MAE/NE-577

Homework #2 (5 problems)

due: February 17th, 2023.

Problem 2.1	Problem 2.2	Problem 2.3	Problem 2.4	Problem 2.5	Total

Problem 2.1 10 points:

Demonstrate that

$$\frac{\partial S}{\partial t} + \underline{w} \cdot \underline{\nabla} S = 0 \; on \; S = 0$$

can be obtained from $S(\underline{x}, t) = 0$ evaluated at $t + dt$ (using Taylor's series expansion in a vector form).

Problem 2.2 15 points:

Consider a single bubble (D=7 mm) in a standing fluid with large temperature gradient ($\frac{dT}{dx} = G$) in zero gravity conditions.

Assume that Eotvos rule can be used to estimate water surface tension as a function of temperature:

$$\gamma = 0.07275 N/m \cdot (1 - 0.002 \cdot (T - 291K))$$

(a) Compute the pressure drop between gas inside and liquid outside the bubble (Assume that the bubble is perfectly spherical with diameter D if placed in constant temperature conditions at T = 320 K.) (3 points)

> Note: use Eq. (33) from the notes to compute the pressure drop. Note that the curvature value can be assumed as $\kappa = 2/R$.

(b) For a given pressure drop, the curvature of the bubble surface will locally change due to different local surface tension. Derive an expression for the curvature if the temperature of the fluid changes (6 points):

- from 270K to 320K across the bubble
- from 270K to 370K across the bubble

> Note: use the pressure drop obtained in (a) and derive the equation for the bubble shape in 2D for demonstrating the temperature increase with θ, i.e., from T_1 to T_2:
>
> for $\theta \in [0, \pi]$, $T(\theta) = T_1 + (T_2 - T_1)\frac{\theta}{\pi}$;
>
> for $\theta \in [\pi, 2\pi]$, $T(\theta) = T_2 - (T_2 - T_1)\frac{\theta - \pi}{\pi}$.

(c) Use your favorite math tool to plot the 2D bubble shape for both cases in part **(b)** as well as compare to the spherical counterpart. **Discuss** the results (6 points).

Problem 2.3 15 points:

You are tasked to design a multiphase flow experiment which will help develop a cooling/heating system for a laboratory on a Moon surface. The experiment is to be performed in Earth gravity ($g = 9.81\ m/s^2$) while the results to be used on Moon ($g_M = 1.62\ m/s^2$)

If the water/vapor coolant in the heat exchanger will operate at 240 kPa and saturation temperature on Moon, provide the fluid properties necessary for the corresponding Earth-based experiment to maintain the same dimensionless parameters (assume the bubble characteristic length ratio between Earth and Moon is $\mathrm{L_M} = 3\mathrm{L_E}$):

(a) Eo number (5 points)

(b) Mo number (5 points)

(c) Find a fluid and pressure/temperature conditions which will work for your experiment on Earth with the closest possible match of either (or both) of the dimensionless numbers. (5 points)

Problem 2.4 10 points:

Derive Stokes' momentum equation from N-S equation by assuming the Re << 1. Specifically demonstrate why the external force term cannot be neglected while the advection term can.

Note: N-S equation:

$$\frac{\partial \mathrm{u}}{\partial \mathrm{t}} + (\underline{\mathrm{u}} \cdot \nabla)\underline{\mathrm{u}} = -\frac{1}{\rho}\nabla \mathrm{p} + \upsilon \nabla^2 \underline{\mathrm{u}} + f$$

Problem 2.5 15 points:

Estimate the number of bubbles which can be simulated on the fastest supercomputer in the world (see https://www.top500.org/lists/top500/2022/11/).

Assume the following parameters:

1) Each computing core can process:

   (a) 2,048 **(b)** 4,096 and **(c)** 8,192 nodal points.

   (d) Assume that to compute every 1,000,000 nodal points, about 3.25 TFLOPS of "Rmax" power is required as listed in the Top500.org. Note that 1PFLOPS (peta-**fl**oating-point **o**perations **p**er **s**econd) = 1,000 TFLOPS (tera-) = 1,000,000 GFLOPS (giga-) = 10^{15} FLOPS.

2) Mesh resolution for each bubble is **(a)** 10 **(b)** 20 **(c)** 30 nodal points across the diameter.

3) The computational domain is a cube with uniform grid resolution in each direction: domain size is dictated by available computing power.

4) The bubble concentration is **(a)** 2% **(b)** 4% and **(c)** 8%.

5) Only: **(a)** 25% **(b)** 50% and **(c)** 75% **(d)** 100% of computing cores of the fastest machine are utilized for the computation.

(a) Provide the equations you used to make your estimates. Use the notation listed in the following table: (3 points)

n_{bub}	number of bubbles
N_c	total cores of a supercomputer
n_0	nodal points each computing core can process
n_d	nodal points across the diameter of each bubble
v_f	bubble concentration
a_v	availability of computing cores

(b) Results for all combinations of 1(a,b,c,d) – 2(a,b,c) – 4(a,b,c) – 5(a,b,c,d) must be determined. (6 points)

(c) Discuss the results and think of real multiphase flow systems (and their size) which can be simulated today using DNS approach. How is this affected by the choice of the parameters in 1)-5)? (6 points)

Extra credit (5 points): repeat the calculation for the fastest machine in top500 in June 2013 for case 1d – 2b – 4a – 5d and use it to estimate the computation power (Cores, Rmax, Number of nodal points in 1d, and the number of bubbles) in 2033 (assuming that the rate of growth in the next 10 years is the same as that in the last 10 years).