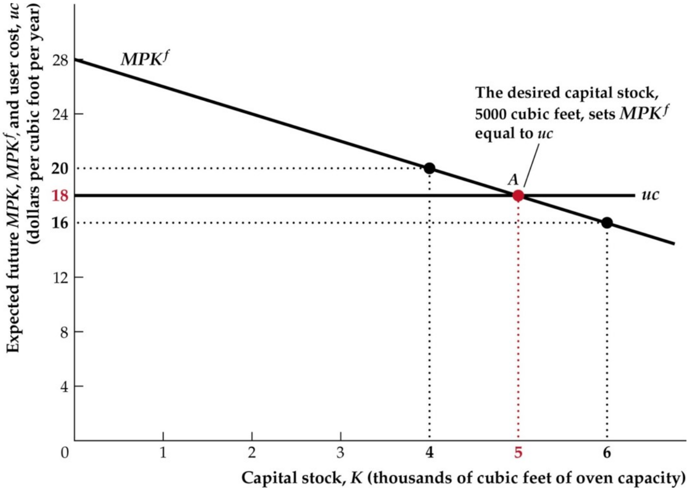
- Textbook example: Kyle's bakery
- Kyle is considering purchasing a solar-powered oven for his bakery
- Oven costs \$100, depreciates at 10% yearly, and the prevailing interest rate is 8%
- Kyle could invest \$100 in the bank for a year and earn

$$\$100 \times (1 + 8\%) - \$100 = \$8$$

- At the same time, if Kyle wants to sell the oven after a year of use, it will only be worth $\$100 \times (1 - 10\%) = \$90$
- The total user cost of the oven, for a year, is then $\$8 + \$10 = \$18$

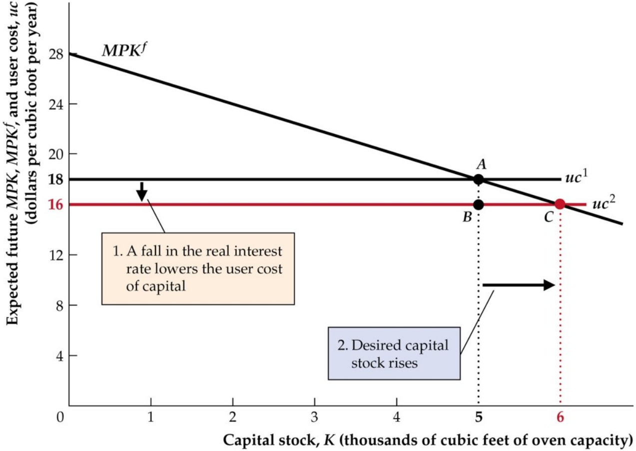
Desired Capital Stock

$$MPK^f = uc$$



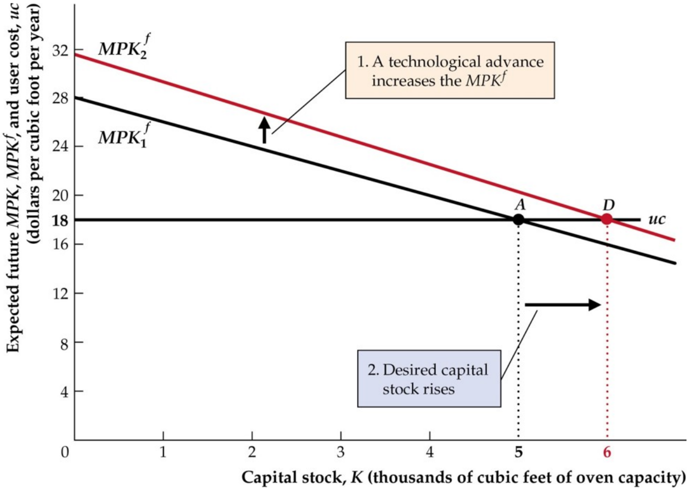
Desired Capital Stock

- Analysis is very similar to that of the demand for labor
- *MPK* is downward sloping due to decreasing marginal productivity
- Changes in the user cost of capital shift the *uc* line up and down, affecting the firm's demand for capital



Desired Capital Stock

Any factor that changes *MPK* changes the desired capital stock for a constant *uc*



Effect of Taxes on Desired Capital

- Productivity and taxes are among the factors that affect the MPK curve and, therefore, the desired capital stock and firm investment
- Assume that the firm pays a proportional tax τ on its revenue, so that its problem is now given by

$$\max_{K,N}(1-\tau)AF(K,N) - uc \times K - w \times N$$

- The optimal choice of capital is given by

$$(1-\tau)MPK = uc$$

- A decrease in τ is equivalent to an expansion of MPK, and thus results in more desired capital

Investment

- Investment is the change in the desired capital stock
- The total amount spent in new capital goods is called **gross investment** and denoted by I_t
- However, part of the existing capital stock also depreciates every period, and so part of the amount spent in capital goods is not really creating new capital but rather compensating for depreciating capital

$$\text{Depreciation}_t = \delta K_t$$

Investment

- **Net investment** is the actual change in the total capital stock of the economy, and is equal to gross investment minus depreciation

$$K_{t+1} - K_t = I_t - \delta K_{t-1}$$

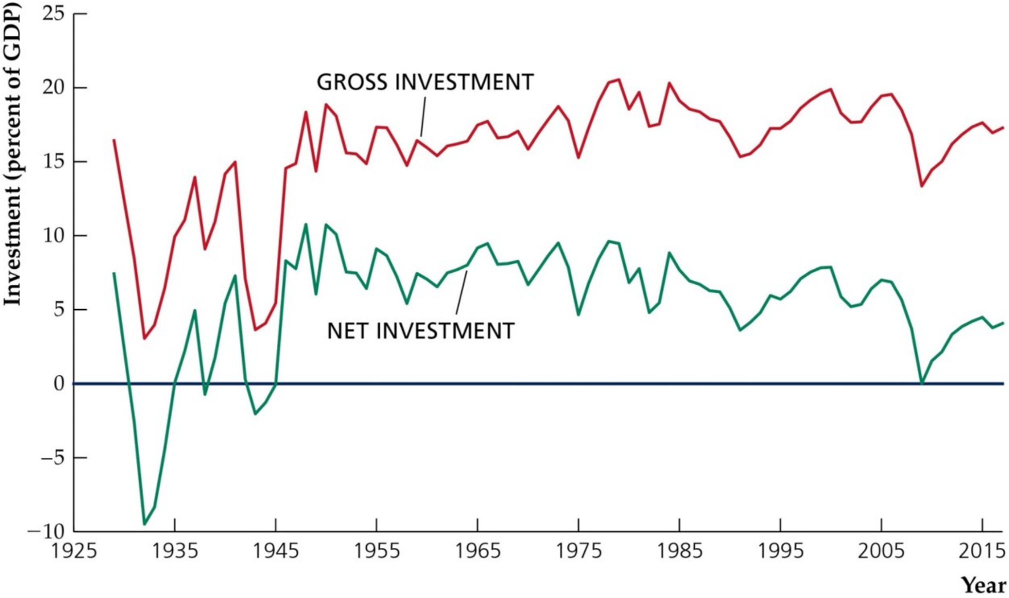
- This is usually written as

$$I_t = K_{t+1} - (1 - \delta) K_t$$

- If firms can freely adjust capital stock to their desired level, then

$$I_t = K^* - (1 - \delta) K_t$$

Gross and Net Investment in the US



Determinants of Desired Investment

Investment rises with

1. A fall in the real interest rate, which leads to a fall in the user cost of capital
2. A fall in the tax rate, which raises the effective MPK
3. A future increase in productivity that raises the future MPK

3. Goods market equilibrium

Goods Market Equilibrium

- Equilibrium in the goods (and services) market is attained when total production Y equals total demand for goods and services

$$Y = C^d + I^d + G$$

- We have analyzed the determinants of desired consumption C^d and desired investment I^d
- Note that we can rewrite the equilibrium condition as

$$Y - G - C^d = I^d$$

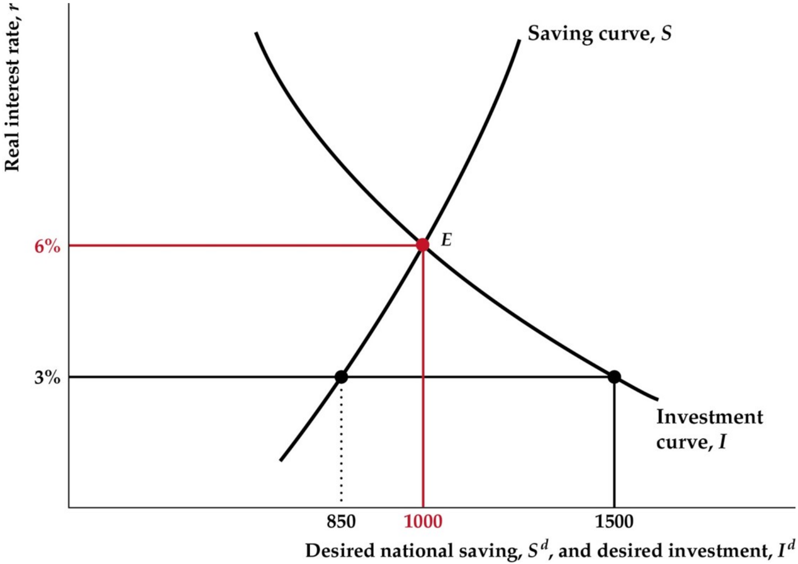
- We have seen that desired savings can be written as $S^d = Y - G - C^d$
- So equilibrium in the goods market is equivalent to equilibrium in the savings (or capital) market

$$S^d = I^d$$

- We know that S^d is increasing in the real interest rate and I^d is decreasing in the real interest rate

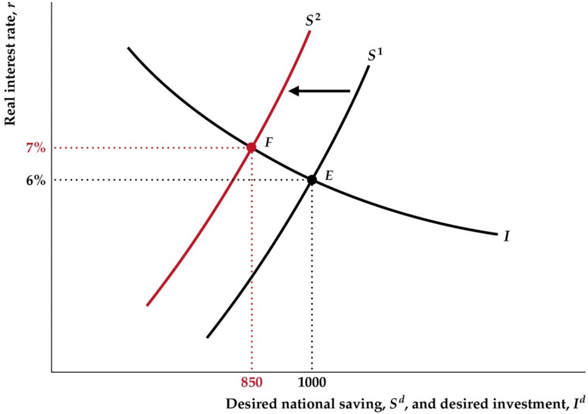
Saving-Investment Diagram

Equilibrium in the goods market is achieved by an equilibrium real interest rate r^*

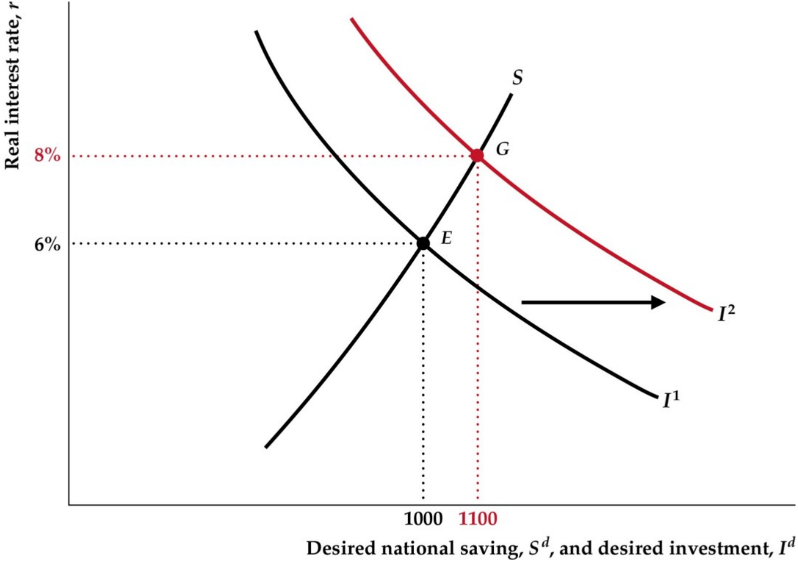


Shifts in the Saving Curve

- Any factor other than r that affects desired saving and investment leads to a shift in the respective curve, affecting r^*
- Savings curve contracts with (i) fall in current output, (ii) rise in future output, (iii) rise in wealth, (iv) rise in government purchases, (v) rise in taxes (if Ricardian Equivalence does not hold)



Shifts in the Investment Curve



Consumption and Stock Prices