Problem 5.1 10 points:

Derive boundary conditions for staggered grid arrangement for velocities and pressure at a corner node of 2D problem. Assume same value velocity at both walls adjacent to the corner. [8 points]

Please sketch the staggered grid. [2 points]

Note: substitute (2.19) written for $u^*_{i-\frac{1}{2},j}$, $v^*_{i,j+\frac{1}{2}}$ into (2.20) and using $u^{n+1}_{i-\frac{1}{2},j} = u_{b,j}$, $v^{n+1}_{i,j+\frac{1}{2}} = v_{i,b}$.

Solution:

The staggered grid used here is:

| ——— $V_{i,b}$ ——————————————

| | | |

$U_{b,j}$ ——— $P_{i,j}$ ——— $U_{i+1/2,j}$ ——— $P_{i+1,j}$

| | | |

| ——— $V_{i,j-1/2}$ ———   ——— $V_{i+1,j-1/2}$

The predictor step for $u^*_{i-\frac{1}{2},j}$, and $v^*_{i,j+\frac{1}{2}}$:

$$u^{n+1}_{i-\frac{1}{2},j} = u^*_{i-\frac{1}{2},j} - \frac{1}{\rho}\frac{\Delta t}{h}\left(P^{n+1}_{i,j} - P^{n+1}_{i-1,j}\right)$$

$$v^{n+1}_{i,j+\frac{1}{2}} = v^*_{i,j+\frac{1}{2}} - \frac{1}{\rho}\frac{\Delta t}{h}\left(P^{n+1}_{i,j+1} - P^{n+1}_{i,j}\right)$$

Plug above into eq. 2.20:

$$\frac{P^{n+1}_{i+1,j} + P^{n+1}_{i-1,j} + P^{n+1}_{i,j+1} + P^{n+1}_{i,j-1} - 4P^{n+1}_{i,j}}{h^2} = \frac{1}{\rho}\frac{\Delta t}{h}\left(u^*_{i+\frac{1}{2},j} - u^{n+1}_{i-\frac{1}{2}} - \frac{1}{\rho}\frac{\Delta t}{h}\left(P^{n+1}_{i,j} - P^{n+1}_{i-1,j}\right) + v^{n+1}_{i,j+\frac{1}{2}} + \frac{1}{\rho}\frac{\Delta t}{h}\left(P^{n+1}_{i,j+1} - P^{n+1}_{i,j}\right) - v^*_{i,j-\frac{1}{2}}\right)$$

Given $u^{n+1}_{i-\frac{1}{2},j} = u_{b,j} = 0$, $v^{n+1}_{i,j+\frac{1}{2}} = v_{i,b} = 0$, rearrange above, we obtain:

$$\Rightarrow \qquad \frac{P^{n+1}_{i+1,j} + P^{n+1}_{i,j-1} - 2 \cdot P^{n+1}_{i,j}}{h^2} = \frac{\rho}{\Delta t}\frac{u^*_{i+\frac{1}{2},j} - v^*_{i,j-\frac{1}{2}}}{h}$$