Problem 8.2 10 points:

Consider a two-phase air/water bubbly flow at atmospheric conditions. Assume that there are three groups of spherical bubbles:

1. Group 1: Mean diameter of 1 mm and volume fraction of 3%
2. Group 2: Mean diameter of 1.5 mm and volume fraction of 1%
3. Group 3: Mean diameter of 2.0 mm and volume fraction of 1%

Use the two-phase turbulent viscosity contribution proposed by Sato & Sekoguchi (1975)

$$\upsilon_{2\phi} = 0.6 D_{dv} \alpha_{dv} |v_r|$$

to evaluate:

a) Two-phase viscosity contribution from each group. And normalize $\upsilon_{2\phi}$ by kinematic viscosity of the water, i.e. $\upsilon_{2\phi}/\upsilon_{cl}$. Use $\nu_{cl} = 1.0 \cdot 10^{-6}\ m^2/s$.

b) Assume there is a single group with void fraction of 5% containing all the bubbles from Groups 1,2 and 3. Estimate the equivalent diameter and corresponding two-phase viscosity contribution. Discuss if $\upsilon_{2\phi}$ should be the same as those in part a).

Solution:

(a)

$$v_r = \sqrt{\frac{4(\rho_l - \rho_g) D_{dv} \cdot \mathrm{g}}{3 C_D \rho_l}}$$

Under the condition of room temperature and 1 atm,

$$\rho_\mathrm{l} = 998.21\ \frac{\mathrm{kg}}{\mathrm{m}^3}$$

$$\rho_\mathrm{g} = 1.205\ \frac{\mathrm{kg}}{\mathrm{m}^3}$$

$$\mathrm{v}_{r1} = \sqrt{\frac{4(\rho_l - \rho_g) D_{dv1} \cdot \mathrm{g}}{3 C_D \rho_l}} = 0.1666\ m/s$$

$$\mathrm{v}_{r2} = \sqrt{\frac{4(\rho_l - \rho_g) D_{dv2} \cdot \mathrm{g}}{3 C_D \rho_l}} = 0.2041\ m/s$$

$$\mathrm{v}_{r3} = \sqrt{\frac{4(\rho_l - \rho_g) D_{dv3} \cdot \mathrm{g}}{3 C_D \rho_l}} = 0.2357\ m/s$$

The Kinematic viscosity of the liquid: $\nu_\mathrm{cl} = 1.0 \cdot 10^{-6}\ \mathrm{m}^2/\mathrm{s}$

Using the assumption by Sato & Sekoguchi (1975)

$\nu_{2\phi 1} = 0.6 \cdot \mathrm{D_{dv1}} \cdot \alpha_{\mathrm{dv1}} \cdot \mathrm{v}_{r1} = 2.999 \cdot 10^{-6}\ \mathrm{m^2/s}$

$\nu_{2\phi 2} = 0.6 \cdot \mathrm{D_{dv2}} \cdot \alpha_{\mathrm{dv2}} \cdot \mathrm{v}_{r2} = 1.837 \cdot 10^{-6}\ \mathrm{m^2/s}$

$\nu_{2\phi 3} = 0.6 \cdot \mathrm{D_{dv3}} \cdot \alpha_{\mathrm{dv3}} \cdot \mathrm{v}_{r3} = 2.828 \cdot 10^{-6}\ \mathrm{m^2/s}$

The normalized viscosity contributions are then just:

$$\frac{\nu_{2\phi 1}}{\nu_{\mathrm{cl}}} = 2.9941$$

$$\frac{\nu_{2\phi 2}}{\nu_{\mathrm{cl}}} = 1.8335$$

$$\frac{\nu_{2\phi 3}}{\nu_{\mathrm{cl}}} = 2.8229$$

(b)

- Method 1:

$\alpha = 5\%$

$D_{dve} = \frac{3}{5} \cdot \mathrm{D_{dv1}} + \frac{1}{5} \cdot \mathrm{D_{dv2}} + \frac{1}{5} \cdot \mathrm{D_{dv3}}$

$= 1.3\ \mathrm{mm}$

$$\mathrm{v_{re}} = \sqrt{\frac{4(\rho_l - \rho_g) D_{dve} \cdot \mathrm{g}}{3 C_D \rho_l}} = 0.19\ \mathrm{m/s}$$

$$\nu_{2\phi \mathrm{e}} = 0.6 \cdot \mathrm{D_{dve}} \cdot \alpha_{\mathrm{dve}} \cdot \mathrm{v}_{r3} = 7.41 \cdot 10^{-6}\ \mathrm{m^2/s}$$

$$\frac{\nu_{2\phi \mathrm{e}}}{\nu_{\mathrm{cl}}} = 7.3966$$

- Method 2:

Use the volume average to find the equivalent bubble diameter.

Assume the domain volume to be V, the bubble volume in each group to be $\mathrm{V_i}(i = 1, 2{,}3)$, the number of bubbles in each group to be N_i, and the equivalent bubble volume to be V_{eq}.

$$N_1 V_1 + N_2 V_2 + N_3 V_3 = (N_1 + N_2 + N_3) V_{eq} = 5\% V$$

and $N_1V_1 = 3\%V$, $N_2V_2 = 1\%V$, and $N_3V_3 = 1\%V$

thus $\frac{3\%}{V_1} + \frac{1\%}{V_2} + \frac{1\%}{V_3} = \frac{5\%}{V_{eq}}$

therefore $V_{eq} = 7.65 \cdot 10^{-10}\ \mathrm{m}^3$

$$D_{eq} = \sqrt[3]{\frac{6V_{eq}}{\pi}} = 1.13mm$$

$$\mathrm{v_{req}} = \sqrt{\frac{4(\rho_l - \rho_g)D_{eq} \cdot \mathrm{g}}{3C_D\rho_l}} = 0.1775$$

$$\nu_{2\phi\mathrm{eq}} = 0.6 \cdot \mathrm{D_{eq}} \cdot \alpha_{\mathrm{dveq}} \cdot \mathrm{v}_{req} = 6.043 \cdot 10^{-6}\ \mathrm{m^2/s}$$

$$\frac{\nu_{2\phi\mathrm{eq}}}{\nu_{\mathrm{cl}}} = 6.0326$$

Discussion: the equivalent diameter and viscosity contribution evaluated by either Method 1 or Method 2 is not the same as those from part a). From the formula given, it can be seen that the production of the bubble diameter and the void fraction has a great influence on the viscosity of the two-phase flow. Therefore, the resulting viscosity contribution is larger than the group 1, group 2, and group 3.