Venn Diagrams

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History

Aristotelian syllogism: Aristotle

- All men are mortal.
- Socrates is a man.
- Therefore, Socrates is mortal.

[1]Aristotle, Prior Analytics, 350 BC.

[2]https://en.wikipedia.org/wiki/Syllogism

[3]https://plato.stanford.edu/entries/aristotle-logic/, Stanford Encyclopedia of Philosophy

- If the premises of a demonstration are scientifically known, then they must be demonstrated.
- The premises from which each premise are demonstrated must be scientifically known.
- Either this process continues forever, creating an infinite regress of premises, or it comes to a stop at some point.
- If it continues forever, then there are no first premises from which the subsequent ones are demonstrated, and so nothing is demonstrated.
- On the other hand, if it comes to a stop at some point, then the premises at which it comes to a stop are undemonstrated and therefore not scientifically known; consequently, neither are any of the others deduced from them.
- Therefore, nothing can be demonstrated.

- There are five houses.
- The Englishman lives in the red house.
- The Spaniard owns the dog.
- Coffee is drunk in the green house.
- The Ukrainian drinks tea.
- The green house is immediately to the right of the ivory house.
- The Old Gold smoker owns snails.
- Kools are smoked in the yellow house.
- Milk is drunk in the middle house.
- The Norwegian lives in the first house.
- The man who smokes Chesterfields lives in the house next to the man with the fox.
- Kools are smoked in the house next to the house where the horse is kept.
- The Lucky Strike smoker drinks orange juice.
- The Japanese smokes Parliaments.
- The Norwegian lives next to the blue house.

Who drinks water? Who owns the zebra?

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Aristotelian syllogism

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1880 **John Venn** (Venn diagrams)

Aristotelian syllogism: Raymond Llull

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Raymond Llull (1232—1315.5)

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Wikipedia
He invented a philosophical system known as the **Art**, conceived as a type of universal logic to prove the truth of Christian doctrine to interlocutors of all faiths and nationalities. The Art consists of a set of general principles and combinatorial operations. It is illustrated with diagrams.

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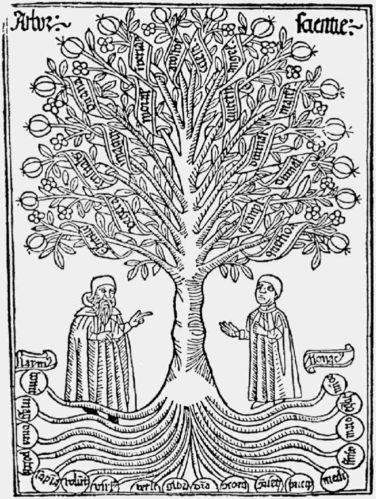
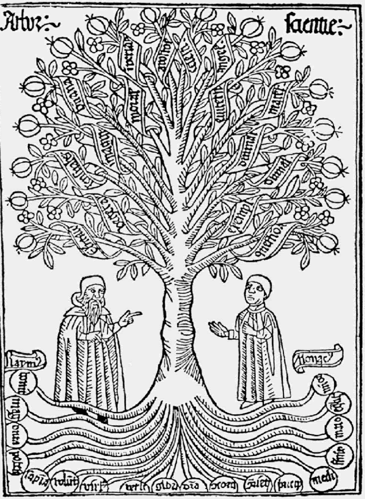


Raymond Llull’s Tree of Science

Aristotelian syllogism: Raymond Llull



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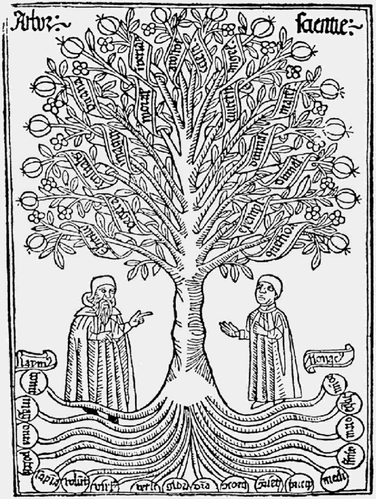

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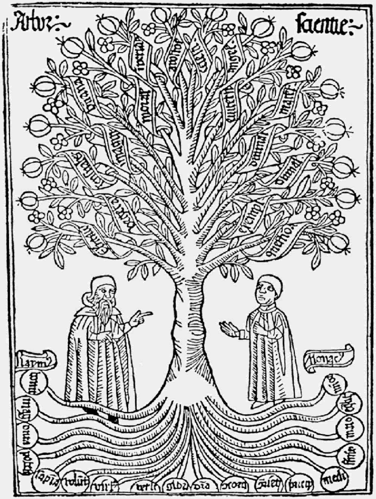

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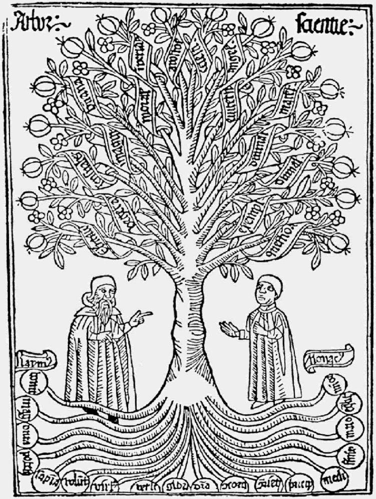

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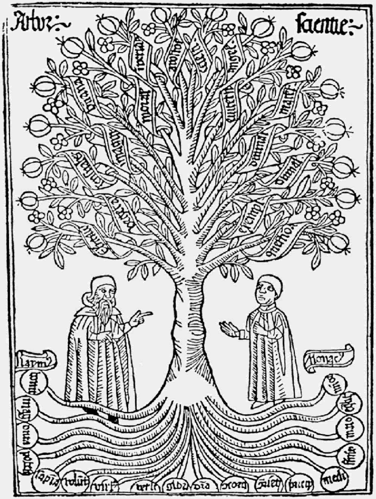

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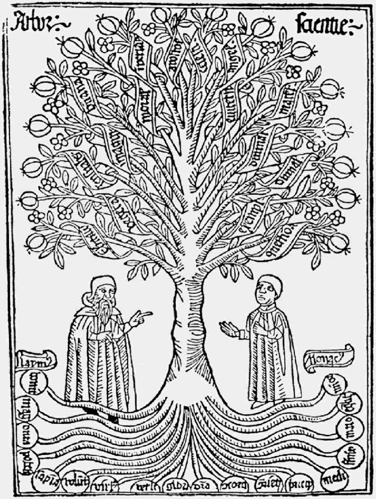

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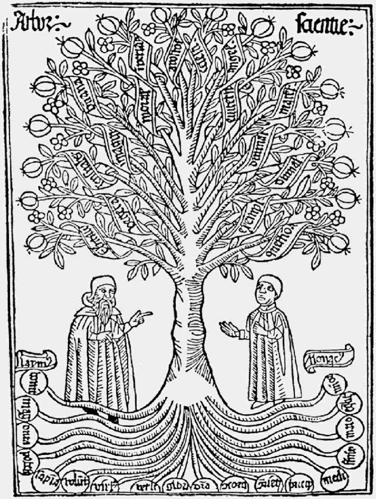

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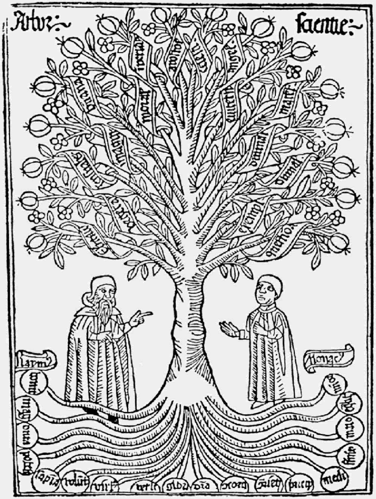

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Also, a **pioneer** of

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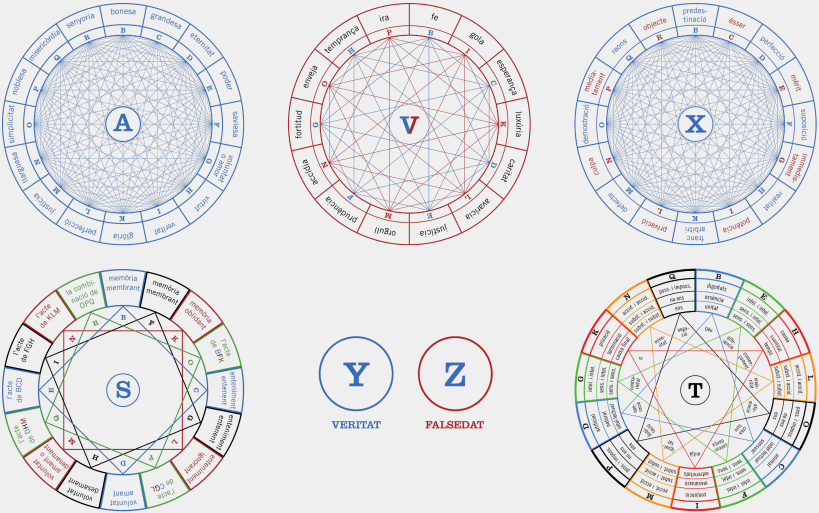
- Artificial Intelligence
- Information Technology
- Electoral System



[1]R. Llull, *Ars demonstrativa* , 1284.
[2]A. Bonner, *The Art and Logic of Ramon Llull: A User's Guide*, 2007.

Aristotelian syllogism: Juan Luis Vives

Juan Luis Vives (1493–1540)

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A **pioneer** of

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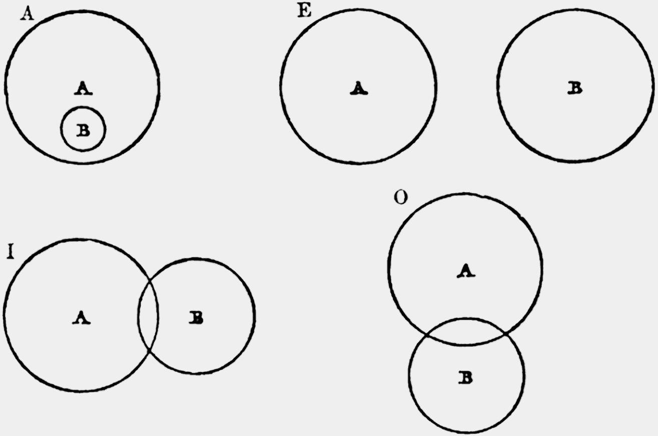
- Psychology
- Cognitive Science
- Memory
- Social Legislation
- Education

All A is B.
All C is A.
Therefore, All C is B.

[1] J. L. Vives, *De Censura Veri*, 1535.

ARISTOTELIAN SYLLOGISM: WEISE CIRCLES

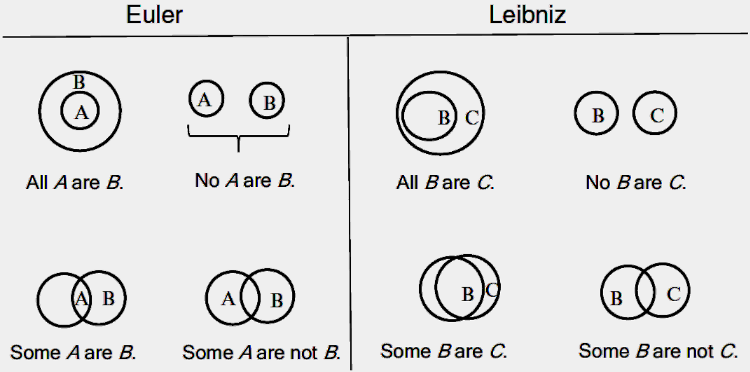


A proposition: All are
I proposition: Some are

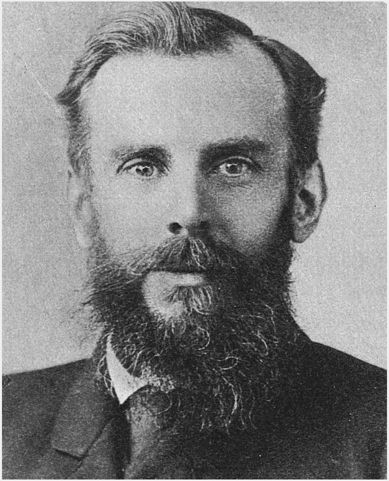
E proposition: None are
O proposition: Some are not

[1]C. Weise, *Nucleus Logicae*, 1691.

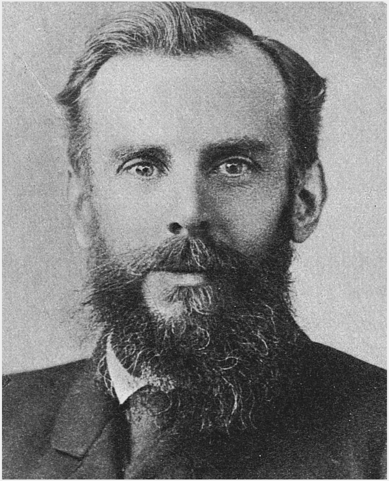
Aristotelian syllogism: Euler's/Leibniz's circles



Aristotelian syllogism: John Venn



John Venn (1834–1923)



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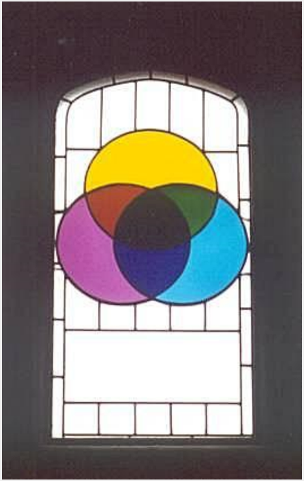
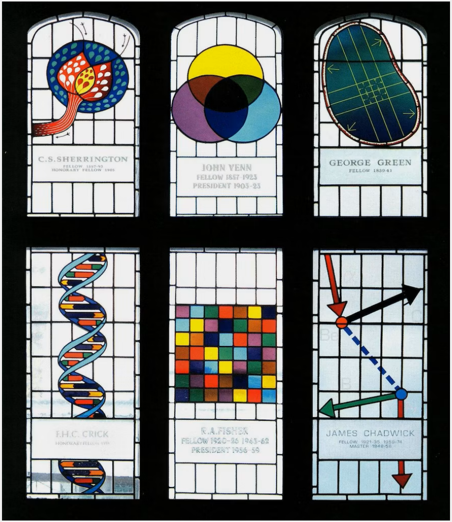


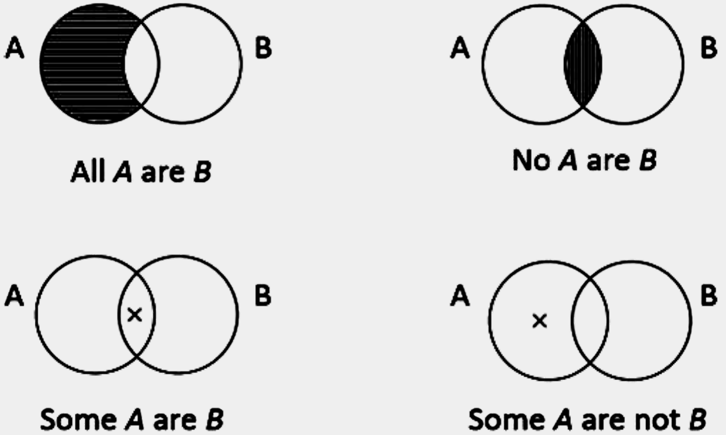
Gonville and Caius College

Charles S. Sherrington (Synapses)
John Venn (Venn Diagrams)
George Green (Green's Theorem)



Francis H. C. Crick (DNA)
Ronald Fisher (Statistics)
James Chadwick (Neutron)

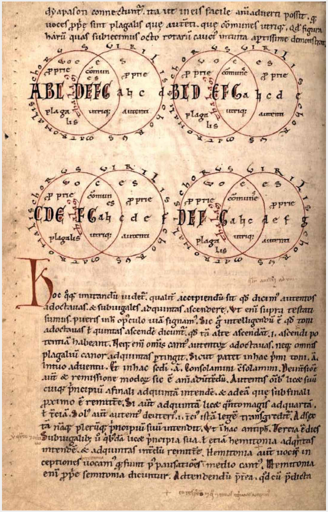
Gonville and Caius College, Cambridge University



[1] J. Venn, On the diagrammatic and mechanical representation of propositions and reasonings, *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* **9**(59) (1880), 1–18.

[2] J. Venn, *Symbolic Logic*, 1881.

Music Theory: Aribo's/John's diagrams



[1]Aribo Scholasticus, De musica, 1068–1078.
[2]John of Afflighem, De musica cum tonario, 1100.

Construction of Venn diagrams

Definition

A family $V = \{C_1, \ldots, C_n\}$ of closed Jordan curves in the pland is an *n*-Venn diagram if

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- the intersection of any two curves is a union of finitely many points.

What is a Venn diagram?

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Definition

A Venn diagram is simple if no three curves intersect.

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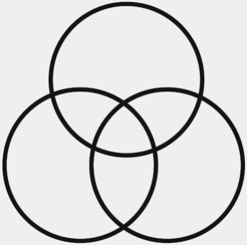
[2] B. Grünbaum, Venn diagrams I, *Geombinatorics* **1**(4) (1992), 5–12.



3-Venn diagram[1]

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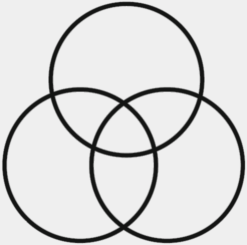
Construction: Venn circles/ellipses



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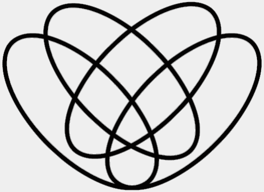

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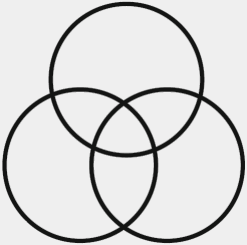

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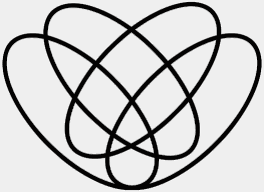

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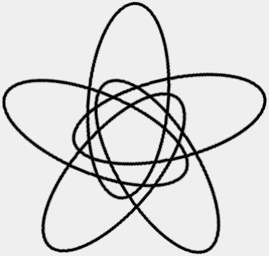

5-Venn diagram[2]

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Definition

A family $V = \{C_1, \ldots, C_n\}$ of Jordan curves in the plan is **independent** if $X_1 \cap \cdots \cap X_n \neq \emptyset$, where X_i is the interior or exterior region of C_i, for all $i = 1, \ldots, n$.

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Theorem (Grünbaum, 1975[1])

If an independent family of n curves is such that each two curves meet in at most k points, then

$$k \geq \frac{2^n - 2}{\binom{n}{2}}.$$

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Corollary

There is no Venn diagrams with four circle or six ellipses.

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Theorem (Carroll, Ruskey, and Weston, 2007[1])

There exists an n-Venn diagram of k-gons only if

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[1] J. Carroll, F. Ruskey, and M. Weston, Which *n*-Venn diagrams can be drawn with convex *k*-gons? *Discrete Comput. Geom.* **37**(4) (2007), 619–628.

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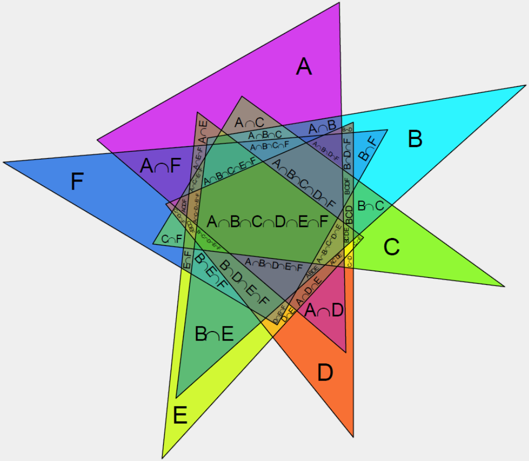
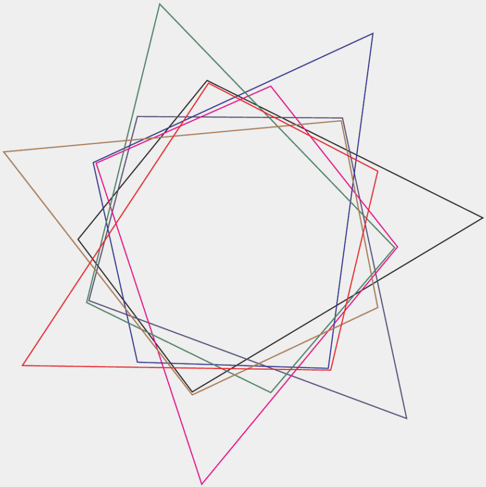
There is no Venn diagram with seven triangles or eight quadrilaterals.

[1] J. Carroll, F. Ruskey, and M. Weston, Which n-Venn diagrams can be drawn with convex k-gons? *Discrete Comput. Geom.* **37**(4) (2007), 619–628.

CONSTRUCTION: 6-VENN DIAGRAM OF TRIANGLES



[1] J. Carroll, Drawing Venn triangles, Technical Report HPL-2000-73, *HP Labs*, 2000.



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Theorem (Pakula, 1989[1])

Let V be an n-dimensional vector space of real functions on $\mathbb{R}^2$ and $U = V + \mathbb{R}$. Then no collection of $n+1$ functions chosen from U define boundaries for an $(n+1)$-Venn diagram.

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- $V = \langle x^2 + y^2,\ x,\ y\rangle$ for circles
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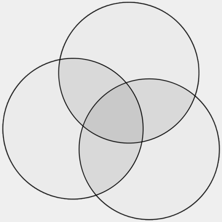
CONSTRUCTIONS: VENN, 1880[1]



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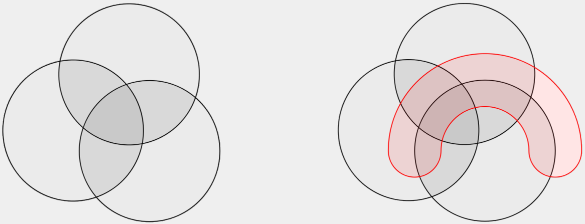

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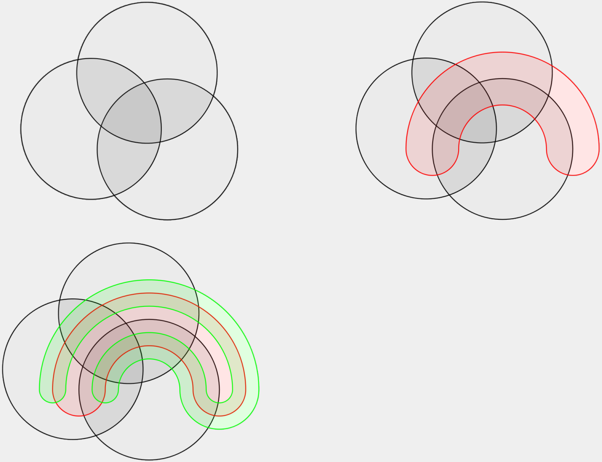

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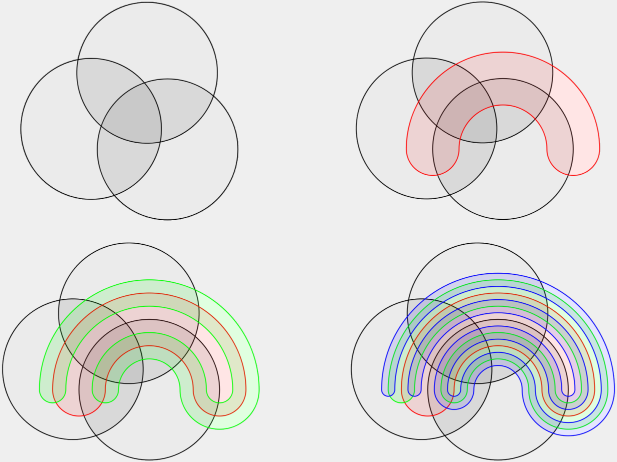
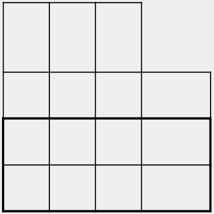


[1] J. Venn, On the diagrammatic and mechanical representation of propositions and reasonings, *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* **10** (1880), 1–18.

Constructions: Anderson-Cleaver, 1965[1]



[1] D. E. Anderson and F. L. Cleaver, Venn-type diagrams for arguments of n terms, *J. Symb. Logic* **30** (1965), 113–118

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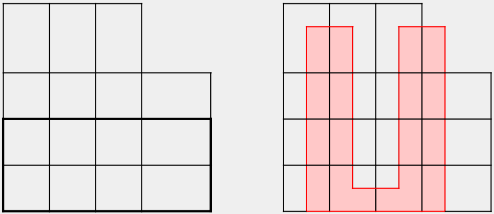

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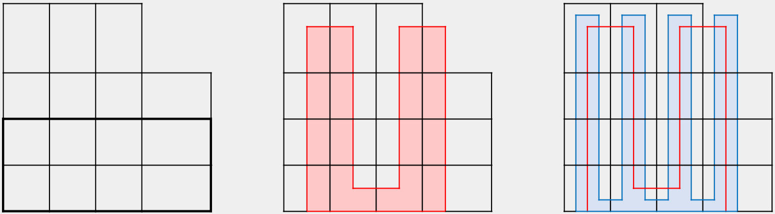
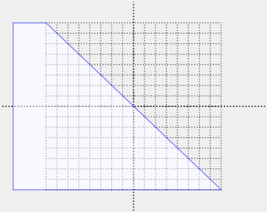


[1] D. E. Anderson and F. L. Cleaver, Venn-type diagrams for arguments of *n* terms, *J. Symb. Logic* **30** (1965), 113–118

Constructions: Fisher-Koh-Grünbaum, 1988[1]



[1] J. C. Fisher, E. L. Koh, B. Grünbaum, Diagrams Venn and how, *Math. Mag.* **61**(1) (1988), 36–40.

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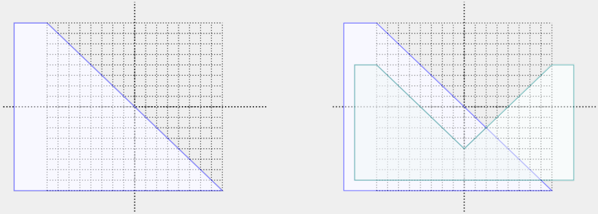

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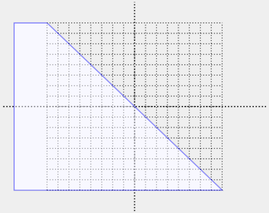

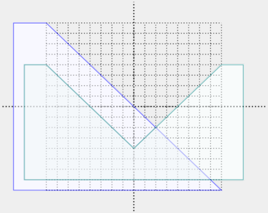

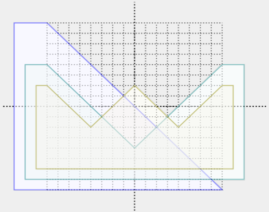

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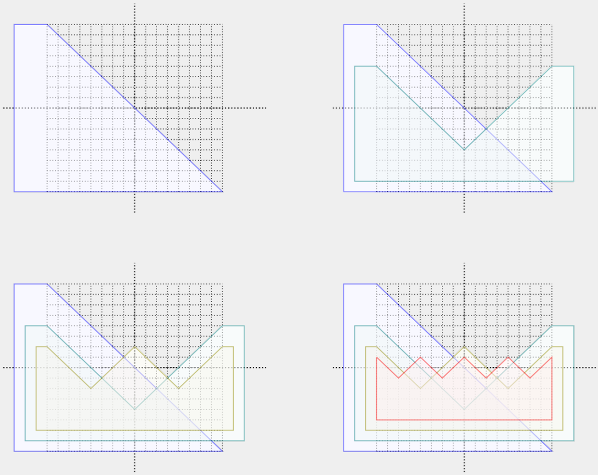

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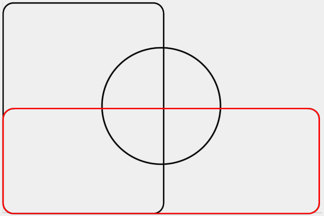
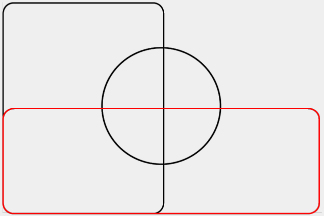
Constructions: Edwards, 1989[1]



[1] A. W. F. Edwards, Venn diagrams for many sets, *New Scientist* **7** (1989), 51–56.

CONSTRUCTIONS: EDWARDS, 1989[1]



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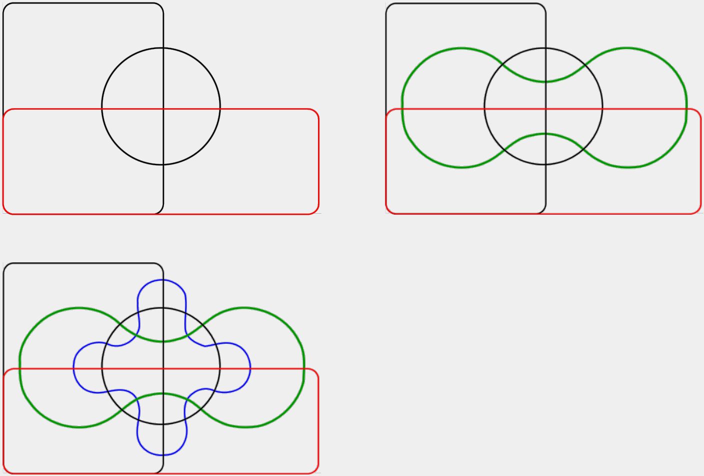

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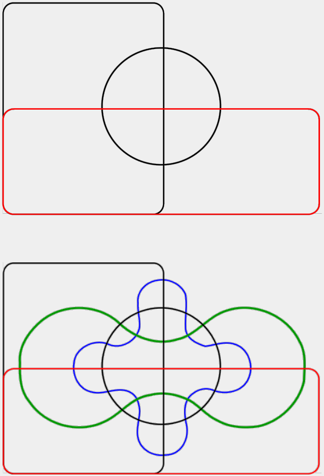

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Congruent, convex, monotone, and exposed Venn diagrams

Definition

An n-Venn diagram $C_1, \ldots, C_n$ is congruent if all C_i's are congruent.

[1] B. Grünbaum, Venn diagrams II, *Geombinatorics* **2**(2) (1992), 25–32.

Definition

An n-Venn diagram $C_1, \ldots, C_n$ is congruent if all C_i's are congruent.

Conjecture (Grünbaum, 1992[1])

There exists a congruent n-Venn diagram with congruent curves for all $n \geq 1$.

[1]B. Grünbaum, Venn diagrams II, *Geombinatorics* **2**(2) (1992), 25–32.

Definition

An n-Venn diagram $C_1, \ldots, C_n$ is convex if C_i is convex for all i.

[1] A. Rényi, C. Rényi, and J. Surányi, Sur l'independance des domaines simples dans l'espace Euclidien à n dimensions, *Colloquium Math.* **2** (1951), 130–135.

Definition

An n-Venn diagram $C_1, \ldots, C_n$ is convex if C_i is convex for all i.

Theorem (Rényi, Rényi, and Surányi, 1951[1])

There exists a convex n-Venn diagram for all $n \geq 1$.

[1] A. Rényi, C. Rényi, and J. Surányi, Sur l'independance des domaines simples dans l'espace Euclidien à n dimensions, *Colloquium Math.* **2** (1951), 130–135.

Definition

A convex n-Venn diagram is strongly convex if the complement of the unbounded region is convex.

[1] B. Grüunbaum, Venn diagrams and independent families of sets, *Math. Mag.* **48** (1975), 12–23.

Definition

A convex n-Venn diagram is strongly convex if the complement of the unbounded region is convex.

Theorem (Grüunbaum, 1975[1])

There exists a strongly convex n-Venn diagram for all $n \geq 1$.

[1] B. Grüunbaum, Venn diagrams and independent families of sets, *Math. Mag.* **48** (1975), 12–23.

Definition

A Venn diagram is monotone if every face of weight k is adjacent to a face of weight $k-1$ (if $k > 0$) and a face of weight $k+1$ (if $k < n$).

[1]B. Bultena, B. Grünbaum, and F. Ruskey, Convex drawings of intersecting families of simple closed curves, 1998.

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Theorem (Bultena, Grünbaum, and Ruskey, 1998[1])

A Venn diagram is isomorphic to a convex Venn diagram iff it is monotone.

[1] B. Bultena, B. Grünbaum, and F. Ruskey, Convex drawings of intersecting families of simple closed curves, 1998.

Theorem (Bultena and Ruskey, 1998[1])

A monotone n-Venn diagram has has least $\binom{n}{\lfloor n/2 \rfloor}$ vertices.

[1] B. Bultena and F. Ruskey, Venn diagrams with few vertices, *Electron. J. Combin.* **5** (1998), R44.

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A Venn diagram is exposed if the unbounded region has an edge in common with every curve.

[1] K. B. Chilakamarri, P. Hamburger, and R. E. Pippert, Analysis of Venn diagrams using cycles in graphs, *Geom. Dedicata* **82**(1-3) (2000), 193–223.
[2] K. B. Chilakamarri, P. Hamburger, and R. E. Pippert, Venn diagrams and planar graphs, *Geom. Dedicata* **62**(1) (1996), 73–91.

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Theorem (Chilakamarri, Hamburger, and Pippert, 1996[2])

For every positive integer n, there exists a simple n-Venn diagram having no k-face for $k > 5$.

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Theorem (Chilakamarri, Hamburger, and Pippert, 2000[1])

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[1] K. B. Chilakamarri, P. Hamburger, and R. E. Pippert, Analysis of Venn diagrams using cycles in graphs, *Geom. Dedicata* **82**(1-3) (2000), 193–223.

[2] K. B. Chilakamarri, P. Hamburger, and R. E. Pippert, Venn diagrams and planar graphs, *Geom. Dedicata* **62**(1) (1996), 73–91.

Symmetric Venn diagrams

Definition

An n-Venn diagram $C_1, \ldots, C_n$ is symmetric if there exists a point O rotation around it by angle of $2\pi/n$ maps every curve C_i to the curve C_{i+1}, for all $1 \leq i < n$.

[1] D. W. Henderson, Venn diagrams for more than four classes, *Amer. Math. Monthly* **70**(4) (1963), 424–426.

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Theorem (Henderson, 1963[1])

Symmetric n-Venn diagrams exists only if n is a prime.

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Theorem (Henderson, 1963[1])

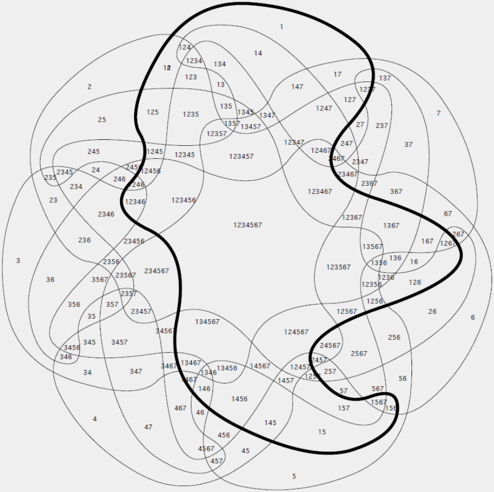
Symmetric n-Venn diagrams exists only if n is a prime.

Conjecture

There exists a symmetric simple p-Venn diagram for all primes p.

[1]D. W. Henderson, Venn diagrams for more than four classes, *Amer. Math. Monthly* **70**(4) (1963), 424–426.

Symmetric Venn diagrams: Example



A symmetric simple 7-Venn diagram (M2)

[1]B. Grünbaum, Venn diagrams II, *Geombinatorics* **2**(2) (1992), 25–32.

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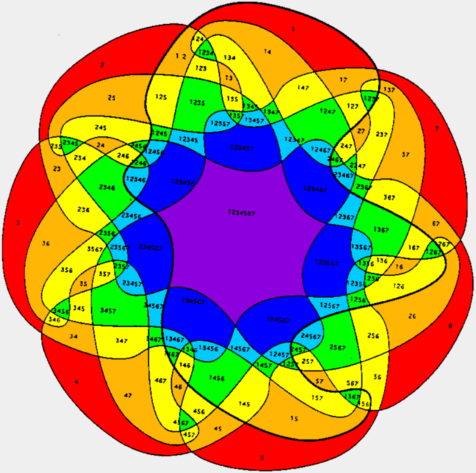

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Theorem (Griggs, Killian, and Savage, 2004[1])

For any prime p, there exists a symmetric monotone p-Venn diagram with minimum number of vertices, namely $\binom{p}{\lfloor p/2 \rfloor}$.

[1] J. Griggs, C. E. Killian, C. D. Savage, Venn diagrams and symmetric chain decompositions in the Boolean lattice, *Electron. J. Combin.* **11**(1) (2004), #R2, 30 pp.

[2] C. E. Killian, F. Ruskey, C. D. Savage, and M. Weston, Mark. Half-simple symmetric Venn diagrams, *Electron. J. Combin.* **11**(1) (2004), R86.

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Theorem (Killian, Ruskey, Savage, and M. Weston, 2004[2])

For any prime p, there exists a symmetric p-Venn diagram with $2^{p-1}(1-o(1))$ vertices.

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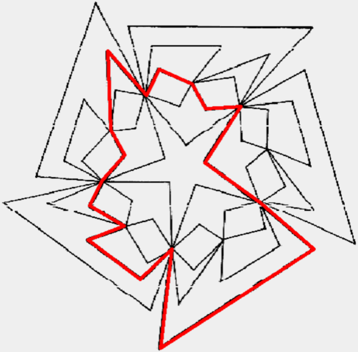
Symmetric Venn diagrams: Examples

[1] P. Hamburger, Pretty drawings. More doodles and doilies, symmetric Venn diagrams, *Util. Math.* **67** (2005), 229–254.

[2] P. Hamburger and R. Pippert, A symmetrical beauty. A non-simple 7-Venn diagram with a minimum vertex set, *Ars Combin.* **66** (2003), 129–137.



Symmetric 5-Venn diagram[1]

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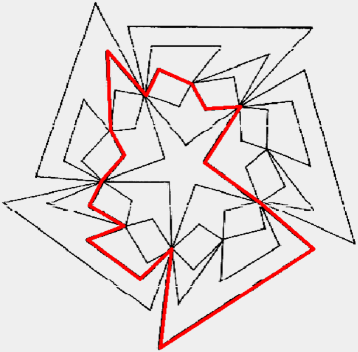

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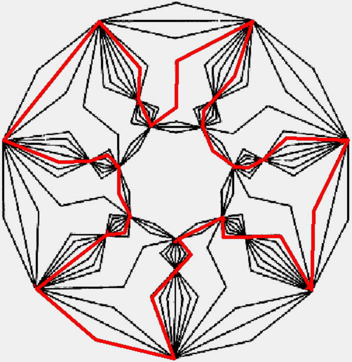

Symmetric 7-Venn diagram[2]

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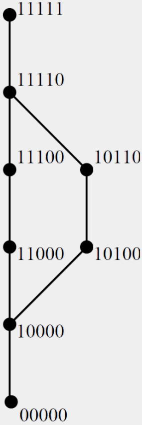
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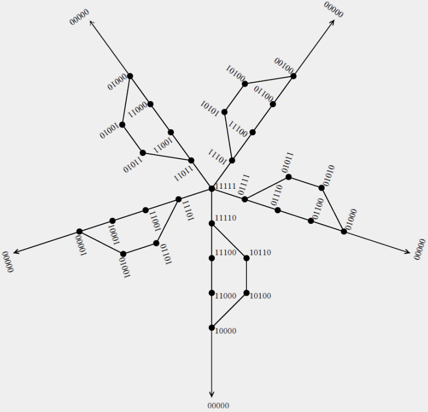
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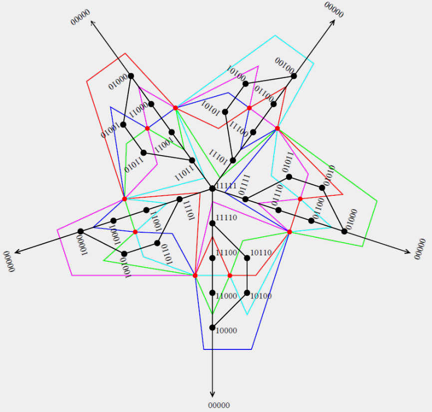
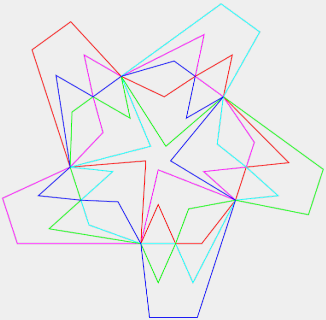
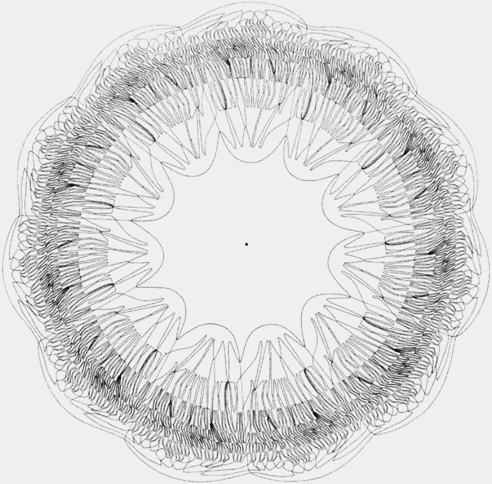
Symmetric Venn diagrams: Proof





Symmetric Venn diagrams: 11-doily with 1837 vertices



[1] P. Hamburger and E. Hepp, Symmetric Venn diagrams in the plane: the art of assigning a binary bit string code to planar regions using curves, *Leonardo* **38**(2) (2005), 125–132.

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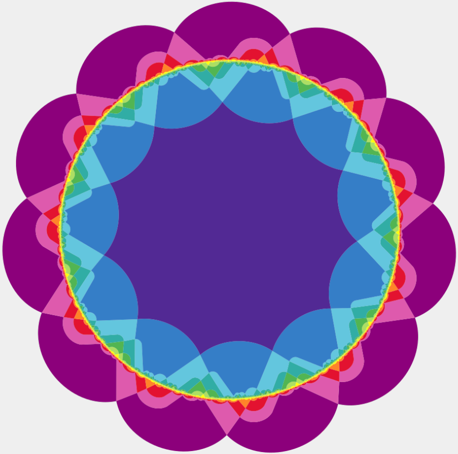
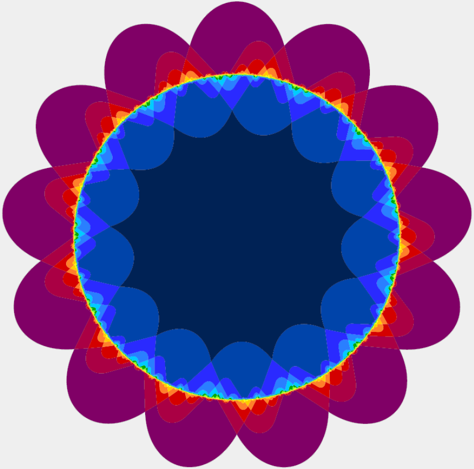


[1]K. Mamakani and F. Ruskey, New roses: simple symmetric Venn diagrams with 11 and 13 curves, *Discrete Comput. Geom.* **52**(1) (2014), 71–87.



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Theorem (Grünbaum, 1999[1])

A symmetric independent family of curves has at least

$$2 + n(N_2(n) - 2)$$

regions, where $N_2(n)$ is the number of n-bead necklaces with two colors. Note that

$$N_2(n) = \sum_{d|n} M_2(n)$$

with

$$M_2(n) = \frac{1}{n} \sum_{d|n} \varphi(d) 2^{\frac{n}{d}}.$$

[1] B. Grünbaum, The search for symmetric Venn diagrams, *Geombinatorics* **8**(4) (1999), 104–109.

Conjecture (Grünbaum, 1999[1])

If n is composite, then a symmetric independent family with $2 + n(N_2(n) - 2)$ regions exists and it is non-simple.

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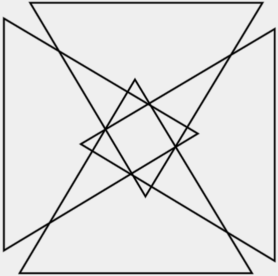

Symmetric independent family of 4 curves with
$2 + 4(N_2(4) - 2) = 18$ regions

[1] B. Grünbaum, The search for symmetric Venn diagrams, *Geombinatorics* **8**(4) (1999), 104–109.

Definition

A Venn diagram is antipodal if it is fixed by antipidal symmetry.

[1]F. Ruskey and M. Weston, Spherical Venn diagrams with involutory isometries, *Electron. J. Combin.* **18**(1) (2011), P191.

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Theorem (Ruskey and Weston, 2011[1])

There exists an antipodal n-Venn diagram, for all $n \geq 1$.

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Theorem (Ruskey and Weston, 2011[1])

There exists an antipodal n-Venn diagram, for all $n \geq 1$.

Corollary

There exists an n-Venn diagram fixed by a given involutary isometry of the sphere, for all $n \geq 1$.

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Definition

A completely symmetric Venn diagram is a spherical symmetric Venn diagram with congruent north and south hemispheres.

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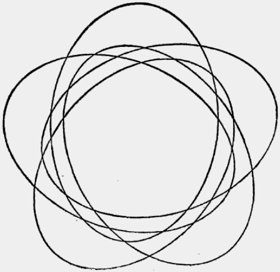
Example

The 1-, 2-, and 3-Venn diagrams are completely symmetric.



A completely symmetric 5-Venn diagram

[1]B. Grünbaum, Venn diagrams and independent families of sets, *Math. Mag.* **48** (1975), 12–23.

Reducibility

Definition

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[1] B. Grünbaum, The construction of Venn diagrams, *College Math. J.* **15**(3) (1984), 238–247.

Definition

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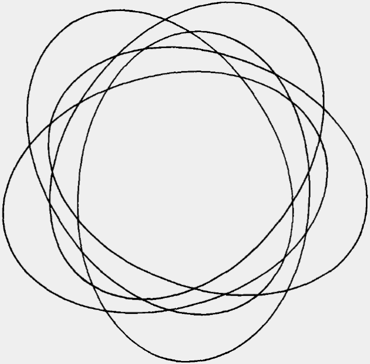
Theorem (B. Grünbaum, 1984[1])

The exists simple irreducible Venn diagrams for all $n \geq 5$ sets.

[1]B. Grünbaum, The construction of Venn diagrams, *College Math. J.* **15**(3) (1984), 238–247.

Irreducible Venn diagrams: Example



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Conjecture (Winkler, 1984[3])

Every simple Venn diagram can be extended to a new simple Venn diagram by the addition of a suitable curve.

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[2] B. Grünbaum, Venn diagrams I, *Geombinatorics* **1**(4) (1992), 5–12.

[3] P. Winkler, Venn diagrams: Some observations and an open problem, *Congr. Numer.* **45** (1984), 267–274.

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Conjecture (Winkler, 1984[3])

Every simple Venn diagram can be extended to a new simple Venn diagram by the addition of a suitable curve.

Conjecture (Grünbaum, 1992[2])

Every Venn diagram can be extended to a new Venn diagram by the addition of a suitable curve.

Theorem (Chilakamarri, Hamburger, and Pippert, 1996[1])

Grünbaum's conjecture is true.

[1] K. B. Chilakamarri, P. Hamburger, R. E. Pippert, Hamilton cycles in planar graphs and Venn diagrams, *J. Combin. Theory Ser. B* **67**(2) (1996), 296–303.

[2] B. Grünbaum, Venn diagrams I, *Geombinatorics* **1**(4) (1992), 5–12.

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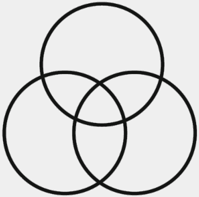
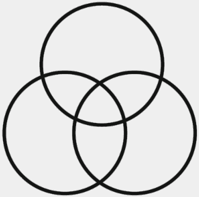
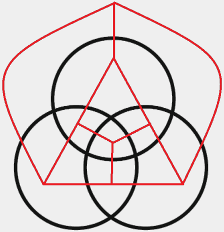


Chilakamarri, Hamburger, Pippert's Proof





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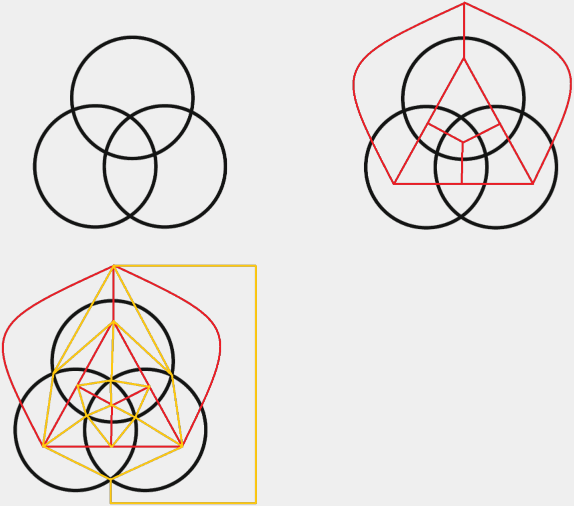

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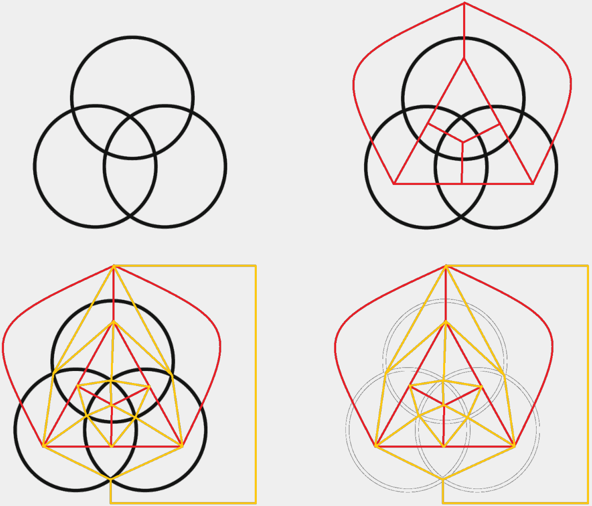

Classification of Venn diagrams

Isomorphism

Definition

Two Venn diagrams are isomorphic if a suitable deformation of the plane converts one to the other modulo a mirror symmetry.

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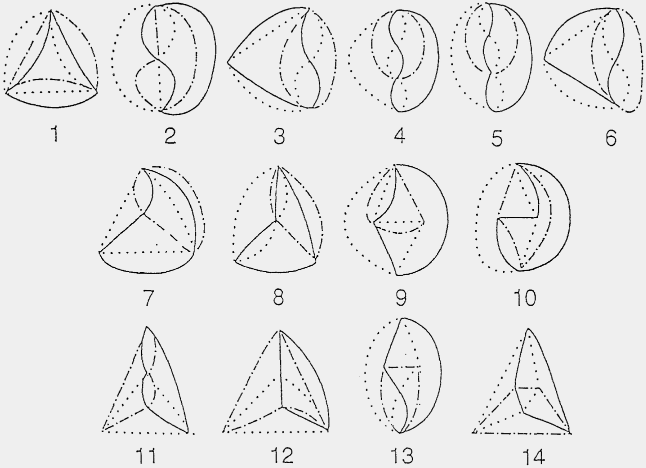
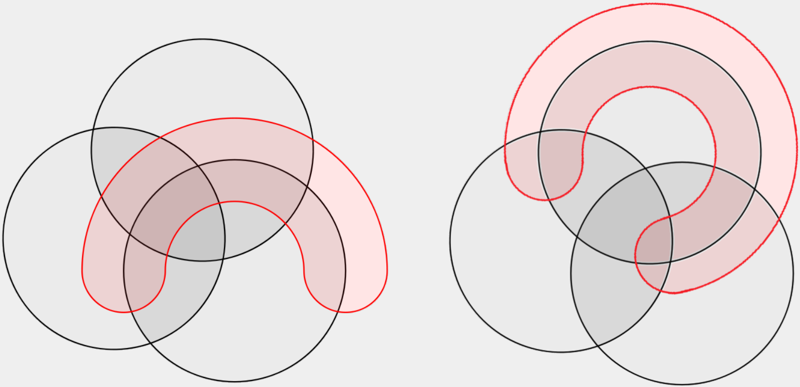
Problem

Classify isomorphism classes of n-Venn diagrams.

3-Venn diagrams



[1] K. B. Chilakamarri, P. Hamburger, and R. E. Pippert, Venn diagrams and planar graphs, *Geom. Dedicata* **62**(1) (1996), 73–91.



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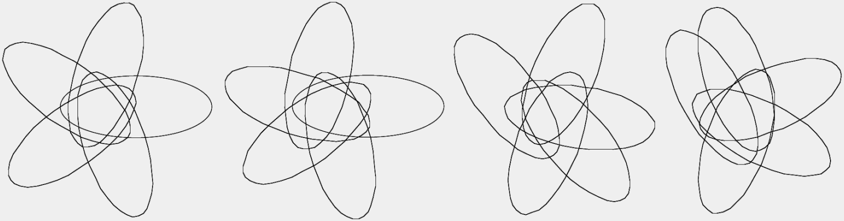

Conjecture (Grünbaum, 1992[1])

Every simple convex 5-Venn diagram of ellipses is isomorphic to one of the above diagrams.

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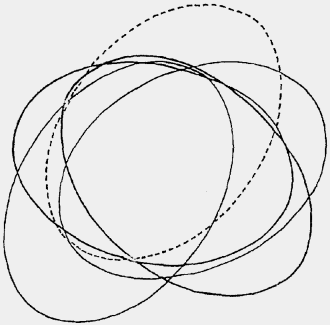
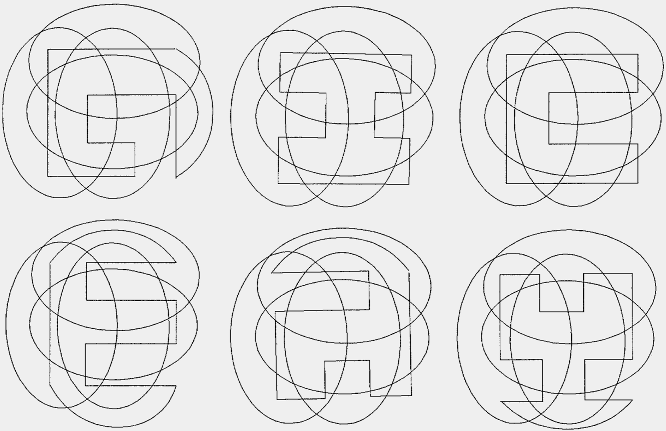


Conjecture

Every simple convex 5-Venn diagram of ellipses is isomorphic to one of the above five diagrams.

[1]K. B. Chilakamarri, P. Hamburger, and R. E. Pippert, Venn diagrams and planar graphs, *Geom. Dedicata* **62**(1) (1996), 73–91.

Simple reducible spherical 5-Venn diagrams



[1]P. Hamburger and R. E. Pippert, Simple, reducible Venn diagrams on five curves and Hamiltonian cycles, *Geom. Dedicata* **68**(3) (1997), 245–262.

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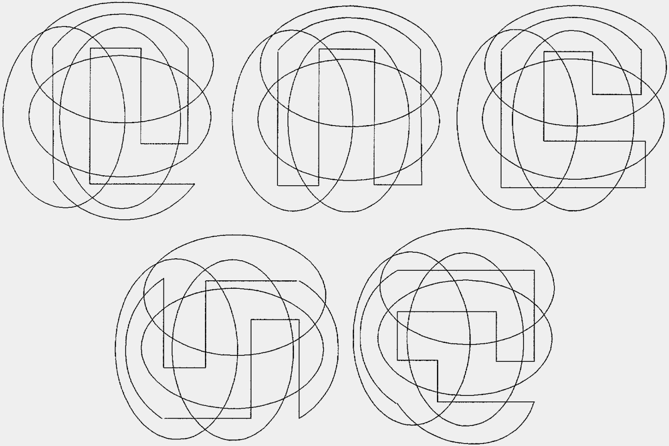

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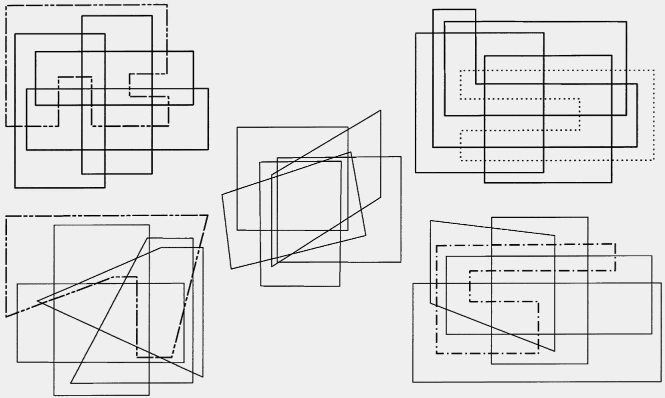
Simple irreducible spherical 5-Venn diagrams



[1] K. B. Chilakamarri, P. Hamburger, and R. E. Pippert, Analysis of Venn diagrams using cycles in graphs, *Geom. Dedicata* **82**(1-3) (2000), 193–223.

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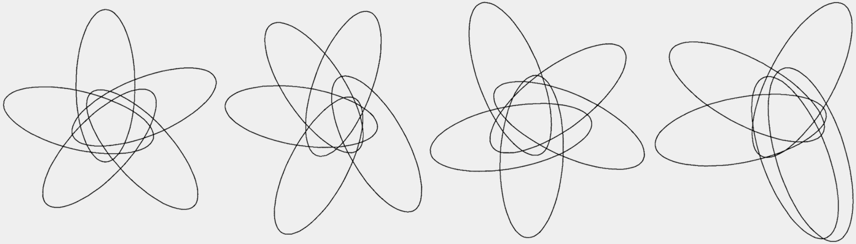

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Theorem (Carroll, 2000[1])

There are 126 triangular 6-Venn diagrams.

[1] J. Carroll, Drawing Venn triangles, Technical Report HPL-2000-73, *HP Labs*, 2000.

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---

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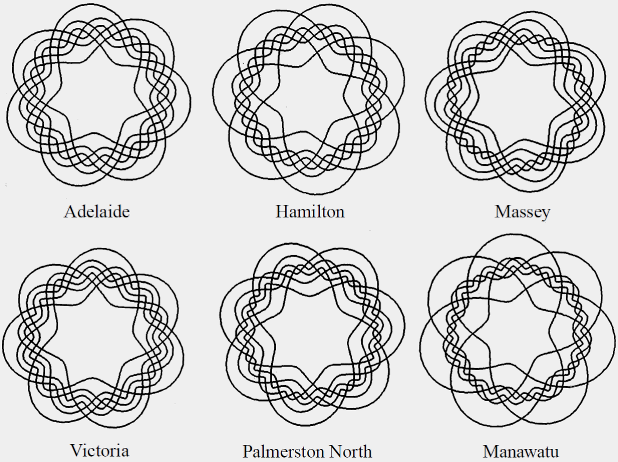
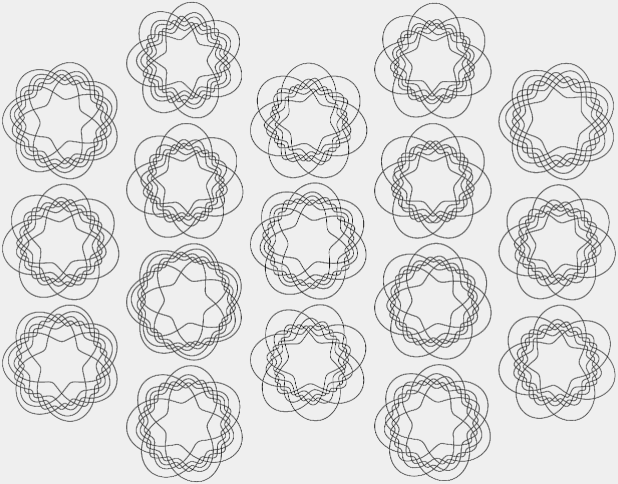
[2] K. Mamakani, W. Myrvold, and F. Ruskey, Generating simple convex Venn diagrams, *J. Discrete Algorithms* **16** (2012), 270–286.

Completely symmetric monotone simple 7-Venn diagrams



[1] A. W. F. Edwards, Seven-set Venn diagrams with rotational and polar symmetry, *Combin. Probab. Comput.* **7**(2) (1998), 149–152.

Non-polar symmetric monotone simple 7-Venn diagrams



[1]K. Mamakani, W. Myrvold, and F. Ruskey, Generating simple convex Venn diagrams, *J. Discrete Algorithms* **16** (2012), 270–286.

Polynomio Venn diagrams

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Definition

- The family $V = \{C_1, \ldots, C_n\}$ of curves is a polyVenn diagram if the intersection $X_1 \cap \cdots \cap X_n$ is a non-empty connected region, where X_i is the interior or exterior of C_i, for $i = 1, \ldots, n$.

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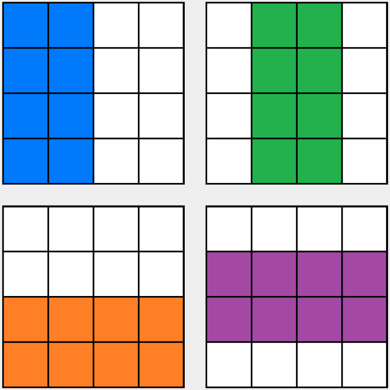
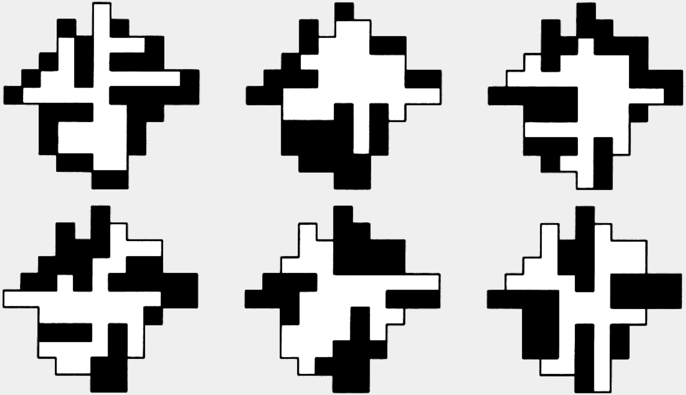
- The polyVenn diagram V is a minimum area polyVenn if all the regions $X_1 \cap \cdots \cap X_n$ are unit squares.



4-polyVenn diagram



6-polyVenn diagram

[1]S. Chow and F. Ruskey, Minimum area Venn diagrams whose curves are polyominoes, *Math. Mag.* **80**(2) (2007), 91–103.

Definition

An (r, c)-polyVenn diagram is a minimum area n-polyVenn diagram ($n = r + c$) inside a $2^r \times 2^c$ rectangle.

[1] B. Bultena, M. Klimesh, and F. Ruskey, Minimum area polyomino Venn diagrams, *J. Comput. Geom.* **3**(1) (2012), 154–167.

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Theorem (Bultena, Klimesh, and Ruskey, 2012[1])

There exists an (r, c)-polyVenn diagram for all $r, c \geq 2$.

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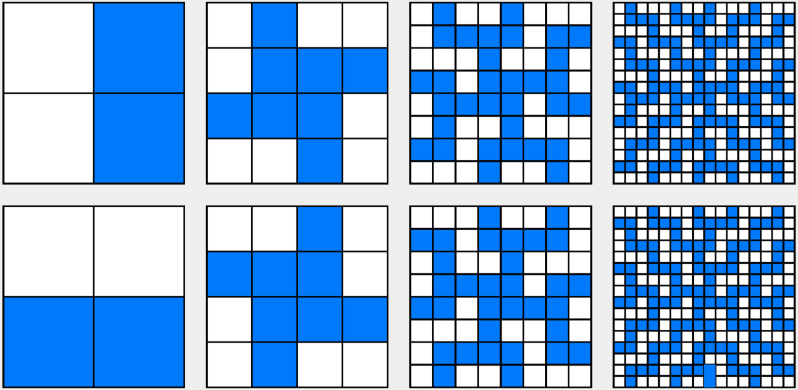

2-, 4-, 6-, 8-polyVenn diagrams

Venn diagrams in any dimension

Definition

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- An *m*-space is a subset of a Euclidian space homeomorphic to an m-dimensional ball or subspace.

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- An *m-surface* is a subset of an m-space that is homeomorphic to an $(m-1)$-dimensional sphere.
- S^0 and S^1 denote the interior and exterior of a m-surface S, respectively.

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- the intersection of any two surfaces is a union of finitely many $(m-1)$-surfaces.

Definition

An m-dimensional Venn diagram is simple if the intersection of any k surfaces is a union of finitely many $(m-k+1)$-surfaces.

[1] M. Farrokhi D. G., Fully reducible Venn diagrams, arXiv:2206.03323.

Definition

An m-dimensional Venn diagram is simple if the intersection of any k surfaces is a union of finitely many $(m-k+1)$-surfaces.

Theorem (MFDG, 2005[1])

There exists a simple m-dimensional n-Venn diagram for each $m \geq 2$ *and* $n \geq 1$.

[1] M. Farrokhi D. G., Fully reducible Venn diagrams, arXiv:2206.03323.

Definition

A Venn diagram $V = \{S_1, \ldots, S_n\}$ is fully reducible if V' is a Venn diagram for every subset V' of V.

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Let $V = \{S_1, \ldots, S_n\}$ be a simple Venn diagram and $1 < r < n$. Then V is fully reducible iff every subset of V of size r is a Venn diagram.

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Theorem (MFDG, 2005[1])

Let $V = \{S_1, \ldots, S_n\}$ be a simple Venn diagram. Then $|E(V)| \leq n2^{n-1}$ and the equality holds iff V is fully reducible.

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Theorem (MFDG, 2005[1])

Let $V = \{S_1, \ldots, S_n\}$ be a simple m-dimensional Venn diagram. If V is fully reducible, then $n \leq m + 1$.

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Conjecture (MFDG, 2005[1])

If V is a simple m-dimensional n-Venn diagram with $n \leq m + 1$, then V is fully reducible.

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Conjecture (MFDG, 2005[1])

If V is a simple m-dimensional n-Venn diagram, then

$$|E(V)| \leq m2^n + a_0 + a_1 n + \cdots + a_{m-2} n^{m-2},$$

where the coefficients $a_0, a_1, \ldots a_{m-2}$ satisfy the equation

$$\begin{bmatrix} 1 & 2 & 2^2 & \ldots & 2^{m-2} \\ 1 & 3 & 3^2 & \ldots & 3^{m-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & m & m^2 & \ldots & m^{m-2} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{m-2} \end{bmatrix} = \begin{bmatrix} 2 \cdot 2^1 - m \cdot 2^2 \\ 3 \cdot 2^2 - m \cdot 2^3 \\ \vdots \\ m \cdot 2^{m-1} - m \cdot 2^m \end{bmatrix}.$$

[1]M. Farrokhi D. G., Fully reducible Venn diagrams, arXiv:2206.03323.

k-fold Venn diagrams

Definition

Let $V = \{C_1, \ldots, C_n\}$ be a family of n curves on the plane such that $\mathbb{R}^2 \setminus C_i$ is a disjoint union of k connected open subsets $C_i^1, \ldots, C_i^k$, for all $i = 1, \ldots, n$. Then V is a **k-fold n-Venn diagram** if $C_1, \ldots, C_n$ divide the plan into k^n regions such that every intersection $C_1^{\varepsilon_1} \cap \cdots \cap C_n^{\varepsilon_n}$ is a non-empty connected region, for all $1 \leq \varepsilon_1, \ldots, \varepsilon_n \leq n$.

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Theorem (B. Grünbaum, 1984[1])

There exist k-fold n-Venn diagrams for all $n \geq 1$ and $k \geq 2$.

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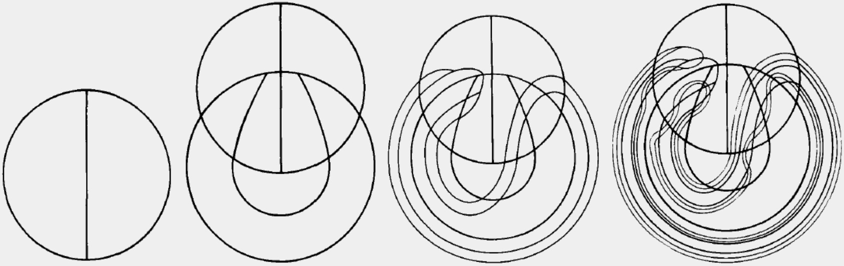

3-Fold 1-, 2-, 3-, and 4-Venn diagrams

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Topics not covered in this talk

- Connections of Venn diagrams to knots and links

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Thanks!