Homework #8 (4 problems; 60 points)

due: April 21st, 2023.

Problem 8.1 [10]	Problem 8.2 [20]	Problem 8.3 [20]	Problem 8.4 [10]	Total [60]

Problem 8.1(extra credits) 10 points:

Consider a steady-state, fully developed channel flow. At the wall (y=0), the velocity is zero and pressure is p_w. Show that mean axial pressure gradient is uniform across the flow: $\frac{\partial\langle p\rangle}{\partial x} = \frac{dp_w}{dx}$.

Hint: write the lateral mean-momentum equation and integrate it in y -direction analytically.

Problem 8.2 20 points:

Derive the exact transport equation for turbulent kinetic energy (TKE), k (Eq. (10.35) in lecture notes).

Use the following steps:

a) Subtract the Reynolds equations (momentum written for mean velocities) from N.S. momentum equations, thus obtain the equation for fluctuating velocity [10 points].

b) Obtain a scalar product of fluctuating velocity and the vector-equation you got in part a) and apply Reynolds averaging to the result [10 points].

The following parameters are given for both Problem 8.3 and Problem 8.4:
$C_D = 0.49$ for spherical bubble, $\rho_l = 999.21\ \frac{kg}{m^3}$, $\rho_g = 1.205\ \frac{kg}{m^3}$, $\mu_l = 1.0 \cdot 10^{-3}\ \mathrm{Pa \cdot s}$ and, $\gamma = 7.27 \cdot 10^{-2}\ \mathrm{N/m}$

Problem 8.3 20 points:

Consider a steady-state air bubble rise in infinite standing water.

a) Develop an expression for the rise velocity (also the relative velocity) v_r based on drag/buoyancy force balance. [5 points]

Expressions for drag force and buoyancy are:

$$\vec{F}_D = \frac{1}{2} C_D A_x \rho_l v_r^2$$

$$\vec{F}_B = (\rho_l - \rho_g) g V$$

where C_D, A_x, v_r, g, V are drag coefficient, cross sectional area of the spherical bubble, relative velocity, gravitational acceleration, and volume of the bubble, respectively.

b) Assuming the bubble preserved spherical shape, plot the rise velocity dependence on bubble diameter. Use the bubble diameter range from 0.5 mm to 30 mm. [5 points]

c) Assume variable drag coefficient according to this expression:

$$C_D = \sqrt{\left[\frac{16}{Re_b}\left(1 + \frac{2}{1 + \frac{16}{Re_b} + \frac{3.315}{\sqrt{Re_b}}}\right)\right]^2 + \left(\frac{4\,Eo}{Eo + 9.5}\right)^2}$$

Use bubble Reynolds number (Re_b) and Eotvos number based on bubble diameter and air/water surface tension. Re-plot the rise velocity dependence. [5 points]

Expression for Re_b:

$$Re_b = \frac{\rho_l D_b v_r}{\mu_l}$$

where D_b is the bubble diameter

Note: use v_r in Re_b and solve the implicit equation iteratively. Submit your code.

d) Compare the results in b) and c) and **discuss** them. [5 points]

Problem 8.4 10 points:

Consider a two-phase air/water bubbly flow at atmospheric conditions. Assume that there are three groups of spherical bubbles:

1. Group 1: Mean diameter of 1 mm and volume fraction of 3%
2. Group 2: Mean diameter of 1.5 mm and volume fraction of 1%
3. Group 3: Mean diameter of 2.0 mm and volume fraction of 1%

Use the two-phase turbulent viscosity contribution proposed by Sato & Sekoguchi (1975)

$$v_{2\phi} = 0.6 D_{dv} \alpha_{dv} |v_r|$$

$$v_r = \sqrt{\frac{4(\rho_l - \rho_g) D_{dv} g}{3 C_D \rho_l}}$$

to evaluate:

a) Two-phase viscosity contribution from each group. And normalize $v_{2\phi}$ by kinematic viscosity of the water, i.e., $v_{2\phi}/v_{cl}$. Use $\nu_{cl} = 1.05 \cdot 10^{-6}\ m^2/s$. [5 points]

b) Assume there is a single group with void fraction of 5% containing all the bubbles from Groups 1,2 and 3. Estimate the equivalent diameter and corresponding two-phase viscosity contribution. **Discuss** if $v_{2\phi}$ should be the same as those in part a). [5 points]