MAE/NE-577

Homework #2 (6 problems)

due: 11:45am, February 10, 2020.

Problem 2.1	Problem 2.2	Problem 2.3	Problem 2.4	Problem 2.5	Problem 2.6	Total

Problem 2.1 10 points:

Based on Reynolds number dependency of DNS meshing requirements provided in the notes, **derive** the following expressions in **3D** simulation:

a) Mesh size dependence on Reynolds number

b) Time-dependent computational cost dependence on Reynolds number. Assume that the time step size is proportional to the mesh spacing ($\Delta t \sim \Delta x$)

Assume that mesh size of Re = $5\text{x}10^4$ is **one non-dimensional unit**. Computation cost of this simulation is also one unit with the same Re. Provide plots for both a) and b) with Re up to $5\text{x}10^5$. Use both linear and log-scale representations.

Note 1: in your plot, the spacing on x axis should be $\Delta x=5\text{x}10^4$, and y=1 corresponds to Re = $5\text{x}10^4$.
Note 2: please plot with math tools. Hand-drawn plots are not accepted.

Problem 2.2 10 points:

Using **dimensional analysis** derive the expressions for Kolmogorov's length, time, and velocity scales.

Note: the turbulence dissipation rate (ε) has the dimensions of $\left[\frac{m^2}{s^3}\right]$ and kinematic viscosity has the dimensions of $\left[\frac{m^2}{s}\right]$.

Problem 2.3 10 points:

If the interface is represented by $S(\underline{x}, t) = 0$, demonstrate that

$$\frac{\partial S}{\partial t} + \underline{w} \cdot \underline{\nabla} S = 0$$

can be obtained from $S(\underline{x}, t) = 0$ evaluated at *t+dt* (using Taylor's series expansion in a **vector form** provided in class notes).

Hint: expand space and time in $S(\underline{x} + \underline{w}dt, t + dt)$ separately, and then assume dt → 0

Problem 2.4 20 points:

Consider a single bubble (D=8mm) in a standing fluid with large temperature gradient ($\frac{dT}{d\theta} = G$) in zero gravity conditions.

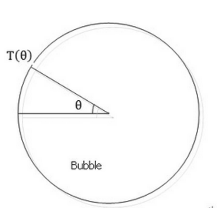
Assume that Eotvos rule can be used to estimate water surface tension as a function of temperature:

$$\gamma = 0.07275 N/m \cdot (1 - 0.002 \cdot (T - 291K))$$

(a) Compute the pressure drop between gas inside and liquid outside the bubble (Assume that the bubble is perfectly spherical with 8mm diameter if placed in constant temperature conditions at T = 320 K.)

> Note: use Eq. (33) from the course notes to compute the pressure drop. Note that the curvature value can be assumed as $\boldsymbol{\kappa} = \mathbf{2}/\boldsymbol{R}$.

(b) For a given pressure drop, the curvature of the bubble surface will change locally due to different local surface tension. Use the pressure drop obtained in (a) and determine the 2D bubble shape when temperature changes with θ:



From T_1 to T_2:
for $\theta \in [0, \pi]$, $T(\theta) = T_1 + (T_2 - T_1)\frac{\theta}{\pi}$;
for $\theta \in [\pi, 2\pi]$, $T(\theta) = T_2 - (T_2 - T_1)\frac{\theta - \pi}{\pi}$.

Derive the expression of **R(θ)** in two conditions:

(i) Temperature changes from 270K to 320K across the bubble (ii)Temperature changes from 270K to 370K across the bubble

(c) Use your favorite math tool to plot the 2D bubble shape for both cases in part (b). **Discuss** the results.

Note: plot deformed bubble and the initial spherical bubble together for comparison.

Problem 2.5 15 points:

You are tasked to design a multiphase flow experiment which will help develop a cooling/heating system for a laboratory on a Mars surface. The experiment is to be performed in Earth gravity ($g = 9.81\ m/s^2$) while the results to be used on Mars ($g_M = 3.71\ m/s^2$).

(a) Assume the water/vapor coolant in the heat exchanger on Mars will operate at 180 kPa and saturation temperature, look for the corresponding fluid properties (ρ_{fM}, σ_M, and μ_M).

(b) The characteristic length ratio between Earth and Mars is $L_M = 2L_E$. To maintain the **same** dimensionless parameters, what conditions of fluid properties should satisfy in Earth-based experiment?

(i) Eo number

(ii) Mo number

(c) Find a fluid and pressure/temperature conditions which will work for your experiment on Earth with the closest possible match of either Eo or Mo.

Note 1: do not use the surface tension function in Problem 2.4. Look into fluid property tables.

Note 2: include **units** of all the fluid properties you find.

Extra credits [5 points]: find a fluid which matches both of the dimensionless numbers.

Problem 2.6 10 points:

Derive Stokes' momentum equation from N-S equation by assuming the Re << 1. Specifically demonstrate why the **external force term** cannot be neglected while the **advection term** can.

Note: Stat with N-S equation $\frac{\partial u}{\partial t} + \left(\underline{u} \cdot \nabla\right)\underline{u} = -\frac{1}{\rho}\nabla p + \upsilon\nabla^2\underline{u} + f$