NC STATE UNIVERSITY

ENGINEERING ONLINE

Lecture Notes

Course Number: 577

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① In the projection method we split the momentum equation in 2 parts by introducing temp. velocity:

$\underline{u}^*$ such that: $\underline{u}^{n+1} - \underline{u}^n = \underbrace{\underline{u}^{n+1} - \underline{u}^*} + \underbrace{\underline{u}^* - \underline{u}^n}$

The first part is a predictor step where the temp. velocity is found by ignoring effect of pressure:

$$\frac{\underline{u}^* - \underline{u}^n}{\Delta t} = -\underline{A}_h(\underline{u}^n) + \nu \underline{D}_h(\underline{u}^n) + \underline{f}_\ell^n \quad (2.3)$$

Second step: (projection step): we add the pressure gradient:

$$\frac{\underline{u}^{n+1} - \underline{u}^*}{\Delta t} = -\frac{1}{\rho} \nabla_h p^{n+1} \quad (2.4)$$

② To find the pressure, we use (2.2) to eliminate $\underline{u}^{n+1}$ from (2.4):

$$\frac{1}{\rho}\nabla_h^2 p^{n+1} = \frac{1}{\Delta t}\nabla_h \cdot \underline{u}^* \qquad (2.5)$$

Sequence: (2.3); (2.5); (2.4)

$$\begin{array}{cccc} & (2.3) & (2.5) & (2.4) \\ \nearrow & \downarrow & \downarrow & \downarrow \\ \underline{u}^n & \underline{u}^* & p^{n+1} & \underline{u}^{n+1} \end{array}$$

Note: we do not assume that $\nabla_h \cdot \underline{u}^n = 0$

This time integration algorithm is subject to time step limitations.

(3) If spatial derivatives are approximated using centered second-order approach (will discuss later), stability analysis considering only viscous terms requires:

$$\Delta t < C_\nu \frac{h^2}{\nu} \qquad (2.6)$$

where $C_\nu = \frac{1}{4}$ for 2d flows

$C_\nu = \frac{1}{6}$ for 3d flows

h - grid spacing

5 The advection scheme is unstable on it's own, but it is stabilized by viscosity if:

$$\Delta t < \frac{2\nu}{q^2} \qquad (2.7)$$

where $q^2 = \underline{u} \cdot \underline{u}$

More advanced advection schemes are stable w/o viscosity are generally subject to the Courant-Friedrich-Lewy (CFL) condition [1928]:

$$\Delta t < \frac{h}{|u|} \qquad (2.8)$$

for 1d flow.

$$CFL = \frac{\Delta t\, |u|}{h}$$

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- 2d & 3d → the condition applies in each direction
- for unsteady flows, CFL condition is not the most severe
- (2.6) is more stringent for slow flows
- when s.t. is important, it is necessary to limit the t/s so a capillary wave travels less than a grid size in 1 t/s.

6 The simple explicit forward algorithm is only first order accurate.

For most problems it is desirable to employ at least a second order accurate integration method.

In such methods advection is treated explicitly

Viscous are handled implicitly.