Homework #6 (2 problem, 40 points)

<u>due</u>: 11:45am on April 10th Friday 2020.

Problem 6.1 [10]	Problem 6.2 [30]

Problem 6.1 10 points:

Derive the $D_x^+ D_x^- (\phi_d)_{i,j}$ term in Eq.(3.52), using the definition in Eq.(3.46). Then derive the $D_y^+ D_y^- (\phi_d)_{i,j}$ term.

$$D_x^+ \phi_{i,j} = \phi_{i+1,j} - \phi_{i,j} \qquad D_x^- \phi_{i,j} = \phi_{i,j} - \phi_{i-1,j}. \tag{3.46}$$

$$\begin{aligned} \tilde{D}_x^+ &= D_x^+ (\phi_d)_{i,j} - \frac{1}{2} M(D_x^+ D_x^- (\phi_d)_{i,j}, D_x^+ D_x^- (\phi_d)_{i+1,j}) \\ \tilde{D}_x^- &= D_x^- (\phi_d)_{i,j} + \frac{1}{2} M(D_x^+ D_x^- (\phi_d)_{i,j}, D_x^+ D_x^- (\phi_d)_{i-1,j}). \end{aligned} \tag{3.52}$$

<u>Note:</u> This derivation will help with the coding in Problem 6.2.

Problem 6.2 30 points:

Use the code you have created for Problem 3.4 and improved for 4.4 and 5.2.

a) Keep the advection steps in Problem 5.2 unchanged, add the re-initialization steps into the solver. Use M=3 in Eq. (3.53) and (3.54). Similar to Problem 5.2, advect the droplet through ~20% of the domain length, i.e. ~0.004m with 20 and 60 elements across the domain height respectively. Compare the distance fields without and with re-initialization. Discuss your result.

<u>Note:</u> refer to Eq.(3.48)-Eq.(3.53), Problem 5.1, Problem 6.1, and Pages 60-61 of the textbook.

b) Implement the surface tension force into the solver. Use surface tension coefficient = 0.06 N/m. Still advect the droplet ~0.004m. Start with using 20 elements across the domain height and then increase the number of elements until 120 at your own pace. Plot the distance fields with at least 2 element numbers and discuss your result.

Note: refer to Eq.(3.55), Eq.(3.56), as well as Eq.(3.8), Eq.(3.11), and Pages 61-62 of the textbook.

c) Use variable properties in Table 1 for liquid and gas and update the solver (you will obtain a two-phase solver using the "one-fluid" approach). Still advect the droplet ~0.004m, using 2 element numbers you tested in b). Plot at least 4 distance fields with different combinations of two-phase properties, and discuss how highly contrast properties affect your results.

Table 1 Two-phase properties

	Density (ρ)	Viscosity (μ)
Liquid (droplet)	1000	1×10^{-3}
Gas	999, 100, 10, 1	9×10^{-4}, 1×10^{-4}, 1×10^{-5}

Notes:

1) Refer to Eq.(3.12)-Eq.(3.13) , and Pages 61-62 of the textbook.

2) In Table 1, the corresponding kinematic viscosity (ν) of droplet is 1×10^{-6}, which is the value you have been using since Problem 3.4. However, you need to use dynamic viscosity (μ) in c).

3) If you need to further increase the properties in Table 1, you can increase viscosity first, and then density.

General notes:

1) To expedite your coding and save trouble for debugging, write functions for all the operators (strongly recommend!).
2) When submitting your code, if you prefer not to copy code and paste into HW document, you can zip all scripts into a file and submit it as an email attachment.