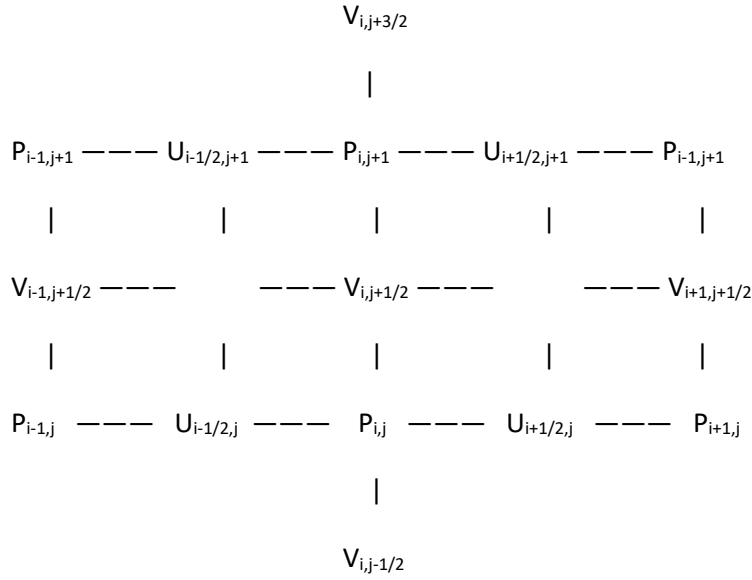


### Problem 3.2 15 points:

Using staggered grid approach (when pressure, x-velocity (u) and y-velocity(v) are computed at different locations) derive the approximation of the y-component of the advection term. Make sure all the steps are explained and detailed schematics of the control volume is shown.

Note that the vector form of the advection term is:  $\underline{A}(\underline{u}^n) = \frac{1}{\Delta V} \oint_S \underline{u}^n (\underline{u}^n \cdot \underline{n}) dS$

Solution:



According to the pattern of control volume which centered at  $V_{i,j+1/2}$ ,

Advection term is  $\underline{A}(\underline{u}^n) = \frac{1}{\Delta V} \oint_S \underline{u}^n (\underline{u}^n \cdot \underline{n}) dS$

$$\text{Right: } \left( \frac{u_{i+\frac{1}{2},j}^n + u_{i+\frac{1}{2},j+1}^n}{2} \right) \left( \frac{v_{i,j+\frac{1}{2}}^n + v_{i+1,j+\frac{1}{2}}^n}{2} \right) h \quad (1)$$

$$\text{Left: } - \left( \frac{u_{i-\frac{1}{2},j}^n + u_{i-\frac{1}{2},j+1}^n}{2} \right) \left( \frac{v_{i-1,j+\frac{1}{2}}^n + v_{i,j+\frac{1}{2}}^n}{2} \right) h \quad (2)$$

$$\text{Above: } \left( \frac{v_{i,j+\frac{3}{2}}^n + v_{i,j+\frac{1}{2}}^n}{2} \right)^2 h \quad (3)$$

$$\text{Below: } - \left( \frac{v_{i,j-\frac{1}{2}}^n + v_{i,j+\frac{1}{2}}^n}{2} \right)^2 h \quad (4)$$

$$(A_y)_{i,j+\frac{1}{2}}^n = \frac{1}{h^2} \left[ \left( \frac{u_{i+\frac{1}{2},j}^n + u_{i+\frac{1}{2},j+1}^n}{2} \right) \left( \frac{v_{i,j+\frac{1}{2}}^n + v_{i+1,j+\frac{1}{2}}^n}{2} \right) h - \left( \frac{u_{i-\frac{1}{2},j}^n + u_{i+\frac{1}{2},j}^n}{2} \right) \left( \frac{v_{i-1,j+\frac{1}{2}}^n + v_{i,j+\frac{1}{2}}^n}{2} \right) h \right. \\ \left. + \left( \frac{v_{i,j+\frac{3}{2}}^n + v_{i,j+\frac{1}{2}}^n}{2} \right)^2 h - \left( \frac{v_{i,j-\frac{1}{2}}^n + v_{i,j+\frac{1}{2}}^n}{2} \right)^2 h \right]$$

$$\Rightarrow (A_y)_{i,j+\frac{1}{2}}^n = \frac{1}{h} \left[ \left( \frac{u_{i+\frac{1}{2},j}^n + u_{i+\frac{1}{2},j+1}^n}{2} \right) \left( \frac{v_{i,j+\frac{1}{2}}^n + v_{i+1,j+\frac{1}{2}}^n}{2} \right) - \left( \frac{u_{i-\frac{1}{2},j}^n + u_{i+\frac{1}{2},j}^n}{2} \right) \left( \frac{v_{i-1,j+\frac{1}{2}}^n + v_{i,j+\frac{1}{2}}^n}{2} \right) \right. \\ \left. + \left( \frac{v_{i,j+\frac{3}{2}}^n + v_{i,j+\frac{1}{2}}^n}{2} \right)^2 - \left( \frac{v_{i,j-\frac{1}{2}}^n + v_{i,j+\frac{1}{2}}^n}{2} \right)^2 \right]$$