Department of

Electrical & Electronics Engineering

Abdullah Gül University

Project Report for Archimedean Spiral

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Project Report

EE1100 Computation and Analysis (COMA) Capsule

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OBJECTIVE

The aim of the project is to examine the trajectory curve drawn by a line starting from the center and rotating with constant angular velocity in a 2-dimensional experimental plane.

In this experiment, it was checked whether the experimental and theoretical results of the spiral drawn by the pen on a rotating disk were the same. If the results are not the same, the reason why the theoretical formulas and the results are not the same will be explained below.

BACKGROUND

For the project, a fixed-speed drill (as a motor) and a 52-centimeter disc fixed on the motor were attached to the middle point of the pen, and the curve drawn as a result of its rotation was examined.

ANALYTICAL AND SIMULATION PROCEDURES

a. Steps of Experiment

In this project

- Fixed-speed drill
- Cardboard disk
- Pencil
- Screw
- String

are used.

1. A hard material such as cardboard was used to keep the floor stable while moving. In order to reduce the error margin of the experiment, papers were placed on the cardboard disc as the system would run again and again.
2. As a motor the drill is used because it has high torque and rotates at a constant speed. The motor was mounted from the center of the disc to the bottom of the disc with the help of screws.

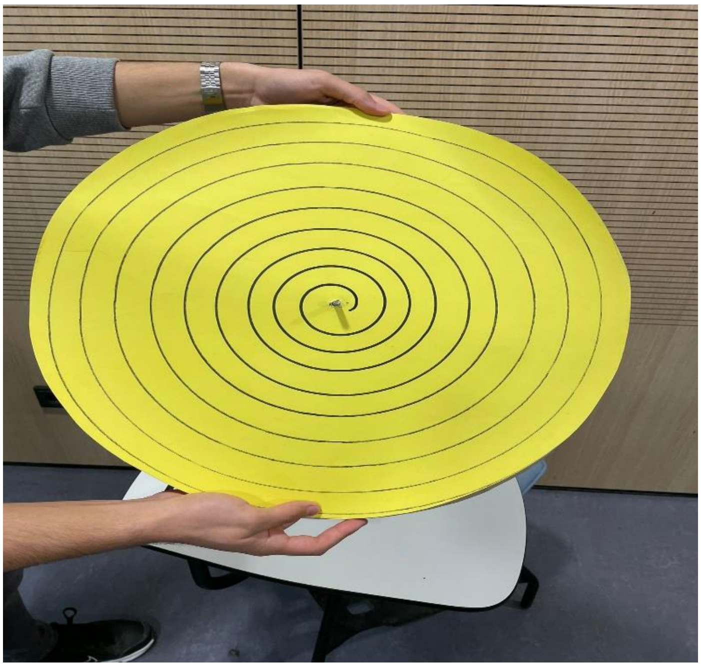
3. In order to draw the Archimedean spiral in a certain route, a string was attached to the screw coming out of the middle of the disk and to the pencil to rotate at the same speed as the disk. The string was wrapped around the screw to draw the spiral.
4. We used a straight stick to advance the pen in a linear road, as a result it can be understood to be drawn in a linear path.
5. A straight stick was used as a reference to advance the pen in a linear path, as a result it can be understood that it was drawn in a linear path.
6. We tried our system over and over to make sure we got the correct data. We took a video recording during the spiral formation (YouTube video link is attached. https://www.youtube.com/watch?v=k3WeKjIHUAw)

7. As a result of the experiment, a spiral disk of 504 cm in length and 22 cm in diameter was obtained by turning the rope 7 turns. The margin of error resulting from environmental factors was calculated.

Abstract Stages of Experiment

- As seen in the pictures, it is set up the mechanism so as to do the experiment and the sizes of the cardboard and disk are fit and mensurated. Ready to do the experiment.

- The Archimedean spiral was drawn with the help of materials. The sizes of spiral made such as the length of the spiral, the number of rotate and the gaps between the trajectories are mensurated.



- Suitable to take other values such as angular velocity, period etc. These values will be used for comments of physical and algebraic process below.

b. Evaluation of Problems

The theoretical formula is specified to evaluate the results obtained in the experiment and to find the margin of error. The velocity-time (V(t)) equation, which we will use in the two-dimensional cartesian system, was obtained as a result of source research [1*].

$$v(x) = v.cos\omega t - \omega(vt + c)sin\omega t$$

$$v(y) = v.sin\omega t + w(vt + c)cos\omega t$$

As a physical expression, the integral of the velocity-time equation gives the distance-time equation**.**

$$\int (v(x)) = x \qquad x = (vt + c).cos\omega t$$

$$\int \left(v(y)\right) = y \qquad y = (vt + c).sin\omega t$$

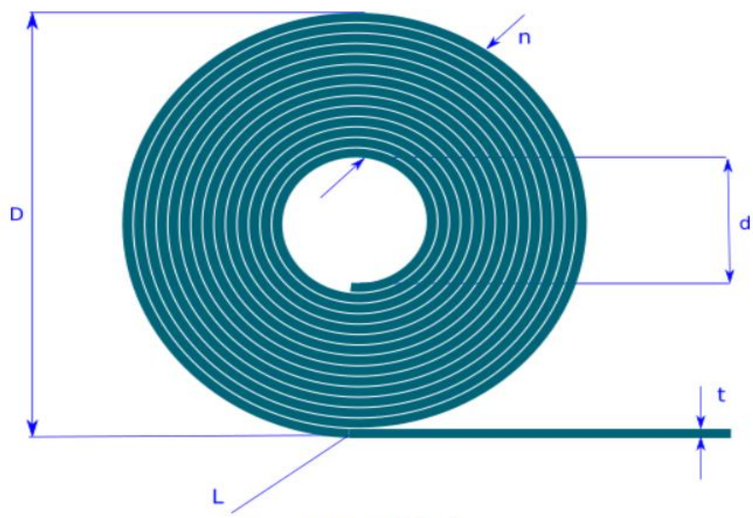
After these finding, the following formula was found on the following website as a result of the resource research conducted to find the length of the spiral theoretically [2*]. {As asked to the instructor (Sergey Borisenok), the equation below is used to find the length of the spiral.}

$$N \rightarrow \ number\ of\ tour$$

$$D \rightarrow \ outer\ diameter$$

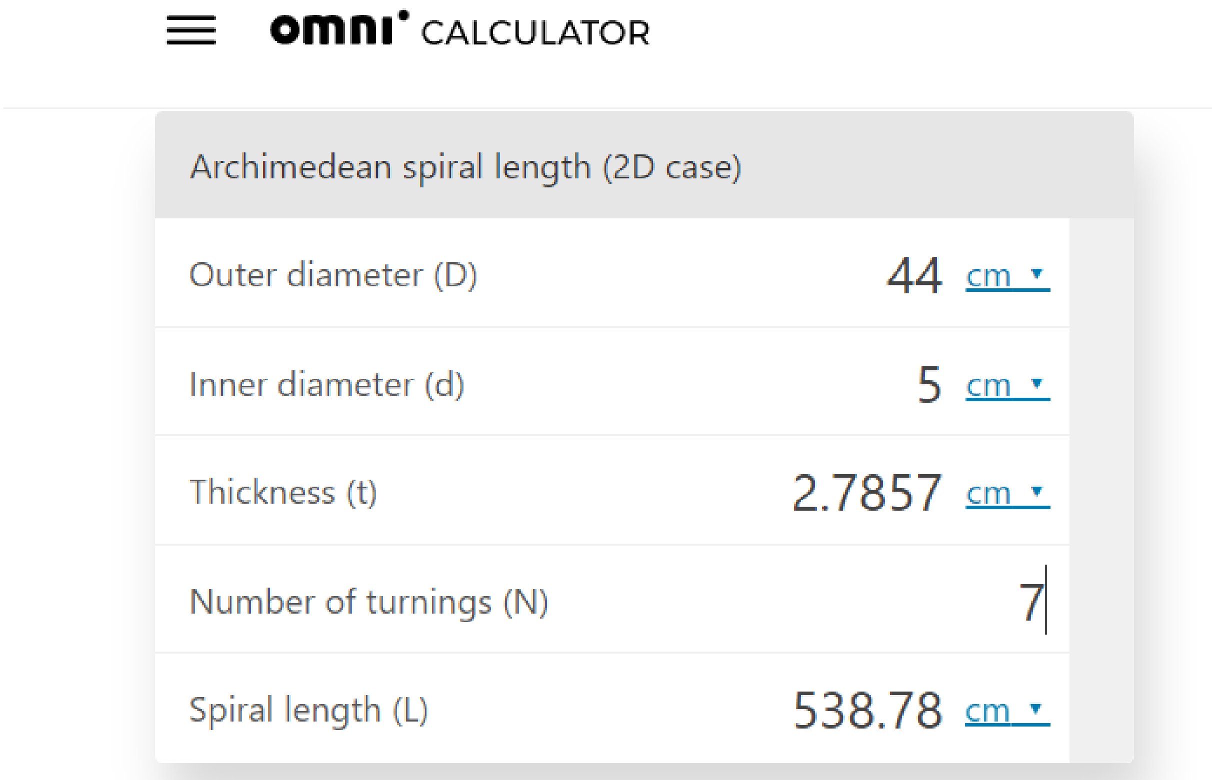
$$d \rightarrow inner\ diameter$$

$$L = \frac{\Pi \times N \times (D + d)}{2} \Rightarrow \frac{\Pi \times 7 \times (44 + 5)}{2} = 538,78314\ cm$$



Archimedean spiral

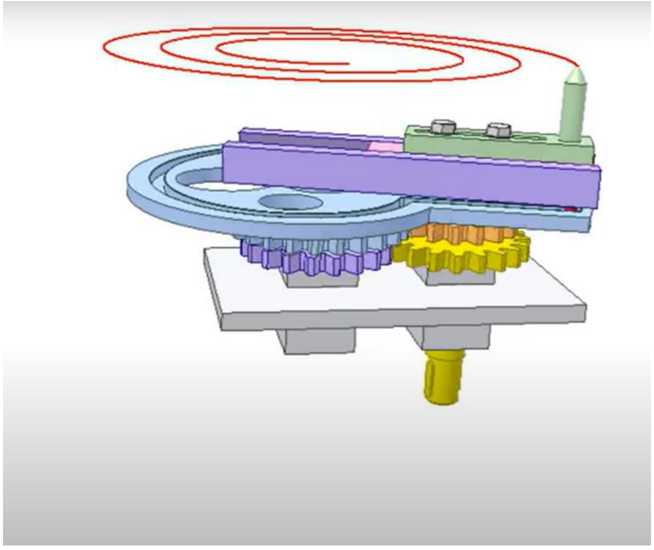
The margin of error was determined by making theoretical calculations from the website [3*].



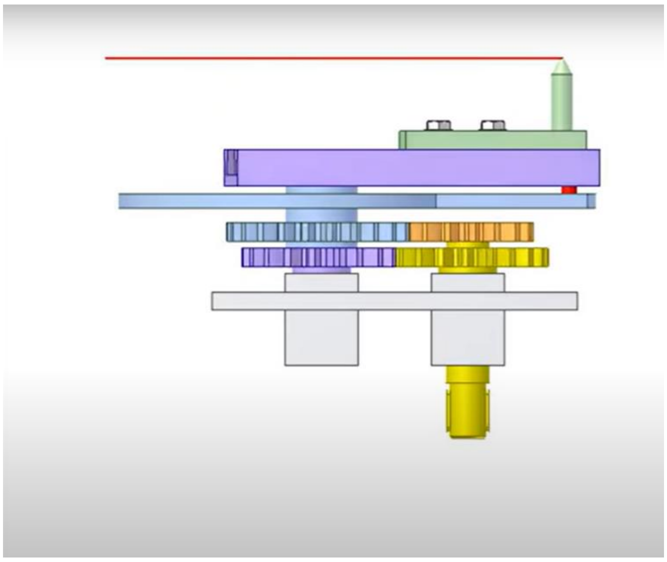
$$\frac{538{,}78314 - 504}{504} \times 100 \cong 6{,}9$$

504 cm is obtained from experimental measurement. Margin of error of the theoretical and experimental results was determined as 6.9%.

As two different reference points, we chose the pen's own trajectory of motion (on the disc) and an observer observing the motion of the pen from the outside. If an outside observer is considered as the second reference point, the possibility of 2 different orbits arises. The first of these is a one-dimensional straight line (Figure1.1) extending from the origin to infinity in the minus and plus directions. To see this straight line, the disc must be viewed from the full side. Another option is to consider the angle of 90 degrees as the disc is viewed from the highest point, and when viewed from an angle of approximately 45 degrees (Figure 1.0) it can be seen that the pen draws an elliptical trajectory. [4*]



(Figure 1.0)



(Figure 1.1)

When it comes to showing all the equations that you've derived for coordinates and velocities, in matrix equation form ($Ax = b$).

$$\begin{bmatrix} \mathrm{v} & -\omega(\mathrm{vt}+\mathrm{c}) \\ \omega(\mathrm{vt}+\mathrm{c}) & \mathrm{v} \end{bmatrix} \begin{bmatrix} \cos\omega \mathrm{t} \\ \sin\omega \mathrm{t} \end{bmatrix} = \begin{bmatrix} \mathrm{v(x)} \\ \mathrm{v(y)} \end{bmatrix}$$

$$\begin{bmatrix} \mathrm{vt}+\mathrm{c} \\ \mathrm{vt}+\mathrm{c} \end{bmatrix} \begin{bmatrix} \cos\omega \mathrm{t} & \sin\omega \mathrm{t} \end{bmatrix} = \begin{bmatrix} \mathrm{x} \\ \mathrm{y} \end{bmatrix}$$

In terms of find the lengths of resultant vectors for coordinate and velocity vectors at a certain time

T(period)=2 s   r(radius)=0,22 m   t(time)=14 s (that we choose)

V= $\omega . r$ v= $\frac{2\Pi}{\mathrm{T}}$.r $v = \frac{2\pi}{2} \times 0{,}22 = 0{,}22\pi\ m/s$ $\omega = \pi$

<u>X-Axis of Distance-Time Graph</u>

$$x = (vt + c).cos\omega t$$

$$x = (0{,}22\pi \times 14) \times \cos(14\pi)$$

$$x = 3.08\pi \text{ m}$$

<u>Y-Axis of Distance-Time Graph</u>

$$y = (vt + c).sin\omega t$$

$$= (0{,}22\pi \times 14) \times sin(14\pi) = 0$$

$$|\underset{r}{\rightarrow}| = \sqrt{\boldsymbol{X^2 + Y^2}} = \sqrt{(\boldsymbol{3,08\pi})^2 + \boldsymbol{0^2}} \ = \mathbf{3{,}08\pi\ m}$$

<u>X-Axis of Velocity-Time Graph</u>

$$v(x) = v.cos\omega t - \omega(vt + c)sin\omega t$$

$$=0{,}22\pi \times cos14\pi - \pi(0{,}22 \times 14) \times sin14\pi$$

$$v(x) \ = 0{,}22\pi \text{ m/s}$$

Y-Axis of Velocity-Time Graph

$$v(y) = v.sin\omega t + w(vt + c)cos\omega t$$

$$= 0{,}22\pi \times sin14\pi + \pi(0{,}22\pi \times 14) \times cos14\pi$$

$$v(y) = 3{,}08\pi^2 \text{ m/s}$$

$$|\underset{v}{\rightarrow}| = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{(0{,}22\pi)^2 + (3{,}08\pi^2)^2} \text{ m/s}$$

To look at evaluating this system from the point of having solutions.

$$\begin{bmatrix} v & -w(vt+c) & \vdots & v(x) \\ w(vt+c) & v & \vdots & v(y) \end{bmatrix} ------\rightarrow \left(-\frac{R1}{v}\right).w(vt+c) + R2 -\rightarrow$$

$$\begin{bmatrix} v & -w(vt+c & \vdots & v(x) \\ 0 & ((w(vt+c))^2/v) + v & \vdots & v(y) + (-v(x)/v.w(vt+c) \end{bmatrix}$$

$$Firstly;\ sinwt: a\ coswt: \text{b}$$

$$(w(vt+c))^2/v + v = v.a + w(vt+c).a - [(vb - w(vt+c).a).w(vt+c)]/v$$

$$va + [w(vt+c)(b - ((vb - w(vt+c).a)/v)] = (w(vt+c))^2/v + v$$

$$w(vt+c)[b - (vb - w(vt+c).a)/v - w(vt+c)/v] = v(1-a)$$

$$w(vt+c)[b - (vb - w(vt+c).(a-1)/v] = v(1-a)$$

$$w(vt+c).[vb - vb - w(vt+c)(a-1)/v] = v(1-a)$$

$$After\ simplifications;$$

$$v^2 = [w(vt+c)]^2 \ --------> \ v = w(vt+c)$$

-The matrix system has infinitely many solutions-

RESULT AND DISCUSSION

We mentioned above that we do this experiment over and over again. As a result of these experiments, we observed that the period did not change when we increased the rotation time. That is, no matter how long or short the upright turns, the time it takes does not change. In another experiment, we increased the velocity and analyzed the results. As a result of the increase in velocity, the duration of one lap, that is, the period time, decreased, and as a result, the number of laps it took during the time it moved increased and we observed that the length of the curve from the above curve length formula increased.

CONCLUSIONS

In this experiment, the movement of the Archimedean spiral in two dimensions was observed. The velocity values were calculated with the help of the data and the position vector in the x and y co-ordinates of the spiral, as well as the velocity equation depending on the coordinates**.** The length of the curve drawn was calculated with the theoretical formula and compared with the experimental length and the margin of error was calculated. We tried to explain mathematical expressions algebraically and physically.

In this project, we compared experimental and theoretical results. As a result of this we got some errors. From this project, we learnt that exactly same results cannot be obtained due to environmental conditions. The most important thing that we learnt organizing a teamwork, researching a topic which we don’t know anything about and sharing information. We are pleased to gain these achievements.

THE VIDEO WE RECORDED

https://www.youtube.com/watch?v=k3WeKjIHUAw

REFERENCES

1. https://en.wikipedia.org/wiki/Archimedean_spiral
2. https://planetcalc.com/9063/
3. https://www.omnicalculator.com/math/spiral-length
4. https://www.youtube.com/watch?v=euvvukuz1TQ