PHYS811 Project 1

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1 Introduction

As neutron scattering involves a large scale of neutrons and interactions, it is a perfect candidate to study using Monte Carlo methods. In this project, we explore reflection, transmission, and capture rates and their dependence on capture probability, plate length, and mean path length. For reference, I have attached my code at the end of the report.

2 Part A

2.1 a

For part a, $L = 1$, $p_c = 2/3$ and N, the number of incident neutrinos, is 10,000. Respective probabilities for different λ values are shown in Table 1.

λ	Reflection	Absorption	Transmission
0.01	17.03	82.97	0.0
0.05	17.30	82.70	0.0
0.1	17.80	82.20	0.0
1	17.86	74.06	8.08
10	17.60	67.49	14.91

Table 1: Reflection, Absorption, and Transmission probability percents for N = 10 000

We clearly see that as λ increases, a neutrino is more likely to transmit through the plate. However, this is only in comparison to the length of the plate. The first three λ values, which represent the average length a neutrino will move after being scattered, are much smaller than the length of the plate. In this case, the neutrino will have many scattering events and is more likely to be captured at each event. As λ nears and surpasses the plate length, the neutrino only has one or two chances to be captured. When $\lambda >$ L, we see that absorption rate is closest to the given probability of capture, which we expect as in this case there is only one event where the neutrino is either captured, reflected, or transmitted. With smaller λ, we expected a higher absorption rate as it has a majority chance of being captured in a large number of scattering events.

3 b

Now, L = 1, $p_c = 0.5$, and $\lambda = 0.05$. With the same N = 10,000, our results are as follows: Reflection - 26.55%, Absorption - 73.45%, Transmission - 0.0%. As in part a, there are no transmitted neutrinos. This is expected as λ is much smaller than the thickness of the plate, so there are many events where the neutrino is likely to be captured. However, we now see more reflected neutrinos. As the neutrino hits the plate it has two options: be captured, or scatter. This scattering can either be forward into the plate, or back towards the beam and be considered reflected. Since our p_c decreased from part a to part b, we expect to see less neutrinos captured on that initial impact and be reflected back towards the direction of the beam.

4 c

Here, L = 2, $p_c = 0.5$, and $\lambda = 0.1$. In part b, if the neutrino was scattered directly to the right, it must be scattered $\frac{1}{0.05} = 20$ times before is it transmitted. Here the neutrino must be scattered $\frac{2}{.1} = 20$ times to be transmitted. Therefore, we should see similar results to part b. After running the simulation, we find: Reflection - 26.53%, Absorption - 73.47%, Transmission - 0.0%. Which are nearly identical to part b, as expected.

4.1 a

Plots of neutrino scattering from part a and part b are depicted in Figure 1. We can clearly see the concepts I described in previous parts. When λ is small compared to the length of the plate, the neutrino has many opportunities to be captured, resulting in a high absorption rate. We see that as the absorption probability decreases, there are more scattering events. Similarly, as the absorption probability increases, there are less scattering events. These graphs make it clear that the transmission probability depends on the ratio between L and λ, not just their values, as explored in part a and part c.

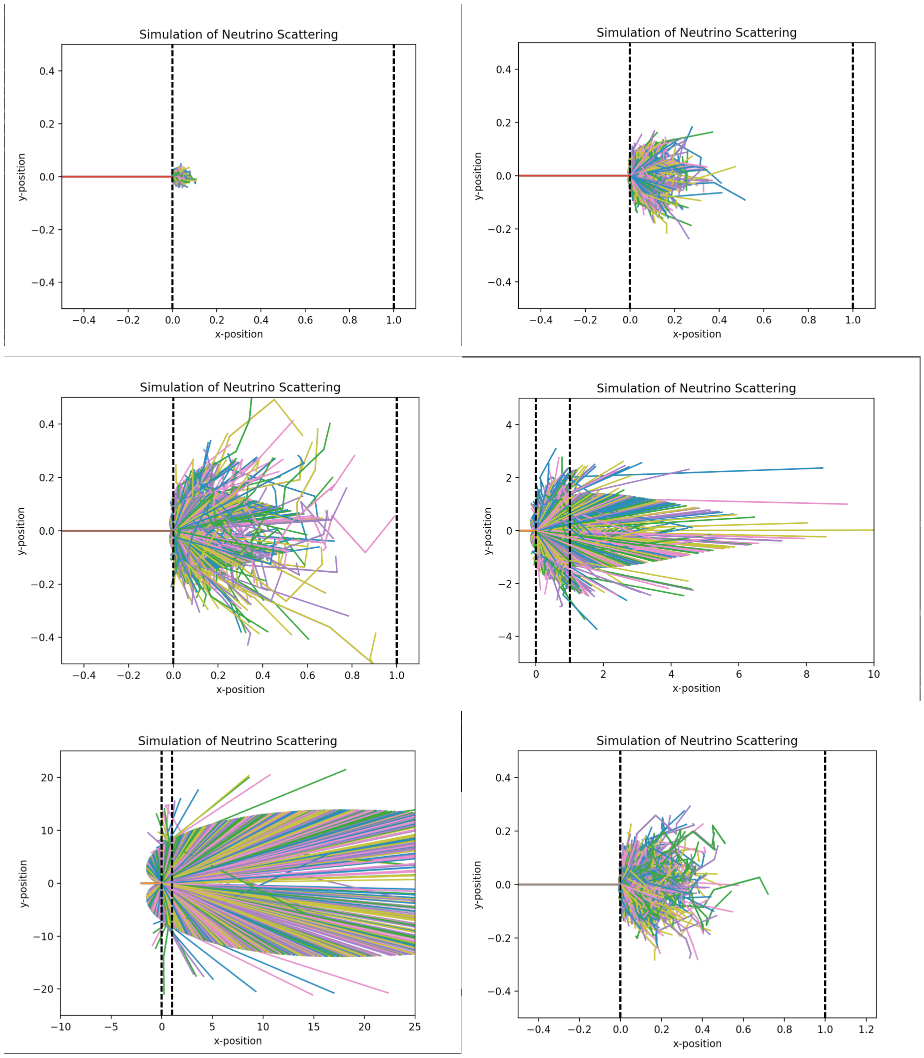


Figure 1: Top row: Part a $\lambda = 0.01$, Part a $\lambda = 0.05$. Middle row: Part a $\lambda = 0.1$, Part a $\lambda = 1$. Bottom Row: Part a $\lambda = 10$, Part b. The dashed vertical black lines represent the thickness of the plate.

5 Part B

6 a

Here, I consider all 5 λ values from Part A.a, calculate the E from those λ values, then see how the four given f values affect the rates for all 5 λ values.

For $\lambda = 0.01$, E = 0.0001. The rates are shown in Table 2:

f	Reflection	Absorption	Transmission
0.01	16.8	83.20	0.0
0.05	17.23	82.77	0.0
0.1	17.10	82.90	0.0
.5	17.77	82.23	0.0

Table 2

For For $\lambda = 0.05$, E = 0.0025. The rates are shown in Table 3:

f	Reflection	Absorption	Transmission
0.01	17.35	82.65	0.0
0.05	17.21	82.79	0.0
0.1	17.37	82.63	0.0
.5	17.34	82.66	0.0

Table 3

For $\lambda = 0.1$, E = 0.01. The rates are shown in Table 4:

f	Reflection	Absorption	Transmission
0.01	17.50	82.50	0.0
0.05	17.91	82.09	0.0
0.1	17.92	82.08	0.0
.5	16.96	83.04	0.0

Table 4

For $\lambda = 1$, E = 1. The rates are shown in Table 5:

f	Reflection	Absorption	Transmission
0.01	17.37	75.03	7.6
0.05	17.59	74.08	8.33
0.1	17.82	74.70	7.48
.5	16.64	76.17	7.19

Table 5

For $\lambda = 10$, E = 100. The rates are shown in Table 6:

f	Reflection	Absorption	Transmission
0.01	17.38	67.61	15.01
0.05	17.70	67.42	14.88
0.1	17.07	67.53	15.40
.5	17.29	67.45	15.26

Table 6

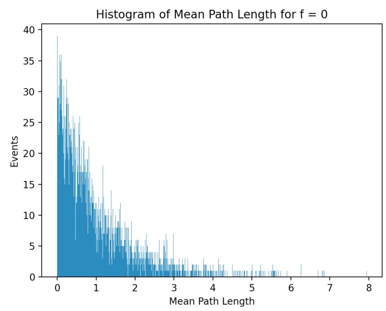
Before we compare to part A, we notice that the highest f value will decrease our mean path length by 1/4. For λ ¡¡ L where there were no transmission, we should see similar values. For λ ¿¿ L, the mean path length will still be longer than the thickness of the plate, so we should see similar numbers there as well. When λ is about equal to L, that is when we should see more absorption and less transmission. As the energy that would have been enough to get the neutrino out has been lost, so the neutrino may have to go through a couple extra scattering events where it is likely to be captured.

That trend is exactly what we see. First, we note that small f values are less impactful than larger ones, as f = 0 in part A. We see the most drastic change in transmission for $\lambda = 1$, as expected. In part A, there was a transmission rate of 8.08% at $\lambda = 1$, but with half the energy being lost, that drops to 7.19 %. This matches our prediction as the neutrino had lost energy, did not travel as far after scattering, then had to go through a couple more collisions, making it less likely to transmit through the plate.

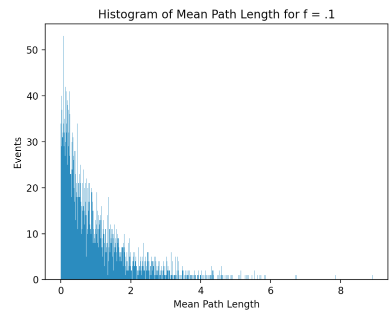
6.1 b

Here, I chose $\lambda = 1$ and $p_c = 2 =$ as it is the most impacted by the different f values. I chose the same parameters as part A.a. The three histograms are shown in Figure 2:

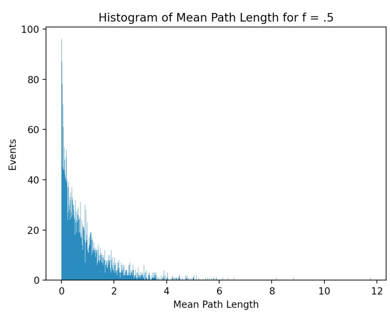
Since f represents the fraction of energy lost by the neutrino, as f increases, we should see the mean path length move towards zero, as less energy means the neutrino cannot move as far. Our data verifies this. As f approaches 1, the mean path length should approach 0 as the neutrino loses all energy and cannot move. To demonstrate this extreme example, I included the histogram for f = 0.99.



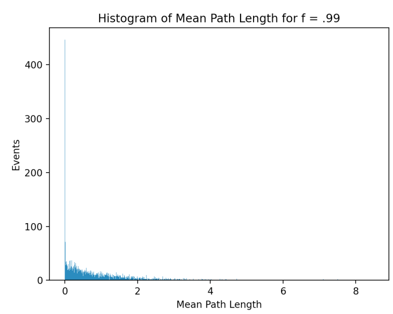
(a) f = 0



(b) f = 0.1



(c) f = 0.5



(d) f = 0.99

Figure 2:

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Overall, this assignment is a perfect example of how Monte Carlo simulations can and should be used to solve physics problems. Attached below is my code

```
import random
import math
import matplotlib.pyplot as plt

L = 2
pc = 0.5
ps = 1 - pc
TotalScatter = 0
TotalCapture = 0
TotalReflect = 0
TotalTransmit = 0
i = 0
posneg = [-1, 1]
```

```
N = 10000

for i in range(N):

    x = 0
    y = 0
    xplt = [0]
    yplt = [0]
    mfp = 0.1
    x_coordinates = [-2, 0]
    y_coordinates = [0, 0]

    while 0 <= x <= L:

        r = random.uniform(0, 1)
        if random.randint(1,100) < (pc*100):
            TotalCapture += 1
            x = L+1
        else:
            cos =  1 - 2 * r
            l = -mfp * math.log(r)
            z = l*cos

            x+=z
            xplt.append(x)

            sin = math.sqrt(1 - cos**2)
            y += random.choice(posneg)*l*sin
            yplt.append(y)


            if x < 0:
                TotalReflect += 1
            elif x > L:
                TotalTransmit += 1


            plt.plot(xplt, yplt, x_coordinates, y_coordinates)
            plt.ylim(-.5,.5)
            plt.xlim(-.5,1.25)
            plt.xlabel('x-position')
            plt.ylabel('y-position')
            xposition = [0.0, 1.0]
            for xc in xposition:
                plt.axvline(x=xc, color='black', linestyle='--')
            plt.title('Simulation of Neutrino Scattering')
```

```
print("After {0} firings there were {1} captured, {2} reflected, and {3} transmi
plt.show()
```

For part B:

```
import random
import math
import matplotlib.pyplot as plt
import numpy as np

L = 1
pc = 2/3
ps = 1 - pc
f = 0
TotalScatter = 0
TotalCapture = 0
TotalReflect = 0
TotalTransmit = 0
i = 0
posneg = [-1, 1]
lplot = []



N = 10000



for i in range(N):

    x = 0
    y = 0
    xplt = [0]
    yplt = [0]
    mfp = 1
    x_coordinates = [-2, 0]
    y_coordinates = [0, 0]

    while 0 <= x <= L:

        r = random.uniform(0, 1)

        if random.randint(1,100) < (pc*100):
            TotalCapture += 1

            x = L+1
```

```
        else:
            cos =  1 - 2 * r
            l = -mfp * math.log(r)
            z = l*cos
            x += z
            mfp -= math.sqrt(f) * mfp

            lplot.append(l)

            xplt.append(x)

            sin = math.sqrt(1 - cos**2)
            y += random.choice(posneg)*l*sin

            yplt.append(y)

            if x < 0:
                TotalReflect += 1
            elif x > L:
                TotalTransmit += 1

plt.hist(lplot, 1000)
plt.xlabel('Mean Path Length')
plt.ylabel('Events')
plt.title('Histogram of Mean Path Length for f = 0')

print("After {0} firings there were {1} captured, {2} reflected, and {3} transmi
plt.show()
```