NC STATE UNIVERSITY

ENGINEERING ONLINE

Lecture Notes

Course Number: 577

Instructor: Bolotnov

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① The momentum eq. (9) becomes:

$$\frac{1}{Sl}\cdot\frac{\partial \underline{u}^*}{\partial t^*} + (\underline{u}^*\cdot\underline{\nabla}_*)\,\underline{u}^* = -\frac{\Delta p}{\rho U^2}\cdot\underline{\nabla}_* p^* + \frac{1}{Re}\nabla_*^2\underline{u}^* + \frac{\mathcal{F}L}{U^2}\underline{f}^* \quad (4)$$

where

$$Sl = \frac{U\tau}{L} \quad (5)$$

is the Strouhal number which represents the ratio of intrinsic time scale τ to the convective timescale: L/U. If no external or imposed t/s present, then $Sl = 1.0$.

$$Re = \frac{\rho L U}{\mu} = \frac{L U}{\nu} \quad (6)$$

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② When $\underline{f}$ is gravity, $\mathcal{F} = |g|$;

$$Fr = \frac{U^2}{gL} \qquad (7)$$

is the Froude number.

Pressure difference depends on a situation.

If fluid inertia is important: $\Delta P \sim \rho U^2$

$$\frac{1}{St} \cdot \frac{\partial \underline{u}^*}{\partial t^*} + (\underline{u}^* \cdot \underline{\nabla}_*)\underline{u}_* = -\underline{\nabla}_* p^* + \frac{1}{Re}\nabla_*^2 \underline{u}_* + \frac{fL}{U^2}\underline{f}_* \qquad (8)$$

Frequently $St = 1$:

(9)

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③ If the flow is dominated by viscosity

$$\Delta P = \frac{\mu U}{L}:$$

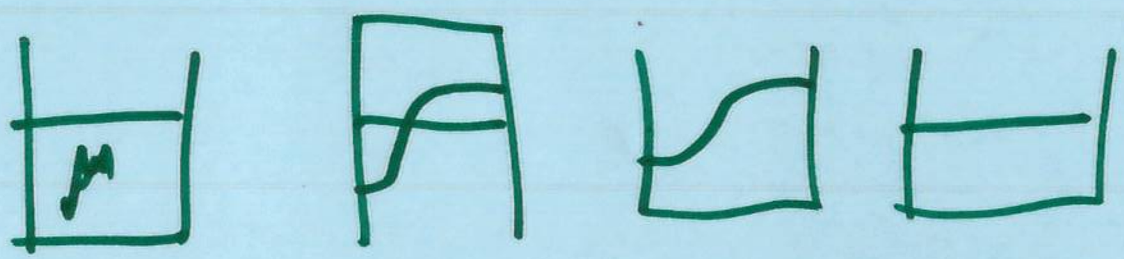
$$\frac{1}{St}\frac{\partial \underline{u}^*}{\partial t^*} + (\underline{u}_*^* \cdot \underline{\nabla}_*)\underline{u}_*^* = -\frac{1}{Re}\underline{\nabla}_* p^* + \frac{1}{Re}\nabla_*^2 u_* + \frac{fL}{V^2}\underline{f}^* \qquad (10)$$

A special equation for $Re \ll 1$;

$$\frac{Re}{St} = \frac{L^2}{\nu\tau} \ll 1:$$

$$-\underline{\nabla} p + \mu \nabla^2 \underline{u} + \rho \underline{f} = 0 \qquad (11)$$

which is known as Stokes equation. (w eq(3))



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4 For inertia dominated pressure scaling ($\Delta P = \rho U^2$) the normal stress condition:

$$-P_2^* + P_1^* + \frac{1}{Re}\underline{n}\cdot\left(\underline{\underline{\tau}}_2^* - \underline{\underline{\tau}}_1^*\right)\cdot\underline{n} = \frac{1}{We}K_* \quad (12)$$

where $K_* = LK$

and $We = \dfrac{\rho L U^2}{\gamma}$ (13)

is the Weber number.

Sometimes U is governed by buoyancy:

$$U \sim \sqrt{\frac{|\rho - \rho'|}{\rho} gL} \quad (13a)$$

Eq. (13):

$$E\ddot{o} = Bo = \frac{|\rho - \rho'| g L^2}{\gamma} \quad (14)$$

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5 If $\rho' \ll \rho$ (air/water):

$$Eö = \frac{\rho g L^2}{\gamma} \qquad (15)$$

Estimating length scale from μ, γ, ρ:

$$Mo = \frac{g \mu^4}{\rho \gamma^3} \qquad (16)$$

If we express Re in terms of characteristic velocity, $\sqrt{gL}$, we get:

$$Mo = \frac{Eo^3}{Re^4}$$

If we use (13a) in Re definition:

$$Ga = \frac{\sqrt{g \rho |\rho - \rho'| L^3}}{\mu} \qquad (17)$$

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(6) In case of viscosity dominated scaling:

$$-p_2^* + p_1^* + \underline{n}\cdot\left(\underline{\underline{\tau}}_2^* - \underline{\underline{\tau}}_1^*\right)\cdot\underline{n} = \frac{1}{Ca}K^* \quad (18)$$

where $Ca = \dfrac{\mu U}{\gamma}$ (19)

is the capillary number.

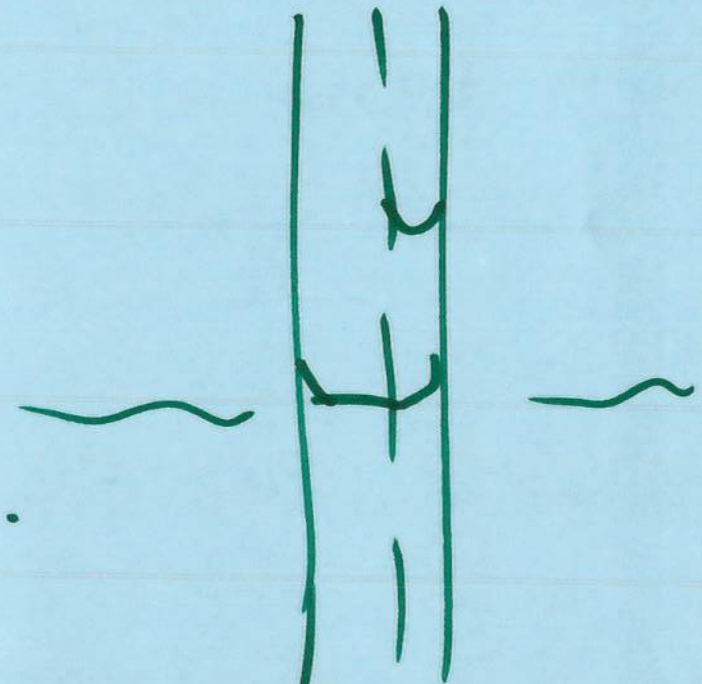
Characteristic time:

$$\frac{L}{U} \sim \sqrt{\frac{\rho L^3}{\gamma}}$$

intrinsic time scale is the diffusion time: $\dfrac{L^2}{\nu}$.

The inverse of Sl is

$$Oh = \frac{\mu}{\sqrt{\rho \gamma L}} \quad (20)$$ "Ohnesorge".



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