Problem 8.1 20 points:

Consider a steady-state air bubble rise in infinite standing water.

a) Develop an expression for the rise velocity (also the relative velocity) v_r based on drag/buoyancy force balance.

b) Assuming the bubble preserved spherical shape, plot the rise velocity dependence on bubble diameter. Use the bubble diameter range from 0.5mm to 25mm.

c) Assume variable drag coefficient according to this expression:

$$C_D = \sqrt{\left[\frac{16}{Re_b}\left(1+\frac{2}{1+\frac{16}{Re_b}+\frac{3.315}{\sqrt{Re_b}}}\right)\right]^2 + \left(\frac{4\,Eo}{Eo+9.5}\right)^2}$$

Use bubble Reynolds number (Re_b) and Eotvos number based on bubble diameter and air/water surface tension. Re-plot the rise velocity dependence.

Note: use v_r in Re_b and solve the implicit equation iteratively. Submit your code.

d) Compare the results in b) and c) and discuss them.

Solution:

(a) Using force balance including drag force and buoyancy force,

$$F_b = (\rho_l - \rho_g)gV$$

$$F_D = \frac{1}{2}C_D A \rho_l {v_r}^2$$

$$F_b = F_D \implies (\rho_l - \rho_g)gV = \frac{1}{2}C_D A \rho_l {v_r}^2$$

where A is the cross-section area of the spherical bubble and V is its volume:

$$\mathrm{A} = \frac{\pi {D_{dv}}^2}{4}$$

$$V = \frac{\pi {D_{dv}}^3}{6}$$

$$v_r = \sqrt{\frac{4(\rho_l - \rho_g)D_{dv}\cdot \mathrm{g}}{3C_D\rho_l}}$$

(b)

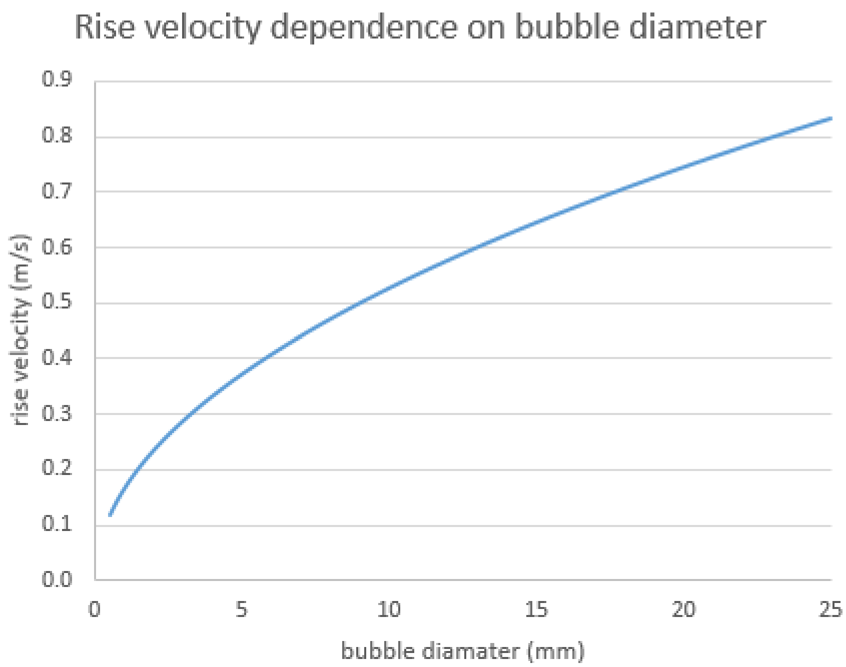
Assume room temperature and 1 atm

C_D = 0.47

$\rho_l = 998.2 \frac{\mathrm{kg}}{\mathrm{m}^3}$

$\rho_g = 1.205 \frac{\mathrm{kg}}{\mathrm{m}^3}$

Set the range of bubble diameter from 0.5mm to 25mm



(c)

$\mu_1 = 1.0 \cdot 10^{-3}\ \mathrm{Pa \cdot s}$

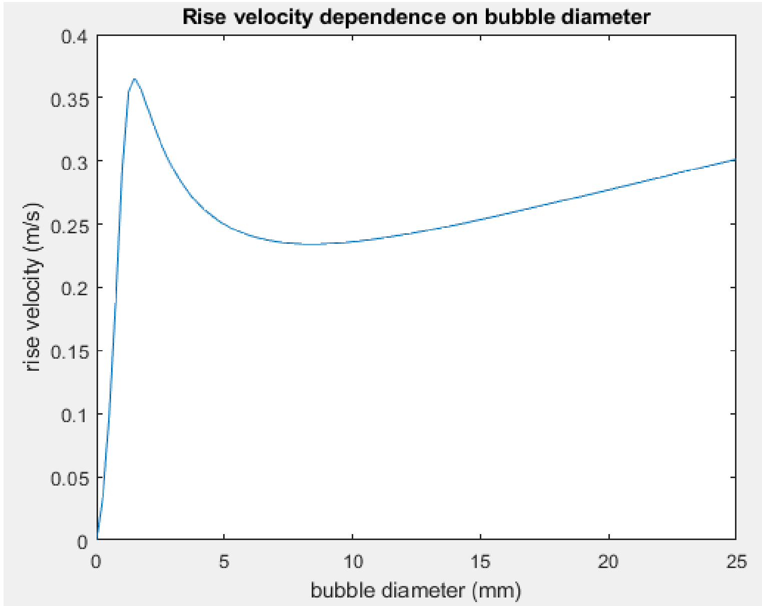
$\gamma = 7.27 \cdot 10^{-2}\ \mathrm{N/m}$

$$Eo = \frac{(\rho_l - \rho_g) \cdot g \cdot D_g^2}{\gamma}$$

$$Re_b = \frac{\rho_l \cdot D_g \cdot v_r}{\mu_1}$$

$$C_D = \sqrt{\left[\frac{16}{Re_b}\left(1 + \frac{2}{1 + \frac{16}{Re_b} + \frac{3.315}{\sqrt{Re_b}}}\right)\right]^2 + \left(\frac{4\,Eo}{Eo + 9.5}\right)^2}$$

Plug into the expression of rise velocity and iterate to solve v_r.



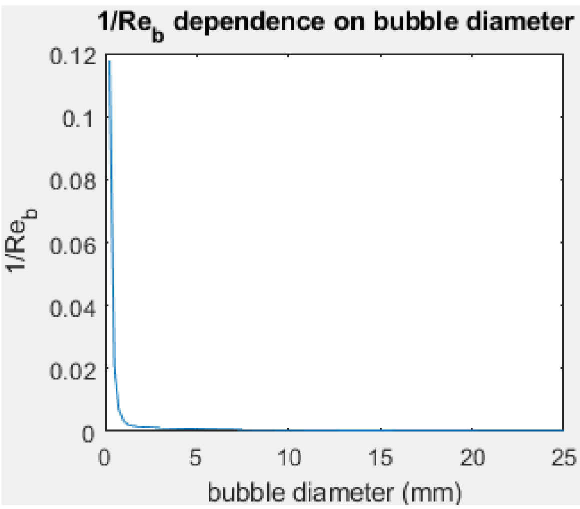
- Programming:

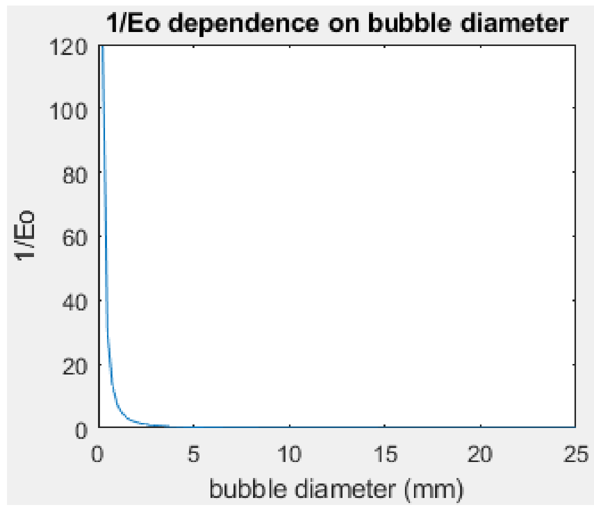
```
R_bubble = linspace(0,12.5,101); % in mm
R_bubble = R_bubble./1000;      % in m
D_bubble = R_bubble.*2;         % in m
D_BUBBLE = D_bubble.*1000;      % in mm
     v_r = linspace(0.2,1.2,length(R_bubble));
     Re  = zeros(1,length(R_bubble));
     EO  = zeros(1,length(R_bubble));
   mu_cl = 1E-6;
   gamma = 0.0727;
     tol = 1E-5;
 gravity = 9.8;
   rho_l = 998.21;
   rho_g = 1.205;

for i = 1:length(R_bubble)
    v_r_old = 0;
    v_r_new = 1;
    while abs(v_r_old-v_r_new)> tol
      v_r_old = v_r_new;
       Re_b = v_r_old*D_bubble(i)/mu_cl;
         Eo = gravity*(rho_l-rho_g)*D_bubble(i)^2/gamma;
        C_D =
sqrt((16/Re_b*(1+2/(1+16/Re_b+3.315/sqrt(Re_b))))^2+(4*Eo/(Eo+9.5))^2);
        v_r_new = sqrt(8/3*R_bubble(i)*gravity*(rho_l-rho_g)/C_D/rho_l);
    end
    v_r(i)= v_r_new;
    Re(i) = Re_b;
    EO(i) = Eo;
end

figure
```

```
plot(D_BUBBLE,v_r)
title('Rise velocity dependence on bubble diameter')
xlabel('bubble diamater (mm)')
ylabel('rise velocity (m/s)')
```

(d)

The reason why the plot in part (b) and part (c) are different is the difference of drag coefficient. In part (b), the C_D is a constant, however, in part (c), it considers about the bubble Reynolds number and Eovtos number. For bubble diameter lager than 8.5mm, $1/Re_b$ approaches 1E-3 and 1/Eo approaches 0.1 and both decreases slowly, thus the rise velocity increase slowly afterwards.





Using the expression to evaluate C_D is more reasonable than simply assigning a constant value on C_D, since the movement of bubble will disturb the fluid field, thus the bubble motion will impact the drag coefficient. The variation in part (c) curve demonstrates the different dynamics between the bubble and the fluid.