

Problem 5.1 10 points:

Derive boundary conditions for staggered grid arrangement for velocities and pressure at a corner node of 2D problem. Assume same value velocity at both walls adjacent to the corner.

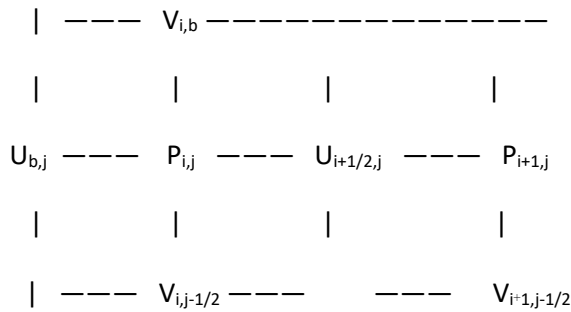
[8 points]

Please sketch the staggered grid. [2 points]

Note: substitute (2.19) written for $u_{i-\frac{1}{2},j}^*$, $v_{i,j+\frac{1}{2}}^*$ into (2.20) and using $u_{i-\frac{1}{2},j}^{n+1} = u_{b,j}$, $v_{i,j+\frac{1}{2}}^{n+1} = v_{i,b}$.

Solution:

The staggered grid used here is:



The predictor step for $u_{i-\frac{1}{2},j}^*$, and $v_{i,j+\frac{1}{2}}^*$:

$$u_{i-\frac{1}{2},j}^{n+1} = u_{i-\frac{1}{2},j}^* - \frac{1}{\rho} \frac{\Delta t}{h} (P_{i,j}^{n+1} - P_{i-1,j}^{n+1})$$

$$v_{i,j+\frac{1}{2}}^{n+1} = v_{i,j+\frac{1}{2}}^* - \frac{1}{\rho} \frac{\Delta t}{h} (P_{i,j+1}^{n+1} - P_{i,j}^{n+1})$$

Plug above into eq. 2.20:

$$\frac{P_{i+1,j}^{n+1} + P_{i-1,j}^{n+1} + P_{i,j+1}^{n+1} + P_{i,j-1}^{n+1} - 4P_{i,j}^{n+1}}{h^2} = \frac{1}{\rho} \frac{\Delta t}{h} \left(u_{i+\frac{1}{2},j}^* - u_{i-\frac{1}{2},j}^{n+1} - \frac{1}{\rho} \frac{\Delta t}{h} (P_{i,j}^{n+1} - P_{i-1,j}^{n+1}) + v_{i,j+\frac{1}{2}}^{n+1} + \frac{1}{\rho} \frac{\Delta t}{h} (P_{i,j+1}^{n+1} - P_{i,j}^{n+1}) - v_{i,j-\frac{1}{2}}^* \right)$$

Given $u_{i-\frac{1}{2},j}^{n+1} = u_{b,j} = 0$, $v_{i,j+\frac{1}{2}}^{n+1} = v_{i,b} = 0$, rearrange above, we obtain:

$$\Rightarrow \frac{P_{i+1,j}^{n+1} + P_{i,j-1}^{n+1} - 2 \cdot P_{i,j}^{n+1}}{h^2} = \frac{\rho}{\Delta t} \frac{u_{i+\frac{1}{2},j}^* - v_{i,j-\frac{1}{2}}^*}{h}$$