Assignment #2
MAE-577

Prob 2 1) $\frac{\partial S}{\partial t} + w \cdot \underline{\nabla} S = 0$ on $S = 0$

From taylor expression

$$f(x+\Delta x, t+\Delta t) = f(x,t) + \Delta x \frac{\partial f}{\partial x} + \Delta t \frac{\partial f}{\partial t} + \cdots$$

we can write

$$S(x+\Delta x, t+\Delta t) \approx S(x,t) + dx \frac{\partial S}{\partial x} + dt \frac{\partial S}{\partial t}$$

$S = 0$

$$0 = dx \frac{\partial S}{\partial x} + dt \frac{\partial S}{\partial t}$$

$$0 = w\,dt \frac{\partial S}{\partial x} + dt \frac{\partial S}{\partial t}$$

Dividing by dt

$$\frac{\partial S}{\partial t} + w \frac{\partial S}{\partial x} = 0$$

In vector form

$$\boxed{\frac{\partial S}{\partial t} + w \cdot \underline{\nabla} S = 0}$$

Prob 2.2

(a) If perfectly spherical in constant temp conditions

$$\Delta P = \gamma K$$

$$\cancel{K = \frac{\Delta P}{\gamma}}$$

$$K = \frac{\Delta P}{0.07275\,(1-0.002\cdot(T-291))}$$

$$\Delta P = \frac{2}{3.5\times10^{-3}} \times 0.07275\,(1-0.002\,(320-291))$$

$$\boxed{\Delta P = 39.16\ Pa}$$

2.2

(b)

$$\theta \in [0,\pi],\ T(\theta) = T_1 + (T_2 - T_1)\frac{\theta}{\pi}$$

$$\theta \in [\pi, 2\pi],\ T(\theta) = T_2 - (T_2 - T_1)\frac{\theta-\pi}{\pi}$$

(i) from 270 - 320 K

$$K = \frac{39.16}{0.07275\,[1-0.002\,(T-291)]}$$

where

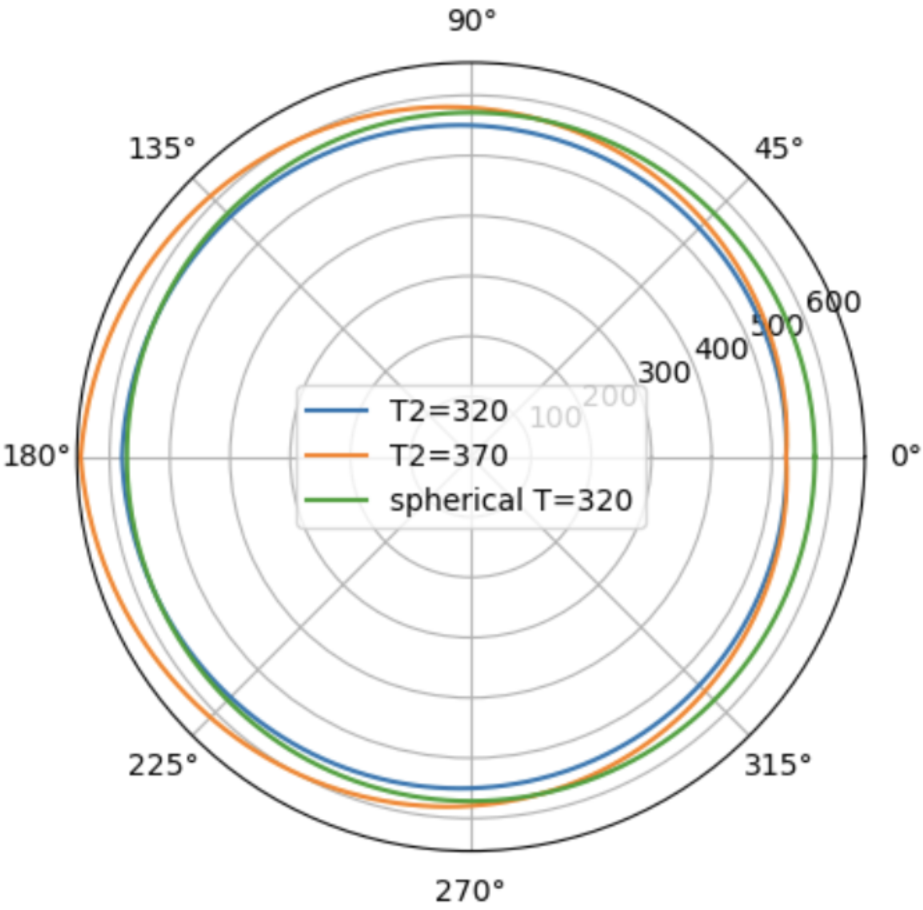
$$T = T\theta = \begin{cases} T_1 + \frac{T_2 - T_1}{\pi}\theta & (0,\pi) \\ T_2 - (T_2 - T_1)\frac{\theta-\pi}{\pi} & (\pi, 2\pi) \end{cases}$$

$$\begin{cases} 270 + 50\frac{\theta}{\pi} & 0,\pi \\ 320 - 50\frac{\theta}{\pi} & \pi, 2\pi \end{cases}$$

(ii) from 270 - 370

$$T(\theta) \begin{cases} 270 + 100\frac{\theta}{\pi} & 0,\pi \\ 270 - 100\frac{\theta}{\pi} & \pi, 2\pi \end{cases}$$

$$K = \frac{39.16}{0.07275(1-0.002(T(\theta)-291))}$$


90°
135°
45°
180°
0°
225°
315°
270°
100
200
300
400
500
600
T2=320
T2=370
spherical T=320

Problem 2.3

Given conditions

$P = 240\ kPa = 2.4\ bar$

From steam tables

$T_{sat} = 126.072\ °C = 399.22\ K$

$\rho_g = 1.3393\ \frac{kg}{m^3}$ $\rho_L = 938.14\ \frac{kg}{m^3}$

$\mu_L = 220.053\ \mu Pa\,s$ $\gamma = 0.0537369$

Now

$g_E = 9.81\ \frac{m}{s^2}$, $g_M = 1.62\ \frac{m}{s^2}$ and $L_M = 3(L_E)$

Part (a)

$$(E\ddot{o})_E = (E\ddot{o})_M$$

$$\left(\frac{|\rho_L - \rho_g|\, 9.81\, (L_E)^2}{\gamma}\right)_E = \left(\frac{|938.14 - 1.3393|\,(1.62)\,(3L_E)^2}{0.0537369}\right)_M$$

$$\left(\frac{|\rho_L - \rho_g|}{\gamma}\right)_E = 25909.74\ \frac{kg}{N m^2} \quad (1)$$

Part (b)

$$(M_o)_E = (M_o)_M$$

$$\frac{9.81\, \mu_L^4}{\rho_L \gamma^3} = \frac{(1.62)(220.055\times10^{-6})^4}{(938.14)(0.0537309)^3}$$

$$\boxed{\left[\frac{\mu_L^4}{\rho_L \gamma^3}\right]_E = 2.66\times10^{-15} \frac{s^2}{m}} \quad — (2)$$

(c) From (1) we have

$$|\rho_L - \rho_g| = 25909.74\,\gamma$$

From (2)

$$\mu_L^4 = 2.66\times10^{-15} \rho_L \gamma^3$$

$$\mu_L^4 = 2.66\times10^{-15} \rho_L \left(\frac{|\rho_L - \rho_g|}{25909.74}\right)^3$$

$$\mu_L^4 = 1.53\times10^{-28} \rho_L (|\rho_L - \rho_g|)^3$$

We have to find a fluid with these properties

Problem 2.4

$$\frac{\partial \underline{u}}{\partial t} + (\underline{u}\cdot\underline{\nabla})\underline{u} = -\frac{1}{\rho}\underline{\nabla}p + \nu\nabla^2\underline{u} + f \quad (B)$$

For derivation of stokes equation we need to find non-dimensionlized form of N.S equation

Expressing variables in form of reference values

$x = Lx^*$, $t = \tau t^*$, $\underline{u} = U\underline{u}^*$

$L \to$ reference length, $\tau \to$ reference time, $U \to$ reference velocity

Also $\underline{\nabla}p = \frac{\Delta p}{L}\underline{\nabla}^* p^*$ $\quad f = Ff^*$

$\Delta p \to$ pressure difference scale $\quad F \to$ reference force

Consider the terms in eq (B)

$$\frac{\partial \underline{u}}{\partial t} = \frac{\partial(U\underline{u}^*)}{\partial(\tau t^*)} = \frac{U}{\tau}\frac{\partial \underline{u}^*}{\partial t^*}$$

$$\& \ (\underline{u}\cdot\underline{\nabla})\underline{u} = u_i\frac{\partial u_j}{\partial x_i} = Uu_i^*\frac{\partial(Uu_j^*)}{\partial(Lx_i^*)}$$

$$= \frac{U^2}{L}u_i^* \cdot \frac{\partial u_j^*}{\partial x_i^*}$$

$$= \frac{U^2}{L}\underline{u}^* \cdot \underline{\nabla}^*\underline{u}^*$$

and $-\frac{1}{\rho}\nabla p = \frac{1}{\rho}\frac{\Delta P}{L}\nabla^* p^*$

and $\nu\nabla^2\underline{U} = \nu\frac{\partial}{\partial x_j}\frac{\partial}{\partial x_j}U_i = \frac{\nu U}{L^2}\frac{\partial^2 U_i^*}{\partial x_j^*\partial x_j^*}$

$= \frac{U}{L^2}\nu\nabla_*^2 U^*$

Substituting all simplified terms into eq (B)

$$\frac{U}{\tau}\frac{\partial U^*}{\partial t^*} + \frac{U^2}{L}(\underline{U}^*\cdot\underline{\nabla}^*)\underline{U}^* = -\frac{\Delta P}{\rho L}\nabla^* p^*$$

$$+\frac{U\nu}{L^2}\nabla_*^2\underline{U}^* + F f^*$$

multiplying above with $\frac{L}{U^2}$

$$\frac{L}{\tau U}\left(\frac{\partial U^*}{\partial t^*}\right) + \left(U^*\cdot\underline{\nabla}^*\right)\underline{U}^* = -\frac{\Delta P}{\rho U^2}\nabla + \frac{\nu}{UL}\nabla_*^2\underline{U}^2$$

$$+\frac{FL}{U^2}f^*$$

introducing dimensionless numbers

$Sl = \frac{\tau U}{L}$, $Re = \frac{UL}{\nu}$ we get

$$\frac{1}{Sl}\left(\frac{\partial U^*}{\partial t^*}\right) + (U^*\cdot\underline{\nabla}^*)\underline{U}^* = -\frac{\Delta P}{\rho U^2}\cdot\nabla^* p^*$$

$$+\frac{1}{Re}\nabla_*^2\underline{U}^* + \frac{FL}{U^2}f^*$$

if $Re << 1 \Rightarrow$ Flow is dominated by viscosity and $\left(\Delta p \sim \frac{\nu U}{L}\right)$ in such cases

inertial forces are not significant
where pressure gradient and viscous forces work against each other (convective time scale is not important)

Also for such cases
$\frac{Re}{St} << 1$ as well

$\Rightarrow$ Since ratio of $\frac{Re}{\nu} << 1 \Rightarrow \frac{\partial u}{\partial t}$ becomes

negligible and ratio of viscous time scale

to convective time scale is small that implies diffusion time scale is dominant that makes term $(\underline{u}^* \cdot \underline{\nabla}^*)\underline{u}^*$ negligible

This can be observed mathematically as well if eq (v) is multiplied with Re

the force fields only play role in the modification of pressure fields (modified reduced pressure)

$\Rightarrow$ their behaviour is not dependent

on diffusive or convective time scale
Therefore it cant be neglected

Therefore eq for stokes flow can be written from non-dimensiolized form as

$$-\nabla p + \mu \nabla^2 \underline{u} + \rho f_1 = 0$$

Problem 2.5

(1a-1c) Algebraic Notation for parameters mentioned

Total nodes a specific super computer can process;

$$\boxed{N_t = N_c \times n_o \times a_v}$$

if V_t is the total volume of the computational domain

Total bubble volume is given by

$$V_{BT} = V_f \times V_t$$

If N is the total number of bubbles

$$\boxed{N V_B = V_f \times V_t}$$

$$N\left(\frac{4}{3}\pi r^3\right) = V_f \times V_t$$

$$N\left(\frac{4}{3}\pi\left(\frac{n_d}{2}\right)^3\right) = V_f \times V_t$$

$$N\left(\frac{\pi}{6} n_d^3\right) = V_f \times V_t$$

$$\boxed{N = \frac{6 V_f \times V_t}{\pi n_d^3}}$$

1.
(d)

Max Computational power

$$E_{max} = 1102 \times 10^{15} \text{ FLOPS}$$

We are given

$$1000000 \text{ Nodes} \longrightarrow 3.25 \times 10^{12} \text{ FLOPS}$$

At max power

$$N_t = \frac{1102 \times 10^{15}}{3.25 \times 10^{12}} \times 1000000$$

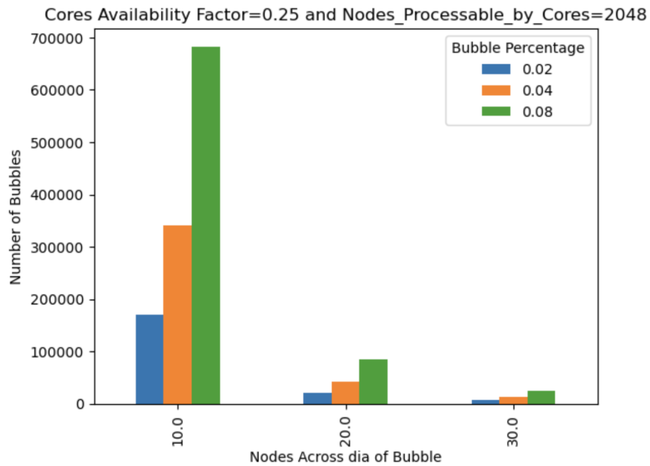
$$\boxed{N_t = 3.39 \times 10^{11}}$$

The rest of the equations remain the same parameters

Cores Availability Factor	Nodes_Processable_by_Cores	Bubble Percentage	Nodes Across dia of Bubble	Number of Bubbles
0.25	2048.0	0.02	10.0	170734.0
0.25	2048.0	0.02	20.0	21342.0
0.25	2048.0	0.02	30.0	6323.0
0.25	2048.0	0.04	10.0	341469.0
0.25	2048.0	0.04	20.0	42684.0
0.25	2048.0	0.04	30.0	12647.0
0.25	2048.0	0.08	10.0	682938.0
0.25	2048.0	0.08	20.0	85367.0
0.25	2048.0	0.08	30.0	25294.0
0.25	4096.0	0.02	10.0	341469.0
0.25	4096.0	0.02	20.0	42684.0
0.25	4096.0	0.02	30.0	12647.0
0.25	4096.0	0.04	10.0	682938.0
0.25	4096.0	0.04	20.0	85367.0
0.25	4096.0	0.04	30.0	25294.0
0.25	4096.0	0.08	10.0	1365876.0
0.25	4096.0	0.08	20.0	170734.0
0.25	4096.0	0.08	30.0	50588.0
0.25	8192.0	0.02	10.0	682938.0
0.25	8192.0	0.02	20.0	85367.0
0.25	8192.0	0.02	30.0	25294.0
0.25	8192.0	0.04	10.0	1365876.0
0.25	8192.0	0.04	20.0	170734.0
0.25	8192.0	0.04	30.0	50588.0
0.25	8192.0	0.08	10.0	2731751.0
0.25	8192.0	0.08	20.0	341469.0
0.25	8192.0	0.08	30.0	101176.0
0.5	2048.0	0.02	10.0	341469.0
0.5	2048.0	0.02	20.0	42684.0
0.5	2048.0	0.02	30.0	12647.0
0.5	2048.0	0.04	10.0	682938.0
0.5	2048.0	0.04	20.0	85367.0
0.5	2048.0	0.04	30.0	25294.0
0.5	2048.0	0.08	10.0	1365876.0
0.5	2048.0	0.08	20.0	170734.0
0.5	2048.0	0.08	30.0	50588.0
0.5	4096.0	0.02	10.0	682938.0
0.5	4096.0	0.02	20.0	85367.0
0.5	4096.0	0.02	30.0	25294.0
0.5	4096.0	0.04	10.0	1365876.0

0.5	4096.0	0.04	20.0	170734.0
0.5	4096.0	0.04	30.0	50588.0
0.5	4096.0	0.08	10.0	2731751.0
0.5	4096.0	0.08	20.0	341469.0
0.5	4096.0	0.08	30.0	101176.0
0.5	8192.0	0.02	10.0	1365876.0
0.5	8192.0	0.02	20.0	170734.0
0.5	8192.0	0.02	30.0	50588.0
0.5	8192.0	0.04	10.0	2731751.0
0.5	8192.0	0.04	20.0	341469.0
0.5	8192.0	0.04	30.0	101176.0
0.5	8192.0	0.08	10.0	5463502.0
0.5	8192.0	0.08	20.0	682938.0
0.5	8192.0	0.08	30.0	202352.0
0.75	2048.0	0.02	10.0	512203.0
0.75	2048.0	0.02	20.0	64025.0
0.75	2048.0	0.02	30.0	18970.0
0.75	2048.0	0.04	10.0	1024407.0
0.75	2048.0	0.04	20.0	128051.0
0.75	2048.0	0.04	30.0	37941.0
0.75	2048.0	0.08	10.0	2048813.0
0.75	2048.0	0.08	20.0	256102.0
0.75	2048.0	0.08	30.0	75882.0
0.75	4096.0	0.02	10.0	1024407.0
0.75	4096.0	0.02	20.0	128051.0
0.75	4096.0	0.02	30.0	37941.0
0.75	4096.0	0.04	10.0	2048813.0
0.75	4096.0	0.04	20.0	256102.0
0.75	4096.0	0.04	30.0	75882.0
0.75	4096.0	0.08	10.0	4097627.0
0.75	4096.0	0.08	20.0	512203.0
0.75	4096.0	0.08	30.0	151764.0
0.75	8192.0	0.02	10.0	2048813.0
0.75	8192.0	0.02	20.0	256102.0
0.75	8192.0	0.02	30.0	75882.0
0.75	8192.0	0.04	10.0	4097627.0
0.75	8192.0	0.04	20.0	512203.0
0.75	8192.0	0.04	30.0	151764.0
0.75	8192.0	0.08	10.0	8195253.0
0.75	8192.0	0.08	20.0	1024407.0
0.75	8192.0	0.08	30.0	303528.0
1.0	2048.0	0.02	10.0	682938.0
1.0	2048.0	0.02	20.0	85367.0

1.0	2048.0	0.02	30.0	25294.0
1.0	2048.0	0.04	10.0	1365876.0
1.0	2048.0	0.04	20.0	170734.0
1.0	2048.0	0.04	30.0	50588.0
1.0	2048.0	0.08	10.0	2731751.0
1.0	2048.0	0.08	20.0	341469.0
1.0	2048.0	0.08	30.0	101176.0
1.0	4096.0	0.02	10.0	1365876.0
1.0	4096.0	0.02	20.0	170734.0
1.0	4096.0	0.02	30.0	50588.0
1.0	4096.0	0.04	10.0	2731751.0
1.0	4096.0	0.04	20.0	341469.0
1.0	4096.0	0.04	30.0	101176.0
1.0	4096.0	0.08	10.0	5463502.0
1.0	4096.0	0.08	20.0	682938.0
1.0	4096.0	0.08	30.0	202352.0
1.0	8192.0	0.02	10.0	2731751.0
1.0	8192.0	0.02	20.0	341469.0
1.0	8192.0	0.02	30.0	101176.0
1.0	8192.0	0.04	10.0	5463502.0
1.0	8192.0	0.04	20.0	682938.0
1.0	8192.0	0.04	30.0	202352.0
1.0	8192.0	0.08	10.0	10927005.0
1.0	8192.0	0.08	20.0	1365876.0
1.0	8192.0	0.08	30.0	404704.0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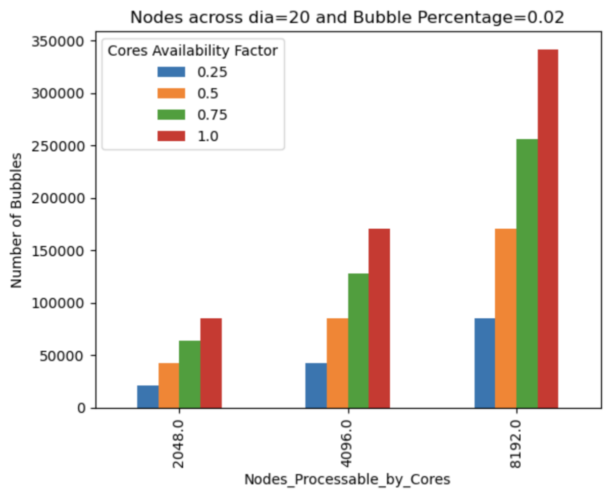
Bubble Percentage
0.02
0.04
0.08
Number of Bubbles
Nodes Across dia of Bubble
10.0
20.0
30.0
0
100000
200000
300000
400000
500000
600000
700000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Cores Availability Factor
0.25
0.5
0.75
1.0
Number of Bubbles
Nodes_Processable_by_Cores
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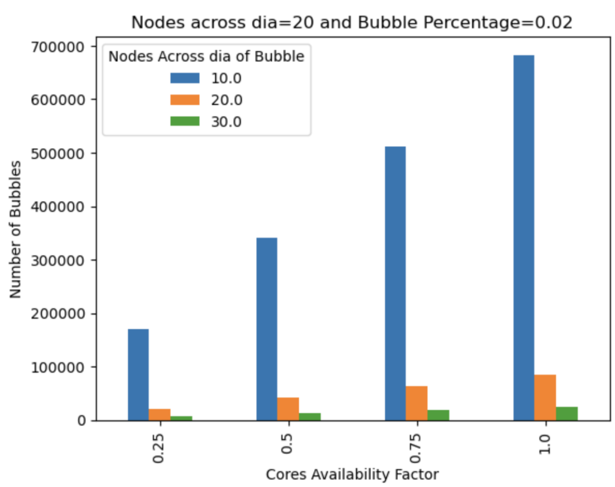
4096.0
8192.0
0
50000
100000
150000
200000
250000
300000
350000


Nodes across dia=20 and Bubble Percentage=0.02
Nodes Across dia of Bubble
10.0
20.0
30.0
Number of Bubbles
Cores Availability Factor
0.25
0.5
0.75
1.0
0
100000
200000
300000
400000
500000
600000
700000