The St. Louis Fed DSGE Model*

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Abstract

This document contains a technical description of the dynamic stochastic general equilibrium (DSGE) model developed and maintained by the Research Division of the St. Louis Fed as one of its tools for forecasting and policy analysis. The St. Louis Fed model departs from an otherwise standard medium-scale New Keynesian DSGE model along two main dimensions: first, it allows for household heterogeneity, in the form of workers and capitalists, who have different marginal propensities to consume (MPC). Second, it explicitly models a fiscal sector endowed with multiple spending and revenue instruments, such as social transfers and distortionary income taxes. Both of these features make the model well-suited for the analysis of fiscal policy counterfactuals, and monetary-fiscal interactions. We describe how the model is estimated using historical data for the US economy and how the COVID-19 pandemic is accounted for. Some examples of model output are presented and discussed.

Keywords: DSGE model; policy analysis; New Keynesian model; TANK model; Bayesian estimation; fiscal policy
JEL Classification: E1, E2, E3, E4, E5

1 Introduction

In this document, I describe a dynamic stochastic general equilibrium (DSGE) model that can be used for policy analysis and forecasting. The model takes mostly standard ingredients found in medium-scale DSGE models (Christiano et al., 2005; Smets and Wouters, 2007) and extends the standard model along a few dimensions that are not typically considered in models used at central banks. In particular, our model contains both household heterogeneity and a more detailed description of fiscal policy.

The base of the model is similar to the current models adopted by different Federal Reserve Banks across the Federal Reserve System: New York (Cocci et al., 2013; Del Negro et al., 2017),

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Philadelphia (Arias et al., 2019), Chicago (Campbell et al., 2023), or Cleveland (Gelain and Lopez, 2023). Each of these models puts more emphasis on certain aspects of modelling and/or measurement: financial frictions, frictional labor markets, etc. Our modeling framework, instead, takes a more conventional approach to these features, but allows us to directly analyze the macroeconomic effects of different types of fiscal policy on the economy and on fiscal variables such as government debt. Features such as an explicit financial intermediation sector or search-and-matching labor markets can easily be added to the current framework.

Following standard practice, we externally calibrate some parameters and estimate the rest using Bayesian methods. Our estimation sample runs from 1964Q1 to 2019Q4, the last full quarter before the economic effects of the COVID-19 pandemic started being felt in the US. While we only estimate the model through 2019Q4, we filter the latest data through the model to obtain estimates of the underlying shocks and decompose the contribution of each shock to the endogenous variables. We use a standard approach to deal with the COVID-19 pandemic, where we add one-time i.i.d. components shocks to some of the exogenous variables (Ferroni et al., 2022). This accounts for extraordinary movements in aggregate demand, labor supply and fiscal policy, among other sources of impulses, that were large but short-lived.

Key differences relative to other models. There are two specific departures from other models. First, we allow for household heterogeneity in the form of two agents with heterogeneous marginal propensities to consume. In order to leverage the methodology and toolkits that have been developed to solve and estimate models of this kind, we assume a limited form heterogeneity: instead of a representative Ricardian agent, we assume that there are two types of agents, capitalists and workers, following the recent work of Cantore and Freund (2021). Capitalists do not work, are the residual claimants to all financial assets in the economy, and otherwise behave in a manner that is similar to that of the representative agent in standard medium-scale models. Workers, on the other hand, invest in risk-free claims on government debt subject to portfolio adjustment costs. This allows the model to do a better job of replicating the dynamics of both aggregate consumption and marginal propensities to consume (MPC) without having to rely on modeling devices such as habits on consumption, for example.

The fact that agents and MPCs are heterogeneous brings us to the second major feature, which is an explicit government budget constraint and different types of fiscal policies. Most medium-scale DSGE models cannot speak to the effects of transfer and redistributive policies given the assumption of a single representative agent who is unconstrained and for whom the Ricardian Equivalence holds. In our model, the assumption of limited heterogeneity allows for the analysis of redistributive policies such as lump-sum transfers. In our model, there is a well-defined government budget constraint. Importantly, and differently from other models, we measure the levels of government consumption and transfers directly from the data, which allows us to model fiscal deficits and therefore analyze the effects of shocks on variables such as government debt. Most medium-scale DSGE models treat government consumption as the residual of the income-expenditure identity in a closed economy setting, $Y = C + I + G$. Since we treat government consumption as an observable, and therefore observe all components of domestic expenditure, we add a new residual that captures deviations of total output from absorption: a reduced-form way of modeling the trade balance.

There is a long tradition of multiple-representative agent New Keynesian models in economics (Galí et al., 2007) as a way to capture heterogeneity in a limited way. The standard approach, which typically consists of two types of agents, Ricardian and hand-to-mouth, has well-known issues that limit its applicability for quantitative work. There is a large, more recent literature that detects, highlights, and proposes solutions to some of these issues (Bilbiie, 2019; Broer et al., 2020; Debortoli and Galí, 2024). Two major issues are: i) the fact that when working agents receive firm profits, the cyclicality of this variable generates implausible income effects over labor supply; and ii) strict hand-to-mouth agents have a MPC equal to 1, which results in implausible responses to fiscal policy shocks such as transfers. We adopt the parsimonious framework proposed by Cantore and Freund (2021), which addresses both of these issues by assuming that the agents who own firms do not supply labor, and that the agents who work face soft constraints on portfolio adjustment, which raises their MPC but does not make them completely constrained.

Purpose and Structure. This document is written more as guide than an academic article, aimed at those who are interested in the more practical and technical aspects of model development and usage. The model is continuously maintained and improved, and the plan is to update this document accordingly.

The rest of the document is structured as follows: Section 2 is a technical description of the model. Section 3 presents the full list of equilibrium conditions. Section 4 describes the calibration and estimation of the model. Finally, Section 5 presents some model results.

2 Model

Time is discrete and infinite, $t = 0, 1, 2, \ldots$. The agents in the economy are: two types of households, capitalists and workers, labor unions, intermediate goods producers, final goods producers, a fiscal authority, a monetary authority, and the rest of the world. The numeraire is a final nondurable consumption good.

2.1 TFP Growth

Labor-augmenting TFP is denoted by Z_t, which grows at gross rate Γ_t:

$$Z_t = \Gamma_t Z_{t-1} \tag{1}$$

2.2 Households

There are two types of households: workers in fixed share $\lambda \in [0,1]$, and capitalists in fixed share $1-\lambda$. Capitalists are similar to the representative agent in medium-scale DSGE models: they do not face any sort of borrowing or portfolio constraints and are the residual owners of financial and real claims in the economy. They are Ricardian due to the absence of borrowing constraints, as they internalize the fact that increases in government debt correspond to future taxation. This implies that their marginal propensity to consume (MPC) will be relatively low. Workers, on the other hand, save in risk-free government debt subject to portfolio adjustment costs. This raises their MPC, but importantly does not make them fully static/constrained as in other TANK models, which would imply a MPC equal to 1.

2.2.1 Capitalists

Capitalists are similar to the representative household in standard DSGE models. These households derive utility from the final consumption good, do not supply any labor, invest in physical capital and government debt, and choose the utilization rate of physical capital.

We can write their problem in recursive form as

$$V^s(B^s_{t-1}, K_{t-1}, I_{t-1}) = \max_{C^s_t, B^s_t, I_t, K_t, \nu_t} \chi_t \log(C^s_t) + \beta \mathbb{E}_t V^s(B^s_t, K_t, I_t) \tag{2}$$

$$\text{s.t.} \qquad P_t C^s_t + P_t \frac{B^s_t}{R_t} + P_t I_t = P_{t-1} B^s_{t-1} + (1-\tau^d_t) P_t (\nu_t R^k_t K_{t-1} + D_t) - P_t A(\nu_t) K_{t-1} \tag{3}$$

$$K_t = (1-\delta) K_{t-1} + \zeta_t [1 - S(I_t / I_{t-1})] I_t \tag{4}$$

where C^s_t is nondurable consumption, B^s_t is government debt held by capitalists, K_t is the physical stock of capital, I_t is investment in physical capital, $\nu_t \geq 0$ is the utilization rate of physical capital, P_t is the price level in terms of the nondurable consumption good, R_t is the nominal interest rate, T_t are transfers from the government, τ^d_t is a linear tax on capital income and profits, and D_t are profits from goods producers.

Equation 2 is the value function of the capitalist. We assume that these agents have log utility over consumption. Typical medium-scale models allow for habits on consumption in order to match the slow response of aggregate consumption to certain types of shocks. One of the advantages of the way we introduce heterogeneity is that this becomes unnecessary.[1] χ_t is a marginal utility shock that can be thought of as an "aggregate demand shock": this shock raises the marginal utility of consumption today, raising household consumption everything else constant. This plays a role similar to discount factor shocks in other DSGE models.

[1] We have tried versions of the model where we allow for consumption habits for capitalists, and estimation results point towards a value of the parameter governing the habits that is close to zero.

Equation 3 is the budget constraint of the capitalist, equating uses of income to sources of income. The household spends in nondurable consumption, invests in government bonds discounted at the one-period nominal interest rate R_t, and invests in physical capital. The household derives income from government debt repayments, as well as from (utilized) capital and firm profits, both of which are taxed at rate τ_t^d. Finally, $A(\nu)$ is a convex function that reflects capital utilization costs.

Equation 4 is the law of motion for physical capital: capital available tomorrow is equal to capital available today net of depreciation δ plus new capital formed through gross investment I_t. Investment is subject to adjustment costs represented by the function S. ζ_t is a shock to the marginal efficiency of investment that affects the economy's efficiency to convert final consumption goods into physical capital. These shocks have been shown to play an important role in the business cycle (Justiniano et al., 2011).

Optimality Conditions. The capitalist stochastic discount factor is given by:

$$m^s_{t,t+k} \equiv \beta \frac{\chi_{t+k}/C^s_{t+k}}{\chi_t/C^s_t} \tag{5}$$

The optimality conditions are as follows. First, there is an Euler equation for government debt:

$$1 = R_t \mathbb{E}_t \left[\frac{m^s_{t,t+1}}{\Pi_{t+1}} \right] \vartheta_t \tag{6}$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate. ϑ_t is a "convenience yield" shock that drives an exogenous wedge between the required return on government debt and capital as in Del Negro et al. (2017).[2]

Second, the FOC for capital is given by:

$$\omega_t = \mathbb{E}_t m^s_{t,t+1} \left[(1 - \tau^d_{t+1}) \nu_{t+1} R^k_{t+1} + (1-\delta)\omega_{t+1} - A(\nu_{t+1}) \right] \tag{7}$$

where ω_t is the marginal value of a unit of physical capital, or Tobin's Q.

The optimality condition for physical investment is given by

$$1 - \omega_t \zeta_t \left[1 - S_t - \frac{I_t}{I_{t-1}} S'_t \right] = \mathbb{E}_t m^s_{t,t+1} \left[\omega_{t+1} \zeta_{t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 S'_{t+1} \right] \tag{8}$$

where I abuse notation as $S_t \equiv S(I_t/I_{t-1})$. Finally, the optimal condition for capital utilization is

$$R^k_t = A'(\nu_t) \tag{9}$$

[2] This can be microfounded with bonds in the utility function, reflecting liquidity benefits of these assets that are not explicitly modeled, such as use as collateral for financial transactions.

2.2.2 Workers

Workers derive utility from the final consumption good, supply labor, receive transfers from the government, and invest in government bonds subject to portfolio adjustment costs. Their value function is given by

$$V^w(B^w_{t-1}) = \max_{C^w_t, B^w_t} \chi_t[\log(C^w_t) - v(N_t)] + \beta \mathbb{E}_t V^w(B^w_t) \tag{10}$$

$$\text{s.t.} \qquad P_t C^w_t + P_t \frac{B^w_t + \Psi(B^w_t)}{R_t} = (1-\tau_t) W_t N_t + P_{t-1} B^w_{t-1} + P_t T_t + P_t F_t \tag{11}$$

where $v(N_t)$ is disutility from labor supplied and N_t are hours worked, τ_t is a linear tax on labor income, W_t is the nominal wage, T_t are government transfers, and $\Psi(B^w_t)$ is a convex function that measures the degree of portfolio adjustment costs. In practice, we will assume that this is a quadratic function of deviations of B^w_t holdings from a baseline value, as described in more detail later. Note that we pre-multiply flow utility by χ_t so that this shock does not affect the relative preference between consumption and leisure, which would distort labor supply. We also assume that these costs are rebated to the households as lump-sum transfers F_t, so as to prevent them from generating income effects.

Optimality Conditions. We define the SDF of the worker in an analogous way to that of the capitalist:

$$m^w_{t,t+k} \equiv \beta \frac{\chi_{t+k}(C^w_{t+k})^{-1}}{\chi_t (C^w_t)^{-1}} \tag{12}$$

The Euler equation for the worker is then given by:

$$1 = R_t \vartheta_t \mathbb{E}\left[\frac{m^w_{t,t+1}}{\Pi_{t+1}} \frac{1}{1+\Psi'(B^w_t)}\right] \tag{13}$$

Notice that the savings decision is distorted by the derivative of Ψ: in an environment without any adjustment costs, the Euler equation of the worker is exactly the same as the capitalist's Euler 6. In an extreme case with infinite adjustment costs, the worker is effectively hand-to-mouth, cannot adjust their asset holdings and consumption is pinned down by the budget constraint 11, generating a MPC of 1. A finite derivative for this function allows for an intermediate case. In particular, it allows the worker to, for example, smooth out income that is received from the government instead of having to spend it right away, which would be the case in a pure TANK model.

2.3 Labor Unions

Notice that the worker does not choose how much labor to supply. That is because this decision is taken by labor unions, a convenient way to introduce nominal wage rigidities.

We begin by assuming that the worker supplies a continuum of varieties of labor indexed by j. We assume that the disutility of labor for the worker takes the form

$$v(N) = \xi \int \frac{(N(j))^{1+\varphi}}{1+\varphi} \mathrm{d}j \tag{14}$$

Each of these labor varieties is supplied to a union, which decides the nominal wage for this specific variety of labor $W_t(j)$. These labor varieties are then supplied to a "labor aggregator firm" that converts them into a labor composite N_t.

Labor Aggregator. The labor aggregator firm has access to a CES technology that converts labor varieties $N_t(j)$ into a final labor composite N_t that is then "sold" to intermediate goods producers. The labor aggregator hires each variety at wage $W_t(j)$ and then "sells" the final labor composite at wage W_t. The problem of the labor aggregator is given by:

$$\max_{N_t, \{N_t(j)\}} W_t N_t - \int W_t(j) N_t(j) \mathrm{d}j \tag{15}$$

$$\text{s.t.} \qquad N_t = \left[\int N_t(j)^{\frac{1}{\mu_t^w}} \mathrm{d}j \right]^{\mu_t^w} \tag{16}$$

where 16 is a CES technology that converts the continuum of labor varieties into the final labor composite. μ_t^w is a transformation of the elasticity of substitution between varieties,which determines the mark-up. This is an exogenous process reflecting fluctuations in labor market markups. The solution to the labor aggregator's problem yields labor variety-specific demand curves of the type

$$N_t(j) = \left[\frac{W_t(j)}{W_t} \right]^{-\frac{\mu_t^w}{\mu_t^w - 1}} N_t \tag{17}$$

The aggregate wage level is also implicitly defined as

$$W_t^{-\frac{1}{\mu_t^w - 1}} = \int W_t(j)^{-\frac{1}{\mu_t^w - 1}} \mathrm{d}j \tag{18}$$

Labor Union Problem. Each union j chooses how much labor to supply $N_t(j)$ and what wage to set $W_t(j)$ subject to the demand function for that specific variety 17 and adjustment costs à la Rotemberg. These adjustment costs make it costly to change the nominal wage for that specific variety.

The union chooses $W_t(j)$ to maximize the present discounted value of wage income, given disutility of labor supply. The recursive formulation of the union's problem is given by:

$$V^u[W_{t-1}(j)] = \max_{W_t(j),N_t(j)} [W_t(j) - P_t MRS_t(j)]N_t(j) - P_t\frac{\eta_w}{2}N_t\left[\frac{W_t(j)}{W_{t-1}(j)}\frac{1}{\Pi_t^{w,index}} - 1\right]^2 + \beta\mathbb{E}_t\frac{V^u[W_t(j)]}{\Pi_{t+1}} \tag{19}$$

$$\text{s.t.} \qquad N_t(j) = \left[\frac{W_t(j)}{W_t}\right]^{-\frac{\mu_t^w}{\mu_t^w-1}} N_t$$

The union's flow payoff is composed of nominal labor income $W_t(j)N_t(j)$ minus disutility of labor expressed in terms of nominal dollars, $P_t MRS_t(j)N_t(j)$. The second term is the Rotemberg menu cost: the union faces convex costs of adjusting $W_t(j)$ relative to the nominal wage from the previous period. $\Pi_t^{w,index}$ is an indexation term that allows for some "free adjustment".

$MRS_t(j)$ is equal to the marginal rate of substitution for the worker:

$$MRS_t(j) = (1-\tau_t)^{-1}C_t^w\xi N_t(j)^\varphi \tag{20}$$

That is, it is equal to $-\nu'(N_t)/u'(C_t)$: the disutility of labor expressed in terms of units of consumption. In order to compress notation, define $MRS_t(j) \equiv \Xi_t N_t(j)^\varphi$. The first-order condition for the union problem in 19 is:

$$N_t(j)\left[1 + \varphi\Xi_t N_t(j)^\varphi\frac{\mu_t^w}{\mu_t^w - 1}\frac{P_t}{W_t(j)}\right] - [W_t(j) - P_t\Xi_t N_t(j)^\varphi]\frac{\mu_t^w}{\mu_t^w - 1}\frac{N_t(j)}{W_t(j)}$$
$$-\eta_w N_t\frac{P_t}{W_{t-1}(j)\Pi_t^{w,index}}\left[\frac{W_t(j)}{W_{t-1}(j)\Pi_t^{w,index}} - 1\right] + \beta\eta_w\mathbb{E}_t N_{t+1}\frac{P_{t+1}W_{t+1}(j)}{W_t(j)^2\Pi_{t+1}^{w,index}\Pi_{t+1}}\left[\frac{W_{t+1}(j)}{W_t(j)\Pi_{t+1}^{w,index}} - 1\right] = 0$$

We now assume that all unions behave symmetrically, setting $W_t(j) = W_t$ and $N_t(j) = N_t$ for all j. Define nominal wage inflation as $\Pi_t^w = W_t/W_{t-1}$. This allows us to simplify the above expression and arrive at the wage Phillips curve that governs wage inflation and labor supply:

$$\frac{W_t}{P_t} + \frac{\mu_t^w}{\mu_t^w - 1}\left[\Xi_t N_t^\varphi(1+\varphi) - \frac{W_t}{P_t}\right] - \eta_w\frac{\Pi_t^w}{\Pi_t^{w,index}}\left(\frac{\Pi_t^w}{\Pi_t^{w,index}} - 1\right) + \beta\eta_w\mathbb{E}_t\frac{N_{t+1}}{N_t}\frac{\Pi_{t+1}^w}{\Pi_{t+1}^{w,index}}\left(\frac{\Pi_{t+1}^w}{\Pi_{t+1}^{w,index}} - 1\right) = 0 \tag{21}$$

2.4 Goods Producers

As with labor supply, there are two types of firms in this economy. First, we have final goods producers, who convert intermediate goods varieties into a final nondurable good using a CES aggregator. Each of those intermediate varieties is produced by monopolistically competitive producers who hire physical capital and labor and set nominal prices subject to menu costs.

Final Goods Producers. The final goods producer combines a continuum of varieties indexed by k using a CES aggregator to produce final nondurable output. The problem of this producer is given by

$$\max_{\{Y_t(k)\},Y_t} P_t Y_t - \int P_t(k) Y_t(k) \mathrm{d}k$$
$$\text{s.t.} \qquad Y_t = \left[\int Y_t(k)^{\frac{1}{\mu_t^p}} \mathrm{d}k\right]^{\mu_t^p}$$

where $P_t(k)$ is the price of each variety. Notice that we allow the markup to be time-varying, μ_t^p. This problem gives rise to variety-specific demand curves

$$Y_t(k) = \left[\frac{P_t(k)}{P_t}\right]^{-\frac{\mu_t^p}{\mu_t^p - 1}} Y_t \tag{22}$$

The solution to this problem also defines the price level as a function of the variety prices:

$$P_t^{-\frac{1}{\mu_t^p - 1}} = \int P_t(k)^{-\frac{1}{\mu_t^p - 1}} \mathrm{d}k$$

Intermediate Goods Producers. A continuum of intermediate goods producers rent capital and hire labor to produce variety k. They also choose the nominal price of their respective variety, subject to menu costs. The recursive formulation of their problem is:

$$V^f[P_{t-1}(k)] = \max_{P_t(k),Y_t(k),N_t(k),K_t^u(k)} P_t(k)Y_t(k) - W_t N_t(k) - P_t R_t^k K_t^u(k) \tag{23}$$
$$-P_t \frac{\eta_p}{2} Y_t \left[\frac{P_t(k)}{P_{t-1}(k)} \frac{1}{\Pi_t^{p,index}} - 1\right]^2 + \mathbb{E}_t m_{t,t+1}^s V^f[P_t(k)]$$
$$\text{s.t.} \qquad Y_t(k) = \left[\frac{P_t(k)}{P_t}\right]^{-\frac{\mu_t^p}{\mu_t^p - 1}} Y_t \tag{24}$$
$$Y_t(k) \le (K_t^u(k))^{\alpha} (Z_t N_t(k))^{1-\alpha} \tag{25}$$

where $K_t^u(k)$ is the quantity of capital rented by producer of variety k, $\Pi_t^{p,index}$ is an index term that allows the firm to undertake some free adjustment of nominal prices, and Z_t is labor-augmenting technological change.

The solution to this problem is standard. It is convenient to first derive the optimal input mix that minimizes input costs for a given level of production. That is given by:

$$K_t^u(k) = \frac{\alpha}{1-\alpha} \frac{W_t / P_t}{R_t^k} N_t(k) \tag{26}$$

This allows us to define the real marginal cost of producing one input of output as

$$MC_t(k) = Z_t^{-(1-\alpha)} \left(\frac{R_t^k}{\alpha}\right)^{\alpha} \left(\frac{W_t/P_t}{1-\alpha}\right)^{1-\alpha} \tag{27}$$

We can then use the expression for the marginal cost to recast 23 as a problem with two control variables: the varietal price and output. Taking first-order conditions and, again, imposing symmetry across firms $P_t(k) = P_t, \forall k$ yields the New Keynesian Phillips curve (NKPC):

$$\frac{\Pi_t}{\Pi_t^{p,index}} \left(\frac{\Pi_t}{\Pi_t^{p,index}} - 1\right) = \mathbb{E}_t \left[m_{t,t+1}^s \frac{Y_{t+1}}{Y_t} \frac{\Pi_{t+1}}{\Pi_{t+1}^{p,index}} \left(\frac{\Pi_{t+1}}{\Pi_{t+1}^{p,index}}\right)\right] + \frac{1}{\eta_p(\mu_t^p - 1)} \left(\mu_t^p MC_t - 1\right) \tag{28}$$

Finally, total profits from the intermediate producers are rebated to the capitalists:

$$D_t = Y_t - \frac{W_t}{P_t} N_t - R_t^k K_t^u - \frac{\eta_p}{2} Y_t \left[\frac{P_t(k)}{P_{t-1}(k)} \frac{1}{\Pi_t^{p,index}} - 1\right]^2$$

2.5 Fiscal Authority

The fiscal component of the model is more detailed than what is usually found in medium-scale DSGEs, and follows closely Faria-e-Castro (2024).

The fiscal authority engages in government consumption G_t, runs a social transfer program targeted at workers T_t, issues nominal debt B_t^g, and levies distortionary labor income taxes τ_t as well as capital income and profit taxes τ_t^d.

The government budget constraint is:

$$P_t G_t + P_t \lambda T_t + P_{t-1} B_{t-1}^g = \tau_t W_t N_t + \tau_t^d P_t (\nu_t R_t^k K_{t-1} + D_t) + P_t \frac{B_t^g}{R_t} \tag{29}$$

Since some agents are non-Ricardian, the timing of deficits matters. We assume that both types of taxes follow a fiscal rule that responses to deviations of debt from a baseline level $\bar{B}^g$:

$$\tau_t = \bar{\tau} \left(\frac{B_{t-1}^g}{\bar{B}^g}\right)^{\phi_\tau} \tag{30}$$

$$\tau_t^d = \bar{\tau}^d \left(\frac{B_{t-1}^g}{\bar{B}^g}\right)^{\phi_\tau} \tag{31}$$

where ϕ_τ governs the speed of tax adjustment. If ϕ_τ is low, taxes do not respond much to fluctuations in the level of debt, and so fiscal expansions will tend to be deficit-financed and result in large fluctuations of public debt. Debt becomes the residual instrument that adjusts to satisfy the government budget constraint.

2.6 Monetary Authority

The central bank follows a standard Taylor rule as in Smets and Wouters (2007) or Del Negro et al. (2017):

$$R_t = R_{t-1}^{\rho_r} \left[\bar{R} \left(\frac{\Pi_t}{\Pi_t^*} \right)^{\phi_\Pi} \left(\frac{Y_t}{Y_t^f} \right)^{\phi_Y} \right]^{1-\rho_r} \left(\frac{Y_t / Y_t^f}{Y_{t-1} / Y_{t-1}^f} \right)^{\phi_\Delta} \mathrm{mp}_t \tag{32}$$

First, the rule contains an autoregressive term that captures policy inertia. Second, there is a "standard static term" through which the policy rate responds positively to deviations of inflation from its target Π_t^* and output from its natural level Y_t^f. We define the natural level of output later, in section 2.11. Third, the interest rate also responds to changes in the output gap. Finally, mp_t is a monetary policy shock that stands for non-systematic deviations of monetary policy from the rule.

2.7 Rest of the World

The rest of the world is modelled as an exogenous process that represents the trade balance, NX_t. We assume that the trade balance is the product between output and a scale shock:

$$NX_t = nx_t \times Y_t$$

where nx_t follows an AR(1) in levels (so that it can be positive or negative). Adding a trade balance subject to exogenous disturbances is necessary to ensure that the income-expenditure identity holds in the model, since we use both output growth as well as the real growth rate of all components of domestic absorption as observables.

2.8 Closing the Model

Summing over the budget constraints of both types of households and government, adding the rest of the world, and simplifying, we arrive at the aggregate resource constraint:

$$C_t + G_t + I_t + NX_t = Y_t \left[1 - \frac{\eta_p}{2} \left(\frac{\Pi_t}{\Pi_t^{p,index}} - 1 \right)^2 \right] - A(\nu_t) K_{t-1}$$

Total sources of domestic expenditure are aggregate consumption $C_t = \lambda C_t^w + (1-\lambda) C_t^s$, government consumption G_t, and investment in physical capital I_t. Additionally, there are utilization costs of capital and product price adjustment costs.

Finally, we allow the economy to be open, and treat the trade balance as an exogenous variable NX_t that is the residual of the aggregate resource constraint. This is different from other DSGE models, which typically treat the economy as closed and treat government consumption as the residual of the resource constraint. In our case, we want to be able to match the dynamics of fiscal variables, and will therefore include the growth rate of government consumption as an observable.

This requires us to define a different residual in order to be able to simultaneously match the growth rates of output and the other components of domestic consumption.

2.9 Functional Forms

Capital Utilization. In equilibrium, capital that is utilized by firms must be consistent with the utilization rate chosen by the owners of capital. Thus we have that

$$K_t^u = \nu_t K_{t-1}$$

For the cost of capital utilization $A(\nu_t)$, we use the following function:

$$A(\nu_t) = \kappa_a(\nu_t - 1) + 0.5\sigma_a(\nu_t - 1)^2$$

Notice that it satisfies the usual desirable properties for such function:

$$\begin{aligned} A(1) &= 0 \\ A'(1) &= \kappa_a (= \bar{R}^k) \\ \frac{A''(1)}{A'(1)} &= \frac{\sigma_a}{\kappa_a} \end{aligned}$$

Portfolio Adjustment Costs. We assume a simple quadratic cost function over deviations of portfolio holdings from a baseline value:

$$\Psi_t(B_t^w) = \frac{\psi_w}{2}(B_t^w - \bar{B}^w)^2$$

Price and Wage Indexation. We assume that the indexation terms are weighted averages of inflation in the past period and the steady state level of inflation:

$$\begin{aligned} \Pi_t^{p,index} &= \Pi_{t-1}^{\iota_p}\bar{\Pi}^{1-\iota_p} \\ \Pi_t^{w,index} &= (\Pi_{t-1}^w)^{\iota_w}(\bar{\Gamma}\bar{\Pi})^{1-\iota_w} \end{aligned}$$

For wages, the indexation term is the product between steady state inflation and steady state labor-augmenting TFP growth $\bar{\Gamma}$. This is required for the model to have a balanced growth path.

2.10 Exogenous Shocks

There are 11 shock series in the model. Unless explicitly noted below, all shocks follow AR(1) processes in logs:

$$\log x_t = (1 - \rho_x)\log \bar{x} + \rho_x \log x_{t-1} + \sigma_x \varepsilon_t^x$$

where $\varepsilon_t^x \sim \mathcal{N}(0, 1)$ are standard Normal innovations.

1. Γ_t is the growth rate of labor-augmenting TFP.

2. ζ_t is the marginal efficiency of investment that governs the rate of transformation of final consumption goods to physical capital.

3. μ_t^p is the product price markup shock. We assume it follows an ARMA(1,1) process in logs following Smets and Wouters (2007), who argue that this is important to capture higher frequency fluctuations in inflation:

$$\log \mu_t^p = (1 - \rho_{\mu^p}) \log \bar{\mu}^p + \rho_{\mu^p} \log \mu_{t-1}^p + \sigma_{\mu^p} \varepsilon_t^{\mu^p} - \eta_{\mu^p} \sigma_{\mu^p} \varepsilon_{t-1}^{\mu^p}$$

4. μ_t^w is the wage markup shock. We also assume it follows an ARMA(1,1) process:

$$\log \mu_t^w = (1 - \rho_{\mu^w}) \log \bar{\mu}^w + \rho_{\mu^w} \log \mu_{t-1}^w + \sigma_{\mu^w} \varepsilon_t^{\mu^w} - \eta_{\mu^w} \sigma_{\mu^w} \varepsilon_{t-1}^{\mu^w}$$

   Note that this shock plays a dual role: it is a shock to wage inflation, everything else constant, and is also the main labor supply shock in the model. A standard way of modeling labor supply shocks in DSGE models is to directly shock the disutility of labor ξ. Such shocks cannot be separately identified from wage markup shocks without further complications and/or assumptions in this model.

5. χ_t is a shock to the marginal utility of both workers and capitalists. It is similar to a discount factor shock in the sense that it changes how households value consumption across time.

6. ϑ_t is the convenience yield or risk premium shock that governs the relative preference for government bonds over physical capital. This shock drives a wedge between the rates of return of these two assets.

7. NX_t is the trade balance shock that serves as the residual for the aggregate resource constraint. We assume that $NX_t = nx_t \times Y_t$ and that nx_t follows an AR(1) in levels as this term can be either positive or negative:

$$nx_t = \rho_{nx} nx_{t-1} + \sigma_{nx} \varepsilon_t^{nx}$$

8. G_t is government consumption. We assume that government consumption is a time-varying fraction of GDP:

$$G_t = (1 + \gamma_t^{-1}) Y_t$$

   where γ_t is the underlying shock that follows an AR(1) in logs.

9. T_t are government transfers to households. To reflect progresivity of the social welfare system, we assume that these transfers are targeted to workers

10. mp_t is the monetary policy shock that drives deviations of the policy rate from what would be implied by the Taylor rule.

11. Π_t^* is the time-varying inflation target of the Fed. This term is helpful to capture the gradual decline in inflation since the 1980s, and can reflect structural changes in the way monetary policy is conducted.

We classify the 11 shocks intok 5 broad categories: supply ($\Gamma_t, \zeta_t, \mu_t^p, \mu_t^w$), demand ($\chi_t, NX_t, \vartheta_t$), fiscal ($G_t, T_t$), and monetary ($\mathrm{mp}_t, \Pi_t^*$). This classification is primarily useful to disentangle movements in variables that are driven by policy vs. non-policy impulses.

2.11 Flexible Price Economy

The flexible price economy is an economy subject to the same shocks and with the same equilibrium conditions as the baseline economy, but with the following restrictions:

1. No nominal rigidities and product markup shocks. The product markup is assumed to be constant $\mu_t^p = \bar{\mu}^p, \forall t$ and the NKPC in 28 is replaced by a static pricing condition for the firm:

$$\bar{\mu}^p MC_t = 1$$

2. No wage rigidities and wage markup shocks. The wage markup is assumed to be constant $\mu_t^w = \bar{\mu}^w, \forall t$ and the wage Phillips curve in 21 is replaced by a static labor supply condition:

$$\frac{W_t}{P_t} = \Xi_t N_t^{\varphi}(1+\varphi)$$

3. Inflation is constant at target, $\Pi_t = \bar{\Pi}$.

Flexible price economy variables are denoted with a superscript f. Of most importance are the natural level of output Y_t^f that enters the central bank's Taylor rule in 32, and the real interest rate r_t^f that corresponds to the neutral level of the interest rate following the definition in Woodford (2003).

3 Full List of Equilibrium Conditions

Due to TFP growth, we define the equilibrium of the model in terms of detrended variables. Let

$$x_t = \frac{X_t}{Z_t}$$

for any variable x. We assume that the adjustment cost functions take the form:

$$\begin{aligned} A(x) &= \kappa_a(x-1) + 0.5\sigma_a(x-1)^2 \\ S(x) &= \frac{\psi_i}{2}(x-\bar{\Gamma})^2 \\ \Psi(x) &= \frac{\psi_w}{2}\left(x-\bar{b}^w\right)^2 \end{aligned}$$

The full list of equilibrium conditions in terms of detrended variables follows.

Worker households:

$$\text{SDF:} \qquad m^w_{t,t+1} = \frac{\beta}{\Gamma_{t+1}} \frac{c^w_t}{c^w_{t+1}} \frac{\chi_{t+1}}{\chi_t} \tag{33}$$

$$\text{Euler eq:} \qquad 1 = R_t \vartheta_t \mathbb{E}\left[\frac{m^w_{t,t+1}}{\Pi_{t+1}} \frac{1}{1+\psi_w(b^w_t - \bar{b}^w)}\right] \tag{34}$$

$$\text{budget const.} \qquad c^w_t + \frac{b^w_t}{R_t} = (1-\tau_t) w_t \frac{N_t}{\lambda} + \frac{b^w_{t-1}}{\Pi_t \Gamma_t} + t_t \tag{35}$$

Capitalist households:

$$\text{SDF:} \qquad m^s_{t,t+1} = \frac{\beta}{\Gamma_{t+1}} \frac{c^s_t}{c^s_{t+1}} \frac{\chi_{t+1}}{\chi_t} \tag{36}$$

$$\text{Euler eq:} \qquad 1 = R_t \vartheta_t \mathbb{E}_t\left[\frac{m^s_{t,t+1}}{\Pi_{t+1}}\right] \tag{37}$$

$$\text{utilization:} \qquad R^k_t = A'(\nu_t) \tag{38}$$

$$\text{Tobin's Q:} \qquad \omega_t = \mathbb{E}_t m^s_{t,t+1}\left[(1-\tau^d_{t+1}) R^k_{t+1} \nu_{t+1} + (1-\delta)\omega_{t+1} - A(\nu_{t+1})\right] \tag{39}$$

$$\text{investment:} \qquad 1 - \omega_t \zeta_t \left[1 - S_t - \Gamma_t \frac{i_t}{i_{t-1}} S'_t\right] = \mathbb{E}_t m^s_{t,t+1}\left[\omega_{t+1}\zeta_{t+1}\left(\Gamma_{t+1}\frac{i_{t+1}}{i_t}\right)^2 S'_{t+1}\right] \tag{40}$$

(41)

Labor markets:

$$\text{wage NKPC:} \quad w_t + \frac{\mu_t^w}{\mu_t^w - 1}\left[\Xi_t (N_t/\lambda)^{\varphi}(1+\varphi) - w_t\right] - \eta_w \frac{\Pi_t^w}{\Pi_t^{w,index}}\left(\frac{\Pi_t^w}{\Pi_t^{w,index}} - 1\right) + \beta\eta_w \mathbb{E}_t \frac{N_{t+1}}{N_t}\frac{\Pi_{t+1}^w}{\Pi_{t+1}^{w,index}}\left(\frac{\Pi_{t+1}^w}{\Pi_{t+1}^{w,index}} - 1\right) = 0 \tag{42}$$

$$\text{MRS:} \quad \Xi_t = (1-\tau_t)^{-1} c_t^w \xi \tag{43}$$

$$\text{wage indexation:} \quad \Pi_t^{w,index} = (\Pi_t^w)^{\iota_w}(\Gamma\Pi)^{1-\iota_w} \tag{44}$$

$$\text{wage inflation:} \quad \Pi_t^w = \frac{w_t}{w_{t-1}}\Gamma_t \Pi_t \tag{45}$$

Capital and investment:

$$\text{LoM capital:} \quad k_t = (1-\delta)\frac{k_{t-1}}{\Gamma_t} + \zeta_t[1 - S(\Gamma_t i_t / i_{t-1})]i_t \tag{46}$$

$$\text{utilized capital:} \quad k_t^u = \frac{\nu_t}{\Gamma_t} k_{t-1} \tag{47}$$

Firms

$$\text{NKPC:} \quad \frac{\Pi_t}{\Pi_t^{index}}\left(\frac{\Pi_t}{\Pi_t^{index}} - 1\right) = \mathbb{E}_t\left[m_{t,t+1}^s \frac{y_{t+1}}{y_t}\Gamma_{t+1}\frac{\Pi_{t+1}}{\Pi_{t+1}^{index}}\left(\frac{\Pi_{t+1}}{\Pi_{t+1}^{index}} - 1\right)\right] + \frac{1}{\eta_p(\mu_t^p - 1)}\left(\mu_t^p mc_t - 1\right) \tag{48}$$

$$\text{marginal cost:} \quad mc_t = \left(\frac{R_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \tag{49}$$

$$\text{indexation:} \quad \Pi_t^{index} = \Pi_{t-1}^{\iota_\Pi}\Pi^{1-\iota_\Pi} \tag{50}$$

$$\text{production:} \quad y_t = (k_t^u)^{\alpha} N_t^{1-\alpha} \tag{51}$$

$$\text{input mix:} \quad k_t^u = \frac{\alpha}{1-\alpha}\frac{w_t}{R_t^k} N_t \tag{52}$$

$$\text{resource constraint:} \quad c_t + g_t + i_t + A(\nu_t)\frac{k_{t-1}}{\Gamma_t} = y_t\left[1 - 0.5\eta_p\left(\frac{\Pi_t}{\Pi_t^{index}} - 1\right)^2 - nx_t\right] \tag{53}$$

$$\text{aggregate consumption:} \quad c_t = \lambda c_t^w + (1-\lambda)c_t^s \tag{54}$$

Government:

$$\text{govt bc:} \quad g_t + \lambda t_t + \frac{b^g_{t-1}}{\Pi_t \Gamma_t} = \tau_t w_t N_t + \tau^d_t (R^k_t k^u_t + d_t) + \frac{b^g_t}{R_t} \tag{55}$$

$$\text{fiscal rule I:} \quad \tau_t = \bar{\tau} \left(\frac{b^g_{t-1}}{\bar{b}^g} \right)^{\phi_\tau} \tag{56}$$

$$\text{fiscal rule II:} \quad \tau^d_t = \bar{\tau}^d \left(\frac{b^g_{t-1}}{\bar{b}^g} \right)^{\phi_\tau} \tag{57}$$

$$\text{govt spending:} \quad g_t = \left(1 - \frac{1}{\gamma_t} \right) y_t \tag{58}$$

$$\text{Taylor rule:} \quad R_t = R^{\rho_r}_{t-1} \left[\bar{R} \left(\frac{\Pi_t}{\Pi^*_t} \right)^{\phi_\Pi} \left(\frac{y_t}{y^f_t} \right)^{\phi_Y} \right]^{1-\rho_r} \left(\frac{y_t / y^f_t}{y_{t-1} / y^f_{t-1}} \right)^{\phi_\Delta} \mathrm{mp}_t \tag{59}$$

Other:

$$\text{firm profits:} \quad d_t = y_t - w_t N_t - R^k_t k^u_t - 0.5 \eta_p y_t \left(\frac{\Pi_t}{\Pi^{index}_t} - 1 \right)^2 \tag{60}$$

$$\text{real interest rate:} \quad 1 + r_t = \frac{R_t}{\mathbb{E}_t \Pi_{t+1}} \tag{61}$$

Shocks:

$$\text{TFP:} \quad \log \Gamma_t = (1 - \rho_\Gamma) \log \bar{\Gamma} + \rho_\Gamma \log \Gamma_{t-1} + \sigma_\Gamma \varepsilon^\Gamma_t \tag{62}$$

$$\text{MEI:} \quad \log \zeta_t = (1 - \rho_\zeta) \log \bar{\zeta} + \rho_\zeta \log \zeta_{t-1} + \sigma_\zeta \varepsilon^\zeta_t \tag{63}$$

$$\text{price markup:} \quad \log \mu^p_t = (1 - \rho_{\mu^p}) \log \bar{\mu}^p + \rho_{\mu^p} \log \mu^p_{t-1} + \sigma_{\mu^p} \varepsilon^{\mu^p}_t - \eta_{\mu^p} \sigma_{\mu^p} \varepsilon^{\mu^p}_{t-1} \tag{64}$$

$$\text{wage markup:} \quad \log \mu^w_t = (1 - \rho_{\mu^w}) \log \bar{\mu}^w + \rho_{\mu^w} \log \mu^w_{t-1} + \sigma_{\mu^w} \varepsilon^{\mu^w}_t - \eta_{\mu^w} \sigma_{\mu^w} \varepsilon^{\mu^w}_{t-1} \tag{65}$$

$$\text{marginal utility:} \quad \log \chi_t = (1 - \rho_\chi) \log \bar{\chi} + \rho_\chi \log \chi_{t-1} + \sigma_\chi \varepsilon^\chi_t \tag{66}$$

$$\text{convenience yield:} \quad \log \vartheta_t = (1 - \rho_\vartheta) \log \bar{\vartheta} + \rho_\vartheta \log \vartheta_{t-1} + \sigma_\vartheta \varepsilon^\vartheta_t \tag{67}$$

$$\text{net exports:} \quad nx_t = \rho_{nx} nx_{t-1} + \sigma_{nx} \varepsilon^{nx}_t \tag{68}$$

$$\text{govt spending:} \quad \log \gamma_t = (1 - \rho_\gamma) \log \bar{\gamma} + \rho_\gamma \log \gamma_{t-1} + \sigma_\gamma \varepsilon^\gamma_t \tag{69}$$

$$\text{fiscal transfers:} \quad \log t_t = (1 - \rho_T) \log \bar{t} + \rho_T \log t_{t-1} + \sigma_T \varepsilon^T_t \tag{70}$$

$$\text{monetary policy:} \quad \log \mathrm{mp}_t = (1 - \rho_{\mathrm{mp}}) \log \bar{\mathrm{mp}} + \rho_{\mathrm{mp}} \log \mathrm{mp}_{t-1} + \sigma_{\mathrm{mp}} \varepsilon^{\mathrm{mp}}_t \tag{71}$$

$$\text{inflation target:} \quad \log \Pi^*_t = (1 - \rho_\Pi) \log \bar{\Pi} + \rho_\Pi \log \Pi^*_{t-1} + \sigma_\Pi \varepsilon^\Pi_t \tag{72}$$

The equilibrium conditions for the flexible price economy are the same as above, (33)-(72), but with the product and wage NKPC replaced by the respective static optimality conditions, and the Taylor rule replaced with constant inflation at target, $\Pi^f_t = \bar{\Pi}$. These assumptions imply that neither markup nor monetary policy shocks affect the equilibrium of the flexible price economy. The neutral rate of interest is computed from the Euler equation for capitalists:

$$1 + r_t^f = [\vartheta_t \mathbb{E}_t m_{t,t+1}^{s,f}]^{-1} \tag{73}$$

4 Estimation and Calibration

We calibrate some parameters and estimate others on US data using standard Bayesian estimation techniques (Adjemian et al., 2024). This section describes the data, the set of calibrated parameters, and details of the model estimation.

4.1 Observation Equations.

The model contains 11 shocks, and so the model can accommodate up to 11 observable series while avoiding stochastic singularity. Our observation equations are the following:

$$\text{GDP growth}_t = 400 \times (\log y_t - \log y_{t-1} + \log \Gamma_t) \tag{74}$$

$$\text{Cons. growth}_t = 400 \times (\log c_t - \log c_{t-1} + \log \Gamma_t) \tag{75}$$

$$\text{Inv. growth}_t = 400 \times (\log i_t - \log i_{t-1} + \log \Gamma_t) \tag{76}$$

$$\text{Govt. cons. growth}_t = 400 \times (\log g_t - \log g_{t-1} + \log \Gamma_t) \tag{77}$$

$$\text{Wage growth}_t = 400 \times (\log w_t - \log w_{t-1} + \log \Gamma_t) \tag{78}$$

$$\text{FFR}_t = 100 \times (R_t^4 - 1) \tag{79}$$

$$\text{Inflation}_t = 400 \times \log \Pi_t \tag{80}$$

$$\text{Govt. spending to output}_t = \frac{g_t + \lambda t_t}{y_t} \tag{81}$$

$$\text{log of hours}_t = \log N_t \tag{82}$$

$$\text{TFP growth demeaned}_t = 100 \times (1-\alpha)(\Gamma_t^4 - \bar{\Gamma}^4 + \sigma_{me,tfp} \varepsilon_t^{me,tfp}) \tag{83}$$

$$\text{10y inflation expectations}_t = 100 \times \left[\left(\prod_{i=1}^{40} \Pi_{t+i} \right)^{1/10} - 1 + \sigma_{me,infl} \varepsilon_t^{me,infl} \right] \tag{84}$$

where $\varepsilon_t^{me,tfp}, \varepsilon_t^{me,infl} \sim \mathcal{N}(0,1)$ are measurement error terms on TFP growth and inflation expectations. We therefore have 11 observables for a total of 11 structural shocks plus 2 measurement error shocks.

4.2 Data

We use quarterly data for the US. Our sample begins in 1959Q1, which is the first date for which we can compute Core PCE inflation. The sample used to estimate the model ends in 2019Q4, the last full quarter before the beginning of the COVID pandemic. We use data up to 2024Q2 (as of this writing) to infer shocks and the state of the US economy, but we do not use the post-2020 period to estimate model parameters due to the high degree of volatility during 2020. We update the series for smoothed shocks as newer data is released. We now describe each of the series in more detail. Most data is extracted from FRED and we include the FRED mnemonics in parentheses. Figure 1 plots the observable series.

1. GDP growth is per-capita real GDP growth. We compute the annualized quarterly growth rate of nominal GDP (GDP) divided by the GDP deflator ($GDPDEF$) and civilian population over the age of 16 ($CNP16OV$). We apply a HP filter to population to remove small discontinuities that arise around census years.

2. Consumption growth is computed the same way as GDP. As is standard in the literature (Arias et al., 2019), we focus on nondurable consumption, and so the base series is equal to nominal consumption (PCE) minus nominal durable consumption ($PCEDG$).

3. Investment growth is computed as above. We include durable consumption as investment. Therefore, the base series is equal to nominal investment ($GPDI$) plus nominal durable consumption ($PCEDG$).

4. Government consumption growth is computed as above, using nominal government consumption (GCE) as the base series.

5. Wage growth is the annualized quarterly growth rate of a measure of real wages. This measure is computed by taking an index of hourly compensation for all workers in the nonfarm business sector ($COMPNFB$) and dividing it by the GDP deflator ($GDPDEF$).

6. The FFR is the shadow federal funds rate from Wu and Xia (2016), downloaded from the FRB Atlanta website. Since the model is linearized, it does not accommodate the zero lower bound and does not feature quantitative easing explicitly. The shadow FFR is the implied measure of conventional monetary policy that tries to account for the effects of unconventional stimulus, and is allowed to be negative during ZLB periods. The shadow FFR is not published away from the ZLB, and so we simply splice it with the regular series for the FFR during these periods ($FEDFUNDS$).

7. Inflation is the quarterly growth rate of the Core PCE price index, i.e. the PCE price index minus food and energy ($PCEPILFE$).

8. Government spending to output is computed as follows. g_t and y_t are computed as previously described, by taking GDP and GCE, dividing them by the GDP deflator and by the smoothed measure of population. t_t is computed similarly, using as the base series nominal social contributions ($BOGZ1FA366404005Q$) plus other current transfers ($BOGZ1FA366403005Q$) plus subsidies ($BOGZ1FA366402005Q$).

9. The log of hours is the natural logarithm of a measure of total hours, constructed following Del Negro et al. (2017). This is equal to average hours worked ($AWHNONAG$) times total employment ($CE16OV$), divided by the smoothed measure of population. This series is only

Parameter	Description	Value	Target
β	Discount factor	0.9996	-
$\bar{\Gamma}$	Long-run TFP growth	$1.017^{0.25}$	1959-2019 average
δ	Depreciation rate	2.5%	Standard
μ^p	Price Markup	1.2	20% markup at steady state
μ^w	Wage Markup	1.2	20% markup at steady state
$\bar{\Pi}$	SS inflation	$1.02^{0.25}$	2% inflation target
τ^d	Tax rate on profits	20%	Maximum capital income tax
$\bar{b^g}/\bar{y}$	Average govt. debt/GDP	4×0.5774	1959-2019 average
$\bar{g}/\bar{y}$	Average govt. cons./GDP	0.2048	1959-2019 average

Table 1: Summary of Calibration

available from 1964Q1 onwards and so we use the Kalman filter to infer its values in the early parts of the sample.

10. TFP growth is taken from the series constructed by Fernald (2012). We use non-utilization adjusted TFP and demean it for the estimation period. We assume that TFP is subject to some measurement error.

11. We use 10-year CPI inflation expectations from the Survey of Professional Forecasters, downloaded from the FRB Philadelphia. We follow the procedure in Del Negro et al. (2017) and subtract 0.5 from the annualized SPF series, which is the average difference between CPI and PCE inflation over the sample. Note that this series is only available from 1979Q4 onwards; we use the Kalman filter to infer its values in the early part of the sample, between 1964Q1 and 1979Q3. Since the model is solved under the assumption of rational expectations, and inflation expectations are plausibly subject to deviations from rationality, we assume that inflation expectations are also subject to measurement error.

4.3 Calibrated Parameters

We calibrate a few parameters and estimate the rest. Table 1 summarizes the calibration. Some of these parameters are set to facilitate policy experiments (i.e. the long-run inflation target of 2%), while others are not well identified from the observable series (i.e. the steady state government debt-to-GDP ratio).

4.4 Estimated Parameters

All other model parameters are estimated. Table 2 summarizes the prior distributions as well as the estimation results. We mostly follow Smets and Wouters (2007) in our choice of prior distributions. As in Del Negro et al. (2017) and Gelain and Lopez (2023), we impose slightly different priors for the autocorrelation coefficients of the inflation target and convenience yield shocks: we

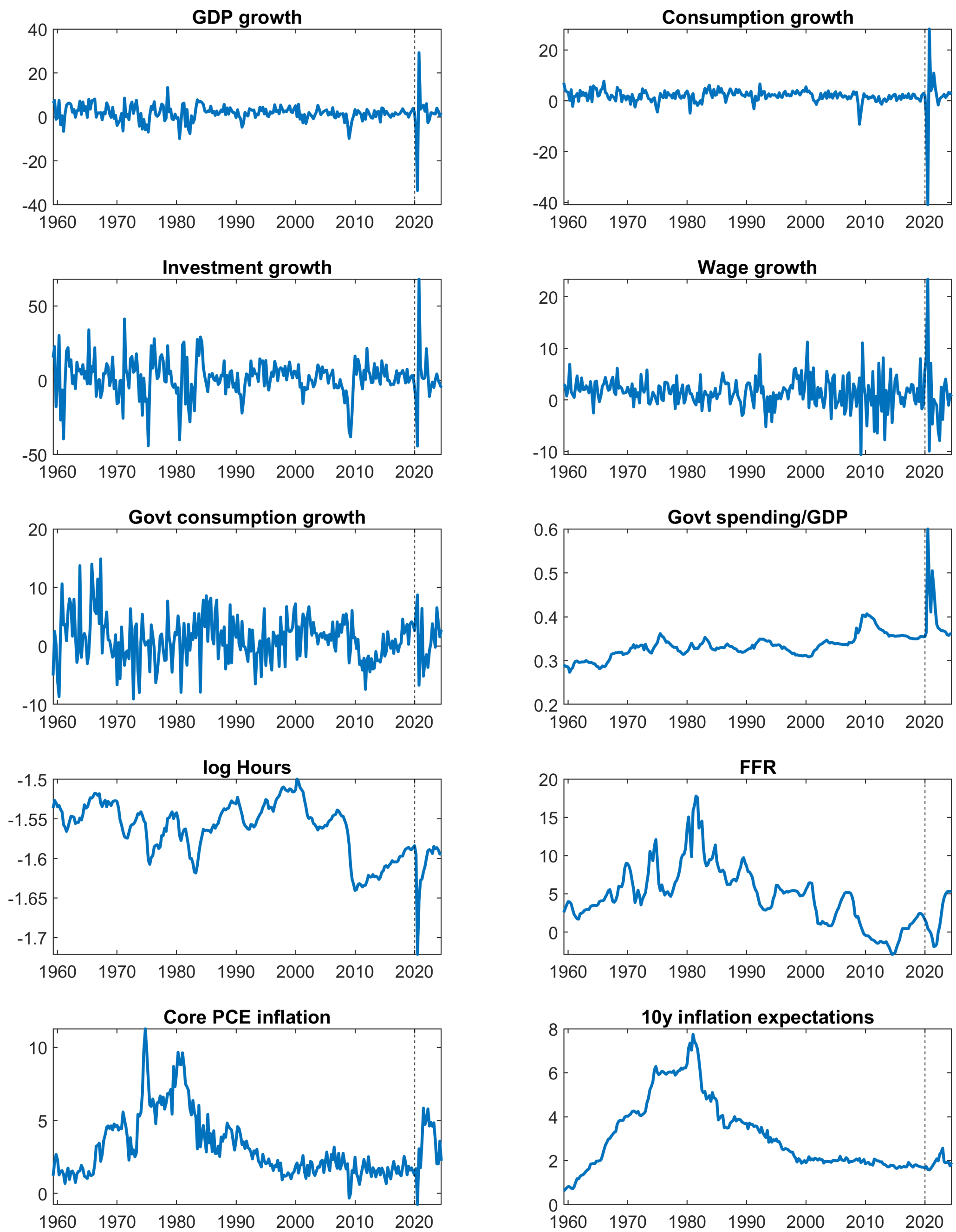


Figure 1: Time series for model observables, 1959Q1-2024Q1. The vertical dashed line corresponds to 2019Q4, the last period of estimation. Hours are only observable from 1964Q1 onwards, and inflation expectations from 1979Q4 onwards. See text for data sources and description.

impose tight priors around very high persistence (0.99). This is important to capture slow-moving fluctuations in the Fed's inflation target, as well as to capture low frequency secular movements in real interest rates. We use a random walk Metropolis Hastings algorithm to generate 250,000 draws from 4 parallel Markov Chain-Monte Carlo chains. We discard the first 100,000 draws and use the following draws to approximate the posterior distribution. We tune the algorithm to achieve an acceptance rate of 20 to 30 percent.

4.5 Filtering Shocks and the COVID period

While we focus on the 1964Q1-2019Q4 period to estimate the model, we use the estimated model to filter/infer shocks hitting the US economy throughout the entire sample period, through 2024Q1. This encompasses the years of 2020-21, when the US economy was arguably subject to unprecedented large shocks. Due to their mean-reverting nature and solution method based on first-order approximations around the steady state, it is not easy for standard DSGE and VAR models to capture these large fluctuations (Primiceri and Tambalotti, 2020).

To handle 2020, we follow an approach inspired by FRB New York (2022) and Ferroni et al. (2022): we assume that the economy was hit by a series of special i.i.d. shocks in 2020-21 that are zero in every other year. While these shocks are not active in other years, they may still have an effect post-2021 due to their impact on the endogenous state variables. To choose these shocks, we adopt the following heuristic approach: first, we run the Kalman smoother without any COVID shocks and estimate innovations for 2020-21. We then select all exogenous variables for which the smoother infers a realization for innovations that is larger than 4 in absolute value, which is a virtually zero probability event for any variable that follows a standard normal distribution.

Using this procedure, we introduce special COVID disturbances on the following exogenous variables: $\chi_t, \mu_t^p, \mu_t^w, \zeta_t, \gamma_t, t_t, \Gamma_t, mp_t$ (marginal utility of consumption, product markup, wage markup, MEI, government spending, fiscal transfers, TFP growth, and monetary policy). We introduce these shocks by replacing exogenous variable x_t with

$$\tilde{x}_t = x_t \times \exp(\varepsilon_t^{covid,x})$$

The idea is that the realizations of the special COVID shocks $\varepsilon_t^{covid,x}$ do not affect the next period value of the exogenous variable x_{t+1} via the lagged autoregressive term. Any persistence arising from these shocks is purely due to their effect on endogenous state variables of the model.

We estimate these shocks by feeding them as observables to the smoother with the model parameters evaluated at the posterior mean. We assume that these shocks are equal to zero for all periods except 2020Q1-2021Q3, and their values are missing for these seven quarters, and so the smoother infers their value.[3] This allows us to capture large fluctuations in 2020 and 2021 that do not necessarily affect the values of the potentially highly persistent exogenous state variables. The

[3] We have experimented with different values for the terminal date for these shocks. 2021Q3 allows us to capture residual effects from the third round of Economic Impact Payments triggered by the American Rescue Plan Act of 2021.

Parameter	Description	Prior distr.	Prior mean	Prior SD	Post. mean	Post. 10%	Post. 90%
φ	Labor elast.	G	2.0	0.75	2.317	1.8376	2.7844
η_p	Price menu cost	G	60.0	20.00	105.4068	80.0714	131.4528
η_w	Wage menu cost	G	60.0	20.00	156.9017	113.9238	198.7891
ψ_i	Inv. adj. costs	IG	5.0	1.000	2.6951	2.2576	3.0857
ι_Π	Price indexation	B	0.5	0.15	0.3049	0.1751	0.4418
ι_w	Wage indexation	B	0.5	0.15	0.0963	0.0324	0.1554
σ_a	Utilization costs	IG	0.5	0.1	0.7259	0.4623	1.0052
α	Capital share	N	0.3	0.05	0.2905	0.2767	0.3043
$\log \bar{N}$	log hours	N	-1.563	0.25	-1.5704	-1.587	-1.5534
ψ_w	portfolio adj. costs	IG	0.1500	0.5	0.1025	0.0579	0.1449
λ	share of workers	B	0.7000	0.2	0.7526	0.6733	0.8286
$\bar{\vartheta}$	mean risk premium	N	1.0031	0.0005	1.003	1.0022	1.0038
ϕ_Π	Taylor rule	N	1.5	0.25	2.9733	2.8488	3.0903
ϕ_Y	Taylor rule	N	0.12	0.05	0.1352	0.0885	0.1839
ϕ_Δ	Taylor rule	N	0.12	0.05	0.3298	0.2858	0.3737
ρ_r	Taylor rule	B	0.75	0.10	0.8024	0.7754	0.83
$\frac{\bar{g}+\bar{t}}{\bar{y}}$	govt. spend. to GDP	N	0.3431	0.01	0.3471	0.3329	0.3614
$\frac{\bar{b}^w}{\bar{y}}$	worker debt to GDP	N	0.7500	0.25	1.4867	1.186	1.7825
ϕ_τ	fiscal rule	IG	0.02	0.05	0.0127	0.0055	0.0205
η_{μ^p}	ARMA term, prices	B	0.5	0.2	0.7716	0.7033	0.8484
η_{μ^w}	ARMA term, wages	B	0.5	0.2	0.7536	0.662	0.8481
ρ_Γ	AR TFP	B	0.5	0.2	0.0362	0.0051	0.0653
ρ_γ	AR govt.	B	0.5	0.2	0.9862	0.9798	0.9925
ρ_{μ^p}	AR price markup	B	0.5	0.2	0.8954	0.8541	0.9352
ρ_{μ^w}	AR wage markup	B	0.5	0.2	0.9306	0.9166	0.9456
ρ_χ	AR mg. util.	B	0.5	0.2	0.99	0.9857	0.9947
ρ_T	AR transfers	B	0.5	0.2	0.9897	0.9854	0.9939
ρ_ζ	AR MEI	B	0.5	0.2	0.4173	0.3095	0.5307
ρ_{nx}	AR trade balance	B	0.5	0.2	0.9938	0.9902	0.9977
ρ_Π	AR infl. target	B	0.99	0.005	0.9914	0.986	0.9974
ρ_ϑ	AR conv. yield	B	0.99	0.005	0.9875	0.9795	0.9962
ρ_{mp}	AR mon. pol.	B	0.5	0.2	0.0814	0.015	0.1435
σ_Γ	SD TFP	IG	0.01	0.05	0.0091	0.0084	0.0098
σ_γ	SD govt. cons.	IG	0.01	0.05	0.0029	0.0027	0.0032
σ_{μ^p}	SD price markup	IG	0.01	0.05	0.1982	0.1429	0.2545
σ_{μ^w}	SD wage markup	IG	0.01	0.05	0.0544	0.044	0.0645
σ_χ	SD mg. util.	IG	0.01	0.05	0.0372	0.0295	0.0452
σ_T	SD transfers	IG	0.01	0.05	0.0018	0.0016	0.002
σ_ζ	SD MEI	IG	0.01	0.05	0.024	0.0208	0.027
σ_{nx}	SD trade balance	IG	0.01	0.05	0.0664	0.0553	0.0771
σ_Π	SD infl. target	IG	0.001	0.05	0.0029	0.0027	0.0031
σ_ϑ	SD conv. yield	IG	0.01	0.05	0.0003	0.0003	0.0004
σ_{mp}	SD mon. pol.	IG	0.01	0.05	0.0012	0.0012	0.0013
$\sigma_{me,infl}$	SD ME infl. exp.	IG	0.005	0.05	0.0012	0.001	0.0015
$\sigma_{me,tfp}$	SD ME TFP	IG	0.01	0.05	0.0275	0.0254	0.0298

Table 2: Prior and posterior distributions for estimated parameters. B = beta, G = gamma, N = normal, IG = inverse gamma.

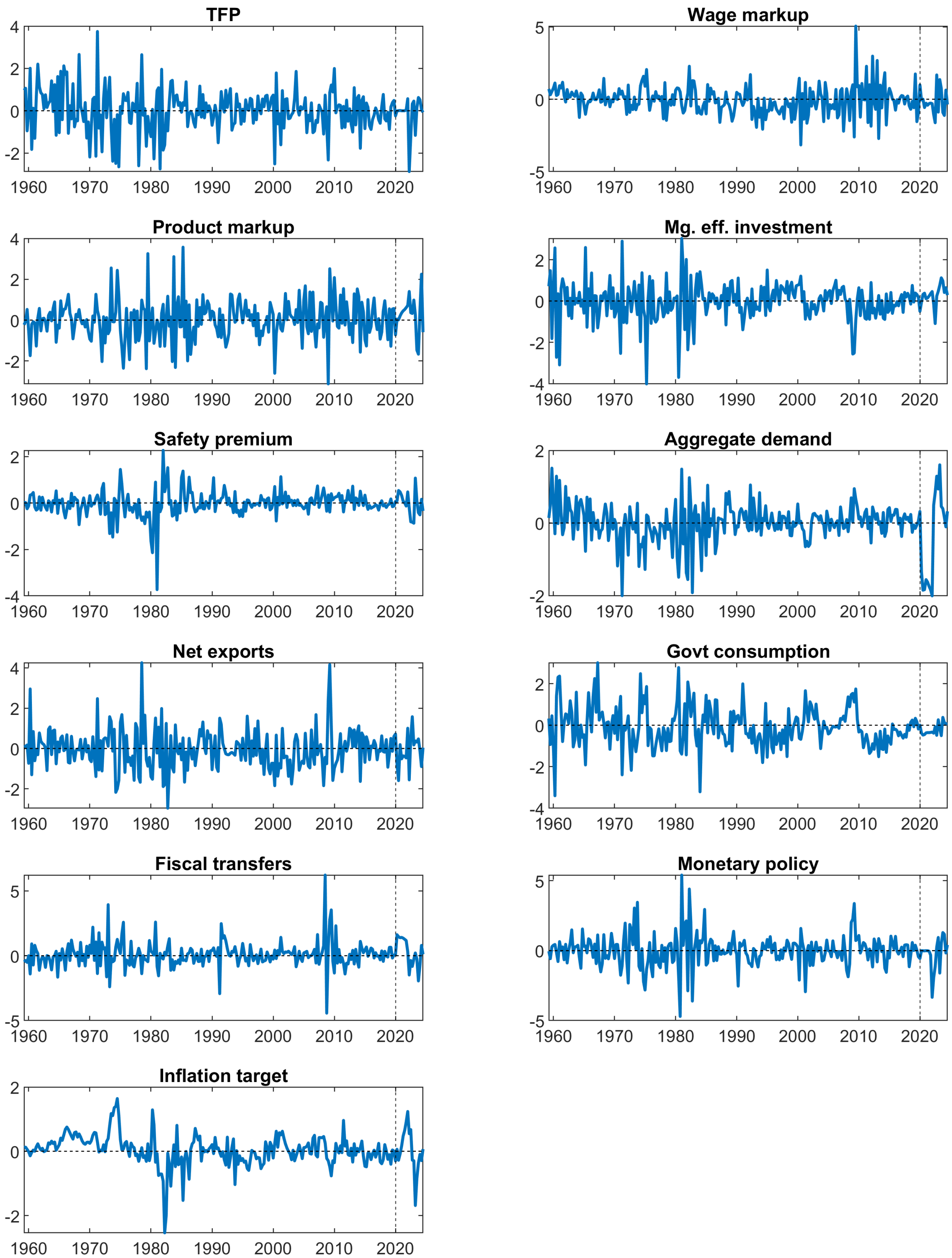


Figure 2: Time series for estimated model shocks, 1959Q1-2024Q1. The vertical dashed line corresponds to 2019Q4, the last period of estimation.

estimates for the innovations that result from the filtering procedure are presented in Figure 2.

Figure 3 plots the COVID shocks between 2019 and 2021. The model rationalizes the large fluctuations in 2020 primarily as very negative demand shock and a large increase in the wage markup, which in our model would be isomorphic to a shock to the disutility of labor supply. The model also captures large increases in government spending: the path of the transfer shock, in particular, is shaped by the three rounds of economic impact payments in 2020 and 2021.

5 Model Results

This section describes some of the model results that are treated as inputs for policy analysis. Appendix A reports the impulse response functions of selected endogenous variables to the different exogenous shocks.

5.1 Historical Decompositions

One of the advantages of using a structural model is that by giving a structural interpretation to the shocks, it allows researchers and policymakers to perform historical decompositions that describe the relative contribution of the different exogenous variables to observed movements in endogenous variables of interest. Figure 4 present simplified historical shock decompositions for year-over-year core PCE inflation, quarter-over-quarter core PCE inflation, and for the output gap. In these simplified decompositions, we group shocks into the aforementioned groups: demand, supply, fiscal, and monetary. Figure 7 in Appendix B presents the full decomposition.

The model decomposition attributes a significant role to monetary and fiscal policy in driving the recent inflationary wave. Fiscal support in the form of transfers triggered a significant demand expansion, while monetary policy kept interest rates below what was warranted by the Taylor rule estimated on historical data. Another relevant component of the "monetary" group is the inflation target, which the model estimates as being higher than usual during this period. This should not be literally interpreted as an inflation target above 2%, but rather as a way for a linearized model to rationalize a higher tolerance for deviations of inflation from its target.[4] These fiscal and monetary impulses were responsible for closing the output gap and maintaining output above its flexible price level since early 2021.

As of 2024Q2, the model identifies fiscal pressures on inflation as having vanished. Strong aggregate demand keeps inflation above the 2% target, while monetary policy has become restrictive, pushing inflation downwards. YoY inflation is significantly affected by base effects, and so it is also instructive to look at the behavior of quarter-over-quarter inflation, in panel (b). This figure reveals that both demand and supply pressures have contributed to keeping inflation above target, while monetary policy has been playing a more restrictive role.

[4] In a fully nonlinear model, this could be captured via a time-varying Taylor rule parameter ϕ_{Π}, but fluctuations in this parameter are second-order and therefore vanish due to linearization.

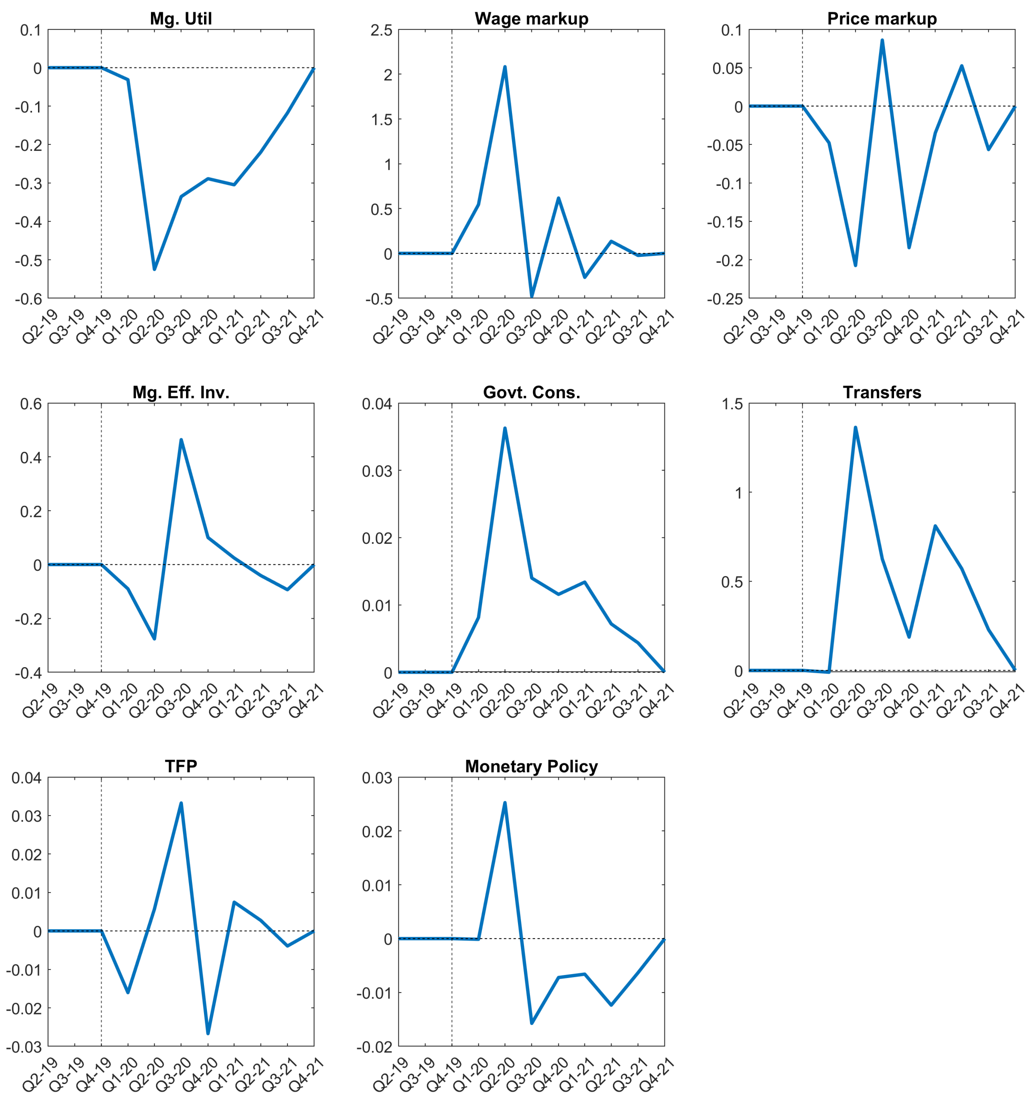
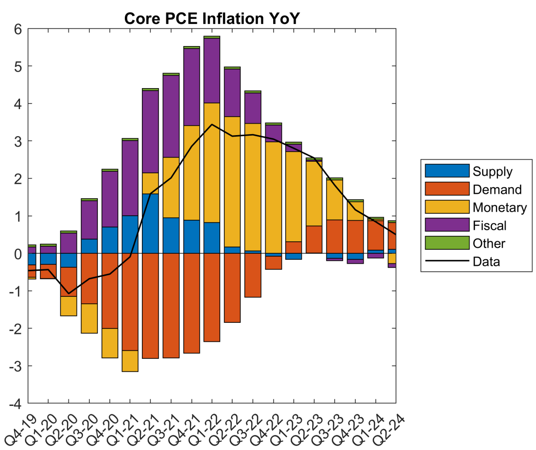
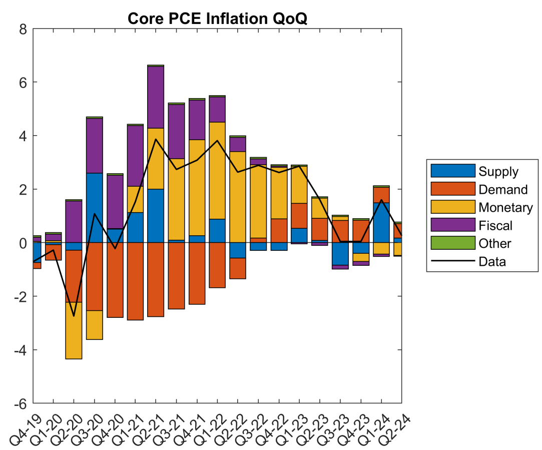


Figure 3: Time series estimated for COVID shocks, 2019Q1-2021Q4. The vertical dashed line corresponds to 2019Q4.

(a) YoY Core PCE Inflation



(b) QoQ Core PCE Inflation



(c) Output Gap

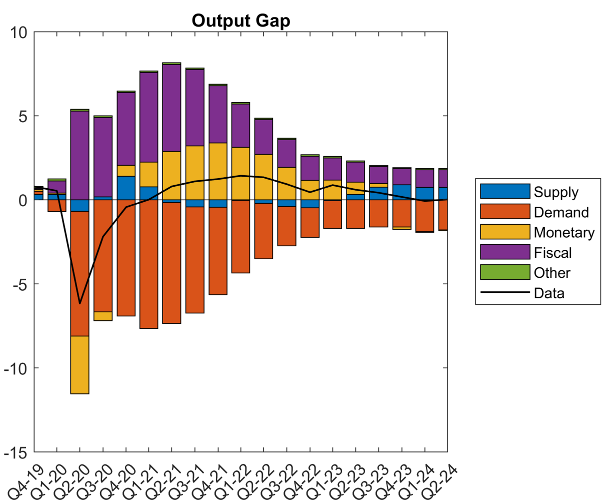


Figure 4: Simplifed historical decompositions 2019Q4-2024Q2, relative to steady state value of the variable.

5.2 Model Forecasts

Figure 5 presents unconditional model forecasts through 2027Q4, using available data as of June 2024. These forecasts are generated by simulating the model forward, and take into account uncertainty emanating from the innovations to the exogenous variables. The shaded areas correspond to a 68% confidence interval.

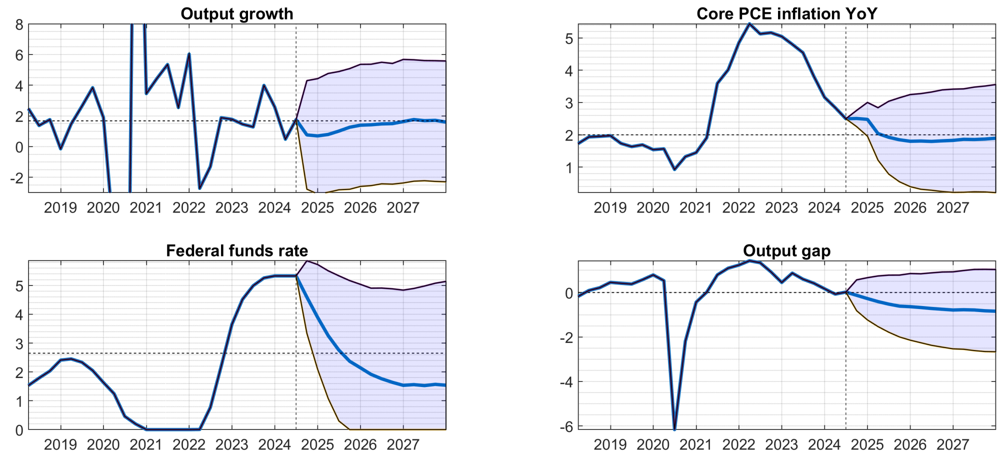


Figure 5: Unconditional model forecasts. The vertical dashed line is 2024Q2, the last quarter of observable data. Shaded areas correspond to 68% confidence intervals.

Table 3 presents point estimates for a few variables of interest, from 2024 to 2027. Output growth and core PCE inflation refer to annual Q4/Q4 rates, while the point estimates for all other variables refer to their levels as of Q4 of the respective year.

	GDP growth	Core PCE inflation	Federal funds rate	Natural rate	Output gap
2024	0.9%	2.4%	3.9%	3.0%	-0.3%
2025	1.1%	1.8%	2.1%	1.0%	-0.7%
2026	1.5%	1.8%	1.5%	0.4%	-0.8%
2027	1.7%	1.8%	1.5%	0.2%	-0.8%

Table 3: Model forecasts: point estimates. GDP growth and inflation are Q4/Q4, all other variables are Q4.

5.3 Natural rate of interest

The model produces estimates for the natural rate of interest that are in line with those in the literature. Model estimates for r^* tend to be more volatile and lower in terms of their level than standard estimates. Figure 6 compares the 1-year forward implied neutral rate that is estimated in the model to some of the most widely reported measures of the natural rate: those of Laubach and Williams (2003), Holston et al. (2023), and Lubik and Matthes (2015). The DSGE-based measure

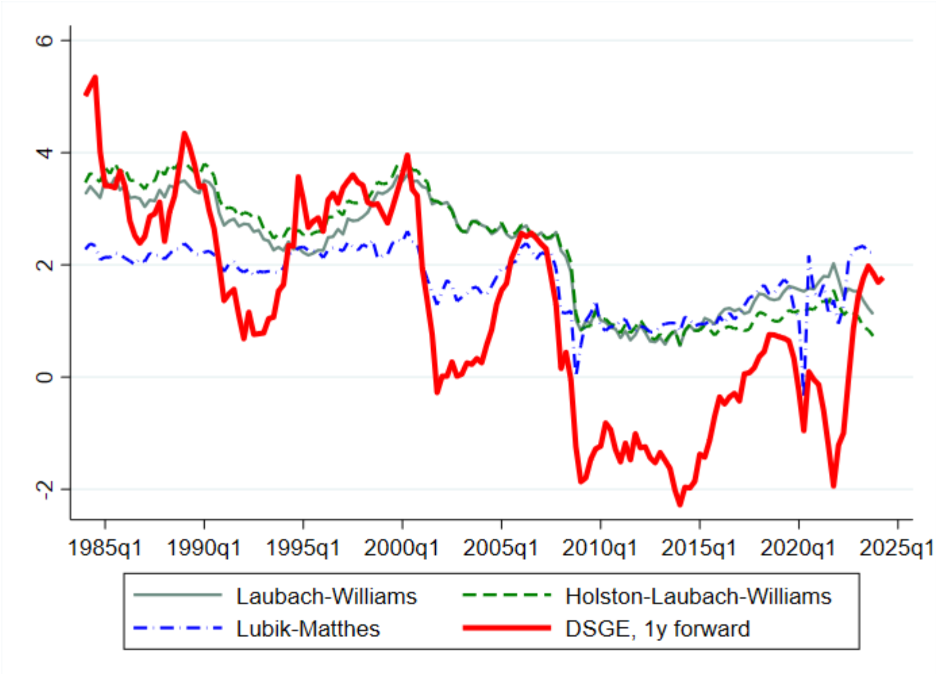


Figure 6: Standard measures of r^* versus model-implied $\mathbb{E}_t r^f_{t+4}$. Sources: FRB New York and FRB Richmond.

	Laubach-Williams	Holston-Laubach-Williams	Lubik-Matthes
Spot rate, r^*_t	0.54	0.57	0.59
1y forward, $\mathbb{E}_t r^*_{t+4}$	0.77	0.78	0.65
5y forward, $\mathbb{E}_t r^*_{t+20}$	0.76	0.80	0.43
10y forward, $\mathbb{E}_t r^*_{t+40}$	0.71	0.76	0.44

Table 4: Correlations between model r^* and standard measures

displays fluctuations that are broadly in line with the other measures, displaying a decreasing trend that seems to have stabilized around the post-Great Financial Crisis period. After a brief resurgence during the 2015-20 period, it fell again with the onset of the COVID-19 pandemic and associated economic disturbances. More recently, it has risen considerably to around 2%, the first time that it reaches this level since the early 2000s. These recent movements are particularly consistent with the Lubik-Matthes measure, which also records a recent increase in the estimated natural rate of interest.

Table 4 presents correlations between these measures and model-based objects: the spot rate, and the 1, 5, and 10-year forward neutral rates. The table shows that the model-based r^* tends to be more correlated with LM at shorter horizons, and presents a higher correlation with LW/HLW at longer horizons. Interpreted through the lens of the model, this suggests that the LM measure is perhaps better able to capture higher frequency movements in the neutral rate, while the LW/HLW measures place more weight on longer-term trends.

6 Conclusion

This document serves as a technical description of the DSGE model that is used as one of several inputs for forecasting and policy analysis at the St. Louis Fed. The present DSGE model extends a medium-scale New Keynesian DSGE model to allow for limited type of household heterogeneity and an explicit fiscal sector, which accounts for heterogeneous marginal propensities to consume and a role for fiscal policies such as social transfers.

These model extensions fall within the scope of standard methodological approaches to DSGE modeling, which allows us to leverage existing toolkits to easily solve and estimate the model to match US data. The model can easily be extended along several dimensions that may allow the analysis of different phenomena and sectors of the US economy. It is possible to add a financial accelerator along the lines of FRB New York (2022) or a frictional labor market that explicitly accounts for unemployment and vacancy rates as in Arias et al. (2019) or Gelain and Lopez (2023). Another natural direction is to include an explicit financial intermediation sector as in Gertler and Karadi (2011), which would allow financial factors to play a larger role in explaining macroeconomic fluctuations.

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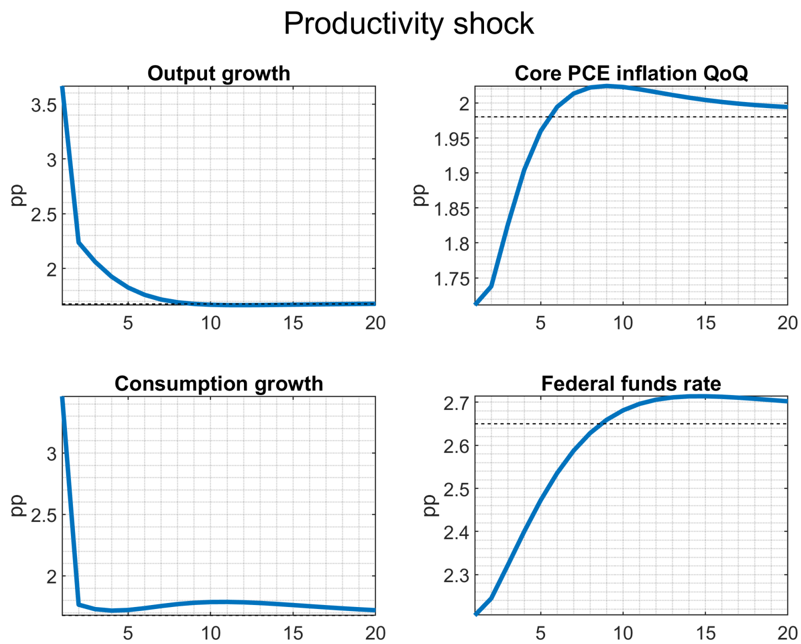
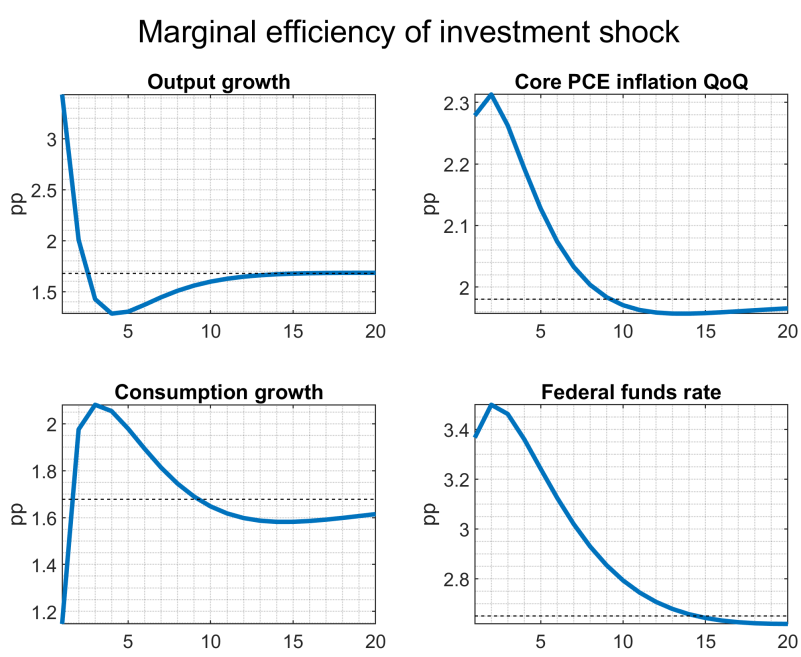
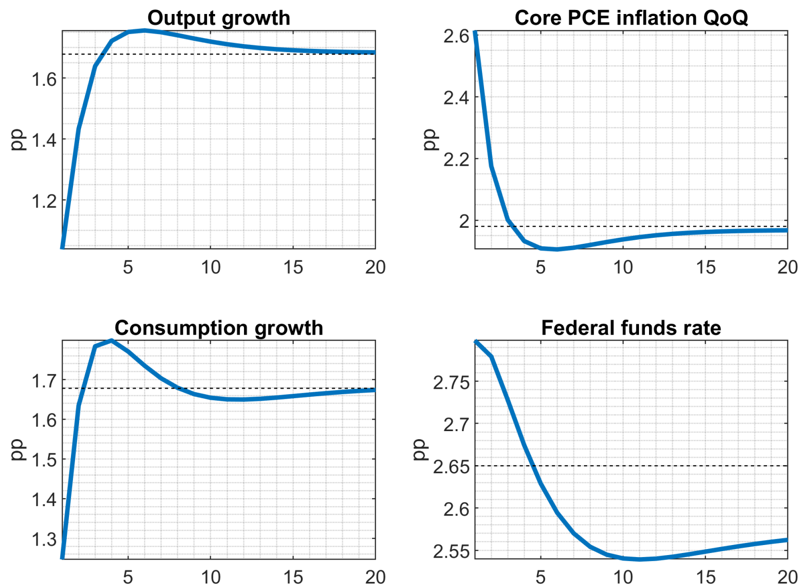
A Impulse Response Functions

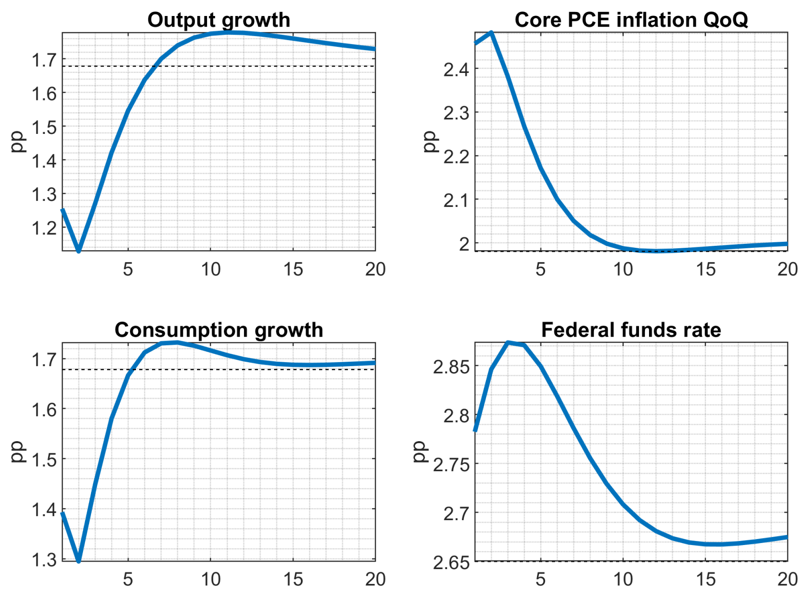


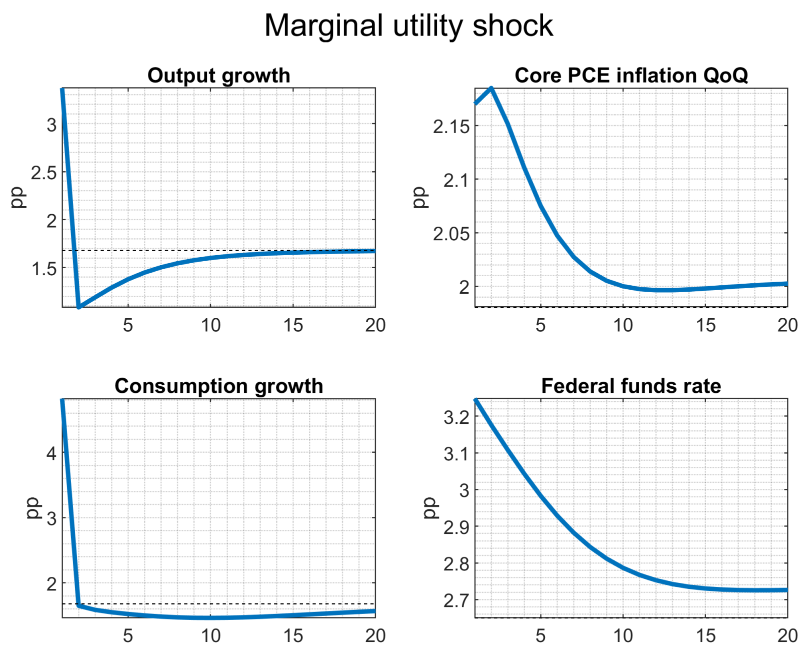


Price markup shock



Wage markup shock




Marginal utility shock
Output growth
Core PCE inflation QoQ
Consumption growth
Federal funds rate
pp
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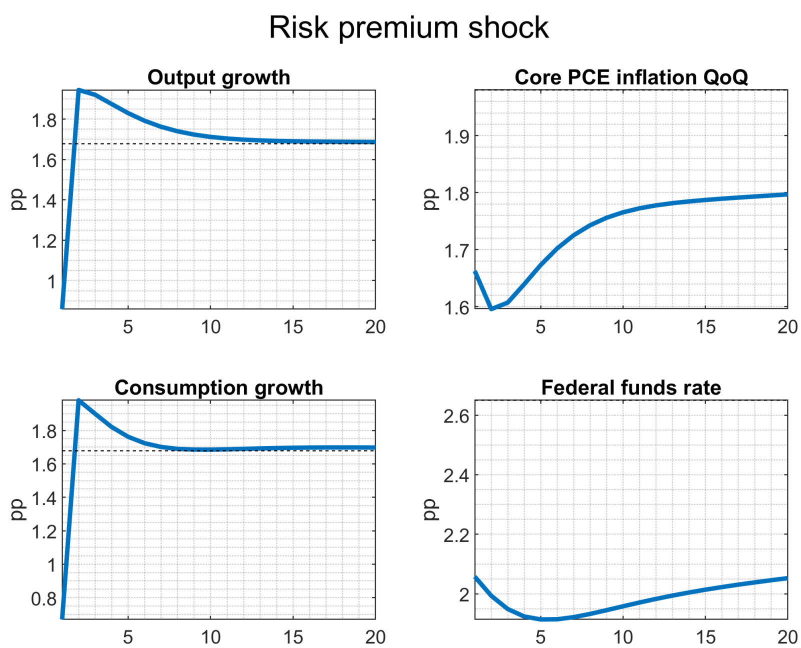
2
1.5
2.15
2.1
2.05
2
4
3
3.2
3.1
3
2.9
2.8
2.7
5
10
15
20


Risk premium shock
Output growth
Core PCE inflation QoQ
Consumption growth
Federal funds rate
pp
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1.4
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1
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1.7
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2.6
2.4
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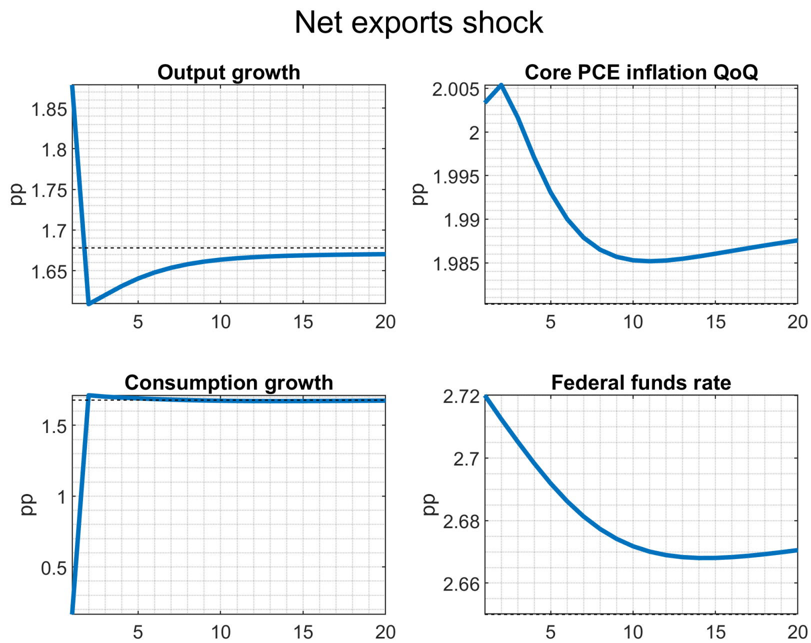
2
5
10
15
20


Net exports shock
Output growth
Core PCE inflation QoQ
Consumption growth
Federal funds rate
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2
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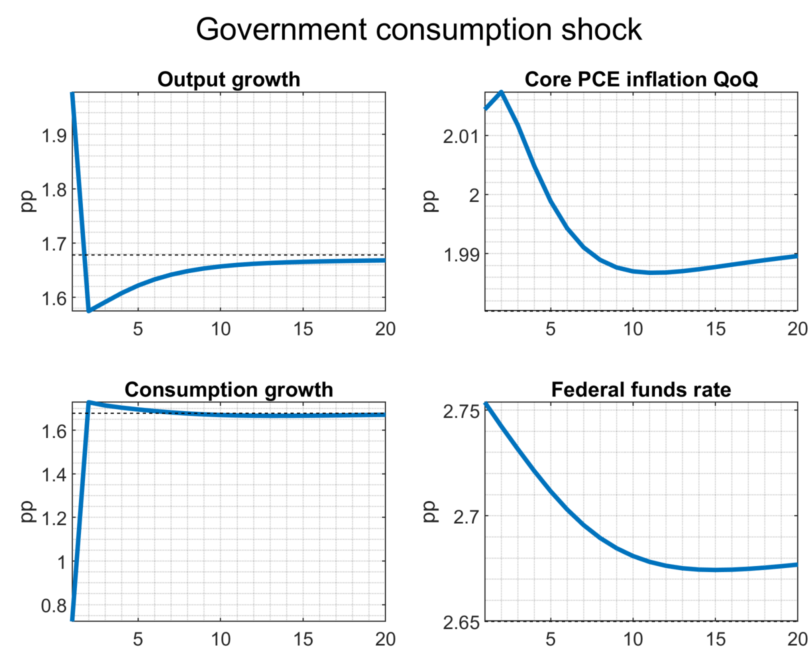
20


Government consumption shock
Output growth
Core PCE inflation QoQ
Consumption growth
Federal funds rate
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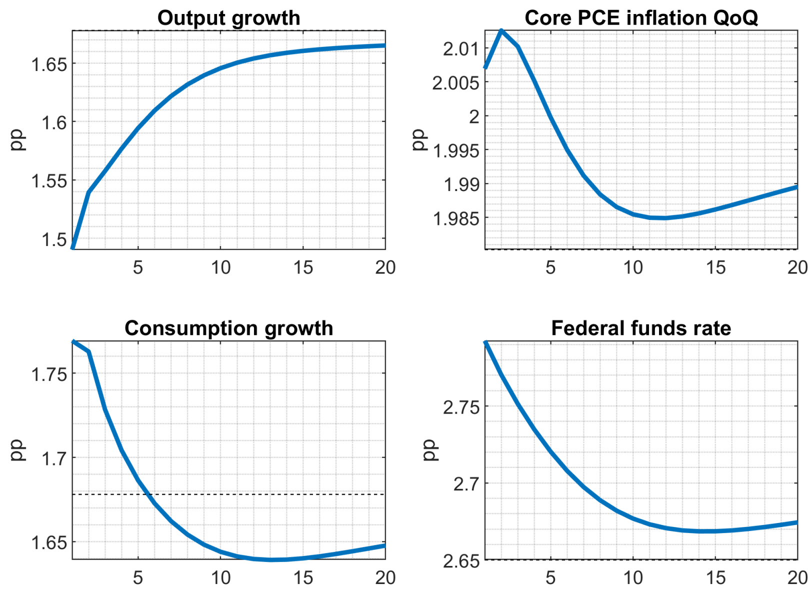
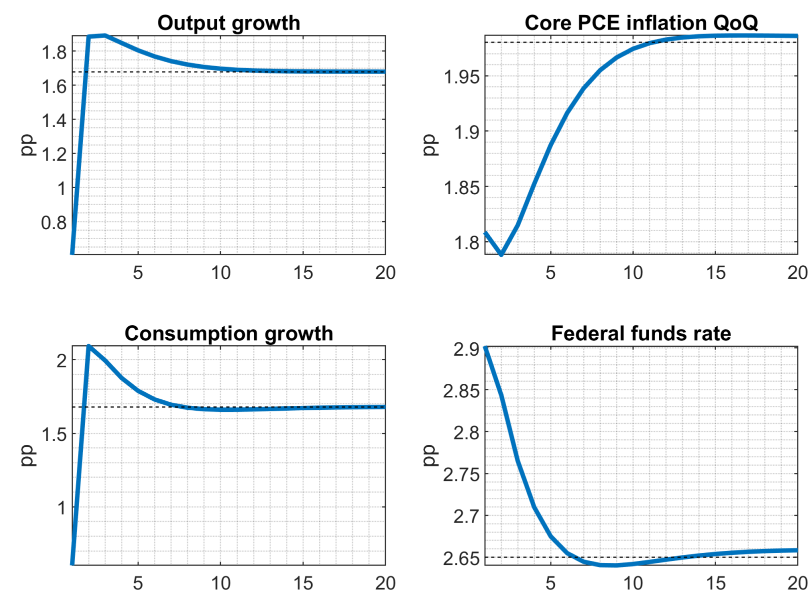
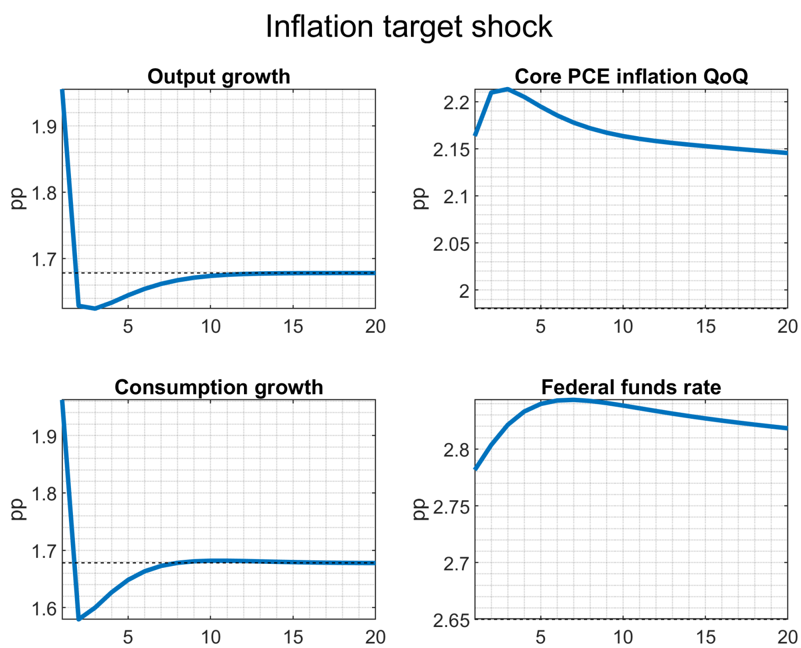
5
10
15
20

Transfers shock



Monetary policy shock




Inflation target shock
Output growth
Core PCE inflation QoQ
Consumption growth
Federal funds rate
pp
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1.7
1.6
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2.15
2.1
2.05
2
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2.75
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5
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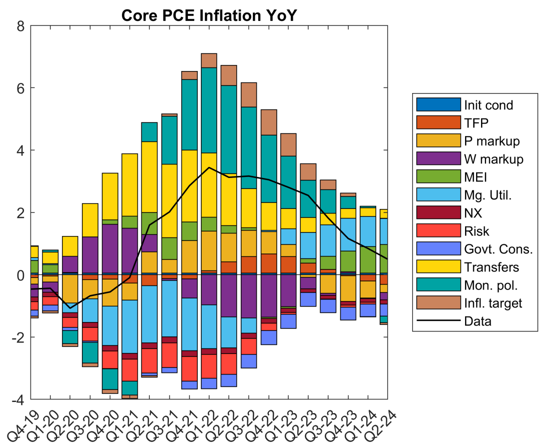
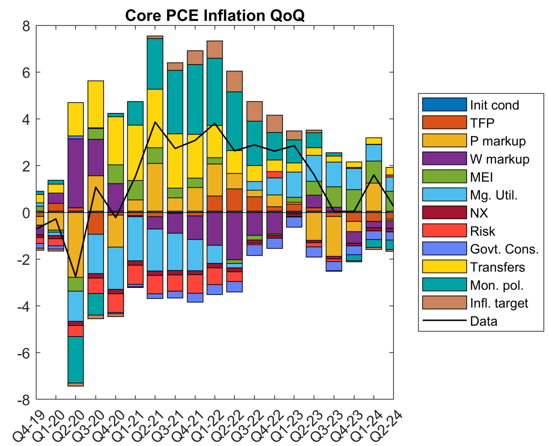
20

B Detailed Historical Decompositions

(a) YoY Core PCE Inflation



(b) QoQ Core PCE Inflation



(c) Output Gap

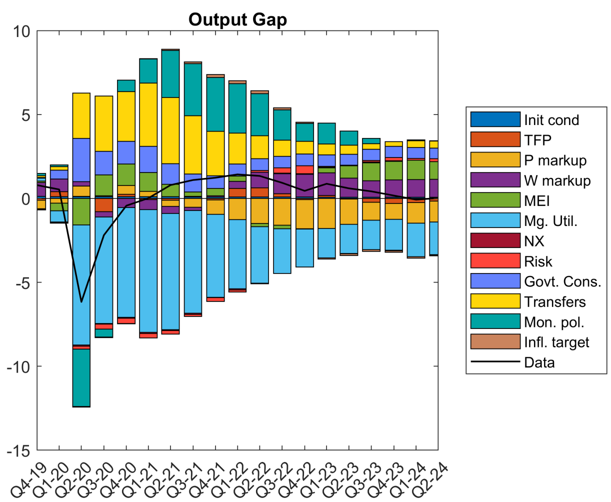


Figure 7: Full historical decompositions 2019Q4-2024Q2, relative to steady state value of the variable