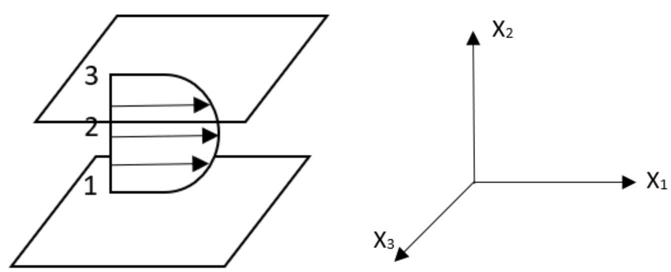
Homework #4 (1 problem, 30 points)

due: March 8th, 2023.

Problem 4.1	Total

Problem 4.1 30 points:

Consider a laminar flow in a channel (between two parallel plates) depicted in below. The streamwise direction is x or x_1 axis, and the centerline between the two plates is $y = 0$ or $x_2 = 0$.



The distance between the plates is $\boldsymbol{L_2 = 0.02\ m}$**.** Use the coordinates for the 3 points in the figure:

Point 1: $x_2 = -0.01\ m;$

Point 2: $x_2 = 0\ m;$

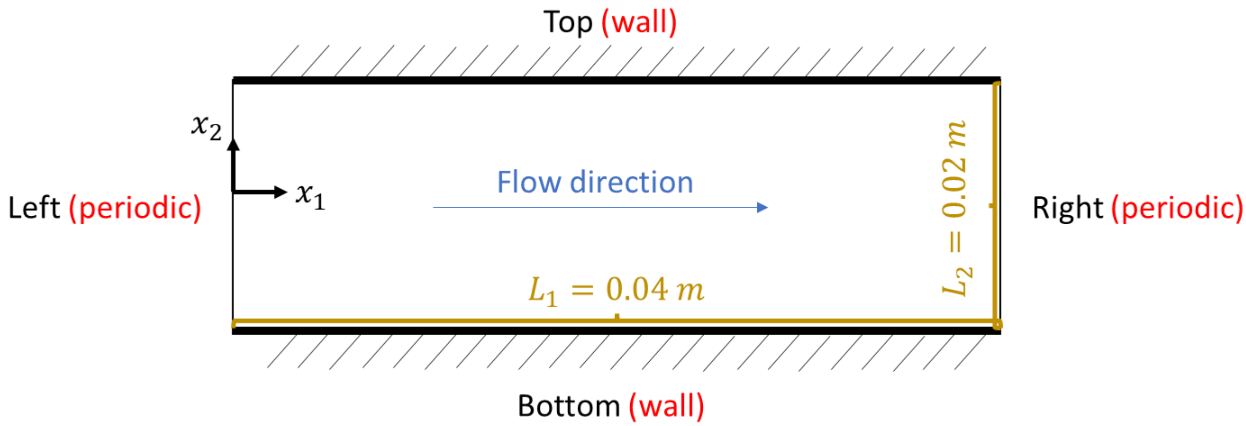
Point 3: $x_2 = 0.01\ m.$

Assume the maximum (peak) velocity is $0.01875\ m/s$ and perform the following analysis:

a) Solve the incompressible N.S. equation $\rho \frac{Du_i}{Dt} = -p_{,i} + \mu\nabla^2 u_i$ for this problem **analytically** at steady state with the body force ignored, given kinematic viscosity $\nu = 10^{-6}\ m^2/s$ and density $\rho = 10^3 kg/m^3$. Compute the **pressure gradient** $\boldsymbol{p_{,1}}$ and **velocity profile** $\boldsymbol{u_1(x_2)}$, and then compute the average velocity $\overline{\boldsymbol{u_1}(\boldsymbol{x_2})}$. [5 points]

<u>Note:</u> pay attention to the units.

b) Develop a staggered grid for this problem using the domain **length of $L_1 = 0.04\ m$**. Assume that grid is 2D, isotropic. Boundaries are denoted by inlet, outlet, top, and bottom. [5 points]



Boundary conditions of this fluid domain are:

Left	Right	Top	Bottom
Periodic		Wall	Wall

c) Using a programming tool (Fortran is encouraged, but not required), create a simple N.S. solver for the problem under consideration. **Submit your code.** [10 points]

Notes:

- Do **NOT** solve for pressure, simply use the pressure distribution obtained from the pressure gradient in a).
- Assume that normal to the wall velocity is zero and do NOT solve for it. Only solve for the x-velocity component.
- Assign initial condition to be a uniform velocity profile equal to the average velocity $\overline{u_1(x_2)}$ from the analytic solution.

d) Perform iteration until convergence/steady state (no further change in velocity is observed) on 3 different mesh resolutions: 30, 50 and 70 pressure cells **across the channel**. Plot and compare the results with the analytical solution obtained in part a). Then use the normalized L^2 norm in below to estimate the solution error for each mesh resolution: [5 points]

- The normalized L^2 norm to estimate the solution error (based on the error of the numerical solution relative to the analytic solution at each cell):

$$L^2 = \sqrt{\frac{1}{N}\sum_{j=1}^{N}(u_{numerical} - u_{exact})^2}$$

e) Do you expect the steady state solution to change with initial conditions? To test it, initialize your velocity field to be $0\ m/s$, peak velocity $0.01875\ m/s$, and $\overline{u_1(x_2)}$. Compare steady state solutions using 30 pressure cells across the channel. **Discuss the result**. [5 points]

Note: only initial condition is changed, boundary conditions should remain the same.