

The Cost of Capital and Misallocation in the United States*

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Abstract

We develop a method to estimate the cost of capital using credit registry microdata, and apply it to study capital allocation efficiency in the United States. Our measure incorporates the contractual interest rate, expected default probability, recovery rate, and expectations of future rates. We estimate three distinct rates: (i) the lender's discount rate, (ii) the firm's cost of capital, and (iii) the social cost of capital. We derive a sufficient statistic for misallocation based on the first and second moments of the social cost of capital. Dispersion in this rate captures both heterogeneity in lender discounting and the presence of financial frictions. Normal times feature modest amounts of misallocation, corresponding to an output loss of 0.9%, but this increased to 1.8% during the 2020-21 period.

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1 Introduction

How much does it cost a firm to obtain capital? Economic models often simplify by assuming that all firms can borrow in a competitive market at a common rate. In reality, however, the cost of capital varies significantly across firms. This variation stems not only from differences in contractual interest rates but also from firm-specific factors such as default probabilities, loan terms, and lender cost of funds. Such heterogeneity in the cost of capital has profound implications: it can distort the allocation of capital across firms, leading to inefficiencies in economic output (Gilchrist et al., 2013; Hsieh and Klenow, 2009). Understanding these inefficiencies is critical for policymakers and researchers seeking to design effective financial and economic policies.

This paper makes two contributions to the literature. First, we develop a novel method that leverages a dynamic corporate finance model and uses credit registry microdata to measure the dispersion of the cost of capital. This method allows us to quantify how these variations contribute to capital misallocation. The theory implies a sufficient statistic for misallocation that can be directly measured with credit registry data. Second, we apply this method to U.S. data and uncover two primary insights. While the cost of capital is heterogeneous across firms, the implied misallocation is small. This misallocation, however, exhibits a substantial amount of variation over time, and, in particular, it doubled in 2020-21, around the COVID-19 pandemic.

The method we develop offers several advantages. Unlike traditional approaches that require strong assumptions about production functions as well as detailed data on firms financials such as sales or value added, our approach relies on credit registry data that is commonly collected by financial regulators around the developed and developing world, and uses sufficient statistics derived directly from moments of the data. This not only simplifies implementation but also provides more robust identification of the sources of misallocation without heavy reliance on calibration assumptions.

Section 2 describes the dynamic corporate finance model that provides the foundation

for the derivation of the sufficient statistic for misallocation and our empirical measurement of the cost of capital. The model captures firm-level borrowing, investment, and default decisions in the presence of idiosyncratic shocks, such as productivity fluctuations or stochastic fixed operating costs. Each firm borrows from a single lender who discounts future cash flows at a match-specific rate ρ . While we refer to ρ as the lender’s discount rate, it more broadly reflects variation in loan pricing that is not fully explained by observable loan terms. Using the firm’s optimality conditions, we show how ρ influences the firm’s internal cost of capital, which in turn determines its expected marginal revenue product of capital. This link forms the basis for our measure of misallocation.

In Section 3, we theoretically map heterogeneity in the cost of capital to economic efficiency costs. We compare the decentralized equilibrium to the allocation chosen by a planner who reallocates capital across firms to maximize aggregate output, subject to the same aggregate capital stock and taking firms’ default decisions as given. We define the social cost of capital, which is the marginal value of allocating an extra unit of capital for each firm from the point of view of the planner. We show that this measure is approximately equal to the sum of two terms: the lender’s discount rate plus a term that reflects financial frictions related to limited liability and recovery in case of default in the spirit of Cooley and Quadrini (2001). At the optimum, the planner would like to equate the social cost of capital across firms. This insight allows us to derive a sufficient statistic for the output loss from misallocation that depends only on the mean and variance of the social cost of capital. This statistic is robust to firm-level heterogeneity in production technologies and does not rely on structural estimation.

Section 4 describes how we use credit registry data for measurement. We define the lender’s discount rate as the internal rate of return that satisfies the lender’s break-even condition, accounting for both repayment probabilities and expected losses in default. To compute this rate at the firm level, we require loan-level data on contractual interest rates, loan maturities, borrower-specific probabilities of default, and loss given default (LGD), as well as the expected inflation term structure. In the case of floating-rate loans, we also

need expectations for forward benchmark interest rates. Using these variables as well as the equations of the model, we estimate three distinct rates: the lender’s discount rate, the firm’s cost of capital, and the social cost of capital. We then apply the sufficient statistic to the social cost of capital to estimate the output loss from misallocation.

Section 5 presents our empirical findings. Using data from over sixty thousand loans originated between 2014 and 2024, we show that the average measures of cost of capital closely track the five-year U.S. Treasury rate. We estimate three distinct rates. First, the *lender’s discount rate* is specific to each borrower-lender pair and captures the efficiency of the credit match. It has a mean of 1.9% and a standard deviation of 1.6% over the period in analysis. Second, the *firm’s cost of capital*—defined as the expected payment by the firm conditional on no default—has a mean of 0.9% and a standard deviation of 2.8%. This rate is lower than the lender’s discount rate because the firm does not internalize recoveries in the case of default, unlike the lender. Finally, the *social cost of capital* reflects the total return on capital from a social perspective, incorporating expected recovery in the event of default. It has a mean of 1.7% and a variance of 1.8%. For comparison, the average five-year U.S. Treasury rate over this period was 0.4%, in real terms.

At the optimum, the planner seeks to equalize the social cost of capital across firms. Our sufficient statistic provides a mapping from the variance of the social cost of capital—1.8% on average—to output losses due to misallocation. We estimate that, under normal conditions, the implied output loss from capital misallocation is modest, around 0.9%. However, this loss increased significantly during the COVID-19 pandemic (2020–2021), rising to 1.8% at its peak.

We investigate the drivers of this increase in misallocation. First, we show that the social cost of capital can be decomposed into the sum of two terms: one that reflects lender discount rates and another that reflects financial frictions related to default (limited liability and recovery rates). We then show that the increase in misallocation was driven by rising dispersion in lender discount rates, rather than by a worsening of the financial frictions term. Recall that the lender discount rate reflects fluctuations in pricing that do not reflect

fundamental observable characteristics of the loan. We find that this increase dispersion in lender discount rates was driven by an increase in the dispersion of expected losses (the product of the probability of default and loss given default) that was not reflected in increased dispersion in contractual interest rates. Thus interest rates did not “move enough” to reflect fluctuations in loan-level risk, which led to dispersion in discount rates and thus inefficient allocation of credit.

Finally, we show that our measure of social cost of capital is correlated with measures of ARPK that are commonly used in the literature. The social cost of capital tends to display stronger correlation with ARPK measures that are based on value added, rather than sales or measures of earnings. The social cost of capital implies lower overall levels of capital misallocation than those that would be implied by the ARPK-based measures.

Literature Review. Our paper contributes to the broader literature on measuring misallocation. Following seminal work by [Restuccia and Rogerson \(2008\)](#) and [Hsieh and Klenow \(2009\)](#), there has been significant progress in quantifying misallocation across various settings (see [Hopenhayn \(2014\)](#) and [Restuccia and Rogerson \(2017\)](#) for comprehensive reviews). A key challenge in this literature is measuring misallocation without imposing strong assumptions on firms’ production technologies. [Haltiwanger et al. \(2018\)](#) emphasize that standard approaches are only valid under restrictive assumptions, such as a common Cobb-Douglas production function with firm-specific productivity shifters.

One strand of the literature focuses on specific sources of distortions. For instance, [Kaymak and Schott \(2024\)](#) study corporate tax asymmetries and find that heterogeneity in effective marginal tax rates can distort capital and labor allocation, reducing aggregate productivity. Alternatively, recent work has sought to directly estimate marginal products using (quasi-)experimental variation, allowing for richer production heterogeneity (e.g., ([Carrillo et al., 2023](#); [Hughes and Majerovitz, 2025](#))). However, such approaches have only been applied in narrow contexts where experimental variation is available.

Our paper measures heterogeneity in the marginal product of capital by exploiting firm-

level variation in the cost of capital, allowing us to assess misallocation across a much broader set of firms while remaining agnostic to functional form assumptions. Closest to our approach, [Gilchrist et al. \(2013\)](#) develop a tractable framework to quantify misallocation arising from dispersion in borrowing costs. We emphasize two key differences. First, a portion of the credit spread dispersion in [Gilchrist et al. \(2013\)](#) reflects variation in default probabilities and recovery rates, whereas our framework explicitly models corporate default. Second, their analysis relies on corporate bond data, which restricts attention to large firms. In contrast, we use bank loan data that encompasses a significantly wider range of firms, including small, medium, and large-sized enterprises.

We also contribute to a literature that estimates heterogeneity across firms in interest rates and/or the cost of capital. [Banerjee and Duflo \(2005\)](#) summarize early evidence for substantial heterogeneity in interest rates across borrowers in developing countries, arguing that this heterogeneity implies significant misallocation. Recent work by [Gormsen and Huber \(2023, 2024\)](#) analyzes transcripts of firm earnings calls to extract information on the discount rates and cost of capital that firms use. [Cavalcanti et al. \(2024\)](#) use credit registry data to study heterogeneity in interest rates for borrowing firms in Brazil. They find substantial heterogeneity across firms and use a dynamic structural model with financial frictions to infer the cost of capital. This paper also builds on the findings of [Faria-e-Castro et al. \(2024\)](#), who analyze the dispersion in borrowing rates for U.S. firms using a comprehensive database of loans and bonds. Their study highlights significant heterogeneity in borrowing costs, even within firms, and demonstrates the persistent impact of borrowing costs on firm-level investment and borrowing behaviors.

Relative to this previous literature, our paper makes two key methodological contributions. First, we provide a method to estimate a firm’s cost of capital from credit registry data. This is not as simple as measuring the interest rate because the cost of capital depends on the ex-ante repayment probability and expected losses given default. Second, we show how to use moments of the distribution of the cost of capital to develop sufficient statistics that allow us to measure the cost of misallocation non-parametrically in a dynamic,

stochastic model.

2 Corporate Finance Model

This section outlines the core components of the model, which we use as a measurement device. We demonstrate how the model’s optimality conditions can be derived and integrated with microdata on loan characteristics to estimate the lender’s discount rate and the firm’s cost of capital. These rates are used to infer the firm’s expected marginal product of capital, which is a key input for our measure of misallocation.

Time is discrete and indexed by $t = 0, 1, \dots$. The economy is populated by firms that borrow and invest, and by lenders who finance those firms. There is a unit mass of firms, indexed by i , who exit over time. We assume that every firm that exits is replaced by a firm with identical characteristics that does not produce in the current period, such that the mass of firms is constant and equal to 1. Each firm is matched with a lender. There is no aggregate risk. We now describe the decision problem of the borrowing firm, and its interaction with the lenders.

Borrowers. The borrowers in the model are firms operating in the nonfinancial sector. These firms operate under limited liability and make decisions regarding production, investment, and borrowing. Output (net of non-capital costs) is generated using a production function $f(k_i, z_i)$, where k_i represents capital and z_i denotes a vector of shocks that affect firm net output. We allow z_i to be a vector; this accommodates productivity shocks, stochastic fixed costs, as well as rich heterogeneity in the production function. To sustain or expand their operations, firms invest in capital and issue long-term defaultable debt b_i . In the event of default, lenders recover an amount $\phi_i(k_i)$ that depends on the stock of firm assets k_i .

Lenders. Lenders finance firms, with each firm matched to a single lender. Upon matching, the borrower-lender pair draws a realization of ρ_i , which represents the discount rate that the

lender uses to price debt.¹ For this reason, we refer to ρ_i as the lender's discount rate. Loans are priced so that lenders break even using ρ_i as their discount rate, taking into account firm-specific characteristics and risk assessments.

Firm's Problem. Firms determine their investment and borrowing strategies to maximize their value, taking into account the possibility of future default. The value of repayment for a firm is expressed as:

$$V_i(k_i, b_i, z_i) = \max_{k'_i, b'_i} \pi(k_i, b_i, z_i, k'_i, b'_i) + \beta \mathbb{E} [\max \{V_i(k'_i, b'_i, z'_i), 0\} | z_i], \quad (1)$$

where $\pi(k_i, b_i, z_i, k'_i, b'_i)$ denotes the firm's profit function, and β represents the discount factor. The profit function captures the firm's net return from production and financing decisions:

$$\pi(k_i, b_i, z_i, k'_i, b'_i) = f(k_i, z_i) + (1 - \delta)k_i - k'_i - \theta_i b_i + Q_i(k'_i, b'_i, z_i)[b'_i - (1 - \theta_i)b_i].$$

Here, $f(k_i, z_i)$ represents the firm's net output as a function of capital k_i and productivity z_i , $(1 - \delta)k_i$ accounts for the depreciated value of current capital, and k'_i denotes new capital investment. We model long-term debt as a geometrically decaying perpetuity with rate θ_i . Thus $\theta_i b_i$ reflects repayment on existing debt, while $Q_i(k'_i, b'_i, z_i)$ captures the price of newly issued debt, with $b'_i - (1 - \theta_i)b_i$ representing the net new borrowing.

Debt Pricing. Lenders are risk-neutral and price debt based on their cost of capital, ρ_i . The price of debt $Q_i(k'_i, b'_i, z_i)$ is determined as:

$$Q_i(k'_i, b'_i, z_i) = \frac{\mathbb{E} \left\{ \mathcal{P}_i(k'_i, b'_i, z'_i) [\theta_i + (1 - \theta_i)Q_i(k''_i, b''_i, z'_i)] + (1 - \mathcal{P}_i(k'_i, b'_i, z'_i)) \frac{\phi_i(k'_i)}{b'_i} \middle| k'_i, b'_i, z_i \right\}}{1 + \rho_i}, \quad (2)$$

where $\mathcal{P}_i(k'_i, b'_i, z'_i)$ is an indicator function that is equal to 1 if the firm repays, and 0 otherwise, and $\phi_i(k'_i)/b'_i$ is the recovery rate in the event of default, per dollar lent.

¹This variable captures both lender- and borrower-specific factors that lie outside the scope of the model, such as lender financing costs, risk appetite, or the dynamics of relationship lending. While we do not provide a specific microfoundation for the heterogeneity in ρ_i , we focus on analyzing its implications.

Solution to the Firm's Problem. The solution to the firm's problem in (1) is characterized by two first-order conditions, with respect to capital k'_i and debt b'_i :

$$\begin{aligned} [k'_i] : & -1 + \frac{\partial Q_i(k'_i, b'_i, z_i)}{\partial k'_i} [b'_i - (1 - \theta_i)b_i] + \beta \mathbb{E} \{ \mathcal{P}_i(k'_i, b'_i, z'_i) [f_k(k'_i, z'_i) + 1 - \delta] | z_i \} = 0 \\ [b'_i] : & \frac{\partial Q_i(k'_i, b'_i, z_i)}{\partial b'_i} [b'_i - (1 - \theta_i)b_i] + Q_i(k'_i, b'_i, z_i) - \beta \mathbb{E} \{ \mathcal{P}_i(k'_i, b'_i, z'_i) [\theta_i + (1 - \theta_i)Q_i(k''_i, b''_i, z'_i)] | z_i \} = 0 \end{aligned}$$

where we use the envelope conditions $\frac{\partial V_i}{\partial k_i} = f_k(k_i, z_i) + 1 - \delta$ and $\frac{\partial V_i}{\partial b_i} = -[\theta_i + (1 - \theta_i)Q_i]$.

Throughout, we assume that the firm's discount factor β is low enough such that the firm chooses an interior solution for debt.² Simplifying notation, so that \mathcal{P}'_i is a shorthand for $\mathcal{P}_i(k'_i, b'_i, z'_i)$, we can combine the two first-order conditions to write:

$$\underbrace{\frac{\mathbb{E} [\mathcal{P}'_i(\theta_i + (1 - \theta_i)Q'_i) | z_i]}{Q_i}}_{(1)} \times \underbrace{\left[\frac{1 - \frac{\partial Q_i}{\partial k'_i} [b'_i - (1 - \theta_i)b_i]}{1 + \frac{\partial Q_i}{\partial b'_i} \frac{[b'_i - (1 - \theta_i)b_i]}{Q_i}} \right]}_{(2)} = \underbrace{\mathbb{E} [\mathcal{P}'_i(f_k(k'_i, z'_i) + 1 - \delta) | z_i]}_{(3)} \quad (3)$$

Equation (3) is going to be key to our analysis. It relates a measure of the firm's perceived marginal cost of funds (first term), multiplied by an adjustment factor that reflects the impact of firm decisions on its price of debt (second term), to the firm's expected marginal product of capital (third term). Our empirical strategy is based on measuring the first term in the data, and using it to infer the third term.

The Firm's Cost of Capital. We define the firm's cost of capital, r_i^{firm} , as the ratio of the expected value of future repayments adjusted for the probability of repayment, $\mathbb{E} [\mathcal{P}'_i(\theta_i + (1 - \theta_i)Q'_i)]$, relative to the current price of borrowing, Q_i .³ It corresponds to the first term in equation (3). The firm's cost of capital is the expected implicit interest rate that it pays on its debt. Formally, it is expressed as:

$$1 + r_i^{firm} = \frac{\mathbb{E} [\mathcal{P}'_i(\theta_i + (1 - \theta_i)Q'_i) | k'_i, b'_i, z_i]}{Q_i}. \quad (4)$$

This equation captures how the firm's borrowing cost depends on repayment probabilities and debt maturity. The firm's cost of capital is one of the key components of the firm's first

² $\beta < \frac{1}{1+\rho_i}$ is sufficient but not necessary, since lenders value recovery in default states.

³We use \mathcal{P}'_i as a shorthand for $\mathcal{P}_i(k'_i, b'_i, z'_i)$, and the same for Q'_i .

order condition with respect to capital. In what follows, we show how to measure r_i^{firm} in the data, and this will give us information about the marginal revenue product of capital.

Proposition 1 characterizes the firm's cost of capital. All proofs are in Appendix A.

Proposition 1 (Firm's Cost of Capital). *The firm's cost of capital can be written as:*

$$1 + r_i^{firm} = \frac{1 + \rho_i}{1 + \Lambda_i}, \quad \Lambda_i := \frac{\mathbb{E}[(1 - \mathcal{P}'_i) \phi_i(k'_i)/b'_i | k'_i, b'_i, z_i]}{\mathbb{E}[\mathcal{P}'_i(\theta + (1 - \theta_i)Q'_i) | k'_i, b'_i, z_i]}.$$

The term Λ_i represents the wedge between the borrower's cost of capital, r_i^{firm} , and the lender's discount rate, ρ_i . This wedge arises due to lender recovery in the event of default. When there is no recovery ($\phi_i = 0$), the wedge disappears ($\Lambda_i = 0$), and the firm's cost of capital equals the lender's discount rate ($r_i^{firm} = \rho_i$). On the other hand, when the lender can recover some value after default ($\phi_i > 0$), the wedge becomes positive ($\Lambda_i > 0$), and the firm's cost of capital r_i^{firm} is lower than ρ_i . This reduction in perceived borrowing cost occurs because the borrower only accounts for states where repayment occurs. Thus the firm expects to pay a lower rate than the lender expects to recover.

Marginal Revenue Product of Capital. The firm's investment decision follows a standard first-order condition, which equates the firm's cost of capital with its expected marginal revenue product of capital. Combining equation (3) with the definition of firm cost of capital in (4), we obtain:⁴

$$(1 + r_i^{firm})\mathcal{M}_i = \mathbb{E}[\mathcal{P}'_i(f_k(k'_i, z'_i) + 1 - \delta) | k'_i, b'_i, z_i]. \quad (5)$$

The left-hand-side of equation (5) represents the cost of raising more capital. This includes the firm's cost of capital, r_i^{firm} , adjusted by the price feedback multiplier, \mathcal{M}_i , which captures the effect of the firm's borrowing and investment on the price of debt. The price feedback multiplier \mathcal{M}_i is the second term in equation (3). In order to map this object

⁴We provide a derivation of this equation in Appendix A.

to the data, it is useful to rewrite it as:

$$\mathcal{M}_i := \frac{1 - \gamma_i \times \frac{Q_i \cdot b'_i}{k'_i} \times \frac{\partial \log Q_i}{\partial \log k'_i}}{1 + \gamma_i \times \frac{\partial \log Q_i}{\partial \log b'_i}}, \quad \gamma_i := \frac{b'_i - (1 - \theta_i)b_i}{b'_i},$$

where γ_i measures the share of debt tomorrow that will be newly purchased. The numerator of \mathcal{M}_i reflects the feedback from changes in capital on the price of debt, while the denominator incorporates the feedback from changes in borrowing. Together, these terms provide a comprehensive characterization of how price dynamics influence the firm's cost of capital.

The right-hand-side of equation (5) represents the expected marginal revenue product of capital. This term includes the marginal productivity of capital, $f_k(k'_i, z'_i)$, and the depreciation factor, $1 - \delta$, weighted by the states of the world in which the firm repays, \mathcal{P}'_i .

3 Measuring Misallocation

When financial markets are efficient, all firms face the same cost of capital. However, in the data we find that the cost of capital varies across firms. How does this inefficiency in financial markets translate into an inefficiency in the real economy? We now consider the aggregation of output and investment across firms in order to study the steady-state costs of misallocation arising from dispersion in the cost of capital.

3.1 The Aggregate Economy and Welfare

We begin by setting up the aggregate environment in order to study both the decentralized equilibrium and the planner's problem. The firm's problem will be the same as before. There is no aggregate risk, so aggregates are not stochastic. Firms make undifferentiated products and take the price of their output as given. There is some initial stock of capital K_0 , and future capital depends on investment and depreciation through the standard law of motion.

We introduce the notation $\omega_{i,t}$, which is equal to one if firm i is still operating at time t ,

and zero if it has exited. Note that $\mathbb{E}_{t-1}[\omega_{i,t}] = \mathcal{P}_{i,t}$. Aggregate output is given by:

$$Y_t = \int_0^1 \underbrace{\omega_{i,t} \cdot f(k_{i,t}, z_{i,t})}_{\text{Output if Operates}} - \underbrace{(1 - \omega_{i,t}) \cdot ((1 - \delta) k_{i,t} - \phi_i(k_{i,t}))}_{\text{Losses if Defaults}} di \quad (6)$$

Note that we have defined output, Y_t , so that it includes both the firm's output in the event of production, $f(k_{i,t}, z_{i,t})$, and the losses from liquidation, $(1 - \delta) k_{i,t} - \phi_i(k_{i,t})$, in the event of default. This allows us to define aggregate investment simply:

$$I_t = K_{t+1} - (1 - \delta) K_t \quad (7)$$

Finally, aggregate capital is given by:

$$K_t = \int_0^1 k_{i,t} di \quad (8)$$

The planner wishes to maximize welfare, U , controlling each firm's capital and exit decision. However, the planner is subject to the same information constraints as the firm: $k_{i,t}$ must be decided in period $t - 1$, without yet knowing the productivity or operating costs that will prevail in that period. Exit decisions are made after $z_{i,t}$ is revealed, but with the values for future periods still unknown.

There is a representative household that obtains utility from consumption: we abstract from inequality to focus on productive efficiency. The household's utility is additively separable over time. Consumption is equal to aggregate output minus investment. Thus, welfare in this economy is given by:

$$U = \sum_{t=0}^{\infty} \beta^t \cdot u(Y_t - I_t)$$

where β is the household's discount rate and u is the utility it gets from consumption.

3.2 The Planner's Problem

Let $S_i^t := \{z_{is}\}_{s=0}^t$ denote the entire history of states, through period t .⁵ Define $S^t := \{S_i^t\}_{i \in [0,1]}$ as the collection of all firms' histories. We can use this notation to set up the appropriate constraints to the planner's problem: the planner must set $k_{i,t}$ as a function of S^{t-1} , and $\omega_{i,t}$ as a function of S^t .⁶ The planner's problem is:

$$\begin{aligned}
 U^* = & \max_{\left\{ \{k_{i,t}(S^{t-1}), \omega_{i,t}(S^t)\}_{i \in [0,1]} \right\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u(Y_t - I_t) \\
 \text{s.t.} \\
 & \omega_{i,t}(S^t) \in \{0, 1\} \forall i \\
 & \omega_{i,t+1}(S^{t+1}) \leq \omega_{i,t}(S^t) \quad \forall S^t \subset S^{t+1}, \forall i \\
 & \text{and Equations (6), (7), and (8) hold}
 \end{aligned}$$

where the inequality $\omega_{i,t+1}(S^{t+1}) \leq \omega_{i,t}(S^t)$ notes that if the firm exits, it cannot subsequently re-enter. In period $t = 0$, all firms operate and capital is set exogenously.

We can rewrite the planner's problem as a nested maximization problem, to isolate the intensive-margin choice of capital, holding aggregate capital and the extensive margin fixed. Note that $I_t = K_{t+1} - (1 - \delta)K_t$, and so it depends only on aggregate capital (not the allocation across firms). We can thus rewrite the planner's problem in the following nested form:

$$U^* = \max_{\left\{ K_t, \{\omega_{i,t}(S^t)\}_{i \in [0,1]} \right\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u \left(\left(\max_{\left\{ \{k_{i,t}(S^{t-1})\}_{i \in [0,1]} \right\}_{t=1}^{\infty}} Y_t \right) - I_t \right)$$

with the same constraints as before.

⁵Note that the only shock in our model is $z_{i,t}$, so this is the full history of states.

⁶In practice, since there is no aggregate risk, the planner will only need to use the individual firm's state histories to make decisions.

3.3 The Cost of Misallocation

We can now turn our attention to the inner problem. Note that the inner problem is separable across time periods, allowing us to separate it into a sequence of static problems. We focus on the cost of misallocation in terms of output. Simplifying our notation, we can rewrite the problem as follows:

$$Y_t^* \left(K_t, \{\omega_{i,t}\}_{i \in [0,1]} \right) = \max_{\{k_{i,t}\}_{i \in [0,1]}} \int_0^1 \mathbb{E}_{t-1} [\omega_{i,t} \cdot f(k_{i,t}; z_{i,t}) - (1 - \omega_{i,t}) \cdot ((1 - \delta) k_{i,t} - \phi_i(k_{i,t}))] di$$

s.t.

$$K_t = \int_0^1 k_{i,t} di$$

This problem is now a special case of the environment in [Hughes and Majerovitz \(2025\)](#). We can use their main proposition to derive the cost of misallocation, up to a second-order approximation. Define

$$g_i(k_i) := \mathbb{E}_{t-1} [\omega_{i,t} \cdot f(k_{i,t}; z_{i,t}) - (1 - \omega_{i,t}) \cdot ((1 - \delta) k_{i,t} - \phi_i(k_{i,t}))].$$

Proposition 2 shows the cost of intensive-margin misallocation.

Proposition 2 ((Special Case of [Hughes and Majerovitz \(2025\)](#))). *The cost of intensive-margin misallocation is given by*

$$\underbrace{\log Y_t^* \left(K_t, \{\omega_{i,t}(S^t)\}_{i \in [0,1]} \right) - \log Y_t}_{\text{Cost of Intensive-Margin Misallocation}} \approx \frac{1}{2} \cdot \underbrace{\mathbb{E}_{g_i(k_i)}[\mathcal{E}_i]}_{\text{Sales-Weighted Elasticity}} \cdot \underbrace{\text{Var}_{g_i(k_i)\mathcal{E}_i} \left(\log \left(\frac{\partial}{\partial k_{i,t}} g_i(k_i) \right) \right)}_{\text{Weighted Variance of Log Expected MPK}}$$

where $g_i(k_i)$ is the expected output of the firm as a function of k_i , \mathcal{E}_i is the elasticity of expected output with respect to the cost of capital, $\mathbb{E}_{g_i(k_i)}[\cdot]$ denotes the weighted average, weighting by $g_i(k_i)$, $\text{Var}_{g_i(k_i)\mathcal{E}_i}(\cdot)$ denotes the weighted variance, weighting by $g_i(k_i)\mathcal{E}_i$. All moments are computed for the set of firms that are operating at time $t - 1$. The formulas for the expected output of the firm and the elasticity of expected output with respect to the cost

of capital are given by:

$$g_i(k_i) = \mathbb{E}_{t-1} [\omega_{i,t} \cdot f(k_{i,t}; z_{i,t}) - (1 - \omega_{i,t}) \cdot ((1 - \delta) k_{i,t} - \phi_i(k_{i,t}))]$$

$$\mathcal{E}_i = - \frac{\left(\frac{\partial}{\partial k_i} g_i(k_i) \right)^2}{g_i(k_i) \cdot \frac{\partial^2}{(\partial k_i)^2} g_i(k_i)}$$

Note that in a Cobb-Douglas setting, with $f(k, z) = z \cdot k^\alpha$ and no default, the elasticity simplifies to $\mathcal{E} = \frac{\alpha}{1-\alpha}$. In our quantitative analysis, we will calibrate $\mathcal{E} = \frac{1}{2}$, consistent with $\alpha = \frac{1}{3}$. Moreover, note that although the proposition above provides a second-order approximation, it becomes exact in a setting where production is Cobb-Douglas and where productivity and distortions are jointly log-normal (the weights also fall out in that special case).

3.4 The Social Cost of Capital

We have already introduced the notion of the lender's discount rate, ρ , and the firm's cost of capital, r^{firm} . We now introduce the notion of the social cost of capital, r^{social} . This will reflect the social marginal product of capital at firm i . We define $r_{i,t}^{social}$ as the derivative of aggregate consumption ($Y_t - I_t$) at time $t + 1$ with respect to k_{it+1} , taking expectations at time t (when the investment decision is made).⁷ We have:

$$r_{i,t}^{social} := \frac{\partial \mathbb{E}_t [Y_{t+1} - I_{t+1}]}{\partial k_{i,t+1}}$$

$$= \mathbb{E}_t [\mathcal{P}_{i,t+1} (f_k(k_{i,t+1}; z_{i,t+1}) + 1 - \delta)] + (1 - \mathcal{P}_{i,t+1}) \cdot \phi'_i(k_{i,t+1})$$

The social cost of capital for firm i is the shadow value for the planner of allocating an extra unit of capital to that firm. Combining this with the firm's first-order condition for investment in Equation (5) yields:

$$1 + r_{i,t}^{social} = \left(1 + r_{i,t}^{firm} \right) \mathcal{M}_{i,t} + (1 - \mathcal{P}_{i,t+1}) \cdot \phi'_i(k_{i,t+1}) \quad (9)$$

⁷Proposition 2 is in terms of gross output, rather than consumption. Nevertheless, we define r^{social} in this way to parallel our definitions of ρ and r^{firm} .

Note that $1 + r_{i,t-1}^{social} = \frac{\partial}{\partial k_i} g_i(k_i) + 1 - \delta$. This will allow us to use the distribution of r^{social} to measure the cost of misallocation. When we bring this result to the data, we will focus on measuring the variance of r^{social} , and use standard values to calibrate \mathcal{E} . Moreover, we will make two further simplifying assumptions. First, we will focus on the unweighted variance, since the weights are difficult to observe in practice. Second, we will use the log-normal approximation $\text{Var}(\log(r_{i,t-1}^{social} + \delta)) \approx \log\left(1 + \frac{\text{Var}(r_{i,t-1}^{social} + \delta)}{\mathbb{E}[r_{i,t-1}^{social} + \delta]^2}\right)$

To measure misallocation in our data, we combine this with our derivation of r^{social} to yield the following corollary:

Corollary 1. *Assume that $(r_{i,t-1}^{social} + \delta)$ is log-normally distributed, and also assume that weighted moments can be replaced with unweighted moments. The cost of intensive-margin misallocation is given by*

$$\begin{aligned} & \log Y_t^* \left(K_t, \{\omega_{i,t}(S^t)\}_{i \in [0,1]} \right) - \log Y_t \\ & \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log \left(1 + \frac{\text{Var}(r_{i,t-1}^{social})}{\mathbb{E}[r_{i,t-1}^{social} + \delta]^2} \right) \end{aligned}$$

This corollary allows us to connect dispersion in r^{social} , an object that we will be able to measure in the microdata, with the cost of intensive-margin misallocation of capital. Intuitively, the planner would like to equalize the shadow value of capital across all firms: this would eliminate dispersion in the social return of capital and result in zero misallocation. We next turn to how to measure ρ , r^{firm} , and r^{social} using credit registry data.

4 Empirical Methodology

This section describes the main data sources that we use, as well as the procedures we follow to map model objects to the data in order to estimate the three rates: the lender discount rate, ρ , the firm's cost of capital, r^{firm} , and the social cost of capital, r^{social} .

4.1 Data Sources

Our main data source is the Schedule H.1 of the FR Y-14Q dataset (Y-14 for short). This is a quarterly regulatory dataset maintained by the Federal Reserve for stress testing purposes, which contains information on individual loan facilities held in the books of the top 30 to 40 bank holding companies (BHCs) in the US. The Y-14 includes all loan facilities exceeding \$1 million and we consider data in the period ranging from 2014Q4 to 2024Q4. Importantly for the purposes of our analysis, the Y-14 contains detailed characteristics of credit facilities such as facility size, origination date and maturity, interest rate or spread, interest rate variability, and the type of loan. Additionally, the Y-14 also covers BHC’s risk assessments for each borrower, which include estimates for the 1-year probability of default and loss given default. The probability of default is typically estimated using internal default models that have to be approved by regulators. While there is scope for some discretion in the assignment of these default probabilities (Plosser and Santos, 2018), these models have been subject to standardized guidelines following Basel II (BCBS, 2001). We focus on term loans issued to non-governmental and nonfinancial companies based in the US. Our unit of observation is a loan origination. We do not include credit lines due to lack of information about the fee structure, which would be needed to price these facilities. Appendix B contains a detailed description of the data cleaning procedure and sample restrictions.

In terms of coverage, Faria-e-Castro et al. (2024) show that the FR Y-14Q Schedule H.1 accounts for 91% of Commercial & Industrial lending undertaken by the 25 largest banks in the US (FRED mnemonic: CIBOARD), and 55% of all Commercial & Industrial lending undertaken by all commercial banks in the US (FRED mnemonic: BUSLOANS). Our focus in term loans and relatively stringent cleaning procedures leave us with a total of 65,284 loans.

4.2 Mapping the Model to the Data

An important difference between the model and the data is the payment structure of loans. In the model, for tractability, we assume that firms borrow in long-term debt that is modeled as a perpetuity with geometrically decaying coupons. In the data, on the other hand, we focus our analysis on term loans with a fixed maturity. This section shows how we map model objects to the data, and how we exploit the Y-14 data to retrieve estimates of the lender's discount rate, ρ , the firm's cost of capital, r^{firm} , and the social cost of capital, r^{social} .

Let (i, t) denote a term loan originated at period t . The loan has principal value $B_{i,t}$, maturity $T_{i,t}$, payment schedule $\{D_{i,t,s}\}_{s=1}^{T_{i,t}}$, repayment probability $P_{i,t}$ assumed to be constant over time, and loss given default $LGD_{i,t}$, also constant over time. The break-even condition at origination for a lender with discount rate $\rho_{i,t}$ is given by:

$$B_{i,t} = \mathbb{E}_t \sum_{s=1}^{T_{i,t}} \left[\frac{P_{i,t}^s \cdot D_{i,t,s} + P_{i,t}^{s-1} \cdot (1 - P_{i,t}) \cdot (1 - LGD_{i,t}) \cdot B_{i,t}}{(1 + \rho_{i,t})^s \cdot \Pi_{t,s}} \right],$$

where $\Pi_{t,s} \equiv \prod_{j=0}^s (1 + \pi_{t+j})$ is the gross cumulative inflation rate from loan origination at period t through s . The lender forms expectations over two types of risks. On one hand, there is idiosyncratic loan-level default risk: at each period s , the loan repays with probability $P_{i,t}$, in which case the lender earns the payment $D_{i,t,s}$, or it defaults with probability $1 - P_{i,t}$, in which case the lender earns the principal reduced by the loss given default, $(1 - LGD_{i,t})B_{i,t}$. On the other hand, the payments themselves are risky due to aggregate uncertainty, both due to inflation realized between origination and the period the payment is received $\Pi_{t,s}$ and, in the case of floating rate loans, to fluctuations in the underlying reference rate.

Assume now that the loan is a non-amortizing term loan, with each payment consisting of interest over the life of the loan, and the final payment consisting of a lump-sum principal repayment. Thus $D_{i,t,s} = r_{i,t,s} \cdot B_{i,t}$ for $s < T_{i,t}$ and $D_{i,t,T_{i,t}} = (1 + r_{i,t,T_{i,t}})B_{i,t}$. The interest rate $r_{i,t,s}$ is either a constant number, in the case of fixed-rate loans, or a fixed spread over a floating benchmark rate for floating rate loans. We can then rewrite the approximate

break-even condition at origination as:⁸

$$1 = \sum_{s=1}^{T_{i,t}} \left[\frac{P_{i,t}^s \cdot \mathbb{E}_t(r_{i,t,s}) + P_{i,t}^{s-1} \cdot (1 - P_{i,t}) \cdot (1 - LGD_{i,t})}{(1 + \rho_{i,t})^s \cdot \mathbb{E}_t(\Pi_{t,s})} \right] + \frac{P_{i,t}^{T_{i,t}}}{(1 + \rho_{i,t})^{T_{i,t}} \cdot \mathbb{E}_t(\Pi_{t,T_{i,t}})}, \quad (10)$$

This equation balances the present value of expected payments from the borrower against the lender's opportunity cost, ensuring that the lender breaks even. For a fixed-rate term loan, data on $(P_{i,t}, LGD_{i,t}, T_{i,t}, r_{i,t}, \{\mathbb{E}_t(\Pi_{t,s})\}_{s=1}^{T_{i,t}})$ allows us to solve this equation for the match-specific lender's discount rate $\rho_{i,t}$.

Floating rate loans. The data contains both fixed and floating rate loans. To estimate $\rho_{i,t}$ for floating rate loans, it is necessary to obtain estimates of $\mathbb{E}_t(r_{i,t,s})$, the expected interest rate. Floating rate loans typically charge a reference rate plus a spread. For our analysis, we use smoothed daily yield curve estimates provided by the Federal Reserve Board, based on the methodology described in [Gürkaynak et al. \(2007\)](#). Under the expectations hypothesis, long-term interest rates are assumed to reflect the market's expectations of future short-term rates. For each floating rate loan, we compute the sequence of forward short-term interest rates at the date of origination, and add the (fixed) loan spread to obtain a sequence of interest rates that are used to price that loan. Using this framework, we back out $\mathbb{E}_t(r_{i,t,s})$ for each loan by combining the treasury forward rate with the loan's spread.⁹ It is worth noting majority of floating rate loans in our sample are indexed to the LIBOR/SOFR rather than Treasury rates. However, for the period in analysis, the spread between the SOFR and short-term Treasury rates is negligible. In the absence of readily available forward curve estimates for the LIBOR or SOFR, we treat them as identical to the Treasury curve.

⁸This is an approximation, as we abstract from the covariance between the expected floating rate and expected inflation, and from the Jensen's inequality term that arises because inflation appears in the denominator. Accounting for these terms would significantly complicate the estimation $\rho_{i,t}$, as it would require information about the expected term structure of the covariance between reference rates and inflation.

⁹More specifically, the estimate for the reference rate n years ahead at time t is given by $f_t(n, 0) = \beta_0 + \beta_1 \exp(-n/\tau_1) + \beta_2(n/\tau_1) \exp(-n/\tau_1) + \beta_3(n/\tau_2) \exp(-n/\tau_2)$ (equation 21 in [Gürkaynak et al. \(2007\)](#)), where estimates for $(\beta_1, \beta_2, \beta_3\tau_1, \tau_2)$ at each date are regularly updated by the Board of Governors of the Federal Reserve and published at <https://www.federalreserve.gov/data/nominal-yield-curve.htm>. We compute the sequence of forward rates at loan origination, and add the fixed spread to obtain an estimate for the interest rate at each repayment point in time.

Interpreting loan values and loss given default. How should loan values in the data map to our model? It is straightforward to map between the model and data, but only at loan origination. A firm that sells debt for the first time receives $Q \cdot b'$ in exchange. Thus, loan value recorded at origination corresponds to $Q \cdot b'$ in the model. Another important variable from the data is the expected loss given default (LGD), which is given as a percentage. At origination, we interpret LGD as the expected losses relative to the loan's book value, so that $1 - LGD = \phi(k')/(Q \cdot b')$.

After origination, banks do not update loan values on the books to reflect changes in market prices. Instead, they use historical-cost accounting, recording loans at their original value and adjusting over time based on the stated interest rate and repayment schedule (Begenau et al., Forthcoming). As a result, observed loan values after origination no longer correspond to the market value of debt, $Q_t \cdot b_{t+1}$. For this reason, we focus only on newly originated loans.

Lender's discount rate. For each loan origination, the lender's discount rate $\rho_{i,t}$ can be estimated by numerically solving equation (10). This requires credit registry data on: i) loan maturity $T_{i,t}$, ii) repayment probability $P_{i,t}$, iii) loss given default $LGD_{i,t}$, and iv) interest rate or spread. For floating rate loans, data on spreads is combined with the Treasury forward curve to obtain $\mathbb{E}_t(r_{i,t,s})$. Finally, we compute $\{\mathbb{E}_t \Pi_{t,s}\}_{s=1}^{T_{i,t}}$ for each loan using estimates for the expected inflation term structure from Federal Reserve Bank of Cleveland (2025).

An instructive case is that of a hypothetical case of a fixed real interest rate loan; this serves as a useful approximation and is described in the following proposition:

Proposition 3 (Lender's discount rate for a fixed real rate). *For a fixed real interest rate loan:*

$$1 + \rho_{i,t} = P_{i,t} (1 + r_{i,t}) + (1 - P_{i,t}) (1 - LGD_{i,t}).$$

where $r_{i,t}$ is the fixed real interest rate on the loan. This expression reflects the lender's return, accounting for repayment in non-default states and recovery in default states. In

this case, $\rho_{i,t}$ is independent of the loan's maturity $T_{i,t}$, which simplifies its calculation and interpretation.

Firm's Cost of Capital. Returning to the firm's cost of capital of Proposition 1, we can estimate Λ for term loans, and then solve for r^{firm} . Proposition 4 provides an equation to estimate Λ directly from the data.

Proposition 4 (Firm's Cost of Capital). *We can solve for $\Lambda_{i,t}$ as:*

$$\Lambda_{i,t} = \frac{(1 - P_{i,t})(1 - LGD_{i,t})}{1 + \rho_{i,t} - (1 - P_{i,t})(1 - LGD_{i,t})}.$$

This allows us to write the firm's cost of capital as:

$$1 + r_{i,t}^{firm} = \frac{1 + \rho_{i,t}}{1 + \Lambda_{i,t}} = (1 + \rho_{i,t}) - \underbrace{(1 - P_{i,t})(1 - LGD_{i,t})}_{\text{Expected Recoveries}}.$$

In this expression, $(1 - P_{i,t})(1 - LGD_{i,t})$ represents expected recoveries, capturing the key difference between $\rho_{i,t}$ and $r_{i,t}^{firm}$: lenders benefit from expected recoveries, but borrowers get zero payoff in the default state, regardless of whether the lender recovers anything on the loan. Thus the expected cost of the loan for the firm is lower than the expected return for the lender. This formula allows us to measure the firm's cost of capital for each loan in the data, at origination.

Social Cost of Capital. Finally, we explain how we use data to estimate the social cost of capital, $r_{i,t}^{social}$. For measurement, we specialize and assume that the liquidation technology is linear, meaning it takes the form $\phi_i(k_i) = \phi_i \cdot k_i$. Combining with equation (9) and proposition 4, this yields a formula for $r_{i,t}^{social}$ in terms of objects in the data.

Proposition 5. *Assume a linear liquidation technology. The social cost of capital is then:*

$$\begin{aligned} 1 + r_{i,t}^{social} &= \left(1 + r_{i,t}^{firm}\right) \mathcal{M}_{i,t} + (1 - P_{i,t}) \cdot (1 - LGD_{i,t}) \cdot lev_{i,t} \\ &= (1 + \rho_{i,t}) \mathcal{M}_{i,t} + (lev_{i,t} - \mathcal{M}_{i,t}) \cdot (1 - P_{i,t}) \cdot (1 - LGD_{i,t}) \end{aligned}$$

where $lev_{i,t} := \frac{Q_{i,t} \cdot b_{i,t+1}}{k_{i,t+1}}$ is the firm's leverage ratio.

In our empirical analysis, we set the price feedback multiplier $\mathcal{M}_{i,t} = 1$.¹⁰ Under that calibration, the social cost of capital simplifies further:

$$1 + r_{i,t}^{social} = \underbrace{1 + \rho_{i,t}}_{\text{Matching efficiency}} + \underbrace{(\text{lev}_{i,t} - 1) \cdot (1 - P_{i,t}) \cdot (1 - LGD_{i,t})}_{\text{Financial Frictions}} \quad (11)$$

The social cost of capital is thus equal to the lender's discount rate (i.e., matching efficiency), plus a term that reflects financial frictions related to default and recovery. This latter term reflects the tension between lenders and borrowers. The presence of this term also implies that whether the social return on capital exceeds the lender's discount rate or not is a function of whether firm leverage exceeds 1 or not. This is due to the following: lenders care about average recovery per dollar of debt, $\phi_i(k_i)/b_i$, which is equal to $1 - LGD_i$ in the data. The planner, on the other hand, cares about the marginal recovery $\phi'_i(k_i)$, which is equal to $(1 - LGD_i) \cdot \text{lev}_i$ in the data. The two coincide when $\text{lev}_i = 1$, and so the social cost of capital equals the lender's discount rate in that case. In general, for firms with $\text{lev}_{i,t} \in (0, 1)$, we obtain the following ranking: $r_{i,t}^{firm} < r_{i,t}^{social} < \rho_{i,t}$.

5 Empirical Results

We now describe the results of applying the measurement exercise described in the previous section to the U.S. Y-14 data.

5.1 Summary Statistics

We provide summary statistics for key variables in Table 1. Since our unit of observation is a loan origination, and so all reported firm financials correspond to the financials reported in the quarter in which that origination took place. The average annual loan interest rate in our sample is 4.18%. Adjusted for expected inflation at origination, this results in an average real rate of 2.39%. We compute the real interest rate at origination by subtracting (annualized) expected inflation over the maturity of the loan, using the expected inflation

¹⁰We show that this is a good assumption in Appendix B.2, where we estimate this variable in the data and find a distribution extremely concentrated around 1.

term structure that prevailed at time of origination. These loans have an average expected default probability of 1.45% over the next year, and banks expect to lose, on average, 34.41% of the outstanding value of the loan in the event of default. As a result, the lender's discount rate, ρ , averages 1.87%.¹¹ The social cost of capital and firm's cost of capital respect the aforementioned ranking and are even lower, at 1.66% and 0.92% respectively.

Real interest rates vary across loans, with a standard deviation of 1.2% (1.7% for nominal rates), reflecting heterogeneity both within and across time. The lender's discount rate shows similar heterogeneity, with a standard deviation of 1.6%. In contrast, the social cost of capital and firm's cost of capital have higher heterogeneity, with standard deviations of 1.8% and 2.8%, respectively.

Why are the variances of the lender's discount rate and the contractual interest rate similar? To build intuition, we can focus on the formula for the lender's discount rate, ρ , for fixed real rate loans. With some rearrangement, the lender's discount rate can be expressed as $\rho = r - (1 - P)(r + LGD)$. Since r is small at annual frequencies compared to LGD , we can further approximate $\rho \approx r - (1 - P) \cdot LGD$. This yields the following variance decomposition:

$$\mathbb{V}[\rho] \approx \mathbb{V}[r] + \mathbb{V}[(1 - P) \cdot LGD] - 2 \cdot \mathbb{C}[r, (1 - P) \cdot LGD] \quad (12)$$

The variance of ρ is similar to the variance of r because the variance of expected losses, $(1 - P) \cdot LGD$, is offset by the covariance term: interest rates are higher when the lender's expected losses are high.

We view these results as a vindication for our method of estimating the cost of capital. If our observed measures of default probabilities and recovery rates were just noise, then the variance of the lender's discount rate would be substantially greater than the variance of interest rates: the covariance term would be zero, and the term $\mathbb{V}[(1 - P) \cdot LGD]$ would

¹¹The negative covariance of r and P means that the average ρ is lower than we might have expected from the raw averages of P , r , and LGD . To see this, consider the approximation for real fixed rate loans, $1 + \rho = P(1 + r) + (1 - P)(1 - LGD)$; the average value of $P(1 + r)$ will be brought down by the fact that P is low when $(1 + r)$ is high.

push the variance of ρ substantially above the variance of r . Instead, the variance of the cost of capital is similar to that of the interest rate, suggesting that default probabilities and recovery rates covary with interest rates in the way that we would expect in a financial market that is close to efficient.

Table 1: Summary Statistics

	mean	sd	p10	p50	p90
Interest rate	4.18	1.69	2.21	3.94	6.60
Maturity (yrs)	6.83	4.65	3.00	5.00	10.25
Real interest rate	2.39	1.24	0.88	2.33	4.00
ρ (%)	1.87	1.55	0.41	1.88	3.62
r^{firm} (%)	0.92	2.80	-0.86	1.26	3.03
r^{social} (%)	1.66	1.78	0.12	1.73	3.47
Prob. Default (%)	1.45	2.53	0.19	0.85	2.88
LGD (%)	34.41	13.17	16.00	35.60	50.00
Loan amount (M)	10.75	67.58	1.11	2.57	22.92
Sales (M)	1,269.93	6,051.48	2.16	58.50	1,560.10
Assets (M)	1,760.37	8,894.15	1.07	35.55	1,782.22
Leverage (%)	72.17	24.68	42.68	71.29	100.00
Return on assets (%)	27.60	58.51	4.56	15.76	47.81
N Loans	65,284				
N Firms	38,751				
N Fixed Rate	32,592				
N Variable Rate	32,692				

5.2 Averages by Quarter of Origination

We begin by analyzing the time series of average values, by quarter of origination. The key inputs into our measures of the cost of capital are the interest rate, default probability, and loss given default. We first analyze the behavior of these averages over time, in Figure 1. The first panel plots real rates at origination for all loans, while the second panel plots contractual interest rate spreads for floating-rate loans only. During the time period we study, interest rates fall and then rise concurrent with the movements of monetary policy; average spreads are very stable, ranging from 1.9% to 2.3%. Default probabilities show a modest upward secular trend, along with a temporary spike around the time of the COVID-

19 pandemic. Expected losses given default fall around the onset of the pandemic, implying that banks expect larger recoveries in the event of default. Note, however, that the magnitude of the change in recoveries is sufficiently small that it has little effect on ρ , since this change is multiplied by the (small) probability of default.

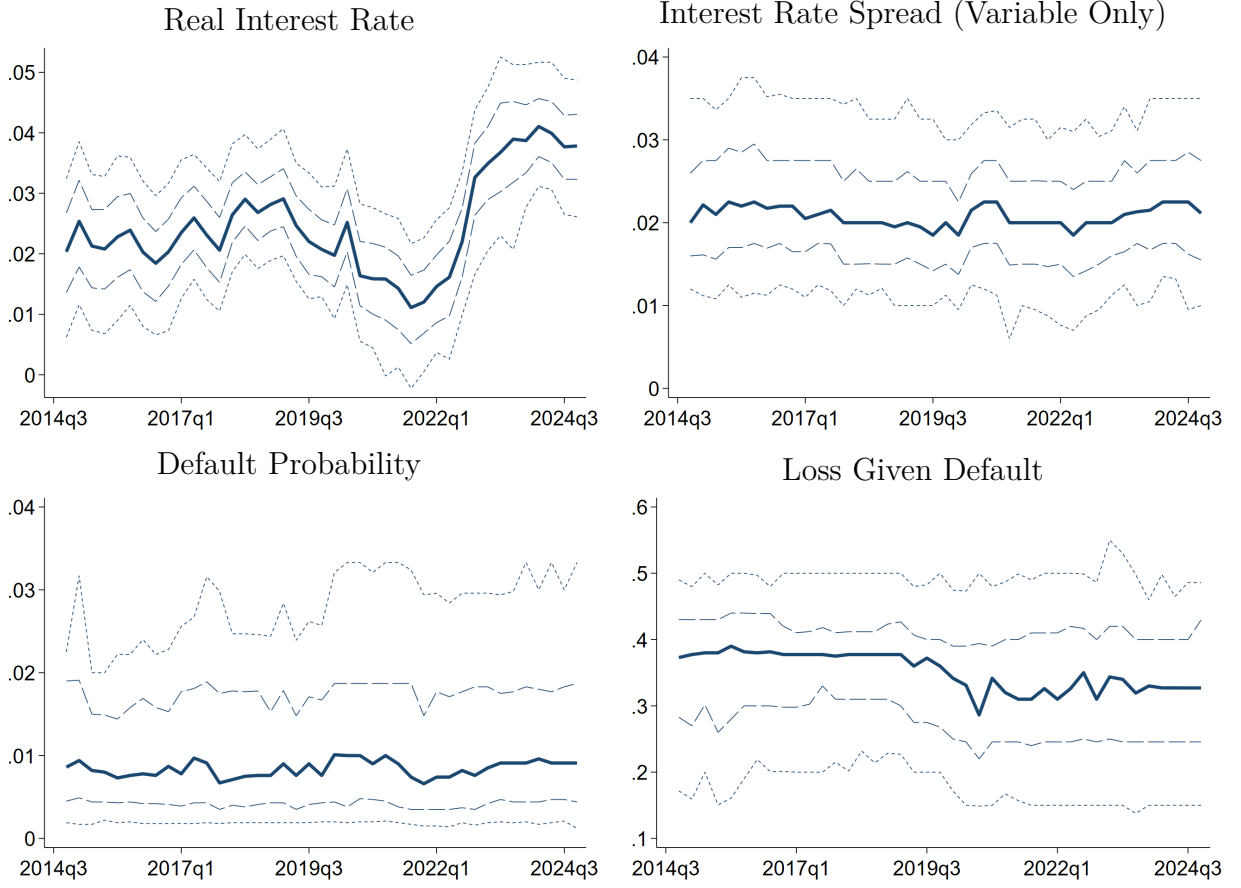


Figure 1: The solid line corresponds to the median values of key inputs by quarter of origination: real interest rate at origination, interest rate spread at origination (floating-rate loans only), probability of default, and loss given default. Dashed lines correspond to p10, p25, p75, and p90.

Next, in Figure 2, we plot the lender's discount rate, ρ , the firm's cost of capital, r^{firm} , and the social cost of capital, r^{social} , against the real five-year treasury rate. All rates covary strongly with the real five-year Treasury rate. The average lender's discount rate is similar to the average r^{social} , with a roughly 20 basis points spread. There is an average spread of roughly 156 basis points between the lender's discount rate and the treasury rate, although

it has a delayed reaction to movements in treasury rates: the spread is initially stable at 150 basis points, then rises above the average when treasury rates fall and falls below the average once treasury rates rise again. Note that the lender's discount rate is already adjusted for default risk, and so this cannot explain the spread relative to treasuries. While the social cost of capital, r^{social} is quite close to ρ , the firm's cost of capital, r^{firm} , tracks the treasury rate more closely with a small spread.

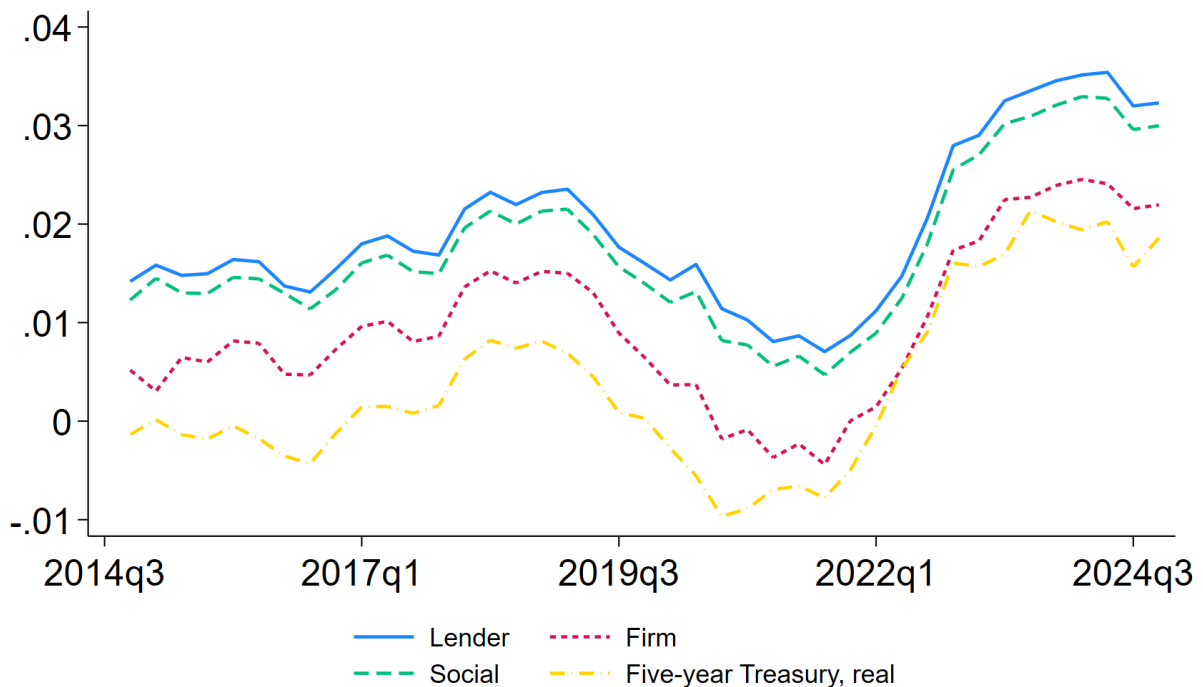


Figure 2: Averages by quarter of origination for the lender's discount rate (ρ), firm cost of capital (r_{firm}), social cost of capital (r_{social}), and time series for the real five-year Treasury rate.

While our analysis takes into account the maturity of the loan, there are potential concerns that loans of different maturities may face different rates, even if they reflect a constant spread on a (time-varying) risk-free rate. To mitigate these concerns, in Appendix B.5, we redo our analysis focusing on loans with a 5-year maturity. This is the most common maturity for fixed rate loans. Five-year loans are convenient because they allow direct comparison to the five-year treasury rate. The average cost of capital for five-year loans is very similar to

the overall sample: there is a roughly 150 basis point spread relative to five-year treasuries, following the same dynamics as in the overall sample.

5.3 Cross-Sectional Heterogeneity

While the means display similar movements, they mask a substantial amount of cross-sectional heterogeneity in these measures. In this section, we show that this heterogeneity is substantial across measures even after controlling for time, lender and firm fixed effects. To this end, we follow the variance decomposition of [Daruich and Kozlowski \(2023\)](#). To ensure that we can estimate firm level fixed effects we subset our sample to the set of firms with five or more distinct loans. We then progressively add time, lender-time, and firm-lender-time fixed effects, building to the fixed-effects specification displayed in equation (13) below, where i indexes the firm, τ represents the quarter of origination, b indexes the lender (BHC), and l represents the particular loan.

$$r_{\tau bil} = \alpha_{\tau} + \gamma_{\tau b} + \delta_{\tau bi} + \varepsilon_{\tau bil} \quad (13)$$

The results are in Table 2, for the nominal interest rate, real rate, lender’s discount rate, firm cost of capital, and social cost of capital. The time fixed effect explains 69% of the variance in nominal interest rates, 49% of the variance in real interest rates, and 43% of the variance in the lender’s discount rate. For all five variables, adding in lender-time fixed effects explains a negligible share of the variance (at most 4.21% for r^{social}), suggesting that heterogeneity across lenders is not an important source of heterogeneity in interest rates or the different measures of cost of capital. Adding in firm-lender-time fixed effects explains an additional 15% to 31% of the variance of these measures. Finally, notice that the loan-level variance, after controlling for firm-lender-time fixed effects, remains substantial, ranging from 15% for nominal interest rates up to 49% for the firm cost of capital. To the extent that our measure of misallocation depends on the variance of r^{social} , these results show that a significant share of that variance is loan-specific and cannot be accounted for by either time, lender, or borrower effects.

	Time	Bank	Firm	Loan
Contractual rate	69.08	1.68	14.72	14.52
Real rate	49.35	3.62	25.32	21.71
ρ	43.07	3.61	22.93	30.39
r^{firm}	16.5	3.73	30.88	48.9
r^{social}	34.72	4.21	24.94	36.13
N Firms	1844			
N Loans	16088			

Table 2: Variance decomposition for contractual interest rates and different measures of the cost of capital (ρ , r^{firm} , and r^{social}) using equation 13.

Appendix B.3 further explores the correlation between the cost of capital and firm-level covariates such as leverage, return on assets, and assets. While the effects of leverage and size vary across the specific measure, there is a consistent positive correlation between all measures of cost of capital and the return on assets: the cost of capital is consistently higher at firms with high return on assets, even though the return on assets is negatively correlated with the real interest rate once leverage and size are controlled for. Although we cannot attach a causal interpretation to the estimated coefficients, this would be consistent with a model where causality runs from the cost of capital to firm decisions: firms with a higher cost of capital will demand a high return on their investments.

5.4 Misallocation

What does the heterogeneity in the cost of capital imply for the cost of misallocation? To answer this we use our formula for misallocation from Corollary 1, setting $\delta = 0.06$.¹² We compute this statistic by quarter of origination, in order to focus on within-period misallocation. Our model does not contain aggregate shocks, and we would thus need a richer model to study misallocation across time. We interpret our results below as reflecting what misallocation would be for an economy that remained in the same steady state.

¹²Note that in steady state $\delta = \frac{I/Y}{K/Y}$. In the data, at the annual frequency, the capital-output ratio is about 3 while the investment-output ratio is about 0.18 (we measure capital as BEA Current-Cost Net Stock of Fixed Assets and investment is GDPI in FRED). Hence, at an annual frequency, $\delta = 0.06$. Additionally, recall that, as previously explained, we set $\mathcal{E} = 1/2$ and $\mathcal{M} = 1$.

We plot our estimates in Figure 3. We find that, overall, the misallocation of capital resulting from heterogeneity in the cost of capital, plus the financial friction term, is small. In the period before the COVID pandemic the implied misallocation is low and decreasing: reallocating capital across firms would increase aggregate output by 0.85%, on average, during this period. This number rises with the onset of the pandemic, averaging 1.78% during 2020 and 2021, before falling back to a somewhat elevated 1.23% starting in 2022.

Our model of misallocation studies an economy in steady-state, which complicates the interpretation of short-run changes in the distribution of the cost of capital. A temporary shock to the dispersion of r^{social} among newly originated loans will have only limited effects on the dispersion of r^{social} in the full population of firms. Moreover, a steady-state model with no aggregate shocks is not well suited to studying aggregate dynamics in response to a shock. Thus, we caution against over-interpreting the transitory rise in implied misallocation during the pandemic: if the increased dispersion of r^{social} were permanent, then misallocation would rise by about 1% in steady state, but it is not obvious how much misallocation actually rose in response to the transitory shock. Instead, our main takeaway from the analysis is that in “normal times” (e.g. before the pandemic), heterogeneity in r^{social} implies a small cost of misallocation in steady state.

5.5 Decomposing Misallocation

To understand the drivers of misallocation, we decompose our measure into the component coming from heterogeneous cost of capital, ρ , and the component coming from heterogeneity in the financial frictions term. We perform two counterfactuals. In the first, we replace ρ with its average value for that quarter, which is informative about how much misallocation arises from heterogeneity in the financial frictions term. In the second counterfactual we set the financial frictions term equal to its average value within the quarter, allowing us to measure the misallocation that arises from heterogeneity in ρ . Note that neither counterfactual changes the within-quarter average of r^{social} , and thus the results are driven by changes in the variance of r^{social} .

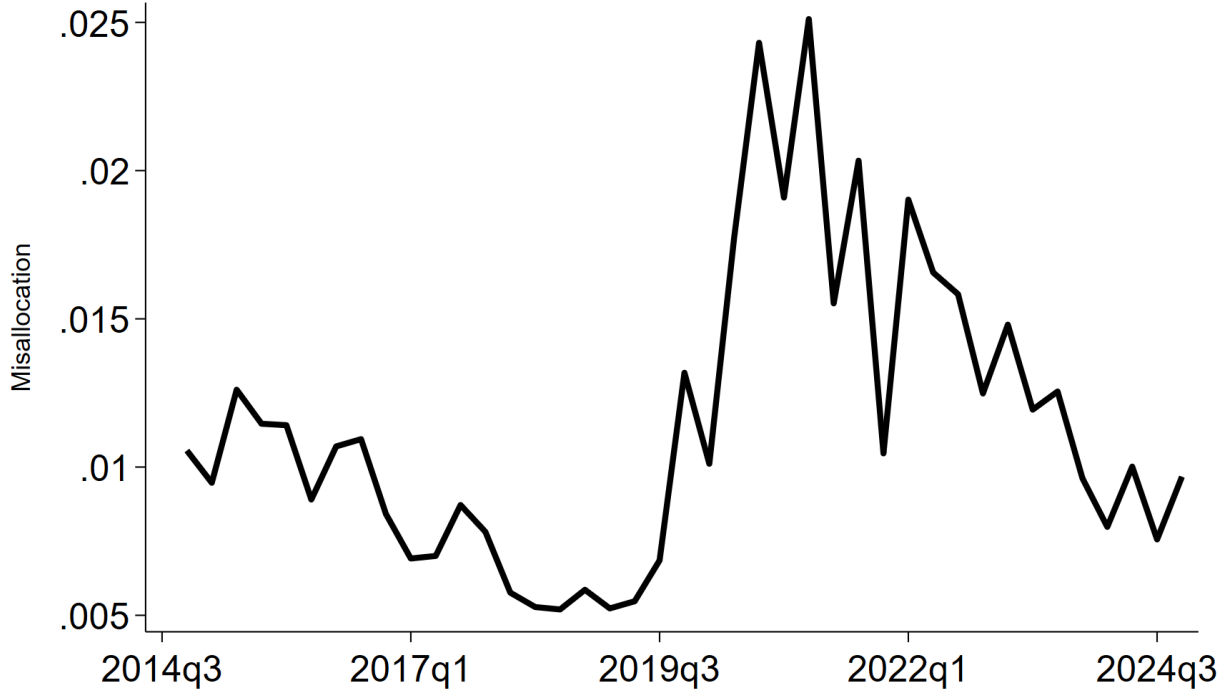


Figure 3: Time series for the cost of misallocation, using the formula in Corollary 1. The cost of misallocation is the percentage difference between actual output and output in a counterfactual economy where the planner is free to reallocate physical capital across firms, keeping exit decisions fixed.

We show the results of these decompositions in Figure 4, for three distinct time periods. The main takeaway is that misallocation is mostly driven by heterogeneity in lender discount rates. In the pre-pandemic period, if ρ is equalized across firms then the cost of misallocation falls to just 0.11% instead of 0.85%. In contrast, the cost of misallocation in the counterfactual with a constant financial friction is 0.61%. Note that there is an interaction between ρ and the financial frictions term as total misallocation exceeds the sum of the two counterfactuals. During the pandemic period (2020-2021), there is a significant increase in total misallocation, which more than doubles. While still small, the financial frictions term also doubles in importance. Quantitatively, the large increase comes from an increased dispersion in ρ . In the post-pandemic period (2022-2024), misallocation remains over 40% more elevated than in the pre-pandemic period. Once again, ρ plays the dominant role:

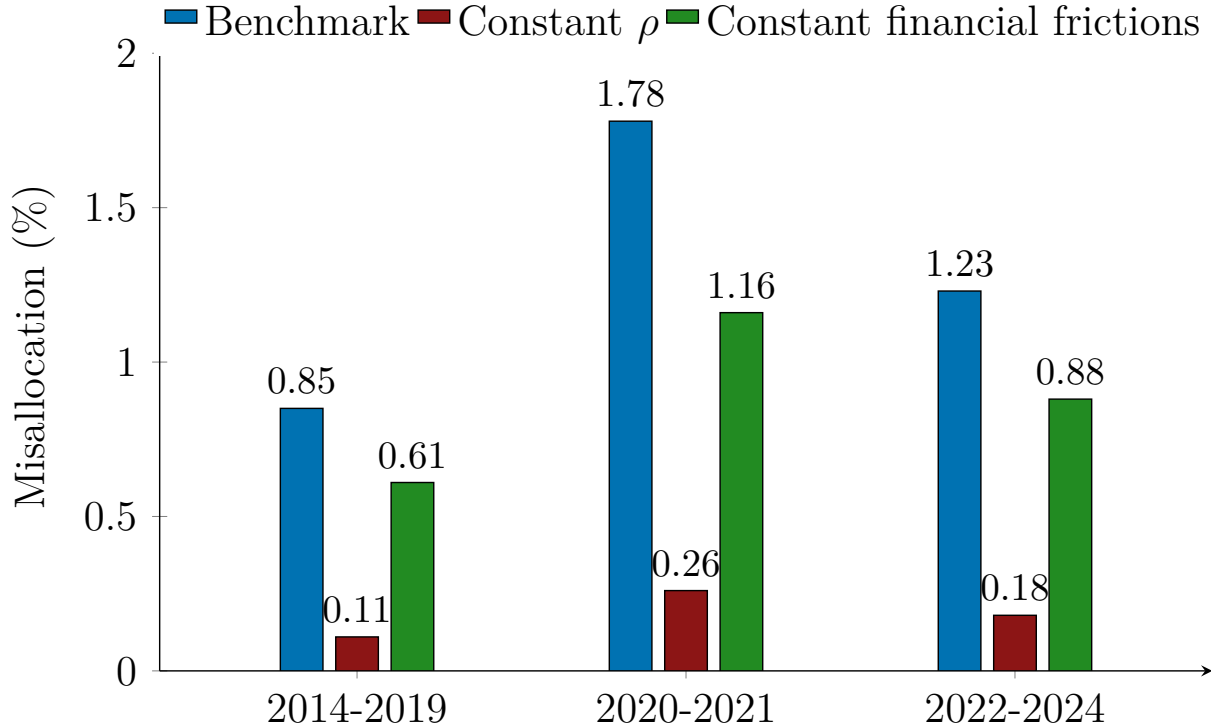


Figure 4: Decomposing the cost of misallocation. The “benchmark” columns correspond to the averages for the cost of misallocation in Figure 3 in the respective time period. “Constant ρ ” is the average cost of misallocation in a counterfactual exercise where we set the estimated ρ for each newly originated loan to be equal to the average ρ for that quarter. “Constant financial frictions” repeats the exercise for a counterfactual where we keep the second term in equation 11 equal to its average value for each quarter, for all newly originated loans.

misallocation would be only 0.18% in the constant ρ counterfactual.

In Appendix B.5, we repeat our misallocation analysis, focusing on loans with a maturity of five years. We find that the results are very similar to those of our main analysis. This reinforces the robustness of our results, confirming that heterogeneity in the cost of capital across firms is not driven by differences in maturity or term structure.

5.6 The 2020-21 increase in misallocation

Our analysis in Figure 4 reveals that while both components of the social cost of capital contribute to an increase in misallocation in the 2020-21, the heterogeneity in ρ is quantita-

tively the most important factor. Figure 5 plots the time series for the mean and standard deviation of ρ . First, as already shown in Figure 2 the average ρ follows the risk-free interest rate. Hence, as risk-free rates decreased during 2020-2021, the average ρ also decreased. Second, the standard deviation of ρ increases during this period. Both these movements contribute to an increase in the coefficient of variation for ρ . The increased dispersion in ρ translates into an increased dispersion of r^{social} which directly affects our measure of misallocation.



Figure 5: Mean and standard deviation of the lender's discount rate, ρ .

As explained in section 4, the key inputs for the calculation of $\rho_{i,t}$ are loan-level characteristics and thus movements in its volatility must reflect changes in the volatility of the underlying components. As highlighted in the previously discussed approximation, $\rho \simeq r - (1 - P) \cdot LGD$, fluctuations in the variance of ρ reflect fluctuations in underlying real

interest rates, expected losses, and the covariance of these two terms. To study this formally, we consider the following decomposition:

$$\rho_{i,t} = \underbrace{\rho_{i,t}|_{P_{i,t}=1}}_{\text{real yield}} + \underbrace{[\rho_{i,t} - \rho_{i,t}|_{P_{i,t}=1}]}_{\text{expected losses}} \quad (14)$$

where $\rho_{i,t}|_{P_{i,t}=1}$ is the $\rho_{i,t}$ that solves the no arbitrage equation (10) for $P_{i,t} = 1$. We call this object the “real yield” as it is analogous to the yield to maturity of a bond: the implicit discount rate at which a loan is being priced under the assumption of no default on its payments. The second term, the difference between the lender’s discount rate and the real yield, reflects expected losses. We can thus decompose the variance of ρ into the sum of the variance of the real yield (that reflects fluctuations in real rates only), variance of expected losses (that reflects fluctuations in probabilities of default and losses given default), and a covariance term.

Figure 6 plots the results of such variance decomposition, period by period. The covariance term is close to zero throughout. The variance of the real yield is relatively stable, reflecting the fact that the variance of real interest rates remained stable throughout the sample, with a slight increase towards the end of the sample. Thus, while this term can help explain why misallocation has remained more elevated in the post-pandemic period, it cannot account for the sharp increase in 2020-21. This, in turn, is primarily explained by a sharp increase in the variance of expected losses. One possible interpretation is that the 2020-21 period brought about an increase in the heterogeneity of expected losses increased without a commensurate increase in the dispersion of interest rates, which would have been offset by a more negative covariance term. That is, the dispersion of expected losses increased, but without an increase in the dispersion of interest rates.

We hypothesize that this pattern may have emerged due to broad-based fiscal and monetary interventions implemented during 2020–2021 to support the economy. In particular, many of these programs involved lending to or rescuing firms and sectors under financial distress (such as the Paycheck Protection Program, the Main Street Lending Program, Primary Market Corporate Credit Facility, or the Secondary Market Corporate Credit Facility,

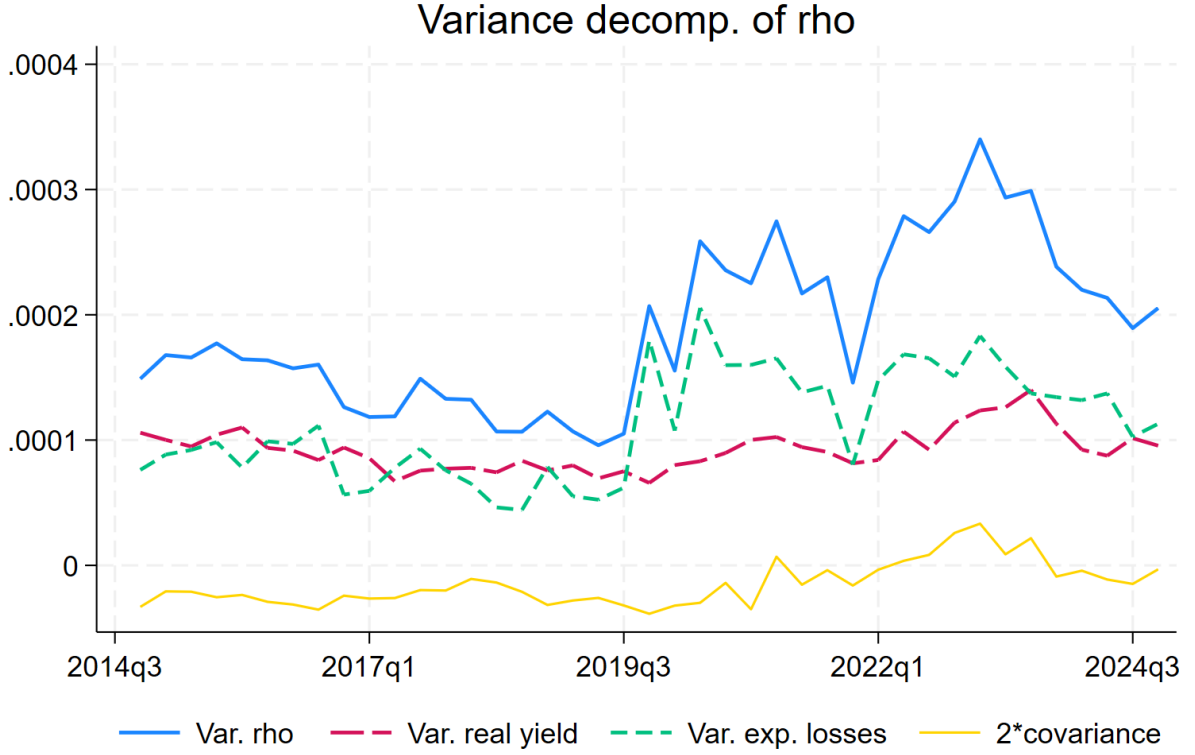


Figure 6: Decomposition of the variance of ρ .

among others). Our hypothesis is that lenders inferred an implicit government guarantee for many of these loans: if risky borrowers were to default, lenders believed the government would likely intervene to cover the losses. This underpricing of risk may have ultimately led to increased misallocation of credit.

Risk premia and aggregate shocks. Alternatively, a plausible explanation for the rise in the dispersion of ρ is that it reflects risk premia, as lenders require an increased compensation for risk during an extremely uncertain and volatile period. Furthermore, since different firms may be differentially exposed to aggregate shocks, the existence of heterogeneity in risk premia may not necessarily imply the presence of misallocation (David et al., 2022). Our framework is set in steady state and cannot accommodate aggregate shocks that would

trigger increases in risk premia. An increase in risk premia for some firms, say firms whose cash flows were more exposed to COVID-19 disruptions, should raise the skewness of the distribution of ρ . What we find is the opposite: not only the average ρ falls from 1.8% (2014-19) to 1.0% (2020-21), but its skewness also becomes slightly more negative (from -3.55 to -3.60). Thus, if anything, the “left tail” becomes more pronounced. This is at odds with an explanation related to the rise in risk premia and, if anything, suggests that risk premia seem to have fallen during this period, potentially due to explicit and implicit guarantees.

5.7 Relation to ARPK-based measures

The dominant approach to measuring misallocation builds on [Hsieh and Klenow \(2009\)](#) and typically consists of using data on firm financials to approximate the marginal revenue product of capital (MRPK) at the firm-level. Under certain assumptions about the structure of the model, researchers can approximate MRPK with the average revenue product of capital (ARPK). Aggregate measures of misallocation are then compute based on moments of the distribution for ARPK. Our approach, instead, uses data on the cost of borrowing to measure r^{social} , which we argue is informative about the measure of MRPK that a planner who seeks to minimize misallocation cares about. In this subsection, we show that r^{social} is correlated to traditional measures of ARPK, which we view as a form of validation of our measuring framework.

Table 3 reports the results for regressions of the type:

$$\log ARPK_{i,t} = \alpha_{j,t} + \beta \log(r_{i,t}^{social} + \delta) + \epsilon_{i,t}$$

where $ARPK_{i,t}$ is a measure of the ARPK, $\alpha_{j,t}$ are sector-quarter fixed effects (NAICS4, as commonly used in the misallocation literature), and $\epsilon_{i,t}$ is the error term. Each column corresponds to a different measure of ARPK and/or sample. Columns (1)-(2) refer to the Y-14 sample, while (3)-(5) focus on a restricted sample of Y-14 loans that we are able to match to Compustat firms. We consider different measures of ARPK that have been used in the literature: columns (1) and (3) compute ARPK as sales over fixed assets for the

Y-14 and Compustat, respectively. Columns (2) and (4) use earnings before interest, taxes and depreciation over fixed assets. Finally, column (5) uses a measure of value-added, only available for Compustat firms.¹³ The table shows that the social cost of capital is significantly correlated with all different measures of ARPK, even after controlling for industry and time, with this correlation being stronger and more statistically significant for the VA-based measure in column (5). The remaining lines of the table report the variance of the log of each ARPK measure, and the implied amount of misallocation (on average) that would be obtained by using that respective measure in our sufficient statistic for misallocation. Use of the ARPK measures in the Y-14 data results in extremely large losses from misallocation: 64% and 46% for sales and EBITDA, respectively. Specializing to the Compustat sample results in lower values, but still an order of magnitude above those implied by r^{social} .

Why these differences? One caveat of our measure of misallocation is that it is a measure of capital misallocation only, abstracting from the efficiency in the allocation of other inputs, as well as product market misallocation (i.e., misallocation due to markup dispersion). On the other hand, our measure relies on basic data elements that are typically available in credit registries maintained by financial regulators all around the world, and does not require detailed information on firm financials that is typically needed to compute ARPK, itself just an approximation of MRPK.

5.8 Cross-Country Comparison

Finally, in Table 4, we compare our results to related values for other countries. Our method for deriving the distribution of r^{social} incorporates the joint distribution of interest rates, default probabilities, expected losses given default, and leverage. Although prior work has not combined all of these variables in the ways necessary to apply our method we can still

¹³Computing value added requires information on the cost of intermediates, which is available neither in the Y-14 nor Compustat. For Compustat firms, it is possible to infer a measure of value added by using the method described in Donangelo et al. (2019): some firms report total annual labor expenses. This can be combined with the number of employees to compute an average wage by industry and year. We then use the average wage to impute total labor costs to other firms in the industry in a given year. Value added is then calculated as the sum of EBITDA to labor expenses, implicitly assuming that all non-labor costs are related to intermediates.

Table 3: Relation to ARPK measures

	(1)	(2)	(3)	(4)	(5)
	logARPK, Sales	logARPK, EBITDA	logARPK, Sales	logARPK, EBITDA	logARPK, VA
$\log(r^{social} + \delta)$	0.15*** (0.03)	0.24*** (0.04)	0.16** (0.07)	0.15* (0.08)	0.39*** (0.07)
Observations	59294	57334	4184	4072	3432
Adj. R2	0.27	0.22	0.68	0.52	0.61
NAICS4, Quarter FE	yes	yes	yes	yes	yes
Sample	Y-14	Y-14	Compustat	Compustat	Compustat
Var(log(ARPK))	1.97	1.52	0.19	0.24	0.21
Misalloc., ARPK, %	63.63	46.08	4.75	6.20	5.28
Var(log(rsocial))	0.04	0.04	0.01	0.01	0.01
Misalloc., rsocial, %	0.96	0.96	0.36	0.36	0.36

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Cross-Country Comparison of Cost of Capital

	Aleem 1990 Pakistan	Khwaja & Mian 2005 Pakistan	Cavalcanti et al. 2024 Brazil	Beraldi 2025 Mexico	This paper 2025 United States
Years of data	1980–1981	1996–2002	2006–2016	2003–2022	2014–2024
$\mathbb{E}[r_i]$, %	66.8	8.00	83.0	12.4	1.4
$SD(r_i)$, %	38.1	2.9	93.3	5.2	1.2
$\mathbb{E}[1 - P_i]$, %	2.7	16.9	4.0	8.9	1.5
$\mathbb{E}[1 - LGD_i]$, % (World Bank)	42.8	42.8	18.2	63.9	66.6
Implied misallocation, %	6.5	13.5	21.5	2.8	0.8

Notes: Data for Pakistan (1980–1981) are from [Aleem \(1990\)](#), and for 1996–2002 from [Khwaja and Mian \(2005\)](#). Brazilian data are from [Cavalcanti et al. \(2024\)](#), and Mexican data from [Beraldi \(2025\)](#). Recovery rates ($1 - LGD$) are country-level estimates obtained from the World Bank’s Doing Business database.

learn something from more commonly reported statistics. We focus on papers that report summary statistics for a representative sample of bank loans, or moneylenders in the case of [Aleem \(1990\)](#). We are able to find five such papers that report the mean and standard deviation of interest rates as well as the mean probability of default. For comparison, we also use our data to provide the same statistics for the United States.¹⁴ Relative to less developed countries, the United States stands out for a low mean and low standard deviation of interest

¹⁴[Cavalcanti et al. \(2024\)](#) report the spread relative to deposit rates; we compute the mean interest rate by adding the mean deposit rate, which they report in the paper, to the mean spread. [Beraldi \(2025\)](#) reports the spread relative to the Mexican 28-day interbank rate; we compute the mean interbank rate over this time period and add this to the mean spread. For both papers, we use the distribution of spreads to get the standard deviation of the spread. [Beraldi \(2025\)](#) reports quantiles instead of the standard deviation, so we use the 90/10 percentile range and a normal approximation to infer the standard deviation. We convert reported nominal rates to real rates by subtracting average annual inflation in the respective country during the period of analysis.

rates. However there is significant heterogeneity among the results for other countries, which does not appear to simply track the level of development. Bank loans in Pakistan and Mexico have a standard deviation of interest rates that is only modestly higher than in the United States, while bank loans in Brazil and loans from moneylenders in Pakistan have an extremely high standard deviation of interest rates.

We can attempt to use these statistics to compute misallocation, at the cost of strong assumptions. We use recovery rates from the World Bank’s Doing Business database to provide information on expected losses given default. Since we do not have information on firm leverage, we use the lender’s cost of capital, ρ , in place of the social cost of capital, r^{social} . We then use the fixed rate formula for ρ and assume that the probability of default and the losses given default do not vary across firms. This allows us to compute a cost of misallocation, which we show in the last column. The cost of misallocation we compute for the United states is similar to the actual cost that we computed earlier, although this is no guarantee that the same is true for other countries.

There are two main takeaways from our misallocation analysis. First, the United states seems to have the most efficient credit markets of the countries in our table: misallocation is moderately higher for bank loans in Mexico and moneylenders in Pakistan, and substantially higher in Brazil and among Pakistani banks. However the relationship between development and credit market efficiency varies across settings, even for countries like Brazil and Mexico that are at similar levels of economic development.

6 Conclusion

This paper develops a novel methodology to estimate the cost of capital using credit registry microdata, and examines the implications of dispersion in the cost of capital for misallocation. We show, in a dynamic corporate finance model, the connection between the lender’s cost of capital, the firm’s cost of capital, and the social cost of capital, and how to measure these objects in the data. We also show how the mean and variance of the social cost

of capital can be used as sufficient statistics to measure the output losses from misallocation that arise from credit market imperfections.

After developing this general methodology, we apply it to credit registry data for the United States. We find that although the cost of capital varies across firms, the resulting misallocation is modest in normal times, resulting in output losses of about 1%. However, dispersion in the social cost of capital among newly originated loans rose dramatically during the COVID-19 pandemic, driven by a rise in the dispersion of lender discount rates. Understanding the causes of this rise in dispersion, as well as the consequences for aggregate productivity, is an important area for future research. Moreover, comparing the distribution of the cost of capital in the United States to the distribution in other economies, especially less developed economies, will help us better understand how financial markets contribute to development.

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Appendix

A Proofs

Proof of Proposition 1.

$$\begin{aligned}
\mathbb{E}_t \left[\frac{\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})}{Q_t} \right] &= (1 + \rho) \frac{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]}{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})] + \mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi(k')/b']} \\
&= (1 + \rho) \left(1 + \frac{\mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi(k')/b']}{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]} \right)^{-1} \\
&= (1 + \rho) (1 + \Lambda)^{-1}
\end{aligned}$$

where

$$\Lambda \equiv \frac{\mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi(k')/b']}{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]}$$

□

Derivation of Equation 5. We combine the first-order conditions for capital and for debt. Recall the recursive formulation of the model.

$$\begin{aligned}
V(k, b, z) &= \max_{k', b'} \pi(k, b, z, k', b') + \beta \mathbb{E} [\max \{V(k', b', z'), 0\} \mid z] \\
\pi(k, b, z, k', b') &= f(k, z) + (1 - \delta)k - k' - \theta b + Q(k', b', z, \rho)(b' - (1 - \theta)b)
\end{aligned}$$

The firm's maximization yields the first-order conditions for tomorrow's capital, k' , and tomorrow's debt, b' .

$$\begin{aligned}
0 &= \frac{\partial \pi(k, b, z, k', b')}{\partial k'} + \beta \mathcal{P}(k', b', z) \mathbb{E} \left[\frac{\partial}{\partial k'} V(k', b', z') \mid z, V > 0 \right] \\
0 &= \frac{\partial \pi(k, b, z, k', b')}{\partial b'} + \beta \mathcal{P}(k', b', z) \mathbb{E} \left[\frac{\partial}{\partial b'} V(k', b', z') \mid z, V > 0 \right]
\end{aligned}$$

where $\mathcal{P}(k', b', z)$ is the probability of not defaulting, and $V > 0$ indicates that the firm did not default.

We next use the Envelope Theorem to note that $\frac{\partial V(k', b', z')}{\partial k'} = \frac{\partial}{\partial k'} \pi(k', b', z', k'', b'')$ and

similarly to note that $\frac{\partial V(k', b', z')}{\partial b'} = \frac{\partial}{\partial b'} \pi(k', b', z', k'', b'')$. Our first-order conditions become:

$$\begin{aligned} 0 &= \frac{\partial \pi(k, b, z, k', b')}{k'} + \beta \mathcal{P}(k', b', z) \mathbb{E} \left[\frac{\partial}{\partial k'} \pi(k', b', z', k'', b'') \mid z, V > 0 \right] \\ 0 &= \frac{\partial \pi(k, b, z, k', b')}{b'} + \beta \mathcal{P}(k', b', z) \mathbb{E} \left[\frac{\partial}{\partial b'} \pi(k', b', z', k'', b'') \mid z, V > 0 \right] \end{aligned}$$

Next, we take derivatives of the profit function to plug into our first-order conditions.

We have:

$$\begin{aligned} \frac{\partial \pi(k, b, z, k', b')}{k} &= f_k(k, z) + (1 - \delta) \\ \frac{\partial \pi(k, b, z, k', b')}{b} &= -\theta - (1 - \theta) Q(k', b', z) \\ \frac{\partial \pi(k, b, z, k', b')}{k'} &= -1 + \frac{\partial Q(k', b', z)}{\partial k'} (b' - (1 - \theta) b) \\ \frac{\partial \pi(k, b, z, k', b')}{b'} &= Q(k', b', z) + \frac{\partial Q(k', b', z)}{\partial b'} (b' - (1 - \theta) b) \end{aligned}$$

Plugging these expressions in, our first-order conditions now become:

$$\begin{aligned} 0 &= -1 + \frac{\partial Q(k', b', z)}{\partial k'} (b' - (1 - \theta) b) + \beta \mathcal{P}(k', b', z) \mathbb{E}[f_k(k', z') + (1 - \delta) \mid z, V > 0] \\ 0 &= Q(k', b', z) + \frac{\partial Q(k', b', z)}{\partial b'} (b' - (1 - \theta) b) + \beta \mathcal{P}(k', b', z) \mathbb{E}[-\theta - (1 - \theta) Q(k'', b'', z') \mid z, V > 0] \end{aligned}$$

We next combine these two first-order conditions. Rather than thinking about investment that is financed through earnings, we want to instead imagine that the firm is financing a marginal unit of capital through borrowing. To do this, we multiply the first-order condition for debt by

$$-\frac{1 - \frac{\partial Q(k', b', z)}{\partial k'} (b' - (1 - \theta) b)}{Q(k', b', z) + \frac{\partial Q(k', b', z)}{\partial b'} (b' - (1 - \theta) b)}$$

which reflects the amount of new debt needed to finance a marginal unit of capital. The denominator reflects the amount raised by selling a unit of debt, $Q(k', b', z)$, plus an adjustment factor, $\frac{\partial Q}{\partial b'} (b' - (1 - \theta) b)$, that reflects how the change in the price of debt affects the cost of borrowing. Similarly, the numerator reflects how an increase in capital lowers is partly self-financing, because it lowers the cost of borrowing.

Combining the two equations then yields:

$$\begin{aligned} \beta \mathcal{P}(k', b', z) \mathbb{E}[f_k(k', z') + (1 - \delta) \mid z, V > 0] &= \frac{1 - \frac{\partial Q(k', b', z)}{\partial k'} (b' - (1 - \theta) b)}{Q(k', b', z) + \frac{\partial Q(k', b', z)}{\partial b'} (b' - (1 - \theta) b)} \\ &\quad \times \beta \mathcal{P}(k', b', z) \mathbb{E}[\theta + (1 - \theta) Q(k'', b'', z') \mid z, V > 0] \end{aligned}$$

Further manipulation then yields:

$$\begin{aligned} \mathcal{P}(k', b', z) \mathbb{E}[f_k(k', z') + (1 - \delta) \mid z, V > 0] &= \frac{1 - \frac{\partial Q(k', b', z)}{\partial k'} (b' - (1 - \theta) b)}{1 + \frac{\partial Q(k', b', z)}{\partial b'} \cdot (b' - (1 - \theta) b) / Q(k', b', z)} \\ &\quad \times \mathcal{P}(k', b', z) \mathbb{E}\left[\frac{\theta + (1 - \theta) Q(k'', b'', z')}{Q(k', b', z)} \mid z, V > 0\right] \\ &= \mathcal{M} \cdot \left(1 + r_t^{firm}\right) \end{aligned}$$

where \mathcal{M} is given by the following formula:

$$\begin{aligned} \mathcal{M} &= \frac{1 - \frac{\partial Q}{\partial k'} (b' - (1 - \theta) b)}{1 + \frac{\partial Q}{\partial b'} \cdot (b' - (1 - \theta) b) / Q} \\ &= \frac{1 - \frac{\partial \log Q}{\partial \log k'} \cdot \frac{Q}{k'} (b' - (1 - \theta) b)}{1 + \frac{\partial \log Q}{\partial \log b'} \cdot \frac{Q}{b'} (b' - (1 - \theta) b) / Q} \\ &= \frac{1 - \frac{\partial \log Q}{\partial \log k'} \cdot \frac{Q \cdot b'}{k'} \frac{(b' - (1 - \theta) b)}{b'}}{1 + \frac{\partial \log Q}{\partial \log b'} \cdot \frac{(b' - (1 - \theta) b)}{b'}} \\ &= \frac{1 - \gamma \cdot \frac{Q \cdot b'}{k'} \cdot \frac{\partial \log Q}{\partial \log k'}}{1 + \gamma \cdot \frac{\partial \log Q}{\partial \log b'}} \end{aligned}$$

where $\gamma := \frac{(b' - (1 - \theta) b)}{b'}$. This completes the proof. \square

Proof of Proposition 3.

$$1 = \sum_{t=1}^T \left(\frac{P}{1 + \rho} \right)^t \left[r + \frac{(1 - P)}{P} (1 - LGD) \right] + \left(\frac{P}{1 + \rho} \right)^T$$

Let $x = \frac{P}{1 + \rho}$ so

$$1 = \left(r + \frac{1 - P}{P} (1 - LGD) \right) \frac{x}{1 - x} (1 - x^T) + x^T$$

Guess that $1 + \rho = (1 + r)P + (1 - P)(1 - LGD)$

$$\frac{1 - x}{x} = \frac{1}{x} - 1 = \frac{(1 + r)P + (1 - P)(1 - LGD)}{P} - 1 = r + \frac{1 - P}{P}(1 - LGD)$$

And, therefore

$$1 = 1(1 - x^T) + x^T$$

which validates the guess. □

Proof of Proposition 4. Rearranging Equations 2 and 4, we have

$$\begin{aligned} 1 + \rho &= \frac{\mathbb{E} \left[\mathcal{P}_{t+1} (\theta + (1 - \theta)Q_{t+1}) + (1 - \mathcal{P}_{t+1}) \frac{\phi(k_{t+1})}{b_{t+1}} \middle| k_{t+1}, b_{t+1}, z_t \right]}{Q_t} \\ &= 1 + r_t^{firm} + \mathbb{E} \left[(1 - \mathcal{P}_{t+1}) \frac{\phi(k_{t+1})}{Q_t \cdot b_{t+1}} \middle| k_{t+1}, b_{t+1}, z_t \right] \\ &= 1 + r_t^{firm} + (1 - P) \cdot (1 - LGD) \end{aligned}$$

with the last line using our formula for LGD at origination. □

Proof of Proposition 5. We start with Equation 9, then we use the fact that, by assumption, $\phi'(k_{t+1}) = \phi(k_{t+1})/k_{t+1}$, and then we plug in the definitions of lev and LGD . This yields:

$$\begin{aligned} 1 + r^{social} &= (1 + r^{firm})\mathcal{M} + (1 - P) \cdot \phi'(k_{t+1}) \\ &= (1 + r^{firm})\mathcal{M} + (1 - P) \cdot \frac{\phi(k_{t+1})}{k_{t+1}} \cdot \frac{Q_t b_{t+1}}{Q_t b_{t+1}} \\ &= (1 + r^{firm})\mathcal{M} + (1 - P) \cdot (1 - LGD) \cdot lev \end{aligned}$$

Plugging in our formula for r^{firm} from Proposition 4 yields

$$\begin{aligned}
1 + r^{social} &= (1 + \rho - (1 - P)(1 - LGD))\mathcal{M} + (1 - P) \cdot (1 - LGD) \cdot lev \\
&= (1 + \rho)\mathcal{M} + (lev - \mathcal{M}) \cdot (1 - P)(1 - LGD)
\end{aligned}$$

□

B Data

B.1 Details on Data Cleaning and Construction

While the FR Y-14Q Schedule H.1 data goes back to 2011, we keep only data from 2014Q4 due to data quality and consistency of reporting issues.

Borrowers. We drop all loans to borrowers without a Tax Identification Number. We keep only Commercial & Industrial loans to nonfinancial U.S. addresses, i.e. lines reported on FR Y-9C equal to 3, 4, 8, 9, and 10. We drop all borrowers with NAICS codes 52 (Finance and Insurance), 92 (Public Administration), 5312 (Offices of Real Estate Agents and Brokers), and 551111 (Offices of Bank Holding Companies), as some financial companies are classified under the later two NAICS codes in our sample.

Loans. We drop all loans with a negative committed exposure, or for which the utilized exposure exceeds the committed exposure as these are likely to be mistakes. We drop all observations for which the origination date exceeds the current date, and all those for which the maturity date precedes the current date.

We keep only “vanilla” term loans (Facility type equal to 7), and we thus exclude Type A, B, and C term loans, as well as bridge term loans. We keep only loans that are classified as fixed or variable rate, and drop mixed interest rate variability loans. We keep only loans with maturity between 1 and 10 years, thus excluding very short-term and very long-term

loans. We keep only loans with interest rates in the 1st-99th percentiles for fixed rate loans, and spread in the 1st-99th percentiles for variable rate loans, as some of the very high and low rates/spreads are likely to be data errors. Additionally, we drop loans with interest rates higher than 50% at origination. We also drop loans for which the probability of default and the loss given default are either missing or outside of the $[0, 1]$ intervals. We also drop loans for which the probability of default is equal to 1, as that is an indicator that the loan is in default.

B.2 Estimating \mathcal{M}

In this section, we argue that our calibration of $\mathcal{M} = 1$ is a good approximation. To this end, we provide estimates of this object in the data. Recall that this object was defined as

$$\mathcal{M}_t := \frac{1 - \gamma \times \frac{Qb'}{k'} \times \frac{\partial \log Q}{\partial \log k'}}{1 + \gamma \times \frac{\partial \log Q}{\partial \log b'}}$$

Given estimates for the function Q , γ , and firm leverage Qb'/k' we can compute \mathcal{M} for every observation (loan origination) in our data. The main challenge is to estimate Q as a function of firm borrowing and investment. This function can either be obtained by solving a calibrated version of our model, or estimated non-parametrically in the data. In this subsection, we present results for the latter approach.

First, we compute Q for every loan origination in the data. In a model setting such as ours, where loans are modeled as perpetuities that decay at a geometric rate θ , we can write Q as the present value of all future payments, discounted at the contractual interest rate r :

$$Q = \frac{\theta + (1 - \theta)Q}{1 + r} = \frac{\theta}{r + \theta}$$

r is directly observed in the data, and we can apply the common approximation that θ is equal to the inverse of the loan maturity, $\theta = 1/T$. This allows us to compute Q for every loan origination in the data.

The model establishes that Q is a function of firm investment k' , firm borrowing b' , as well as the current level of productivity z . Additionally, Q should also depend on the lender's cost

of capital ρ . We therefore approximate (the log of) Q as a polynomial of these four variables. We measure firm investment as (the log of) tangible assets at loan origination, firm borrowing as (the log of) total debt owed by the firm at loan origination, firm productivity as the (the log of) sales over tangible assets (a measure of TFPR following [Hsieh and Klenow \(2009\)](#)). The lender's cost of capital ρ is measured as in the main text. We therefore estimate:

$$\begin{aligned}\log Q_i = & \alpha + \beta_k \log k_i + \beta_b \log b_i + \beta_z \log z_i + \beta_\rho \rho_i \\ & + \beta_{k,k}(\log k_i)^2 + \beta_{k,b} \log k_i \times \log b_i + \beta_{k,z} \log k_i \times \log z_i + \beta_{k,\rho} \log k_i \times \rho_i \\ & + \beta_{b,b}(\log b_i)^2 + \beta_{b,z} \log b_i \times \log z_i + \beta_{b,\rho} \log b_i \times \rho_i \\ & + \beta_{z,z}(\log z_i)^2 + \beta_{z,\rho} \log z_i \times \rho_i + \beta_{\rho,\rho}(\rho_i)^2 + \epsilon_i\end{aligned}$$

The resulting estimates can be used to compute the partial derivatives of $\log Q$ with respect to investment and borrowing. Qb'/k' is measured in a consistent manner, as the sum of total liabilities plus new borrowings divided by total assets plus new borrowings. Finally, we take advantage of the fact that at the steady state, $\gamma = \theta = 1/T$.

Figure 7 presents the histogram for the estimated \mathcal{M}_i in our sample. Clearly, the distribution is extremely concentrated around 1. The mean is equal to 0.996 and the median to 0.997, with a standard deviation of 0.006. Figure 8 replicates our measure of misallocation, when computed accounting for heterogeneity in \mathcal{M} , and compares it to our baseline, showing that the two measures are extremely similar, both in terms of magnitudes and dynamics. Taken together, these results suggest that our assumption that $\mathcal{M} = 1$ is a good one.

B.3 Cross-sectional Heterogeneity

We also explore the correlation between the cost of capital and firm-level covariates. We regress $\log(1+r)$ separately on log leverage, log return on assets, and log assets. We conduct this analysis for interest rates, ρ , r^{firm} , and r^{social} . The results are shown in Table 5. Of the three covariates, the best predictor is the return on assets; interest rates and the cost of capital are consistently higher at firms with high return on assets. Although we cannot attach a causal interpretation to the estimated coefficients, this would be consistent with a

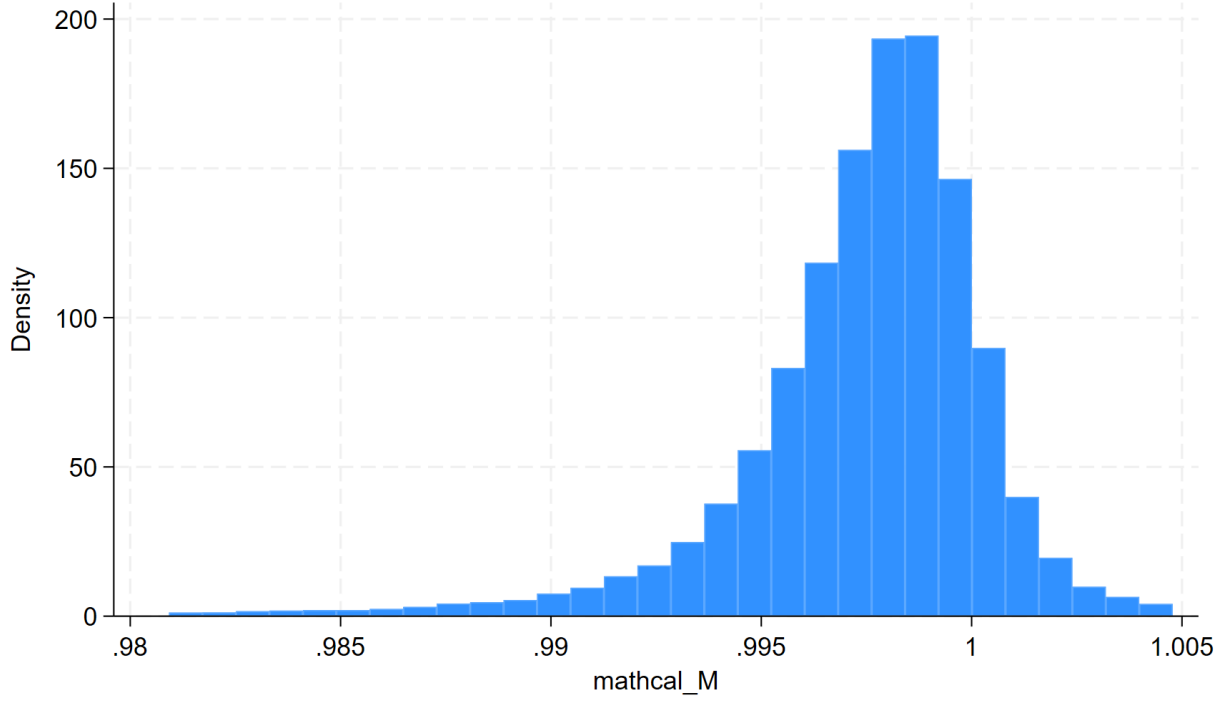


Figure 7: Histogram for estimated \mathcal{M}_i

model where causality runs from the cost of capital to firm decisions: firms with a higher cost of capital will demand a high return on their investments. Yet perhaps more notable is the very low R^2 . The return on assets explains between 2 and 3% of the variance, depending on the measure of the cost of capital, with other covariates explaining less than 1%. Firm-level covariates explain approximately none of the variance in the cost of capital.

Table 5: Determinants of Capital Costs and Spreads

Panel A: Real Contractual Rate				
	(1)	(2)	(3)	(4)
log leverage	0.100*** (0.00)			0.081*** (0.00)
log roa		0.010*** (0.00)		-0.056*** (0.00)
log assets			-0.174*** (0.00)	-0.182*** (0.00)
Observations	63189	60156	60913	60153
Adj. R2	0.47	0.46	0.48	0.49
NAICS4, Quarter FE	yes	yes	yes	yes
Panel B: Lender Discount Rate				
	(1)	(2)	(3)	(4)
log leverage	0.005 (0.00)			0.010*** (0.00)
log roa		0.038*** (0.00)		0.028*** (0.00)
log assets			-0.042*** (0.00)	-0.029*** (0.00)
Observations	63189	60156	60913	60153
Adj. R2	0.35	0.34	0.35	0.34
NAICS4, Quarter FE	yes	yes	yes	yes
Panel C: Firm's Cost of Capital				
	(1)	(2)	(3)	(4)
log leverage	-0.072*** (0.00)			-0.058*** (0.00)
log roa		0.052*** (0.00)		0.086*** (0.00)
log assets			0.068*** (0.00)	0.092*** (0.00)
Observations	63189	60156	60913	60153
Adj. R2	0.21	0.20	0.20	0.21
NAICS4, Quarter FE	yes	yes	yes	yes
Panel D: Social Cost of Capital				
	(1)	(2)	(3)	(4)
log leverage	0.103*** (0.00)			0.110*** (0.00)
log roa		0.066*** (0.00)		0.062*** (0.00)
log assets			-0.029*** (0.00)	0.018*** (0.00)
Observations	63189	60156	60913	60153
Adj. R2	0.29	0.27	0.27	0.28
NAICS4, Quarter FE	yes	yes	yes	yes

Standardized beta coefficients; Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

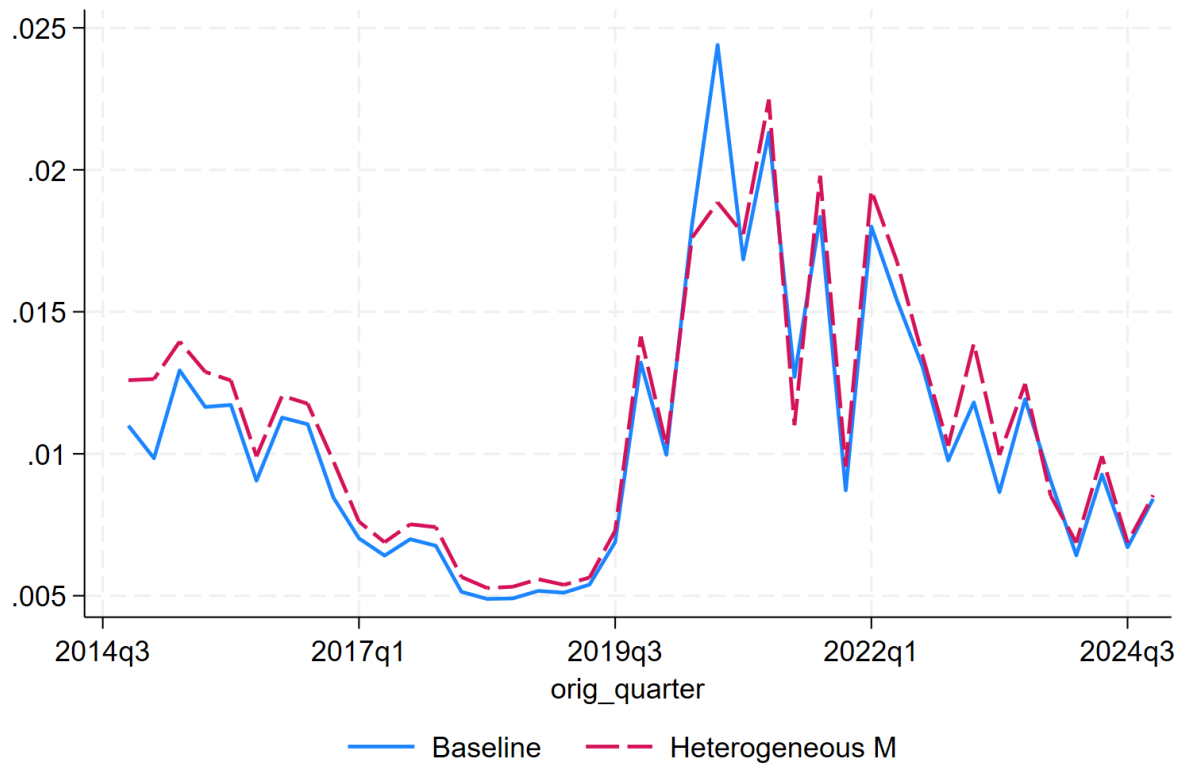


Figure 8: Misallocation measure with $\mathcal{M} = 1$ vs. estimated \mathcal{M}_i

B.4 Misallocation weighted by loan size

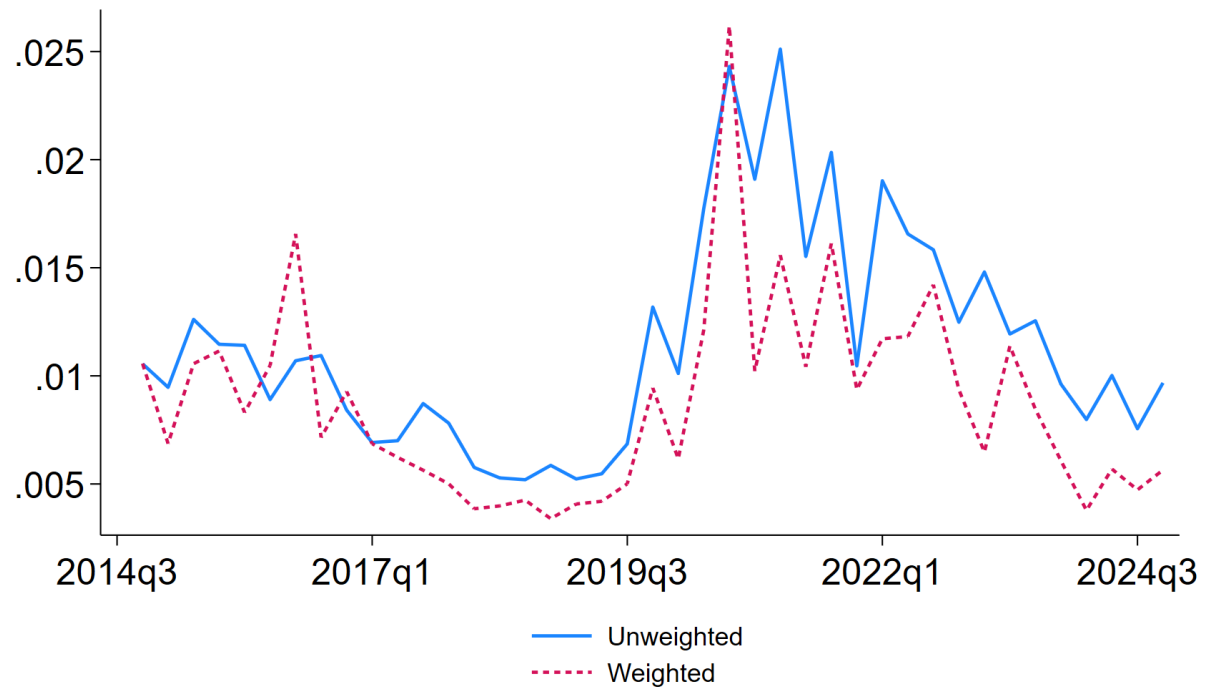


Figure 9: Misallocation, unweighted and weighted by loan size

B.5 Robustness: Results for Five-Year Loans

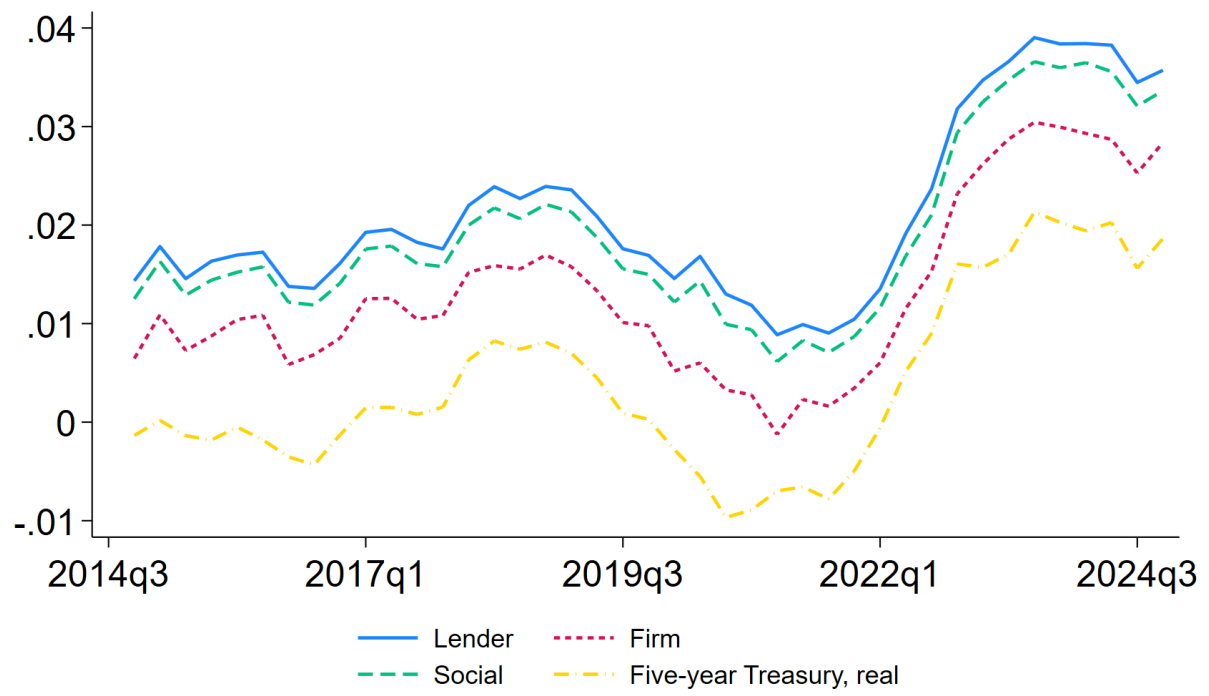


Figure 10: Averages by Quarter of Origination (Five-Year Sample)

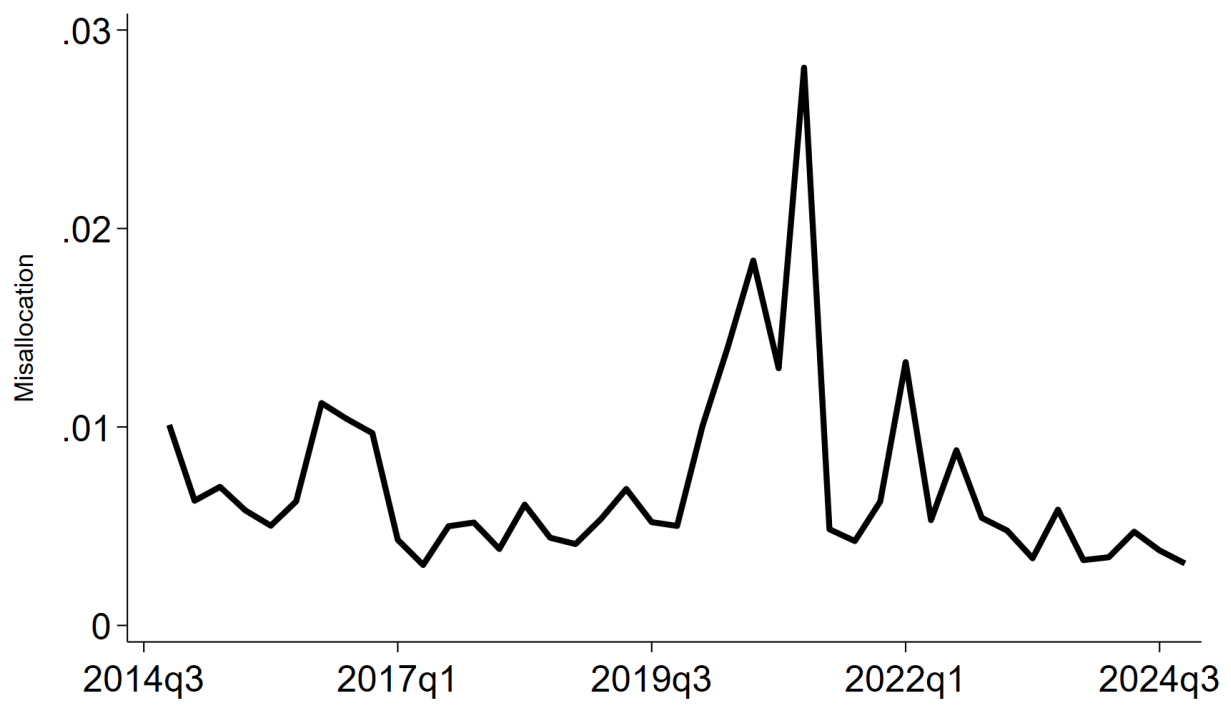


Figure 11: Cost of Misallocation (Five-Year Sample)

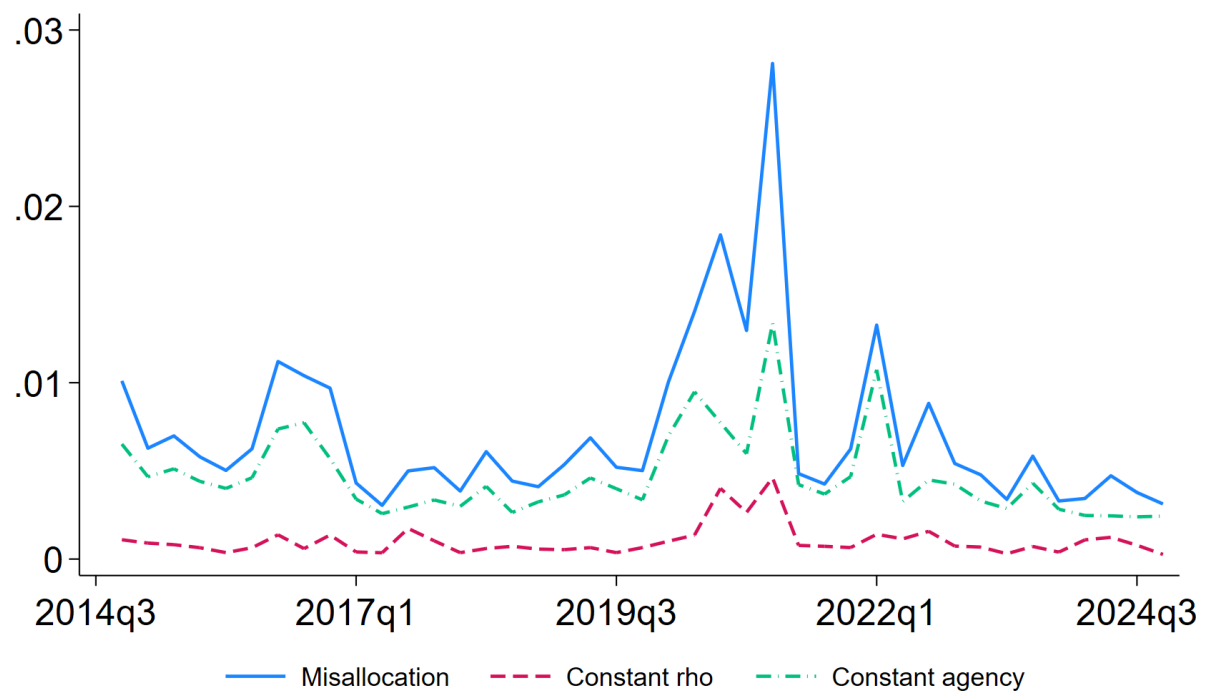


Figure 12: Decomposing Misallocation (Five-Year Sample)