Homework #7 (3 problems; 40 points)

due: 11:45am on April 17th Friday 2020.

Problem 7.1 [10]	Problem 7.2 [20]	Problem 7.3 [10]	Total [40]

Problem 7.1 10 points:

Consider a steady-state, fully developed channel flow. At the wall (y=0), the velocity is zero and pressure is p_w. Show that mean axial pressure gradient is uniform across the flow: $\frac{\partial \langle p \rangle}{\partial x} = \frac{dp_w}{dx}$.

Hint: write the lateral mean-momentum equation and integrate it in y-direction analytically.

Problem 7.2 20 points:

Derive the exact transport equation for turbulent kinetic energy (TKE), k (Eq. (10.35) in lecture notes).

Use the following steps:

a) Subtract the Reynolds equations (momentum written for mean velocities) from N.S. momentum equations, thus obtain the equation for fluctuating velocity;

b) Obtain a scalar product of fluctuating velocity and the vector-equation you got in part a) and apply Reynolds averaging to the result.

Problem 7.3 10 points:

Show that the transport equation for turbulence dissipation rate (ε, Eq. (10.53) in notes) can be obtained from the equation for TKE (Eq. (10.41)).

Note: the *P* in Eq. (10.41) is not pressure, but the production term in Eq. (10.35)