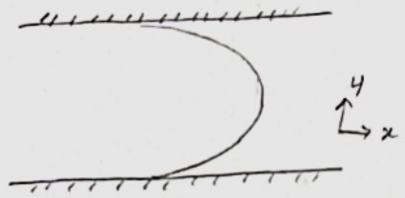
Problem 8.1

Assumptions

Fully developed flow through channel:



Mean momentum eq:

$$\frac{\bar{D}\langle U_j\rangle}{\bar{D}t} = \nu \nabla^2 \langle U_j\rangle - \frac{\partial \langle u_i u_j\rangle}{\partial x_i} - \frac{1}{\rho}\frac{\partial \langle p\rangle}{\partial x_j}$$

writing mean momentum eq in y-direction, we get:

$$\cancel{\frac{\partial \langle U_y\rangle}{\partial t}} + \cancel{(\langle U\rangle\cdot\nabla)(\langle U_y\rangle)} = \nu \nabla^2 U_y - \cancel{\frac{\partial \langle u_y u_y\rangle}{\partial x}} - \frac{1}{\rho}\frac{\partial \langle p\rangle}{\partial y}$$

simplifying we get

$$\frac{\partial \langle p(x,y)\rangle}{\partial y} = 0$$

integrating above eq shows that there will be no change in pressure across y-direction

therefore $\langle p(x,y)\rangle = \langle p(x)\rangle$ (y becomes irrelevant)

also $\langle p(x)\rangle|_{y=0} = p_w$

$$\therefore \frac{\partial \langle p\rangle}{\partial x} = \frac{p_w}{\partial x}$$