

Assignment #2
MAE-577

Prob 2) $\frac{\partial S}{\partial t} + \mathbf{w} \cdot \nabla S = 0$ on $S = 0$

From taylor expression

$$f(x+\Delta x, t+\Delta t) = f(x, t) + \Delta x \frac{\partial f}{\partial x} + \Delta t \frac{\partial f}{\partial t} + \dots$$

We can write

$$S(x+\Delta x, t+\Delta t) \approx S(x, t) + \Delta x \frac{\partial S}{\partial x} + \Delta t \frac{\partial S}{\partial t}$$

$$S = 0$$

$$0 = \Delta x \frac{\partial S}{\partial x} + \Delta t \frac{\partial S}{\partial t}$$

$$0 = w \Delta t \frac{\partial S}{\partial x} + \Delta t \frac{\partial S}{\partial t}$$

Dividing by Δt

$$\frac{\partial S}{\partial t} + w \frac{\partial S}{\partial x} = 0$$

In vector form

$$\boxed{\frac{\partial S}{\partial t} + \mathbf{w} \cdot \nabla S = 0}$$

Prob 2.2

(a) If perfectly spherical in constant temp conditions

$$\Delta P = YK$$

$$K = \frac{\Delta P}{Y}$$

$$K = \frac{\Delta P}{0.07275(1 - 0.002(T - 291))}$$

$$\Delta P = \frac{2}{3.5 \times 10^5} \times 0.07275(1 - 0.002(320 - 291))$$

$$\Rightarrow \boxed{\Delta P = 39.16 \text{ Pa}}$$

2.2

(b)

$$\theta \in [0, \pi], T(\theta) = T_1 + (T_2 - T_1) \frac{\theta}{\pi}$$

$$\theta \in [\pi, 2\pi], T(\theta) = T_2 - (T_2 - T_1) \frac{\theta - \pi}{\pi}$$

- (i) from 270 - 320 K

$$K = \frac{39.16}{0.07275(1 - 0.002(T - 291))}$$

where

$$T = T\theta = \begin{cases} T_1 + \frac{T_2 - T_1}{\pi} \theta & (0, \pi) \\ T_2 - (T_2 - T_1) \frac{\theta - \pi}{\pi} & (\pi, 2\pi) \end{cases}$$

$$\begin{cases} 270 + \frac{50\theta}{\pi} & 0, \pi \\ 320 - \frac{50\theta}{\pi} & \pi, 2\pi \end{cases}$$

(ii) from $270 - 370$

$$T(\theta) \begin{cases} 270 + \frac{100\theta}{\pi} & 0, \pi \\ 270 - \frac{100\theta}{\pi} & \pi, 2\pi \end{cases}$$

$$K = \frac{39.16}{0.07275(1 - 0.002(T(\theta) - 291))}$$

Problem 2.3

Given conditions

$$P = 240 \text{ kPa} = 2.4 \text{ bar}$$

From steam tables

$$T_{sat} = 126.072^\circ\text{C} = 399.22 \text{ K}$$

$$f_g = 1.3393 \frac{\text{kg}}{\text{m}^3} \quad f_L = 938.14 \frac{\text{kg}}{\text{m}^3}$$

$$N_L = 220.053 \text{ N Pas} \quad \gamma = 0.0537369$$

No air

$$g_E = 9.81 \frac{\text{m}}{\text{s}^2} \rightarrow g_m = 1.62 \frac{\text{m}}{\text{s}^2} \text{ and } L_m = 3(L_E)$$

Part (a)

$$(E_o^{\circ})_E = (E_o^{\circ})_m$$

$$\left(\frac{|f_L - f_g| 9.81 (L_E)^2}{\gamma} \right)_E = \left(\frac{[938.14 - 1.3393] (1.62) (3L_E)^2}{0.0537369} \right)_m$$

$$\left(\frac{|f_L - f_g|}{\gamma} \right)_E = 25909.74 \frac{\text{kg}}{\text{Nm}^2} \quad \text{--- (1)}$$

Part(b)

$$(M_0)_E = (M_0)_M$$

$$\frac{9.81 N_L^4}{\int_L V^3} = (1.62)(220.055 \times 10^{-6})^4$$
$$\frac{(938.14)(0.0537369)^3}{}$$

$$\left(\left[\frac{N_L^4}{\int_L V^3} \right]_E = 2.66 \times 10^{-15} \frac{s^2}{m} \right) - \textcircled{2}$$

(c) From ① we have

$$|\int_L - \int_g| = 25909.74 \gamma$$

From ②

$$N_L^4 = 2.66 \times 10^{-15} \int_L V^3$$

$$N_L^4 = 2.66 \times 10^{-15} \int_L \left(\frac{|\int_L - \int_g|}{25909.74} \right)^3$$

$$N_L^4 = 1.53 \times 10^{-28} \int_L (|\int_L - \int_g|)^3$$

We have to find a fluid with these properties

Notebook - hw2

Problem 2.4

$$\frac{\partial \underline{U}}{\partial t} + (\underline{U} \cdot \nabla) \underline{U} = -\frac{1}{\rho} \nabla P + \nu \nabla^2 \underline{U} + \underline{f} \quad (B)$$

For derivation of Stokes equation we need to find non-dimensionalized form of N-S equation

Expressing variables in form of reference values

$$x = L x^* \rightarrow t = T t^* \rightarrow \underline{U} = U \underline{U}^*$$

$L \rightarrow$ reference length $T \rightarrow$ reference time $U \rightarrow$ reference velocity

$$\text{Also } \nabla P = \frac{\Delta P}{L} \nabla^2 \underline{U}^* \quad \underline{f} = \underline{F} f^*$$

$\Delta P \rightarrow$ pressure difference scale

$F \rightarrow$ reference force

Consider the terms in eq (B)

$$\frac{\partial \underline{U}}{\partial t} = \frac{\partial (U \underline{U}^*)}{\partial (T t^*)} = \frac{\underline{U}}{T} \frac{\partial \underline{U}^*}{\partial t^*}$$

$$\text{if } (\underline{U} \cdot \nabla) \underline{U} = U_i \frac{\partial U_j}{\partial x_i} = U_i U_j^* \frac{\partial (U U_j^*)}{\partial (L x_i^*)}$$

$$\begin{aligned} &= \frac{U^2}{L} U_i \cdot \frac{\partial U_j^*}{\partial x_i^*} \\ &= \frac{U^2}{L} \underline{U} \cdot \nabla^* \underline{U}^* \end{aligned}$$

$$\text{and } -\frac{1}{f} \nabla p = \frac{1}{f} \frac{\Delta P}{L} \nabla^* p^*$$

$$\text{and } \nu \nabla^2 \underline{U} = \nu \frac{\partial}{\partial x_j} \frac{\partial}{\partial x_j} U_0 = \nu \frac{U}{L^2} \frac{\partial^2 U^*}{\partial x_j \partial x_j^*}$$

$$= \frac{U}{L^2} \nu \nabla^2 U^*$$

Substituting all simplified terms into eq(B)

$$\frac{U}{\nu} \frac{\partial U^*}{\partial t^*} + \frac{U^2}{L} (U^* \cdot \nabla^*) U^* = -\frac{\Delta P}{f L} \nabla^* p^*$$

$$+ \frac{\nu U}{L^2} \nabla^2 U^* + f f^*$$

Multiplying above with $\frac{L}{U^2}$

$$\frac{L}{U} \left(\frac{\partial U^*}{\partial t^*} \right) + (U^* \cdot \nabla^*) U^* = -\frac{\Delta P}{f U^2} D + \frac{\nu}{U L} \nabla^2 U^*$$

$$+ \frac{f L}{U^2} f^*$$

introducing dimensionless numbers

$$SL = \frac{\tau U}{L}, \quad Re = \frac{UL}{\nu} \text{ we get}$$

$$\frac{1}{SL} \left(\frac{\partial U^*}{\partial t^*} \right) + (U^* \cdot \nabla^*) U^* = -\frac{\Delta P}{f U^2} \nabla^* p^*$$

$$+ \frac{L}{Re} \nabla^2 U^* + \frac{f L}{U^2} f^*$$

if $Re \ll 1 \Rightarrow$ Flow is dominated by viscosity and $\Delta P \sim \nu U$ in such cases

Inertial forces are not significant where pressure gradient and viscous forces work against each other (convective time scale is not important)

Also for such cases

$$\frac{Re}{Sc} \ll 1 \text{ as well}$$

\Rightarrow Since ratio of $\frac{Re}{N} \ll 1 \Rightarrow \frac{\partial U}{\partial t}$ becomes negligible and ratio of viscous time scale to convective time scale is small that implies diffusion time scale is dominant that makes term $(U^*, D^*) U^*$ negligible

This can be observed mathematically as well if eq(1) is multiplied with Re

The force fields only play role in the modification of pressure fields (modified reduced pressure)

\Rightarrow their behaviour is not dependent

on diffusive or convective time scale
Therefore it can't be neglected

Therefore eq for stokes flow can be written from non-dimensionalized form as

$$-\nabla p + N D^2 \underline{u} + f_f = 0$$

Problem 2.5

(1a)
 $\underline{-1c})$ Algebraic Notation for parameters mentioned

Total nodes a specific supercomputer can process;

$$N_t = N_c \times n_o \times a_v$$

if V_f is the total volume of the computational domain

Total bubble volume is given by

$$V_{BT} = V_p \times V_t$$

if N is the total number of bubbles

$$NV_B = V_p \times V_t$$

$$N \left(\frac{4}{3} \pi r^3 \right) = V_p \times V_t$$

$$N \left(\frac{4}{3} \pi \left(\frac{\pi d}{2} \right)^3 \right) = V_p \times V_t$$

$$N \left(\frac{1}{6} \pi r_d^3 \right) = V_p \times V_t$$

$$N = \frac{6 V_p \times V_t}{\pi r_d^3}$$

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(d)

Max Computational power

$$\bar{E}_{\max} = 1102 \times 10^{15} \text{ FLOPS}$$

We are given

$$1000000 \text{ nodes} \rightarrow 3.25 \times 10^{12} \text{ FLOPS}$$

At max power

$$N_t = \frac{1102 \times 10^{15}}{3.25 \times 10^{12}} \times 1000000$$
$$N_t = 3.39 \times 10^{11}$$

The rest of the equations remain the same

parameters

Cores Availability Factor	Nodes_Processable_by_Cores	Bubble Percentage	Nodes Across dia of Bubble	Number of Bubbles
0.25	2048.0	0.02	10.0	170734.0
0.25	2048.0	0.02	20.0	21342.0
0.25	2048.0	0.02	30.0	6323.0
0.25	2048.0	0.04	10.0	341469.0
0.25	2048.0	0.04	20.0	42684.0
0.25	2048.0	0.04	30.0	12647.0
0.25	2048.0	0.08	10.0	682938.0
0.25	2048.0	0.08	20.0	85367.0
0.25	2048.0	0.08	30.0	25294.0
0.25	4096.0	0.02	10.0	341469.0
0.25	4096.0	0.02	20.0	42684.0
0.25	4096.0	0.02	30.0	12647.0
0.25	4096.0	0.04	10.0	682938.0
0.25	4096.0	0.04	20.0	85367.0
0.25	4096.0	0.04	30.0	25294.0
0.25	4096.0	0.08	10.0	1365876.0
0.25	4096.0	0.08	20.0	170734.0
0.25	4096.0	0.08	30.0	50588.0
0.25	8192.0	0.02	10.0	682938.0
0.25	8192.0	0.02	20.0	85367.0
0.25	8192.0	0.02	30.0	25294.0
0.25	8192.0	0.04	10.0	1365876.0
0.25	8192.0	0.04	20.0	170734.0
0.25	8192.0	0.04	30.0	50588.0
0.25	8192.0	0.08	10.0	2731751.0
0.25	8192.0	0.08	20.0	341469.0
0.25	8192.0	0.08	30.0	101176.0
0.5	2048.0	0.02	10.0	341469.0
0.5	2048.0	0.02	20.0	42684.0
0.5	2048.0	0.02	30.0	12647.0
0.5	2048.0	0.04	10.0	682938.0
0.5	2048.0	0.04	20.0	85367.0
0.5	2048.0	0.04	30.0	25294.0
0.5	2048.0	0.08	10.0	1365876.0
0.5	2048.0	0.08	20.0	170734.0
0.5	2048.0	0.08	30.0	50588.0
0.5	4096.0	0.02	10.0	682938.0
0.5	4096.0	0.02	20.0	85367.0
0.5	4096.0	0.02	30.0	25294.0
0.5	4096.0	0.04	10.0	1365876.0

0.5	4096.0	0.04	20.0	170734.0
0.5	4096.0	0.04	30.0	50588.0
0.5	4096.0	0.08	10.0	2731751.0
0.5	4096.0	0.08	20.0	341469.0
0.5	4096.0	0.08	30.0	101176.0
0.5	8192.0	0.02	10.0	1365876.0
0.5	8192.0	0.02	20.0	170734.0
0.5	8192.0	0.02	30.0	50588.0
0.5	8192.0	0.04	10.0	2731751.0
0.5	8192.0	0.04	20.0	341469.0
0.5	8192.0	0.04	30.0	101176.0
0.5	8192.0	0.08	10.0	5463502.0
0.5	8192.0	0.08	20.0	682938.0
0.5	8192.0	0.08	30.0	202352.0
0.75	2048.0	0.02	10.0	512203.0
0.75	2048.0	0.02	20.0	64025.0
0.75	2048.0	0.02	30.0	18970.0
0.75	2048.0	0.04	10.0	1024407.0
0.75	2048.0	0.04	20.0	128051.0
0.75	2048.0	0.04	30.0	37941.0
0.75	2048.0	0.08	10.0	2048813.0
0.75	2048.0	0.08	20.0	256102.0
0.75	2048.0	0.08	30.0	75882.0
0.75	4096.0	0.02	10.0	1024407.0
0.75	4096.0	0.02	20.0	128051.0
0.75	4096.0	0.02	30.0	37941.0
0.75	4096.0	0.04	10.0	2048813.0
0.75	4096.0	0.04	20.0	256102.0
0.75	4096.0	0.04	30.0	75882.0
0.75	4096.0	0.08	10.0	4097627.0
0.75	4096.0	0.08	20.0	512203.0
0.75	4096.0	0.08	30.0	151764.0
0.75	8192.0	0.02	10.0	2048813.0
0.75	8192.0	0.02	20.0	256102.0
0.75	8192.0	0.02	30.0	75882.0
0.75	8192.0	0.04	10.0	4097627.0
0.75	8192.0	0.04	20.0	512203.0
0.75	8192.0	0.04	30.0	151764.0
0.75	8192.0	0.08	10.0	8195253.0
0.75	8192.0	0.08	20.0	1024407.0
0.75	8192.0	0.08	30.0	303528.0
1.0	2048.0	0.02	10.0	682938.0
1.0	2048.0	0.02	20.0	85367.0

1.0	2048.0	0.02	30.0	25294.0
1.0	2048.0	0.04	10.0	1365876.0
1.0	2048.0	0.04	20.0	170734.0
1.0	2048.0	0.04	30.0	50588.0
1.0	2048.0	0.08	10.0	2731751.0
1.0	2048.0	0.08	20.0	341469.0
1.0	2048.0	0.08	30.0	101176.0
1.0	4096.0	0.02	10.0	1365876.0
1.0	4096.0	0.02	20.0	170734.0
1.0	4096.0	0.02	30.0	50588.0
1.0	4096.0	0.04	10.0	2731751.0
1.0	4096.0	0.04	20.0	341469.0
1.0	4096.0	0.04	30.0	101176.0
1.0	4096.0	0.08	10.0	5463502.0
1.0	4096.0	0.08	20.0	682938.0
1.0	4096.0	0.08	30.0	202352.0
1.0	8192.0	0.02	10.0	2731751.0
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1.0	8192.0	0.02	30.0	101176.0
1.0	8192.0	0.04	10.0	5463502.0
1.0	8192.0	0.04	20.0	682938.0
1.0	8192.0	0.04	30.0	202352.0
1.0	8192.0	0.08	10.0	10927005.0
1.0	8192.0	0.08	20.0	1365876.0
1.0	8192.0	0.08	30.0	404704.0



