Pro

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Problem 1.1

(a)

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{U}) = 0$$

Expanding second term using chain rule

$$\frac{\partial \rho}{\partial t} + \rho\vec{\nabla}\cdot\vec{U} + \vec{U}\cdot\vec{\nabla}\rho = 0$$

$$\frac{\partial \rho}{\partial t} + \vec{U}\cdot\vec{\nabla}\rho + \rho\vec{\nabla}\cdot\vec{U} = 0$$

$$\frac{D\rho}{Dt} + \rho\vec{\nabla}\cdot\vec{U} = 0 \qquad \left(\because \frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + \vec{U}\cdot\vec{\nabla}(\ )\right)$$

(b) $$\frac{\partial(\rho U_i)}{\partial t} + \nabla\cdot(\rho U_i\vec{U}) = -P_{,i} + \tau_{ji,i} + \rho f_i$$

L.H.S

$$\frac{\partial(\rho U_i)}{\partial t} + \nabla\cdot(\rho U_i\vec{U})$$

$$= \rho\frac{\partial U_i}{\partial t} + U_i\frac{\partial \rho}{\partial t} + \rho U_i\vec{\nabla}\cdot\vec{U} + \vec{U}\cdot\vec{\nabla}(\rho U_i)$$

$$= \rho\frac{\partial U_i}{\partial t} + U_i\frac{\partial \rho}{\partial t} + \rho U_i\vec{\nabla}\cdot\vec{U} + \vec{U}\cdot\left[U_i\vec{\nabla}\rho + \rho\vec{\nabla}U_i\right]$$

$$= \rho\frac{\partial U_i}{\partial t} + U_i\frac{\partial \rho}{\partial t} + \rho U_i\vec{\nabla}\cdot\vec{U} + U_i\vec{\nabla}\rho\cdot\vec{U} + \rho\vec{\nabla}U_i\cdot\vec{U}$$

$$= U_i\left(\frac{\partial \rho}{\partial t} + \vec{\nabla}\rho\cdot\vec{U}\right) + \rho\left(\frac{\partial U_i}{\partial t} + U_i\vec{\nabla}\cdot U + \vec{\nabla}U_i\cdot\vec{U}\right)$$

$$= U_i\left(\frac{\partial \rho}{\partial t} + \vec{U}\cdot\vec{\nabla}\rho\right) + \rho\left(\frac{\partial U_i}{\partial t} + \vec{U}\cdot\vec{\nabla}U_i + \vec{U}_i\nabla\cdot\vec{U}\right)$$

$$= U_i\left(-\rho\vec{\nabla}\cdot\vec{U}\right) + \rho\left(\frac{\partial U_i}{\partial t} + \vec{U}\cdot\vec{\nabla}U_i + U_i\vec{\nabla}\cdot\vec{U}\right)$$

$$= \rho\left(\cancel{-U_i\vec{\nabla}\cdot\vec{U}} + \frac{\partial U_i}{\partial t} + \vec{U}\cdot\vec{\nabla}U_i + \cancel{U_i\vec{\nabla}\cdot\vec{U}}\right) = \rho\left[\frac{DU_i}{Dt}\right]$$

$$\rho\frac{DU_i}{Dt} = -P_{,i} + \tau_{ji,i} + \rho f_i$$

(c) L.H.S

$$\frac{\partial}{\partial t}\left[\rho\left(e+\frac{u^2}{2}\right)\right]+\vec{\nabla}\cdot\left[\rho\left(e+\frac{u^2}{2}\right)\vec{u}\right]=\rho\dot{q}+(kT_{,i})_{,i}-(u_i p)_{,i}+(u_j\tau_{ij})_{,i}+\rho\vec{f}\cdot\vec{u}$$

$$\frac{\partial}{\partial t}\left[\rho\left(e+\frac{u^2}{2}\right)\right]+\left[\rho\left(e+\frac{u^2}{2}\right)\vec{\nabla}\cdot\vec{u}+\vec{u}\cdot\vec{\nabla}\left(\rho\left(e+\frac{u^2}{2}\right)\right)\right]$$

$$=\frac{\partial}{\partial t}\left[\rho\left(e+\frac{u^2}{2}\right)\right]+\vec{u}\cdot\vec{\nabla}\left(\rho\left(e+\frac{u^2}{2}\right)\right)+\rho\left(e+\frac{u^2}{2}\right)\vec{\nabla}\cdot\vec{u}$$

$$=\frac{D\left[\rho\left(e+\frac{u^2}{2}\right)\right]}{Dt}+\rho\left(e+\frac{u^2}{2}\right)\vec{\nabla}\cdot\vec{u}$$

$$=\left(e+\frac{u^2}{2}\right)\frac{D\rho}{Dt}+\rho\frac{D\left(e+\frac{u^2}{2}\right)}{Dt}+\rho\left(e+\frac{u^2}{2}\right)\vec{\nabla}\cdot\vec{u}$$

$$=\left(e+\frac{u^2}{2}\right)\left[\frac{D\rho}{Dt}+\rho\vec{\nabla}\cdot\vec{u}\right]+\rho\frac{D\left(e+\frac{u^2}{2}\right)}{Dt}$$

eq of continuity

$$\cancel{=\left(e+\frac{u^2}{2}\right)\left[\frac{D\rho}{Dt}+\right]}$$

$$=\left(e+\frac{u^2}{2}\right)(0)+\rho\frac{D\left(e+\frac{u^2}{2}\right)}{Dt}$$

$$\Rightarrow\rho\frac{D\left(e+\frac{u^2}{2}\right)}{Dt}=\rho\dot{q}+(kT_{,i})_{,i}-(u_i p)_{,i}+(u_j\tau_{ij})_{,i}+\rho\vec{f}\cdot\vec{u}$$

Problem 1.2

$$\frac{\partial \vec{U}}{\partial t} + \vec{\nabla}.(\vec{U}\vec{U}) = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \vec{U} + \vec{f}$$

Continuity eq: $\frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{U}) = 0$ —①

Mom eq: $\frac{\partial(\rho U_i)}{\partial t} + \vec{\nabla}.(\rho U_i \vec{U}) = -p_{ii} + \tau_{ij} + \rho \bar{f_i}$ —②

the given equation can be expressed as

$$\frac{\partial \vec{U}}{\partial t} + (\vec{U}.\vec{\nabla})\vec{U} = -\frac{1}{\rho}\vec{\nabla} p + \nu \nabla^2 \vec{U} + \vec{f}$$

Cylindrical coordinates r, ψ, z

$\vec{U} = U_r e_r + U_\psi e_\psi + U_z e_z$

(e_r, e_ψ and e_z are unit vectors)

Also: $\frac{\partial e_r}{\partial t} = \frac{\partial e_\psi}{\partial t} = \frac{\partial e_z}{\partial t} = 0$ —(i)

$\frac{\partial e_r}{\partial r} = \frac{\partial e_\psi}{\partial r} = \frac{\partial e_z}{\partial r} = 0$ —(ii)

$\frac{\partial e_r}{\partial \psi} = e_\psi$, $\frac{\partial e_\psi}{\partial \psi} = -e_r$, $\frac{\partial e_z}{\partial \psi} = 0$ —(iii)

$\frac{\partial e_r}{\partial z} = \frac{\partial e_\psi}{\partial z} = \frac{\partial e_z}{\partial z} = 0$

Now $\frac{\partial \vec{U}}{\partial t} = \frac{\partial}{\partial t}(U_r e_r + U_\psi e_\psi + U_z e_z)$

$$\frac{\partial \vec{U}}{\partial t} = \frac{\partial U_r}{\partial t} e_r + \frac{\partial U_\psi}{\partial t} e_\psi + \frac{\partial U_z}{\partial t} e_z \quad - (4)$$

Consider convective term

$$(\vec{U}\cdot\vec{\nabla})\vec{U} = \left\{ (U_r e_r + U_\psi e_\psi + U_z e_z)\cdot\left(e_r \frac{\partial}{\partial r} + \frac{1}{r} e_\psi \frac{\partial}{\partial \psi} + e_z \frac{\partial}{\partial z}\right)\right\} (U_r e_r + U_\psi e_\psi + U_z e_z)$$

$$= \left(U_r \frac{\partial}{\partial r} + \frac{U_\psi}{r}\frac{\partial}{\partial \psi} + U_z \frac{\partial}{\partial z}\right)(U_r e_r + U_\psi e_\psi + U_z e_z)$$

$$= U_r \frac{\partial U_r}{\partial r} e_r + U_r U_r \cancel{\left(\frac{\partial e_r}{\partial r}\right)}^{0} + \frac{U_\psi}{r}\frac{\partial U_\psi}{\partial \psi} e_\psi + \frac{U_\psi}{r} U_\psi \cancel{\left(\frac{\partial e_\psi}{\partial \psi}\right)}^{-e_r}$$

$$+ U_z \frac{\partial U_z}{\partial z} e_z + U_z U_z \cancel{\left(\frac{\partial e_z}{\partial z}\right)}^{0} + U_r U_\psi \cancel{\left(\frac{\partial e_\psi}{\partial r}\right)}^{0} + U_r \frac{\partial U_\psi}{\partial r} e_\psi$$

$$+ U_r \frac{\partial U_z}{\partial r} e_z + U_r U_z \cancel{\left(\frac{\partial e_z}{\partial r}\right)}^{0} + \frac{U_\psi}{r}\frac{\partial U_r}{\partial \psi} e_r + \frac{U_\psi}{r} U_r \cancel{\left(\frac{\partial e_r}{\partial \psi}\right)}^{e_\psi}$$

$$+ \frac{U_\psi}{r}\frac{\partial U_z}{\partial \psi} e_z + \frac{U_\psi}{r} U_z \cancel{\left(\frac{\partial e_z}{\partial \psi}\right)}^{0} + U_z \frac{\partial U_r}{\partial z} e_z + U_z U_r \cancel{\left(\frac{\partial e_r}{\partial z}\right)}^{0}$$

$$+ U_z \frac{\partial U_\psi}{\partial z} e_\psi + U_z U_\psi \cancel{\left(\frac{\partial e_\psi}{\partial z}\right)}^{0}$$

$$= U_r \frac{\partial U_r}{\partial r} e_r + U_r \frac{\partial U_\psi}{\partial r} e_\psi + \frac{U_\psi}{r}\frac{\partial U_r}{\partial \psi} e_r + \frac{U_\psi}{r} U_r e_\psi$$

$$+ \frac{U_\psi}{r}\frac{\partial U_\psi}{\partial \psi} e_\psi - \frac{U_\psi^2}{r} e_r + U_z \frac{\partial U_r}{\partial z} e_r + U_z \frac{\partial U_\psi}{\partial z} e_\psi$$

$$+ U_z \frac{\partial U_z}{\partial z} e_z + U_r \frac{\partial U_z}{\partial r} e_z + \frac{U_\psi}{r}\frac{\partial U_z}{\partial \psi} e_z$$

Combining terms

$$(\vec{U}\cdot\vec{\nabla})\vec{U} = e_r\left(U_r \frac{\partial U_r}{\partial r} + \frac{U_\psi}{r}\frac{\partial U_r}{\partial \psi} - \frac{U_\psi^2}{r} + U_z \frac{\partial U_r}{\partial r}\right)$$
$$+ e_\psi\left(U_r \frac{\partial U_\psi}{\partial r} + \frac{U_r U_\psi}{r} + \frac{U_\psi}{r}\frac{\partial U_\psi}{\partial \psi} + U_z \frac{\partial U_\psi}{\partial z}\right) \quad — (5)$$
$$+ e_z\left(U_r \frac{\partial U_z}{\partial r} + \frac{U_\psi}{r}\frac{\partial U_z}{\partial \psi} + U_z \frac{\partial U_z}{\partial z}\right)$$

Now

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \psi^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2\vec{U} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \vec{U}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \vec{U}}{\partial \psi^2} + \frac{\partial^2 \vec{U}}{\partial z^2} \quad — (6)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \vec{U}}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}\left(e_r U_r + e_\psi U_\psi + e_z U_z\right) \quad — (7)$$
$$= e_r \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_r}{\partial r}\right) + e_\psi \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_\psi}{\partial r}\right) \quad — (8)$$
$$+ e_z \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_z}{\partial r}\right)$$

$$\frac{1}{r^2}\frac{\partial^2}{\partial \psi^2}\left(e_r U_r + e_\psi U_\psi + e_z U_z\right) = \frac{1}{r^2}\frac{\partial}{\partial \psi}\left[U_r \frac{\partial e_r}{\partial \psi} + e_r \frac{\partial U_r}{\partial \psi} + U_\psi \overset{-e_r}{\cancel{\frac{\partial e_\psi}{\partial \psi}}}\right.$$
$$\left. + e_\psi \frac{\partial U_\psi}{\partial \psi} + U_z \frac{\partial e_z}{\partial \psi} + e_z \frac{\partial U_z}{\partial \psi}\right]$$

$$\frac{1}{r^2}\frac{\partial^2 \vec{U}}{\partial \psi^2} = \frac{1}{r^2}\frac{\partial}{\partial \psi}\left[e_\psi U_r + e_r \frac{\partial U_r}{\partial \psi} - e_r U_\psi + e_\psi \frac{\partial U_\psi}{\partial \psi} + e_z \frac{\partial U_z}{\partial \psi}\right] — (9)$$

$$\frac{1}{r^2}\frac{\partial^2 \vec{U}}{\partial \psi^2} = \frac{1}{r^2}\Bigg[\overset{-e_r}{\frac{\partial e_\psi}{\partial \psi}} U_r + \frac{e_\psi U_r}{\partial \psi} + \frac{\partial U_r}{\partial \psi}\overset{e_\psi}{\cancel{\frac{\partial e_r}{\partial \psi}}} + e_r \frac{\partial^2 U_r}{\partial \psi^2} - \overset{e_\psi}{\cancel{\frac{\partial e_r}{\partial \psi}}} U_\psi$$

$$- e_r \frac{\partial U_\psi}{\partial \psi} + \frac{\partial U_\psi}{\partial \psi}\overset{-e_r}{\cancel{\frac{\partial e_\psi}{\partial \psi}}} + e_\psi \frac{\partial^2 U_\psi}{\partial \psi^2} + \overset{0}{\cancel{\frac{\partial e_z}{\partial \psi}}}\frac{\partial U_z}{\partial \psi}$$

$$+ e_z \frac{\partial^2 U_z}{\partial \psi^2}\Bigg]$$

$$\frac{1}{r^2}\frac{\partial^2 \vec{U}}{\partial \psi^2} = \frac{1}{r^2}\Bigg[e_r\left(-U_r + \frac{\partial^2 U_r}{\partial \psi^2} - 2\frac{\partial U_\psi}{\partial \psi}\right) + e_\psi\left(2\frac{\partial U_r}{\partial \psi} - U_\psi + \frac{\partial^2 U_\psi}{\partial \psi^2}\right)$$

$$+ e_z \frac{\partial^2 U_z}{\partial \psi^2}\Bigg] \quad — (10)$$

Now

$$\frac{\partial^2}{\partial z^2}(e_r U_r + e_\psi U_\psi + e_z U_z) = e_r \frac{\partial^2 U_r}{\partial z^2} + e_\psi \frac{\partial^2 U_\psi}{\partial z^2} + e_z \frac{\partial^2 U_z}{\partial z^2} \quad — (11)$$

For r-direction

$$[\nabla^2 U]_r = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_r}{\partial r}\right) + \frac{1}{r^2}\left(-U_r + \frac{\partial^2 U_r}{\partial \psi^2} - 2\frac{U_\psi}{\partial \psi}\right) + \frac{\partial^2 U_r}{\partial z^2}$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_r}{\partial r}\right) - \frac{U_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 U_r}{\partial \psi^2} - \frac{2}{r^2}\frac{\partial U_\psi}{\partial \psi} + \frac{\partial^2 U_r}{\partial z^2}$$

$$[\nabla^2 \vec{U}]_r = \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r U_r)\right] + \frac{1}{r^2}\frac{\partial^2 U_r}{\partial \psi^2} - \frac{2}{r^2}\frac{\partial U_\psi}{\partial \psi} \quad — (12)$$

for ψ direction

$$[\nabla^2 \vec{U}]_\psi = \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r U_\psi)\right] + \frac{1}{r^2}\frac{\partial^2 U_\psi}{\partial \psi^2} + \frac{2}{r^2}\frac{\partial U_r}{\partial \psi} + \frac{\partial^2 U_\psi}{\partial z^2}$$

For z-direction

$$[\nabla^2 \vec{U}]_z = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 U_z}{\partial \psi} + \frac{\partial^2 U_z}{\partial z^2} \quad - (14)$$

$\Rightarrow$ for r direction

$$\frac{\partial U_r}{\partial t} + U_r\frac{\partial U_r}{\partial r} + \frac{U_\psi}{r}\frac{\partial U_r}{\partial \psi} + U_z\frac{\partial U_r}{\partial z} - \frac{\psi^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_r}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 U_r}{\partial \psi^2} + \frac{\partial^2 U_r}{\partial z^2} - \frac{U_r}{r^2} - \frac{2}{r^2}\frac{\partial U_\psi}{\partial \psi}\right] + f_r$$

$\Rightarrow$ for ψ direction

$$\frac{\partial U_\psi}{\partial t} + U_r\frac{\partial U_\psi}{\partial r} + \frac{U_\psi}{r}\frac{\partial U_\psi}{\partial \psi} + U_z\frac{\partial U_\psi}{\partial z} + \frac{U_r U_\psi}{r}$$

$$= -\frac{1}{\rho}\frac{\partial p}{\partial \psi} + \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 U_\psi}{\partial \psi^2} + \frac{\partial^2 U_\psi}{\partial z^2} + \frac{2}{r^2}\frac{\partial U_r}{\partial \psi} - \frac{U_\psi}{r^2}\right] + f_\psi$$

for z-direction

$$\frac{\partial U_z}{\partial t} + U_r\frac{\partial U_z}{\partial r} + \frac{U_\psi}{r}\frac{\partial U_z}{\partial \psi} + U_z\frac{\partial U_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 U_z}{\partial \psi^2} + \frac{\partial^2 U_z}{\partial z^2}\right)\Big] + f_z$$

Problem 1.3

Canonical form of PDE

$$a\phi_{xx} + b\phi_{xy} + c\phi_{yy} = h$$

where $D = b^2 - 4ac$

if $D > 0$ hyperbolic

if $D = 0$ parabolic

if $D < 0$ elliptic

(a) heat equation

$$\frac{\partial U}{\partial t} = a\frac{\partial^2 U}{\partial x^2}$$

$$U_t = a\,U_{xx}$$

$$a\,U_{xx} + 0\,U_{xt} + 0\,U_{tt} = U_t$$

$$D = (0)^2 - 4(a)(0) = 0 \quad \text{(Parabolic)}$$

(b) N.S 2D in space

$$\frac{\partial \vec{U}}{\partial t} + \rho U_x \frac{\partial U}{\partial x} + \rho U_y \frac{\partial U}{\partial y} = -\frac{\partial P}{\partial x} + N\left[\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right]$$

$$h = N[U_{xx} + U_{yy}]$$

$$D = (0)^2 - 4(N)(N) = -4N^2$$

$\Rightarrow$ Equation is elliptic in space

Problem 1.4

We have

$$\frac{\partial U_j}{} + \frac{\partial (U_i U_j)}{\partial x_i} = -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + \nu \frac{\partial^2 U_j}{\partial x_i} + \frac{1}{\rho} f_j \quad -①$$

the given equation is in indical form if we transform this eq into vector form we get

$$\frac{\partial U}{\partial t} + (\vec{U}\cdot\vec{\nabla})\vec{U} = -\frac{1}{\rho}\vec{\nabla} p + \nu \nabla^2 \vec{U} + \frac{\vec{f}}{\rho} \quad -②$$

taking divergence of ② & considering L.H.S

$$\underline{\underline{L.H.S}} = \vec{\nabla}\cdot\left[\frac{\partial \vec{U}}{\partial t} + (\vec{U}\cdot\vec{\nabla})\vec{U}\right]$$

$$= \frac{\partial}{\partial t}\cancel{(\vec{\nabla}\vec{U})}^{0} + \vec{\nabla}\cdot(\vec{U}\cdot\vec{\nabla})\vec{U}$$

$$= \vec{\nabla}\cdot(\vec{U}\cdot\vec{\nabla})\vec{U}$$

Consider $\underline{\underline{R.H.S}}$ Now

$$\underline{\underline{R.H.S}} = \vec{\nabla}\cdot\left[-\frac{1}{\rho}\vec{\nabla} p + \nu \nabla^2 \vec{U} + \frac{\vec{f}}{\rho}\right]$$

$$= -\frac{1}{\rho}\nabla^2 p + \nu \nabla^2 \cancel{(\nabla\cdot\vec{U})}^{0} + \nabla\cdot\frac{\vec{f}}{\rho}$$

Therefore modified eq becomes

$$\vec{\nabla}\cdot(\vec{U}\cdot\vec{\nabla})\vec{U} = -\frac{1}{\rho}\nabla^2 p + \frac{1}{\rho}\vec{\nabla}\cdot\vec{f}$$

Assuming $\vec{f}=0$ and transform this equation back into indical form we get

$$\boxed{\therefore \vec{\nabla}\cdot(\vec{U}\cdot\vec{\nabla})\vec{U} = [(U_i U_j)_{,j}]_{,i}}$$

$$\nabla^2 p = -\rho\,[(U_i U_j)_{,j}]_{,i}$$

Problem 1.5

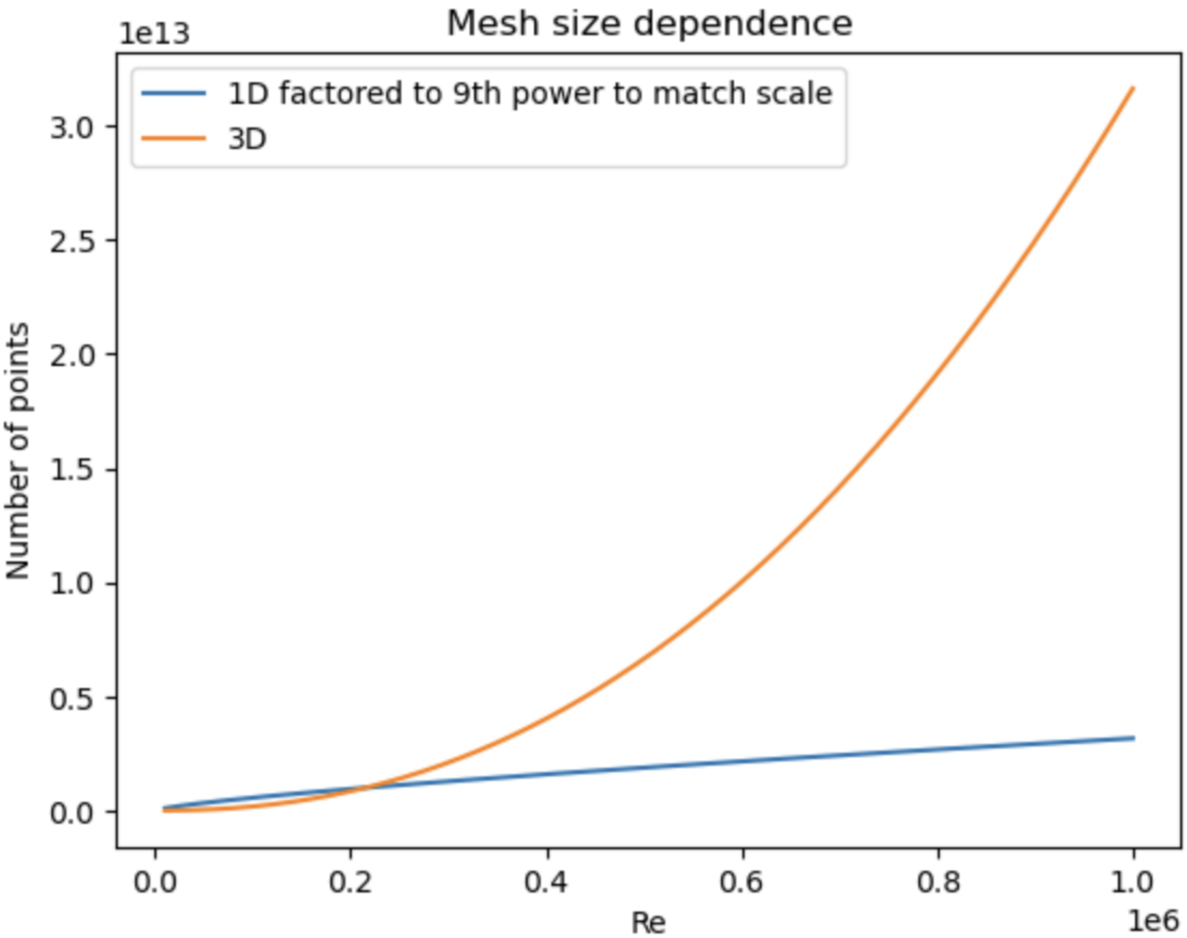
For no of points N along a given mesh direction with uniform mesh increments

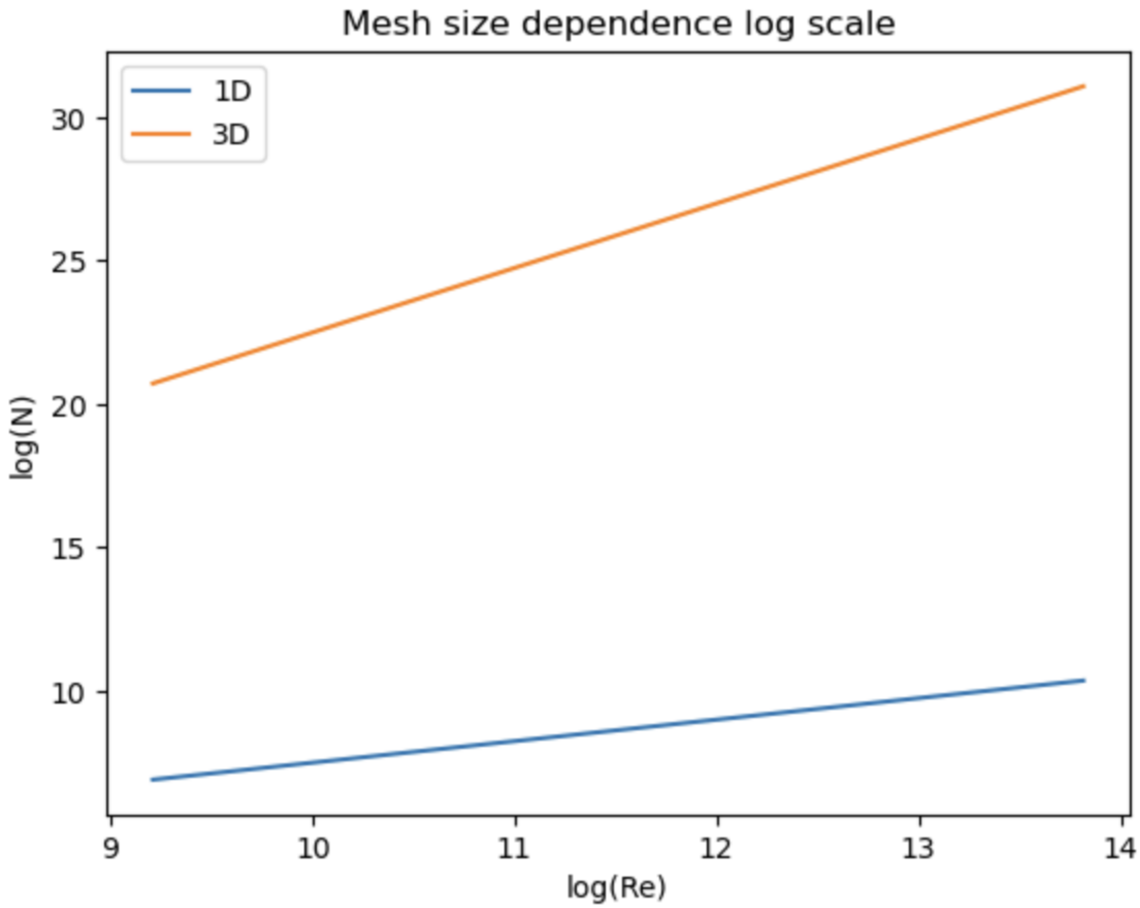
$$N \geq Re^{3/4}$$

a) for 3D case $\rightarrow N \geq Re^{2.25}$

b) given that

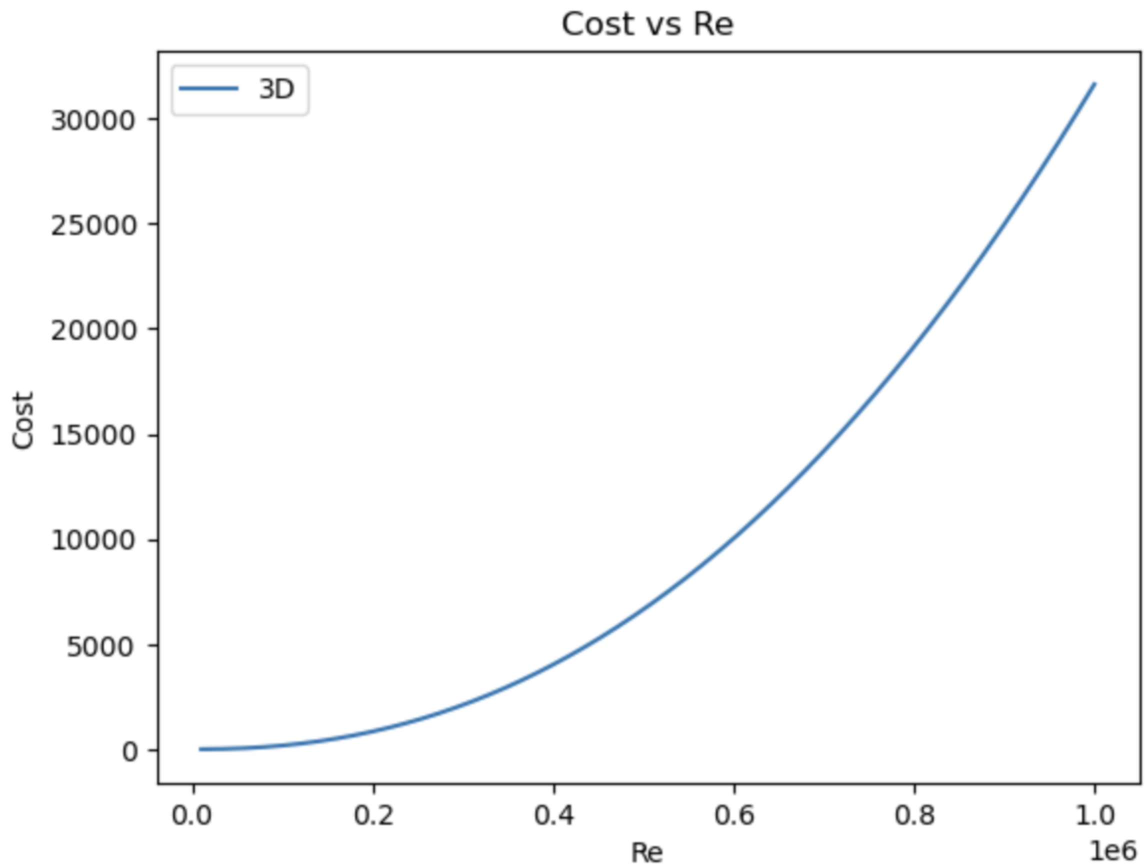
$Re = 10^4$ is one dimensional unit and computation cost of summation is also one unit with same Re graph is plotted.


Mesh size dependence
1e13
1D factored to 9th power to match scale
3D
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Number of points
0.0
0.2
0.4
0.6
0.8
1.0
Re
1e6

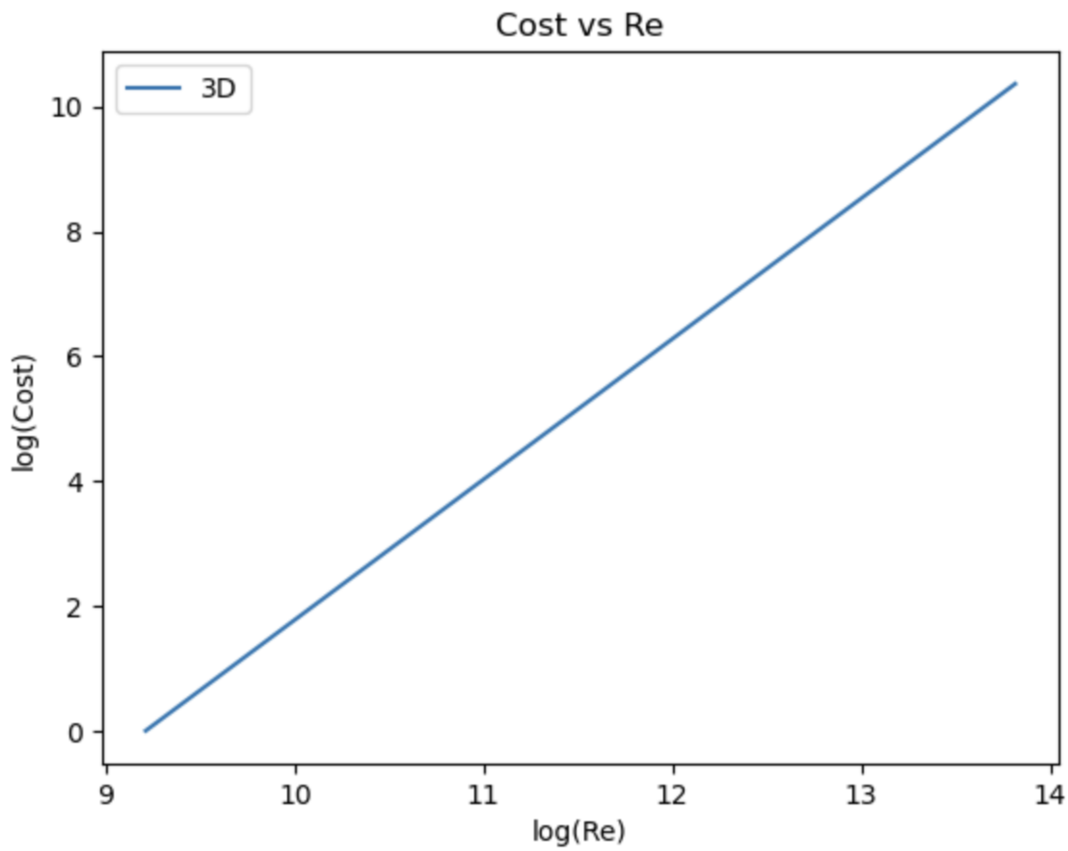


Mesh size dependence log scale
1D
3D
30
25
20
15
10
log(N)
9
10
11
12
13
14
log(Re)

COST



Cost vs Re
3D
30000
25000
20000
15000
10000
5000
0
Cost
0.0
0.2
0.4
0.6
0.8
1.0
1e6
Re


Cost vs Re
3D
log(Cost)
log(Re)
0
2
4
6
8
10
9
10
11
12
13
14