Problem 2.1 10 points:

Demonstrate that

$$\frac{\partial S}{\partial t} + \underline{w} \cdot \underline{\nabla} S = 0 \; on \; S = 0$$

can be obtained from $S(\underline{x}, t) = 0$ evaluated at *t+dt* (using Taylor's series expansion in a vector form).

Solution:

Using Taylor's series expansion in a vector form

(Eq.36 in http://mathworld.wolfram.com/TaylorSeries.html)

Let $\underline{a} = (\underline{w}\mathrm{dt}, \mathrm{dt})$ and $\nabla_{\mathrm{r}'} = (\underline{\nabla}, \frac{\partial}{\partial \mathrm{t}})$

$$S(\underline{x} + \underline{w}dt, t + dt) = S(\underline{x}, t) + (\underline{w}dt \cdot \underline{\nabla})S + \left(dt \cdot \frac{\partial}{\partial t}\right) S + \cdots$$

Truncate the higher order terms:

$$S(\underline{x} + \underline{w}dt, t + dt) = S(\underline{x}, t) + \underline{w} \cdot \underline{\nabla} S dt + \left(dt \cdot \frac{\partial S}{\partial t}\right)$$

Since $S(\underline{x}, t) = 0$

$$S(\underline{x} + \underline{w}dt, t + dt) = \left(\underline{w} \cdot \underline{\nabla} S + \frac{\partial S}{\partial t}\right) dt$$

Assume $dt \to 0$,

$$S(\underline{x} + \underline{w}dt, t + dt) \to S(\underline{x}, t)$$

Therefore $S(\underline{x} + \underline{w}dt, t + dt) = 0$, thus

$$\left(\underline{w} \cdot \underline{\nabla} S + \frac{\partial S}{\partial t}\right) dt = 0$$

$$\Longrightarrow \frac{\partial S}{\partial t} + \underline{w} \cdot \underline{\nabla} S = 0$$