

Problem 8.4

group 1: $\bar{D} = 1 \times 10^{-3} \text{ m}$, $\alpha = 0.03$

group 2: $\bar{D} = 1.5 \times 10^{-3} \text{ m}$, $\alpha = 0.01$

group 3: $\bar{D} = 2.0 \times 10^{-3} \text{ m}$, $\alpha = 0.01$

Part (a)

for continuous liquid $v_{cl} = 1.05 \times 10^{-6} \frac{\text{m}^2}{\text{s}}$

and $v_{2\phi} = 0.6 \bar{D} \alpha_{dr} |v_r|$

now normalizing $v_{2\phi}$ i.e. $v_{2\phi} / v_{cl}$

$$\left(\frac{v_{2\phi}}{v_{cl}} \right)_1 = \frac{0.6 \times (1 \times 10^{-3}) \times 0.03 |v_r|}{1.05 \times 10^{-6}} = \frac{120}{7} |v_r| = 17.143 |v_r|$$

$$\left(\frac{v_{2\phi}}{v_{cl}} \right)_2 = \frac{0.6 \times (1.5 \times 10^{-3}) \times 0.01 |v_r|}{1.05 \times 10^{-6}} = \frac{60}{7} |v_r| = 8.571 |v_r|$$

$$\left(\frac{v_{2\phi}}{v_{cl}} \right)_3 = \frac{0.6 \times (2.0 \times 10^{-3}) \times 0.01 |v_r|}{1.05 \times 10^{-6}} = \frac{80}{7} |v_r| = 11.439 |v_r|$$

$$\sum_{n=1}^3 \left(\frac{v_{2\phi}}{v_{cl}} \right)_n = \frac{260}{7} |v_r| = 37.143 |v_r|$$

Contribution from each group will be

group 1: $(120/7) / (260/7) = 46.15\%$

group 2: $(60/7) / (260/7) = 23.08\%$

group 3: $(80/7) / (260/7) = 30.77\%$

Part (b)

single group containing bubbles from group 1, group 2 and group 3. with $\alpha = 0.04$

estimating equivalent bubble diameter

$$\bar{D}_{eq} = \frac{(\alpha_1 \bar{D}_1) + (\alpha_2 \bar{D}_2) + (\alpha_3 \bar{D}_3)}{\sum \alpha}$$

$$\bar{D}_{eq} = \frac{(1 \times 10^{-3})(0.03) + (1.5 \times 10^{-3})(0.01) + (2 \times 10^{-3})(0.01)}{0.03 + 0.01 + 0.01}$$

$$\bar{D}_{eq} = 1.3 \times 10^{-3} \text{ m}$$

$$\left(\frac{v_{2g}}{v_a} \right)_{eq} = \frac{0.6 \times (1.3 \times 10^{-3}) \times 0.04 |v_r|}{1.05 \times 10^{-6}} = \frac{208}{7} |v_r| = 2$$

$\left(\frac{v_{2g}}{v_a} \right)_{eq}$ is different. this is because summation of void fraction of group 1, group 2 and group 3 in part (a) is not equal to void fraction of single group in part (b).

therefore $\frac{v_{2g}}{v_a}$ may be different depending on weightage factors.