Problem 2.2 15 points:

Consider a single bubble (D=7 mm) in a standing fluid with large temperature gradient ($\frac{dT}{dx} = G$) in zero gravity conditions.

Assume that Eotvos rule can be used to estimate water surface tension as a function of temperature:

$$\gamma = 0.07275N/m \cdot (1 - 0.002 \cdot (T - 291K))$$

(a) Compute the pressure drop between gas inside and liquid outside the bubble (Assume that the bubble is perfectly spherical with diameter D if placed in constant temperature conditions at T = 320 K.) (3 points)

Note: use Eq. (33) from the notes to compute the pressure drop. Note that the curvature value can be assumed as $\kappa = 2/R$.

(b) For a given pressure drop, the curvature of the bubble surface will locally change due to different local surface tension. Derive an expression for the curvature if the temperature of the fluid changes (6 points):

- from 270K to 320K across the bubble
- from 270K to 370K across the bubble

Note: use the pressure drop obtained in (a) and derive the equation for the bubble shape in 2D for demonstrating the temperature increase with θ, i.e., from $\mathrm{T_1}$ to $\mathrm{T_2}$:

for $\theta \in [0, \pi]$, $\mathrm{T}(\theta) = \mathrm{T_1} + (\mathrm{T_2} - \mathrm{T_1})\frac{\theta}{\pi}$;

for $\theta \in [\pi, 2\pi]$, $\mathrm{T}(\theta) = \mathrm{T_2} - (\mathrm{T_2} - \mathrm{T_1})\frac{\theta - \pi}{\pi}$.

(c) Use your favorite math tool to plot the 2D bubble shape for both cases in part **(b)** as well as compare to the spherical counterpart. **Discuss** the results (6 points).

Solution:

(a) $\Delta p = \frac{2\gamma}{R}$

where $\gamma = 0.07275N/m \cdot (1 - 0.002 \cdot (T - 291K))$

Reference for the Eötvös rule: https://en.wikipedia.org/wiki/E%C3%B6tv%C3%B6s_rule

As T=320k and R=0.004m,

$$\Delta p = \frac{2 \cdot 0.07275 \cdot (1 - 0.002 \cdot (320 - 291))}{0.0035} Pa$$

$$\Longrightarrow \Delta p = 39.16\ Pa$$

(b)

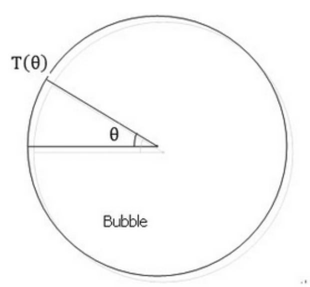


Fig. 1 Illustration for temperature changing with θ

(i) from 270K to 320K

$$\mathrm{T}(\theta) = 270 + 50\frac{\theta}{\pi} \qquad \theta \in [0, \pi]$$

$$\mathrm{T}(\theta) = 320 - 50\frac{\theta - \pi}{\pi} \qquad \theta \in [\pi, 2\,\pi]$$

$$\mathrm{R}_1(\theta) = \frac{2\gamma}{\Delta \mathrm{p}} = \frac{2 \cdot 0.07275 \cdot (1 - 0.002 \cdot (270 + 50\frac{\theta}{\pi} - 291))}{39.16} \qquad \theta \in [0, \pi]$$

$$\mathrm{R}_1(\theta) = \frac{2\gamma}{\Delta \mathrm{p}} = \frac{2 \cdot 0.07275 \cdot (1 - 0.002 \cdot (320 - 50\frac{\theta - \pi}{\pi} - 291))}{39.16} \qquad \theta \in [\pi, 2\,\pi]$$

(ii) from 270K to 370K

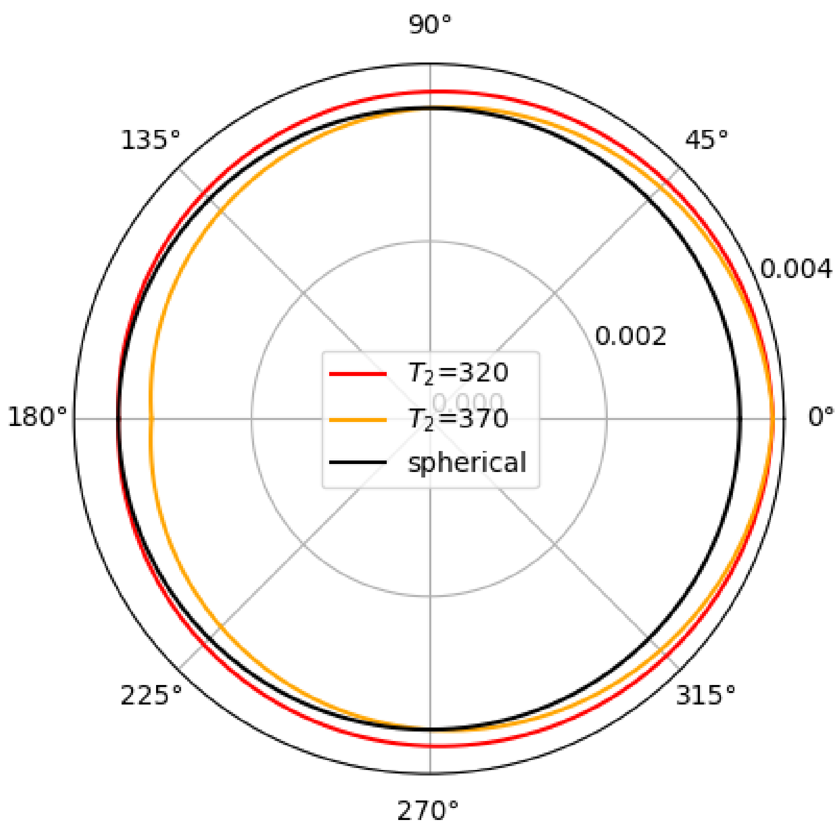
$$\mathrm{T}(\theta) = 270 + 100\frac{\theta}{\pi} \qquad \theta \in [0, \pi]$$

$$\mathrm{T}(\theta) = 370 - 100\frac{\theta - \pi}{\pi} \qquad \theta \in [\pi, 2\,\pi]$$

$$\mathrm{R}_2(\theta) = \frac{2\gamma}{\Delta \mathrm{p}} = \frac{2 \cdot 0.07275 \cdot (1 - 0.002 \cdot (270 + 100\frac{\theta}{\pi} - 291))}{39.16} \qquad \theta \in [0, \pi]$$

$$R_2(\theta) = \frac{2\gamma}{\Delta p} = \frac{2 \cdot 0.07275 \cdot \left(1 - 0.002 \cdot \left(370 - 100\frac{\theta - \pi}{\pi} - 291\right)\right)}{39.16} \qquad \theta \in [\pi, 2\pi]$$

(c):



<u>Discussion:</u>

As temperature decreases, the surface tension coefficient increases. If with the **same pressure drop** across the interface, the bubble needs to become larger to balance with the surface coefficient incensement.

As the temperature difference across the bubble increases, the bubble becomes more deformable compared with the original bubble.