Problem 1.3 10 points:

Determine the classification (e.g. whether they are elliptic, parabolic or hyperbolic) of:

a) heat equation ($\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2},\ a > 0$) (3 points);

b) incompressible 2-D N.S. equations (in space (4 points) and in time (3 points)).

Solution:

a) Rearrange the heat equation into:

$$a\frac{\partial^2 u}{\partial x^2} + \cdots = 0$$

$$A = a, B = 0, C = 0$$

$$B^2 - AC = 0$$

Hence the heat equation is **parabolic.**

b) 2-D NS equation:

$$\rho\frac{Du_i}{Dt} = -p_{,i} + \mu\left(\frac{\partial^2 u_i}{\partial x_j^2} + \frac{\partial^2 u_j}{\partial x_i^2}\right) + \rho f_i$$

For space, the second order terms are in the viscous stress term. $A = C = \mu > 0$ and $B = 0$. Hence $B^2 - AC < 0$. NS is **elliptic** in space.

For time, there is no second order time derivative, and it is not applicable for 2nd order PDE classification. Nevertheless, one can still show that $A = B = C = 0$ and $B^2 - AC = 0$. The 2D NS is **parabolic** in time.