Beyond mean modelling: multi-parameter GAMs

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Material available at:

`https://github.com/mfasiolo/GAM_Workshop_Enbis_EDF_23`

Recall GAM model structure:

$$y|\boldsymbol{x} \sim \text{Distr}\{y|\theta_1 = \mu(\boldsymbol{x}), \theta_2, \dots, \theta_p\},$$

where

$$g\{\mu(\boldsymbol{x})\} = \sum_{j=1}^{m} f_j(\boldsymbol{x}).$$

Multi-parameter GAM structure (Wood et al., 2016):

$$y|\boldsymbol{x} \sim \text{Distr}\{y|\theta_1 = \mu_1(\boldsymbol{x}), \theta_2 = \mu_2(\boldsymbol{x}), \dots, \theta_p = \mu_p(\boldsymbol{x})\},$$

where

$$g_1\{\mu_1(\boldsymbol{x})\} = \sum_{j=1}^{m} f_j^1(\boldsymbol{x}), \quad \dots \quad g_p\{\mu_p(\boldsymbol{x})\} = \sum_{j=1}^{m} f_j^p(\boldsymbol{x}).$$

Special case are **Generalized Additive Models for Location Scale and Shape** (GAMLSS) (Rigby and Stasinopoulos, 2005).

See appendix for complete list of distributions in `mgcv`.

Example: **Gaussian location-scale model**

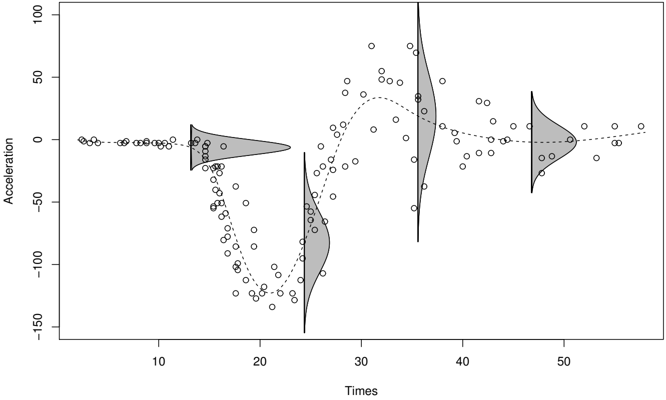
Model is

$$y|\mathbf{x} \sim N\{y|\mu(\mathbf{x}), \sigma^2(\mathbf{x})\}$$

where

$$\mu(\mathbf{x}) = \sum_{j=1}^{m} f_j^1(\mathbf{x})$$

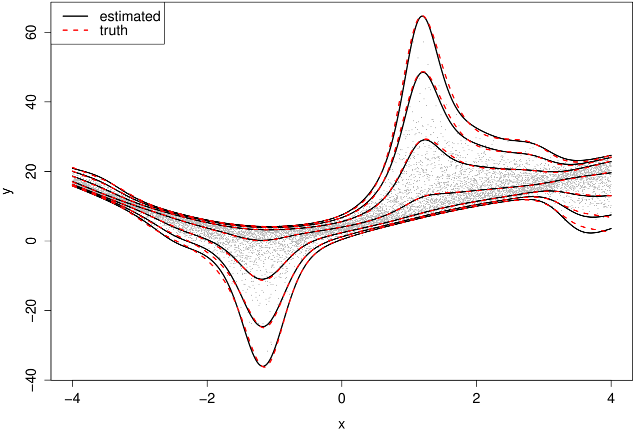
$$\log \sigma(\mathbf{x}) = \sum_{j=1}^{m} f_j^2(\mathbf{x})$$



In mgcv: `gam(list(y ~ s(x), ~ s(x)), family=gaulss)`.

NOTE: `bam()` can not be used with multi-parameter GAMs.

Example: **Sinh-arcsinh (shash) distribution** (Jones and Pewsey, 2009)



`gam(list(y~s(x), ~s(x), ~s(x), ~s(x)), family=shash).`

Non-GAMLSS example: **multivariate normal GAMs**

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_d \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} \mu_1(\boldsymbol{x}) \\ \mu_2(\boldsymbol{x}) \\ \mu_3(\boldsymbol{x}) \\ \vdots \\ \mu_d(\boldsymbol{x}) \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} & \cdots & \cdots & \Sigma_{1d} \\ \Sigma_{12} & \Sigma_{22} & \cdots & \cdots & \Sigma_{2d} \\ \Sigma_{13} & \Sigma_{23} & \cdots & \cdots & \Sigma_{3d} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Sigma_{1d} & \Sigma_{2d} & \cdots & \cdots & \Sigma_{dd} \end{pmatrix} \right).$$

In `mgcv`:

```
gam(list(y1~s(x1),y2~s(x3),y3~s(x3)), family = mvn(3))
```

With the `SCM` package we can model $\boldsymbol{\Sigma}$ as well

$$\boldsymbol{y} \sim N(\boldsymbol{\mu}(\boldsymbol{x}), \boldsymbol{\Sigma}(\boldsymbol{x})).$$

$\boldsymbol{\Sigma}(\boldsymbol{x})$ must be positive definite so we can **not** write $\Sigma_{jk} = \sum_j f_j(\boldsymbol{x})$.

One option is the modified Cholesky decomposition (Pourahmadi, 1999)

$$\boldsymbol{\Sigma}^{-1} = \boldsymbol{T}^{\top}\boldsymbol{D}^{-2}\boldsymbol{T}\,,$$

where $\boldsymbol{D}^2$ is a diagonal matrix and $\boldsymbol{T}$ is upper triangular.

Available via the `SCM` package

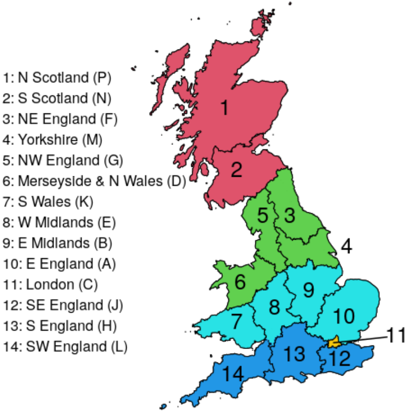
`https://github.com/VinGioia90/SCM`

To install type:

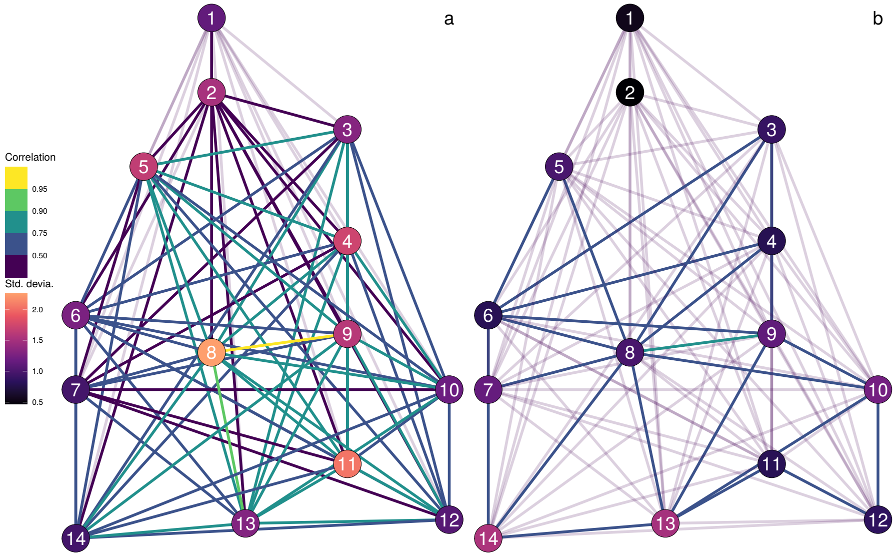
```
library(devtools)
install_github("VinGioia90/SCM")
```

See Gioia et al. (2022) for details.

The UK electricity grid is divided into 14 grid supply groups (GSP).

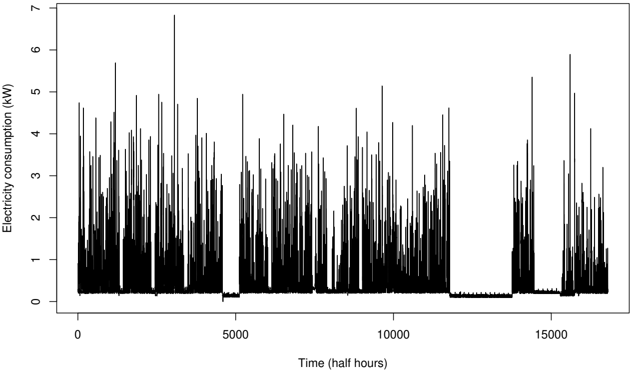


Gioia et al. (2022) produce **joint** probabilistic forecasts of net-demand.



Left: 7am 31/12/2018. Right: midnight 20/08/2018.

Non-GAMLSS example: **additive stacking** or **aggregation of experts**



Two models for log-demand at time t or day d:

1. $\log \text{dem}_{td} \sim N(\mu = f(\text{time\_of\_day}_t), \sigma^2)$
2. $\log \text{dem}_{td} \sim N(\mu = \text{mean}(\log \mathbf{dem}_{d-1}), \sigma^2 = \text{var}(\log \mathbf{dem}_{d-1}))$

where $\mathbf{dem}_d = \{\log \text{dem}_{1d}, \dots, \log \text{dem}_{48d}\}$.

We want to predict $y|\boldsymbol{x}$ and we have models $p_1(y|\boldsymbol{x}), \dots, p_K(y|\boldsymbol{x})$.

Build mixture with covariate-dependent weights

$$p_{\text{mix}}(y|\boldsymbol{x}) = \sum_{k=1}^{K} w_k(\boldsymbol{x}) p_k(y|\boldsymbol{x});$$

where $0 \leq w_k \leq 1$ and $\sum_k w_k = 1$.

Capezza et al. (2021) use the multinomial parametrisation

$$w_k = \frac{e^{\eta_k}}{\sum_j e^{\eta_j}},$$

with $\eta_1 = 0$ and $\eta_k = \sum_l f_{lk}(\boldsymbol{x})$ for $k = 2, \dots, K$.

Available via the gamFactory:

```
install_github("mfasiolo/gamFactory")
```

Upcoming gamstackr package by Euan Enticott (EDF-sponsored PhD):

euan.enticott@bristol.ac.uk

will provide more methods.

Email him if interested!

Now continue example on 2_multi_gams.html.

For related approaches see Yao et al. (2022) and Rügamer et al. (2022).

References I

Capezza, C., B. Palumbo, Y. Goude, S. N. Wood, and M. Fasiolo (2021). Additive stacking for disaggregate electricity demand forecasting. *The Annals of Applied Statistics* 15(2), 727–746.

Gioia, V., M. Fasiolo, J. Browell, and R. Bellio (2022). Additive covariance matrix models: modelling regional electricity net-demand in great britain. *arXiv preprint arXiv:2211.07451*.

Jones, M. and A. Pewsey (2009). Sinh-arcsinh distributions. *Biometrika 96*(4), 761–780.

Pourahmadi, M. (1999). Joint mean-covariance models with applications to longitudinal data: Unconstrained parameterisation. *Biometrika* 86(3), 677–690.

Rigby, R. A. and D. M. Stasinopoulos (2005). Generalized additive models for location, scale and shape. *Journal of the Royal Statistical Society: Series C (Applied Statistics) 54*(3), 507–554.

Rügamer, D., F. Pfisterer, B. Bischl, and B. Grün (2022). Mixture of experts distributional regression: Implementation using robust estimation with adaptive first-order methods. *arXiv preprint arXiv:2211.09875*.

References II

Wood, S. N., N. Pya, and B. Säfken (2016). Smoothing parameter and model selection for general smooth models. *Journal of the American Statistical Association 111*(516), 1548–1575.

Yao, Y., G. Pirš, A. Vehtari, and A. Gelman (2022). Bayesian hierarchical stacking: Some models are (somewhere) useful. *Bayesian Analysis 17*(4), 1043–1071.

Youngman, B. D. (2022). evgam: An r package for generalized additive extreme value models. *Journal of Statistical Software 103*(3), 1–26.

List of distributions in mgcv

Type `?mgvc::family` on the R console to see a list of distributions from the exponential family.

Wood et al. (2016) allows Distr $\notin$ exponential family (**extended GAMs**):

1. `scat` $\rightarrow$ scaled Student-t;
2. `betar` $\rightarrow$ beta for $y \in (0, 1)$;
3. `ziP` $\rightarrow$ zero-inflated Poisson;
4. `tw` $\rightarrow$ Tweedie;
5. `ocat` $\rightarrow$ order categorical;
6. `nb` $\rightarrow$ negative binomial.

Note that these EGAMs have one linear predictor, i.e.

$$y_i|\boldsymbol{x}_i \sim \text{Distr}\{y_i|\theta_1(\boldsymbol{x}), \theta_2, \dots, \theta_p\}.$$

hence are not GAMLSSs.

Available GAMLSS families:

1. `gammals` $\rightarrow$ 2-par gamma;
2. `gaulss` $\rightarrow$ 2-par Gaussian;
3. `shash` $\rightarrow$ 4-par sinh-arsinh;
4. `ziplss` $\rightarrow$ 2-par zero-inflated Poisson;
5. `gevlss` $\rightarrow$ 3-par generalised extreme value distribution (GEV);
6. `gumbls` $\rightarrow$ 2-par Gumbel (special case of GEV);
7. `twlss` $\rightarrow$ 3-par Tweedie.

For extreme value GAMs, see also the `evgam` package (Youngman, 2022).

Further models are:

1. `multinom` $\rightarrow$ multinomial categorical;
2. `cox.ph` $\rightarrow$ Cox Proportional Hazards model;
3. `mvn` $\rightarrow$ multivariate Gaussian model (fixed covariance).