Homework #8 (2 problems, 30 points)

due: 11:45am on April 24th Friday 2020.

Problem 8.1 [20]	Problem 8.2 [10]	Total

The following parameters are given for both Problem 8.1 and Problem 8.2:
C_D = 0.47 for spherical bubble, $\rho_l = 998.21\ \frac{kg}{m^3}$, and $\rho_g = 1.205\ \frac{kg}{m^3}$.

Problem 8.1 20 points:

Consider a steady-state air bubble rise in infinite standing water.

a) Develop an expression for the rise velocity (also the relative velocity) v_r based on drag/buoyancy force balance.

b) Assuming the bubble preserved spherical shape, plot the rise velocity dependence on bubble diameter. Use the bubble diameter range from 0.5mm to 25mm.

c) Assume variable drag coefficient according to this expression:

$$C_D = \sqrt{\left[\frac{16}{Re_b}\left(1 + \frac{2}{1 + \frac{16}{Re_b} + \frac{3.315}{\sqrt{Re_b}}}\right)\right]^2 + \left(\frac{4\,Eo}{Eo + 9.5}\right)^2}$$

Use bubble Reynolds number (Re_b) and Eotvos number based on bubble diameter and air/water surface tension. Re-plot the rise velocity dependence.

Note: use v_r in Re_b and solve the implicit equation iteratively. Submit your code.

d) Compare the results in b) and c) and discuss them.

Problem 8.2 10 points:

Consider a two-phase air/water bubbly flow at atmospheric conditions. Assume that there are three groups of spherical bubbles:

1. Group 1: Mean diameter of 1 mm and volume fraction of 3%
2. Group 2: Mean diameter of 1.5 mm and volume fraction of 1%
3. Group 3: Mean diameter of 2.0 mm and volume fraction of 1%

Use the two-phase turbulent viscosity contribution proposed by Sato & Sekoguchi (1975)

$$\upsilon_{2\phi} = 0.6 D_{dv} \alpha_{dv} |v_r|$$

to evaluate:

#8.1

(a)

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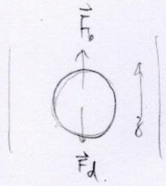
#8.1

(a) bubble of radius r_b rising in stagnant water: accelerated by buoyancy and drag force. assume it a perfect sphere. $\vec{g} = -9.8\ m/s^2$ ($-\hat{z}$ dir)



$$V_b = \frac{4}{3}\pi r_b^3$$

$$\vec{F}_b = (\rho_g - \rho_\ell) V_b \vec{g}$$

$$\vec{F}_d = -\frac{1}{8}\rho_\ell C_D |\vec{v}_r| \vec{v}_r A_x, \quad A_x = \pi r_b^2, \quad \vec{v}_r = \vec{v}_b - \vec{v}_\ell = \vec{v}_b \ (\because \vec{v}_\ell = 0)$$

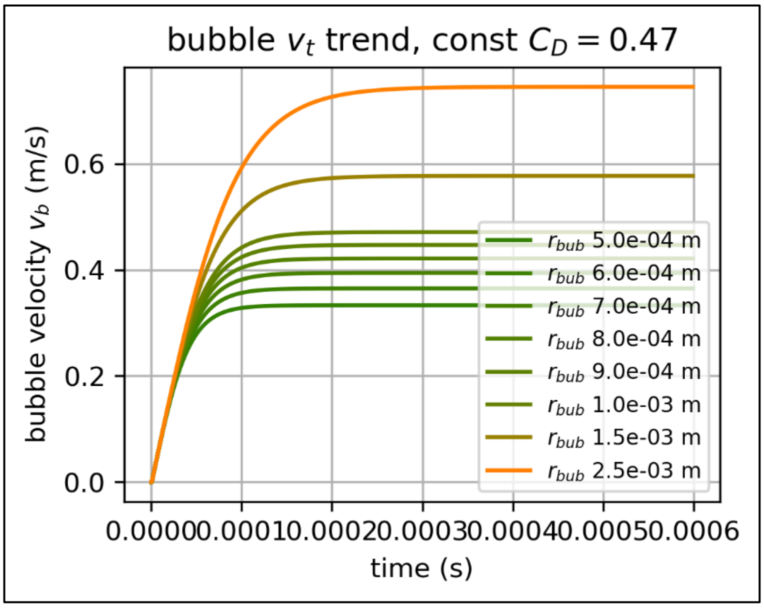
by $\vec{F} = m\vec{a}$, $m = \rho_g V_b = \frac{4}{3}\rho_g \pi r_b^3$, $\vec{F} = \vec{F}_b + \vec{F}_d$.

$$\therefore \vec{a} = \frac{d\vec{v}_b}{dt} = \frac{\vec{F}}{m} = \frac{(\rho_g - \rho_\ell) V_b \vec{g} + \left(-\frac{1}{8}\rho_\ell C_D |\vec{v}_r| \vec{v}_r A_x\right)}{\frac{4}{3}\rho_g \pi r_b^3} = \frac{\Delta\rho V_b \vec{g} + \left(-\frac{1}{8}\rho_\ell C_D |\vec{v}_b| \vec{v}_b A_x\right)}{\frac{4}{3}\rho_g \pi r_b^3}$$

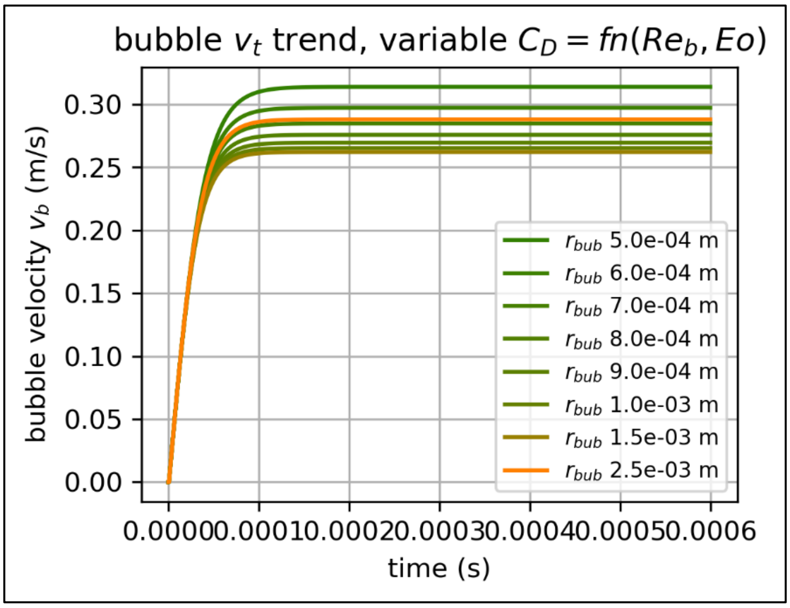
$$\therefore \vec{v}_b = \int_0^t \vec{a}(t')\,dt' = \int_0^t \frac{\Delta\rho V \vec{g} - \frac{1}{8}\rho_\ell C_D |\vec{v}_b| \vec{v}_b A_x}{\frac{4}{3}\rho_g \pi r_b^3}\,dt.$$ an implicit equation of $\vec{v}_b$.

(b), (c), (d), see attached figures and code.

(b)



(c) (d)



In the case (b) where drag coefficient has no dependence on relative velocity and bubble deformation, the time needed for bubbles to reach terminal velocity increases as size increases. Moreover, the magnitude of terminal velocity is also positively correlated to bubble size. In case (d), where drag coefficient has dependency on relative velocity, bubble size and shape, the time required for bubbles to reach terminal velocity became more uniform. Magnitude of terminal velocity also became more concentrated between 0.25 to 0.30 m/s. Larger bubbles are the group effected the most. The effect is expected to be caused by the dependence of C_D on bubble size. C_D is roughly proportional to D_{bub}^n, where n is between due to the $\frac{1}{Re}$ and

$\left(\frac{4Eo}{Eo+9.5}\right)^2$ dependence in the equation. Therefore, larger bubbles tend to have larger drag coefficient and results in faster arrival of terminal velocity.

#8.2

given 2ϕ turbulence viscosity : $\nu_{2\phi} = 0.6\, D_{dv} \alpha_{dv} |\nu_r|$ $\frac{m^2}{s}$

(a) given $\nu_{cl} = 1\times10^{-6}$ $\frac{m^2}{s}$, 3 bubble groups:

$$\begin{cases} g1: \bar{D} = 1\times10^{-3}m, & \alpha = 0.03 \\ g2: \bar{D} = 1.5\times10^{-3}m & \alpha = 0.01 \\ g3: \bar{D} = 2\times10^{-3}m & \alpha = 0.01 \end{cases}$$

, define $\tilde{\nu}_{2\phi} = \frac{\nu_{2\phi,g}}{\nu_{cl}}$ as normalized $\nu_{2\phi}$.

$$\tilde{\nu}_{2\phi,1} = \frac{0.6\times1\times10^{-3m}\times0.03|\nu_r|}{10^{-6}\frac{m^2}{s}} = 18|\nu_r|$$

$$\tilde{\nu}_{2\phi,2} = \frac{0.6\times1.5\times10^{-3m}\times0.01|\nu_r|}{10^{-6}\frac{m^2}{s}} = 9|\nu_r|$$

$$\tilde{\nu}_{2\phi,3} = \frac{0.6\times2\times10^{-3m}\times0.01|\nu_r|}{10^{-6}\frac{m^2}{s}} = 12|\nu_r|$$

$\therefore$ total $\tilde{\nu}_{2\phi} = \sum_g \tilde{\nu}_{2\phi,g} = 39|\nu_r|$. , contribution from each g:

$$\begin{cases} g1 = \frac{18}{39} = 46.15\% \\ g2 = \frac{9}{39} = 23.075\% \\ g3 = \frac{12}{39} = 30.775\% \end{cases}$$

(b) assume single group, $\alpha = 0.05$, containing g1 g2 & g3.

the equivalent mean bubble diameter $\bar{D}_{eqv}$ is obtained by use α_g as weighting

$$\therefore \bar{D}_{eqv} = \frac{\alpha_1\bar{D}_1 + \alpha_2\bar{D}_2 + \alpha_3\bar{D}_3}{\alpha_{tot}} = \frac{0.03\times10^{-3m} + 0.01\times(1.5\times10^{-3} + 2\times10^{-3})^m}{0.05} = 1.3\times10^{-3m}$$

$$\therefore \tilde{\nu}_{2\phi,eqv} = \frac{0.6\times1.3\times10^{-3m}\times0.05|\nu_r|}{10^{-6}\frac{m^2}{s}} = 39|\nu_r|$$

the $\tilde{\nu}_{2\phi,eqv}$ is same as (a) since the calculation in (b) is essentially identical to (a). But this also means different weighting measures can yield diff. $\tilde{\nu}_{2\phi,eqv}$