Problem 5.3 20 points:

Consider a set of 2D two-phase flow problems to be solved using the Level-Set method. Analytically derive the initial conditions for the level set distance field in the following scenarios (note that you have to specify an expression in the form $\varphi(x, y) =$?):

(a) A single circular liquid drop with radius r_d and center at (x_d, y_d) in the domain. Note that the level set field should be positive in the liquid and negative in the gas phase. [3 points]

(b) A stratified flow initial condition represented by a straight horizontal line (parallel to the x axis). Assume that the liquid level is y_w and the liquid is on the bottom of the domain. [3 points]

(c) Develop an initial condition for the following case (using the results from (a) and (b)): [6 points]

- Include the stratification line described in (b)
- Add N_d droplets with coordinates (x_d^i, y_d^i), $i = 1, N_d$ and radii r_d
- Add N_b bubbles with coordinates (x_b^j, y_b^j), $j = 1, N_b$ and radii r_b
- Note that it is assumed that the droplets are located in the gas part of the domain and the bubbles are in the liquid part.

(d) Assume that there are 5 bubbles and 5 droplets. Randomly choose coordinates of the centers. Assume that the bubble/droplet diameter is 1/10 of the domain height. Use any software to plot a 2D level-set distribution for a domain size of 10.0 x 5.0. Assume that the liquid level is located at y = 2.0. Discuss why the distribution is correct. [8 points]

Solution:

a) The distance function is:

$$\varphi_d(x, y) = r_d - \sqrt{(x - x_d)^2 + (y - y_d)^2}$$

b) The distance function is:

$$\varphi_w(y) = y_w - y$$

c) For a single droplet:

$$\varphi_d(x,y) = r_d - \sqrt{(x-x_d)^2 + (y-y_d)^2}$$

For a single bubble:

$$\varphi_b(x,y) = \sqrt{(x-x_b)^2 + (y-y_b)^2} - r_b$$

From b), initial condition for stratified flow is:

$$\varphi_w = y_w - y$$

For a set of droplets in the gas, when the contours of distance function intersect with each other or the stratified flow interface:

$$\varphi(x,y) = \max\{\varphi_w, \varphi_{d,1}, \varphi_{d,2}, \dots \varphi_{d,5}\}, y \geq y_w$$

For a set of bubbles in the liquid, when the contours of distance function intersect with each other or intersect with the stratified flow interface:

$$\varphi(x,y) = \min\{\varphi_w, \varphi_{b,1}, \varphi_{b,2}, \dots, \varphi_{b,5}\}, y \leq y_w$$

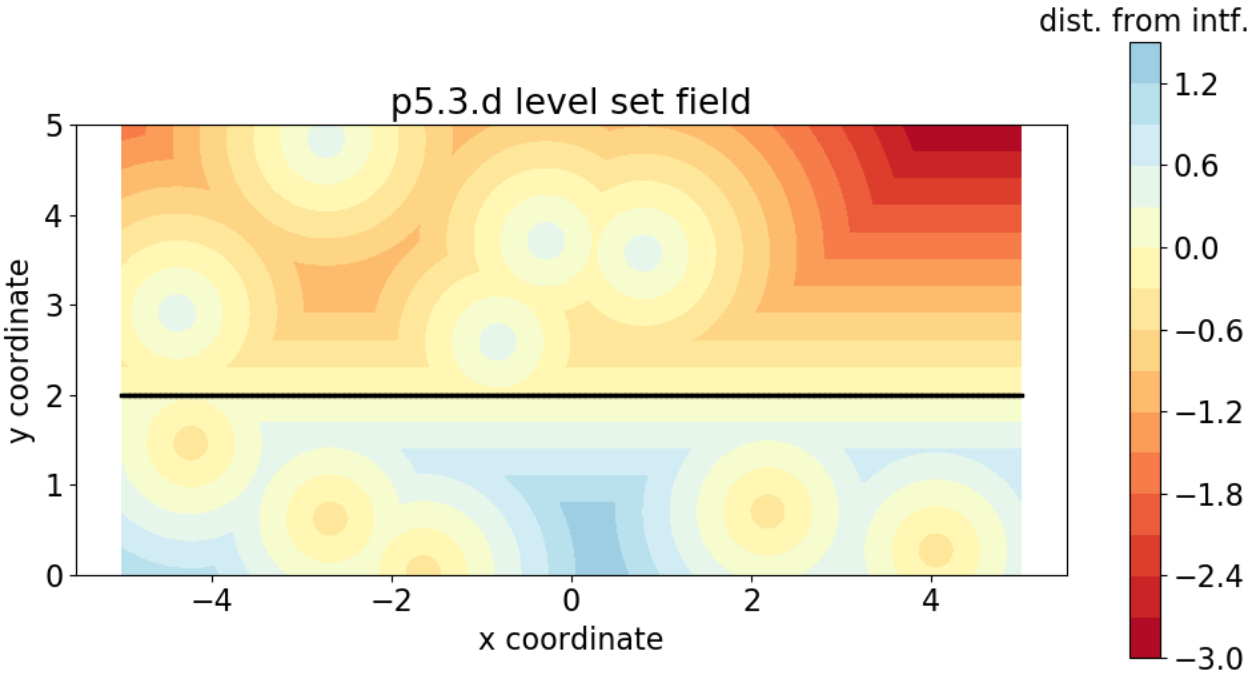
d) The bubble/droplet diameter is:

$$d = \frac{1}{10} \times 5.0 = 0.5$$

The stratified flow interface is at:

$$y_w = 2.0$$

The resulting level set:



The distribution is correct because:

(1) $\varphi = 0$ on the interfaces of droplets, bubbles and the water level.
(2) The φ around the bubbles and droplets is circular
(3) In the liquid ($y_w < 2$), $\varphi > 0$ outside bubbles; and in the gas ($y_w > 2$), $\varphi < 0$ outside the droplets.
(4) The gradient of φ aligns with the normal of the nearest interface location.
(5) There is no overlap of φ contours, and the transitions among bubbles, droplets and the line are smooth.