MAE/NE577

Homework #6 (2 problem, 40 points)

due: 11:45am on April 10th Friday 2020.

Problem 6.1 [10]	Problem 6.2 [30]

Problem 6.1 10 points:

Derive the $D_x^+ D_x^- (\phi_d)_{i,j}$ term in Eq.(3.52), using the definition in Eq.(3.46). Then derive the $D_y^+ D_y^- (\phi_d)_{i,j}$ term.

$$D_x^+ \phi_{i,j} = \phi_{i+1,j} - \phi_{i,j} \qquad D_x^- \phi_{i,j} = \phi_{i,j} - \phi_{i-1,j}. \tag{3.46}$$

$$\begin{aligned} \tilde{D}_x^+ &= D_x^+(\phi_d)_{i,j} - \frac{1}{2} M(D_x^+ D_x^- (\phi_d)_{i,j}, D_x^+ D_x^- (\phi_d)_{i+1,j}) \\ \tilde{D}_x^- &= D_x^-(\phi_d)_{i,j} + \frac{1}{2} M(D_x^+ D_x^- (\phi_d)_{i,j}, D_x^+ D_x^- (\phi_d)_{i-1,j}). \end{aligned} \tag{3.52}$$

Note: This derivation will help with the coding in Problem 6.2.

Problem 6.2 30 points:

Use the code you have created for Problem 3.4 and improved for 4.4 and 5.2.

a) Keep the advection steps in Problem 5.2 unchanged, add the re-initialization steps into the solver. Use M=3 in Eq. (3.53) and (3.54). Similar to Problem 5.2, advect the droplet through ~20% of the domain length, i.e. ~0.004m with 20 and 60 elements across the domain height respectively. Compare the distance fields without and with re-initialization. Discuss your result.

Note: refer to Eq.(3.48)-Eq.(3.53), Problem 5.1, Problem 6.1, and Pages 60-61 of the textbook.

b) Implement the surface tension force into the solver. Use surface tension coefficient = 0.06 N/m. Still advect the droplet ~0.004m. Start with using 20 elements across the domain height and then increase the number of elements until 120 at your own pace. Plot the distance fields with at least 2 element numbers and discuss your result.

NE 577. Hwk 6.

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#6-1.

given: $D_x^+ \phi_{i,j} = \phi_{i+1,j} - \phi_{i,j}$; $D_x^- \phi_{i,j} = \phi_{i,j} - \phi_{i-1,j}$

$D_x^+ D_x^- (\phi_d)_{i,j} = D_x^+ \left[(\phi_d)_{i,j} - (\phi_d)_{i-1,j} \right] = D_x^+ (\phi_d)_{i,j} - D_x^+ (\phi_d)_{i-1,j}$, expand individual term.

$= \left[(\phi_d)_{i+1,j} - (\phi_d)_{i,j} \right] - \left[(\phi_d)_{i,j} - (\phi_d)_{i-1,j} \right] = (\phi_d)_{i+1,j} - 2(\phi_d)_{i,j} + (\phi_d)_{i-1,j}$.

Likewise:

$D_y^+ D_y^- (\phi_d)_{i,j}$, with $D_y^+ \phi_{i,j} = \phi_{i,j+1} - \phi_{i,j}$; $D_y^- \phi_{i,j} = \phi_{i,j} - \phi_{i,j-1}$

$\therefore D_y^+ D_y^- (\phi_d)_{i,j} = D_y^+ \left[(\phi_d)_{i,j} - (\phi_d)_{i,j-1} \right] = D_y^+ (\phi_d)_{i,j} - D_y^+ (\phi_d)_{i,j-1}$

$= \left[(\phi_d)_{i,j+1} - (\phi_d)_{i,j} \right] - \left[(\phi_d)_{i,j} - (\phi_d)_{i,j-1} \right]$

$= (\phi_d)_{i,j+1} - 2(\phi_d)_{i,j} + (\phi_d)_{i,j-1}$

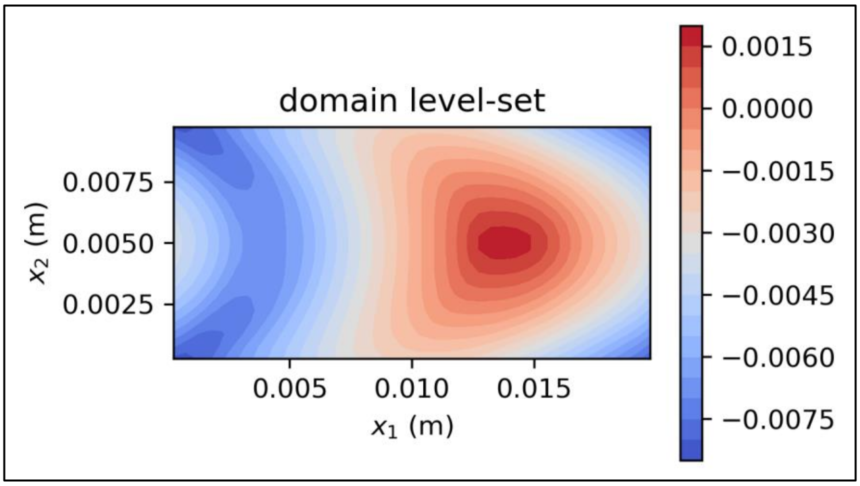
#Problem 6.2

(a)

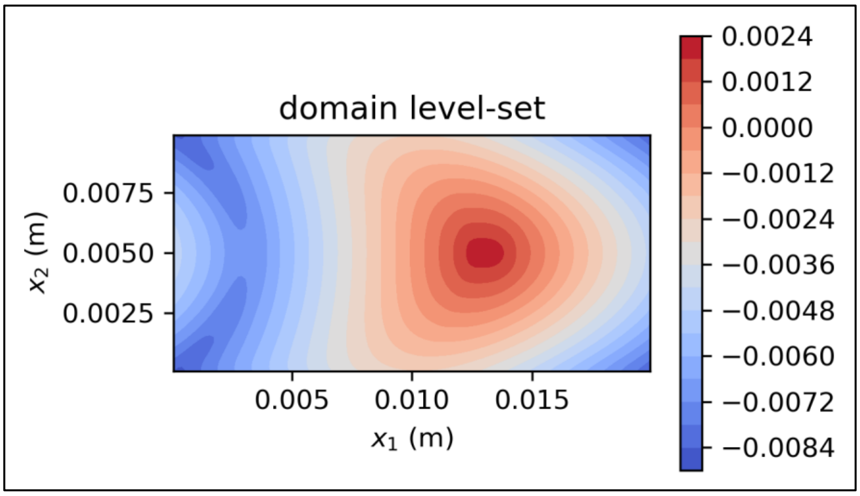
INS solver with level-set redistancing feature:

Compare with the results from hw5, with re-distancing, the level-set field near domain boundary is more physical. Moreover, re-initialized level-set is elongated at the center of the bubble, which is consistent with the shape of the interface.

Mesh with 20 cells across the domain height



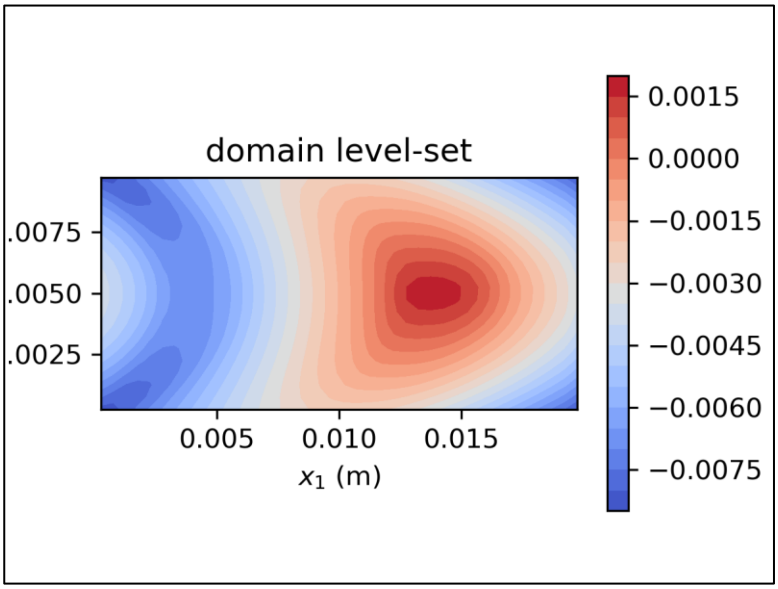
Mesh with 60 cells across the domain height



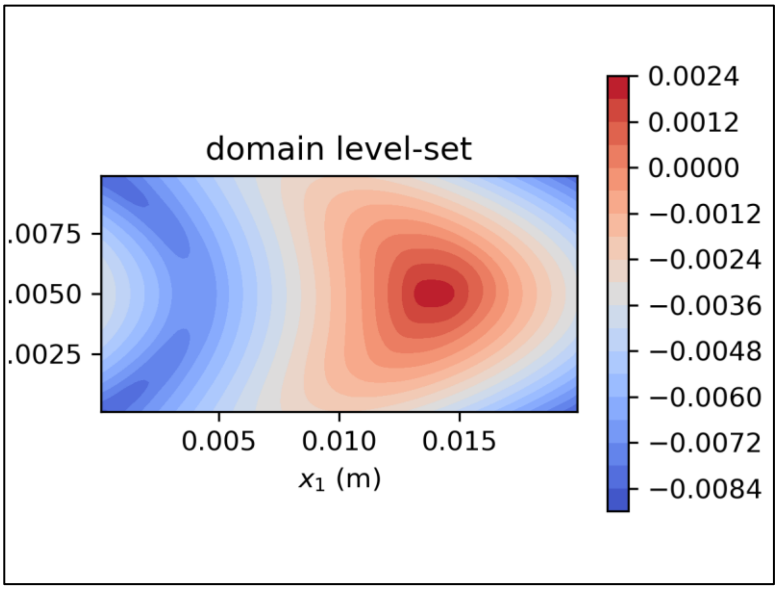
(b)

INS solver with level-set redistancing, and surface tension treatment:

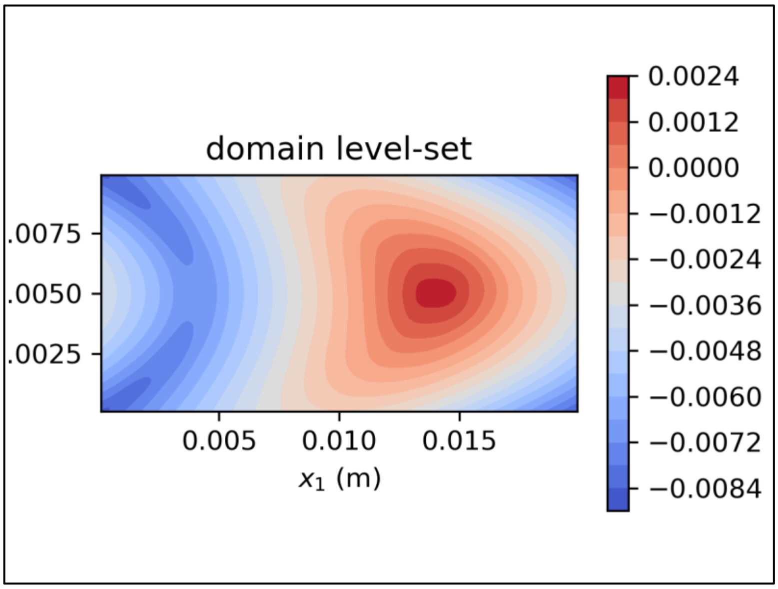
Level-set field of mesh with 20 cells across the domain height



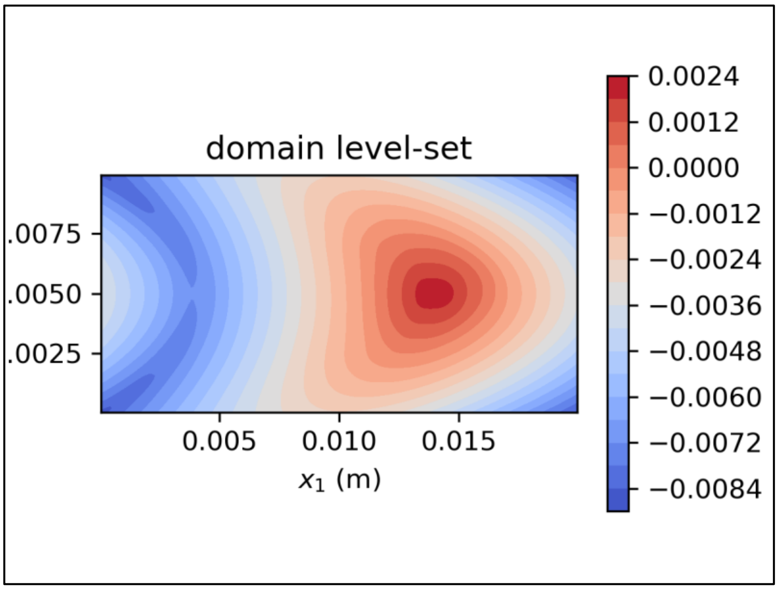
Level-set field of mesh with 50 cells across the domain height



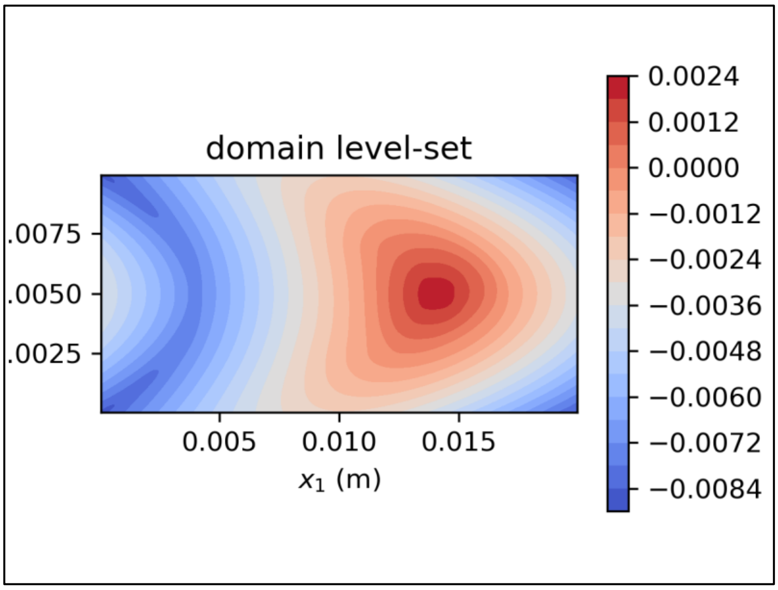
Level-set field of mesh with 60 cells across the domain height



Level-set field of mesh with 80 cells across the domain height



Level-set field of mesh with 120 cells across the domain height

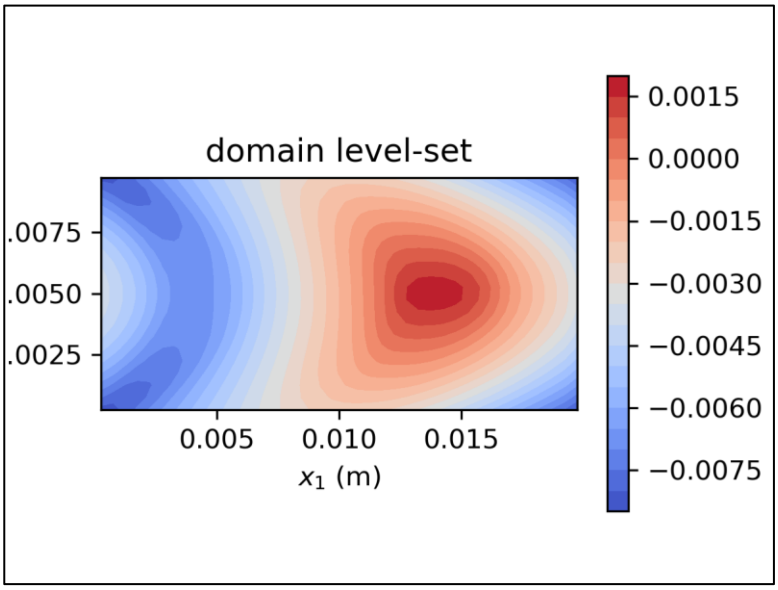


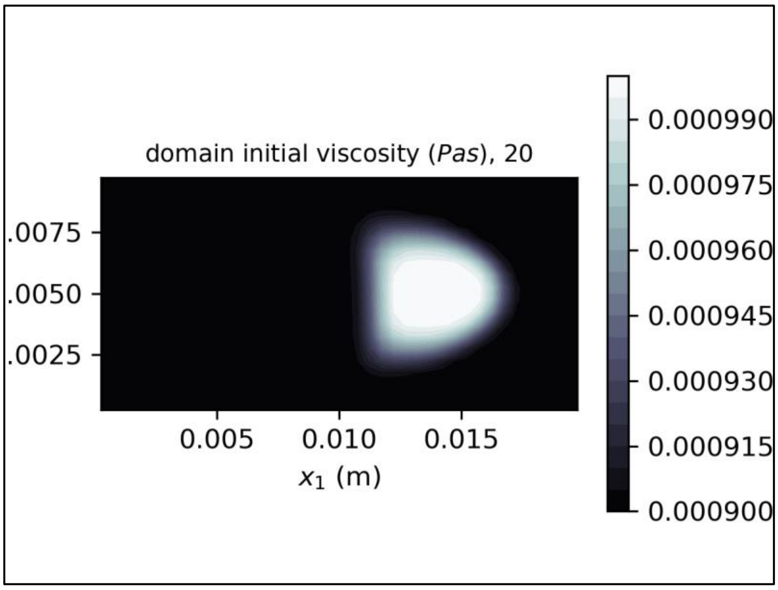
As the refinement of the mesh goes, the shape of the interface gets smoother. The case with only 20 meshes across shows mere difference compared to result from (a). As mesh gets finer, the bubble shape gets rounder and rounder. The area with largest level-set also becomes less elongated.

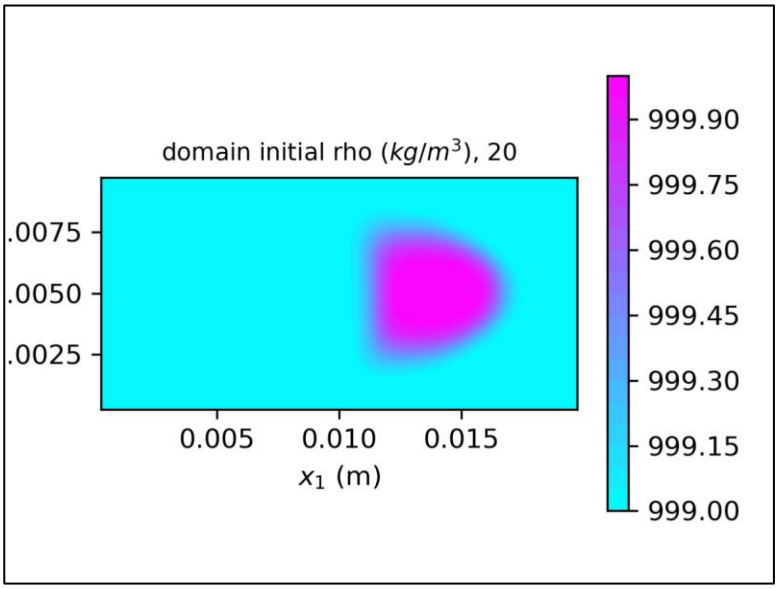
(c)

Attempted to run the code using $\rho_{liq} = 1000\frac{kg}{m^3}$ and $\rho_{gas} = 1,100\frac{kg}{m^3}$ but diverged. Following runs uses $\rho_{gas} = 999\frac{kg}{m^3}$ with different gas viscosity.

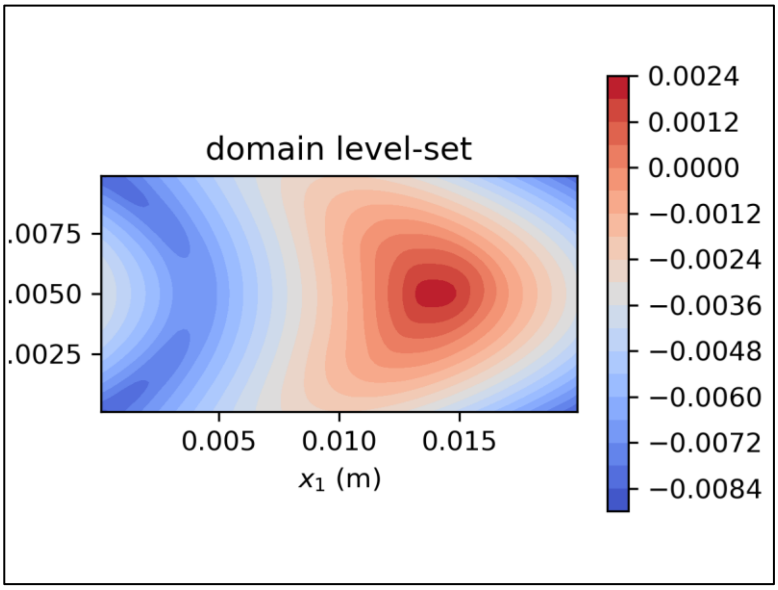
- Level-set field, density and viscosity distribution, $\rho_l = 1000\frac{kg}{m^3}, \rho_g = 999\frac{kg}{m^3}, \mu_l = 10^{-3}\ Pa\ s,$ $\mu_g = 9 \times 10^{-4} Pa\ s$:
  - 20 Meshes across the domain height:

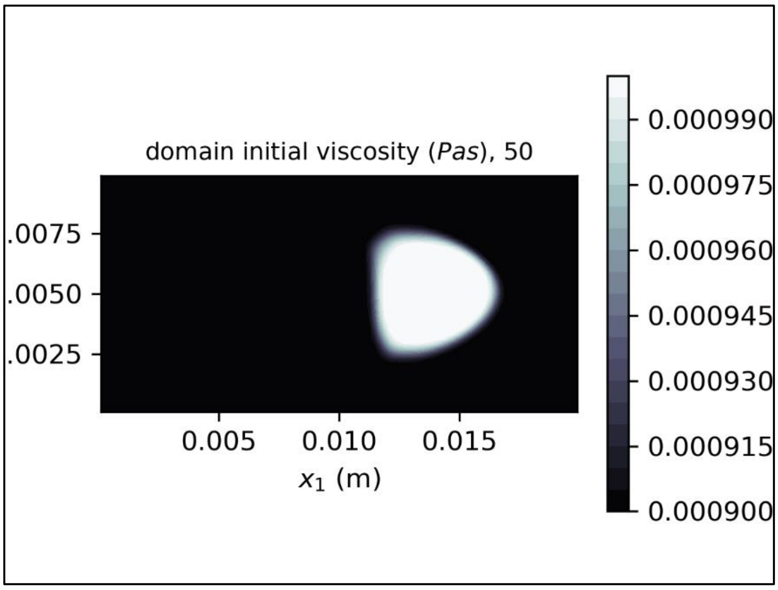


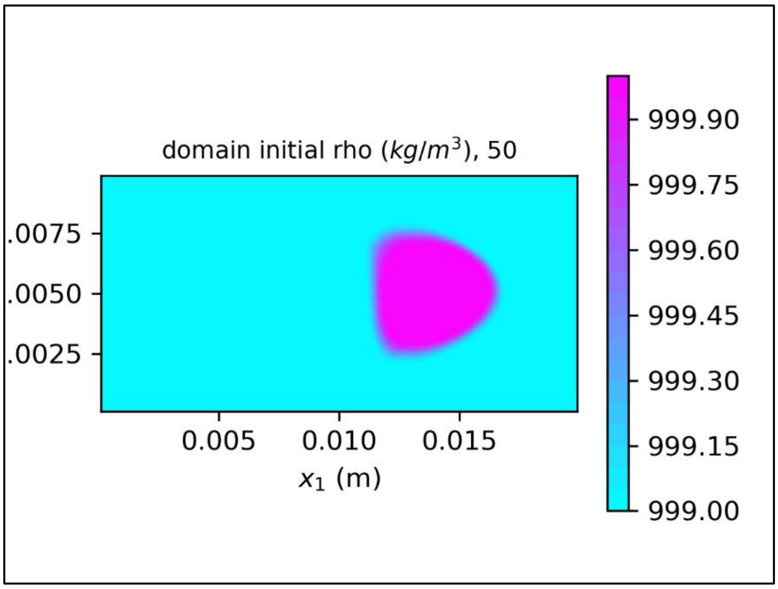




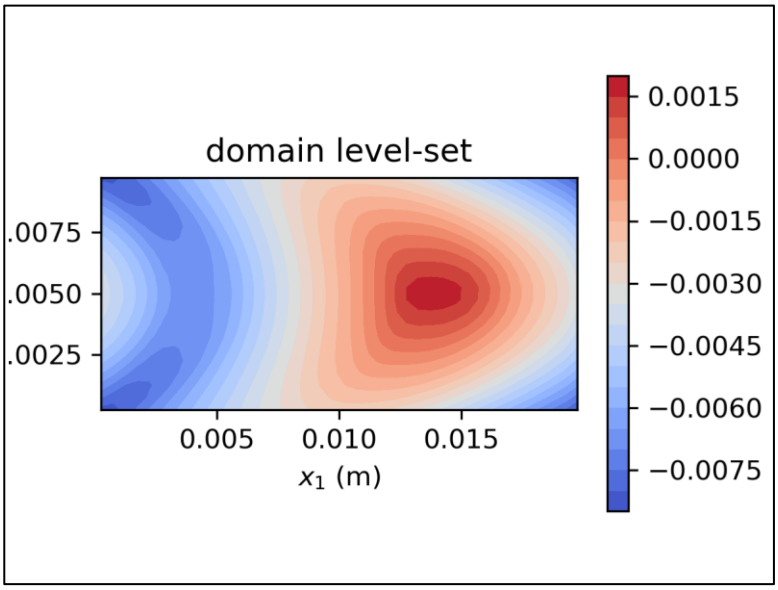
- 50 Meshes across the domain height:

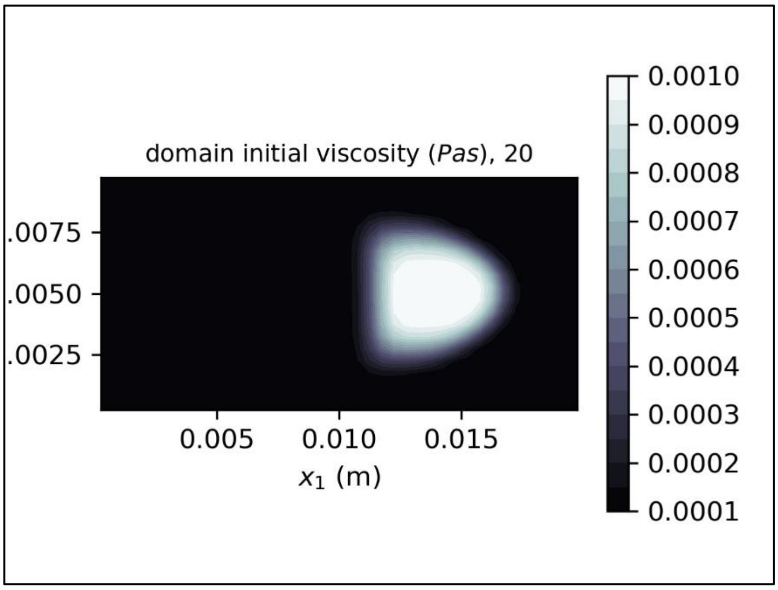


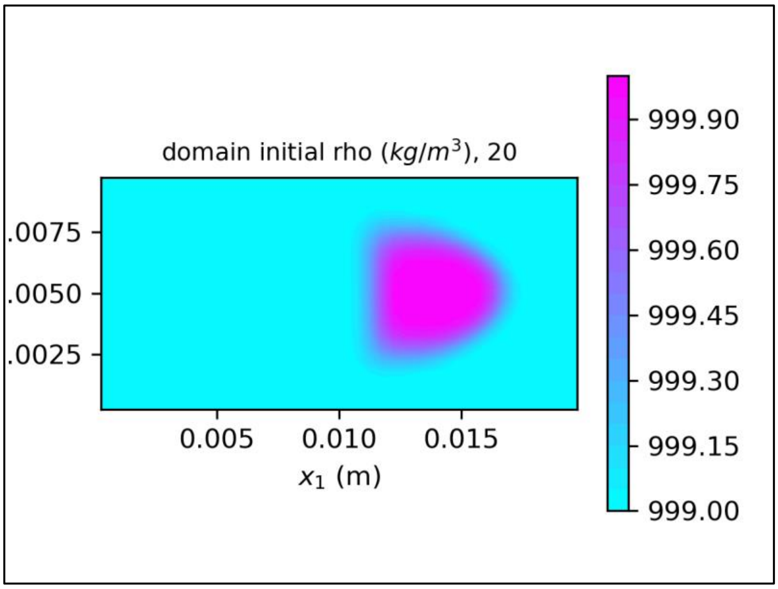




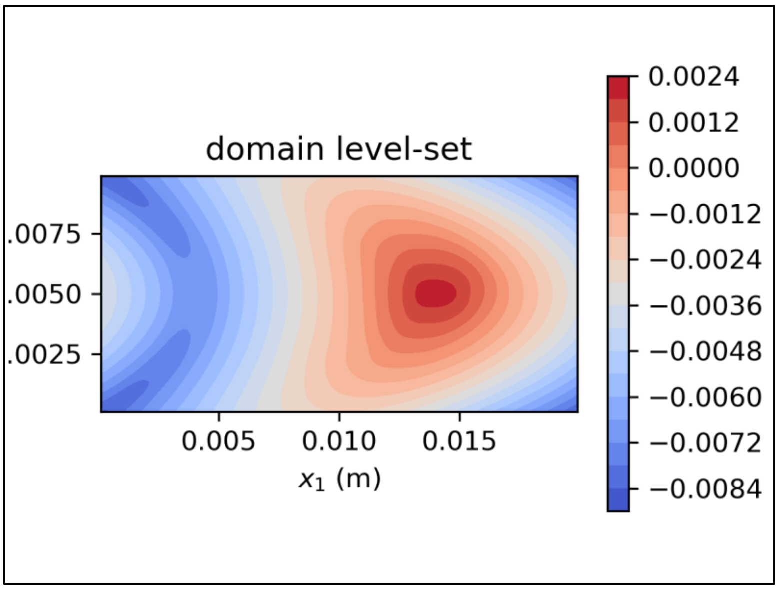
- Level-set field, density and viscosity distribution, $\rho_l = 1000\frac{kg}{m^3}, \rho_g = 999\frac{kg}{m^3}, \mu_l = 10^{-3}\ Pa\ s,$ $\mu_g = 1 \times 10^{-4} Pa\ s$:
  - 20 Meshes across the domain height:

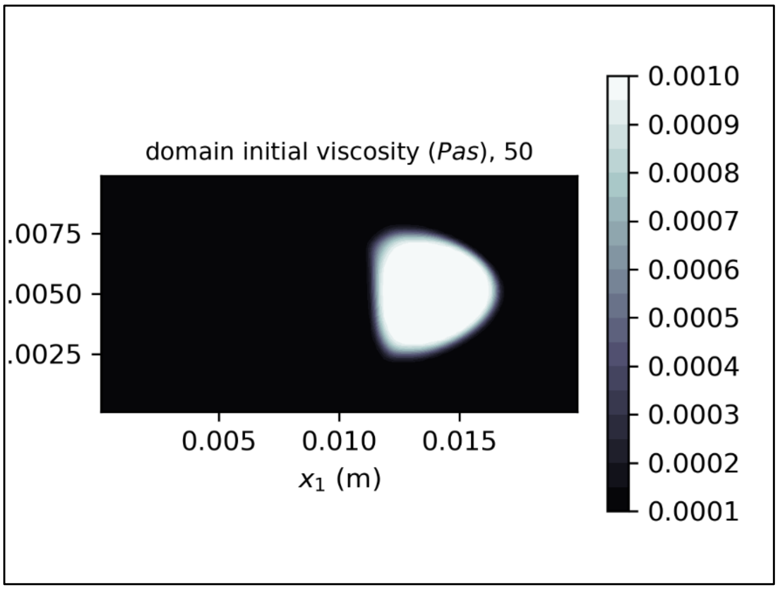


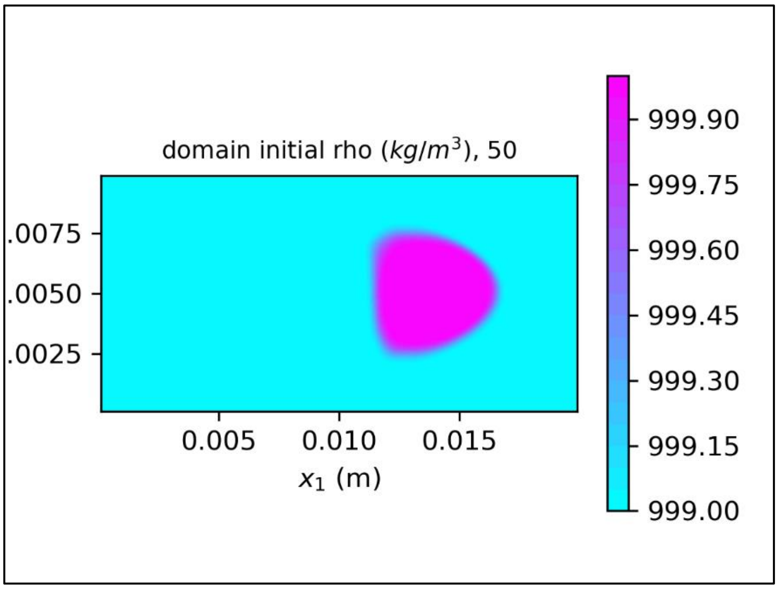




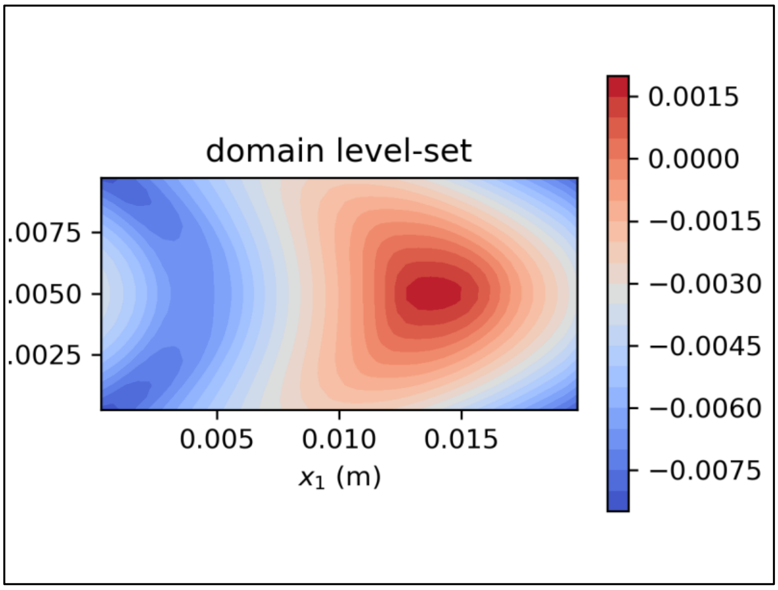
■ 50 Meshes across the domain height:

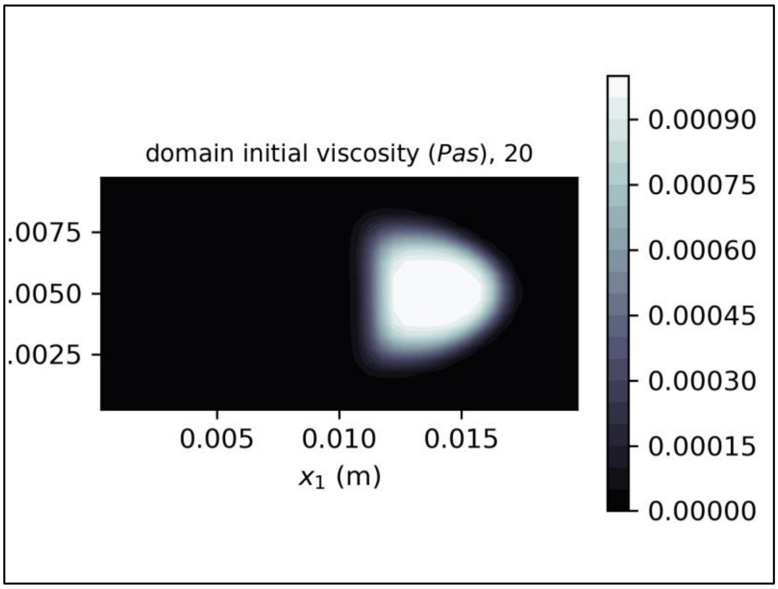


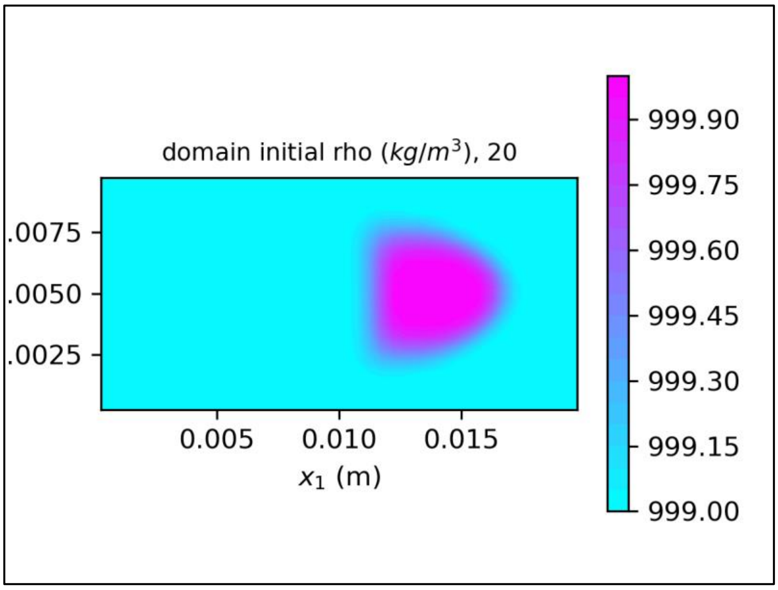




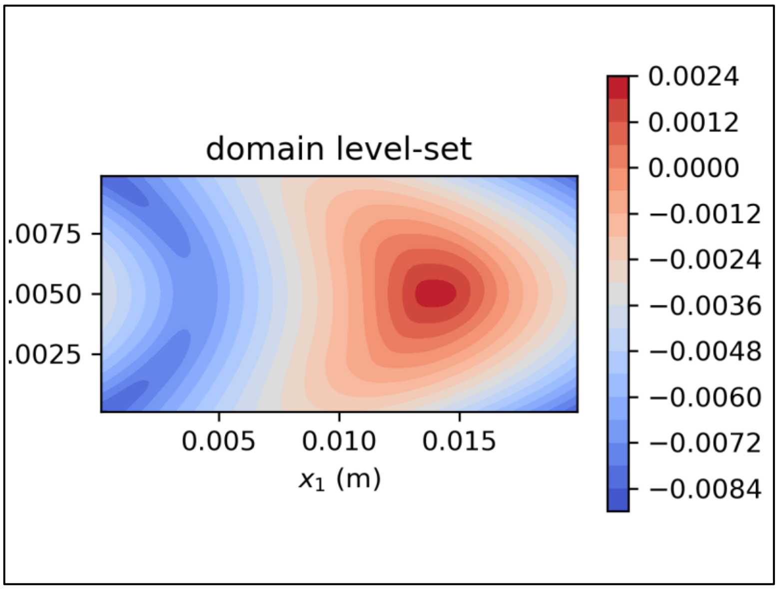
- Level-set field, density and viscosity distribution, $\rho_l = 1000 \frac{kg}{m^3}, \rho_g = 999 \frac{kg}{m^3}, \mu_l = 10^{-3}\ Pa\ s$, $\mu_g = 1 \times 10^{-5} Pa\ s$:
  - 20 Meshes across the domain height:

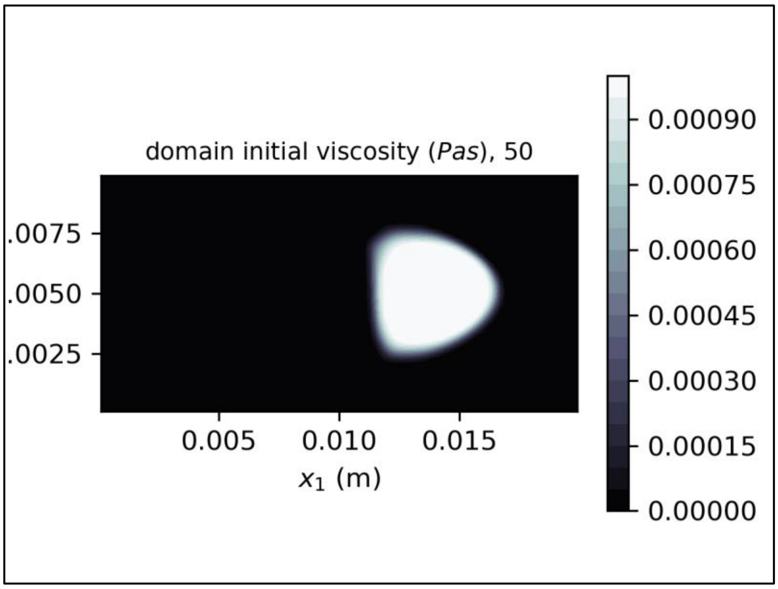




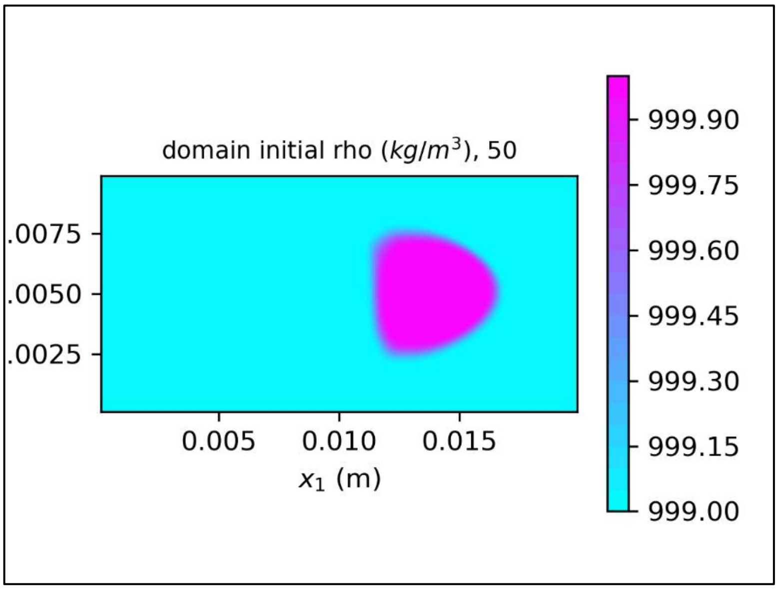


- 50 Meshes across the domain height:





domain initial viscosity (Pas), 50
.0075
.0050
.0025
0.005
0.010
0.015
x_1 (m)
0.00090
0.00075
0.00060
0.00045
0.00030
0.00015
0.00000



domain initial rho (kg/m^3), 50
.0075
.0050
.0025
0.005
0.010
0.015
x_1 (m)
999.90
999.75
999.60
999.45
999.30
999.15
999.00

Additional tests are conducted with high viscosity contrast conditions ($\rho_l = 1000\frac{kg}{m^3}, \rho_g = 999\frac{kg}{m^3}$ $\mu_l = 1\ Pa\ s, \mu_g = 10^{-3}\ Pa\ s$). The pressure field failed to converge: