Problem 6.2 15 points:

Initialize the level-set distance field for a simple **1D** domain sketched in below. Blue denotes a liquid object (imagine it is an 1D droplet) in the middle of gas (black). Domain length is $0.015\ m$ resolved by 80 elements. The object length is half of the domain length.

Plot the 1D level-set distance field. Then follow these steps to compute the object mass.

a) Implement density $\rho(\phi)$ varied with the smoothed Heaviside function $f(\phi)$. Use $\rho_l = 1000\frac{kg}{m^3}$ and $\rho_g = 1\frac{kg}{m^3}$. [10 points]

$$f(\phi) = \begin{cases} 0, & if\ \phi < -Mh \\ \frac{1}{2}\left(1 + \frac{\phi}{Mh} + \frac{1}{\pi} sin\left(\frac{\pi\phi}{Mh}\right)\right), & if\ |\phi| \le Mh, \\ 1 & if\ \phi > Mh. \end{cases} \quad (3.54)$$

$$\rho(\phi) = \rho_l f(\phi) + \rho_g(1 - f(\phi))$$

Plot the density distributions $\rho(\phi)$ in the two-phase domain with M=1, 2, 3 respectively.

Note: this problem requires an individual programming script, but the code for $f(\phi)$ and $\rho(\phi)$ can be recycled in next homework. Submit your code.

b) Calculate the numerical mass of the liquid object with different interface thickness (M=1, 2, 3) with the following three options: [5 points]

Option 1: for any element with $\phi > 0$,

$$m_{droplet} = \sum_{element} (\rho_l \cdot length_{element})$$

Option 2: for any element with $\phi > 0$,

$$m_{droplet} = \sum_{element} (\rho(\phi)_{element} \cdot length_{element})$$

Option 3: for all the elements,

$$m_{droplet} = \sum_{element} (\rho_l \cdot density\ weight \cdot\ length_{element})$$

where $density\ weight = \frac{\rho(\phi)_{element} - \rho_g}{\rho_l - \rho_g}$

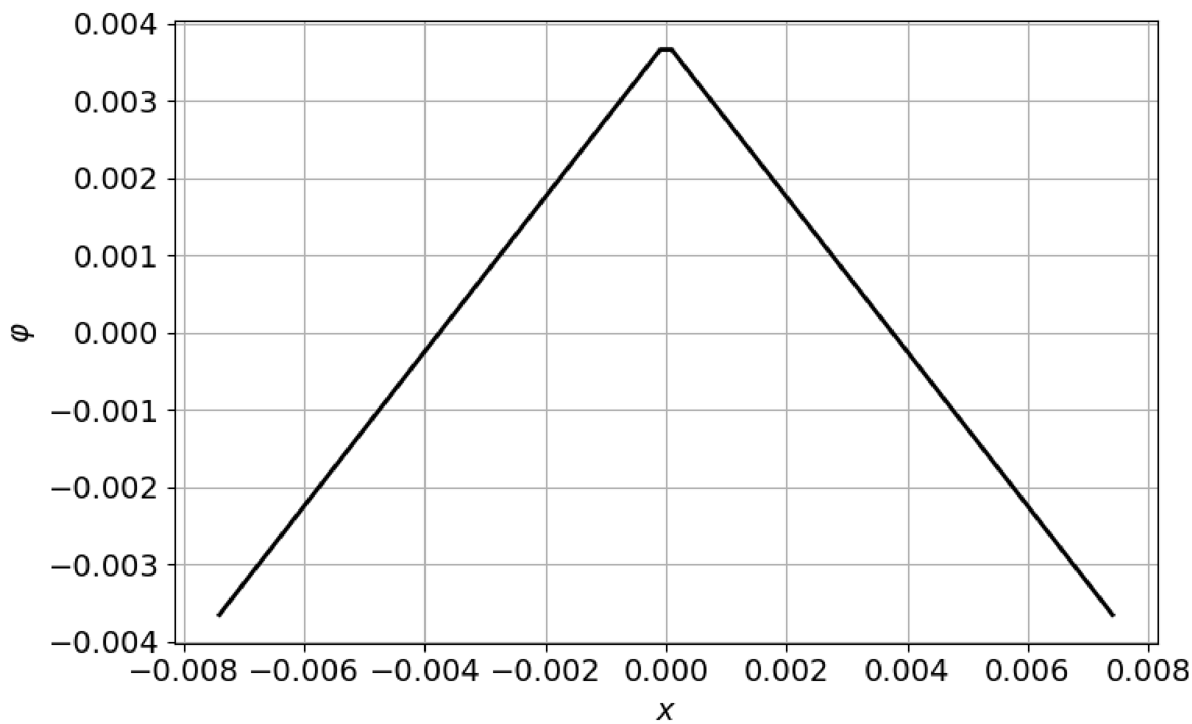
Compare the 3 numerical object masses varying with interface thickness (M=1, 2, 3) and the analytical mass. **Discuss** your results and clarify **which option** is the most appropriate to compute the numerical mass.

Note: the analytical mass is $\rho_l \cdot$ object length.

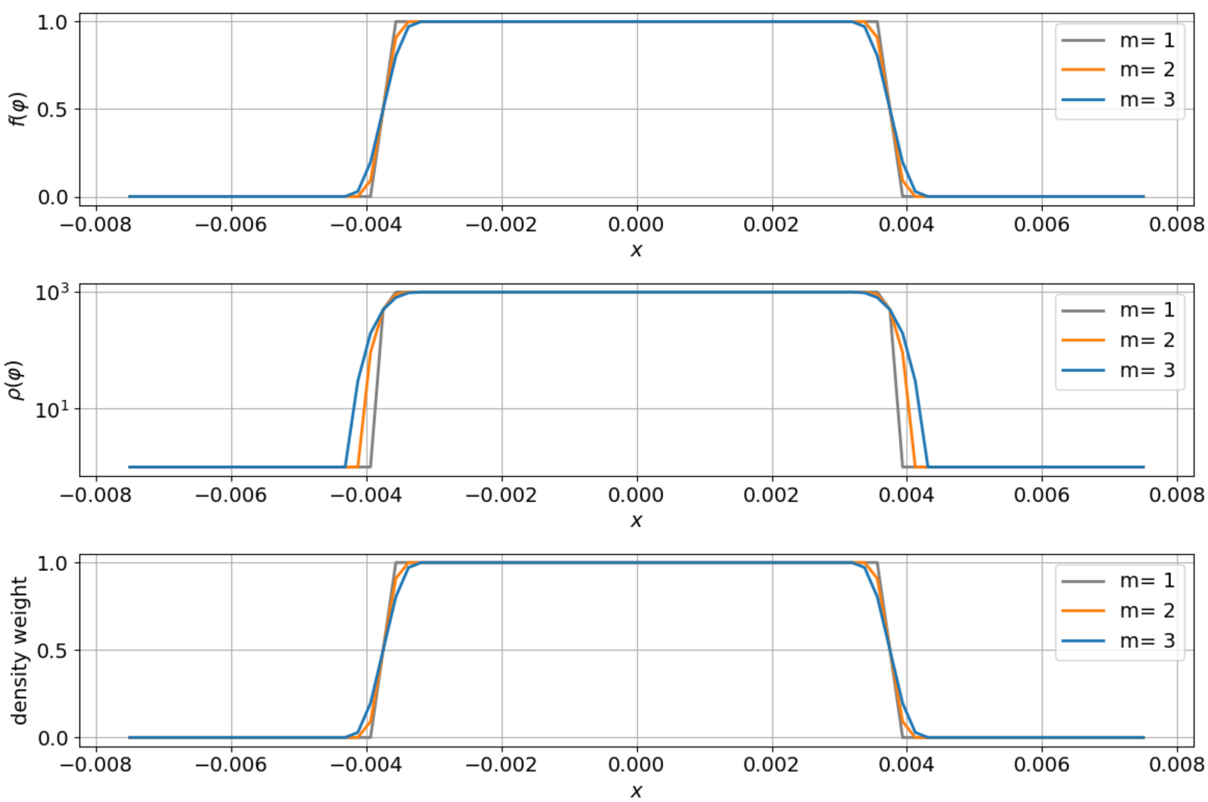
Solution:

The following results are generated using $\varphi, f(\varphi), \rho(\varphi)$ based on the x coordinate of the cell center.

The initial level-set distance field is:



a) 1D smoothed Heaviside function, density weight and density with $M = 1{,}2$ and 3:



Smoothed Heaviside function produces density variation along the interface. As M increases from 1 to 3, the transition range will be widened.

b) Analytical 1D droplet mass: $m = \rho_l L = 1000 \frac{kg}{m^2} * 0.075\ m = 7.5kg/m^2$

M	m_op1	m_op2	m_op3
1	7.5	7.465967	7.5
2	7.5	7.397008	7.5
3	7.5	7.338278	7.5

Results of Option 1 are the same as the analytical mass and do not change with M. With Option 2, more mass loss is observed as we increase M. This is because that this method does not account for the mass in the half interface thickness. Hence, the higher the M, the more unaccounted mass is. Option 3 overcame this issue by taking the "tail" of the transition profile into consideration. The outcome agrees with the analytical results. Hence, both Option 1 and 3 are considered robust. From the numerical implementation aspect, Option 3 is regarded the most suitable one to incorporate with level set.