Productivity, Output, and Employment

Econ 4021, Washington University in St. Louis

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Introduction

- Previous lectures: how to measure economic variables
- We now switch gears to try to understand how those variables are determined
- Output, inflation, unemployment, etc. are all jointly determined in a collection of markets where different agents interact (firms, households, government, etc.)
- We begin by trying to understand what determines and motivates the behavior of those agents in these markets

Introduction

This series of lectures:

1. The Production Function
2. The demand for labor
3. The supply of labor
4. Labor market equilibrium
5. Unemployment
6. Okun's Law

1. The Production Function

The Production Function

- The production function is a mathematical representation of how the economy transforms inputs into output

$$Y = AF(K, N)$$

- K is capital and N is labor; these are the most important inputs or **factors of production** (in practice there can be others, such as energy or raw materials)
- F is a function that maps quantities of inputs into output
- A is **total factor productivity** (TFP), a term that affects the relationship between inputs and outputs in a multiplicative manner
  - Quality of management
  - Natural conditions
  - Available technology

The US Production Function

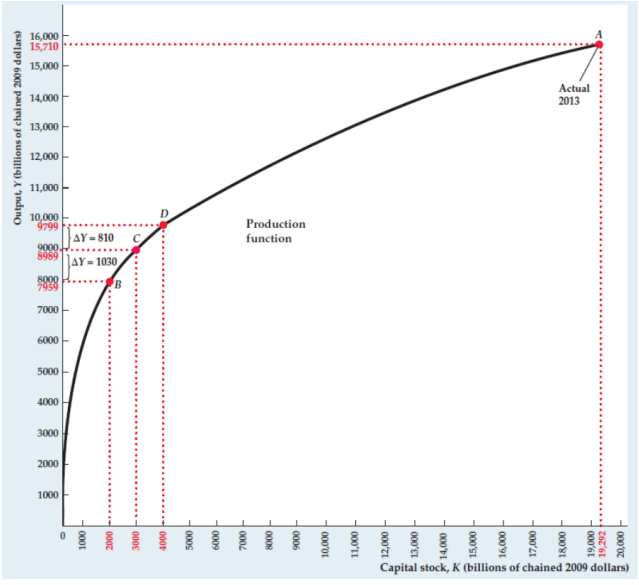
- A common type of production function is the **Cobb-Douglas production function**, which takes the form

$$Y = AK^{\alpha}N^{1-\alpha}$$

- The way that the US economy converts inputs to output is reasonably well approximated by a Cobb-Douglas production function with parameter $\alpha = 0.3$

$$Y = AK^{0.3}N^{0.7}$$

The Production Function



Marginal Product

- An important concept is that of **marginal product** of capital and labor

$$MPK = \frac{\partial Y}{\partial K} = A\alpha K^{\alpha-1} N^{1-\alpha}$$
$$MPN = \frac{\partial Y}{\partial N} = AK^{\alpha}(1-\alpha)N^{-\alpha}$$

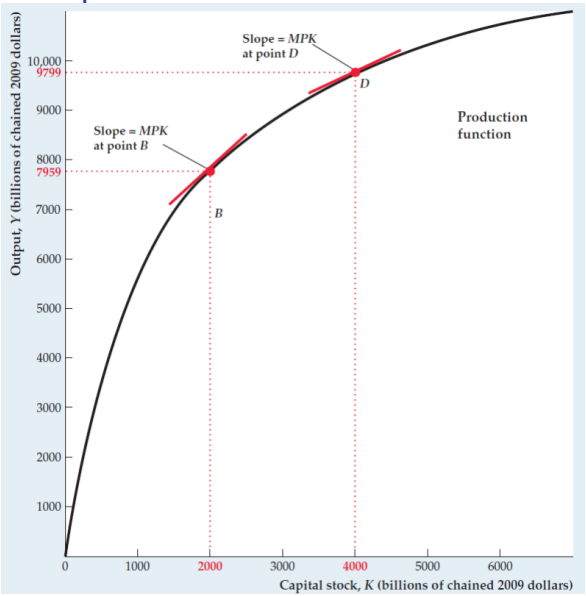
- The marginal product of an input is the change in output that results from a one-unit increase in that input
- We typically assume two things about the production function
  1. Marginal products are positive

$$\frac{\partial Y}{\partial K} > 0, \frac{\partial Y}{\partial N} > 0$$

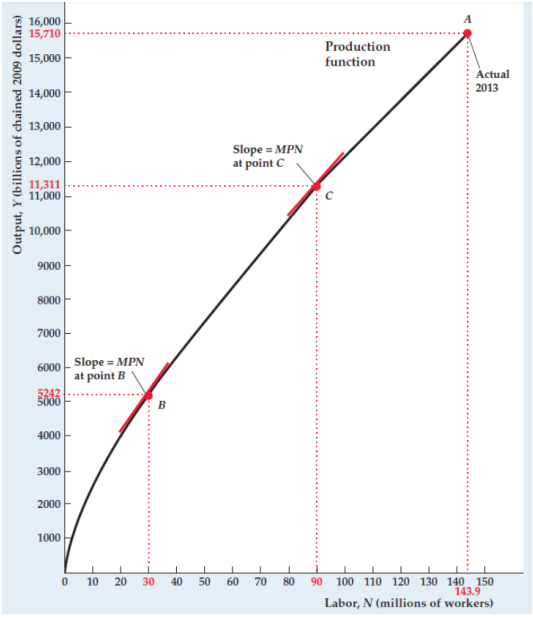
  2. Marginal products decline as the quantity of the respect inputs increase

$$\frac{\partial^2 Y}{\partial K^2} < 0, \frac{\partial^2 Y}{\partial N^2} < 0$$

Marginal Product of Capital



Marginal Product of Labor

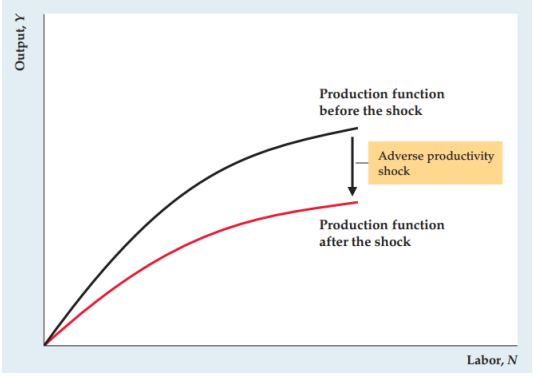


Supply Shocks

- Supply shocks, or **productivity shocks** are changes in the economy's production function
- A supply shock occurs when A changes, for example, which leads to changes in output for the same quantity of inputs'
- These shocks can be positive or negative
  - Technological progress is usually represented by an increase in A, which allows the economy to produce more output with the same inputs
  - Natural disasters may interfere with the way inputs are combined to produce output, thereby lowering A

Supply Shocks

- Supply shocks are shifts of the production function
- Contrast with changes in the quantity of inputs which are movements along the production function



2. The Demand for Labor

Demand for Labor

- Firm inputs: capital K and labor N
- Capital is relatively slow to adjust: firms need to invest in new capital, or scrap/sell old capital, and this takes time
- Labor can be adjusted relatively quickly: ask workers to work overtime, fire workers, etc.
- In the **short-run** we treat capital as fixed and labor as flexible

Simplifying assumptions:

1. Workers are all alike (no differences in skill, willingness to work, etc.)
2. Labor markets are competitive and firms take the prevailing wage as given
3. Firms maximize profits

Problem of the Firm

$$\max_N P \times AF(K, N) - WN - RK$$

- Capital is fixed: K is taken as given
- Competitive labor markets: nominal wage W is taken as given
- First-order condition with respect to labor:

$$P \times A\frac{\partial F(K, N)}{\partial N} = W$$

- Marginal revenue of labor is equated to nominal wage
- Marginal revenue of labor = price $\times$ MPN
- Alternatively, we can say that the MPN is equal to the **real wage** $w = \frac{W}{P}$
- Firm keeps hiring workers until the revenue of hiring an extra worker is equal to its cost

Analysis at the Margin

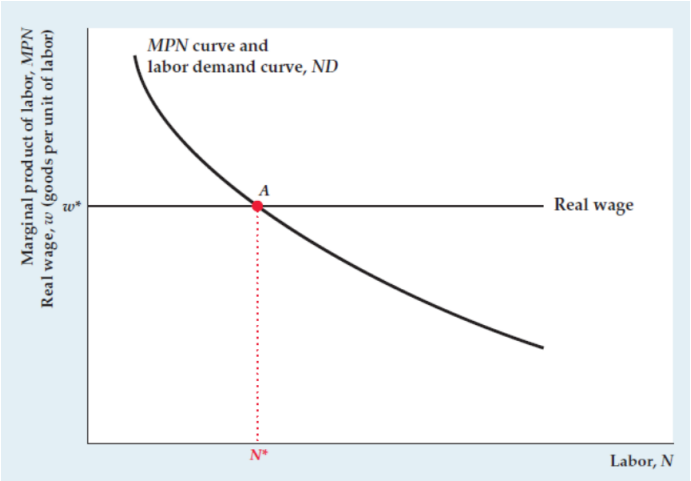
- Recall that we assumed **diminishing marginal productivity of labor**

$$\frac{\partial^2 Y}{\partial N^2} < 0$$

- Second-derivative wrt labor is negative $\Rightarrow$ first derivative wrt labor is **decreasing**
- In other words, MPN is **decreasing** in labor
- If $MPN > w$, hiring an extra worker produces more revenue than it costs, so $N \uparrow$
- If $MPN < w$, then the firm could fire a worker and increase its profit
- Firm profits are maximized when $MPN = w$

Labor demand

The MPN curve is the labor demand curve for the firm



The optimal amount of labor N^* is the point where the labor demand curve intercepts the real wage

Example: Cobb-Douglas Production Function

Recall that the Cobb-Douglas production function is

$$Y = AK^{\alpha}N^{1-\alpha}$$

The firm's problem is

$$\max_{N} AK^{\alpha}N^{1-\alpha} - wN - rK$$

The firm's optimality condition is then

$$\frac{\partial AK^{\alpha}N^{1-\alpha}}{\partial N} = AK^{\alpha}(1-\alpha)N^{-\alpha} = w$$

We can then solve for the optimal number of workers

$$N^{*} = \left[\frac{AK^{\alpha}(1-\alpha)}{w}\right]^{\frac{1}{\alpha}}$$

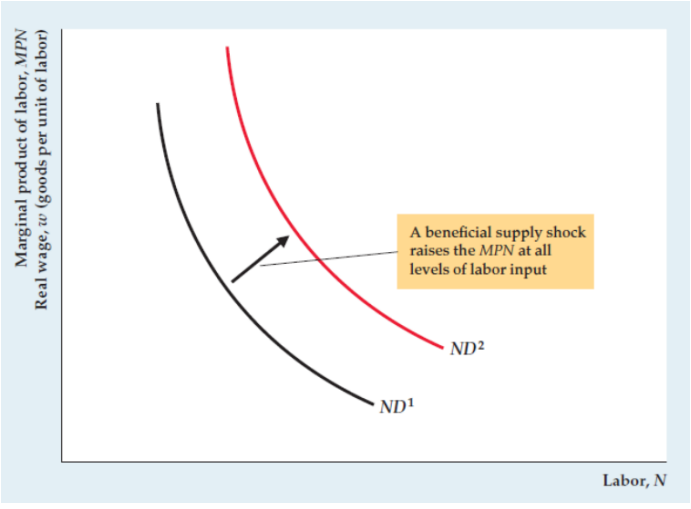
Shifts in Labor Demand

- A change in the wage causes a movement **along** the labor demand curve, not a shift of the curve
- A change in any other variable that affects the MPN **shifts** labor demand
- Recall Cobb-Douglas example:

$$MPN = AK^{\alpha}(1-\alpha)N^{-\alpha}$$

- Changes in TFP A or in capital K shift the labor demand curve
- Increase in A or K are **positive supply shocks**, shift curve to the right
- Firm willing to hire more workers at the same real wage w

Shifts in Labor Demand



Aggregate Labor Demand

- In macroeconomics, we typically work with the **aggregate demand for labor**
- This is the sum of the labor demand of all individual firms
- In an economy with J firms, aggregate labor demand is given by

$$N^d(w) = \sum_{j=1}^{J} N_j^d(w)$$

- In the Cobb-Douglas case:

$$N^d(w) = \sum_{j=1}^{J} \left[\frac{A_i K_i^{\alpha} (1-\alpha)}{w} \right]^{\frac{1}{\alpha}}$$

- If all firms have the same productivity and capital:

$$N^d(w) = J \times \left[\frac{A K^{\alpha} (1-\alpha)}{w} \right]^{\frac{1}{\alpha}}$$

3. The Supply of Labor

The Supply of Labor

- The labor demand is determined by firms, while the supply of labor is determined by workers (or unions)
- Households/individuals observe the equilibrium wage w and decide how much to work/how much labor to supply to firms
- The cost of working is the **opportunity cost** of alternative uses of working time: leisure, home production, going to school, etc.
- The **aggregate supply of labor** is the combined supply of labor by everyone in the economy

The Income-Leisure Trade-off

- Economists typically assume that individuals' happiness can be measured using a concept called **utility**
- Utility is typically assumed to increase with the consumption of goods and services as well as with leisure time $U = u(C, L)$ where u is a **utility function** that is increasing in consumption C and leisure L
- Utility levels are irrelevant - what matters is how the utility of different bundles (C, L) compare
- Bundle (C_1, L_1) is preferred to bundle (C_2, L_2) if and only if

$$u(C_1, L_1) > u(C_2, L_2)$$

- Common assumptions:
  1. More is preferred to less (i.e. marginal utility is positive)
  2. Diversity is good (i.e., preferences are convex)
  3. Consumption and leisure are normal goods, i.e. desired quantities increase with income

The Income-Leisure Trade-off

- Households want as much consumption and leisure as possible
- However, individuals need income in order to consume, and that income must be earned by working
- Individuals have limited time in a day, and so working takes time away from leisure
- These trade-offs can be represented via mathematical constraints
- The **time constraint** states that the individual has limited time that must be allocated between work N and leisure L

$$N + L = 24$$

The Income-Leisure Trade-off

- The **budget constraint** states that consumption expenditure must not exceed income, which is earned by working

$$C = wN$$

where w is the real wage per hour worked

- Recall that $N = 24 - L$

- We can combine the two constraints into a single budget constraint that illustrates the income-leisure trade-off:

$$C = w(24 - L)$$

The Income-Leisure Trade-off

- We assume that the marginal utility of consumption and leisure is positive

$$\frac{\partial u(C,L)}{\partial C} > 0, \frac{\partial u(C,L)}{\partial L} > 0$$

  This means that individuals always like more consumption and more leisure

- We also assume **decreasing marginal utility** of consumption and leisure

$$\frac{\partial^2 u(C,L)}{\partial C^2} < 0, \frac{\partial^2 u(C,L)}{\partial L^2} < 0$$

- Each additional unit of consumption and/or leisure generates a decreasing amount of utility

The Income-Leisure Trade-off

- The household's decision problem can be expressed as

$$\max_{C,L} u(C, L)$$

subject to

$$C = w(24 - L)$$

- Replace the constraint in the utility function to obtain

$$\max_{L} u[w(24 - L), L]$$

- Univariate problem $\Rightarrow$ easy to solve!

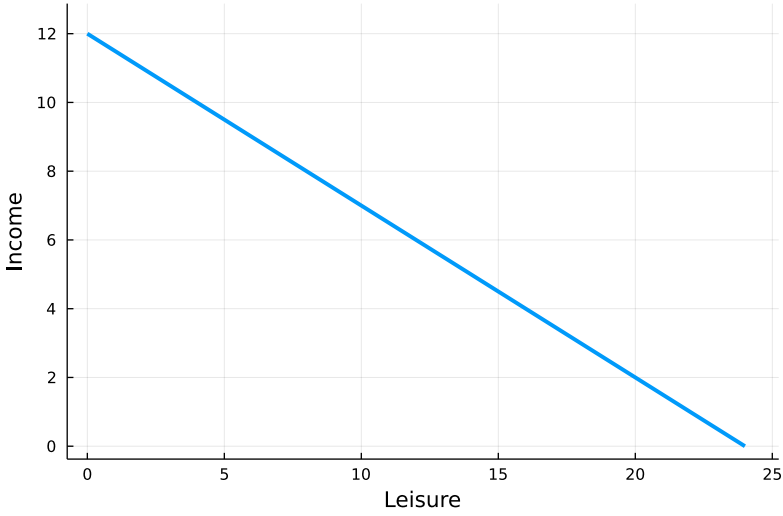
$$\frac{\partial u(C, L)}{\partial C}(-w) + \frac{\partial u(C, L)}{\partial L} = 0$$

- Familiar condition: MRS equals relative prices

$$MRS_{C,L} = \frac{\frac{\partial u(C,L)}{\partial L}}{\frac{\partial u(C,L)}{\partial C}} = w$$

The Income-Leisure Trade-off

Example with $w = 0.5$



The Income-Leisure Trade-off

- The wage is the relative price of leisure
- By enjoying an extra hour of leisure, the household foregoes w units of real consumption
- Choose a combination of consumption and leisure such that the ratio of marginal utilities equals the wage
- This takes into account the consumption benefit of working an extra hour (consume an extra w units) and the leisure cost of working that extra hour

Example

Consider the following utility function

$$u(C, L) = \sqrt{C} + \eta\sqrt{L}$$

The household problem is then

$$\max_{L} \sqrt{w(24 - L)} + \eta\sqrt{L}$$

Taking the first-order conditions:

$$\sqrt{w}\frac{1}{2}\frac{1}{\sqrt{24 - L}}(-1) + \eta\frac{1}{2}\frac{1}{\sqrt{L}} = 0$$

We can solve for optimal leisure as

$$L^* = \frac{24\eta^2}{w + \eta^2}$$

$$\Rightarrow N^* = 24 - L = 24\left[1 - \frac{\eta^2}{w + \eta^2}\right]$$

Example

Optimal leisure and labor are

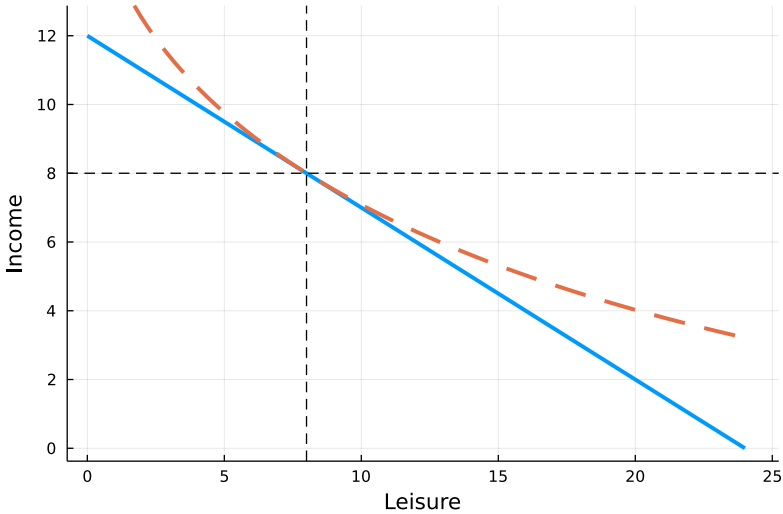
$$L^* = \frac{24\eta^2}{w + \eta^2}$$

$$N^* = 24\left[1 - \frac{\eta^2}{w + \eta^2}\right]$$

We can then use the budget constraint to solve for optimal consumption

$$\begin{aligned} C^* &= w \times N^* \\ &= w \times 24\left[1 - \frac{\eta^2}{w + \eta^2}\right] \end{aligned}$$

Example



Real Wages and Labor Supply

- The **real wage** is defined as the real payment for supplying an hour of work

$$w = \frac{W}{P}$$

- What happens when the real wage increases?
  1. The benefit of working an additional hour increases, which induces people to supply more labor. This is the **substitution effect**.
  2. But for the same amount of hours worked, workers earn a higher income and could thus afford the same amount of consumption by working less hours and enjoying more leisure. This induces people to supply less labor and is called the **income effect**.

- The two forces have opposite effects on labor supply

Substitution Effect

- If $w \uparrow$, working an extra hour leads to a greater benefit in terms of consumption
- This induces people to work more, everything else constant
- Temporary increases in wages are likely to generate "pure substitution effects"

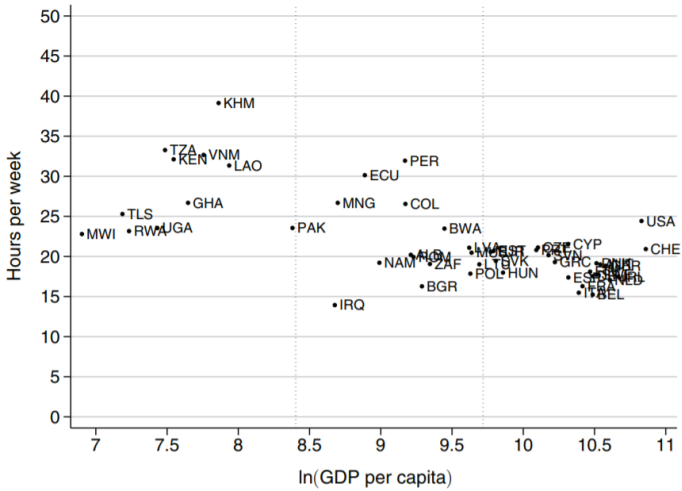
Income Effect

- An increase in income (unrelated to labor, for example) means that a worker can afford the same level of consumption and work fewer hours
- Ex: inheritance, lottery wins
- This is called the income effect, and is also called the wealth effect
- It could also be related to labor: a future increase in the wage, for example

Combining the Two Effects

- An increase in the real wage increases hours worked via the substitution effect, and decreases hours worked via the income effect
- The net effect is in principle uncertain and depends on several factors
  1. The magnitude of the increase in the wage
  2. The elasticity of labor supply
  3. etc
- Empirical evidence: substitution effects tend to dominate for temporary changes in real wages, while income effects dominate for permanent changes

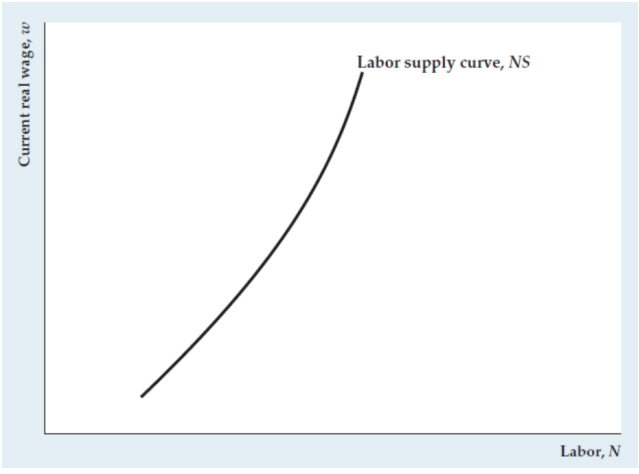
Hours worked vs. GDP per capita



Source: Bick et al., 2018 "How do Hours worked vary with income? Cross-country evidence and implications", American Economic Review

Labor Supply

Amount of labor N that a worker supplies at a given real wage w



The optimal amount of labor is the point where the labor supply curve intercepts the prevailing real wage.

Example

Recall our utility function example

$$N^s = 24\left[1 - \frac{\eta^2}{w + \eta^2}\right]$$

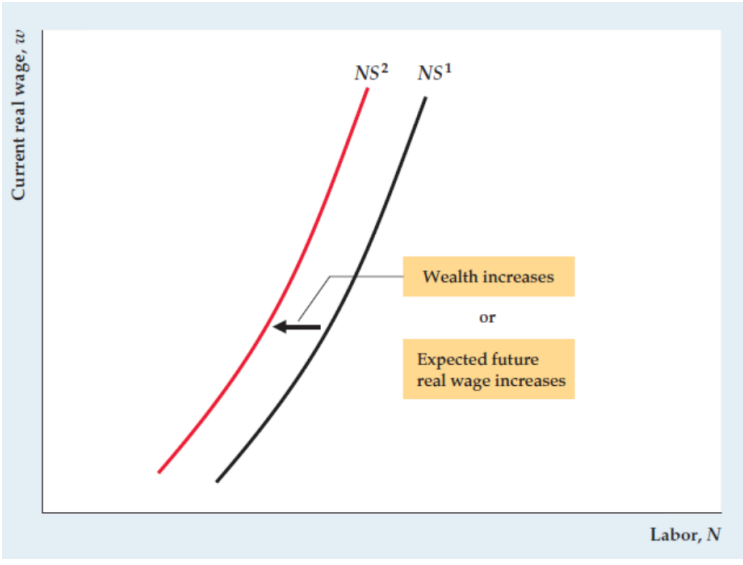
Note that labor supply is *increasing* in the real wage.

- What does this mean regarding income vs. substitution effects?

Shifts in Labor Supply

- A change in the wage causes a movement **along** the labor supply curve, not a shift of the curve
- Changes in any other variables that affect the leisure-consumption trade-off lead to a shift in the curve
- For ex: non-wage related changes in wealth trigger an income effect that shift the labor supply curve
- An increase in wealth means that the worker supplies less hours for the same wage $\Rightarrow$ labor supply shifts to the left

Shifts in Labor Supply



Aggregate Labor Supply

- The **aggregate labor supply** is the total amount of labor supplied by all workers in an economy for a given real wage
- In an economy with I workers, aggregate labor supply is given by

$$N^s(w) = \sum_{i=1}^{I} N_i^s(w)$$

- In our previous example:

$$N^s(w) = \sum_{i=1}^{I} 24 \left[1 - \frac{\eta_i^2}{w + \eta_i^2}\right]$$

- If all workers have the same $\eta_i = \eta, \forall i$:

$$N^s(w) = I \times 24 \left[1 - \frac{\eta^2}{w + \eta^2}\right]$$

Aggregate Labor Supply

Economy-wide increases in the real wage tend to increase the aggregate quantity of labor that is supplied in two ways:

1. For people who are currently working, the substitution effect may dominate and they may supply more hours. This is called the **intensive margin**.

2. Some people may have dropped out of the labor force but may be enticed to return if wages are high enough. This is called the **extensive margin**.

People who are out of the labor force include:

- Retirees
- Adults who attend school full time
- Adults who engage in domestic production full time

Shifts in Aggregate Labor Supply

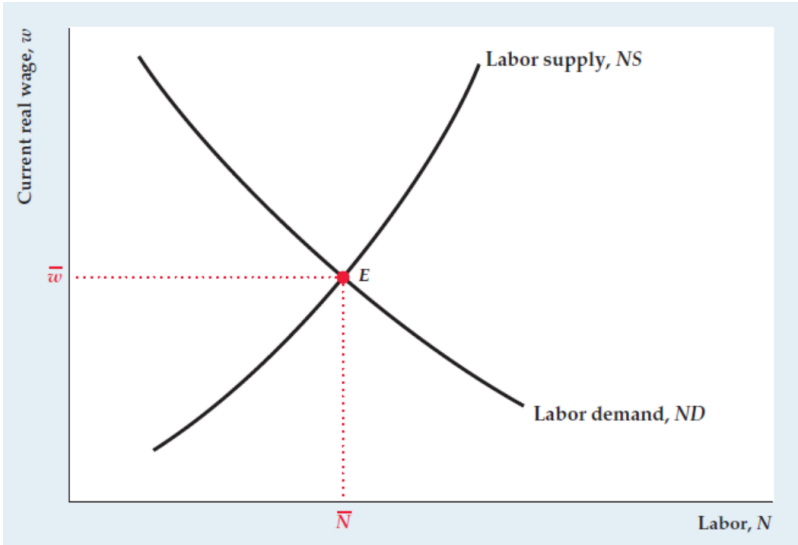
Factors That Shift the Aggregate Labor Supply Curve

An increase in	Causes the labor supply curve to shift	Reason
Wealth	Left	Increase in wealth increases amount of leisure workers can afford.
Expected future real wage	Left	Increase in expected future real wage increases amount of leisure workers can afford.
Working-age population	Right	Increased number of potential workers increases amount of labor supplied.
Participation rate	Right	Increased number of people wanting to work increases amount of labor supplied.

4. Labor Market Equilibrium

(Classical) Labor Market Equilibrium

Combine supply and demand to obtain the labor market equilibrium:

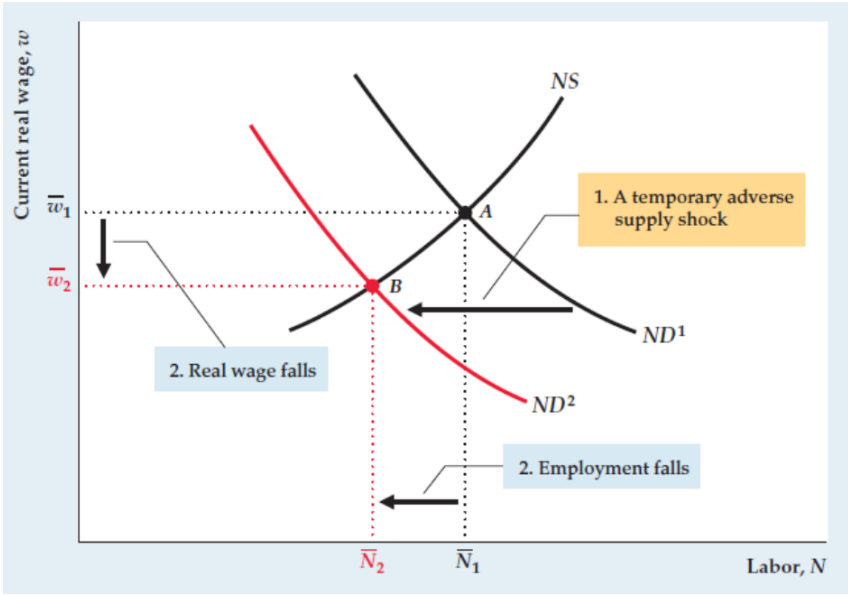


(Classical) Labor Market Equilibrium

Why "classical"?

- Quantity of labor and real wage fully determined by a labor demand curve and a labor supply curve
- Assumes that wages and quantities respond quickly to shocks
- Workers are always responding optimally to the prevailing real wage $\Rightarrow$ there is no notion of unemployment
- Everyone is working the exact number of hours they want to work at the equilibrium real wage
- In practice, wages may take time to adjust in response to shocks, and some people who would like to work at the current wage may not find a job

Labor Demand Shock



Full Employment

- Classical model is better suited to study **full employment**
- This is the equilibrium level of employment $\bar{N}$ that arises in the classical model, where wages adjust instantly
- The level of output that is consistent with full employment is called **full employment output** or **potential output**

$$\bar{Y} = AF(K, \bar{N})$$

- It is the level of output that would be produced under full employment, i.e. if everyone who wanted to work could work the number of hours they wanted to at the equilibrium real wage $\bar{w}$

Full Employment

- Full employment and potential output fluctuate with factors that shift labor demand and labor supply

1. Shocks to productivity A
2. Shocks to capital K
3. Shifts in labor supply

5. Unemployment

Unemployment

- Classical model: everyone who wants to work at wage rate $\bar{w}$ can find a job
- This is not how economies work in practice: often, many people would like to work at prevailing wages and cannot find a job
- Thus the economy's factors of production (specifically, labor) may be underutilized at times, and output may be below its potential level
- What is an unemployed person?
  - Anyone who does not have a job but is looking for one

Measuring Unemployment

- Every month, the Bureau of Labor Statistics (BLS) surveys 60,000 households via the **Current Population Survey**
- The BLS asks many questions about demographic characteristics and labor force status
- Every person who is 16 years of age or older is assigned to one of three categories:
  1. Employed, if that person worked full- or part-time during the past week
  2. Unemployed, if that person didn't work during the past week but looked for work in the past four weeks
  3. Our of the labor force, if that person did not work during the past week and did not look for work during the past four weeks

Employment Status of US Adult Population

Employment Status of the U.S. Adult Population, May 2015

Category	Number (millions)	Share of labor force (percent)	Share of adult population (percent)
Employed workers	148.8	94.5	59.4 (employment ratio)
Unemployed workers	8.7	5.5 (unemployment rate)	3.5
Labor force (employed + unemployed workers)	157.5	100.0	62.9 (participation rate)
Not in labor force	93.0		37.1
Adult population (labor force + not in labor force)	250.5		100.0

Note: Figures may not add up because of rounding.
Sources: The Employment Situation, May 2015, Table A-1.

Measuring Unemployment

Let

- E denote the number of employed people
- U denote the number of unemployed people
- NLF denote the number of people not in the labor force

$$\text{Adult Population} = E + U + NLF$$

- The **labor force** is the sum of employed and unemployed people.

$$LF = E + U$$

- The **participation rate** is the fraction of the adult population in the labor force

$$PR = \frac{LF}{\text{Adult Population}}$$

Measuring Unemployment

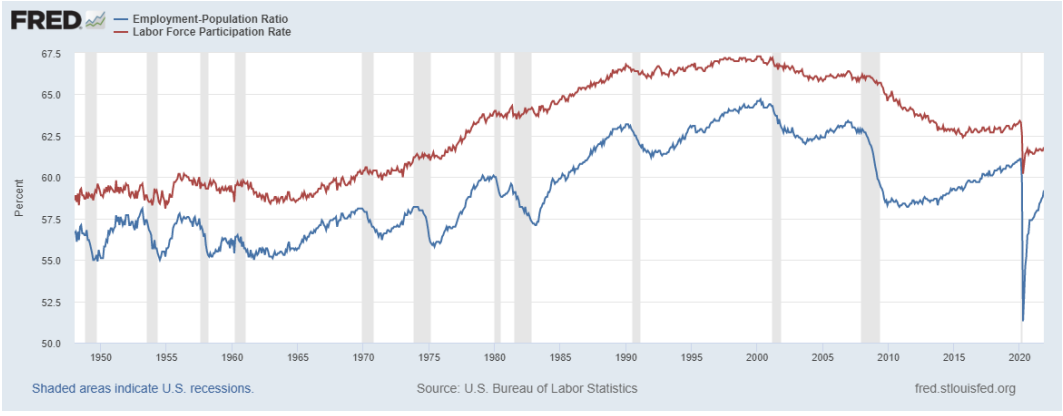
- The **unemployment rate** is the percentage of adults in the labor force that are unemployed

$$u = \frac{U}{LF}$$

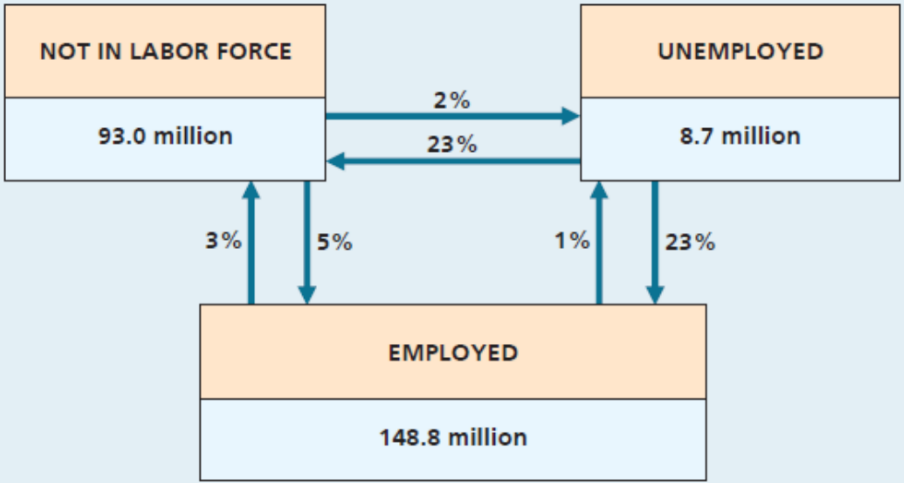
- The **employment rate** is the percentage of adults that are employed

$$EPOP = \frac{E}{\text{Adult Population}}$$

EPOP and LFP


FRED
Employment-Population Ratio
Labor Force Participation Rate
Percent
67.5
65.0
62.5
60.0
57.5
55.0
52.5
50.0
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020
Shaded areas indicate U.S. recessions.
Source: U.S. Bureau of Labor Statistics
fred.stlouisfed.org

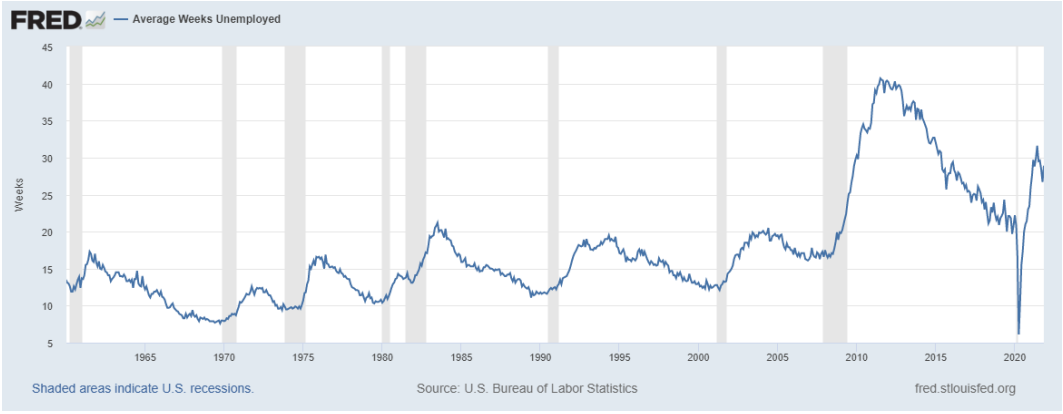
Labor Market Flows in May 2015



Unemployment Duration

- Some unemployed workers find jobs, others leave the labor force
- Some of the unemployed workers who leave the labor force are called **discouraged workers**: they looked for jobs, but became discouraged and stopped searching
- Not all unemployed workers who leave the LF are discouraged: others simply retire or go to school
- The period of time during which a person is continuously unemployed is called an **unemployment spell**
- The duration of unemployment is the length of unemployment spells and is of particular interest to economists
  - When unemployment duration is low, this means that unemployed workers are able to find new jobs relatively quickly
  - When unemployment duration is high, this means that it takes a long time for unemployed workers to find new jobs.
- Long unemployment spells have many social costs: they force people to dissave, may lead to the erosion of skills/human capital, and may be costly for taxpayers
- Reducing unemployment duration is typically an important policy goal

Unemployment Duration



Why is unemployment never equal to zero?

- Unemployment typically rises during recessions
- But even during expansions the unemployment rate is positive
- There are two types of unemployment:
  1. **Frictional Unemployment** arises from the normal functioning of the economy. People quit jobs all the time to look for better jobs. The process of finding a new job ("matching") takes time for many reasons.
  2. **Structural Unemployment** refers to longer-term unemployment that does not necessarily arise due to the dynamism of the economy but rather due to long-term skill mismatches and others social issues
- Frictional unemployment can be a sign of a healthy and dynamic economy, while structural unemployment generally reflects problematic social and economic issues.
- Factors such as deindustrialization and offshoring tend to contribute to structural unemployment

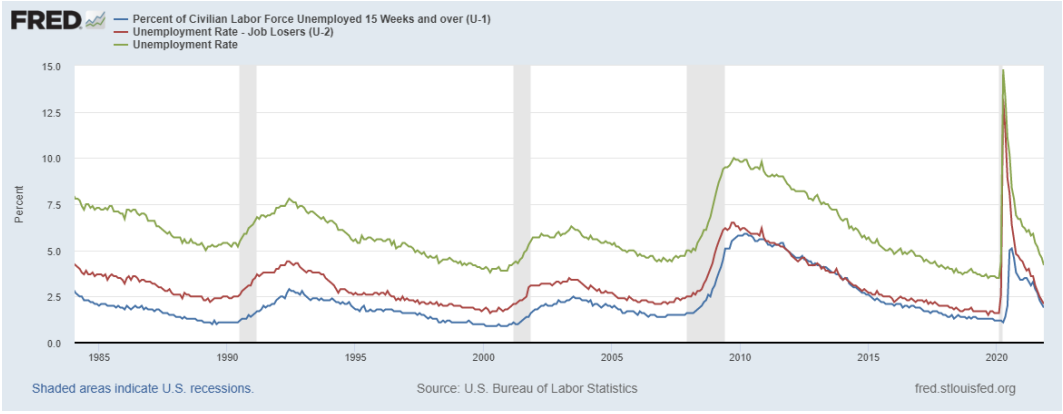
The Natural Rate of Unemployment

- The NRU, $\bar{u}$, is the unemployment rate that prevails when the economy is at full employment
- As output fluctuates around its potential level, so does the unemployment rate fluctuate around the NRU
- The difference between the unemployment rate and the NRU is called **cyclical unemployment**

$$u_t - \bar{u}_t$$

- Cyclical unemployment is positive during recessions and negative during expansions

Measures of Unemployment



6. Okun's Law

Output and Unemployment

- Most short-run fluctuations in output are associated with fluctuations in employment
- Productivity and capital are typically fixed in the short-run, meaning that deviations in output from its potential level are associated with deviations of unemployment from its natural level
- Okun's "Law" is an empirical relationship between the output gap and cyclical unemployment

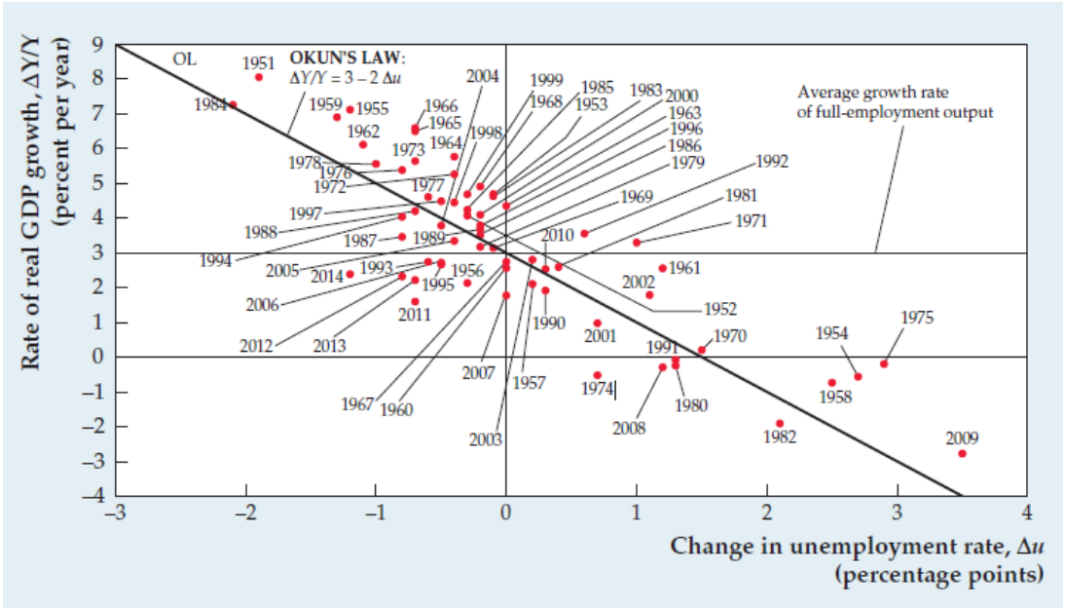
$$\frac{\bar{Y}_t - Y_t}{\bar{Y}_t} = 2 \times (u_t - \bar{u}_t)$$

- Named after Arthur Okun, chairman of the CEA in the 1960s, during LBJ's presidency
- The output gap increases by 2 percentage points for each percentage point cyclical unemployment exceeds the natural rate

Okun's Law

- Why is the coefficient equal to 2 and not 1?
- A rise in unemployment tends to understate the extent to which labor input is underutilized
- Labor force participation is **procyclical**: it rises when output is above potential, and falls when output is below potential
- Since the LFPR falls during recessions, movements in the unemployment rate understate the true change in hours worked/persons employed
- Recall that the denominator of the unemployment rate is the LF, not the adult population

Okun's Law


OL
OKUN'S LAW:
$\Delta Y/Y = 3 - 2\,\Delta u$
Average growth rate of full-employment output
Rate of real GDP growth, ΔY/Y (percent per year)
Change in unemployment rate, Δu (percentage points)
1951
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