Department of

Electrical & Electronics Engineering

Abdullah Gül University

Project Report for Ballistic Pendulum

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Electrical & Electronics Engineering

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Project Report

EE1100 Computation and Analysis (COMA) Capsule

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Submitted by: SÜMEYYE YILDIRIM

GROUP MEMBERS

MEHMET FATİH GÖĞÜŞ

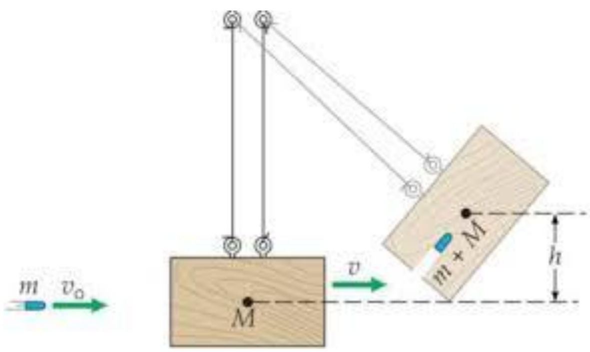
SÜMEYYE YILDIRIM

ZEYNEP DOLGUN

OBJECTIVE

The purpose of the ballistic pendulum experiment is to calculate the initial velocity at which the ball was launched and to observe the laws of conservation of momentum and mechanical energy. The ball thrown with a fixed 90° angle from the shooting apparatus in the mechanism hits the ballistic pendulum and the pendulum moves forward with the effect of this impact. The height reached with this movement is calculated with the angle indicator on the system.

In this experiment, using the height reached by the thrown ball, the conservation of mechanical energy and momentum, it is checked whether the experimental and theoretical results are the same as the velocity at which the ball was first thrown. If the results are not the same, it will be explained below why the theoretical formulas and the results are not the same.



Ballistic Pendulum Diagram

Figure 1

BACKGROUND

For the project, the initial velocity of the ball launched with the system created with the launcher mechanism and the ballistic pendulum mechanism and the maximum height it reached were examined.

ANALYTICAL AND SIMULATION PROCEDURES

a. Steps of Experiment

In this project

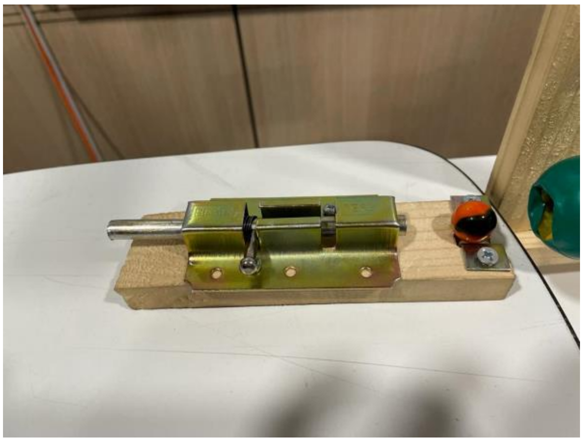
- Launcher Mechanism (with 3 different spring types)
- Ballistic Pendulum Mechanism
- Protractor
- Ball

are used.

1. A hard material such as wood is used to keep the floor stable while being transported. In the experiment, ball which is weight of 0,005 kg was shot 5 times for each spring in such a way that the system would operate repeatedly.
2. A thin stick which is length of 0,2 m was used as the pendulum rope to reduce the amount of swing. One end of this pendulum rope is fixed to the system and the other end to the ball holding mechanism which is weight of 0,03 kg.
3. The door bolt was used as the launching mechanism. Different types of springs were placed on this door slider to enable shooting.
4. 3 different springs were used in the experiment. 5 different shots were made for each spring types. Video recording was made during the shooting. (YouTube video link is attached in reference list.)
5. As a result of the experiment, 15 different data were obtained from 5 shots made with 3 different springs. With these data, momentum value, mechanical energy, initial velocity, and height of the ball were determined. The margin of error due to environmental factors has been calculated.

Abstract Stages of Experiment

- As seen in the pictures **(Figure 2, Figure 3, Figure 4)** , the mechanism is set up to perform the experiment, and the ball and gripper mechanism are appropriate and measured. Ready to do the experiment.



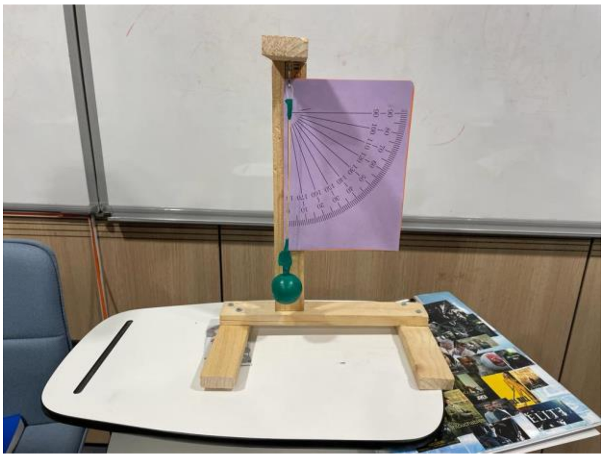


Figure 2 **Figure 3**

- Ballistic pendulum experiment was done with the help of materials. With the springs used in 3 different types, 5 different measurements (a total of 15 different values were taken) are made and values such as the maximum height of the ball, the change in the angle of the pendulum are taken.
- The maximum height at which the ball comes out, initial velocity, etc. it is suitable for getting other values. These values will be used in the interpretation of physical and algebraic operations below.

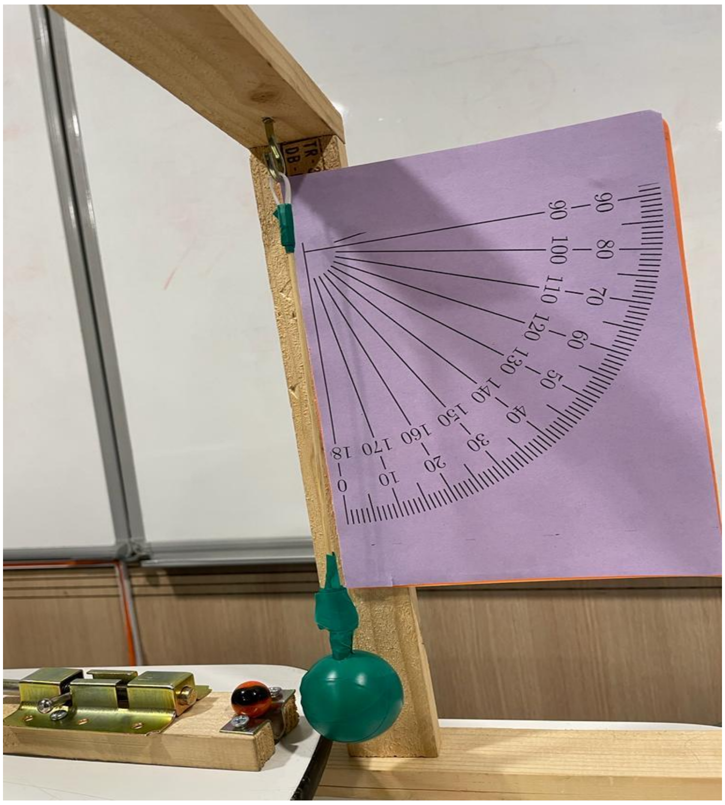


Figure 4

a. Evaluation of Problems

Reference point was taken as the line where the ball was thrown. Thus, the potential energy of the ball was initially accepted as zero.

The weight of the ball used in the experiment is 5 grams. The weight of the pendulum is 30 grams. Conservation of momentum is used to find the initial velocity (V_0) of the ball. The steps taken are shown below.

$$\mathbf{m_1V_0+m_2V_1=(m_1+m_2)V}$$

m_1= the mass of the ball

V_0= the initial velocity of the ball

m_2= the mass of the pendulum

V_1= the initial velocity of the pendulum

V= the common velocity of pendulum and ball

According to equation of $0{,}005.V_0+0{,}03.0=0{,}035.V$ the following is obtained: $5V_0=35V$ and $V_0=7V$.

Three different springs are used in this experiment. Moreover, five shots were made for each spring. Therefore, since different springs have different k constants, there are differences in the initial velocity of the ball. The $\mathbf{V_0=\sqrt{2gL(1-\cos\alpha)}(1+\frac{m_2}{m_1})}$ formula is used to find V_0 velocity **[*1]**. L is the length of the thin stick. g is the gravitational constant. cos α is the angle between last location of thin stick and first location of thin stick.

FIRST SPRING	ANGLES
1. Shot	**32°**
2. Shot	**33°**
3. Shot	**32°**
4. Shot	**33°**
5. Shot	**32°**

FIRST SPRING:

First shot: $V_0=\sqrt{2.9{,}8.0{,}2.(1-cos32)}\,.\,(1+\frac{0{,}03}{0{,}005})$= 5,40249… m/s

Since $V_0=7V$; V=0,771785… m/s

Second shot: $V_0=\sqrt{2.9{,}8.0{,}2.(1-cos33)}\,.\,(1+\frac{0{,}03}{0{,}005})$=5,56670… m/s

Since $V_0=7V$; V= 0,7952… m/s

Third shot: $V_0=\sqrt{2.9{,}8.0{,}2.(1-cos32)}\ (1+\frac{0{,}03}{0{,}005}) = 5{,}40249...$ m/s

Since V_0=7V ; V=0,771785… m/s

Fourth shot: $V_0=\sqrt{2.9{,}8.0{,}2.(1-cos33)}\ .\ (1+\frac{0{,}03}{0{,}005})=5{,}56670...$ m/s

Since V_0=7V ; V= 0,7952… m/s

Fifth shot: $V_0=\sqrt{2.9{,}8.0{,}2.(1-cos32)}\ (1+\frac{0{,}03}{0{,}005}) = 5{,}40249...$ m/s

Since V_0=7V ; V=0,771785… m/s

SECOND SPRING	ANGLES
1. Shot	$\mathbf{26°}$
2. Shot	$\mathbf{25°}$
3. Shot	$\mathbf{27°}$
4. Shot	$\mathbf{25°}$
5. Shot	$\mathbf{24°}$

SECOND SPRING:

First shot: $V_0=\sqrt{2.9{,}8.0{,}2.(1-cos26)}\ (1+\frac{0{,}03}{0{,}005}) = 4{,}40904...$ m/s

Since V_0=7V ; V=0,629863… m/s

Second shot: $V_0=\sqrt{2.9{,}8.0{,}2.(1-cos25)}\ (1+\frac{0{,}03}{0{,}005}) =4{,}24222...$ m/s

Since V_0=7V ; V=0,606031… m/s

Third shot: $V_0=\sqrt{2.9{,}8.0{,}2.(1-cos27)}\ (1+\frac{0{,}03}{0{,}005}) = 4{,}57553...$ m/s

Since V_0=7V ; V= 0,653647… m/s

Fourth shot: $V_0=\sqrt{2.9{,}8.0{,}2.(1-cos25)}\ (1+\frac{0{,}03}{0{,}005}) = 4{,}24222...$ m/s

Since V_0=7V ; V=0,606031… m/s

Fifth shot: $V_0=\sqrt{2.9{,}8.0{,}2.(1-cos24)}\ (1+\frac{0{,}03}{0{,}005}) = 4{,}07507...$ m/s

Since V_0=7V ; V=0,582153… m/s

THIRD SPRING	ANGLES
1. Shot	$\mathbf{33°}$
2. Shot	$\mathbf{33°}$
3. Shot	$\mathbf{36°}$
4. Shot	$\mathbf{34°}$
5. Shot	$\mathbf{36°}$

THIRD SPRING:

First shot: $V_0=\sqrt{2.9{,}8.0{,}2.(1-cos33)}\ (1+\frac{0{,}03}{0{,}005})$ = 5,56670… m/s

Since $V_0=7V$; V= 0,7952… m/s

Second shot: $V_0=\sqrt{2.9{,}8.0{,}2.(1-cos33)}\ (1+\frac{0{,}03}{0{,}005})$ =5,56670… m/s

Since $V_0=7V$; V= 0,7952… m/s

Third shot: $V_0=\sqrt{2.9{,}8.0{,}2.(1-cos36)}\ (1+\frac{0{,}03}{0{,}005})$ =6,05673… m/s

Since $V_0=7V$; V= 0,865248… m/s

Fourth shot: $V_0=\sqrt{2.9{,}8.0{,}2.(1-cos34)}\ (1+\frac{0{,}03}{0{,}005})$ =5,73049… m/s

Since $V_0=7V$; V=0,818641… m/s

Fifth shot: $V_0=\sqrt{2.9{,}8.0{,}2.(1-cos36)}\ (1+\frac{0{,}03}{0{,}005})$ =6,05673… m/s

Since $V_0=7V$; V= 0,865248… m/s

Conservation of energy was used to find the height H theoretically. The following equation is used to find H: $\mathbf{\frac{1}{2}mV^2=mgH}$. m is the total mass of pendulum and ball. V is the common velocity.

1. **SPRING**

First Shot:

$$\frac{1}{2}(0{,}005+0{,}03).(0{,}771785)^2 = (0{,}005+0{,}03).9{,}8.H$$

H=0,030390 m

Second Shot

$$\frac{1}{2}(0{,}005+0{,}03).(0{,}7952)^2 = (0{,}005+0{,}03).9{,}8.H$$

H=0,0322624 m

Third Shot:

$$\frac{1}{2}(0{,}005 + 0{,}03).(0{,}771785)^2 = (0{,}005 + 0{,}03).9{,}8.H$$

H=0,030390 m

Fourth Shot:

$$\frac{1}{2}(0{,}005 + 0{,}03).(0{,}7952)^2 = (0{,}005 + 0{,}03).9{,}8.H$$

H=0,0322624 m

Fifth Shot:

$$\frac{1}{2}(0{,}005 + 0{,}03).(0{,}771785)^2 = (0{,}005 + 0{,}03).9{,}8.H$$

H=0,030390 m

2.SPRING

First Shot:

$$\frac{1}{2}(0{,}005 + 0{,}03).(0{,}629863)^2 = (0{,}005 + 0{,}03).9{,}8.H$$

H= 0,020241 m

Second Shot:

$$\frac{1}{2}(0{,}005 + 0{,}03).(0{,}606031)^2 = (0{,}005 + 0{,}03).9{,}8.H$$

H=0,0187384477 m

Third Shot:

$$\frac{1}{2}(0{,}005 + 0{,}03).(0{,}653647)^2 = (0{,}005 + 0{,}03).9{,}8.H$$

H=0,021798694 m

Fourth Shot:

$$\frac{1}{2}(0{,}005 + 0{,}03).(0{,}606031)^2 = (0{,}005 + 0{,}03).9{,}8.H$$

H=0,0187384477 m

Fifth Shot:

$$\frac{1}{2}(0{,}005+0{,}03).(0{,}582153)^2 = (0{,}005+0{,}03).9{,}8.H$$

H=0,0172909243 m

3.SPRING

First Shot:

$$\frac{1}{2}(0{,}005+0{,}03).(0{,}7952)^2 = (0{,}005+0{,}03).9{,}8.H$$

H=0,0322624 m

Second Shot:

$$\frac{1}{2}(0{,}005+0{,}03).(0{,}7952)^2 = (0{,}005+0{,}03).9{,}8.H$$

H=0,0322624 m

Third Shot:

$$\frac{1}{2}(0{,}005+0{,}03).(0{,}865248)^2 = (0{,}005+0{,}03).9{,}8.H$$

H=0,0381966379 m

Fourth Shot:

$$\frac{1}{2}(0{,}005+0{,}03).(0{,}818641)^2 = (0{,}005+0{,}03).9{,}8.H$$

H=0,03419255045 m

Fifth Shot:

$$\frac{1}{2}(0{,}005+0{,}03).(0{,}865248)^2 = (0{,}005+0{,}03).9{,}8.H$$

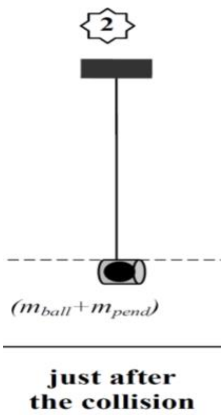
H=0,0381966379 m

When the experiment is examined in terms of momentum conservation, the following conclusions are reached:

The momentum lost by one of the colliding objects is gained by the other. That is, the momentum changes are equal and opposite in magnitude. According to this relation, the total momentum of a system is constant. This is called conservation of momentum. It comes from Newton's second law: **F=ma ; a=V/t ; Ft=mV**. When calculated theoretically, it is observed that momentum must be conserved. However, it can be seen some rate of non-conservation experimentally because of air resistance which is the most important effect of environmental factors.

When mechanical energy is analyzed theoretically and experimentally, the following data were obtained:

This experiment is an example of inelastic collision. An inelastic collision is one in which kinetic energy is lost. In an inelastic collision, the momentum of the system is conserved, but the kinetic energy is not. This is because some of the kinetic energy has been transferred to other energy types such as sound energy, heat energy etc. Mechanical energy is divided into kinetic energy and potential energy. To find height H of the mechanism was found by using the energy conservation law. The height H can also be found from the formula **H=L(1-cos α) [*1]**. **L** is the length of the thin stick and α is the angle between last location and first location of thin stick. When we set the degree as 33 for the first string H=L(1-cos33), the result will be 0,0322659. The result which was obtained is the real value of height in experimentally. But the theoretical value is 0,0322624. Due to the fact that these values are not equal, initial velocities would be different. That's why kinetic energy is not conserved. Kinetic and potential energy both are not conserved. As a consequence of these, mechanical energy is not conserved.



The distance between the pendulum mechanism and the ground is 5 centimeters in experiment. The mechanical energy is most clear in the Figure 5 state. Because, as can be seen in the Figure 5, it is the only case where it has both kinetic energy and potential energy. Mechanical energy is found by the sum of kinetic energy and potential energy. It can be formulated like this:

$$U=\frac{1}{2}mV^2+mgH$$

Figure 5

The mechanical energy was not conserved. To find the average % of the lost mechanical energy, the most frequently obtained height and velocity values were used. When we evaluate the average % of the lost for mechanical energy in experiment, for 3 different springs the results will be like these:

FIRST SPRING: $\mathbf{\frac{0.02744-0.01715}{0.02744}}$**x100 =37.5**

SECOND SPRING: $\mathbf{\frac{0.02345-0.022295}{0.02345}}$**x100** $\cong$ **4.95198**

THIRD SPRING: $\mathbf{\frac{|0.025725-0.02744|}{0.02744}}$**x100=6.25**

To construct a matrix equation in the form Ax=b to calculate three different H heights, the next steps were done:

In this experiment, 5 different H heights were obtained because 3 different springs were used and 5 different shots were made for each type of springs. Since 3 different H heights were requested, the average of 5 different H heights for each spring was taken.

First Height:

$\frac{(0,030390+0,0322624+0,030390+0,0322624+0,030390)}{5}$=0,03113896 m

Second Height:

$\frac{(0,020241+0,018738+0,021798694+0,0187384477+0,0172909243)}{5}$= 0,0193615415 m

Third Height:

$\frac{(0,0322624+0,0322624+0,0381966379+0,03419255045+0,0381966379)}{5}$=0,035022116 m

A=$[\mathbf{0,03113896} \quad \mathbf{0,0193615415} \quad \mathbf{0,035022116}]$ → NOT INVERTIBLE (One of the conditions for a matrix to be invertible is that it is a square matrix. Since this matrix is not a square matrix, it is not invertible.)

$$\mathbf{Ax=}\ [\mathbf{0,03113896} \quad \mathbf{0,0193615415} \quad \mathbf{0,035022116}] \begin{bmatrix} \boldsymbol{x_1} \\ \boldsymbol{x_2} \\ \boldsymbol{x_3} \end{bmatrix} = [\mathbf{0}]$$

The first row of A is multiplied by 1/0,03113896 in order to obtain 1 ($a_{11}=1$):

$$\mathbf{Ax=}\ [\mathbf{1} \quad \mathbf{0,621778682} \quad \mathbf{1,1247041}] \begin{bmatrix} \boldsymbol{x_1} \\ \boldsymbol{x_2} \\ \boldsymbol{x_3} \end{bmatrix} = [\mathbf{0}]$$

$$0{,}03113896.x_1 \quad = -\ 0{,}0193615415.x_2 \quad - \quad 0{,}035022116.x_3$$

$$x_1 \quad = \ -0{,}621778682.x_2 - \ 1{,}1247041.x_3$$

$$\boldsymbol{x_1\ is\ pivot.}$$

$$\boldsymbol{x_2 and\ x_3\ are\ free\ variables.}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -0{,}621778682.x_2 - \ 1{,}1247041.x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \ x_2 \begin{bmatrix} -0{,}621778682 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1{,}1247041 \\ 0 \\ 1 \end{bmatrix}$$ → **NULL SPACE N(A)**

A= $[0{,}03113896 \quad 0{,}0193615415 \quad 0{,}035022116]$

C(A)= $x_1[0{,}03113896]$ + $x_2[0{,}0193615415]$ + $x_3[0{,}035022116]$ → **COLUMN SPACE C(A)**

The formula is $\mathbf{x_c=x_p+x_s}$ to find complete solution.

$\mathbf{x_c}$ **:** complete solution $\mathbf{x_p}$: particular solution $\mathbf{x_s}$: special solution

When finding a particular solution, free variables are assumed to be zero.

$$0{,}03113896.x_1 \quad = -\ 0{,}0193615415.x_2 \quad - \quad 0{,}035022116.x_3$$

$$x_1 \quad = \ -0{,}621778682.x_2 - \ 1{,}1247041.x_3$$

Since x_2 and x_3 is zero, x_1 would be 0. So, particular solution will be like that:

$$\mathbf{x_p} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

When finding null space, also special solution was obtained.

To make it clear, when finding a special solution, while one of free variables is assumed to be one, the other ones are assumed to be zero and this process is done for each free variable.

When x_2 is one and x_3 is zero:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -0{,}621778682 \\ 1 \\ 0 \end{bmatrix} x_2$$

When x_2 is zero and x_3 is one:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1{,}1247041 \\ 0 \\ 1 \end{bmatrix} x_3$$

$$\mathbf{x_s} = \begin{bmatrix} \mathbf{-0,621778682} \\ \mathbf{1} \\ \mathbf{0} \end{bmatrix} \mathbf{x_2} + \begin{bmatrix} \mathbf{-1,1247041} \\ \mathbf{0} \\ \mathbf{1} \end{bmatrix} \mathbf{x_3}$$

$$\mathbf{x_c} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \mathbf{x_1} + \begin{bmatrix} \mathbf{-0,621778682} \\ \mathbf{1} \\ \mathbf{0} \end{bmatrix} \mathbf{x_2} + \begin{bmatrix} \mathbf{-1,1247041} \\ \mathbf{0} \\ \mathbf{1} \end{bmatrix} \mathbf{x_3}$$

To construct a matrix C whose first row is composed of three different momentum, second row is composed of three various potential energy and third row is composed of three different H heights that we calculated. When C matrix is factorized into LU form:

$$\begin{pmatrix} \frac{5468057}{200000000} & \frac{861763}{40000000} & \frac{28976759}{1000000000} \\ \frac{106806633}{10000000000} & \frac{664100873}{100000000000} & \frac{60062929}{5000000000} \\ \frac{389237}{12500000} & \frac{38723083}{2000000000} & \frac{8755529}{250000000} \end{pmatrix} \times \left(\frac{-106806633}{273402850}\right)$$

$$\equiv$$

$$R_2 - \left(\frac{106806633}{273402850}\right) \cdot R_1 \to R_2$$

$$\begin{pmatrix} \frac{5468057}{200000000} & \frac{861763}{40000000} & \frac{28976759}{1000000000} \\ 0 & \frac{-19811404007861}{1115930000000000} & \frac{27052161363869}{390575500000000} \\ \frac{389237}{12500000} & \frac{38723083}{2000000000} & \frac{8755529}{250000000} \end{pmatrix} \times \left(\frac{-6227792}{5468057}\right)$$

$$\equiv$$

$$R_3 - \left(\frac{6227792}{5468057}\right) \cdot R_1 \to R_3$$

$$\begin{pmatrix} \frac{5468057}{200000000} & \frac{861763}{40000000} & \frac{28976759}{1000000000} \\ 0 & \frac{-19811404007861}{1115930000000000} & \frac{27052161363869}{390575500000000} \\ 0 & \frac{-1155183893981}{223186000000000} & \frac{394346380803}{195287750000000} \end{pmatrix} \times \left(\frac{-57759194699050}{19811404007861}\right)$$

$$\equiv$$

$$R_3 - \left(\frac{57759194699050}{19811404007861}\right) \cdot R_2 \to R_3$$

$$\begin{pmatrix} \frac{5468057}{200000000} & \frac{861763}{40000000} & \frac{28976759}{1000000000} \\ 0 & \frac{-19811404007861}{1115930000000000} & \frac{27052161363869}{390575500000000} \\ 0 & 0 & \frac{6914300950651}{13867982805502700000000} \end{pmatrix}$$

$$\equiv$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{106806633}{273402850} & 1 & 0 \\ \frac{6227792}{5468057} & \frac{57759194699050}{19811404007861} & 1 \end{pmatrix}, \; U =$$

$$\equiv$$

$$\begin{pmatrix} \frac{5468057}{200000000} & \frac{861763}{40000000} & \frac{28976759}{1000000000} \\ 0 & \frac{-19811404007861}{1115930000000000} & \frac{27052161363869}{390575500000000} \\ 0 & 0 & \frac{6914300950651}{13867982805502700000000} \end{pmatrix}$$

Reduced row echelon form of C matrix:

$$\begin{bmatrix} \frac{5468057}{200000000} & \frac{861763}{40000000} & \frac{28976759}{1000000000} \\ \frac{106806633}{10000000000} & \frac{664100873}{100000000000} & \frac{60062929}{5000000000} \\ \frac{389237}{12500000} & \frac{38723083}{2000000000} & \frac{8755529}{250000000} \end{bmatrix}$$

Solution

Multiply row 1 by $\frac{200000000}{5468057}$: $R_1 = \frac{200000000 R_1}{5468057}$.

$$\begin{bmatrix} 1 & \frac{87935}{111593} & \frac{4139537}{3905755} \\ \frac{106806633}{10000000000} & \frac{664100873}{100000000000} & \frac{60062929}{5000000000} \\ \frac{389237}{12500000} & \frac{38723083}{2000000000} & \frac{8755529}{250000000} \end{bmatrix}$$

Subtract row 1 multiplied by $\frac{106806633}{10000000000}$ from row 2: $R_2 = R_2 - \frac{106806633 R_1}{10000000000}$.

$$\begin{bmatrix} 1 & \frac{87935}{111593} & \frac{4139537}{3905755} \\ 0 & -\frac{19811404007861}{1115930000000000} & \frac{27052161363869}{390575500000000} \\ \frac{389237}{12500000} & \frac{38723083}{2000000000} & \frac{8755529}{250000000} \end{bmatrix}$$

Subtract row 1 multiplied by $\frac{389237}{12500000}$ from row 3: $R_3 = R_3 - \frac{389237 R_1}{12500000}$.

$$\begin{bmatrix} 1 & \frac{87935}{111593} & \frac{4139537}{3905755} \\ 0 & -\frac{19811404007861}{1115930000000000} & \frac{27052161363869}{390575500000000} \\ 0 & -\frac{1155183893981}{223186000000000} & \frac{394346380803}{195287750000000} \end{bmatrix}$$

Multiply row 2 by $-\frac{1115930000000000}{19811404007861}$: $R_2 = -\frac{1115930000000000 R_2}{19811404007861}$.

$$\begin{bmatrix} 1 & \frac{87935}{111593} & \frac{4139537}{3905755} \\ 0 & 1 & -\frac{54104322727738}{138679828055027} \\ 0 & -\frac{1155183893981}{223186000000000} & \frac{394346380803}{195287750000000} \end{bmatrix}$$

Subtract row 2 multiplied by $\frac{87935}{111593}$ from row 1: $R_1 = R_1 - \frac{87935 R_2}{111593}$.

$$\begin{bmatrix} 1 & 0 & \frac{948073427614699}{693399140275135} \\ 0 & 1 & -\frac{54104322727738}{138679828055027} \\ 0 & -\frac{1155183893981}{223186000000000} & \frac{394346380803}{195287750000000} \end{bmatrix}$$

Add row 2 multiplied by $\frac{1155183893981}{223186000000000}$ to row 3: $R_3 = R_3 + \frac{1155183893981R_2}{223186000000000}$.

$$\begin{bmatrix} 1 & 0 & \frac{948073427614699}{693399140275135} \\ 0 & 1 & -\frac{54104322727738}{138679828055027} \\ 0 & 0 & \frac{6914300950651}{138679828055027000000000} \end{bmatrix}$$

Multiply row 3 by $\frac{138679828055027000000000}{6914300950651}$: $R_3 = \frac{138679828055027000000000R_3}{6914300950651}$.

$$\begin{bmatrix} 1 & 0 & \frac{948073427614699}{693399140275135} \\ 0 & 1 & -\frac{54104322727738}{138679828055027} \\ 0 & 0 & 1 \end{bmatrix}$$

Subtract row 3 multiplied by $\frac{948073427614699}{693399140275135}$ from row 1:
$R_1 = R_1 - \frac{948073427614699R_3}{693399140275135}$.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{54104322727738}{138679828055027} \\ 0 & 0 & 1 \end{bmatrix}$$

Add row 3 multiplied by $\frac{54104322727738}{138679828055027}$ to row 2: $R_2 = R_2 + \frac{54104322727738R_3}{138679828055027}$.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

RESULT AND DISCUSSION

As a result of this experiment, when different springs were used, the firing speed of the ball also changed according to the k which is spring constant . Different angles were obtained as a result of the change in the firing speed of the ball. The angles obtained were used while finding the throwing speed of the ball. Momentum conservation was used while finding the common velocity of the ball and the mechanism. While finding the height H, energy conservation laws were used. We observed that the k constant is higher when we use a thick spring, therefore the ball's throwing speed is higher. However, when we used a thinner spring, we concluded that the spring constant k took lower values and the ball was ejected with less velocity. The change in these velocities caused the difference in angles between last location of thin stick and first location of thin stick.

CONCLUSIONS

In this project, we compared experimental and theoretical results for conservation of momentum and conservation of energy. As a result of this we got some errors and therefore, we found average % of the lost for mechanical energy in our experiment. From this project, we learnt that exactly same results cannot be obtained due to environmental conditions. The most important thing that we learnt organizing a teamwork, researching a topic which we don't know anything about and sharing information. We are pleased to gain these achievements.

THE VIDEO WE RECORDED

https://youtu.be/CDhEx5LLTq4

REFERENCES

***1. https://physicstasks.eu/377/ballistic-pendulum-1**