Homework #7 (1 problem; 55 points)
<u>due</u>: April 10th, 2023.

Problem 7.1 [55]	Total

Problem 7.1 55 points:

Start with the code you have created for Problem 5.4. Keep the domain size ($L_x = 0.04\ m$ and $L_y = 0.02\ m$) boundary conditions unchanged.

a) Initialize one spherical droplet in the center of the domain and the corresponding level-set distance field (x coordinate of the droplet center is $x_d = 0.02\ m$). Droplet diameter is half domain height ($D_d = \frac{L_y}{2}$). Plot the initial droplet and the level set field with $N_y = 30$ and 50 elements across the domain height respectively. Use $M = 3$ in Eq. (3.53) and (3.54). [5 points]

b) Implement variable property in your solver (ρ and μ are now arrays instead of just constants, and you will also need to modify your predictor & corrector steps accordingly to incorporate variable density and viscosity). Set liquid density, viscosity as $\rho_l = 10^3 \frac{kg}{m^3}, \mu_l = 10^{-3}\ Pa\ s$, and gas density, viscosity as $\rho_g = 1 \frac{kg}{m^3}, \mu_g = 10^{-5}\ Pa\ s$. Plot your initial density and viscosity distribution with $N_y = 30$ and 50. [10 points]

c) Initialize the velocity field to be the **analytical solution** and **advect** the droplet through $\sim 20\%$ of the domain length, i.e., $0.008\ m$. The advection procedure is described in Eqns. (3.41) – (3.46). Plot the advected droplet and the distance field with $N_y = 30$ and 50 elements across the domain height respectively. Compare and discuss the advected droplet shape with the initial one. [10 points]

<u>Note:</u>

- Reduce your time step size if needed.
- Do not implement the re-initialization, you will implement it in d)
- Do not implement surface tension, you will implement it in e)

d) Add the re-initialization steps into the solver. **Advect** the droplet through $\sim 20\%$ of the domain length with $N_y = 50$ elements across the domain. **Compare and discuss** the distance fields with and without re-initialization. [10 points]

Note: refer to Eq. (3.48)-Eq. (3.53), Problem 6.1, Problem 6.2, and p. 60-61 of the textbook.

e) Implement the surface tension force in your solver. Use surface tension coefficient $\gamma = 0.06\ N/m$. **Advect** the droplet through $\sim 20\%$ of the domain length. Start with using $N_y = 30$ elements across the domain height and then increase the number of elements until $N_y = 120$ at your own pace. Plot the distance and pressure fields with at least two different element numbers besides 30 and 120 and **discuss** your result. [10 points]

Note: refer to Eq. (3.55), Eq. (3.56), as well as Eq. (3.8), Eq. (3.11), and Pages 61-62 of the textbook.

Extra Credit Use variable properties in Table 1 for liquid and gas and update the solver (you will obtain a two-phase solver using the "one-fluid" approach). **Advect** the droplet through $\sim 20\%$ of the domain length and pick two N_y element numbers you tested in e). Plot distance fields with at least 4 different combinations of two-phase properties and discuss how the contrast of properties affect your results. [10 points]

Table 1 Two-phase properties

	Density (ρ) kg/m^3	Dynamic viscosity (μ) $Pa\ s$
Liquid (droplet)	10^3	1×10^{-3}
Gas	9.99×10^2, 10^2, 10, 1	9×10^{-4}, 1×10^{-4}, 1×10^{-5}

Notes:

1) Refer to Eq. (3.12)-Eq. (3.13) and Pages 61-62 of the textbook.

2) If you need to further increase the properties in Table 1, you can increase viscosity first, and then density.

General notes:

1) To expedite your coding and save trouble for debugging, write functions for all the operators (**strongly recommended!**).
2) Submit your code.