① Using a second-order Adams-Bashforth scheme for the advection terms, and a second order Crank-Nicholson scheme for the viscous term, the predictor step (Wesseling, 2001):

or

$$\frac{\underline{u}^* - \underline{u}^n}{\Delta t} = -\frac{3}{2}\underline{A}_h(\underline{u}^n) + \frac{1}{2}\underline{A}_h(\underline{u}^{n-1}) + \frac{\nu}{2}\left(\underline{D}_h(\underline{u}^n) + \underline{D}_h(\underline{u}^*)\right) + \text{force term} \quad (2.9)$$

and the correction step is:

$$\frac{\underline{u}^{n+1} - \underline{u}^*}{\Delta t} = -\nabla_h \Phi^{n+1} \quad (2.10)$$

Φ is not exactly pressure

(2) It can be shown:

$$-\nabla\phi^{n+1} = -\nabla p^{n+1} + \frac{\nu}{2}\left(\underline{D}_h(\underline{u}^{n+1}) - \underline{D}_h(\underline{u}^{*})\right)$$

$\underline{u}^{*}$ is not div.-free, so pseudo-pressure can be found as: (2.11)

$$\nabla_h^2 \phi^{n+1} = \frac{\nabla_h \cdot \underline{u}^{*}}{\Delta t} \qquad (2.12)$$

(2.12) must be solved before $\underline{u}^{n+1}$ is obtained.

3 Re-arrange (2.9); to get a Helmholtz eq. for u^*:

$$D_h(\underline{u}^*) - \frac{2}{\nu\Delta t}\underline{u}^* = \frac{3}{\nu}\underline{A}_h(\underline{u}^n) - \frac{1}{\nu}A_h(\underline{u}^{n-1}) - D_h(\underline{u}^n) - \frac{2}{\nu\Delta t}\underline{u}^n \equiv R.H.S. \quad (2.13)$$

4 First find the average over each control volume:

$$\underline{A}(\underline{u}^n) = \frac{1}{\Delta V}\int_V \underline{\nabla}\cdot(\underline{u}^n\underline{u}^n)\,dV$$

$$= \frac{1}{\Delta V}\oint_S \underline{u}^n(\underline{u}^n\cdot\underline{n})\,dS \qquad (2.14)$$

and $$\underline{D}(\underline{u}^n) = \frac{1}{\Delta V}\int_V \nabla^2\underline{u}^n\,dV \qquad (2.15)$$

ΔV is the volume of C.V.

S is the surface of C.V.

Div. theorem: $\iiint_V (\underline{\nabla}\cdot\underline{F})\,dV = \oiint_S (\underline{F}\cdot\underline{n})\,dS$

(5) Continuity eq., (2.2) can be numerically approximated, $\nabla_h \cdot \underline{u}^{n+1} = 0$ through integration over volume & converting to surface integral:

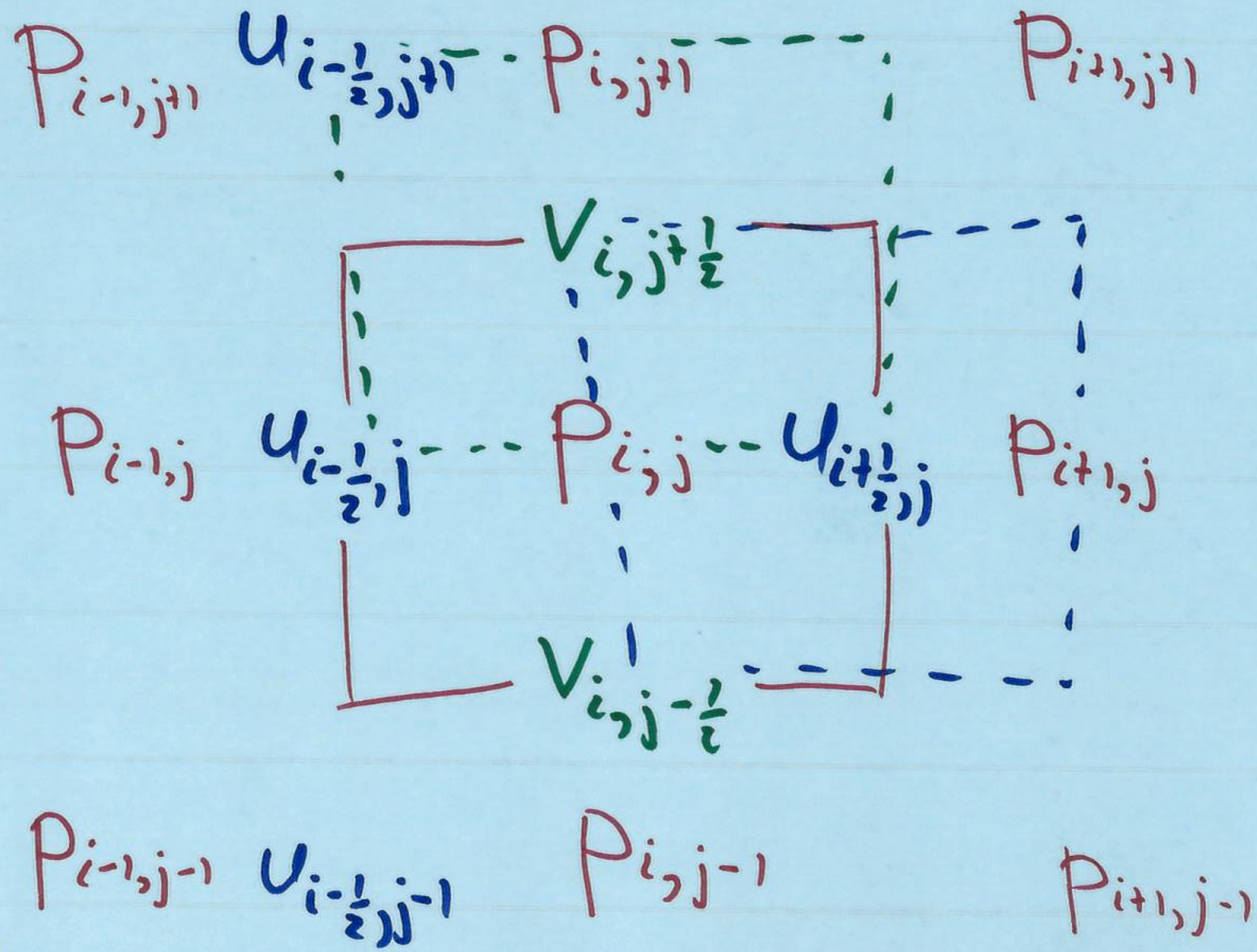
$$\frac{1}{\Delta V} \int_V \nabla \cdot \underline{u}^{n+1} dV = \frac{1}{\Delta V} \oint_S \underline{u}^{n+1} \cdot \underline{n}\, dS \quad (2.16)$$

Incomp:

$$u^{n+1}_{i+\frac{1}{2},j} - u^{n+1}_{i-\frac{1}{2},j} + v^{n+1}_{i,j+\frac{1}{2}} - v^{n+1}_{i,j-\frac{1}{2}} = 0 \quad (2.17)$$

Assumes same grid spacing in both directions.

6


$P_{i-1,j+1}$
$U_{i-\frac{1}{2},j+1}$
$P_{i,j+1}$
$P_{i+1,j+1}$
$V_{i,j+\frac{1}{2}}$
$P_{i-1,j}$
$U_{i-\frac{1}{2},j}$
$P_{i,j}$
$U_{i+\frac{1}{2},j}$
$P_{i+1,j}$
$V_{i,j-\frac{1}{2}}$
$P_{i-1,j-1}$
$U_{i-\frac{1}{2},j-1}$
$P_{i,j-1}$
$P_{i+1,j-1}$

(7) Discrete forms of eq. (2.3) & (2.4) for u vel. in a C.V. centered at $(i+\frac{1}{2}, j)$ and v vel. centered @ $(i, j+\frac{1}{2})$ are:

$$u^{*}_{i+\frac{1}{2},j} = u^{n}_{i+\frac{1}{2},j} + \Delta t\Big(-(A_x)^{n}_{i+\frac{1}{2},j} + \nu (D_x)^{n}_{i+\frac{1}{2},j} + (f_x)_{i+\frac{1}{2},j}\Big)$$

$$v^{*}_{i,j+\frac{1}{2}} = \qquad (2.18)$$

- predictor step.