Problem 7.3 10 points:

Show that the transport equation for turbulence dissipation rate (ε, Eq. (10.53) in notes) can be obtained from the equation for TKE (Eq. (10.41)).

Note: the P in Eq. (10.41) is not pressure, but the production term in Eq. (10.35)

Solution:

From Eq.(10.41)

$$\frac{\bar{D}k}{Dt} = \nabla \cdot \left[\frac{\nu_T}{\sigma_k} \cdot \nabla k\right] + P - \varepsilon$$

Let $\tau = \frac{k}{\varepsilon} \implies k = \tau\varepsilon$

$$\frac{\bar{D}\tau\varepsilon}{Dt} = \nabla \cdot \left[\frac{\nu_T}{\sigma_k} \cdot \nabla\tau\varepsilon\right] + P - \varepsilon$$

Since τ is dependent on t,

$$\tau\frac{\bar{D}\varepsilon}{Dt} = \tau\nabla \cdot \left[\frac{\nu_T}{\sigma_k} \cdot \nabla\varepsilon\right] + P - \varepsilon$$

thus

$$\frac{\bar{D}\varepsilon}{Dt} = \nabla \left[\frac{\nu_T}{\sigma_k} \cdot \nabla\varepsilon\right] + \frac{P}{\tau} - \frac{\varepsilon}{\tau}$$

$$\frac{\bar{D}\varepsilon}{Dt} = \nabla \left[\frac{\nu_T}{\sigma_k} \cdot \nabla\varepsilon\right] + \frac{P\varepsilon}{k} - \frac{\varepsilon^2}{k}$$

Since the model is empirical, we must modify the coefficients based on Launder and Sharma

$$\frac{\bar{D}\varepsilon}{Dt} = \nabla \left[\frac{\nu_T}{\sigma_\varepsilon} \cdot \nabla\varepsilon\right] + C_{\varepsilon 1}\frac{P\varepsilon}{k} - C_{\varepsilon 2}\frac{\varepsilon^2}{k} \qquad \text{Eq (10.53)}$$