

A Quantitative Theory of Relationship Lending

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What are the macro effects of relationship lending?

- Large literature on **relationship lending** in banking
 - Information advantage of banks (Diamond 91; Petersen & Rajan 94; Berger & Udell 95)
 - “Informational lock-in” (Sharpe 90, Rajan 92)
 - Price dispersion and sourcing persistence
- Literature on bank customer capital mostly focused on the **liability** side
 - Egan, Hortacsu & Matvos 17; Drechsler, Savov & Schnabl 17; Li, Loutskina & Strahan 23
- **What are the macroeconomic consequences of relationship lending?**
 1. For the dynamics of individual relationships
 2. For the distribution of banks in the economy (interest rates, capital, risk...)
 3. For how the economy responds to aggregate shocks

1. Quantitative Model of Relationship Lending

- Multiple lenders and sourcing adjustment costs give rise to “relationships”
- 2-tier demand system, amenable to estimation
- Banks internalize relationship formation \Rightarrow dynamic pricing
- Financial frictions interact with motives to manage customer capital

2. Estimation and Validation

- Semi-structural estimation of new parameters using micro data on US bank loans
- Model matches lender switching patterns and “relationship life cycle” pricing patterns

3. Model Results

- Relationship lending generates interest rate dispersion, provides insurance for banks
- Customer capital as a substitute for financial capital
- Passthrough of aggregate shocks nonlinear in the degree of competition
- Models w/ high market power can “mimic” competitive economies

What we contribute to the literature

We combine insights from 2 main literatures:

1. **financial accelerator/banking frictions:** Kiyotaki & Moore 97; BGG 99; Corbae & D'Erasmus 21
 - novel competition structure with long-horizon pricing
 - heterogeneous bank “block” integrates with economy-wide loan market
2. **customer capital / habits:** Ravn et al 06; Gourio & Rudanko 14; Gilchrist et al 17
 - banks internalize habit formation, relationships pin down demand elasticity

towards a quantitative framework with credit market relationships.

- **empirics:** e.g. Rajan & Petersen 94; Drechsler, Savov & Schnabl 17; Atkeson et al 19
- **equilibrium models:** e.g. Boualam (18), ...

Environment and Markets

- Time is discrete and infinite, $t = 0, 1, 2, \dots$
- Two types of agents:
 - A continuum of **identical firms** $i \in [0, 1]$ that hire labor and borrow to produce
 - A continuum of **heterogeneous banks** $j \in [0, 1]$ that fund lending with deposits and retained earnings
 - Banks exit (and are replaced) at rate $1 - \pi$
- Agents interact in imperfectly competitive **lending markets**
- Firms form persistent relationships w/ banks that are costly to adjust
- Partial equilibrium: risk-free rate \bar{r} , wage \bar{w} , and deposit price \bar{q}^d taken as given

Banks' problem

States: net worth n , relationship intensity s , return shock z

$$V(n, s, z; \mu) = \max_{q, e, n', \ell', d', s'} e + \bar{q}\pi \mathbb{E}_{z'} [V(n', s', z'; \mu)]$$

subject to:

[budget constraint] $q\ell' + \psi(e) \leq n + z + \bar{q}^d d'$

[net worth dynamics] $n' = \ell' - d'$

[capital requirement] $\chi q\ell' \leq q\ell' - \bar{q}^d d'$

[loan demand] $\ell' = \ell'(q, s)$

[relationship formation] $s' = \rho_q \frac{q\ell'}{L'(\mu)} + \rho_s s$

$\mu(q, s)$ is the joint distribution of interest rates and relationships

Dynamic Loan Pricing

Define the net period return on a dollar loan

$$\Pi_t = \underbrace{\frac{\bar{q}}{q_t} \pi \mathbb{E}_t \left[\frac{(\psi^{-1})'(e_{t+1})}{(\psi^{-1})'(e_t)} \right]}_{\text{loan return}} - \underbrace{1}_{\text{funding cost}} + \underbrace{\lambda_t(1 - \chi)}_{\text{SV ease cap. req.}}$$

The bank's optimal choice is given by

$$\underbrace{\Pi_t + \overbrace{\bar{q}\pi\rho_q\mathbb{E}_t \sum_{i=1}^{\infty} (\bar{q}\pi(\rho_q + \rho_s))^i \Pi_{t+i}}^{\text{dynamic relationships}}}_{\text{discounted lifetime net profits}} = \underbrace{\overbrace{\epsilon^{-1}(q\ell', q)}^{\text{static market power}} \times \frac{\bar{q}}{q_t} \pi \mathbb{E}_t [(\psi^{-1})'(e_{t+1})]}_{\text{excess return (from today's market power)}}$$

$\epsilon^{-1}(q\ell', q)$ is the inverse elasticity of loan demand ► special cases

Borrowers and Loan Demand

- Working capital constraint motivates borrowing (Christiano, Eichenbaum and Evans 05)
- Continuum of identical firms \Rightarrow focus on representative borrower
- Borrow (in principle) from **all banks** $j \in [0, 1]$, choose sourcing given:
 - q_j : loan price offered by j , implies interest rate $r(q_j)$
 - s_j : (relative) relationship with $j \rightarrow$ weighted average of past loan shares
 - $\mu(q, s)$: joint distribution of prices and relationships
 - borrower does not internalize current loan choices on $\{s'\}$, μ'
 - “external habits” in the spirit of Ravn, Schmitt-Grohe & Uribe, 06
- **Loan share adjustment** subject to quadratic costs with level ϕ

Representative borrower problem

$$\begin{aligned} W(\mathcal{L}; \mu) = & \max_{n, L', \mathcal{L}' = \{\ell'(q, s)\}} \underbrace{zn^\alpha - \bar{w}n}_{\text{op. profits}} + \underbrace{L' - \int \ell(q, s) d\mu(q, s)}_{\text{borrowing, net repayments}} \\ & - \underbrace{\frac{\phi}{2} L' \int \left(\frac{q\ell'(q, s)}{L'} - 1 - (s - S) \right)^2 d\mu(q, s)}_{\text{adjustment costs}} + \bar{q} \mathbb{E} [W(\mathcal{L}'; \mu')] \end{aligned}$$

subject to:

[working cap.]

$$L' \geq \kappa \bar{w} n$$

[sourcing]

$$\int q\ell'(q, s) d\mu(q, s) \geq L'$$

2-part equilibrium loan demand system

1. Bank-specific loan demand

$$\underbrace{\frac{q\ell'(q, s; \mu)}{L'(\mu)}}_{\text{relative loan demand}} = 1 + \underbrace{(s - S)}_{\text{relationship shifter}} - \underbrace{\frac{\bar{q}}{\phi}[r(q) - R(\mu)]}_{\text{elasticity} \times \text{IR spread}}$$

2. Aggregate loan demand

$$L'(\mu) = \kappa \bar{w} \left[\frac{\alpha z / \bar{w}}{1 + \kappa \left(\bar{q} \tilde{R}(\mu) - 1 \right)} \right]^{\frac{1}{1-\alpha}}$$
$$\underbrace{\tilde{R}(\mu)}_{\text{"effective" IR}} = \underbrace{R(\mu)}_{\text{avg. IR}} + \underbrace{\mathbb{C}_{\mu}(r, s)}_{\text{cov. term}} - \underbrace{\frac{1}{2} \frac{\bar{q}}{\phi} \mathbb{V}_{\mu}(r)}_{\text{var. term}}$$

A **stationary recursive competitive equilibrium** in this model consists of:

- loan demand functions $\ell'(q, s; \mu)$ and $L'(\mu)$;
- bank policies $g_q(n, s, z; \mu)$ and $g_d(n, s, z; \mu)$;
- distribution of prices and relationships $\mu(q, s)$; and
- distribution of bank states $m(n, s, z; \mu)$

which satisfy (i) borrower optimality; (ii) bank optimality; (iii) stationarity of bank distribution m given policies g ; and (iv) **consistency of distributions m and μ given g** :

$$\mu(q, s) = \int \mathbf{1}[q = g_q(n, s, z; \mu)] m(\mathrm{d}n, s, \mathrm{d}z) \text{ for all } q, s$$

Strategy for quantifying the model

1. **externally assign** subset of “standard” parameters
2. **directly estimate** key relationship parameters ϕ , ρ_s , and ρ_q
3. **internally calibrate** remaining parameters to match moments related to bank financing and pricing ▶ internal calibration

Goal: tie our hands on ϕ , ρ_q , ρ_s using semi-structural approach on micro data (2), then match other key features of banking industry (3).

Externally set parameters

	Description	Value	Target / Reason
\bar{r}_{ann}	Annualized risk-free rate	2%	Quarterly discount price $\bar{q} = (1 + \bar{r}_{\text{ann}})^{-\frac{1}{4}}$
ν_{ann}	Deposit liquidity premium	0.17%	Quarterly deposit price $\bar{q}^d = (1 + \bar{r}_{\text{ann}} - \nu_{\text{ann}})^{-\frac{1}{4}}$
χ	Capital requirement	8%	Current US bank regulation
π	Bank survival rate	0.9928	Quarterly bank exit rate of 0.72%
α	Returns to scale	0.75	Profit share of 20-30%
\bar{w}	Wage rate	1	Normalization
\bar{A}	Aggregate TFP	1	Normalization

Goal: estimate model-implied demand to retrieve ϕ

$$\frac{q\ell'(q, s; \mu)}{L'(\mu)} = 1 + (s - S) - \frac{\bar{q}}{\phi}[r(q) - R(\mu)]$$

Need data on quantities and prices of credit.

FR Y-14Q (Schedule H.1)

- Regulatory dataset maintained by the Federal Reserve for stress testing
- Quarterly loan-level panel on universe of loan facilities $> \$1\text{M}$
- Covers top 30/40 BHCs, 2014:Q1-2022:Q4
- Detailed information on features of credit facilities

Estimating ϕ : bank-specific demand curves

With data on quantities and prices, we can estimate

$$\frac{\ell_{fbt}}{L_{ft}} = \underbrace{\alpha_{ft} + \alpha_b + \Gamma X_{bt}}_{\text{FEs and controls}} + \underbrace{\beta(r_{fbt} - r_{ft})}_{\text{spread term}} + \underbrace{u_{fbt}}_{\text{s term}}$$

f = firm, b = bank, t = quarter

Classic **simultaneity problem**: follow Amiti & Weinstein 18 and estimate

$$r_{fbt} - r_{ft} = \gamma_{ft} + \gamma_{bt} + v_{fbt}$$

- use $\hat{\gamma}_{bt}$ to instrument spread term
- measures “pure” credit supply shock

Estimating ϕ : results

$$\frac{\ell_{fbt}}{L_{ft}} = \alpha_{ft} + \alpha_b + \Gamma X_{bt} + \beta(r_{fbt} - r_{ft}) + u_{fbt}$$

	(1)	(2)	(3)	(4)
$\hat{\beta}$	-14.084*** (4.121)	-30.932*** (3.928)	-12.191*** (1.767)	-26.505*** (7.998)
Firm identifier	TIN	TIN	ISL cell	ISL cell
Observations	57,346	57,245	218,866	218,827
Model	OLS	IV	OLS	IV
Implied $\hat{\phi}$	0.070	0.033	0.082	0.038

- TIN: tax identification number (individual firm)
- ISL: industry/size/location cell (Degryse et al. 19)

Estimating ρ_s and ρ_q : bank-level dynamics

- Demand regressions: s terms were subsumed into residual u_{fbt}
- Use \hat{u}_{fbt} to proxy s_{fbt} and estimate law of motion with OLS

$$\hat{u}_{fbt} = \alpha_f + \alpha_b + \alpha_t + \underbrace{\rho_q \frac{\ell_{fbt}}{L_{ft}}}_{\text{loan term}} + \underbrace{\rho_s \hat{u}_{fbt-1}}_{\text{lag term}} + \nu_{fbt}$$

- Generated regressor: need to bootstrap standard errors

Estimating ρ_s and ρ_q : results

$$\hat{u}_{fbt} = \alpha_f + \alpha_b + \alpha_t + \rho_q \frac{\ell_{fbt}}{L_{ft}} + \rho_s \hat{u}_{fbt-1} + \nu_{fbt}$$

	(1)	(2)
$\hat{\rho}_q$	0.771*** (0.012)	0.791*** (0.005)
$\hat{\rho}_s$	0.178*** (0.011)	0.141*** (0.005)
Firm identifier	TIN	ISL cell
Observations	36,651	132,290
R-squared	0.91	0.89

Internal Calibration

- Net worth shock: $z_t = \rho_z z_{t-1} + \sigma_z \epsilon_t^z$
- Equity issuance costs:

$$\psi(e) = \begin{cases} -\bar{\psi} \ln \left(1 - \frac{e}{\bar{\psi}} \right) & \text{if } e < 0 \\ e & \text{if } e \geq 0 \end{cases}$$

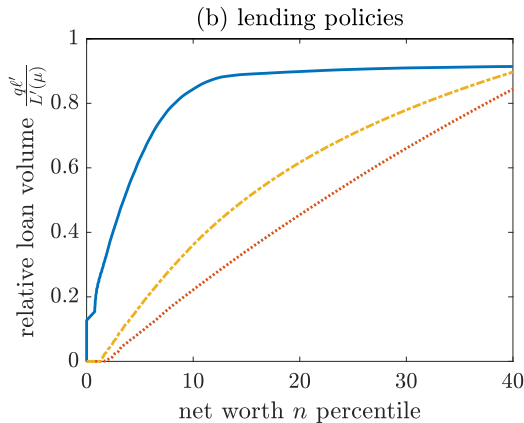
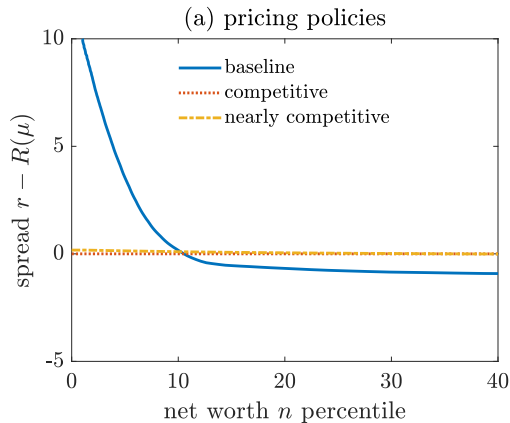
	Description	Value	Target / Reason	Data	Model
κ	Working capital constraint	0.9581	Business debt to GDP ratio	71.5%	71.6%
$\bar{\psi}$	Equity issuance cost curvature	0.0094	Gross equity issuance / NW	1.1%	1.2%
ρ_z	persistence of net worth shocks	0.2619	Net dividend payouts / NW	5.8%	4.4%
σ_z	iid bank shock variance	0.0026	Average net interest margin	1.8%	1.7%
			Average bank leverage	92.0%	91.8%

Quantitative Analysis

Compare three economies:

1. Baseline, with estimated $\hat{\phi}$
2. Perfectly competitive economy
3. Nearly competitive economy, $\phi \rightarrow 0$

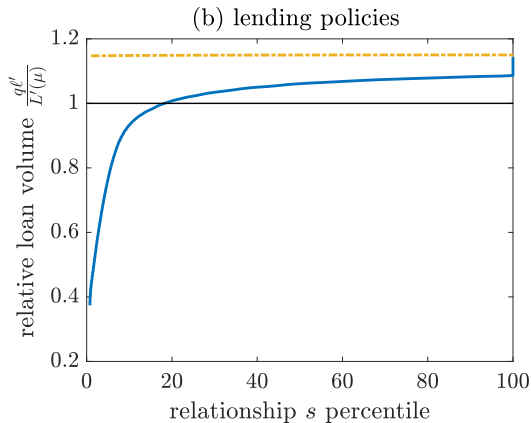
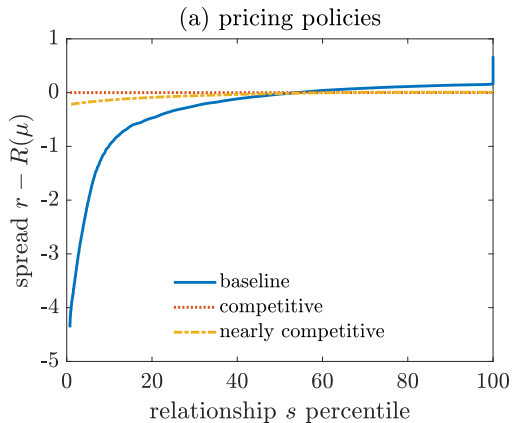
Policies by net worth



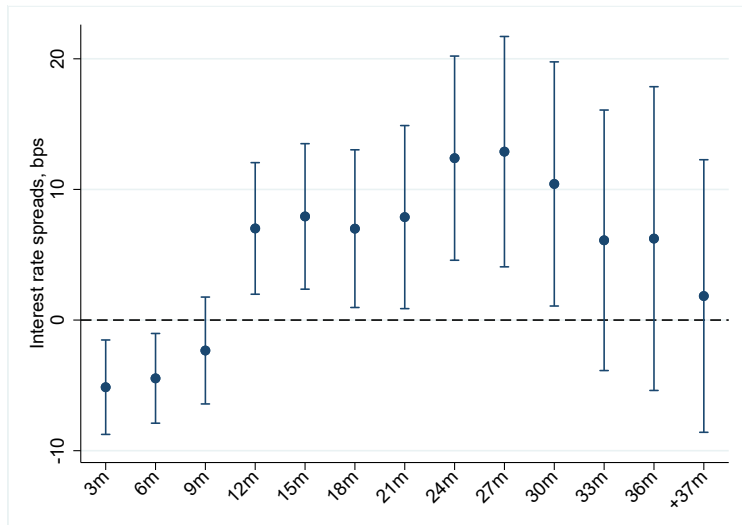
Low $n \implies$ price “above market” to cut loan supply when net worth falls

- financial and customer capital are **substitutes**

Policies by relationship intensity



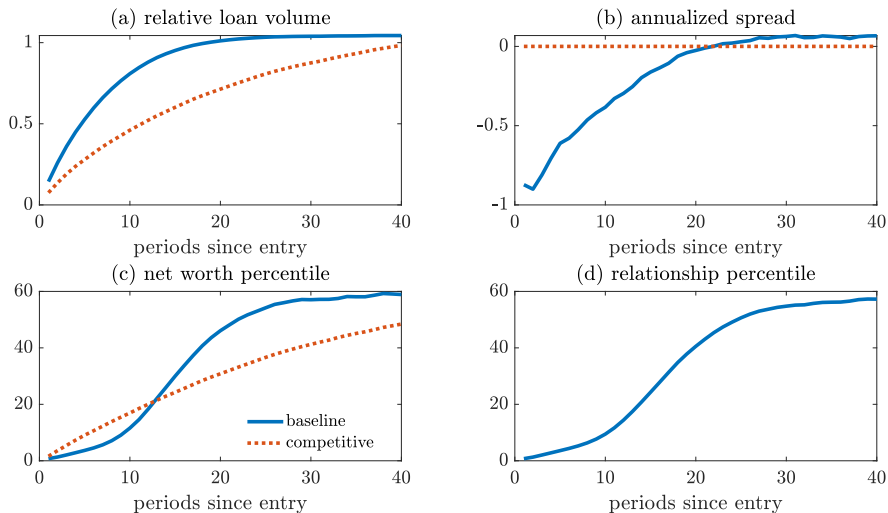
- banks with no existing relationships need to price below market
- doesn't immediately translate into loan volume given demand system



Exercise: match similar loans in Y-14Q, compare terms for switching and non-switching

1. “honeymoon:” upon switching banks, firms pay lower interest rates
2. “holdup:” over time with bank, firms end up paying higher rates

Validation: relationship lifecycle in the model



Model also matches share of switching loans in the data [▶ data on switching](#)

Pricing outcomes across model variants

		baseline	level		% diff rel to base	
			near comp.	comp.	near comp.	comp.
effective IR (pp, ann.)	\tilde{R}	3.51	2.18	2.07	-38.0	-41.0
= average rate	R	3.49	2.18	2.07	-37.6	-37.6
+ covariance term	$\mathbb{C}_{\mu}(r, s)$	0.03	0.00	-	-103.2	-
+ variance term	$\mathbb{V}_{\mu}(r)$	-0.01	0.00	-	-93.4	-
loan-weighted avg. IR	\overline{R}_L	3.49	2.07	2.07	-40.8	-40.8
loan volume	L'	0.30	0.30	0.30	1.2	1.2

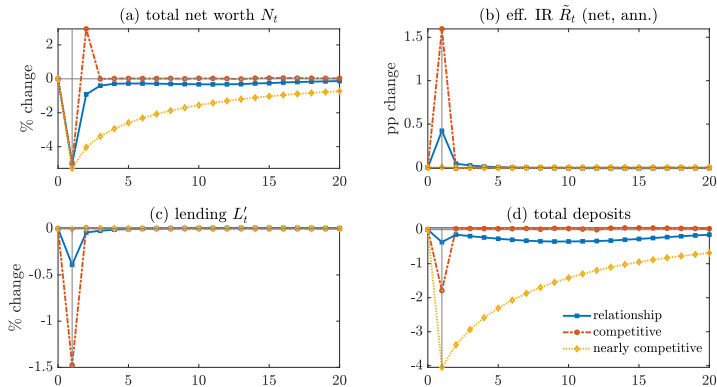
- higher effective IR, mostly driven by average rate
- covariance term raises rate, dispersion term attenuates

Banking industry moments across model variants

	baseline	level		% diff rel to base	
		near comp.	comp.	near comp.	comp.
average net worth	0.026	0.023	0.024	-9.5	-9.5
std dev, net worth	0.006	0.009	0.012	55.7	55.7
std dev, relationships	0.171	0.294	-	72.2	-
corr, net worth and spread	-0.021	-0.005	-	-75.4	-
corr, relationships and spread	0.062	-0.002	-	-102.8	-
corr, net worth and relationships	0.869	0.945	-	8.7	-
share of switches (pp)	8.84	10.32	-	16.7	-

- more competitive model \implies less net worth on average [▶ distributions](#)
 - (s, n) substitutability vs. franchise value effect
- weak negative correlation between spreads and net worth
 - financial constraints vs positive correlation between types of capital

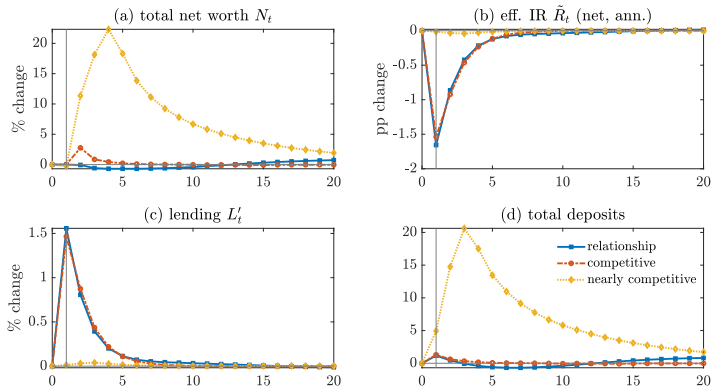
Dynamic experiment 1: aggregate bank net worth shock



Shock: wipe out 5% of net worth at each bank

- fast recaps in competitive and baseline economies (for different reasons)
- low passthrough to credit markets in nearly competitive economy

Dynamic experiment 2: real interest rate shock



Shock: drop \bar{r} from 2% to 0%, persistence of $\rho_{\bar{q}} = 0.5$

- credit markets: competitive and baseline economies observationally equivalent
- nearly competitive economy features almost no passthrough
- degree of competition matters for MP transmission

Conclusion and future directions

Model: imperfect competition via relationships + financial frictions

- **CC** \implies today's pricing decisions affect tomorrow's loan demand
- **frictions** \implies banks can expend CC to smooth shocks
- aggregate demand depends on joint distribution of prices and relationships

Quantitative analysis: estimate demand parameters using micro-data

- **cross-section:** endogenous life cycle, corr. b/w net worth, markups, CC
- **dynamics:** sluggish recovery, muted impact, greater persistence
- Extent of passthrough & dynamics nonlinear in the degree of competition

On deck: hone in on validation, GE, implications for financial stability

Thank you!

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Appendix

1. Fixed Relationship Intensity: $\rho_q = 0$, “local monopolist”

$$\Pi_t = \epsilon^{-1}(q\ell', q) \times \frac{\bar{q}}{q_t} \pi \mathbb{E}_t [(\psi^{-1})'(e_{t+1})]$$

2. Perfect Competition: $\epsilon^{-1} = \rho_q = 0$

$$\Pi_t = 0$$

Let the distribution of banks over states be denoted $m(x)$. This distribution evolves according to

$$T^* m(n', s') = \pi \int \mathbf{1} [n' = z' g_\ell(n, s) + g_a(n, s), s' = (1 - \rho) g_q(n, s) g_\ell(n, s) + \rho s] f(z') dm(n, s)$$

for continuing firms and

$$T^* m(x) = (1 - \pi) \bar{m}(x),$$

where $\bar{m}(x)$ is the distribution of entering banks (0 net worth, 0 customer capital)

- borrowers are indifferent about loan sourcing: care only about L'

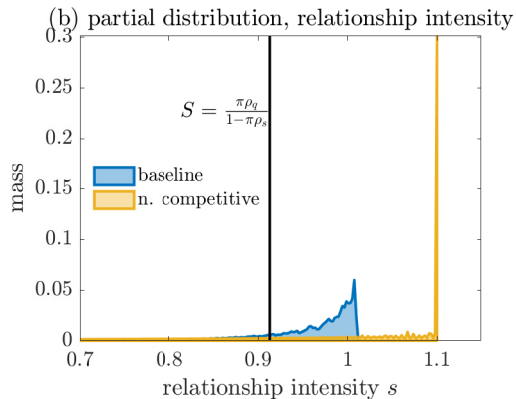
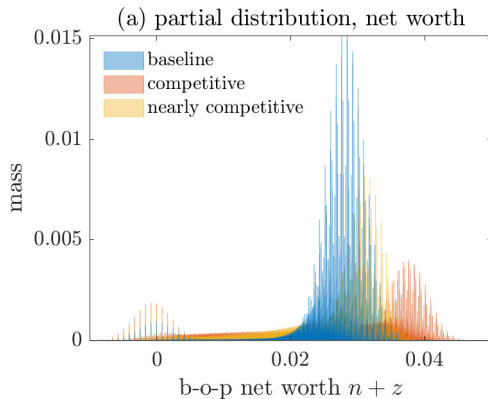
$$L'(R) = \kappa w \left[\frac{\alpha/w}{1 + \kappa(\bar{q}R - 1)} \right]^{\frac{1}{1-\alpha}}$$

Note that this is the same as baseline with $R = \tilde{R}$

- banks choose ℓ' taking $q = 1/R$ as given:

$$\begin{aligned} V(n, z) &= \max_{e, \ell', d'} e + \bar{q}\pi \mathbb{E}[V(n', z')] \\ \text{subject to: [budget]} & \quad q\ell' + \psi(e) \leq n + z + \bar{q}^d d' \\ \text{[net worth dynamics]} & \quad n' = \ell' - d' \\ \text{[capital requirement]} & \quad \bar{q}^d d' \leq (1 - \chi)q\ell' \end{aligned}$$

Distributions across models



All models have lots of compression in both net worth and customer capital

- low ϕ : more dispersion in both n (to left) and s distributions

Data: FR Y-14Q, schedule H.1

- Focus on new loans only (originated in the last 4 quarters)
- Criteria for inclusion:
 - Non-syndicated
 - US dollars
 - Non-missing TIN with US address
 - Not in NAICS 52 (finance) or 92 (government)
 - Loan has positive interest rate and committed exposure
- Three definitions of a “firm”:
 1. Baseline: TIN
 2. Degryse et al. (2019): ISL, CBSA \times size decile \times 3-digit NAICS

Procedure: switching vs. non-switching loans

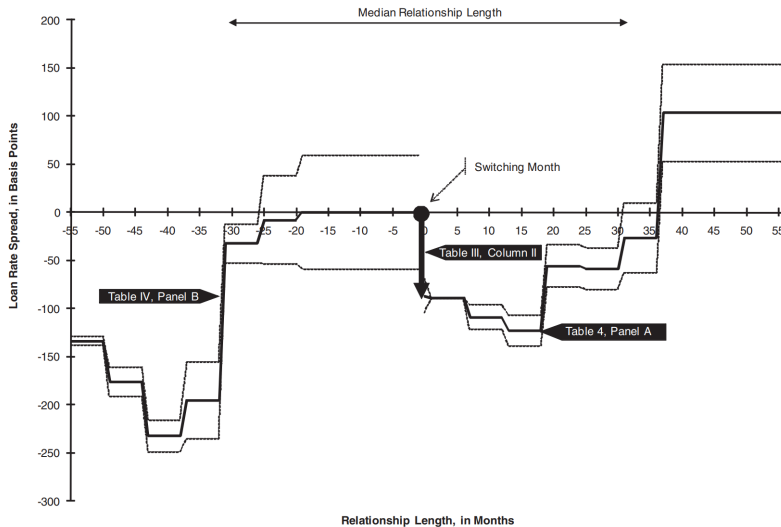
Goal: **match** switching vs. non-switching loans on a set of observables and compare spreads, following Ioannidou and Ongena (2010)

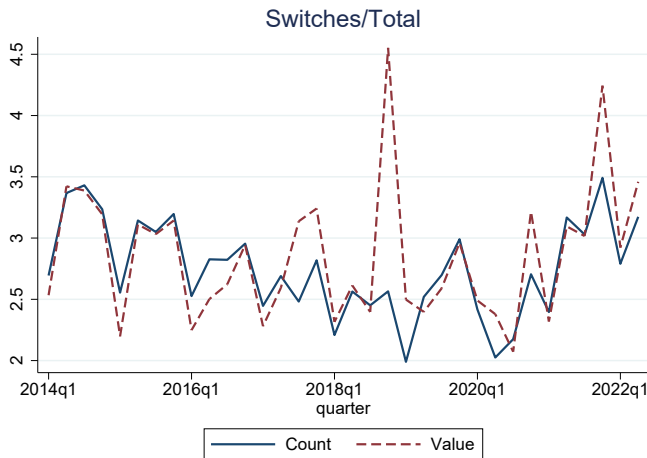
- 1. identify switches:** new loan from bank j from whom firm i has not borrowed in past $N = 4$ quarters (may overstate: unbalanced panel, 1\$ M threshold, loan sales)
- 2. form matched pairs:** match switching and non-switching loans on: (i) quarter; (ii) bank; (iii) quarter of origination; (iv) loan maturity; (v) loan size (percentile); (vi) default probability (percentile); (vii) loan type; (viii) variable v. fixed IR
 - more non-switches than switches \implies resample non-switches to pair each switch
- 3. compare spreads:** for each matched pair k , regress

$$\text{spread}_{kt} = \sum_{q=-Q}^Q \alpha_q \mathbf{1}[t = q] + \varepsilon_{kt} \text{ where } q \text{ is time since switch}$$

Ioannidou and Ongena (2010 JF) Figure 4

► back





Source: Y-14Q. Switches defined in terms of number of loans.

Loan is a switch if it's (i) new and (ii) from a bank with which the firm has had no relationship in past year

- definition follows Ioannidou & Ongena (2010)

Nature of the data \Rightarrow likely an upper bound:

- unbalanced panel: do not observe loans w/ balance < \$1M
- no small firms or small banks, where switching is less likely
- loans may enter/exit panel for many reasons

Compute Herfindahl-Hirschman Indices for local lending markets

- loan market defined as CBSA-quarter pair k
- The HHI is defined as

$$HHI_k = \sum_{i=1}^{N_k} \mu_{i,k}$$

where N_k is the number of banks present in market k and $\mu_{i,k}$ is the market share of bank i

- The DOJ considers an industry with a HHI above 0.18 to denote a “highly concentrated industry”

