Homework #5 (4 problems; 65 points)
due: March 22nd, 2023.

5.1 [10]	5.2 [15]	5.3 [20]	5.4 [20]	Total [65]

Problem 5.1 10 points:

Derive boundary conditions for staggered grid arrangement for velocities and pressure at a corner node of 2D problem. Assume same value velocity at both walls adjacent to the corner. [8 points]

Please sketch the staggered grid. [2 points]

Note: substitute (2.19) written for $u^*_{i-\frac{1}{2},j}$, $v^*_{i,j+\frac{1}{2}}$ into (2.20) and using $u^{n+1}_{i-\frac{1}{2},j} = u_{b,j}$, $v^{n+1}_{i,j+\frac{1}{2}} = v_{i,b}$.

$$
\begin{aligned}
u^{n+1}_{i+1/2,j} &= u^*_{i+1/2,j} - \frac{1}{\rho}\frac{\Delta t}{h}\left(p^{n+1}_{i+1,j} - p^{n+1}_{i,j}\right) \\
v^{n+1}_{i,j+1/2} &= v^*_{i,j+1/2} - \frac{1}{\rho}\frac{\Delta t}{h}\left(p^{n+1}_{i,j+1} - p^{n+1}_{i,j}\right)
\end{aligned} \tag{2.19}
$$

$$
\begin{aligned}
&\frac{p^{n+1}_{i+1,j} + p^{n+1}_{i-1,j} + p^{n+1}_{i,j+1} + p^{n+1}_{i,j-1} - 4p^{n+1}_{i,j}}{h^2} = \\
&= \frac{\rho}{\Delta t}\left(\frac{u^*_{i+1/2,j} - u^*_{i-1/2,j} + v^*_{i,j+1/2} - v^*_{i,j-1/2}}{h}\right)
\end{aligned} \tag{2.20}
$$

Problem 5.2 15 points:

Review the derivation done in class (Eqns. 3.1 – 3.7).

(a) Derive the gradient of the Heaviside function in 2D in terms of 1D delta function. [6 points]

(b) Repeat the derivation in 3D. [5 points]

(c) Express the continuous expressions for dynamic viscosity (μ) and thermal conductivity (k) and their gradients based on the Heaviside function and its 3D gradient. [4 points]

Problem 5.3 20 points:

Consider a set of 2D two-phase flow problems to be solved using the Level-Set method. Analytically derive the initial conditions for the level set distance field in the following scenarios (note that you have to specify an expression in the form $\varphi(x, y) = ?$):

(a) A single circular liquid drop with radius r_d and center at (x_d, y_d) in the domain. Note that the level set field should be positive in the liquid and negative in the gas phase. [3 points]

(b) A stratified flow initial condition represented by a straight horizontal line (parallel to the x axis). Assume that the liquid level is y_w and the liquid is on the bottom of the domain. [3 points]

(c) Develop an initial condition for the following case (using the results from (a) and (b)): [6 points]

- Include the stratification line described in (b)
- Add N_d droplets with coordinates (x_d^i, y_d^i), $i = 1, N_d$ and radii r_d
- Add N_b bubbles with coordinates (x_b^j, y_b^j), $j = 1, N_b$ and radii r_b
- Note that it is assumed that the droplets are located in the gas part of the domain and the bubbles are in the liquid part.

(d) Assume that there are 5 bubbles and 5 droplets. Randomly choose coordinates of the centers. Assume that the bubble/droplet diameter is $1/10$ of the domain height. Use any software to plot a 2D level-set distribution for a domain size of 10.0×5.0. Assume that the liquid level is located at $y = 2.0$. Discuss why the distribution is correct. [8 points]

Problem 5.4 20 points:

Use the code you have created for Problem 4.1. If you have not got nearly full credit (e.g. below 18 points) for that problem and did not improve/fix the code, you are allowed to ask TA for the code (laminar solution N.S. solver).

a) Implement the pressure solver using the iterative methods discussed in class. Demonstrate that the solver shows the correct pressure gradient compared to the analytical value of the single-phase laminar flow solved in Problem 4.1 a). Submit your code. [15 points]

b) Do you expect the steady state solution to change with initial conditions? To test it, initialize your velocity field to be $0\ m/s$, peak velocity $0.01875\ m/s$, and $\overline{u_1(x_2)}$, and compare steady state solutions using 30 pressure cells across the channel. Discuss the result. [5 points]