Dynamic Modeling and Altitude Control of Parrot Rolling Spider using LQR

Ashis C K[1], Rahul Sharma K.[2]
Department of Electrical and Electronics Engineering
Amrita School of Engineering, Coimbatore
Amrita VishwaVidyapeetham, India
[1]*ashis.1ck@gmail.com*, [2]*k_rahulsharma@cb.amrita.edu*

***Abstract*—Parrot Rolling Spider/minidrone is one of the most favoured quadcopters around the world which is used for academic and experimental purposes. This paper deals with the mathematical modeling of kinematics and dynamics of the parrot minidrone, employing linear feedback controller for stabilizing the system and a comparative analysis of altitude control of Parrot minidrone using Proportional Integral Derivative (PID) and Linear Quadratic Regulator (LQR) control techniques. Matlab/SIMULINK software was used for simulating the plant response. The Simulink support package for parrot minidrones was used for obtaining the parameters of the system. All the studies and simulations conducted in this paper are based on kinematics and dynamics of parrot minidrone. The highly nonlinear plant model is linearized around equilibrium points and a linear feedback controller is employed for stabilizing the system.**

Keywords*—*PID, LQR, Matlab, SIMULINK, Parrot Rolling spider

I. Introduction

Researches on Unmanned Aerial Vehicles (UAVs) are increasing day by day. Because of its huge popularity, UAVs are a hot topic nowadays. A UAV does not carry a human pilot on it. These aerial vehicles are really helpful to test and evaluate new ideas in the fields like flight control theory, robotics, GPS navigation etc. The UAV market is growing everyday with variety of UAVs. Among them, quadcopters are the widely used ones because of its huge popularity and symmetry. Quadcopters are more vulnerable to environmental disturbances because of its compact structure. They make use of Vertical Take off and Landing (VTOL) technology [15]. Many online delivery companies are on their development stage to deliver products using drones at the doorstep of their customers. A stable hovering should be achieved by the system for this process. LQR has been used over many years in robotics and it is one of the most trusted control algorithms. This paper focuses on detailed mathematical modeling of a quadcopter UAV. For simulation studies, a support package from Simulink for parrot minidrones was used and was studied in detail. A practical approach was needed for students to test their knowledge on flight dynamics, control theory etc. The Simulink support package for parrot minidrones is a good initiative from Matlab for this. The lab experiments done in [4] using parrot minidrone had achieved a stable hovering drone. The nonlinear model of the quadcopter was linearised using Jacobian method in [10]. A full state feedback controller was designed here for measuring all the 12 states of the quadcopter. The PID controller was tested on a hardware prototype [14]. The primary aim of selecting parrot minidrone for the simulation studies is its availability. Unlike most of the drones, this minidrone is compatible with Simulink working environment. This work has a step by step approach for studying the system in detail and it has been organized into different sections. The next section describes the properties of the system under study in detail. Successive sections include the modelling of the system, controller design, estimation and simulation studies.

II. System Description

The Parrots Rolling Spider is a palm size quadcopter which works with Bluetooth Low Energy (BLE) technology. It can be operated with a smartphone which supports BLE. Since the minidrone weighs only 55 grams, it is more sensitive to environmetal disturbances. It includes a 3 axis accelerometer, 3 axis gyroscope, 0.3 megapixel 60fps downward facing camera and a pressure sensor. It is compatible with Windows and Linux operating systems. It has an operating range of 20 meters and is powered by a 550 mAh Lithium-Polymer (Li-Po) removable battery which can give a flight time of 8 minutes/6minutes (with wheels). The four BLDC motors mounted above the drone rotates the propellers. The motherboard of the rolling spider features Parrot SIP6 chipset with an 800 MHz Arm A9 processor. The Fig 1 shows the Parrot Rolling Spider [16].

Fig. 1. Parrot Rolling Spider

A generic block diagram showing the altitude control of minidrone using altitude controller is shown in Fig 2. The sensors can sense the altitude output and it can feedback the altitude commands to the controller. The controller can give control outputs as thrust commands to the four motors of the system. For controlling the roll angle, pitch angle, yaw anglw and thrust command seperately, seperate PID controllers can be implemented in the system.

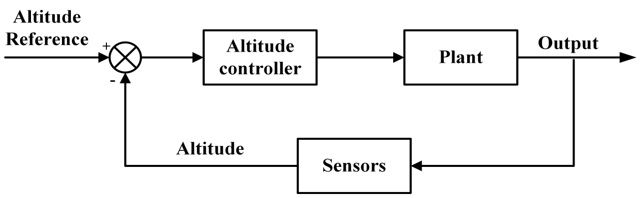

Fig. 2. Block diagram for altitude control

III. System modeling

This section explains the kinematics and dynamics of the quadcopter. A nonlinear state space model has been derived after study. A schematic showing the quadcopter control frames is shown in Fig 3. The earth inertial frame consists of unit vectors along x, y and z directions. The alignment of the body reference frame keeps on changing based on the attitude of the quadcopter. The kinematic model of the system explains about the rotational matrix and matrix of angular transformation.

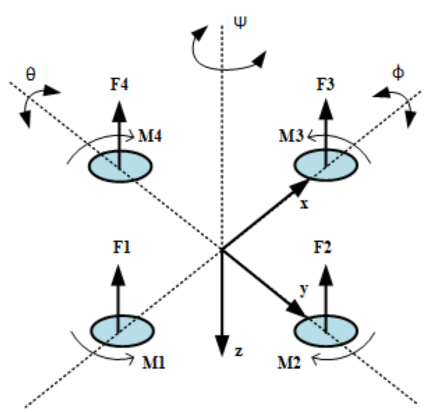

Fig. 3. Schematic of quadcopter reference frames

A. Nonlinear model

The physical properties of the quadcopter are measured using the two reference frames. Some of the physical properties of the quadcopter such as roll, pitch, yaw angles and the angular velocities are measured in earth frame. Some other properties such as linear accelarations are measured in body frame. The linear and angular position vectors of the quadrotor in earth frame is $\begin{bmatrix} x & y & z & \phi & \theta & \psi \end{bmatrix}^T$ where as $\begin{bmatrix} u & v & w & p & q & r \end{bmatrix}^T$ is the linear and angular velocity vectors in the body frame. The relation between the two reference frames is,

$$\nu = R \cdot \nu_b \tag{1}$$

$$\omega = T \cdot \omega_b \tag{2}$$

where $\nu = \begin{bmatrix} \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T$, $\omega = \begin{bmatrix} \dot{\phi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T$, $\nu_b = \begin{bmatrix} u & v & w \end{bmatrix}^T$, $\omega_b = \begin{bmatrix} p & q & r \end{bmatrix}^T$ and R is the rotation matrix which rotates the points from one frame to the other frame. The matrix of angular transformation T is given by,

$$T = \begin{bmatrix} 1 & s(\phi)t(\theta) & c(\phi)t(\theta) \\ 0 & c(\phi) & -s(\phi) \\ 0 & \dfrac{s(\phi)}{c(\theta)} & \dfrac{c(\phi)}{c(\theta)} \end{bmatrix} \tag{3}$$

The total force experienced by the quadcopter is the sum of all individual forces acting on x, y and z directions which is given by,

$$F_B = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = m(\omega_b \cdot \nu_b + \dot{\nu_b}) \tag{4}$$

where m gives the mass of the system and f_x, f_y and f_z are the forces acting on x, y, and z directions respectively. The total torque on the quadcopter is,

$$M_B = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \omega_b \times (I \cdot \omega_b) + I \cdot \dot{\omega_b} \tag{5}$$

where M_x, M_y and M_z are the torques acting on x, y, and z directions respectively. The inertia matrix I is given by,

$$I = diag(I_x, I_y, I_z) \tag{6}$$

$f_w = \begin{bmatrix} f_{wx} & f_{wy} & f_{wz} \end{bmatrix}$ is the force acting on the quadcopter due to wind effect. The difference in rotor speeds generates a control torque and is given by,

$$\tau_B = \begin{bmatrix} \tau_x & \tau_y & \tau_z \end{bmatrix}^T \tag{7}$$

Torques produced by wind on quadrotor,

$$\tau_w = \begin{bmatrix} \tau_{wx} & \tau_{wy} & \tau_{wz} \end{bmatrix} \tag{8}$$

The squared speeds of the rotors is proportional to the torques and input forces. ie,

$$\begin{bmatrix} f_t \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} b(\delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2) \\ bl(\delta_3^2 - \delta_1^2) \\ bl(\delta_4^2 - \delta_2^2) \\ d(\delta_2^2 + \delta_4^2 - \delta_3^2 - \delta_1^2) \end{bmatrix} \tag{9}$$

where the distance between the drone's centre and the rotor is denoted by l. b represents thrust factor and d represents drag factor and δ_i is the speed of the rotor for $i = \{1, 2, 3, 4\}$. The state variables are,

$$X = \begin{bmatrix} x & y & z & u & v & w & \phi & \theta & \varphi & p & q & r \end{bmatrix}^T \tag{10}$$

After considering the above kinematics and dynamics of the quadcopter, the nonlinear model, $\dot{X} = f(X, U)$ can be written as,

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \dot{\phi}\\ \dot{\theta}\\ \dot{\varphi}\\ u\\ v\\ w\\ p\\ q\\ r\end{bmatrix} = \begin{bmatrix} w[s(\phi)s(\varphi)+c(\phi)c(\varphi)s(\theta)]-v[c(\phi)s(\varphi)-c(\varphi)s(\phi)s(\theta)]+u[c(\varphi)c(\theta)]\\ v[c(\phi)c(\varphi)+s(\phi)s(\varphi)s(\theta)]-w[c(\varphi)s(\phi)-c(\phi)s(\varphi)s(\theta)]+u[c(\theta)s(\varphi)]\\ w[c(\varphi)c(\theta)]-u[s(\theta)]+v[c(\theta)s(\phi)]\\ p+r[c(\phi)t(\theta)]+q[s(\phi)t(\theta)]\\ q[c(\phi)]-r[s(\phi)]\\ r\frac{c(\phi)}{c(\theta)}+q\frac{s(\phi)}{c(\theta)}\\ rv-qw-g[s(\theta)]+\frac{f_{wx}}{m}\\ pw-ru+g[s(\phi)c(\theta)]+\frac{f_{wy}}{m}\\ qu-pv+g[c(\theta)c(\phi)]+\frac{f_{wz}-f_t}{m}\\ \frac{I_y-I_z}{I_x}rq+\frac{\tau_x+\tau_{wx}}{I_x}\\ \frac{I_z-I_x}{I_y}pr+\frac{\tau_y+\tau_{wy}}{I_y}\\ \frac{I_x-I_y}{I_z}pq+\frac{\tau_z+\tau_{wz}}{I_z}\end{bmatrix} \tag{11}$$

The nonlinear model in (11) can further be simplified by neglecting the forces produced by the wind on quadcopter. For introducing optimal control, the nonlinear system need to be linearized.

B. Linearization

The nonlinear state space equations can be linearized around an operating point $\bar{x}$ with a constant input value $\bar{u}$. A zero attitude condition was considered for linearization. Linearization of the quadcopter model has been performed for a hovering configuration and is carried out to obtain a generalized linear state space model of the form,

$$x\dot{(}t) = Ax(t) + Bu(t) \tag{12}$$

where,

$$A = \begin{bmatrix} 0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&-g&0&0&0&0&0&0&0&0&0&0\\ g&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0 \end{bmatrix} \tag{13}$$

$$B = \begin{bmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&\frac{1}{I_x}&0&0\\ 0&0&\frac{1}{I_y}&0\\ 0&0&0&\frac{1}{I_z}\\ 0&0&0&0\\ 0&0&0&0\\ \frac{1}{m}&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{bmatrix} \tag{14}$$

A and B are the state matrix and input matrix respectively. The system must be controllable for the design of controllers. The parameters used for simulating the model in matlab are listed in Table I.

TABLE I
MODEL PARAMETERS

Symbols	Values	Description
m	0.068 kg	Mass of the quadcopter
g	9.81 m/s^2	Gravitational constant
I_x	0.0686*10^{-3}kg.m^2	MOI along x axis
I_y	0.092*10^{-3}kg.m^2	MOI along y axis
I_z	0.1366*10^{-3}kg.m^2	MOI along z axis
rho	1.184 kg/m^3	Density of air
b	0.0107	Thrust factor
l	0.0624 m	Rotor to drone centre length
r	0.033 m	Rotor radius

IV. Controller Design

A. PID control

A linear PID controller can be applied on a nonlinear system without linearizing the system dynamics about the equilibrium point. The controller can still stabilize the system in case of external forces. The Simulink support package for parrot minidrones was utilized for the PID control simulation. This package includes one Simulink model with dynamics of linear and nonlinear model. It also includes sensor dynamics and disturbance. The flight control system bock includes six PID controllers for stabilizing the system. A Proportional Derivative (PD) controller was sufficient enough for the system to stabilize itself at a reference altitude. The PID control output is given as,

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{\mathrm{d}e(t)}{\mathrm{d}t} \tag{15}$$

where $u(t)$ denotes the control input, K_p denotes the proportional gain, K_i denotes the integral gain and K_d denotes the derivative gain. The tracking error,

$$e(t) = y(t) - r(t) \tag{16}$$

where $y(t)$ and $r(t)$ are the output and the desired output of the system respectively.

B. Linear Quadratic Regulator

LQR formulates a state feedback law for a full state feedback control. The state feedback law for LQR is,

$$u = -Kx \tag{17}$$

where u and K are the control input and the state feedback gain respectively. The process is assumed to be a continuous time Linear Time Invariant (LTI). The function to be minimized is given by,

$$J(u) = \int_0^\infty (x^T Qx + u^T Ru + 2x^T Nu)dt \tag{18}$$

where Q, R and and N are the state cost, control cost and an optional cross term matrices respectively subject to the dynamics of the system,

$$\begin{aligned} x(\dot{t}) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{19}$$

The LQR controller generates the control law using A, B, Q and R matrices. Solution of S can be obtained by solving the Algebraic Riccati equation,

$$A^T S + SA - (SB + N)R^{-1}(B^T S + N^T) + Q = 0 \tag{20}$$

K can be derived as,

$$K = R^{-1}(B^T S + N^T) \tag{21}$$

V. Simulation Results

Matlab/SIMULINK software was used for simulating the plant model using PID and Linear Quadratic (LQ) control. The altitude responses with PID and LQR controllers has been plotted. Using equation (1) as the nonlinear model, a PID controller in the form of (15) was considered with a sampling time of $Ts = 0.05$ second. The altitude control of parrot minidrone using PID control is shown in Fig 4. A reference command of 1.5 meters was given to the model. The PD controller for altitude control was able to stabilize the system within 5 seconds. The PD controller made the system to hover at 1.5 meters. Tuning the PD controller for $K_p = 0.8$ and $K_d = 0.3$ will result in a hovering control.

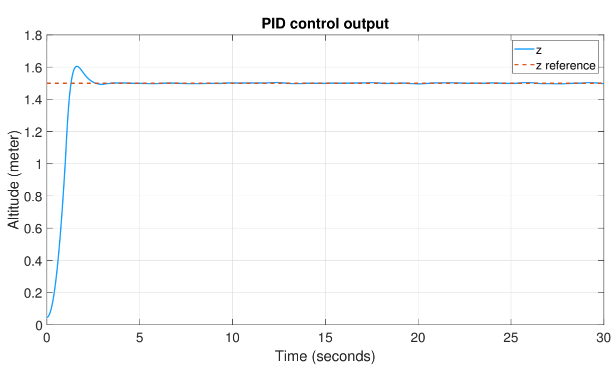

Fig. 4. Altitude control with PID

The LQ control simulation for the minidrone has been shown in Fig 5. The control input given to the drone and the altitude output obtained after simulation has been plotted. For LQ control, equation (17) is used as the controller model, with parameters $R = diag(0.01, 0.01, 0.01, 0.01)$ and $Q = diag(0, 0, 0, 0, 0, 0, 0, 0, 150, 150, 150, 150)$. From the plot, it can be seen that the drone was able to track the reference command.

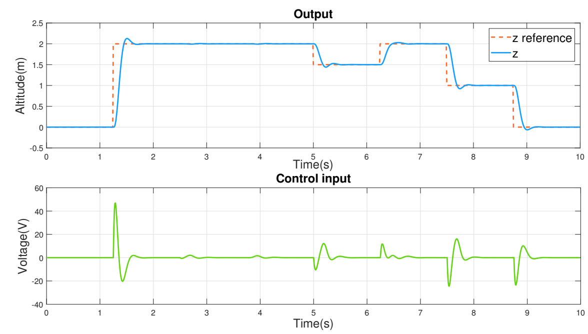

Fig. 5. Altitude control with LQR

The performance evaluation of the controllers has been tabulated in Table II. From the table, it can be observed that the LQR can give a better response than PID.

TABLE II
Performance Evaluation of Controllers

Controller	Settling time	Peak overshoot	Steady state error
PID	3	1.605	0.005
LQR	2.5	1.56	0.005

VI. Conclusion and Future Scope

A nonlinear mathematical model of the Parrot Rolling Spider, considering kinematics and dynamics of the system has been derived. Simulation on altitude control of the quadcopter using PID and LQR control has been done using Matlab/SIMULINK software and simulation outputs has been observed. From the simulation studies it can be observed that the steady state error caused when using PID controller has been considerably reduced when LQ control technique was introduced. Also the LQ control provided a better tracking of the system with minimum settling time. This paper focussed on simulation studies of a stable hovering drone. Tuning the PID control parameters was not that much difficult across the studies. The minidrone was able to achieve stability at hovering condition by using the two controllers discussed in the paper. When compared to the conventional PID control technique, optimal control can ignore the difficulty in tuning the PID control parameters. Instead of using six PID controllers for the whole system, one LQR controller will be enough for multiaxis control of minidrone. Future studies can be carried out on the actual hardware by making use of the Parrot minidrone support package from the Simulink which comes from the Aerospace blockset. The control technique studied in this paper can be verified by deploying that to the minidrone hardware after making modifications in the flight control system block.

REFERENCES

[1] Z. Benic, P. Piljek, and D. Kotarski, Mathematical Modelling of Unmanned Aerial Vehicles with Four Rotors, Interdiscip. Descr. Complex Syst., vol. 14, no. 1, pp. 88100, 2016.
[2] Q. Huang and B. Wingo, Mathematical Modeling of Quadcopter Dynamics Mathematical Modeling of Quadcopter Dynamics, 2016.
[3] J. Li and Y. Li, Dynamic analysis and PID control for a quadrotor, Int. Conf. Mechatronics Autom., pp. 573578, 2011.
[4] R. Mahtani and A. Ollero, Control and Stability Analysis of quadcopter, Int. Conf. Comput. Math. Eng. Technol. iCoMET, pp. 27, 2018.
[5] K. Patel and J. Barve, Modeling, simulation and control study for the quad-copter UAV, 2014 9th Int. Conf. Ind. Inf. Syst., pp. 16, 2014.
[6] M. F. Everett, LQR with Integral Feedback on a Parrot Minidrone, no. 4, p. 6, 2015.
[7] A. Merheb and H. Noura, Emergency Control of AR Drone Quadrotor UAV Suffering a Total Loss of One Rotor, vol. 4435, no. c, pp. 111, 2017.
[8] H. talla M. N. ElKholy, Dynamic Modeling and Control of a Quadrotor Using Linear and Nonlinear Approaches, Master Thesis, Am. Univ. Cairo, pp. 1143, 2014.
[9] J. Ajmera and V. Sankaranarayanan, Trajectory tracking control of a quadrotor, 2015 Int. Conf. Control Commun. Comput. India, no. July, pp. 4853, 2015.
[10] D. Uri and H. Lyu, Multivariable Control of a Rolling Spider Drone, 2017.
[11] B. E. Demir, R. Bayir, and F. Duran, Real-time trajectory tracking of an unmanned aerial vehicle using a self-tuning fuzzy proportional integral derivative controller, Int.J. Micro Air Veh., vol. 8, no. 4, pp. 252268, 2016.
[12] K. A. G. N. A. Tzes, Model predictive quadrotor control : attitude , altitude, vol. 6, no. March, pp. 18121827, 2012.
[13] S. Saha and S. Wadoo, Linear Optimal Control of a Parrot AR Drone 2 . 0, no. 9, 2017.
[14] V. Praveen and Dr. Anju Pillai S, Modelling and simulation of quadcopter using PID controller, International Journal of Control Theory and Applications, vol. 9, pp. 7151-7158 (2016).
[15] Anoop S, K Rahul Sharma, ”Model Predictive Control: Simulation Studies for the Implementation on Vertical Take-Off and Landing Lab Prototype”, 8th International Conference on Advances in Computing and Communication (ICACC-2018), Vol. 143, pp. 663-670
[16] https://www.amainhobbies.com/parrot-rolling-spider-rtf-micro-electric-quadcopter-drone-blue-ptapf723001/p362168