Homework #8 (2 problems, 30 points)

due: 11:45am on April 24th Friday 2020.

Problem 8.1 [20]	Problem 8.2 [10]	Total

The following parameters are given for both Problem 8.1 and Problem 8.2:
C_D = 0.47 for spherical bubble, $\rho_l = 998.21\ \frac{kg}{m^3}$, and $\rho_g = 1.205\ \frac{kg}{m^3}$.

Problem 8.1 20 points:

Consider a steady-state air bubble rise in infinite standing water.

a) Develop an expression for the rise velocity (also the relative velocity) v_r based on drag/buoyancy force balance.

b) Assuming the bubble preserved spherical shape, plot the rise velocity dependence on bubble diameter. Use the bubble diameter range from 0.5mm to 25mm.

c) Assume variable drag coefficient according to this expression:

$$C_D = \sqrt{\left[\frac{16}{Re_b}\left(1 + \frac{2}{1 + \frac{16}{Re_b} + \frac{3.315}{\sqrt{Re_b}}}\right)\right]^2 + \left(\frac{4\,Eo}{Eo + 9.5}\right)^2}$$

Use bubble Reynolds number (Re_b) and Eotvos number based on bubble diameter and air/water surface tension. Re-plot the rise velocity dependence.

Note: use v_r in Re_b and solve the implicit equation iteratively. Submit your code.

d) Compare the results in b) and c) and discuss them.

Problem 8.2 10 points:

Consider a two-phase air/water bubbly flow at atmospheric conditions. Assume that there are three groups of spherical bubbles:

1. Group 1: Mean diameter of 1 mm and volume fraction of 3%
2. Group 2: Mean diameter of 1.5 mm and volume fraction of 1%
3. Group 3: Mean diameter of 2.0 mm and volume fraction of 1%

Use the two-phase turbulent viscosity contribution proposed by Sato & Sekoguchi (1975)

$$\upsilon_{2\phi} = 0.6 D_{dv} \alpha_{dv} |v_r|$$

to evaluate:

a) Two-phase viscosity contribution from each group. And normalize $\upsilon_{2\phi}$ by kinematic viscosity of the water, i.e. $\upsilon_{2\phi}/\upsilon_{cl}$. Use $\nu_{cl} = 1.0 \cdot 10^{-6}\ m^2/s$.

b) Assume there is a single group with void fraction of 5% containing all the bubbles from Groups 1,2 and 3. Estimate the equivalent diameter and corresponding two-phase viscosity contribution. Discuss if $\upsilon_{2\phi}$ should be the same as those in part a).