Problem 1.6 10 points:

Using **dimensional analysis** **derive** the expressions for Kolmogorov's length, time and velocity scales.

Note: the turbulence dissipation rate (ε) has the dimensions of $\left[\frac{m^2}{s^3}\right]$ and kinematic viscosity has the dimensions of $\left[\frac{m^2}{s}\right]$.

Solution:

Let L and T denote unit dimension in length and time, given:

$$\nu = [L^2T^{-1}], \epsilon = [L^2T^{-3}]$$

For Kolmogorov's length scale $\eta[L^1]$:

$$\eta = \nu^a\epsilon^b = [L^1]$$

To find a and b, the following system of equations should be solved:

$$\begin{cases} 2a + 2b = 1 & -(1) \\ -a - 3b = 0 & -(2) \end{cases}$$

$2 \times (2) + (1)$:

$$-4b = 1 \rightarrow b = -0.25$$

Insert it to (2)

$$a = 0.75$$

Thus:

$$\eta = \nu^{0.75}\epsilon^{-0.25} = \left(\frac{\nu^3}{\epsilon}\right)^{\frac{1}{4}}$$

For Kolmogorov's time scale $\tau_\eta[T^{-1}]$:

$$\begin{cases} 2a + 2b = 0 & -(1) \\ -a - 3b = 1 & -(2) \end{cases}$$

Then we obtain:

$$a = 0.5, b = -0.5$$

Hence:

$$\tau_\eta = \nu^{0.25}\epsilon^{-0.25} = \left(\frac{\nu}{\epsilon}\right)^{\frac{1}{2}}$$

For Kolmogorov's velocity scale $u_\eta[L^1T^{-1}]$:

$$\begin{cases} 2a + 2b = 1 & -(1) \\ -a - 3b = -1 & -(2) \end{cases}$$

Then we obtain:

$$a = b = 0.25$$

Therefore,

$$u_\eta = (\nu\epsilon)^{\frac{1}{4}}$$