The Cost of Capital and Misallocation in the United States

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Causes and Consequences of Misallocation

SITE

The Cost of Capital and Misallocation in the United States

Research question: How does dispersion in the cost of capital affect its allocation?

Methodological contribution:

- Adapt a standard dynamic corporate finance model to enable measurement using micro data
- Derive a sufficient statistic for misallocation using credit registry data

Empirical Results (US):

- Low levels of misallocation in normal times ($\approx 1\%$ of GDP)
- Losses from misallocation increased to 1.6% of GDP in 2020-2021
- Possibly tied to mispricing of credit due to credit market interventions

Related literature

- **Measuring misallocation:**
  - Seminal work: Restuccia and Rogerson (2008), Hsieh and Klenow (2009)
  - Contribution: use heterogeneity in funding costs to measure dispersion in MRPK
- **Heterogeneity in the cost of capital:**
  - Developing countries: Banerjee and Duflo (2005), Cavalcanti, Kaboski, Martins, and Santos (2024)
  - US: Gilchrist, Sim, and Zakrajsek (2013), David, Schmid, and Zeke (2022), Gormsen and Huber (2023, 2024), Faria-e-Castro, Jordan-Wood, and Kozlowski (2024)
  - Contribution:
    - Estimate firm cost of capital using credit registry data, correcting for loan characteristics, etc.
    - Derive and estimate sufficient statistic for misallocation

Outline

1. Model
2. Welfare and misallocation
3. Measurement with credit registry data
4. Empirical results

1. Model

Model in one slide

Borrowers

- Produce output $f(k_i, z_i)$
- Invest in capital k_i
- Long-term debt b_i
- Limited liability

Lenders

- Discount rate ρ_i
- Competitive pricing
- Recover $\phi_i k_i$ in default

Key question: how do heterogeneity in ρ_i and financial frictions distort the allocation of capital?

Model in one slide: math

Value of repayment:

$$V_i\left(k_i, b_i, z_i\right) = \max_{k_i', b_i'} \pi_i\left(k_i, b_i, z_i, k_i', b_i'\right) + \beta \mathbb{E} \overbrace{\left[\max\left\{V_i\left(k_i', b_i', z_i'\right), 0\right\} \middle| z_i\right]}^{\text{Limited liability}}$$

Firm profits:

$$\pi_i\left(k_i, b_i, z_i, k_i', b_i'\right) = f\left(k_i, z_i\right) + (1-\delta) k_i - k_i' - \theta b_i + Q_i\left(k_i', b_i', z_i\right)\left[b_i' - \left(1-\theta_i\right) b_i\right]$$

Price of debt:

$$Q_i\left(k_i', b_i', z_i\right) = \frac{\mathbb{E}\left\{\mathcal{P}_i\left(k_i', b_i', z_i'\right)\left[\theta_i + \left(1-\theta_i\right) Q_i\left(k_i'', b_i'', z_i'\right)\right] + \left(1 - \mathcal{P}_i\left(k_i', b_i', z_i'\right)\right) \overbrace{\frac{\phi_i k_i'}{b_i'}}^{\text{recovery}} \middle| z_i\right\}}{\underbrace{1+\rho_i}_{\text{lender discount rate}}}$$

Firm's cost of capital

Define the implicit interest rate paid by the firm as

$$1 + r_i^{firm} = \frac{\mathbb{E}\left[\mathcal{P}_i'(\theta_i + (1-\theta_i)Q_i') \middle| k_i', b_i', z_i\right]}{Q_i}$$

Lemma 1 (Firm's cost of capital)

The firm's cost of capital is:

$$1 + r_i^{firm} = \frac{1+\rho_i}{1+\Lambda_i} \qquad \Lambda_i := \frac{\mathbb{E}\left[(1-\mathcal{P}_i')\,\phi_i k_i'/b_i' \middle| k_i', b_i', z_i\right]}{\mathbb{E}\left[\mathcal{P}_i'\,(\theta + (1-\theta_i)Q_i') \middle| k_i', b_i', z_i\right]}$$

▷ *Proof*

Λ_i: financial frictions wedge that arises due to limited liability and partial recovery ϕ_i

- $\phi_i = 0$: no recovery after default, then $r_i^{firm} = \rho_i$
- If $\phi_i > 0$, then $\Lambda_i > 0$ and $r_i^{firm} < \rho_i$: borrower only takes into account repayment states

Marginal revenue product of capital (MRPK)

$$\underbrace{(1+r_i^{firm})\mathcal{M}_i}_{\text{cost of capital}} = \underbrace{\mathbb{E}[\mathcal{P}_i'(f_k(k_i', z_i') + 1 - \delta) | \, k_i', b_i', z_i]}_{\text{expected marginal revenue product of capital}}$$

where $\mathcal{M}_i$ captures the *price impact* of the firm's actions

$$\mathcal{M}_i := \frac{1 - \gamma_i \times \frac{Q_i \cdot b_i'}{k_i'} \times \frac{\partial \log Q_i}{\partial \log k_i'}}{1 + \gamma_i \times \frac{\partial \log Q_i}{\partial \log b_i'}}, \qquad \gamma_i := \frac{b_i' - (1-\theta_i) b_i}{b_i'}$$

- Heterogeneity in $r_i^{firm} \rightarrow$ heterogeneity in $MRPK_i$
- Approach: measure r_i^{firm} by measuring ρ_i and Λ_i

2. Welfare and misallocation

Aggregate economy and welfare

Decentralized Equilibrium:

$$Y^{DE} + (1-\delta)K^{DE} = \int_0^1 \mathbb{E}_t \left[\mathcal{P}^{DE}_{i,t+1}\left(f(k^{DE}_{i,t+1}, z_{i,t+1}) + (1-\delta)k^{DE}_{i,t+1}\right) + (1-\mathcal{P}^{DE}_{i,t+1})\cdot \phi_i k^{DE}_{i,t+1}\right] \mathrm{d}i$$

Planner's problem:

- Inner problem: redistribute $\{k_{i,t+1}\}_i$ taking exit decisions and K^{DE} as given ▷ full planner problem
- Lower bound on full misallocation:

$$\max_{\left\{k^*_{i,t+1}\right\}_i} \quad \int_0^1 \mathbb{E}_t \left[\mathcal{P}^{DE}_{i,t+1}\left(f(k^*_{i,t+1}, z_{i,t+1}) + (1-\delta)k^*_{i,t+1}\right) + (1-\mathcal{P}^{DE}_{i,t+1})\cdot \phi_i k^*_{i,t+1}\right] \mathrm{d}i$$

$$\text{s.t.} \quad \int_0^1 k^*_{i,t+1}\mathrm{d}i = K^{DE}_{t+1}$$

Social return on capital

- In equilibrium:

$$(1 + r_{i,t}^{firm})\mathcal{M}_{i,t} = \mathbb{E}_t[\mathcal{P}_{i,t+1}^{DE}(f_k(k_{i,t+1}^{DE}, z_{i,t+1}) + 1 - \delta)]$$

- Define the social marginal product of capital at firm i, $r_{i,t}^{social}(k_{i,t+1})$

$$1 + r_{i,t}^{social}(k_{i,t+1}) \equiv \mathbb{E}\left[\mathcal{P}_{i,t+1}^{DE}\left(f_k\left(k_{i,t+1}, z_{i,t+1}\right) + 1 - \delta\right) + \left(1 - \mathcal{P}_{i,t+1}^{DE}\right)\phi_i\right]$$

social return takes into account recovery in case of default

- Planner Optimality: at $\{k_{i,t+1}^{*}\}$ the planner **equalizes** $r_{i,t}^{social}(k_{i,t+1}^{*})$ across firms

- Equilibrium: dispersion on $r_{i,t}^{social}(k_{i,t+1}^{DE}) \rightarrow$ misallocation

Misallocation

Proposition 1 (Misallocation)

Misallocation can be measured with $\mathbb{E}\left[r_i^{social}\right]$ *and* $Var\left(r_i^{social}\right)$ *as*

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2}\cdot\mathcal{E}\cdot\log\left(1+\frac{Var\left(r_i^{social}\right)}{\left(\mathbb{E}\left[r_i^{social}\right]+\delta\right)^2}\right)$$

▷ *Proof*

- Extend Hughes and Majerovitz (2025) to a dynamic economy with default
- Set $\mathcal{E}=\frac{1}{2}$ and $\delta=0.06$ ▷ Calibration
- **Next:** show how to measure r_i^{social} using credit registry data

3. Measurement with credit registry data

Data: FR Y-14Q (Schedule H.1)

▷ cleaning details

- Quarterly loan-level panel on universe of loan facilities $> \$1$M
- Covers top 30/40 BHCs, 2014:Q4-2024Q4
- 91% of C&I undertaken by top 25 banks/ 55% of C&I undertaken by all commercial banks
- Detailed information on features of credit facilities
  - Origination date, size, maturity, interest rate/spread, probability of default, loss given default, fixed vs. floating, type of loan, etc.
- Focus on term loans issued to non-government, non-financial US companies
- Cannot include credit lines due to lack of information on fees.

Summary Statistics

▷ time series

	Mean	St. Dev.	p10	p50	p90
Interest rate	4.17	1.69	2.21	3.93	6.59
Maturity (yrs)	6.85	4.64	3.00	5.00	10.25
Real interest rate	2.38	1.24	0.88	2.33	3.99
Prob. Default (%)	1.42	2.37	0.19	0.82	2.85
LGD (%)	34.50	13.20	16.00	36.00	50.00
Loan amount (M)	10.77	68.82	1.11	2.55	22.64
Sales (M)	1,254.75	5,923.57	2.17	58.79	1,556.69
Assets (M)	1,770.85	8,956.85	1.06	35.51	1,792.00
Leverage (%)	72.03	24.57	42.57	71.17	100.00
Return on assets (%)	27.19	55.25	4.58	15.78	47.58
N Loans	62,686				
N Firms	38,586				
N Fixed Rate	31,540				
N Variable Rate	31,146				

Pricing term loans

The break-even condition for a lender with discount rate ρ_i is

$$1 = \sum_{t=1}^{T_i} \left\{ \frac{P_i^t \mathbb{E}_0\left[r_{i,t}\right] + P_i^{t-1}(1-P_i)\left(1-LGD_i\right)}{\left(1+\rho_i\right)^t \cdot \left(1+\bar{\pi}_t\right)} \right\} + \frac{P_i^{T_i}}{\left(1+\rho_i\right)^{T_i} \cdot \left(1+\bar{\pi}_t\right)}$$

- T_i: maturity
- P_i: repayment probability (constant over time)
- $\mathbb{E}_0[r_{i,t}]$: fixed rate or spread over benchmark rate (Gürkaynak et al., 2007) ▷ forward rates
- LGD_i: loss given default (constant over time)
- $\bar{\pi}_t$: expected inflation, $1+\bar{\pi}_t = \mathbb{E}_0\left[\prod_{j=0}^{t}(1+\pi_j)\right]$ (Cleveland Fed)
- $\Rightarrow$ Solve for lender's discount rate: ρ_i

Firm cost of capital

Lemma 2 (Firm cost of capital)

We can solve for Λ_i as

$$\Lambda_i = \frac{(1 - P_i)(1 - LGD_i)}{1 + \rho_i - (1 - P_i)(1 - LGD_i)}$$

and write the firm cost of capital as

$$1 + r_i^{firm} = (1 + \rho_i) - (1 - P_i)(1 - LGD_i)$$

▷ *Proof*

- $(1 - P_i)(1 - LGD_i) \simeq$ prob. of default event that does not result in a loss for the lender
- Measures pricing wedge between borrower and lender

Social cost of capital

Lemma 3 (Social cost of capital)

The social cost of capital can be written as:

$$\begin{aligned} 1 + r_i^{social} &= (1 + r_i^{firm})\mathcal{M}_i + (1 - P_i)(1 - LGD_i)lev_i \\ &= \underbrace{(1+\rho_i)\,\mathcal{M}_i}_{\text{lender discount rate}} + \underbrace{(lev_i - \mathcal{M}_i)\cdot(1-P_i)\cdot(1-LGD_i)}_{\text{wedge due to financial frictions}} \end{aligned}$$

- social cost of capital $\simeq$ lender discount rate + wedge due to financial frictions
- Wedge due to financial frictions:
  - **Lenders** care about average recovery per dollar of debt: $\phi_i(k_i)/b_i = \mathcal{M}_i(1 - LGD_i)$
  - **Planner** cares about marginal recovery: $\phi_i'(k_i) = (1 - LGD_i) \times lev_i$
  - Coincide when $lev_i = \mathcal{M}_i$

Sufficient statistic for misallocation

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2}\cdot\mathcal{E}\cdot\log\left(1+\frac{\mathsf{Var}\left(r_i^{social}\right)}{\left(\mathbb{E}\left[r_i^{social}\right]+\delta\right)^2}\right)$$

$$1+r_i^{social} = (1+\rho_i)\,\mathcal{M}_i + (lev_i - \mathcal{M}_i)\cdot(1-P_i)\cdot(1-LGD_i)$$

- Set $\mathcal{M}_i = 1$; reasonable approximation given our model ▷ Estimate $\mathcal{M}$
- Can measure misallocation directly with credit registry data!
- Dispersion in r_i^{social} comes from:
  1. Dispersion in lender's discount rate, ρ_i
  2. Dispersion in financial frictions wedge
  3. Covariance between ρ_i and financial frictions wedge

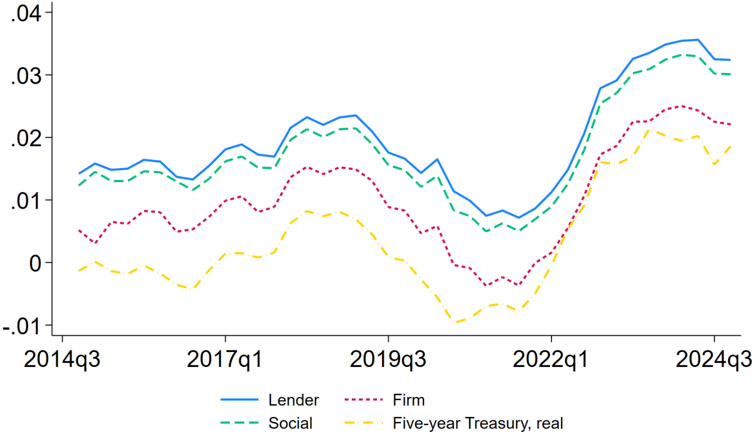
4. Empirical results

Estimates for lender discount rate, firm and social cost of capital

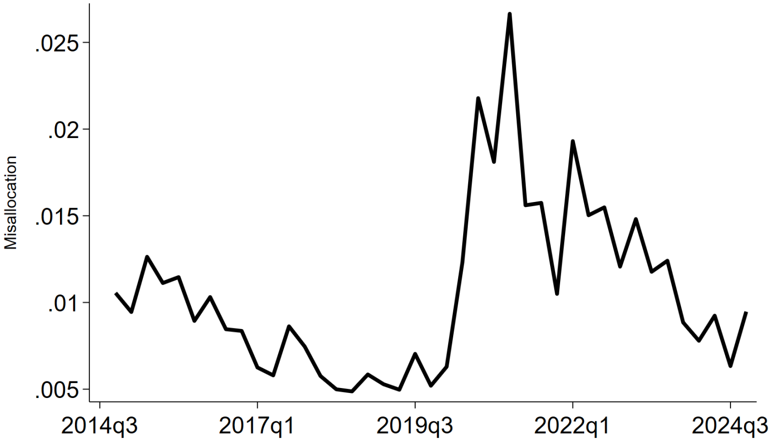
	mean	sd	p10	p50	p90
ρ (%)	1.86	1.53	0.38	1.88	3.60
r^{firm} (%)	0.93	2.65	-0.90	1.26	3.01
r^{social} (%)	1.65	1.73	0.09	1.72	3.45

- $\mathbb{E}\left[r_i^{social}\right] \approx \mathbb{E}\left[\rho_i\right]$
- Financial frictions: $\mathbb{E}\left[r_i^{social}\right] > \mathbb{E}\left[r_i^{firm}\right]$

Time series for average discount rate, firm and social cost of capital



Misallocation in the US, 2014-2024



- About 0.8% before 2020
- ↑ to 1.6% in 2020-2021
- ↓ to 1.2% in 2022-2024

The 2020–2021 increase in misallocation

1. Predominantly explained by changes in dispersion in ρ_i, rather than financial frictions ▷ details

2. Sharp rise in the coefficient of variation of ρ_i ▷ details

3. ρ_i dispersion $\uparrow$ due to increased dispersion of expected losses ▷ details

Relation to measures of ARPK

	(1) $\log(ARPK)$, Sales	(2) $\log(ARPK)$, EBITDA	(3) $\log(ARPK)$, Sales	(4) $\log(ARPK)$, EBITDA	(5) $\log(ARPK)$, VA
$\log(r^{social} + \delta)$	0.17***	0.26***	0.17**	0.15*	0.37***
	(0.03)	(0.04)	(0.07)	(0.08)	(0.07)
Observations	56,908	55,029	4,041	3,933	3,315
Adj. R2	0.28	0.22	0.68	0.52	0.60
NAICS4, Quarter FE	yes	yes	yes	yes	yes
Sample	Y-14	Y-14	Compustat	Compustat	Compustat

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

ARPK-based misallocation

▷ other measures

Focus on Compustat firms to make measures comparable

	$r^{social} + \delta$	$\frac{\text{Sales}}{\text{Capital}}$	$\frac{\text{EBITDA}}{\text{Capital}}$	$\frac{\text{Value Added}}{\text{Capital}}$
$Var(\log)$	0.01	0.18	0.24	0.20
Misallocation (%)	0.37	4.65	6.15	5.23

- Pros: does not require detailed data on firm financials (i.e., value added); applicable to most existing credit registries

- Cons: we measure the gain of reallocating capital only, holding fixed other inputs

Cross-country comparison

▷ details

	Aleem 1990 Pakistan	Khwaja & Mian 2005 Pakistan	Cavalcanti et al. 2024 Brazil	Beraldi 2025 Mexico	This paper 2025 United States
Years of data	1980–1981	1996–2002	2006–2016	2003–2022	2014–2024
$\mu(r_i)$, %	66.8	8.00	83.0	12.4	1.1
$\sigma(r_i)$, %	38.1	2.9	93.3	5.2	1.5
$\mu(1 - P_i)$, %	2.7	16.9	4.0	8.9	1.4
$\mu(1 - LGD_i)$, % (World Bank)	42.8	42.8	18.2	63.9	81.0
Implied misallocation, %	6.5	13.5	21.5	2.8	1.2

- Developing countries: higher mean and standard deviation of real interest rates
- U.S.: lower mean and standard deviation of interest rates, higher recovery
- Brazil: most extreme misallocation: 21.5%.

Conclusion

- Develop a framework to measure misallocation using credit registry data
  1. Standard macrofinance model as measurement device
  2. Sufficient statistic for capital misallocation
  3. Relies on standard credit registry variables as inputs (r, P, LGD, T, etc.)
- Application to U.S. credit registry data (FR Y-14Q)
  1. Estimate lender discount rates, firm-level cost of capital and social cost of capital
  2. Misallocation around 1% in normal times
  3. Rise in 2020-21, driven by increase in variance of expected losses
- **Work in progress:** including aggregate risk

Thank you

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Appendices

Proof: firm cost of capital

▷ back

$$
\begin{aligned}
\mathbb{E}_t\left[\frac{\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)}{Q_t}\right] &= (1+\rho)\,\frac{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]+\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]} \\
&= (1+\rho)\left(1+\frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}\right)^{-1} \\
&= (1+\rho)\,(1+\Lambda)^{-1}
\end{aligned}
$$

where

$$
\Lambda \equiv \frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}
$$

Full planner problem

▷ back

$$U^* = \max_{\{\{k_{i,t}(S^{t-1}),\omega_{i,t}(S^t)\}_i\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u\left(Y_t - I_t\right)$$

$$\text{s.t.} \qquad \omega_{i,t}\left(S^t\right) \in \{0,1\}\,\forall i$$

$$\omega_{i,t+1}\left(S^{t+1}\right) \leq \omega_{i,t}\left(S^t\right)\,\forall S^t \subset S^{t+1}, \forall i$$

Can separate into outer (dynamic) and inner (static) problems:

$$U^* = \max_{\left\{K_t,\{\omega_{i,t}(S^t)\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u\left(\left(\max_{\left\{\{k_{i,t}(S^{t-1})\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} Y_t\right) - I_t\right)$$

Rewrite inner problem as:

$$Y_t^*\left(K_t, \{\omega_{it}\}_{i\in[0,1]}\right) = \max_{\{k_{i,t}\}_{i\in[0,1]}} \int_0^1 \mathbb{E}_{t-1}\left[\omega_{it} \cdot f\left(k_{it}; z_{it}\right) - \left(1-\omega_{it}\right) \cdot \left(\left(1-\delta\right) k_{it} - \phi_i k_{it}\right)\right] di$$

$$\text{s.t.} \qquad K_t = \int_0^1 k_{it} di$$

Proof of Misallocation

▷ back

- Formally, planner's problem is now the same as solving $Y = \max_{\{k_i\}_i} \int_0^1 f_i(k_i) \mathrm{d}i$, where $f_i(k_i)$ is now expected output

- Apply Hughes and Majerovitz (2024), noting $\frac{dY}{dk} = r^{social} + \delta$

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log\left(1 + \frac{\text{Var}\left(r^{social}\right)}{\left(\mathbb{E}\left[r^{social}\right] + \delta\right)^2}\right)$$

- $\mathcal{E}$ is (negative) elasticity of output w.r.t. cost of capital $\left(r^{social} + \delta\right)$

Calibration of $\mathcal{E}$

▷ back

- $\mathcal{E}_i$ is the elasticity of expected output with respect to the cost of capital

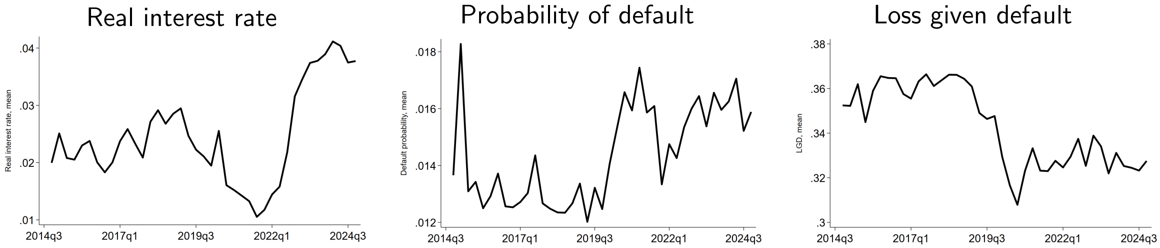
- Assume that $f(k, z) = z \cdot k^{\alpha}$ and there is no default, then

$$\mathcal{E} = \frac{\alpha}{1 - \alpha}$$

- $\alpha = \frac{1}{3}$ implies $\mathcal{E} = \frac{1}{2}$

Time series for averages: real interest rate, PD, LGD

▷ back



Data Cleaning and Sample Construction

▷ back

Sample period: We use FR Y-14Q Schedule H.1 data from 2014Q4 onward **Borrower Filters:**

- Drop loans without a Tax ID
- Keep only Commercial & Industrial loans to nonfinancial U.S. addresses
- Drop borrowers with NAICS codes:
  - 52 (Finance and Insurance), 92 (Public Administration)
  - 5312 (Real Estate Agents), 551111 (Bank Holding Companies)

Data Cleaning and Sample Construction

▷ back

Loan Filters:

- Drop loans with:
  - Negative committed exposure
  - Utilized exposure exceeding committed exposure
  - Origination after or maturity before report date
- Keep only "vanilla" term loans (Facility type = 7)
- Drop loans with:
  - Mixed-rate structures
  - Maturity outside 110 years
  - Implausible interest rates or spreads (outside 1st99th percentile, or $> 50\%$)
  - Missing or invalid PD/LGD values (outside $[0, 1]$)
  - PD $= 1$ (flagged as in default)

Forward interest rate expectations

▷ back

To estimate ρ_i for floating rate loans, need estimates of $\mathbb{E}_0[r_t] + s_i$

- Floating rate loans charge reference rate + spread
- Approximate LIBOR/SOFR using Treasury forward yield curve estimates (Gürkaynak et al., 2007)
- Average spread between SOFR and Treasury rates 2018-2025 $\simeq$ 2 basis points
- Assume expectations hypothesis: long rates reflect expected short rates
- Back out $\mathbb{E}_0[r_t] + s_i$ for each loan, using treasury forward rate plus loan's spread

Firm cost of capital: model

▷ back

$$Q_t = \frac{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta + (1-\theta)\, Q_{t+1}\right) + \left(1-\mathcal{P}_{t+1}\right)\phi k_{t+1}/b_{t+1}\right]}{1+\rho}$$

Note that

$$Q_t = Q_t^P + Q_t^D$$

$$Q_t^P = \frac{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta + (1-\theta)\, Q_{t+1}\right)\right]}{1+\rho}$$

$$Q_t^D = \frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k_{t+1}/b_{t+1}\right]}{1+\rho}$$

That is, we strip the bond into the payment in repay $\left(Q_t^P\right)$ and the payment in default $\left(Q_t^D\right)$. Then:

$$\Lambda = \frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k_{t+1}/b_{t+1}\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta + (1-\theta)\, Q_{t+1}\right)\right]} = \frac{Q_t^D}{Q_t^P}$$

Firm cost of capital: measurement

▷ back

The firm defaults with probability $(1-P)$ and the lender recovers $(1-LGD)$. Hence

$$Q_t^{D,data} = \frac{(1-P)(1-LGD)}{1+\rho}$$

For the payment portion notice that at issuance we have the following condition

$$1 = \sum_{s=1}^{T} \left[\frac{P^s \mathbb{E}_t \left[r_{t+s} \right] + P^{s-1} (1-P)(1-LGD)}{(1+\rho)^s} \right] + \frac{P^T}{(1+\rho)^T}$$

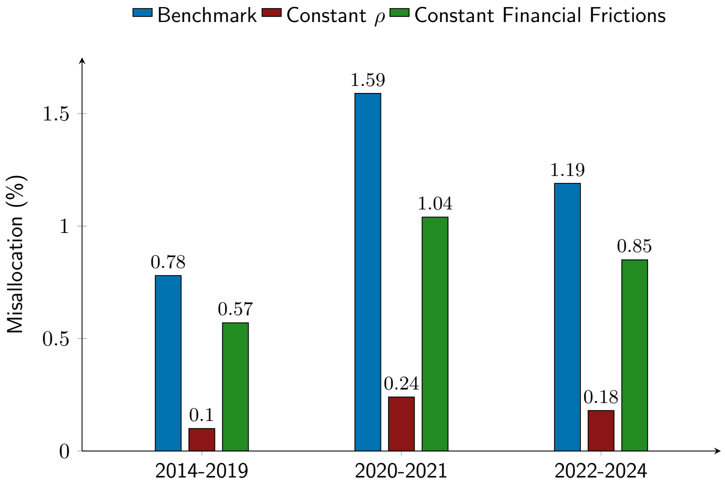
$$1 = \frac{(1-P)(1-LGD)}{1+\rho} + P \frac{\mathbb{E}_t \left[r_{t+1} \right]}{1+\rho} + \left(\sum_{s=2}^{T} \left[\frac{P^s \mathbb{E}_t \left[r_{t+s} \right] + P^{s-1} (1-P)(1-LGD)}{(1+\rho)^s} \right] + \frac{P^T}{(1+\rho)^T} \right)$$

So, we can define $Q_t^{P,data}$ as $1 = Q_t^{P,data} + Q_t^{D,data}$ so $Q_t^{P,data} = 1 - Q_t^{D,data}$. Finally

$$\Lambda^{data} = \frac{Q_t^{D,data}}{Q_t^{P,data}} = \frac{(1-P)(1-LGD)}{1+\rho-(1-P)(1-LGD)}$$

1. The 2020-21 rise in misallocation was driven by $\{\rho_i\}$

▷ details



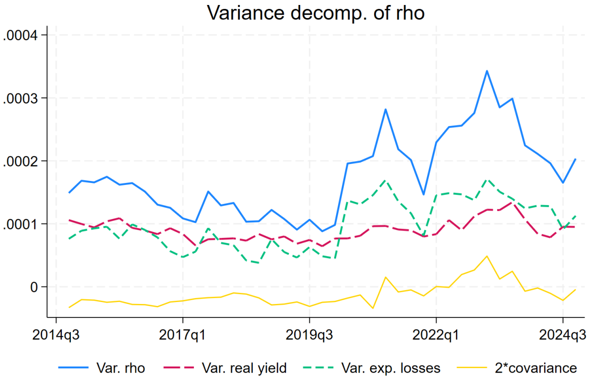
2. The CV of ρ_i increased during 2020-21

- As policy rates decreased in 2020-21, so did mean ρ_i
- Standard deviation of ρ_i increased during this period

3. Variance of ρ related to variance of expected losses

▷ details

$$\rho_i = \underbrace{\rho_i(P_i = 1)}_{\text{real yield}} + \underbrace{[\rho_i - \rho_i(P_i = 1)]}_{\text{exp. losses}}$$



- $\sigma(\rho) \uparrow$ due to $\sigma(\text{exp. losses}) \uparrow$
- $\sigma(\text{exp. losses}) \uparrow$ without $\sigma(r) \uparrow$
- Possibly tied to underpricing of risky loans, implicit guarantees, etc.

Decomposing misallocation

▷ back

Counterfactual I: What if all lenders have the same $\bar{\rho}$?

$$1 + r_{social}^{cf,I} = \overline{(1+\rho)\mathcal{M}} + (lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)$$

Heterogeneity in r_{social}^{cf} $\rightarrow$ Misallocation due to financial frictions

Counterfactual II: what if we equalize financial frictions?

$$1 + r_{social}^{cf,II} = (1+\rho)\,\mathcal{M} + \overline{(lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)}$$

Heterogeneity in r_{social}^{cf} $\rightarrow$ Misallocation due to heterogeneous cost of capital

Details: ρ decomposition

▷ back

- The "real yield" is the implied ρ_i^* when $P_i = 1$

$$1 = \sum_{t=1}^{T_i} \left\{ \frac{\mathbb{E}_0\left[r_{i,t}\right]}{\left(1+\rho_i^*\right)^{t} \cdot \mathbb{E}_0\left[\prod_{j=0}^{t}(1+\pi_j)\right]} \right\} + \frac{1}{\left(1+\rho_i^*\right)^{T_i} \cdot \mathbb{E}_0\left[\prod_{j=0}^{T_i}(1+\pi_j)\right]}$$

- Real yield independent of P_i, LGD_i

- Only affected by losses through the contractual rate r

Variance decomposition

- Decompose total variance in: time, firm, bank, and error
- Keep firms with 5 or more securities

	Time	Bank	Firm	Loan
Contractual rate	71.88	1.63	13.45	13.04
Lender discount rate, ρ	61.94	3.08	14.02	20.96
Firm cost of capital, r^{firm}	33.23	4.25	20.12	42.4
Social cost of capital, r^{social}	53.84	3.87	16.21	26.08
N Firms	1681			
N Loans	14738			

Table: Variance decomposition of interest rates and cost of capital (ρ, r^{firm}, and r^{social})

Estimating $\mathcal{M}$

▷ back

$$\mathcal{M} = \frac{1 - \gamma \times \frac{Qb'}{k'} \times \frac{\partial \log Q}{\partial \log k'}}{1 + \gamma \times \times \frac{\partial \log Q}{\partial \log b'}}$$

Given estimates for the function Q, γ, and firm leverage Qb'/k' we can compute $\mathcal{M}$

1. Loans are modeled as perpetuities that decay at a geometric rate θ, we can write Q as the present value of all future payments, discounted at the real interest rate r:

$$Q = \frac{\theta + (1-\theta)Q}{1+r} = \frac{\theta}{r+\theta}$$

   r is directly observed in the data, and we can approximate $\theta = 1/T$

2. Guess a functional approximation $Q(z, k, b, \rho)$
3. Estimate $\log \hat{Q}(z, k, b, \rho)$ for every loan origination; compute partial derivatives
4. At steady state, $\gamma = \theta = 1/T$

Estimating $\mathcal{M}$: Q elasticities

▷ back

- We approximate (the log of) Q as a polynomial of firm capital, borrowing, productivity and ρ
- Capital: tangible assets
- Borrowing: total debt owed by the firm at loan origination
- Productivity: sales over tangible assets
- Approximation:

$$\begin{aligned}\log Q_i = {} & \alpha + \beta_k \log k_i + \beta_b \log b_i + \beta_z \log z_i + \beta_\rho \rho_i \\ & + \beta_{k,k} (\log k_i)^2 + \beta_{k,b} \log k_i \times \log b_i + \beta_{k,z} \log k_i \times \log z_i + \beta_{k,\rho} \log k_i \times \rho_i \\ & + \beta_{b,b} (\log b_i)^2 \beta_{b,z} \log b_i \times \log z_i + \beta_{b,\rho} \log b_i \times \rho_i \\ & + \beta_{z,z} (\log z_i)^2 \beta_{z,\rho} \log z_i \times \rho_i + \beta_{\rho,\rho} (\rho_i)^2 \epsilon_i\end{aligned}$$

- Compute the partial derivatives of $\log Q$ with respect to investment and borrowing.

Estimating $\mathcal{M}$: results

▷ back

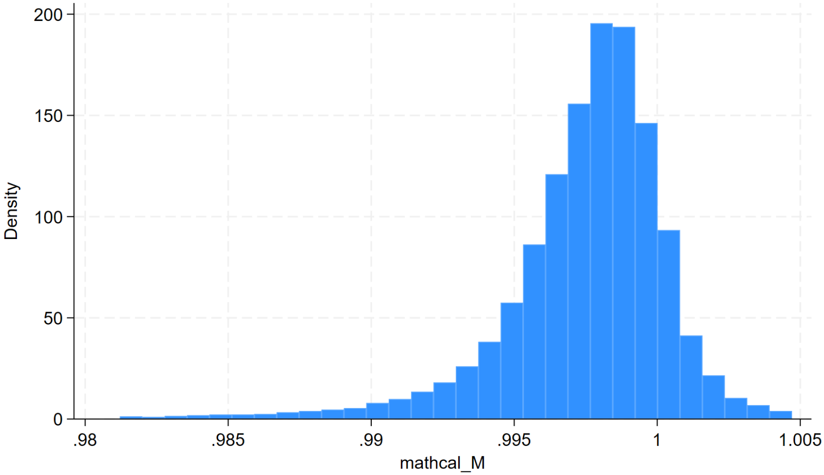


Figure: Histogram for estimated $\mathcal{M}_i$

Risk Premia & Aggregate Shocks

▷ back

- Alternative hypothesis: Rise in ρ reflects higher **risk premia** as lenders demand extra compensation amid extreme uncertainty (e.g. COVID-19).
- Firms differ in exposure to aggregate shocks $\Rightarrow$ heterogeneous risk premia need not imply misallocation (David et al., 2022).
- Our framework is steady-state $\Rightarrow$ cannot model time-varying aggregate shocks or risk-premium spikes.
- Data contradict the risk-premia story:
  - Average ρ **falls** from 3.6% (2014-19) to 2.7% (2020-21).
  - Skewness becomes **more negative**: $-2.6 \rightarrow -3.5$ (left tail thickens).
- Interpretation: Risk premia likely **declined**, perhaps owing to explicit/implicit policy guarantees.

ARPK-based misallocation measures

▷ back

	(1) $\log(ARPK)$, sales	(2) $\log(ARPK)$, EBITDA	(3) $\log(ARPK)$, sales	(4) $\log(ARPK)$, EBITDA
$\log(r^{social} + \delta)$	0.19***	0.26***	0.20***	0.17**
	(0.03)	(0.04)	(0.08)	(0.09)
Observations	56,912	55,033	4,064	3,963
Adj. R2	0.25	0.18	0.62	0.46
NAICS3, Quarter FE	yes	yes	yes	yes
Sample	Y-14	Y-14	Compustat	Compustat
$Var[\log(ARPK)]$	2.17	1.72	0.31	0.37
Misalloc., $ARPK$, %	72.21	53.77	8.03	9.73
$Var[\log(r^{social} + \delta)]$	0.04	0.04	0.02	0.02
Misalloc., r^{social}, %	1.09	1.09	0.41	0.41

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Details on cross-country comparison

▷ back

- Recovery rates and inflation rates from the World Bank
- For a fixed real interest rate, ρ has a closed-form:

$$1 + \rho_i = P_i \left(1 + r_i\right) + \left(1 - P_i\right)\left(1 - LGD_i\right)$$

- Assume all loans have the same maturity:
  1. Obtain mean real rate by subtracting average realized inflation from mean nominal rate
  2. Inflation should not affect standard deviation of nominal rates (or spreads)
- Assume all loans have the same P_i, LGD_i, equal to the average
- Approximate $r_i^{social} \simeq \rho_i$ and compute misallocation using our formula:

$$\log(Y^*/Y^{DE}) = \frac{1}{2}\mathcal{E}\log\left(1 + \frac{Var(\rho_i)}{(\mathbb{E}[\rho_i] + \delta)^2}\right)$$

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