Homework #5 (3 problems; 45 points) **due: 11:45am April 1st, 2020.**

Problem 5.1 [10]	Problem 5.2 [20]	Problem 5.3 [15]	Total

Problem 5.1 10 points:

Derive the value of level-set function on top/bottom cell boundaries, $\phi_{i,j+\frac{1}{2}}$. Follow the lecture example for the left/right cell boundaries $\phi_{i+\frac{1}{2},j}$ in Eq. (3.44).

$$\phi_{i+1/2,j} = \begin{cases} \phi_{i,j} + \frac{1}{2}M(D_x^+\phi_{i,j}, D_x^-\phi_{i,j}), & \frac{1}{2}(u_{i+1,j}+u_{i,j}) > 0 \\ \phi_{i+1,j} - \frac{1}{2}M(D_x^+\phi_{i+1,j}, D_x^-\phi_{i+1,j}), & \frac{1}{2}(u_{i+1,j}+u_{i,j}) < 0. \end{cases} \tag{3.44}$$

Problem 5.2 20 points:

Start with the code you have created for Problem 4.4. Keep the domain size unchanged (0.02m in length and 0.01m in height).

a) Initialize one spherical droplet in the center of the domain and the corresponding level-set distance field (x coordinate of the droplet center is x_d=0.01m). Droplet ***diameter*** is half domain height. Plot the initial droplet and the distance field with 20 and 60 elements across the domain height respectively.

b) Initialize the velocity field to be the **analytical solution** and advect the droplet through ~20% of the domain length, i.e. ~0.004m. The advection procedure is described in Eqs. (3.41) – (3.46). Plot the advected droplet and the distance field with 20 and 60 elements across the domain height respectively. Compared advected droplet shape with the initial shape. **Discuss** your result.

<u>Note:</u> reduce your time step size if needed. <u>Submit your code.</u>

c) Test another droplet initialization. Initialize the droplet at 1/4 of the domain length, i.e. x_d=0.005m. Also advect the droplet for ~0.004m. Plot the **initial** and **advected** droplet and the distance field distributions with 20 elements across the domain height. Compare distributions with the ones in a) and b) and **discuss** whether x_d=0.01m or x_d=0.005m is a better option for current domain length 0.02m.

<u>Hint:</u> periodical boundary condition.

Notes/simplifications:

- Do not implement variable properties
- Do not implement surface tension
- Do not implement the re-initialization

Extra credit [5 points]: Use 100 elements across the domain height to evaluate the advection of the interface using the following analytic technique: (i) pick 3 points on the initial droplet interface: front of the droplet, back of the droplet, and top side of the droplet. (ii) run the simulation for 20 timesteps with time step size 0.003s and check the displacements of the chosen points. (iii) compute analytically the displacement of each point using the analytic laminar velocity value and the simulation time. (iv) compare the displacements between (iii) and (ii) for each point and discuss your results.

Problem 5.3 15 points:

Initialize the level-set distance field for a simple **1D** domain sketched in below. Blue denotes a liquid object (imagine it is an 1D droplet) in the middle of gas (black). Domain length is 0.01m resolved by 60 elements. The object length is half of the domain length.

Plot the 1D level-set distance field. Then follow these steps to compute the object mass.

a) Implement density $\rho(\phi)$ varied with the smoothed Heaviside function $f(\phi)$. Use $\rho_l = 1000\,\frac{kg}{m^3}$ and $\rho_g = 1\,\frac{kg}{m^3}$.

$$f(\phi) = \begin{cases} 0, & \text{if } \phi < -Mh \\ \frac{1}{2}\left(1 + \frac{\phi}{Mh} + \frac{1}{\pi}\sin\left(\pi\frac{\phi}{Mh}\right)\right), & \text{if } |\phi| \leq Mh \\ 1 & \text{if } \phi > Mh. \end{cases} \tag{3.54}$$

$$\rho(\phi) = \rho_l f(\phi) + \rho_g(1 - f(\phi))$$

Plot the density distributions $\rho(\phi)$ in the two-phase domain with M=1, 2, 3 respectively.

Note: this problem requires an individual programming script, but the code for $f(\phi)$ and $\rho(\phi)$ can be recycled in next homework. Submit your code.

b) Calculate the numerical mass of the liquid object with different interface thickness (M=1, 2, 3) with the following three options:

Option 1: for any element with $\phi > 0$,

$$m_{droplet} = \sum_{element} (\rho_l \cdot length_{element})$$

Option 2: for any element with $\phi > 0$,

$$m_{droplet} = \sum_{element} (\rho(\phi)_{element} \cdot length_{element})$$

Option 3: for <u>all the elements</u>,

$$m_{droplet} = \sum_{element} (\rho_l \cdot density\ weight \cdot\ length_{element})$$

where $density\ weight = \frac{\rho(\phi)_{element} - \rho_g}{\rho_l - \rho_g}$

Compare the 3 numerical object masses varying with interface thickness (M=1, 2, 3) and the analytical mass. **Discuss** your results and clarify **which option** is the most appropriate to compute the numerical mass.

<u>Note:</u> the analytical mass is $\rho_l \cdot$ <u>object length</u>.